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Abstract

This document analyses collusion by innovative firms and the role of patents in a continuous-time real options framework. A patent-investment race model is formulated in which innovative firms bargain and reach collusive agreements. It is shown that, while collusion always delays innovation, it does not necessarily delay competition. Depending on a number of factors, collusion can actually accelerate competition.

Keywords: Bargaining, Collusion, Competition, Geometric Brownian motion, Nash axiomatic approach, Stackelberg game.

JEL Classification: C7, D8, K4, L13

Resumen

Este artículo analiza la colusión entre empresas innovadoras y el papel de las patentes en un marco de opciones reales en tiempo continuo. Para ello, se formula un modelo de carrera de patente-inversión en el cual las empresas innovadoras negocian y llegan a acuerdos colusivos. Se demuestra que la colusión siempre retrasa la innovación pero no necesariamente retrasa la competencia. De hecho, dependiendo de ciertos factores, la colusión puede acelerar la competencia.

Palabras Clave: Negociación, Colusión, Competencia, Movimiento geométrico Browniano, Enfoque axiomático de Nash, Juego de Stackelberg.

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1. INTRODUCTION

This paper studies the effects of collusion within a duopoly. In our model, a patent is granted to the first firm to innovate/invest which is then able to license the technology to a second innovative firm.

Unlike previous studies of collusion, we use a stochastic model in which innovators engage in irreversible lump sum investments and hence real options affect the timing of agents' investment decisions. The impact of competition on the timing of investment is then a key issue. This paper investigates whether collusion delays or accelerates innovation and competition.[1]

We find that in the absence of patents, collusion always delays both innovation and competition. When the first to invest can obtain a patent, the answer is more complex and depends on parameter values. Before describing the implications of a patent scheme on the collusive result, it is worth explaining the basic scenario without the patent.

The non-cooperative version of our model was first analysed by Smets (1991) and Grenadier (1996). Smets studies an asymmetric leader-follower equilibrium, where the threat of preemption undercuts a firm's ability to delay entry. Therefore, market entry by the leader is accelerated and the option value of delay is eroded. Dixit and Pindyck (1994) present Smets' model in a simplified framework.

Grenadier (1996) develops a game-theoretic framework to explain actual market behaviour. He finds that development options are not necessarily exercised according to a "smooth pattern". Whether development options are exercised sequentially or simultaneously depends on the underlying conditions of the market. We extend the Smets-Grenadier model by allowing firms to bargain over a long term collusive contract. A leader firm invests and shares monopolistic profits with the idle firm.[2] Collusion continues until the monopoly generates higher profits than duopoly. The terms of the collusive agreement specify i) the leader's investment threshold – the timing of innovation, ii) the sharing rule, and iii) the follower's investment threshold – the competitive stage.

In general terms, cooperation results in an efficient exploitation of the market by the two firms,[3] i.e. the value of the surplus generated by cooperation is maximised. This is achieved because cooperation i) eliminates the preemption threat, and ii) minimises the loss from competition. Also, either in a cooperative environment or a non cooperative one, the follower pays the same investment cost, therefore whether cooperation delays the follower's entry purely depends on the slope of demand. It is intuitive that, if demand has negative slope, cooperation delays the follower's

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[1]As will become clearer later, by innovation we mean investment by the first innovative firm to invest and, by competition, we mean investment by the second innovative firm to invest.

[2]Shapiro (2003, p. 71) refers as *reverse payment* to the cases in which the patent holder makes a cash payment to the challenger, who in turn agrees not to enter the market until some specified date. Leffler and Leffler (2003, p. 77) refer as *lump sum patent settlements* to the cases in which private parties agree to establish a monopoly and share the profits. Gilbert and Tom (2001) mention that quite typical agreements are those where the profits generated from the patent holder are shared with the competitor for a specified period or permanently.

[3]A welfare analysis that takes into account the impact on consumers is beyond the scope of this paper. We say an outcome is 'efficient' if it is Pareto efficient from the private point of view of the two innovative firms.

threshold and, hence, competition.[4] The reason why collusion delays innovation as well as competition should be clear now. Thus, cooperation results in Pareto efficiency.

When a patent is granted to the first innovative firm that files for a patent and invests (the leader)[5] and licenses the technology to the second innovative firm (the follower), the first mover advantage increases proportionally to the licensing fees. The licensing fee is introduced as a lump sum payment from the follower to the leader. Moreover, the proposed patent scheme is flexible enough to take into account legal and/or administrative expenses for enforcing and/or challenging the patent.[6]

To highlight our intuition clearly, consider an extreme scenario in which the patent allows the leader to establish a permanent monopoly. Within our setting, such scenario can be established simply by assuming that the fixed-fee licence is infinite. Ex-ante, both firms have the opportunity to file for the patent, but the first mover gains the project value in its entirety, while the follower loses the investment opportunity and remains empty-handed. With homogeneous firms the benefit of delay is completely eliminated and entry occurs as soon as the investment payoff becomes marginally positive. When, ex-ante, firms can cooperate via a collusive contract, the terms of the agreement will specify again the leader's entry, the sharing rule (the division of monopoly profits), and the follower's entry (if, ex-ante, convenient to both firms). In addition, when a duopoly is more profitable to both firms, the patent holder will allow the follower to enter the market exempt of paying the fixed-fee licence such that the follower's entry is finite. If, for a high enough level of the stochastic demand, the sum of the duopoly profits is higher than the monopoly profits, then the duopoly market will be established via cooperation. In this particular scenario, cooperation, again, delays innovation but competition is restored.

There is a broad range of alternative cases (with finite fixed-fee licences) where cooperative collusion accelerates competition. The model presented here identifies the economic and firm-specific factors – fixed-fee licence, slope of demand, investment costs – that in a collusive scenario accelerate, rather than delay, competition.

Virtually all agree that some level of protection of intellectual property (IP) via a patent is justified. However, it is also recognised that IP protection is a two-edged

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[4]In fact, when players collude and the follower invests, the two firms lose the (shared) monopoly profits and each receive duopoly profits. When demand has a negative slope, the variation of profits (of the joint firms) is always smaller than the duopoly profit of a single firm. Therefore, the cooperative threshold level of the follower must be bigger than the non-cooperative one. Moreover, if the joint variation of profits from the follower's entry is negative, then cooperation would result in a permanent monopoly.

[5]To date, there are two dispute resolution rules, called *first-to-file* and *first-to-invent*, that may determine which firm is granted the patent. The *first-to-file* rule, which applies in all countries except the U.S., means that the patent issues to the first applicant independently of priority of discovery. The *first-to-invent* rule, which applies in the U.S. will issue to the first inventor, provided the date of first invention can be documented (see Scotchmer and Green (1990, p. 133-134)). In our stylised model, as soon as the patent is granted to the first innovator that files an application, investment occurs – they are simultaneously determined.

[6]Amongst others, see Crampes and Langinier (2002) and Lanjouw and Lerner (1997). The administrative/legal costs simply create a gap between what the follower/licensee pays in order to use the technology and what the leader/licensor receives. For a brief review of patent law, see Shy (1995, Chapter 9).

sword. On the one hand, it encourages and rewards innovation. On the other hand, it creates monopolies.[7]

Recognising, as we do, that in a patent framework, collusion does not necessarily delay competition, may be a promising result in lessening the tension between antitrust and IP authorities.[8] This is particularly true for the new economy, well characterised in our setting, where products are based on innovation and typically involve large initial fixed sunk costs and low variable costs.[9]

Furthermore, one may notice that our approach to collusion reestablishes the importance of a real option framework in market entry decision.[10] According to the real option theory, uncertainty creates an option value of delay, but with two competing firms the fear of preemption undermines this approach. Moreover, a patent-race may increase the preemption threat dramatically. For instance, in the limiting case depicted above, when the patent allows the establishment of a permanent monopoly, homogeneous firms will try to invest as soon as the project value is marginally positive. Therefore, under such circumstances, investment occurs according to the traditional net present value (NPV) rule. The core of the problem, as argued by Weeds (2002), is that real options are rarely backed by legal contracts which guarantee the holder's rights in the same terms as financial options. Moreover, investment opportunities are rarely "*de facto* proprietary" (Weeds, 2002).[11] Even when firms can file for a patent, in an ex-ante situation, the investment opportunity is still a non-proprietary option. In these circumstances, the investment race is exacerbated by the patent race and the option value of delay is additionally eroded. By contrast, in our model, ex-ante cooperation establishes a *de facto* proprietary right similar to that granted by financial options.

In our analysis, cooperative collusion could take the explicit form of a horizontal merger between firms facing the same market demand. At the practical level, the model provides a simple and realistic framework which might help to explain the recent wave of consolidations and alliances in the pharmaceutical industry.

As documented in several recent studies,[12] some of the largest mergers in history have involved recent combinations of multinational pharmaceutical firms.[13] Remarkable changes, such as an enormous number of mergers, joint ventures, and

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[7]Within our setting, in the non-cooperative scenario, increases in the cost of the fixed-fee licence paid by the follower delays the follower's entry threshold and, hence, rewards the patent-holder who enjoys monopoly profits over a longer period of time.

[8]See Pitofsky (2001) for a discussion on the Federal Trade Commission (FTC) position with respect to IP laws. For an overview of antitrust policy and related issues, see Kovacic and Shapiro (2000) and Williamson (1987).

[9]For simplicity, it is assumed that there are only fixed-sunk costs and variable costs are zero.

[10]Bloom and Van Reenen (2002) mention that patents represent innovations whose introduction involve sizeable (and irreversible) investments, and when firms face uncertain market conditions they will own patent real options, which reflect the value that a firm places on its ability to choose the timing of its investment.

[11]Weeds (2002, p. 729) refers to cases of strong market position such as natural monopoly and network industry.

[12]See Whitener (1995) and Balto and Mongoven (1999).

[13]For instance, Zeneca's acquisition of Astra, Hoechst's acquisition of Marion Merrell Dow, the merger between Ciba-Geigy and Sandoz and the merger between Glaxo Wellcome and Smith Kline Beecham.

other collusive arrangements are occurring in these markets.[14] One the one hand, the wave of consolidation may be a response to the development of new drugs and other technologies. On the other hand, as mentioned in Balto and Mongoven (1999), a crucial reason is the record number of patent expirations faced by the pharmaceutical industry during these years. According to our framework, the merging scenario takes place before the patent is allocated to any of the two innovators, when both are homogeneous.

It is worth emphasising the applicability of our approach to specific industries, such as the pharmaceutical industry. We model collusion as an ex-ante opportunity available to firms facing the same market. Nevertheless, it is difficult at times to determine whether a firm is a potential competitor in a relevant market. However, the pharmaceutical industry, more than any other one, is uniquely suited for detecting potential competition. As the required U.S. Food and Drug Administration (FDA) approval process for new drugs is transparent, and fairly predictable, pharmaceutical companies are often able to determine the next entrants in the relevant market segment. Although predictions about entry are not perfect, the approval process provides a higher degree of certainty compared to other markets (see Balto and Mongoven (1999)). Therefore the option to collude is usually available to pharmaceutical companies well in advance of market entry.

The remainder of the paper is organised as follows. The next subsection revises very briefly the relevant literature. Section 2 presents the model. In particular, subsection 2.1 provides the general setting and assumptions. Subsection 2.2 shows the non-collusive (competitive) equilibrium of the leader-follower Stackelberg game. Subsection 2.3 formalises and derives the equilibrium of the cooperative scenario by using a Nash axiomatic solution which enables the two firms to sign a long-term collusive contract. Section 3 summarises and concludes.

To the best of our knowledge, this is the first model accounting for cooperative collusion in a real options framework combined with strategic interaction.

1.1. **Literature overview.** Several efforts have been made to analyse the interaction of IP protection and collusion. Unlike our study, vast part of this literature focuses on ex-post collusion arising from post-patent conflicts in a non-stochastic environment. More precisely, collusion takes the form of a patent litigation settlement between a patent holder and a firm infringing and/or challenging the patent.[15] The stochastic scenario, compared to the previous literature, allows us to stress the advantage to collude particularly when firms are still homogeneous (both firms are non-patent holders). That is, ex-ante collusion resolves the uncertainty over the allocation of the patent and hence eliminates the preemption threat of the

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[14]This has raised the concern of the FTC, whose primary task is to guarantee a certain level of competition. See Families USA (2003), FTC (2002a) and FTC (2002b) for an overview of lawsuits against drug manufacturers.

[15]For instance, amongst others, Marshall *et al.* (1994) model a litigation settlement as a bid-and-protest game in non-stochastic contest and shows that the settlement may result in collusion. Crampes and Langinier (2002) model a situation where the patent holder must supervise the market and react in case of infringement, they use a Nash bargaining to determine the collusive agreement. Ex-post collusive agreement through a Nash bargaining is used by Lanjouw and Lerner (1997) and Aoki and Hu (1999) as well. See also Meurer (1989) and Rockett (1990). For a detailed review of theoretical models on patent litigation versus patent settlement see Cooter and Rubinfield (1989).

investment-patent race. Therefore, cooperating, prior to filing for a patent, creates an additional surplus which cannot arise in a deterministic framework.

The literature combining real options with strategic interactions is relatively recent. As argued, our model extends the study of Smets (1991), Dixit and Pindyck (1994) and Grenadier (1996), who examine irreversible market entry and development decisions for a duopoly facing a stochastic demand.[16] Non-cooperative behaviour results in an asymmetric leader-follower equilibrium. This model has been extended in a number of dimensions.[17]

In a two-player game, with private information on each player's exercise cost, Lambrecht and Perraudin (2003) find threshold entry levels located between the monopoly and simple NPV outcomes.

Reiss (1998) uses real-option-based valuation to determine whether and when a firm should patent and adopt an innovation if the arrival time of competitors is stochastic. In particular, he focuses on the optimal investment and patenting strategy for a firm that has developed a new product or a close substitute beforehand. Miltersen and Schwartz (2002) use a real option framework to investigate patent-protected R&D investment projects when there is competition in the development and marketing of the resulting product.[18] In a similar vein, Schwartz (2003) uses a simulation approach to value patents and patent-protected R&D projects based on real options approach with applications to the pharmaceutical industry.

2. THE MODEL

2.1. **Basic Assumptions.** Two innovative firms face an identical investment opportunity consisting of a fixed technology which costs K[19] and generates a single unit of production. As in Dixit and Pindyck (1994), it is assumed that: i) there are no variable costs, ii) the industry demand is sufficiently elastic to ensure capacity production, iii) agents are risk neutral, and iv) the risk-free interest rate is r.

The unit of production guarantees a continuous stream of revenues, with instantaneous level given by the inverse demand function

$$R_t = P_t D\left(Q_t\right), \tag{2.1}$$

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[16]The crucial difference between Dixit and Pindyck (1994) and Grenadier (1996) is due to the fact that Dixit and Pindyck assume firms enter a new market, therefore prior to entry both innovative firms receive zero profits. More precisely, Grenadier models the entry decision as an 'expansion/development decision', that is, both firms are receiving a (non-stochastic) revenue prior to market entry. This creates negative externalities from the leader to the follower.

[17]For an overview, see for instance Grenadier (2000) and Schwartz and Trigeorgis (2001).

[18]They find that competition in R&D not only increases production and reduces prices but also shortens the time of developing the product and raises the probability of a successful development. These benefits to society are countered by increased total investment costs in R&D and lower aggregate value of the R&D investment projects –a medical drug is used as an example for their analysis (see Miltersen and Schwartz (2002, p. 30)).

[19]If production stops, it is assumed that the scrapping value is zero, so the cost K is sunk. In other words, the cash flow will always be positive, hence, it will never be convenient to stop producing after the initial investment. Moreover, as in Dixit and Pindyck (1994), the investment decisions of the model presented here possess three characteristics: i) they are irreversible, ii) there is uncertainty over the future rewards from the investment, and iii) there is some scope about the timing of the investment.

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where Q_t is the total market supply at time t, $D\left(\cdot\right)$ is a differentiable function with $D'\left(\cdot\right) < 0$ and P_t is a multiplicative demand shock. The shock evolves according to a geometric Brownian motion

$$dP = \mu P dt + \sigma P dz, \qquad (2.2)$$

where μ and σ are the drift and the volatility of the process respectively, and dz is a stochastic increment which follows a standard Wiener process.

Assuming that the two innovators are the only ones to access the investment opportunity, the demand (net of the shock) $D\left(Q_t\right)$, takes two values: i) $D\left(Q_t\right) = D(1)$, shortly D_1 if only one entrepreneur invests, or ii) $D\left(Q_t\right) = D(2)$, shortly D_2 if both entrepreneurs invest. As demand is downward sloping, D_2 will be strictly less than D_1.

It is assumed that the firm who innovates and becomes the first to file a patent application is immediately granted a patent – the leader. The patent gives the innovative firm the obligation to license the technology to the second entrant – the follower.[20] The fixed-fee licence paid by the follower, γ_F, is assumed to be a lump-sum payment exogenously given.[21] Usually, there are legal and/or administrative costs for enforcing the patent and overseeing the licensing scheme, we assume that, after paying such costs, the leader – patent-holder – receives the residual part of the licensing fee, namely γ_L. Additionally, it is assumed that the option to charge the fixed-fee licence expires after the follower has invested and therefore cannot be exercised later. When the right to charge for the licence is exercised by the leader, the follower's investment cost becomes $K + \gamma_F$, whilst the leader receives a lump-sum payment γ_L with $\gamma_L \leq \gamma_F$ (being γ_L a share of γ_F). In our case then, both entrepreneurs cannot invest simultaneously since only the patent-holder is allowed to invest first, whilst the rival will pay the fixed-fee licence.[22]

It is quite intuitive that the investment decisions of the two firms are interrelated due to two factors: i) the negative slope of demand – which determines the drop of revenues from the monopoly to the duopoly, and ii) the additional licensing cost, γ_F, paid by the follower. Both factors determine a first mover's advantage in a non-cooperative scenario. Compared to Dixit and Pindyck (1994), the advantage of the first mover is enhanced by the additional benefits from holding the patent. Therefore, our setting extends Dixit and Pindyck's investment race model into a

[20]Such a licensing scheme is known as 'compulsory licensing' and it is typically used as antitrust remedy (see Scherer and Watal (2002)). After the 1994 "Trade Related Aspects of Intellectual Property Regulation" (TRIPS) agreement, compulsory licensing has become an option available to all WTO member nations.

[21]Alternatively, one could have modelled a bargaining over the licensing fee between the patent-holder and the second entrant, where failure to negotiate results into a permanent monopoly. However, it is intuitive that there could be no room for bargaining if the drop in the demand and/or the investment cost are sufficiently high. Moreover, an exogenous licensing fee may be consistent with the TRIPS agreement, where compulsory licensing is triggered by failure to negotiate over the licensing fee – Treaty of Marrakesh TRIPS (Trade-Related Aspects of Intellectual Property Rights) Article 13.

[22]Smets (1991), Dixit and Pindyck (1994) and Grenadier (1996) assume that if both try to invest simultaneously, one randomly selected, is allowed to do so. This assumption is a technical one which can be used to solve coordination problems exogenously; it allows to identify the leader and the follower in the case of simultaneous entry decision and avoid problems of coordination failure resulting in joint investment when it is suboptimal to do so. See Huisman *et al.* (2002) for an endogenous solution of the coordination problem.

(simultaneous) patent/investment race situation. The next subsection solves our non-cooperative entry game leading to an asymmetric leader-follower equilibrium.

2.2. Non-Cooperative Game: Leader-Follower Equilibrium.

First, the players' expected payoffs under our non-cooperative scenario of a leader-follower game are derived. Second, the equilibrium strategy of the game is provided.

In the Stackelberg game, one innovative firm acts as leader – investing first at a cost K and earning an instantaneous (monopolistic) revenue $\frac{D_1 P_t}{(r-\mu)}$ – and the rival will act as a follower. That is, the follower will delay investment and enter when it is optimal to do so, and her/his entry will determine duopoly profits. When the follower invests, the investment cost is either i) $K + \gamma_F$ if the leader exercises the option to charge the fixed-fee licence – at this point the leader would then receive a one-off payment, γ_L; or ii) K if the leader does not exercise the option and allows the follower to use the innovation freely.

An equilibrium strategy consists of an entry threshold, which is a given level of the state variable that triggers the investment decision such that, given the other firm's strategy, neither agent has an incentive, in any state of the world, to deviate from her/his strategy. As in Dixit and Pindyck (1994), it is assumed that an equilibrium exists and an equilibrium strategy consists of two triggers, P_L and P_F, which denote the leader's and the follower's optimal entry, respectively.

As common in timing games, in order to derive an equilibrium, one can use backwards induction. First, the investment problem of the follower after the leader has invested is considered . Notice that, when the follower invests, the leader always exercises the option to charge the follower the licensing fee, γ_F.[23]

Given that the leader has invested and will exercise the option to charge for her/his innovation, the follower, when investing at P_F, has expected revenues equal to $\frac{D_2 P_F}{(r-\mu)}$ and lump-sum costs $K + \gamma_F$. Therefore, the follower's expected value, $F(P)$, can be written as

$$F(P_t) = \begin{cases} F_1(P_t) = \left[\frac{D_2 P_F}{(r-\mu)} - K - \gamma_F\right]\left(\frac{P_t}{P_F}\right)^{\delta} & \text{if } P_t < P_F, \\ F_2(P_t) = \frac{D_2 P_t}{(r-\mu)} - K - \gamma_F & \text{if } P_t \geq P_F, \end{cases} \qquad (2.3)$$

$$\text{with } \delta = \frac{-\left(\mu - \frac{\sigma^2}{2}\right) + \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}}{\sigma^2} > 1, \qquad (2.4)$$

where P_t is the initial level of the state variable and P_F is the follower's entry trigger and δ is the positive root of the quadratic equation[24]

$$r - \mu\delta - \frac{\sigma^2}{2}\delta(\delta - 1) = 0.$$

Solving $F_1(P_t)$ for the optimal P_F yields

$$P_F = \left(\frac{\delta}{\delta - 1}\right)\left[\frac{(K + \gamma_F)(r - \mu)}{D_2}\right]. \qquad (2.5)$$

Working backwards, given the follower's optimal entry, the leader's expected value can be derived as follows. Consider first the scenario where only one firm

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[23]See Appendix 1.

[24]We do not provide the derivation of the firm value through stochastic calculus. See Dixit and Pindyck (1994) for a general analysis of entry and exit decisions under uncertainty.

had the opportunity to invest. In such a scenario, when the single firm invests at P_t, the expected discounted payoff would be equal to $\frac{D_1 P_t}{(r-\mu)} - K$ (i.e. the payoff in a permanent monopoly). In our framework instead, as the follower invests at P_F, the leader will i) lose the expected discounted (monopoly) revenues, $\frac{D_1 P_F}{(r-\mu)}$, ii) earn instead a competitive revenue, $\frac{D_2 P_F}{(r-\mu)}$, and also iii) receive a lump-sum payment γ_L. Therefore, the leader's expected value, $L(P)$, can be written as

$$L(P_t) = \begin{cases} L_1(P_t) = \frac{D_1 P_t}{(r-\mu)} - K + \left[\frac{(D_2 - D_1) P_F}{(r-\mu)} + \gamma_L\right]\left(\frac{P_t}{P_F}\right)^\delta & \text{if } P_t < P_F, \\ L_2(P_t) = F_2(P_t) + \gamma_F + \gamma_L = \frac{D_2 P_t}{(r-\mu)} - K + \gamma_L & \text{if } P_t \geq P_F. \end{cases} \quad (2.6)$$

In order to solve for the leaders' investment trigger, P_L, the argument runs similar to Dixit and Pindyck (1994). If there is any incentive to be a leader, that is, to invest when the state variable is below P_F, it must be the case that the payoff from being the leader is greater or equal to the payoff of being the follower. Then, the optimal entry strategy for the leader must be such that at P_L, the leader's payoff is equal or marginally greater than the follower's payoff. This implies that P_L must be the root of the following equation

$$L_1(P_L) = F_1(P_L). \quad (2.7)$$

Though one cannot provide a closed form solution for P_L,[25] it is easy to see that P_L and P_F are the equilibrium strategies of the Stackelberg game if and only if

$$\begin{cases} L_1(P_t) < F_1(P_t) & \text{for } P_t < P_L, \\ L_1(P_t) \geq F_1(P_t) & \text{for } P_L \leq P_t < P_F, \\ L_2(P_t) > F_2(P_t) & \text{for } P_t \geq P_F, \end{cases} \quad (2.8)$$

The former conditions imply that if, $P_t < P_L$, the value of being the follower exceeds the value of being the leader, therefore both players strictly prefer to delay investment and be the follower. As soon as P_t crosses P_L and remains below P_F, both players prefer to be the leader. Therefore for $P_L \leq P_t < P_F$, both firms would try to invest, but only the patent-holder succeeds and becomes the leader. Hence, the optimal decision for the follower is to delay investment until the state variable has reached the threshold level P_F.

Figure 1 shows the leader's and follower's claim values. The intersection between F_1 and L_1 occurs when $P_t = P_L$, which denotes the leader's entry trigger. Regarding the trigger level P_F, the follower's payoffs, F_1, smooth-pastes the straight line F_2. Moreover, when the state variable starts at or above P_F, both firms have the same expected revenues, $\frac{P_t D_2}{(r-\mu)}$, but the gap between the leader's and the follower's payoff arises from the costs side. The leader's net cost is equal to $K - \gamma_L$, whilst the follower's cost corresponds to $K + \gamma_F$. In fact, when simultaneous investment would occur – the state variable starts above P_F – only one firm wins the patent race and therefore receives γ_L, whilst the rival firm pays the licensing cost, γ_F.

To fully understand the preemptive effect of the patent, Figure 2 presents the investment and trigger values depicted in Figure 1, together with the values of the standard model described in Dixit and Pindyck (1994) – where γ_F is equal to zero. The leader and follower values and their entry triggers, of the model by Dixit and Pindyck, are denoted by the small case letters: l, f, p_l and p_f. As argued, when

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[25]Notice, in fact, that P_L is the root of a polynomial equation of degree δ.

a patent is granted to the first investor and licensed to the second one, the first mover advantage increases proportionally to the licensing fees. Therefore, each firm's ability to delay is additionally undermined by the fear of preemption. Under a patent-licensing scheme, P_L is always below p_l, therefore, the market entry by the leader is accelerated and the innovation is marketed sooner. This result might not seem surprising. As commonly held, by rewarding innovators, IP protection should accelerate innovation. What we want to emphasise here is that it is true that innovation is delivered to the market at an early stage, however, when firms are ex-ante homogeneous, the acceleration results from a preemption war (fear) more than a rewarding scenario. Consistently, in our extension of Dixit and Pindyck's model, the follower's entry and, hence the competitive stage, is delayed due to the additional licensing cost, that is $p_f < P_F$.

2.3. **Cooperative Scenario: Nash Axiomatic Approach.** This subsection analyses an alternative scenario where, instead of competing in a Stackelberg game, players cooperate by signing a long-term state contingent contract which defines the rules of collusion. It is organised as follows. First, the cooperative scenario is formalised through a symmetric Nash axiomatic bargaining; and second, the equilibrium payoffs are derived.

2.3.1. *Nash Axiomatic Bargaining.* Since the technology is fixed, collusion through market quotas or product differentiation is not feasible. Therefore, if limiting the supply to one unit is convenient, collusion can only take the form of a monopoly situation, with one innovator investing and sharing profits with the other one who remains idle.

Moreover, colluding via a monopoly can be beneficial to both innovators as a temporary stage. Intuitively, this is due to the fact that demand might be sufficiently elastic. If this is the case, for sufficiently high levels of the state variable, the sum of the values of the two innovative firms in duopoly might be greater than the value of one monopolistic firm (in which case, a duopoly market might be convenient for high levels of the stochastic demand). For low levels of the state variable the reverse situation applies, which is, monopoly is more convenient than competition and therefore it might be established as a temporary situation.

In order to embody all possible benefits from cooperation, players are allowed to sign an agreement where the terms of the contract define the rule to share the monopolistic profit (whilst one innovative firm invests and the other stays idle), and also the duration of the monopoly scenario, that is, the termination of the contract. Therefore, our cooperative framework is conveniently flexible and able to capture situations where cooperation might lead to a permanent monopoly (where the agreed duration of the contract is infinite) or, a temporary one (where the duration of the contract is finite).

Three features of the contract will be agreed on:

i) the timing of collusion – the level of the state variable at which one innovative firm invests and starts sharing profits with the idle one. This entry trigger is referred to as $\widetilde{P_L}$, meaning that one innovative firm invests as soon as the state variable, P_t, crosses the cooperative trigger $\widetilde{P_L}$;

ii) the partition of the collusive gains, that is, the share of the instantaneous revenues, λ, retained by the monopolist and the residual share, $1 - \lambda$, paid to the idle innovator;

iii) the termination of the contract, that is, the level of the state variable at which the idle innovative firm is allowed to invest by paying the lump-sum costs K without any additional cost. Corresponding to this trigger level, the (former) monopolist stops sharing the revenues. This entry trigger is referred to as $\widetilde{P_F}$.

The environment is cooperative, which implies that players can commit and stick to their promises. Therefore, the bargaining situation is solved by using a symmetric Nash axiomatic approach, where $\widetilde{P_L}$, $\widetilde{P_F}$ and λ, are the unknowns of the bargaining problem.

Briefly, the Nash bargaining can be abstractly understood as a situation where two individuals have the opportunity to share a 'pie' of size one, for instance, by making simultaneous demands to a referee.[26] If the demands are feasible – they sum to one – agreement is reached, the pie is split according to the demands and the game is over. Otherwise, if the demands exceed the size of the pie, the game terminates without an agreement and players receive their disagreement payoff. It has been shown that the Nash bargaining solution, is the unique solution satisfying various axioms, one of which is Pareto efficiency.

In order to define the disagreement payoffs of the Nash axiomatic approach, the equilibrium payoffs of the Stackelberg game, previously derived, are used – they represent the payoffs in the non-cooperative environment.

As argued, the problem consists of solving the bargaining situation for the unknown terms of the contracts, $\widetilde{P_L}$, $\widetilde{P_F}$ and λ. It has been proved that players will agree on a contract which maximises the product of players' surplus – the difference between the agreement and the disagreement payoff – referred to as Nash product.[27]

The Nash product, in our case, corresponding to the symmetric demand game can be written as[28]

$$\left(\widetilde{L} - d\right)\left(\widetilde{F} - d\right), \tag{2.9}$$

where the agreement payoffs for the monopolist and the idle entrepreneur are, respectively

$$\widetilde{L} = \left[\frac{\lambda D_1 \widetilde{P_L}}{(r-\mu)} - K\right]\left(\frac{P_t}{\widetilde{P_L}}\right)^{\delta} + (D_2 - \lambda D_1)\left[\frac{\widetilde{P_F}}{(r-\mu)}\right]\left(\frac{P_t}{\widetilde{P_F}}\right)^{\delta}, \tag{2.10}$$

$$\widetilde{F} = \left[\frac{(1-\lambda)D_1\widetilde{P_L}}{(r-\mu)}\right]\left(\frac{P_t}{\widetilde{P_L}}\right)^{\delta} + \left\{[D_2 - (1-\lambda)D_1]\left(\frac{\widetilde{P_F}}{(r-\mu)}\right) - K\right\}\left(\frac{P_t}{\widetilde{P_F}}\right)^{\delta}, \tag{2.11}$$

and d is the disagreement payoff for each player. As $\widetilde{P_L}$, $\widetilde{P_F}$ and λ are the maximisers of the Nash product, one can write

$$\left(\widetilde{P_L}, \widetilde{P_F}, \lambda\right) = \arg\max\left(\widetilde{L} - d\right)\left(\widetilde{F} - d\right). \tag{2.12}$$

As mentioned, the disagreement payoffs are derived from the non-cooperative scenario where players compete in a Stackelberg game.

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[26]In more detail, each player proposes a partition of the 'pie' to a referee, and he does so without knowing the other player's demand.

[27]See Nash (1950).

[28]An extension of the Nash axiomatic problem to the case where the 'pie' is driven by a geometric Brownian motion is due to Perraudin and Psillaki (1999).

We know that, if players try to file for a patent at the same time, with probability 1/2 one succeeds, whilst the other will (optimally) delay investment and become the follower. Hence, if no agreement is reached at a cooperative stage, with probability 1/2, each player will become the leader or the follower in the Stackelberg game. Therefore, ex-ante, in a pre-patent/investment situation, players are homogeneous and therefore they have identical disagreement payoffs which are given by

$$d = \frac{F(P_t) + L(P_t)}{2}. \tag{2.13}$$

2.3.2. *Equilibrium Payoffs.* Here we provide the equilibrium contract of the cooperative game, that is, the level of $\widetilde{P_L}$, $\widetilde{P_F}$ and λ that maximise the Nash product.

By maximising (2.12) with respect to $\widetilde{P_L}$, $\widetilde{P_F}$ and λ, the following system of equations are obtained from the first order conditions

$$\begin{cases} \frac{\partial \widetilde{L}/\partial \widetilde{P_L}}{(\widetilde{L}-d)} + \frac{\partial \widetilde{F}/\partial \widetilde{P_L}}{(\widetilde{F}-d)} = 0, \\ \frac{\partial \widetilde{L}/\partial \widetilde{P_F}}{(\widetilde{L}-d)} + \frac{\partial \widetilde{F}//\partial \widetilde{P_F}}{(\widetilde{F}-d)} = 0, \\ \frac{\partial \widetilde{L}/\partial \lambda}{(\widetilde{L}-d)} + \frac{\partial \widetilde{F}/\partial \lambda}{(\widetilde{F}-d)} = 0. \end{cases} \tag{2.14}$$

By noticing that

$$\partial \widetilde{L}/\partial \lambda = \left[\frac{D_1 \widetilde{P_L}}{(r-\mu)} \right] \left(\frac{P_t}{\widetilde{P_L}} \right)^\delta - \left[\frac{D_1 \widetilde{P_F}}{(r-\mu)} \right] \left(\frac{P_t}{\widetilde{P_F}} \right)^\delta = -\partial \widetilde{F}/\partial \lambda, \tag{2.15}$$

one can easily rearrange the FOC as follows

$$\begin{cases} \partial \widetilde{L}/\partial \widetilde{P_L} + \partial \widetilde{F}/\partial \widetilde{P_L} = 0, \\ \partial \widetilde{L}/\partial \widetilde{P_F} + \partial \widetilde{F}/\partial \widetilde{P_F} = 0, \\ \widetilde{L} = \widetilde{F}. \end{cases} \tag{2.16}$$

The first two equations capture the Pareto efficiency of the Nash axiomatic solution. In fact, it is evident from the first two conditions that $\widetilde{P_L}$ and $\widetilde{P_F}$ maximise the overall agreement payoff, shortly (AP), where $AP = \widetilde{L} + \widetilde{F}$.[29] Therefore, as argued by Perraudin and Psillaki (1999), one can refer to these two conditions as efficiency conditions. The third condition, $\widetilde{L} = \widetilde{F}$, simply shows that the game is perfectly symmetric in terms of bargaining power and disagreement payoffs. The fact that players are identical in the negotiation as well as in disagreement implies that they will agree on splitting the pie in half. Therefore, not surprisingly, the Nash bargaining provides players with identical payoffs.

Furthermore, it is interesting to notice that the state variable, P_t, drops out of the first order conditions. This result is not surprising in our setting where players bargain ex-ante over future cash flows and do not receive any current cash flow before bargaining. Intuitively, when bargaining occurs ex-ante,[30] looking at the definition of $\widetilde{L}, \widetilde{F}$ and d the current level of the state variable enters in the bargaining problem (2.12) only via stochastic discount factors of the form $(P_t/P)^\delta$ where P represents any of the trigger levels: P_L, P_F (appearing in the definition of d), $\tilde{P}_L$ or $\tilde{P}_F$ (appearing in the definitions of $\widetilde{L}$ and $\widetilde{F}$). Therefore, the maximum

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[29]See Appendix 2.

[30]That is, none of the players has invested yet.

of the Nash product is affected by P_t only via a multiplicative shock of the form P_t^δ and this eliminates the dependence on P_t from the first order conditions.[31] In other words, the stationary nature of the Nash problem in our setting is due to the fact that players are bargaining over option values.

Solving the above system for the three unknowns – $\widetilde{P_L}$, $\widetilde{P_F}$ and λ – yields the following equations

$$\widetilde{P_L} = \left(\frac{\delta}{\delta - 1}\right)\left[\frac{K\,(r - \mu)}{D_1}\right], \tag{2.17}$$

$$\widetilde{P_F} = \left(\frac{\delta}{\delta - 1}\right)\left[\frac{K\,(r - \mu)}{\max\{2D_2 - D_1, 0\}}\right], \tag{2.18}$$

$$\lambda = \frac{1}{2} + \left(\frac{\delta - 1}{2\delta}\right)\left[\frac{1 - (\widetilde{P_L}/\widetilde{P_F})^\delta}{1 - (\widetilde{P_L}/\widetilde{P_F})^{\delta-1}}\right], \tag{2.19}$$

which represent the agreed terms of the equilibrium contract.

First, note that, as long as $D_2 < D_1$, then $\widetilde{P_L} < \widetilde{P_F}$. This implies that a collusive monopoly will always be established and it will persist as long as the state variable (starts and) remains below $\widetilde{P_F}$.

In the limiting case, where demand is infinitely elastic, and, hence, $D_1 = D_2$, then $\widetilde{P_L} = \widetilde{P_F}$. In this scenario, cooperation is still beneficial in that players can agree to allocate ex-ante the patent and not to charge for the licence. In this case, there is simultaneous entry, and the entry threshold becomes

$$\widetilde{P_L} = \widetilde{P_F} = \left(\frac{\delta}{\delta - 1}\right)\left[\frac{K\,(r - \mu)}{D_1}\right]. \tag{2.20}$$

Notice that, according to the above entry trigger, when cooperating, each firm can maximise the option value to invest, and the preemption threat – which is due to the presence of a patent – is fully eliminated.

In addition, one can notice that, if $\max\{2D_2 - D_1, 0\} = 0$, that is, if $2D_2 \leq D_1$, then $\widetilde{P_F} = \infty$. This result implies that the contract establishes a permanent 'collusive monopoly'.

With regard to the sharing rule, λ, one can notice that this is purely determined by the slope of demand curve (D_1 and D_2) and the term δ, whilst the investment cost, K, does not play any role.[32]

Last, by some simple algebra, one can verify that, for D_2 taking values between $(0, D_1)$, the share λ is always $\in \left[\frac{1}{2} + \frac{\delta-1}{2\delta}, 1\right]$. In fact, when

$$D_2 \to D_1 \quad \text{then} \quad \lambda \to \frac{1}{2} + \left(\frac{\delta - 1}{2\delta}\right)\left(\frac{\delta}{\delta - 1}\right) = 1,$$

whilst when

$$D_2 \to 0 \quad \text{then} \quad \lambda \to \frac{1}{2} + \frac{\delta - 1}{2\delta}.$$

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[31]Simply factorise the Nash product with respect to the term P_t^δ.

[32]Formally, the term K cancels out in the ratio $\widetilde{P}_L/\widetilde{P}_F$. Intuitively, K does not affect the sharing rule because both firms pay the same investment cost. Nevertheless, under cooperation, one firm invests and pays K earlier than the idle firm. Therefore, the relative discount factor (which depends on δ and the slope of demand) determines the asymmetry of the sharing rule.

Stackelberg and Cooperative Equilibrium: A Comparison
Here, we provide a comparison between the cooperative equilibrium and the Stackelberg one. The purpose is, first, to stress the (Pareto) inefficiency generated in the leader-follower equilibrium in comparison with the Nash bargaining solution. Second, we will determine under what circumstances collusion accelerates the establishment of a duopoly instead of delaying it.

Given the Nash equilibrium entry triggers, $\widetilde{P_L}$ and $\widetilde{P_F}$, and the Stackelberg ones P_L and P_F, it is immediate to notice that the inefficiency of the non-cooperative scenario simply reflects the fact that the agreement payoff is always greater than the disagreement payoff. A deeper analysis of the agreement and (Stackelberg) disagreement payoff clearly shows the source of inefficiency. The agreement payoff, AP, can be written as the sum of two functions, say $M(\cdot)$ and $C(\cdot, \cdot)$, which are evaluated as follows

$$M(\widetilde{P_L}) = \left(\frac{D_1 \widetilde{P_L}}{r - \mu} - K \right) \left(\frac{P_t}{\widetilde{P_L}} \right)^\delta, \tag{2.21}$$

and

$$C(\widetilde{P_F}, K) = \left[(2D_2 - D_1) \left(\frac{\widetilde{P_F}}{r - \mu} \right) - K \right] \left(\frac{P_t}{\widetilde{P_F}} \right)^\delta. \tag{2.22}$$

Similarly, the overall disagreement payoff, say $DP = 2d$, can be written as the sum of the two functions, $M(\cdot)$ and $C(\cdot, \cdot)$, evaluated as follows

$$M(P_L) = \left(\frac{D_1 P_L}{r - \mu} - K \right) \left(\frac{P_t}{P_L} \right)^\delta, \tag{2.23}$$

and

$$C(P_F, K + \gamma_F - \gamma_L) = \left[(2D_2 - D_1) \left(\frac{P_F}{r - \mu} \right) - K - (\gamma_F - \gamma_L) \right] \left(\frac{P_t}{P_F} \right)^\delta.$$

It is easy now to notice that[33]

$$M(\widetilde{P_L}) \geq M(P_L) \quad \text{and} \quad C(\widetilde{P_F}, K) > C(P_F, K + \gamma_F - \gamma_L). \tag{2.24}$$

The first inequality captures a first source of inefficiency. This is due to early entry of the leader in the Stackelberg game, where an optimal investment decision, like $\widetilde{P_L}$, would be pre-empted.[34]

Another source of inefficiency is captured by the second inequality which refers to the entry in the competitive stage. In the Stackelberg game, the follower's investment decision depends on γ_F and D_2. Regardless of the initial level of demand, the follower will invest at P_F by paying the additional cost γ_F. The follower will invest even if the drop in demand is such that $2D_2 - D_1 \leq 0$, that is, if the value of the two running businesses is lower than the monopolistic one. Therefore, intuitively, the inefficiency results from the fact that D_1 does not play any role in the follower's entry trigger.

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[33]See Appendix 3.

[34]One can see that the Nash entry level $\widetilde{P_L}$ also corresponds to the optimal entry trigger of a Stackelberg game where the role of the leader and that of the follower are exogenously allocated. In this case, the leader would make the optimal investment decision by maximising her/his expected payoff. This is given by $L_1(P_t/P)^\delta$, with L_1 evaluated at P. Therefore, the first order condition yields $\arg\max_P L_1(P_t/P)^\delta = \widetilde{P_L}$. See Dixit and Pindyck (1994).

Lastly, as already argued, all inefficiencies are waived via cooperation by the fact that $\widetilde{P_L}$ and $\widetilde{P_F}$ maximise the agreement payoff AP. This can be re-formalised as

$$\left\{\widetilde{P_L}, \widetilde{P_F}, \gamma = 0\right\} = \arg \max_{P_l, P_f, \gamma} M(P_l) + C(P_f, K + \gamma), \tag{2.25}$$

where the term $\gamma = 0$ simply highlights the fact that in the cooperative framework the additional cost γ_F is not borne. In a way, the former consideration stresses that, through cooperation, players maximise the investment value in both the initial monopolistic stage and in the competitive one.[35] This can be simply rephrased by saying that players maximise the 'project' value by choosing the optimal scale of the investment through a state-contingent contract.

We conclude this section by comparing the entry triggers of the cooperative game and the leader-follower game.

As argued, $\widetilde{P_L}$ corresponds to the entry trigger level of the standard leader-follower model (without patent) where the leadership is exogenously allocated. Therefore, not surprisingly, $\widetilde{P_L}$ must necessarily be greater than P_L which is determined under the threat of preemption. This result simply underlines that cooperation restores the option value of delaying investment until an optimal entry.

Regarding $\widetilde{P_F}$, this cooperative entry level is not necessarily greater than P_F. One can notice that, depending on the investment cost, the slope of the demand and the fixed-fee licence, cooperation can accelerate competition (the establishment of duopoly). When $\widetilde{P_F}$ is finite, that is $2D_2 > D_1$, then, by some simple algebra, one can verify that the following condition holds:

$$\widetilde{P_F} < P_F \text{ if and only if } K\left(\frac{D_1 - D_2}{2D_2 - D_1}\right) < \gamma_F \text{ with } 2D_2 > D_1,$$

which can be rearranged as

$$\widetilde{P_F} < P_F \leftrightarrow \frac{K}{\gamma_F} < \frac{1}{\left(\frac{D_1}{D_2} - 1\right)} - 1 \text{ with } \frac{D_1}{D_2} \in (1, 2). \tag{2.26}$$

We conclude that the above inequality is more likely to hold for $i)$ a low ratio K/γ_F and $ii)$ low ratio D_1/D_2, that is high elasticity of demand.

Figures 3 and 4 show two scenarios in which cooperation results either in a permanent monopoly (Figure 3) or in a temporary one (Figure 4). In particular, Figure 3 compares the Nash bargaining solution with the leader-follower equilibrium. The AP line represents the agreement payoff, whilst the DP line represents the overall disagreement payoff – the sum of the leader's and the follower's payoffs. The cooperative entry trigger, $\widetilde{P_L}$, is delayed by cooperation with respect to the non-cooperative one, P_L. In this case, a monopolistic firm is more profitable than two joint duopolistic firms (i.e. $2D_2 < D_1$); therefore $\widetilde{P_F}$ tends to infinity and cooperation establishes a permanent monopoly. By contrast, Figure 4 shows that for sufficiently high levels of the state variable, it is advantageous to let the idle innovative firm enter the market because $2D_2 > D_1$. Moreover, in this case, condition (2.26) is satisfied, therefore $\widetilde{P_F} < P_F$ and cooperation accelerates competition instead of delaying or eliminating it.

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[35]Or, also, one could say that players minimise the loss from competition. Notice that whenever $2D_2 < D_1$ – competition would generate a loss to the firms collectively – players will commit to establish a permanent monopoly.

3. CONCLUSIONS

This paper has analysed, in a real option framework, the effects of cooperative collusion on a duopoly market protected by a patent. A patent is granted to the first innovator/investor (who files the patent application) and the innovation is licensed to the second entrant. Unlike previous studies on collusion, by introducing uncertainty through a stochastic demand, we find that cooperative collusion does not necessarily delay competition. Collusion always delays market entry and therefore innovation; depending on a number of economic factors, collusion may accelerate the establishment of a duopoly. If firms can cooperate before facing a patent/investment race, they can agree on sharing monopoly profits until one monopolistic firm generates higher profits than two firms. Moreover, when two firms generate higher joint profits than one monopolistic firm, then, ex-ante, players can cooperatively allocate the patent and agree not to enforce the licensing fee. We have shown that depending on the elasticity of demand, the fixed-fee licence and the investment costs, collusion can accelerate duopoly. In particular, this occurs when the elasticity of demand is high and/or the ratio of the investment cost to the licensing fee is low. This finding may be a promising result in lessening the tension between antitrust and IP laws.

Furthermore, the cooperative environment under uncertainty helps to identify two sources of inefficiency generated in the preemption game. First, the preemption threat in the non-cooperative game accelerates the market entry by the leader when sub-optimal. The higher the licensing fee, the more the option value of delay will be eroded. In other words, as the advantage of the first mover increases with the fixed-fee licence, each firm's ability to delay is additionally undermined by the fear of preemption. The second source of inefficiency is determined by the fact that the follower's investment decision is not affected by the level of the revenues generated during monopoly. That is, the follower invests regardless of the drop of revenues (of the two firms collectively) generated by her/his entry decision. Therefore, under cooperation, Pareto efficiency is achieved via: i) eliminating the preemption threat by resolving the uncertainty over the allocation of the patent, which restores the option value of delay and reestablishes the importance of the real option framework in a market entry decision, and ii) maximising the joint project value by choosing the optimal scale of the investment.

Lastly, the model provides a simple and realistic framework, which may help in at explaining the recent wave of consolidations and alliances in the pharmaceutical industry. A crucial reason is the record number of patent expirations faced by the pharmaceutical industry during these years. According to our setting, in the pre-patent situation when firms are still homogeneous, the collusive scenario is most likely to occur, especially if firms can detect potential competitors in advance. Moreover, the transparency of the FDA approval process for new drugs makes the pharmaceutical industry uniquely suited for detecting potential competition. Therefore, the option to collude is usually available to pharmaceutical companies well in advance to market entry.

APPENDIX

Appendix 1   We assume that the leader has already invested. The follower's entry decision and the leader's decision about charging for the use of the innovation

can be described in the following bimatrix game, where the leader is the row player and the follower is the column player.

		Follower		
		Invest		Wait
Leader	Exercise	$\frac{D_2 P_t}{(r-\mu)} + \gamma_L$,	$\frac{D_2 P_t}{(r-\mu)} - K - \gamma_F$	Repeat Game
	Not Exercise	$\frac{D_2 P_t}{(r-\mu)}$,	$\frac{D_2 P_t}{(r-\mu)} - K$	Repeat Game

If the follower delays investment then the game is repeated because the leader retains the option to exercise the licence fee in the future, and the option expires immediately after the follower has invested.

Assume that there exists a finite threshold, $\bar{P} = \sup\{P_f\}$, where P_f is the follower's set of stopping strategies, and at $P_t = \bar{P}$, $\frac{D_2 P_t}{(r-\mu)} - K - \gamma_F > 0$. According to this, there exists a stopping time τ_F, such that $\tau_F = \{\inf t \geq 0; \ P_t \geq \bar{P}\} < \infty$. When the game reaches $t = \tau_F$, the follower can guarantee a payoff equal to zero by never investing. Therefore he/she will strictly prefer to invest regardless of the leader strategy, because in the worst case scenario, he/she can guarantee a payoff greater than zero, that is, $\frac{D_2 P_t}{(r-\mu)} - K - \gamma_F > 0$. When the follower invests the leader always exercises his option as long as $\gamma_L > 0$. $\square$

Appendix 2   Grouping the derivative operator in the first two equations in (2.16)

$$\begin{cases} \partial(\widetilde{L} + \widetilde{F})/\partial\widetilde{P}_L = 0, \\ \partial(\widetilde{L} + \widetilde{F})/\partial\widetilde{P}_F = 0, \end{cases}$$

and using the fact that $\widetilde{L} + \widetilde{F}$ is the overall agreement payoff, implies that in the Nash bargaining, players reach an agreement such that the terms of the contract, $\widetilde{P}_L$ and $\widetilde{P}_F$, maximise the value of the 'pie' they are going to split, therefore guaranteeing the Pareto Efficiency. $\square$

Appendix 3   This appendix proves that

$$M(\widetilde{P}_L) \geq M(P_L) \quad \text{and} \quad C(\widetilde{P}_F, K) > C(P_F, K + \gamma_F - \gamma_L),$$

where $M(\cdot)$ and $C(\cdot, \cdot)$ have been defined as follows

$$M(P_L) = \left(\frac{D_1 P_L}{r - \mu} - K\right)\left(\frac{P_t}{P_L}\right)^\delta,$$

and

$$C(P_F, K + \gamma_F - \gamma_L) = \left[(2D_2 - D_1)\left(\frac{P_F}{r - \mu}\right) - K - (\gamma_F - \gamma_L)\right]\left(\frac{P_t}{P_F}\right)^\delta.$$

It is easy to show that the above inequalities hold by noticing that the triggers $\widetilde{P}_L$ and $\widetilde{P}_F$ are the maximisers of $M(\cdot)$ and $C(\cdot, \cdot)$ respectively. In fact, the first order conditions lead to

$$\frac{\partial M(P)}{\partial P} = 0 \rightarrow P = \left(\frac{\delta}{\delta - 1}\right)\frac{K(r - \mu)}{D_1} = \widetilde{P}_L,$$

$$\frac{\partial C(P, K)}{\partial P} = 0 \rightarrow P = \left(\frac{\delta}{\delta - 1}\right)\frac{K(r - \mu)}{\max\{2D_2 - D_1, 0\}} = \widetilde{P}_F,$$

with second order conditions resulting into

$$\frac{\partial^2 M(P)}{\partial P^2} < 0,$$

$$\frac{\partial^2 C(P,K)}{\partial P^2} < 0 \ \text{ iff } \ 2D_2 - D_1 > 0.$$

Therefore, being $\widetilde{P}_L$ and $\widetilde{P}_F$ the maximisers of $M(\cdot)$ and $C(\cdot,\cdot)$ and noticing that the term $\gamma_F - \gamma_L$ is positive, one can conclude that the following inequalities should hold

$$M(\widetilde{P}_L) = \max_P M(P) \geq M(P_L)$$

$$C(\widetilde{P}_F, K) = \max_P C(P,K) \geq C(P_F, K) > C(P_F, K + \gamma_F - \gamma_L). \ \square$$

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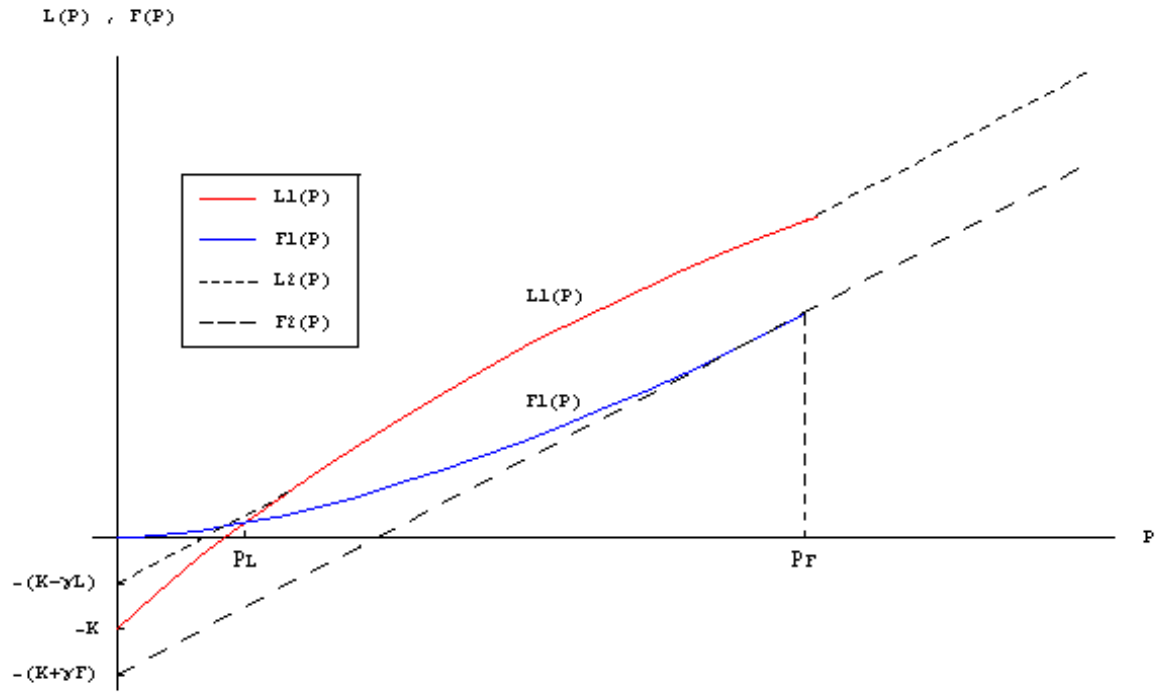
FIGURE 1. Leader-Follower Equilibrium.

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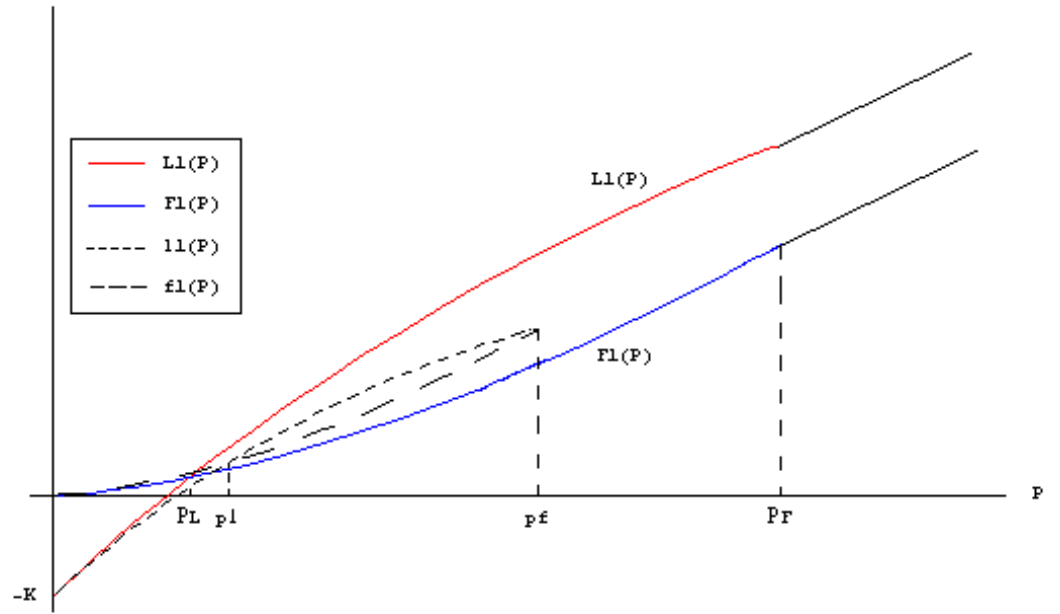FIGURE 2. Leader-Follower Equilibrium: With and Without Patent.

FIGURE 3. Cooperative Collusion via Permanent Monopoly.

L(P) , F(P), AP, DP

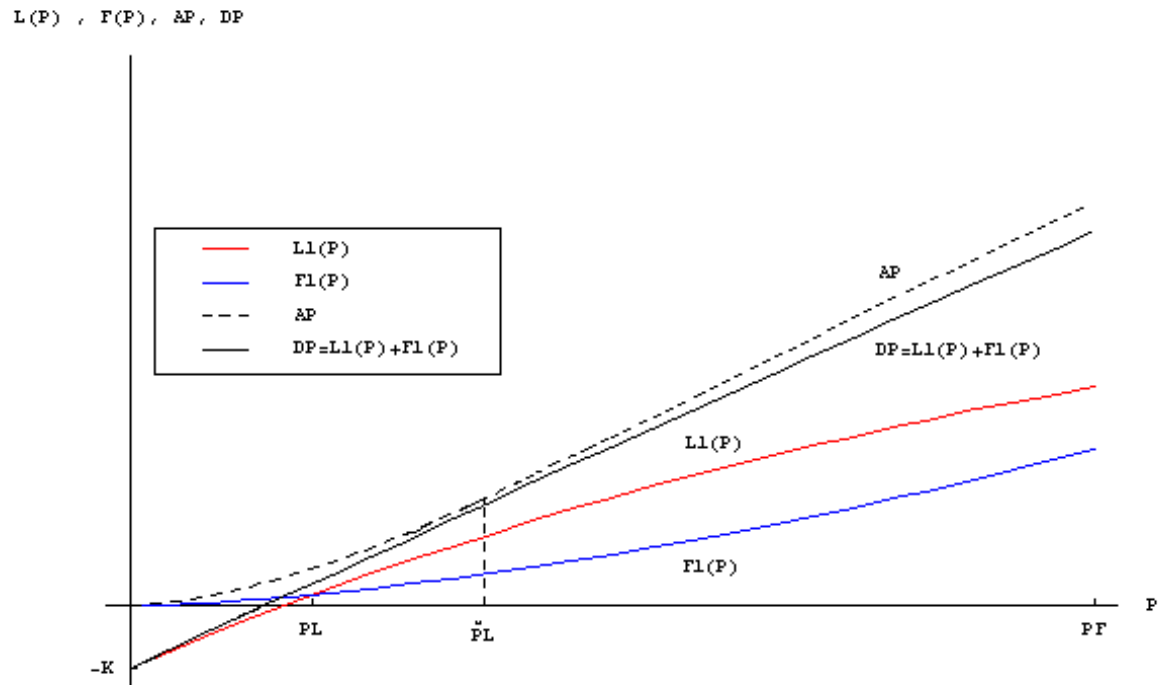FIGURE 4. Cooperative Collusion via Temporary Monopoly.