# «Unanticipated vs. Anticipated Tax Reforms in a Two-Sector Open Economy? » 

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# UNANTICIPATED VS. ANTICIPATED TAX REFORMS IN A TWO-SECTOR OPEN ECONOMY* 

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#### Abstract

We use a two-sector neoclassical open economy model with traded and non-traded goods to investigate the effects of unanticipated and anticipated tax reforms. First, an unanticipated tax reform produces an expansion of GDP, labor, and investment, while an anticipated tax reform has opposite effects before the implementation of the labor tax cut. Quantitatively, if the traded sector is more capital intensive, GDP increases by 1.6 percentage points or declines by 2.8 percentage points after three years, depending on whether the tax cut is unanticipated or anticipated. Second, we find that GDP change masks a wide dispersion in sectoral output responses. Importantly, in all scenarios, a tax reform substantially raises the relative size of the non-traded sector while traded output always drops. Allowing for the markup to depend on the number of competitors, we find that a significant share of GDP change can be attributed to the competition channel while the dispersion of sectoral output responses is amplified. Finally, the workers only benefit from the labor tax cut if the tax change is unanticipated and the traded sector is more capital intensive.


Keywords: Non Traded Goods; Investment; Tax Reform; Anticipation effects.
JEL Classification: F41, E62, E22, F32.

[^0]
## 1 Introduction

Tax reform continues to be a key item on the policy agenda of many industrialized countries. From the early nineties, European tax systems were asked to achieve conflicting targets: reducing the unemployment rate and achieving budget balance over the medium run. As a consequence, several European countries have cut labor tax while consumption tax has been raised in response to the decline in fiscal revenues, due to the unavailability of debt financing. Recently, in response to deficits accumulated by the current crisis, western countries have had to find new revenues while trying to avoid higher labor taxes at the same time. In view of governments' revenue needs and low employment rates, several OECD countries are discussing changing the tax structure, i.e. a tax reform that puts more weight on consumption taxes but reduces labor taxes. ${ }^{1}$ Even though this kind of tax structure change has recently gained popularity, the extent of its beneficial effects is debatable. Further, Mertens and Ravn [2009] and Favero and Giavazzi [2011] find evidence of a fall in GDP before the implementation of a tax cut as a result of fiscal foresight. In this paper, we revisit the effects of unanticipated tax reforms by contrasting those of anticipated tax reforms in a small open neoclassical economy.

One major goal in this paper is to estimate the impact of unanticipated and anticipated tax reforms on the sectoral composition of output. Our study is motivated by recent estimates provided by Bénétrix and Lane [2010] which reveal that fiscal shocks have a significant impact on the sectoral composition of aggregate output and disproportionately benefit the non-traded sector. Furthermore, while it is currently accepted that the labor intensive sector always benefits more from the labor tax cut, our model can test such an assertion. We draw on earlier work by Turnovsky and Sen [1995] who develop an open economy model with a traded and a non-traded sector, but consider elastic labor supply and imperfect competition in product markets. ${ }^{2}$ One attractive feature of a two-sector model with tradable and non- tradable goods is to cover both the closed economy and the open economy dimensions of contemporary industrialized countries. In particular, the empirical evidence shows that the non-tradable content of GDP and employment is substantial, at around two-thirds. ${ }^{3}$ From an analytical point of view, as in a closed economy model, capital accumulation clears the home good market, see e.g., Baxter and King [1993]. In the same way as in a small open economy, external borrowing allows households to smooth consumption intertemporally. Additionally, a two-sector model allows us to investigate the distribution effects, i.e. the movement in the labor share, while in a one-sector model, the

[^1]

Figure 1: Tax Rates in Six European Countries (BEL, FIN, FRA, ITA, ESP, SWE)
labor share is fixed (as long as the elasticity of substitution between capital and labor equal to one).

Since tax reforms take various forms, we consider three types of policy experiments: i) two revenue-neutral tax reforms shifting the tax burden from labor to consumption tax and ii) one labor tax reform shifting the composition of the tax wedge from payroll taxes (i.e. the employer's part of labor taxes) to wage taxes (i.e. the employee's part of labor taxes). ${ }^{4}$ The motivation for considering such revenue-neutral tax reforms draws on the change in tax structure in European countries over the last twenty years. Figure 1 plots changes in labor taxes and consumption taxes over the period 1990-2007 for six European countries featuring high labor taxes and unemployment rates in the early nineties. Figure 1 shows that the labor tax has been lowered by about 4 percentage points and over the same period the consumption tax has been raised substantially in response to the decline in fiscal revenues. Other countries, for example Hungary in the nineties, did not change the consumption tax but instead cut payroll taxes and raised wage taxes. Hence, we also estimate the effects of a shift from payroll taxes to progressive wage taxes as in Heijdra and Lightart [2009]. From a theoretical point of view, focusing on the three tax reforms outlined above is analytically convenient. More specifically, we show that changes in aggregates following a tax reform are simply a scaled-down version of the changes after a labor tax cut associated with a fall in lump-sum transfer.

[^2]Several papers exist, taking the neoclassical and New Keynesian approaches, that analyze the effects of tax reforms in an open economy, see Mendoza and Tesar [1998] and Coenen et al. [2008]. ${ }^{5}$ However, most of the analyses consider unanticipated tax shifts. Recent VAR empirical evidence documented by Mertens and Ravn [2009] shows that a tax cut produces opposite responses in GDP, hours worked and investment during the preimplementation period compared to the responses after the policy implementation. ${ }^{6}$ More specifically, evidence provided by Mertens and Ravn shows that an unanticipated tax cut gives rise to significant increases in output, hours worked, and investment while an anticipated tax cut is associated with pre-implementation declines in these aggregates. By contrast, the real wage increases, whether the tax cut is anticipated or not. Our results contribute to the existing literature on tax reforms by contrasting the impact of unanticipated labor tax cuts with the effects of announced tax labor tax reductions. ${ }^{7}$ Additionally, whereas the previous literature estimates the impacts of tax reforms only numerically, we consider a continuous-time framework which allows us to establish a number of analytical results.

Our paper provides three sets of key findings. First, we find that a shift from payroll taxes to consumption taxes produces an increase in GDP which is about half-way between the large-scale effects after a shift from progressive wage taxes to consumption taxes and the much smaller effects following a shift from payroll taxes to progressive wage taxes. Second, the predictions of our model are broadly in line with the VAR evidence related to the effects of tax shocks. In particular, we find that an unanticipated tax reform involving a labor tax cut stimulates consumption, crowds in investment, increases employment and raises GDP, in line with estimates by Mertens and Ravn [2009]. Moreover, in accordance with the evidence from Romer and Romer [2010] for the US and Cloyne [2011] for the UK, the open economy experiences a trade balance deficit on impact as consumption of the traded good increases while traded output falls. ${ }^{8}$ When considering an anticipated labor tax cut, we find that GDP, hours worked and investment decline during the pre-implementation period, regardless of sectoral capital intensities. The reason is that agents now anticipate that they will be richer in the future. The consequent positive wealth effect induces them

[^3]to raise consumption and lower labor supply. As a result, GDP falls initially while higher private consumption crowds out investment expenditure. Third, numerical results reveal that aggregate effects mask a wide dispersion in sectoral output responses. Following an unanticipated 1 percent decrease in the tax revenue (relative to GDP), we find numerically that GDP increases between 1.6 (1.5) percentage points after three years if the traded (nontraded) sector is more capital intensive than the non-traded (traded) sector. ${ }^{9}$ The sectoral decomposition of GDP growth shows that traded output declines by -1.3 (-1.5) percentage points of initial GDP while non-traded output rises by 2.9 (3.0) percentage points. When the tax reform is anticipated, sectoral capital intensities play a key role in driving the response of aggregate output; specifically, GDP falls by -2.8 (-1.0) percentage points while traded output drops by -2.4 (-6.2) and non-traded output falls (rises) by only -0.4 (5.2) percentage points.

Moreover, we explore quantitatively the role of the competition channel in driving the effects of a tax reform. In particular, a growing amount of literature emphasizes that the variation in the number of competitors and the consequent change in the markup provides an important magnification mechanism, see e.g., Jaimovich and, Floetotto [2008], Wu and Zhang [2000], Zhang [2007]. ${ }^{10}$ We draw on Jaimovich and Floetotto [2008] in allowing for the markup to be endogenous. ${ }^{11}$ Following an unanticipated tax reform, our numerical results show that abstracting from the competition channel would underestimate the three years cumulative response of GDP by $5 \%$. When the shock is anticipated, sectoral capital intensities and endogenous markup interact in determining the extent of the effects. If the traded sector is more capital intensive, a model with a fixed markup would underestimate (overestimate) the drop of GDP after three years by almost $15 \%(10 \%)$ as the rise in the markup during the pre-implementation period triggers a recessionary (expansionary) effect on non-traded output.

Finally, when analyzing the sectoral and distribution effects, we find that our two-sector model produces two counter-intuitive results: i) the more labor intensive sector does not always expand following a labor tax cut, ii) and the workers do not always benefit from the labor tax cut. When the tax reform is unanticipated, non-traded output is above trend while traded output stays below in the short run, regardless of sectoral capital intensities. When the tax cut is anticipated, the output of the more labor intensive sector declines

[^4]substantially and remains below trend. Regarding the distribution effects, the workers only benefit from the labor tax cut if the tax change is unanticipated and the traded sector is more capital intensive. When the fall in the labor tax is anticipated, the fall in hours worked raises the capital-labor ratio which in turn drives the labor share down sharply, and more so when the markup is endogenous, regardless of sectoral capital intensities. However, once the tax cut is implemented, the labor share recovers immediately.

Closely related to our paper is the study by Petrucci and Phelps [2009] who use a lifecycle two-sector setup to analyze the consequences of a labor tax cut compensated by a fall in government spending, or a rise in capital or consumption tax. In a similar spirit, we achieve a better understanding of the aggregate effects of tax reforms by investigating sectoral effects in an open economy. In contrast to our study, they restrict their analysis to the effects of unanticipated tax cuts while we investigate the consequences of both unanticipated and anticipated tax reforms. Additionally, we consider a traded sector alternatively more or less capital intensive than the non-traded sector. Further, we introduce imperfect competition in product markets, contrasting the case of a fixed markup with that of an endogenous markup. Finally, we estimate numerically the short-run and long-run effects and conduct a sensitivity analysis with respect to the anticipation horizon.

The remainder of this paper is organized as follows. Section 2 outlines the specification of a two-sector open economy model with traded and non-traded goods. Sections 3 and 4 provide an analytical exploration of the short-run and long-run effects of unanticipated and anticipated tax reforms. In section 5, we report results from numerical simulations and discuss both the aggregate and sectoral effects of a tax reform, contrasting the case of an unanticipated labor tax cut with that of an anticipated tax cut. Section 6 explores the case of an endogenous markup and discusses the quantitative role of the competition channel. In Section 7, we summarize our main results and present our conclusions.

## 2 The Framework

We consider a small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. ${ }^{12}$ The country is small in terms of both world goods and capital markets, and faces a given world interest rate, $r^{\star}$. A perfectly competitive sector produces a traded good denoted by the superscript $T$ that can be exported and consumed domestically. An imperfectly competitive sector produces a nontraded good denoted by the superscript $N$ which is devoted to physical capital accumulation

[^5]and domestic consumption. ${ }^{13}$ The traded good is chosen as the numeraire. ${ }^{14}$

### 2.1 Households

At each instant the representative agent consumes traded goods and non-traded goods denoted by $C^{T}$ and $C^{N}$, respectively, which are aggregated by a constant elasticity of substitution function:

$$
\begin{equation*}
C\left(C^{T}, C^{N}\right)=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{1}
\end{equation*}
$$

where $\varphi$ is the weight attached to the traded good in the overall consumption bundle $(0<\varphi<1)$ and $\phi$ is the intratemporal elasticity of substitution ( $\phi>0$ ).

The agent is endowed with a unit of time and supplies a fraction $L(t)$ of this unit as labor, while the remainder, $l \equiv 1-L$, is consumed as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Households decide on consumption and worked hours by maximizing lifetime utility:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}-\gamma \frac{1}{1+\frac{1}{\sigma_{L}}} L(t)^{1+\frac{1}{\sigma_{L}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{2}
\end{equation*}
$$

where $\beta$ is the consumer's discount rate, $\sigma_{C}>0$ is the intertemporal elasticity of substitution for consumption, and $\sigma_{L}>0$ is the Frisch elasticity of labor supply.

Factor income is derived by supplying labor $L$ at a wage rate $W$, and capital $K$ at a rental rate $R^{K} .{ }^{15}$ Labor is taxed at rate $\tau^{H}$. The wage tax is levied on households' wage income above a certain threshold $\kappa$, which represents the personal tax allowance. Thus, $W^{A}=W-(W-\kappa) \tau^{H}$ corresponds to the after-tax wage. As long as tax allowances are positive, the tax system is progressive which means that the average tax burden rises with the wage rate. In addition, households accumulate internationally traded bonds, $B(t)$, that yield net interest rate earnings of $r^{\star} B(t)$. Denoting lump-sum transfers by $Z$, the households' flow budget constraint can be written as:

$$
\begin{equation*}
\dot{B}(t)=r^{\star} B(t)+R^{K}(t) K(t)+W^{A}(t) L(t)+Z-P_{C}(P(t))\left(1+\tau^{C}\right) C(t)-P(t) I(t), \tag{3}
\end{equation*}
$$

where $P_{C}$ is the consumption price index which is a function of the relative price of nontraded goods $P$. The last two terms represent households' expenditure which include purchases of consumption goods, inclusive of a consumption $\operatorname{tax} \tau^{C}$, and investment expenditure PI. Aggregate investment gives rise to overall capital accumulation according to the

[^6]dynamic equation
\[

$$
\begin{equation*}
\dot{K}(t)=I(t)-\delta_{K} K(t), \tag{4}
\end{equation*}
$$

\]

where we assume that physical capital depreciates at rate $\delta_{K}$. In the rest of this paper, the time-argument is suppressed for the purposes of clarity.

Denoting the co-state variable associated with eq. (3) by $\lambda$, the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C=\left(P_{C}\left(1+\tau^{C}\right) \lambda\right)^{-\sigma_{C}},  \tag{5a}\\
L=\left(\frac{\lambda}{\gamma_{L}} W^{A}\right)^{\sigma_{L}},  \tag{5b}\\
\dot{\lambda}=\lambda\left(\beta-r^{\star}\right),  \tag{5c}\\
\frac{R^{K}}{P}-\delta_{K}+\frac{\dot{P}}{P}=r^{\star}, \tag{5d}
\end{gather*}
$$

and the appropriate transversality conditions. In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta=r^{\star}$ in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, $\lambda$, will undergo a discrete jump when individuals receive new information and must remain constant over time from then on, i.e. $\lambda=\bar{\lambda}$. Finally, applying Shephard's lemma gives consumption in tradables $C^{T}=\left(1-\alpha_{C}\right) P_{C} C$ and non-tradables $P C^{N}=\alpha_{C} P_{C} C$, where $\alpha_{C}$ denotes the non tradable content of consumption expenditures. ${ }^{16}$

### 2.2 Firms

Both the traded and non-traded sectors use physical capital, $K^{T}$ and $K^{N}$, and labor, $L^{T}$ and $L^{N}$, according to constant returns to scale production functions, $Y^{T}=F\left(K^{T}, L^{T}\right)$ and $Y^{N}=H\left(K^{N}, L^{N}\right)$, which are assumed to have the usual neoclassical properties of positive and diminishing marginal products. Both sectors face two cost components: a capital rental cost equal to $R^{K}$, and a labor cost equal to the wage rate plus the employer's part of labor taxes, i.e. $W^{F} \equiv W\left(1+\tau^{F}\right)$. The traded sector is assumed to be perfectly competitive. The first order conditions derived from profit-maximization in the traded sector state that factors are paid to their respective marginal products, i.e. $F_{K}=R^{K}$ and $F_{L}=W^{F}$. As described in more detail below, the non-traded sector contains a large number of industries and each industry comprises differentiated monopolistically competitive intermediate firms. ${ }^{17}$

[^7]The final non-traded output, $Y^{N}$, is produced in a competitive retail sector with a constant returns to scale production which aggregates a continuum of measure one of sectoral non-traded goods. ${ }^{18}$ We denote the elasticity of substitution between any two different sectoral goods by $\omega>0$. In each sector, there are $N>1$ firms producing differentiated goods that are aggregated into a sectoral non-traded good. The elasticity of substitution between any two varieties within a sector is denoted by $\epsilon>0$, and we assume that this is higher than the elasticity of substitution across sectors, i.e. $\epsilon>\omega$ (see Jaimovich and Floetotto [2008]). Within each sector, there is monopolistic competition; each firm that produces one variety is a price setter. Output $\mathcal{X}_{i, j}$ of firm $i$ in sector $j$ is produced using capital and labor, i.e. $\mathcal{X}_{i, j}=H\left(\mathcal{K}_{i, j}, \mathcal{L}_{i, j}\right)$. Each firm chooses capital and labor by equalizing markup-adjusted marginal products to the marginal cost of inputs, i. e. $H_{K} / \mu=R^{K}$, and $H_{L} / \mu=W^{F}$, where $\mu$ is the markup over the marginal costs. In a symmetric equilibrium, non-traded output is equal to $Y^{N}=N \mathcal{X}=H\left(K^{N}, L^{N}\right)$. We assume that there is a large number of firms within each sector, so that each single intermediate producer is small relative to the economy. In this set up each producer in a sector faces the same price elasticity of demand, $\epsilon$. Hence, the producer of a variety charges a constant markup $\mu=\frac{e}{e-1}$, where $e$ is the price-elasticity of demand. Because the number of competitors is large, $e$ is equal to $\epsilon$. In section 6, we relax this assumption and assume instead that a finite number of firms operate within each sector producing non-tradable varieties. ${ }^{19}$ Whether the markup is fixed or endogenous, we assume instantaneous entry, which implies that the zero profit condition holds at each instant of time.

Denoting by $k^{i} \equiv K^{i} / L^{i}$ the capital-labor ratio for sector $i=T, N$, enables us to express the production functions in intensive form, i.e. $f\left(k^{T}\right) \equiv F\left(K^{T}, L^{T}\right) / L^{T}$ and $h\left(k^{N}\right) \equiv H\left(K^{N}, L^{N}\right) / L^{N}$. Production functions are supposed to take a Cobb-Douglas form: $f\left(k^{T}\right)=\left(k^{T}\right)^{\theta^{T}}$, and $h\left(k^{N}\right)=\left(k^{N}\right)^{\theta^{N}}$, where $\theta^{T}$ and $\theta^{N}$ represent the capital income share in output in the traded and non-traded sectors respectively. Since inputs can move freely between the two sectors, marginal products in the traded and the non-traded
small dispersion across countries whereas for the non-traded sector, the markups average about 1.4 with a large dispersion across countries. Additionally, assuming that the traded sector is imperfectly competitive would not affect the results qualitatively, as long as the markup is fixed. Estimates of the markups charged by the traded sector are available on request, while estimates for the non-traded sector are reported in Table 3.
${ }^{18}$ This setup builds on Jaimovich and Floetotto's [2008] model. Details of its derivation can be found in the Appendix.
${ }^{19}$ As stressed by Yang and Heijdra [1993], departing from the usual assumption made by Dixit and Stiglitz [1977] implies that the price elasticity of demand becomes an increasing function of the number of firms and that the markup is endogenous.
sector equalize:

$$
\begin{align*}
\theta^{T}\left(k^{T}\right)^{\theta^{T}-1} & =\frac{P}{\mu} \theta^{N}\left(k^{N}\right)^{\theta^{N}-1} \equiv R^{K},  \tag{6a}\\
\left(1-\theta^{T}\right)\left(k^{T}\right)^{\theta^{T}} & =\frac{P}{\mu}\left(1-\theta^{N}\right)\left(k^{N}\right)^{\theta^{N}} \equiv W^{F} . \tag{6b}
\end{align*}
$$

These static efficiency conditions mean that the sectoral marginal products of capital and labor must equal the capital rental rate $R^{K}$ and the labor cost $W^{F}$, respectively.

Aggregating labor and capital over the two sectors, gives us the resource constraints for the two inputs:

$$
\begin{equation*}
L^{T}+L^{N}=L, \quad K^{T}+K^{N}=K, \tag{7}
\end{equation*}
$$

where $L_{N}=N \mathcal{L}_{N}$ and $K_{N}=N \mathcal{K}_{N}$.

### 2.3 Government

The final agent in the economy is the government which finances government expenditure on traded and non traded goods, $G=G^{T}+P G^{N}$, and lump-sum transfers to households $Z$ by raising taxes on consumption, $\tau^{C} P_{C} C$, and labor, $\left[\tau^{H}(W-\kappa)+\tau^{F} W\right] L=$ $\left(W^{F}-W^{A}\right) L$, in accordance with the balanced condition: ${ }^{20}$

$$
\begin{equation*}
\tau^{C} P_{C} C+\left(W^{F}-W^{A}\right) L=Z+G^{T}+P G^{N} . \tag{8}
\end{equation*}
$$

Since tax reforms can take various forms, we consider three types of tax restructuring. We explore two revenue-neutral tax reforms which involve simultaneously either cutting payroll taxes by $\mathrm{d} \tau^{F}<0$ or progressive wage taxes by $\mathrm{d} \tau^{H}<0$ and raising consumption taxes by $\mathrm{d} \tau^{C}>0$ so that the government budget is balanced. As long as the tax system is progressive, i.e. $\kappa>0$, cutting either $\tau^{F}$ or $\tau^{H}$ produces different labor outcomes because the payroll tax and wage tax bases are not equal.

These two revenue-neutral tax reforms cause a fall in the marginal tax wedge denoted by $\tau^{M}$ defined as the difference between the producer wage and the after-tax marginal wage expressed as a percentage of the producer cost:

$$
\begin{equation*}
\tau^{M}=1-\frac{1-\tau^{H}}{1+\tau^{F}} . \tag{9}
\end{equation*}
$$

We further analyze a third type of tax restructuring which involves simultaneously cutting payroll taxes by $\mathrm{d} \tau^{F}<0$ and raising progressive wage taxes by $\mathrm{d} \tau^{H}>0$ so that the marginal tax wedge is unchanged.

### 2.4 Short-Run Static Solutions

System (6a)-(6b) can be solved for sector capital intensity ratios: $k^{T}=k^{T}(P)$ and $k^{N}=$ $k^{N}(P)$. Using the fact that $W^{F} \equiv\left(1-\theta^{T}\right)\left(k^{T}\right)^{\theta^{T}}$, the wage rate depends on $P$ and the

[^8]employer's part of labor taxes $\tau^{F}$ as well, i.e. $W=W\left(P, \tau^{F}\right)$, with $W_{P} \gtrless 0$ and $W_{\tau^{F}}<0$. An increase in the relative price $P$ raises or lowers $W$ depending on whether the traded sector is more or less capital intensive than the non-traded sector. A fall in $\tau^{F}$ induces firms to raise the wage $W$ to equalize the labor marginal product with the labor cost.

Plugging sectoral capital-labor ratios into the resource constraints and production functions leads to short-term static solutions for sectoral output: $Y^{T}=Y^{T}(K, L, P)$ and $Y^{N}=Y^{N}(K, L, P)$. According to the Rybczynski theorem, a rise in $K$ raises the output of the sector which is more capital intensive, while a rise in $L$ raises the output of the sector which is more labor intensive. An increase in the relative price of non-tradables $P$ exerts opposite effects on sectoral outputs by shifting resources away from the traded sector towards the non-traded output.

By substituting first $W=W\left(P, \tau^{F}\right)$, eqs. (5a)-(5b) can be solved for consumption and labor supply as follows: $C=C\left(\bar{\lambda}, P, \tau^{C}\right)$ with $C_{\bar{\lambda}}<0, C_{P}<0, C_{\tau^{C}}<0$, and $L=L\left(\bar{\lambda}, P, \tau^{j}\right)$ with $L_{\bar{\lambda}}>0, L_{P} \gtrless 0$, and $L_{\tau^{j}}<0$ (with $\left.j=F, H\right)$. A rise in the shadow value of wealth induces agents to cut their real expenditure and to supply more labor. By raising the consumption price index, an appreciation in the relative price of non-tradables drives down consumption. Depending on whether $k^{T} \gtrless k^{N}$, a rise in $P$ stimulates or depresses labor supply by raising or lowering $W$. Reducing the labor tax $\tau^{j}$ raises the after-tax wage $W^{A}$ and thereby provides an incentive to supply more labor.

### 2.5 Equilibrium Dynamics

The adjustment of the open economy towards the steady-state is described by a dynamic system which comprises two equations. First, the dynamic equation for the relative price of non-traded goods (5d) which equalizes the rates of return on domestic capital (i.e. $R^{K} / P_{-}$ $\left.\delta_{K}+\dot{P} / P\right)$ and foreign bonds $r^{\star}$. Second, the accumulation equation for physical capital clears the non-traded goods market along the transitional path:

$$
\begin{equation*}
\dot{K}=\frac{Y^{N}(K, L, P)}{\mu}-C^{N}\left(\bar{\lambda}, P, \tau^{C}\right)-G^{N}-\delta_{K} K, \tag{10}
\end{equation*}
$$

with $L=L\left(\bar{\lambda}, P, \tau^{j}\right)($ with $j=F, H)$.
Dynamic equations (5d) and (10) form a separate subsystem in $P$ and $K$. Inserting short-run static solutions, linearizing these two equations around the steady-state, and denoting the long-term values with a tilde, we obtain in a matrix form:

$$
\binom{\dot{K}}{\dot{P}}=\left(\begin{array}{cc}
\frac{Y_{K}^{N}}{\mu}-\delta_{K} & \frac{Y_{P}^{N}}{\mu}-C_{P}^{N}  \tag{11}\\
0 & \frac{Y_{K}^{T}}{\dot{P}}
\end{array}\right)\binom{K(t)-\tilde{K}}{P(t)-\tilde{P}}
$$

The determinant of the linearized $2 \times 2$ matrix is unambiguously negative and the trace is equal to $r^{\star} .{ }^{21}$ Hence, the equilibrium yields a unique one-dimensional stable saddle-path,

[^9]irrespective of the relative sizes of the sectoral capital-labor ratios. Denoting the negative eigenvalue by $\nu_{1}$ and the positive eigenvalue by $\nu_{2}$, the general solutions for $K$ and $P$ are
\[

$$
\begin{equation*}
K(t)-\tilde{K}=B_{1} e^{\nu_{1} t}+B_{2} e^{\nu_{2} t}, \quad P(t)-\tilde{P}=\omega_{2}^{1} B_{1} e^{\nu_{1} t}+\omega_{2}^{2} B_{2} e^{\nu_{2} t} \tag{12}
\end{equation*}
$$

\]

where $B_{1}$ and $B_{2}$ are constants to be determined and $\left(1, \omega_{2}^{i}\right)$ is the eigenvector associated with the eigenvalue $\nu_{i}$ (with $i=1,2$ ).

Two features of the two-sector economy's equilibrium dynamics deserve special attention. First, as long as the markup is fixed, if $k^{T}>k^{N}$, the temporal path for the relative price remains flat for the no-arbitrage condition (5d) to be fulfilled. Hence, in this case, $\omega_{2}^{1}=0$. If capital intensities are reversed, then $\omega_{2}^{1}<0$. As a consequence, the relative price exhibits transitional dynamics; $P$ and $K$ move in opposite directions. Second, after an unanticipated permanent tax cut, to ultimately approach the steady-state ( $\tilde{K}, \tilde{P}$ ) and to satisfy the transversality condition $\lim _{t \rightarrow \infty} P(t) K(t) e^{-r^{\star} t}=0$, it is necessary to set the arbitrary constant $B_{2}$ to zero. When the tax reform is announced for time $\mathcal{T}$, new information arrives at time 0 so that agents modify their decisions. Hence, two periods have to be considered, namely a first period (i.e. a pre-implementation period) over which the tax reform is expected, and a second period (i.e. an implementation period) once the tax reform is in effect. While the small country adjusts along a stable path once the tax cut is implemented, i. e. $B_{2}$ must be set to zero, the economy follows unstable paths over the pre-implementation period. These are described by eqs. (12).

Substituting eq. (10) and eq. (8) into eq. (3), we obtain the dynamic equation for the current account (denoted by $C A \equiv \dot{B}$ ):

$$
\begin{equation*}
\dot{B}=r^{\star} B+Y^{T}(K, L, P)-C^{T}\left(\bar{\lambda}, P, \tau^{C}\right)-G^{T}, \tag{13}
\end{equation*}
$$

where $L=L\left(\bar{\lambda}, P, \tau^{F}, \tau^{H}\right)$ and the second term on the RHS, i.e. $Y^{T}-C^{T}-G^{T}$, corresponds to net exports. Eq. (13) states that the current account is equal to the balance of trade denoted by $N X \equiv Y^{T}-C^{T}-G^{T}$ plus interest receipts on outstanding assets. Linearizing (13) around the steady-state and substituting (12), the general solution for the stock of foreign assets is given by: ${ }^{22}$

$$
\begin{equation*}
B(t)=\tilde{B}+\left[\left(B_{0}-\tilde{B}\right)-\Phi_{1} B_{1}-\Phi_{2} B_{2}\right] e^{r^{\star} t}+\Phi_{1} B_{1} e^{\nu_{1} t}+\Phi_{2} B_{2} e^{\nu_{2} t} . \tag{14}
\end{equation*}
$$

When implementation of the tax reform is announced at time $\mathcal{T}$, we must take into account that the open economy accumulates (or decumulates) assets (i.e. domestic capital and foreign bonds) over the pre-implementation period. The time path for net foreign assets is described by eq. (14) during this unstable period. As stocks of assets are modified over

[^10]period 1 (i.e. $(0, \mathcal{T})$ ), we have to take new initial conditions (i.e. $B_{\mathcal{T}}$ and $K_{\mathcal{T}}$ ) into account when the tax reform is in effect. ${ }^{23}$

### 2.6 Steady-State

We now briefly describe the steady-state. Setting $\dot{P}=0$ into eq. ( 5 d ), we obtain equality between the rate of return on domestic capital income and the exogenous world interest rate, i.e.

$$
\begin{equation*}
\frac{h_{k}\left[k^{N}(\tilde{P})\right]}{\mu}-\delta_{K}=r^{\star} \tag{15}
\end{equation*}
$$

This equality states that the steady-state value of the relative price of non-tradables, $\tilde{P}$, remains unaffected by a tax reform (as long as $\mu$ is fixed).

The steady-state level of $P$ determines the wage rate $\tilde{W}=\frac{\theta^{T}\left[k^{T}(\tilde{P})\right]^{\theta^{T}-1}}{1+\tau^{F}}$. Substituting the wage rate into the labor supply decision evaluated at the steady-state, we get $\tilde{L}=\left\{\frac{\bar{\lambda}}{\gamma_{L}}\left[\tilde{W}-(\tilde{W}-\kappa) \tau^{H}\right]\right\}^{\sigma_{L}}$. A labor tax cut exerts two opposite effects on $\tilde{L}$. By producing a positive wealth effect (i.e. a fall in $\bar{\lambda}$ ), agents are induced to supply less labor while the increased after-tax wage counteracts this influence.

Setting $\dot{K}=0$ into eq. (10), the market-clearing condition for the non-traded good is given by:

$$
\begin{equation*}
\frac{Y^{N}(\tilde{K}, \tilde{L}, \tilde{P})}{\mu}=C^{N}(\bar{\lambda}, \tilde{P})+\tilde{I}+G^{N} \tag{16}
\end{equation*}
$$

where $\tilde{I}=\delta_{K} \tilde{K}$.
Setting $\dot{B}=0$ into eq. (13) leads to the market-clearing condition for the traded good:

$$
\begin{equation*}
-r^{\star} \tilde{B} \equiv \tilde{N X}=Y^{T}(\tilde{K}, \tilde{L}, \tilde{P})-C^{T}(\bar{\lambda}, \tilde{P})-G^{T} \tag{17}
\end{equation*}
$$

where $\tilde{N X}$ represents steady-state net exports.
The steady-state stock of foreign bonds $\tilde{B}$ is related to the stock of physical capital through the nation's intertemporal budget constraint which must be imposed for the country to remain ultimately solvent: ${ }^{24}$

$$
\begin{equation*}
\tilde{B}-B_{0}=\Phi_{1}\left(\tilde{K}-K_{0}\right) \tag{18}
\end{equation*}
$$

where $\Phi_{1}<0$ describes the effect of capital accumulation on the external asset position and $K_{0}$ and $B_{0}$ are the initial conditions. ${ }^{25}$

[^11]Before analyzing in detail the effects of a tax reform, we think it would be convenient to build intuition by discussing the long-run effects of a labor tax cut, keeping $\bar{\lambda}$ unchanged. The consequent rise in the wage rate stimulates labor supply which triggers an excess supply or excess demand in the non-traded good market depending on whether $k^{T}>k^{N}$ or $k^{N}>k^{T}$. In either case, the stock of capital must rise to clear the non-traded good market (see eq. (16)). As the open country finances capital accumulation by running current account deficits, the economy decumulates foreign bonds (see eq. (18)) and thereby must run a trade balance surplus in the long run (see eq. (17)).

## 3 Effects of Unexpected Tax Reforms

In this section, we explore analytically the macroeconomic effects of an unexpected tax reform, emphasizing how a labor tax cut modifies the sectoral composition of GDP and its distribution between labor and capital. ${ }^{26}$ Below, we discuss alternatively the case of a traded sector that is more capital intensive than the non-traded sector and the case of a traded sector that is relatively less capital intensive. While we are able to derive impact and steady-state effects in both cases, we present analytical results only in the case $k^{T}>k^{N}$ since the case $k^{N}>k^{T}$ leads to uninteresting complications. When discussing the distribution effects, we will provide analytical results in both cases, i.e. $k^{T} \gtrless k^{N}$, since the direction of the change in the labor share relies heavily on sectoral capital intensities.

### 3.1 Revenue-Neutral Tax Reforms

We analyze first the long-run effects of two revenue-neutral tax reforms. To avoid confusion, we denote by the superscript $\left.\right|^{j, C}$ the effects of a fall in the labor tax by $\mathrm{d} \tau^{j}<0(j=F, H)$ coordinated with a rise in the consumption tax rate by $\left.\mathrm{d} \tau^{C}\right|^{j, C}$ which is endogenously determined so that the government budget constraint is met. Differentiating (8) gives the change in the consumption tax:

$$
\begin{equation*}
\left.\hat{\tau}^{C}\right|^{j, C}=-\frac{\Gamma^{j}}{\Gamma^{C}}=-\frac{\Lambda^{j} \tilde{W}^{A} \tilde{L}\left\{1-\sigma_{L} \frac{\tilde{W}^{F}}{\tilde{W}^{A}}(1-\tilde{\xi})\left[\tau^{C}+\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right)\right]\right\}}{P_{C} \tilde{C}\left(1+\tau^{C}\right)\left\{1-\frac{\sigma_{C}}{\left(1+\tau^{C}\right)} \tilde{\xi}\left[\tau^{C}+\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right)\right]\right\}} \hat{\tau}^{j}>0 \tag{19}
\end{equation*}
$$

where we denote by a hat the percentage deviation from initial steady-state and we set $0<\tilde{\xi} \equiv \frac{\sigma_{L} \tilde{W}^{F} \tilde{L}}{\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}}<1,0<\Lambda^{F} \equiv \frac{\left(1-\tau^{H}\right) \tilde{W}}{\tilde{W}^{A}}<1$, and $0<\Lambda^{H} \equiv 1-\frac{\kappa}{\tilde{W}^{A}}<1 .{ }^{27}$ The numerator of the RHS term of eq. (19) reflects the effect of a labor tax cut on tax revenue while the denominator corresponds to the effect of a rise in the consumption tax. The first term on the RHS simply reflects the relative size of the tax bases while the second term shows the change in tax bases. On the one hand, a labor (consumption) tax cut (rise) lowers

[^12](increases) public revenue, keeping consumption and employment unchanged. On the other hand, a labor (consumption) tax cut (rise) raises (lowers) employment and consumption, and thereby increases (decreases) tax revenues. While analytically, the net overall effect is ambiguous, for reasonable values of $\sigma_{L}$ and $\sigma_{C}$ and the tax rates, we find numerically that $\Gamma^{j}>0$ and $\Gamma^{C}>0$. Hence, following a labor tax cut (i.e. $\hat{\tau}^{j}<0$ ), consumption must increase to balance the budget.

## Steady-State Effects

To analyze the long-run adjustment of macroeconomics aggregates, it is convenient to assume $k^{T}>k^{N} .{ }^{28}$ A tax reform involving a labor tax cut raises after-tax labor income and thereby produces a positive wealth effect. The change in the marginal utility of wealth after a tax reform is given by: ${ }^{29}$

$$
\begin{equation*}
\hat{\bar{\lambda}}^{j, C}=\tilde{\xi} \Lambda^{j} \hat{\tau}^{j}-\left.(1-\tilde{\xi}) \hat{\tau}^{C}\right|^{j, C}<0, \quad j=F, H \tag{20}
\end{equation*}
$$

where $\hat{\tau}^{j}<0,\left.\hat{\tau}^{C}\right|^{j, C}>0,0<\Lambda^{j}<1$, and $0<\tilde{\xi}<1$. As shown by the term on the RHS of eq. (20), both a labor tax cut and a rise in the consumption tax lowers $\bar{\lambda}$ as agents raise labor supply and cut consumption which raises private wealth in both cases. Hence, Eq. (20) implies that the shadow value of wealth falls more after a revenue-neutral tax reform than following a labor tax cut financed by a decline in lump-sum transfer.

Assuming that the stock of financial wealth plus transfers is positive, the labor tax base is smaller than the consumption tax base. ${ }^{30}$ Hence, $\tau^{C}$ must increase by less than the drop in labor tax to balance the budget. As a result, denoting by $X$ the macroeconomic aggregates $C, L, K, N X$, the long-term effect of a tax reform is simply a scaled-down version of the long-term change in the aggregate $X$ after a labor tax cut financed by a fall in lump-sum transfer (denoted by $\frac{\hat{\hat{X}}}{\hat{\tau}^{j}} \hat{\tau}^{j}$ ). Formally, we have:

$$
\begin{equation*}
\left.\hat{\tilde{X}}\right|^{j, C}=\Phi^{j, C} \frac{\hat{\tilde{X}}}{\hat{\tau}^{j}} \hat{\tau}^{j}>0, \quad j=F, H \tag{21}
\end{equation*}
$$

where $0<\Phi^{j, C}=\frac{P_{C} \tilde{C}}{\Gamma^{C}}\left[1-\frac{\tilde{W}^{A} \tilde{L}}{P_{C} \tilde{C}\left(1+\tau^{C}\right)}\right]<1$.
The change in labor supply is central to the propagation mechanism. As the wealth effect is smaller than the positive influence of the increased after-tax wage on hours worked, a shift of the tax burden from labor to consumption induces agents to supply more labor:

$$
\begin{equation*}
\left.\hat{\tilde{L}}\right|^{j, C}=-\Phi^{j, C} \sigma_{L} \Lambda^{j}(1-\tilde{\xi}) \hat{\tau}^{j}>0, \quad j=F, H \tag{22}
\end{equation*}
$$

[^13]where $\hat{\tau}^{j}<0,0<\Phi^{j, C}<1$. By inducing an excess supply in the non-traded good market, higher labor supply triggers long-run capital accumulation (see eq. (16)):
\[

$$
\begin{equation*}
\left.\mathrm{d} \tilde{K}\right|^{j, C}=\Phi^{j, C} \frac{\sigma_{L} \tilde{L}}{\nu_{1}} \Lambda^{j}(1-\tilde{\xi})\left[\alpha_{C} \tilde{k}^{N} \nu_{2}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{1}\right] \hat{\tau}^{j}>0, \quad j=F, H . \tag{23}
\end{equation*}
$$

\]

The accumulation of inputs raises GDP. Noting first that $\tilde{Y}=\tilde{Y}^{T}+(\tilde{P} / \mu) \tilde{Y}^{N}$, and differentiating gives:

$$
\begin{equation*}
\hat{\tilde{Y}}^{j, C}=\left.\left(1-\tilde{\beta}_{L}\right) \hat{\tilde{K}}\right|^{j, C}+\left.\tilde{\beta}_{L} \hat{\tilde{L}}\right|^{j, C}>0, \quad j=F, H, \tag{24}
\end{equation*}
$$

where $\left(1-\tilde{\beta}_{L}\right)=\tilde{P} r^{\star} \tilde{K} / \tilde{Y}$ and $\tilde{\beta}_{L}=\tilde{W}^{F} \tilde{L} / \tilde{Y}$ are the shares of capital and labor income in output, respectively. Two factors influence the extent of the steady-state increase in GDP. The higher the elasticity of labor supply $\sigma_{L}$, the more agents supply labor and accumulate physical capital, and the larger the increase in $\tilde{Y}$. Further, the more progressive the tax scheme is (i.e. the higher the $\kappa$ ) the smaller the increase in the wage rate (as $\Lambda^{H}$ is lower) and thereby the lower the labor and output rise in the long run.

Finally, while the consumption tax must increase to balance the government budget, $\tau^{C}$ must rise less than the labor tax cut as the consumption tax base is larger than the labor tax base. As a result, the positive wealth effect is large enough to raise steady-state consumption.

## Impact Effects

How does the two-sector open economy react to a tax reform in the short run? In the case of $k^{T}>k^{N}$, the dynamics for the relative price of non-tradables degenerate. Hence, consumption and labor immediately jump to their new steady-state levels. Regarding investment, $I$ must clear the non-traded good market (10). Differentiating w.r.t. time the stable solution for capital, evaluating at time $t=0$, the initial response of investment is given by:

$$
\begin{equation*}
\left.I(0)\right|^{j, C}=\Phi^{j, C}\left[\Lambda^{j} \tilde{\xi}\left(\sigma_{C} \tilde{C}^{N}-\nu_{1} \sigma_{L} \tilde{L} \tilde{k}^{T}\right)+\nu_{1} \sigma_{L} \tilde{L} \tilde{k}^{T}\right] \hat{\tau}^{j} \gtrless 0, \quad j=F, H, \tag{25}
\end{equation*}
$$

where $\hat{\tau}^{j}<0$. The first term in brackets on the RHS of eq. (25) represents the wealth effect which exerts a negative impact on investment by inducing agents to consume more and to supply less labor. The second term in brackets corresponds to the tax cut effect which stimulates capital accumulation by raising hours worked. While the response of $I$ is ambiguous, as shown by eq. (25), the open economy accumulates capital in the long run. As a consequence, investment must increase on impact. The same logic applies if $k^{N}>k^{T}$.

The investment boom deteriorates the current account as savings remain unchanged:

$$
\begin{equation*}
\left.C A(0)\right|^{j, C}=-\left.\tilde{P} I(0)\right|^{j, C}<0, \quad j=F, H, \tag{26}
\end{equation*}
$$

where $C A(0)=\dot{B}(0)$ and $\Phi_{1}<0$, and we used the fact that $B(0)=B_{0}$. If $k^{N}>k^{T}$, the current account deteriorates further for a given increase in investment since savings fall as a result of the depreciation in $P$ which lowers the consumption-based real interest rate.

### 3.2 A Labor Tax Reform

As commonly analyzed in the literature considering the macroeconomic effects of a tax reform, we have investigated revenue-neutral tax reforms. We now consider that the policy maker wishes to alter the composition of the marginal tax wedge, but without changing its level. We denote by the superscript $\left.\right|^{F, H}$ the effects of a tax reform which involves simultaneously cutting the employer's part of labor tax and increasing the progressive wage $\operatorname{tax} \mathrm{d} \tau^{H}>0$ so as to leave the marginal tax wedge unchanged (i. e. $\mathrm{d} \tau^{M}=0$, see (9)). A restructuring of labor tax requires a rise in the wage tax by an amount given by:

$$
\begin{equation*}
\left.\mathrm{d} \tau^{H}\right|^{F, H} \equiv-\frac{1-\tau^{H}}{1+\tau^{F}} \mathrm{~d} \tau^{F}>0 . \tag{27}
\end{equation*}
$$

According to (27), the progressive wage tax must be increased by a smaller amount than the fall in $\tau^{F}$ to keep the marginal tax wedge unchanged. Intuitively, since the tax rate on a relatively large base is reduced and the tax rate on a relatively small base is increased, the latter must rise by a smaller proportion than the former decreases so as to leave $\tau^{M}$ unchanged.

The steady-state change of $X=C, L, K, N X$ following a cut in $\tau^{F}$, coordinated with a rise in $\tau^{H}$ by an amount given by (27), reads:

$$
\begin{equation*}
\left.\hat{\tilde{X}}\right|^{F, H}=\Phi^{F, H} \frac{\hat{\tilde{X}}}{\hat{\tau}^{F}} \hat{\tau}^{F}>0, \tag{28}
\end{equation*}
$$

where $\hat{\tau}^{F}<0,0<\Phi^{F, H} \equiv \kappa / \tilde{W}<1$. Setting $\kappa$ to zero implies that such a tax reform will produce no effects after a cut in payroll taxes. Rather, as long as the labor tax scheme is progressive, i. e. $\kappa>0$, the labor tax reform leaving the tax wedge constant produces an expansionary effect on the macroeconomic aggregates $X=C, L, K, N X$. As for revenue-neutral tax reforms, the steady-state changes in $X=C, L, K, N X$ are scaled-down versions of their long-term changes following a labor tax cut financed by a fall in lump-sum transfer. The scaled-down term is equal to $\kappa / \tilde{W}$ and thereby depends on the degree of progressiveness of the tax scheme. The stronger the progressiveness of the tax scheme, the larger the increase in the after-tax wage rate and thereby the greater the beneficial effects on employment and overall economic activity. The impact effects which are similar to that described for revenue-neutral strategies are not discussed further.

### 3.3 Sectoral and Distribution Effects

We now investigate in detail the sectoral and distribution effects of a tax reform. Since the three tax reforms we have considered above produce (qualitatively) similar long-run and short-run effects, we denote from now on by $\left.\right|^{j, k}$ the effects of a fall in the labor tax $\mathrm{d} \tau^{j}<0$ $(j=F, H)$ financed by a rise in $\tau^{k}(k=C, H)$.

## Steady-State Effects

A convenient way to find the direction of steady-state changes of sectoral outputs is to use the market clearing condition for the non-traded and the traded good. Differentiating the market clearing conditions for both goods gives:

$$
\begin{align*}
\left.\frac{1}{\mu} \mathrm{~d} \tilde{Y}^{N}\right|^{j, k} & =\left.\mathrm{d} \tilde{C}^{N}\right|^{j, k}+\left.\mathrm{d} \tilde{I}\right|^{j, k}>0,  \tag{29a}\\
\left.\mathrm{~d} \tilde{Y}^{T}\right|^{j, k} & =\left.\mathrm{d} \tilde{N} X\right|^{j, k}+\left.\mathrm{d} \tilde{C}^{T}\right|^{j, k}>0 . \tag{29b}
\end{align*}
$$

Because agents consume more and investment increases while the balance of trade improves in the long run, both non-traded and traded outputs expand. Interestingly, sectoral outputs are correlated in the long run as a result of the positive link between investment and the balance of trade: the greater the investment boom, the larger the accumulated debt and the more net exports must increase in the long run. Hence, both $Y^{N}$ and $Y^{T}$ rise by a larger amount. ${ }^{31}$

According to the Euler Theorem, GDP is split between capital and labor returns so that the steady-state labor share denoted by $\tilde{\beta}_{L}$ is: ${ }^{32}$

$$
\begin{equation*}
\tilde{\beta}_{L}=\frac{\tilde{W}^{F} \tilde{L}}{\tilde{W}^{F} \tilde{L}+\tilde{R}^{K} \tilde{K}}=\frac{\tilde{\omega}}{\tilde{\omega}+\tilde{k}} \tag{30}
\end{equation*}
$$

where $\tilde{R}^{K}=\tilde{P} r^{\star}$ (since we set $\delta_{K}=0$ for the purposes of clarity); we denoted by $\tilde{\omega}=$ $\tilde{W}^{F} / \tilde{R}^{K}$ the wage-interest ratio and by $\tilde{k}=\tilde{K} / \tilde{L}$ the capital-labor ratio. As long as the relative price of non-tradables $\tilde{P}$ is unaffected by the tax shock, the wage-interest ratio $\tilde{\omega}$ remains unchanged. Hence, the labor share movement is driven only by the capital-labor ratio.

## Impact Effects

How do sectoral outputs react to a tax restructuring on impact? In the case $k^{T}>k^{N}$, the relative price of non-tradables remains unchanged on impact. Because capital stock is initially predetermined, applying the Rybczynski theorem implies that non-traded output rises whereas traded output falls. With the reversal of capital intensities, i.e. if $k^{N}>k^{T}$, the relative price of non-tradables appreciates. As a result, non-traded output always expands, though increased labor moderates the rise in $Y^{N}$.

To analyze the distribution effects of a tax reform in the short run, we first linearize $\beta_{L}=\frac{W(P)\left(1+\tau^{F}\right) L(P)}{Y(K, L)}$ in the neighborhood of the steady-state, evaluate at time $t=0$, and

[^14]differentiate:
\[

$$
\begin{align*}
\left.\mathrm{d} \beta_{L}(0)\right|^{j, k} & =\left.\tilde{\beta}_{L}\left(1-\tilde{\beta}_{L}\right) \hat{\tilde{L}}\right|^{j, C}>0, \quad k^{T}>k^{N},  \tag{31a}\\
\left.\mathrm{~d} \beta_{L}(0)\right|^{j, k} & =\left.\tilde{\beta}_{L}\left(1-\tilde{\beta}_{L}\right) \hat{\tilde{L}}\right|^{j, k}+\left.\tilde{\Theta} \hat{\tilde{K}}\right|^{j, k} \lessgtr 0, \quad k^{N}>k^{T}, \tag{31b}
\end{align*}
$$
\]

where $\tilde{\Theta}=\left\{\frac{\tilde{h} \tilde{k}^{T}}{\tilde{W}^{F} \mu\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}\left[1+\sigma_{L} \Lambda^{j}\left(1-\tilde{\beta}_{L}\right)\right] \omega_{2}^{1}\right\}<0$. As shown by eq. (31a), a labor tax cut unambiguously raises the labor share $\beta_{L}$ on impact if $k^{T}>k^{N}$ as the capital-labor ratio increases while the relative price $P$ remains unaffected (so that $W^{F} / R^{K}$ is unchanged). By contrast, if $k^{N}>k^{T}$, the relative price of non-tradables appreciates on impact which in turn lowers the wage-interest ratio. In this case, the direction of the labor share movement is ambiguous, as shown in eq. (31b).

## 4 Effects of Anticipated Tax Reforms

Until now, we have considered a permanent tax reform by assuming that the tax cut was unexpected. As emphasized by Yang [2005], fiscal policy changes are ordinarily preceded by lengthy debate, and thereby agents often anticipate a planned change several quarters before its realization. Importantly, Mertens and Ravn [2009] and Favero and Giavazzi [2011] find evidence that tax cuts anticipated $\mathcal{T}$ periods before their implementation lead to declines in economic activity during the pre-implementation period. In this section, we investigate analytically the effects of anticipated tax reforms (denoted by the subscript "fut") and assume that at time $t=0$, the government announces credibly a future permanent tax reform for time $\mathcal{T}$. Since the three tax reforms considered above give similar results (qualitatively), we restrict ourselves to the analysis of a tax reform involving a cut in payroll taxes by $\mathrm{d} \tau^{F}<0$ at time $\mathcal{T}$ and a rise in the consumption tax by $\left.\mathrm{d} \tau^{C}\right|_{f u t} ^{F, C}$. At time $\mathcal{T}$, there is no new information and thereby no jump in the marginal utility of wealth at this date. The higher $\mathcal{T}$, the further the implementation of the tax reform. ${ }^{33}$ For reasons of space, we assume that $k^{T}>k^{N}$. However, where necessary, we discuss briefly the case of $k^{N}>k^{T}$.

### 4.1 Steady-State Effects

We investigate the long-run effects of an pre-announced labor tax cut. We discuss the propagation mechanism by providing analytical expressions of steady-state effects on key economic variables, assuming that $k^{T}>k^{N}$. ${ }^{34}$

[^15]A permanent tax reform, which is expected to occur in the future, induces an equilibrium change of the marginal utility of wealth that is smaller than would occur after an unexpected permanent fall in the labor tax:

$$
\begin{equation*}
\left.\hat{\bar{\lambda}}\right|_{f u t} ^{F, C}=\left[\tilde{\xi} \Lambda^{F} \hat{\tau}^{F}-\left.(1-\tilde{\xi}) \hat{\tau}^{C}\right|_{f u t} ^{F, C}\right] e^{-r^{\star} \mathcal{T}}<0 \tag{32}
\end{equation*}
$$

where $\left.\hat{\tau}^{C}\right|_{\text {fut }} ^{F, C}>0$ represents the change in the consumption tax for a given anticipated labor tax cut so as to balance the government budget. As will become clearer when discussing the numerical results, the change in the consumption tax is roughly similar whether the tax reform is anticipated or not. Hence, according to (32), the fall in $\bar{\lambda}$ is smaller after an anticipated tax reform than after an unexpected tax cut, because expected higher income is discounted by $e^{-r^{\star} \mathcal{T}}$. Hence, the farther the decrease in labor tax is expected to occur, the smaller the decline in $\bar{\lambda}$.

In the same way as after an unanticipated tax reform, hours worked increase in the long run since $\tau^{C}$ must increase by a smaller amount than the labor tax cut. Formally, we have:

$$
\begin{equation*}
\left.\hat{\tilde{L}}\right|_{\text {fut }} ^{F, C}=-\sigma_{L} \Lambda^{F}\left(1-\tilde{\xi} e^{-r^{\star} \mathcal{T}}\right) \hat{\tau}^{F}-\left.\sigma_{L}(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}} \hat{\tau}^{C}\right|_{\text {fut }} ^{F, C}>0 . \tag{33}
\end{equation*}
$$

The wealth effect is reflected in the two terms including the scaled-down factor $e^{-r^{\star} \mathcal{T}}$. Because the wealth effect is smaller, agents are induced to supply more labor in the long run whenever the tax cut is anticipated.

Following an anticipated labor tax cut (i.e. $\hat{\tau}^{F}<0$ ), the long-run change of capital stock is the result of a wealth effect and tax changes:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{K}\right|_{f u t} ^{F, C}=-\left.\frac{\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)}{\nu_{1}} \hat{\bar{\lambda}}\right|_{\text {fut }} ^{F, C}-\sigma_{L} \tilde{L} \tilde{k}^{T} \hat{\tau}^{F}-\left.\frac{\sigma_{C} \tilde{C}^{N}}{\nu_{1}} \hat{\tau}^{C}\right|_{\text {fut }} ^{F, C}>0 \tag{34}
\end{equation*}
$$

where $\left.\hat{\bar{\lambda}}\right|_{f u t} ^{F, C}<0$ is given by $(32),\left.\hat{\tau}^{C}\right|_{f u t} ^{F, C}>0$. The decline in the labor tax (second term) and the rise in the consumption tax (third term) exert a positive influence on $\tilde{K}$ by raising labor supply and reducing consumption, respectively. As reflected by the first term on the RHS of eq. (34), the wealth effect exerts a negative impact on $\tilde{K}$ by stimulating $C$ and lowering $L$. Since the wealth effect is smaller after an anticipated tax reform, the capital stock increases by a larger amount than if the tax reform was unanticipated.

Using the same procedure as in section 3.1, we find analytically that GDP unambiguously increases in the long run:

$$
\begin{equation*}
\left.\hat{\tilde{Y}}\right|_{f u t} ^{F, C}=\left.\left(1-\tilde{\beta}_{L}\right) \hat{\tilde{K}}\right|_{f u t} ^{F, C}+\left.\tilde{\beta}_{L} \hat{\tilde{L}}\right|_{f u t} ^{F, C}>0 \tag{35}
\end{equation*}
$$

Since agents supply more labor and accumulate more capital when the labor tax cut is anticipated, GDP rises by a larger amount if the tax reform is anticipated. The farther the tax reform is implemented (i.e. the higher $\mathcal{T}$ ), the larger the increase in steady-state output $\tilde{Y}$.

### 4.2 Impact Effects

We now investigate the impact effects of an anticipated permanent tax reform. We are able to present analytical expressions only if $k^{T}>k^{N}$. At the end of this subsection, we discuss the impact effects when $k^{N}>k^{T}$. Since agents are forward-looking, they know that the present value of disposable income and therefore of wealth is increased at time 0 , while the labor tax cut is implemented only at time $\mathcal{T}$. The positive wealth effect induces agents to supply less labor on impact. This behavior modifies the impact effects considerably compared to those after an unanticipated tax reform.

## Case $k^{T}>k^{N}$

Because the labor tax cut will only be in effect at time $\mathcal{T}$, the positive wealth effect produces a fall in investment at time $t=0$ :

$$
\begin{equation*}
\left.I(0)\right|_{\text {fut }} ^{F, C}=\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)\left[\tilde{\xi} \Lambda^{F} \hat{\tau}^{F}-\left.(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}} \hat{\tau}^{C}\right|_{\text {fut }} ^{F, C}\right] e^{-r^{\star} \mathcal{T}}<0, \tag{36}
\end{equation*}
$$

where $\hat{\tau}^{j}<0$. The reason is that agents supply less labor and consume more which reduces non-traded output (due to the Rybczynski theorem) and raises $C^{N}$. Hence, investment unambiguously declines initially.

The initial response of the current account is ambiguous and is given by:

$$
\begin{align*}
\left.C A(0)\right|_{f u t} ^{F, C}= & \left(\sigma_{L} \tilde{W}^{F} \tilde{L} \hat{\tau}^{F}-\left.\sigma_{C} P_{C} \tilde{C} \hat{\tau}^{C}\right|_{f u t} ^{F, C}\right) e^{-r^{\star} \mathcal{T}} \\
& -\tilde{P}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)\left[\tilde{\xi} \Lambda^{F} \hat{\tau}^{F}-\left.(1-\tilde{\xi}) \tilde{C} \hat{\tau}^{C}\right|_{f u t} ^{F, C}\right] e^{-r^{\star} \mathcal{T}} \gtrless 0,(37 \tag{37}
\end{align*}
$$

where we used the fact that $\Phi_{1}=-\tilde{P}$. The labor tax cut and the rise in the consumption tax have the same effect on the current account. As reflected by the first term on the RHS of eq. (37), as agents expect they will be richer in the future, they immediately raise consumption and lower labor. This smoothing behavior reduces savings and thereby the current account. The second term on the RHS of eq. (37) represents the fall in investment which exerts a positive influence on the current account.

Case $k^{N}>k^{T}$
We now assume that the non-traded sector is more capital intensive than the traded sector. We are unable to provide useful analytical expressions but we can deduce several important results which will be highlighted when discussing the numerical results. In the interests of space, we restrict our attention to the major changes.

When the government pre-announces that a tax reform will be implemented in the future, agents perfectly understand that their wealth will be higher. Hence, they are induced to consume more and to supply less labor at time $t=0$. Since $k^{N}>k^{T}$, for a given $P$, non-traded output now expands on impact as labor shifts towards the more capital intensive sector. At the same time, $C^{N}$ increases more than after an unanticipated tax reform, as the consumption tax rate remains unchanged until time $\mathcal{T}$. Depending on whether or not
the rise in non-traded output exceeds the increase in consumption in non-tradables $C^{N}$, the relative price of tradables depreciates or appreciates. If $P$ depreciates sufficiently on impact, investment is crowded out. Two parameters play a key role in driving the investment response: the anticipation horizon captured by $\mathcal{T}$ and the elasticity of labor supply $\sigma_{L}$. The larger the anticipation horizon (i.e. the higher $\mathcal{T}$ ), the smaller the wealth effect, the less $Y^{N}$ rises, and thereby the more likely investment is crowded out on impact. The more responsive the labor supply, the larger the increase in non-traded output (for given $P$ ), the stronger the depreciation in the relative price of non-tradables and thereby the more likely investment is crowded out. Finally, because traded output falls while $C^{T}$ increases, net exports unambiguously decline so that the open economy experiences a current account deficit.

### 4.3 Sectoral and Distribution Effects

We now discuss the effects of a tax reform pre-announced by the government on sectoral outputs and the labor share. Since long-run effects are qualitatively similar to those prevailing after an unexpected tax reform, we concentrate on the impact effects.

## Sectoral Output Responses

When the tax reform is anticipated, the adjustment of sectoral outputs now relies heavily upon sectoral capital intensities. If $k^{T}>k^{N}$, applying the Rybczynski theorem, as total hours worked decrease, the output of the sector which is more labor intensive falls (i.e. $Y^{N}$ ), while the output of the sector which is more capital intensive rises (i.e. $Y^{T}$ ). If $k^{N}>k^{T}$, as labor shifts towards the more capital intensive sector, $Y^{N}$ should expand whereas $Y^{T}$ should decline. Yet, the relative price of non-tradables also influences the sectoral effects. As stressed previously, the relative price of non-tradables may depreciate rather than appreciate on impact which in turn counteracts the Rybczynski effect. The larger the elasticity of labor supply, the more likely the relative price $P$ depreciates and thereby non-traded output declines.

## Labor Share Response

If $k^{T}>k^{N}$, since the capital-labor ratio unambiguously increases while the ratio $W^{F} / R^{K}$ remains unchanged, the labor share now falls rather than increases. With the reversal of capital intensities, the labor share also decreases as a result of the drop in the capital-labor ratio, though the fall in $\beta_{L}$ can be moderated due to the depreciation in $P$.

## 5 Quantitative Analysis

In this section, we analyze the effects of tax reforms quantitatively. For this purpose we solve the model numerically. ${ }^{35}$ Therefore, first we discuss parameter values before turning to the long-term and short-term consequences of tax reforms.

### 5.1 Benchmark Parametrization

We start by describing the calibration of consumption-side parameters that we use as a baseline. The world interest rate which is equal to the subjective time discount rate $\beta$ is set to $1 \%$. One period of time corresponds to a quarter. The elasticity of substitution between traded and non-traded goods $\phi$ is set to 1.5 (see e.g. Cashin and Mc Dermott [2003]). An additional critical parameter is $\varphi$ which is set to 0.45 in the baseline calibration to target a non-tradable content in total consumption expenditure (i.e. $\alpha_{C}$ ) of $48 \%$, in line with our estimates. ${ }^{36}$ The intertemporal elasticity of substitution for consumption $\sigma_{C}$ is set to 0.5 because empirical evidence overwhelmingly suggest values smaller than one. ${ }^{37}$ One critical parameter is the intertemporal elasticity of substitution for labor supply $\sigma_{L}$. In our baseline parametrization, we set $\sigma_{L}=0.5$, in line with evidence reported by Domeij and Flodén [2006].

We now describe the calibration of production-side parameters. We assume that physical capital depreciates at a rate $\delta_{K}=1.5 \%$ to target an investment-GDP ratio of $20 \%$. The shares of sectoral capital income in output take two different values depending on whether the traded sector is more or less capital intensive than the non-traded sector. In line with our estimates, if $k^{T}>k^{N}, \theta^{T}$ and $\theta^{N}$ are set to 0.38 and 0.3 , respectively. ${ }^{38}$ Alternatively, when $k^{N}>k^{T}$, we choose $\theta^{T}=0.30$ and $\theta^{N}=0.38$. When the markup is fixed, we set $\epsilon$ to 3.8 which yields a markup of 1.36 , which is close to our estimates (see Table 3). When the markup is endogenous, keeping $\epsilon$ fixed, we set the elasticity of substitution between sectoral goods $\omega$ to 1 so that the markup is 1.36 .

We set $G^{N}$ and $G^{T}$ so as to yield a non-tradable share of government spending of $90 \%$, and government spending as a share of GDP of $20 \% .{ }^{39}$ To set $\tau^{C}, \tau^{F}$ and $\tau^{H}$, we estimated

[^16]the effective tax rates for fourteen OECD countries over the period from 1990 to 2004 (see Appendix A.2). Consumption $\operatorname{tax} \tau^{C}$ is set to $14 \%$, the employer's part of labor taxes $\tau^{F}$ to $17 \%$ and the wage $\operatorname{tax} \tau^{H}$ to $33 \%$. Tax allowances captured by $\kappa$ are set to 0.4 to obtain a share of taxable income in the gross wage earnings $(W-\kappa) / W$ of 0.8 (see Table 3 ).

In evaluating the effects of a tax reform quantitatively, we consider a labor tax cut which lowers the tax receipts by 1 percent of GDP. We differentiate between $k^{T}>k^{N}$ and $k^{N}>k^{T}$. If $k^{T}>k^{N}$, we consider two revenue-neutral tax reforms and a restructuring of tax keeping the marginal tax wedge is explored. ${ }^{40}$ When $k^{N}>k^{T}$, we consider only one revenue-neutral tax reform shifting the tax burden from payroll taxes to consumption taxes for reasons of space. ${ }^{41}$ For $k^{T}>k^{N}$ and $k^{N}>k^{T}$, considering a tax reform involving a fall in $\tau^{F}$ coordinated with a rise in $\tau^{C}$, we also conduct a sensitivity analysis with respect to the elasticity of labor supply (i.e. we set $\sigma_{L}$ to 0.2 and 1 ) and explore the role of the competition channel by allowing for the markup to be endogenous.

When investigating numerically the effects of an anticipated tax reform, we choose an implementation lag of six quarters but also experiment with a longer implementation lag of ten quarters. ${ }^{42}$ For either $k^{T}>k^{N}$ or $k^{N}>k^{T}$, we consider a tax reform involving a fall in $\tau^{F}$ coordinated with a rise in $\tau^{C}$. We also conduct a sensitivity analysis with respect to the elasticity of labor supply and the degree of openness (i.e. we set $\varphi=0.8$ ), and we allow for the markup to be endogenous. We discuss below the effects with fixed markup while we investigate the role of endogenous markups in section 6.

### 5.2 Long-Run Aggregate Effects

In panel A of Table 1 we report the numerical results of an unexpected tax reform. Quantitatively, the long-run effects are small and are not sensitive to sectoral capital intensities. In the baseline scenario, agents raise consumption by $0.1 \%$ of GDP as a result of the positive wealth effect. The increase in the after-tax wage raises steady-state labor by $0.15 \%$. The resulting change in non-traded output stimulates capital accumulation; $\tilde{K}$ increases by $0.15 \%$. As shown in the fourth line of Panel A, GDP rises in all scenarios, as a result of the accumulation of inputs. As expected, the elasticity of labor supply plays a substantial role in determining the size of the effects. Raising $\sigma_{L}$ from 0.2 to 1 raises GDP growth from $0.08 \%$ to $0.22 \%$.

When comparing the size of the effects of tax reforms, a first conclusion that emerges from the numerical results reported in Panel A of Table 1 is that the tax structure, and thereby the type of tax reform, matter. We find that a shift from payroll taxes to consump-

[^17]tion taxes produces an increase in GDP which is about half-way between the large effects after a shift from progressive wage taxes to consumption taxes and the much smaller effects following a shift from payroll taxes to progressive wage taxes. Shifting the tax burden from progressive wage taxes to consumption taxes produces the largest effects. ${ }^{43}$ The reason is that the after-tax labor income increases by an amount given by $\tilde{W}-\kappa$ after a fall in $\tau^{H}$ which exceeds its rise given by $\tilde{W} \frac{1-\tau^{H}}{1+\tau^{F}}$ after a drop in $\tau^{F}$. A reform keeping the marginal tax wedge constant produces the smallest effects on $L, K$ and thereby on GDP. The reason is that tax progressiveness is not large enough to raise substantially the after-tax wage which results in a small increase in labor supply. ${ }^{44}$

It is worthwhile noticing that the rise in the consumption $\operatorname{tax} \tau^{C}$ given in Panel B of Table 1, which is adjusted accordingly to balance the government budget, varies substantially, ranging from $0.8 \%$ if labor supply is highly elastic (i.e. $\sigma_{L}$ is set to 1 ) to $1.5 \%$ if the progressive wage $\operatorname{tax} \tau^{H}$ is cut rather than the payroll $\operatorname{tax} \tau^{F}$.

Panel A of Table 2 gives the numerical results for the long-run effects of an anticipated tax reform shifting the tax burden from payroll to consumption taxes. When announced six quarters in advance, the tax reform produces larger affects than if the tax reform was unexpected by agents. For example, in the baseline scenario, output growth increases from $0.15 \%$ to about $0.20 \%$. The explanation is that when the labor tax cut is anticipated, the increased after-tax labor income is expressed in present discounted value terms so that the wealth effect is smaller. As a result, agents are induced to supply more labor which boosts further capital accumulation and thereby GDP. Since the wealth effect is lower, consumption increases less than after an unexpected tax reform.

### 5.3 Long-Run Sectoral and Distribution Effects

When exploring the sectoral effects, in line with our theoretical predictions, numerical results show that both traded and non-traded output expand in the long-run, whether or not the tax reform is announced, and regardless of sectoral capital intensities. Moreover, sectoral outputs are positively correlated. For example, as shown in the two last lines of Panel A of Table 1, raising $\sigma_{L}$ from 0.2 to 1 amplifies traded and non-traded output growth from $0.04 \%$ to $0.10 \%$ and from $0.04 \%$ and $0.11 \%$ of initial GDP, respectively, if $k^{T}>k^{N}$. The explanation is as follows. When agents supply more labor, the open economy accumulates more capital which in turns deteriorates further the current account

[^18]in the short run. Hence, the open economy must run a larger trade balance surplus in the long run. The larger investment boom requires $\tilde{Y}^{N}$ to increase more while $\tilde{Y}^{T}$ must increase further to produce a higher surplus in the balance of trade.

Interestingly, when the tax cut is anticipated, as shown in Panel A of Table 2, traded output growth always exceeds non-traded output growth. The reason is that if the labor tax cut is anticipated rather than being unexpected, agents accumulate more capital and thereby decumulate further foreign assets. As a consequence, net exports and thereby $\tilde{Y}^{T}$ must increase by a larger amount, and more so as trade openness is raised.

Regarding the distribution effects, since the relative price of non-tradables remains unchanged in the long run and because the capital-labor ratio (i.e. $\tilde{K} / \tilde{L}$ ) remains almost constant, the labor share is unaffected in the long run across all scenarios. ${ }^{45}$

### 5.4 Impact Effects

We now turn to the impact effects of a tax reform, contrasting the consequences after an unanticipated labor tax cut with those following an anticipated tax cut. We take a change of tax structure involving a shift from a payroll tax to consumption tax as our baseline scenario. Panel C of Tables 1 and 2 shows the impact effects for this situation, as well as for a number of alternative scenarios.

Before analyzing in detail the role of sectoral reallocation and anticipation in shaping the short-run dynamics in response to a labor tax cut, we should mention the set of empirical evidence established by Mertens and Ravn [2009]. The authors compare the effects of a tax cut depending on whether the tax change is anticipated or not. It is found that an exogenous unexpected labor tax cut raises output, worked hours, and investment. When the tax change is announced, output, hours worked, and investment decline during the preimplementation period and expand only when the tax cut is implemented. Furthermore, the real wage increases whether the tax cut is anticipated or not. Since we consider a fall in labor tax, we find it interesting to compare (qualitatively) the predictions of our model for the behavior of these variables when $k^{T}>k^{N}$ and when $k^{N}>k^{T}$. One major result that emerges from this analysis is that the predictions of our two-sector model are broadly in line with the evidence.

To begin with, note that with a fixed markup, the dynamics for the relative price of non-tradables degenerate if $k^{T}>k^{N}$. Hence, consumption and labor adjust instantaneously to their new steady-state values. Regardless of sectoral capital intensities, when the tax reform is unanticipated, hours worked and thereby GDP increase by $0.15 \%$ and $0.10 \%$, respectively, as shown in the fourth and seventh lines of Panel C of Table 1, due to the

[^19]rise in the after-tax wage. By contrast, when the tax cut is anticipated, hours worked and aggregate output decline sharply on impact by $0.5 \%$ and $0.3 \%$ respectively, as the positive wealth effect provides an incentive to raise consumption and lower labor supply.

In the model, the initial reaction of investment depends on whether the labor tax cut is anticipated or not. In the former case, the response of investment is ambiguous if $k^{N}>k^{T}$. On impact, an unexpected labor tax cut boosts investment by attracting resources towards the non-traded sector, regardless of sectoral capital intensities. As shown in the fifth line of Table 1, investment increases by about $0.4 \%$ of initial GDP in the baseline scenario. When the tax cut is anticipated, investment is crowded out, for either $k^{T}>k^{N}$ or $k^{N}>k^{T}$, though under certain circumstances. If $k^{T}>k^{N}$, the fall in labor supply lowers the output of the sector which is more labor intensive, i.e. $Y^{N}$. Hence investment declines sharply by $1.7 \%$ of initial GDP, as displayed in the fifth line of Panel C of Table 2. If $k^{N}>k^{T}$, in line with the model's predictions, the response of investment relies heavily on the elasticity of labor supply and the anticipation horizon. When labor supply is responsive enough (i.e. $\sigma_{L}$ is set to 1 ), investment declines by $0.3 \%$ of initial GDP, as the depreciation in the relative price of non-tradables is large enough to drive down non-traded output. When the anticipation horizon $\mathcal{T}$ is raised from six to ten, investment declines further from $0.03 \%$ to about $0.5 \%$ of initial GDP.

The response of the current account is shown in the sixth line of panel C of Tables 1 and 2. We obtain a decline of the current account when the tax reform is unanticipated, regardless of sectoral capital intensities, as a result of the investment boom. Moreover, savings fall if $k^{N}>k^{T}$. When the tax cut is anticipated, the open economy experiences a current account surplus if $k^{T}>k^{N}$ and a deficit if sectoral capital intensities are reversed. In both cases, agents dissave as the positive wealth effect provides an incentive to consume more. However, in the former case, the fall in investment is larger which results in a current account surplus by $1.1 \%$ of initial GDP.

In the model, the real wage $W$ is equal to the ratio of the marginal product of labor to the employer's part of labor taxes, i.e. $W \equiv \frac{\left(1-\theta^{T}\right)\left(k^{T}\right)^{\theta^{T}}}{1+\tau^{F}}$ (see eq. (6b)) with $k^{T}=k^{T}(P)$ (as long as the markup is fixed). Hence, the real wage increases only if the employer's part of labor taxes $\tau^{F}$ is lowered and/or sectoral capital intensities increase (which raise the marginal product of labor). The initial reaction of the real wage is shown in the third line of Panel C of Tables 1 and 2. Regardless of sectoral capital intensities, the real wage increases by about $1.5 \%$ in all scenarios, except after a shift from progressive wage taxes to consumption taxes. In the latter case, only the after-tax wage rises. When the shock is anticipated, the real wage remains unaffected if $k^{T}>k^{N}$ since the dynamics for the relative price $P$ degenerate and hence the sectoral capital-labor ratios remain unchanged. In contrast, if $k^{N}>k^{T}$, as the relative price of non-tradables depreciates on impact,
the Stolper-Samuelson effect produces an increase in the real wage, ranging from $0.03 \%$ if $\sigma_{L}=0.2$ to $0.25 \%$ if $\sigma_{L}=1$.

### 5.5 Transitional Adjustment

We now investigate the dynamic effects of a tax reform. The transitional paths of key variables under the baseline scenario are displayed in Figure 2. We consider a revenueneutral tax reform shifting the tax burden from labor (i.e. payroll taxes) to consumption. The responses of GDP, investment and current account are expressed as a percentage of the initial steady-state output, while labor and the relative price of non tradables (i.e. $P$ ) are given as the percentage deviation from the initial steady state. Horizontal axes measure quarters. We also compare the baseline scenario (solid line) to alternative scenarios. The dashed line shows the results for an endogenous markup which will be discussed in section 6. When the tax cut is unanticipated, the dotted line gives the results for a low labor supply elasticity (i.e. $\sigma_{L}=0.2$ ), while it shows those for a longer anticipation horizon (i.e. $\mathcal{T}=10$ ) when the tax cut is anticipated.

While panel C gives the response on impact, panel D displays the cumulative responses over a six-quarter horizon. Since the empirical literature analyzing the effects of tax shocks commonly report the GDP response over a three-year horizon, Panel E of Tables 1 and 2 shows the responses of aggregate and sectoral outputs after three years.

We analyze the dynamic adjustment, contrasting the effects of an unanticipated tax reform with those when the tax cut is anticipated. We start with the adjustment of labor which is displayed in the third line of Figure 2. When the tax reform is not announced, if $k^{T}>k^{N}$, the temporal path for $L$ is flat while with the reversal of sectoral capital intensities, the real wage increases along the transitional path. As the relative price of nontradables depreciates (after an initial appreciation), the resulting increase in the real wage pushes up labor supply. By contrast, when the tax reform is anticipated, hours worked fall sharply six quarters before the policy realization and decline during the pre-implementation period if $k^{N}>k^{T}$.

The dynamics for investment are displayed in the first line of Figure 2. As long as the labor tax cut is unanticipated, the rise in labor supply produces an investment boom, regardless of sectoral capital intensities. Along the transitional path, investment declines monotonically. Note that Mertens and Ravn [2009] find that investment increases by $10 \%$ at peak while our model predicts a smaller response of investment (over a six-quarter horizon) which ranges from $1.4 \%$ (if $k^{N}>k^{T}$ ) to $1.6 \%$ (if $k^{T}>k^{N}$ ) in the baseline scenario. In line with the evidence, an anticipated tax cut gives rise to a contraction of investment, though it recovers quickly if $k^{N}>k^{T}$. As shown in Panel D of Table 2, the cumulative response over a six-quarter horizon of investment is negative only if $k^{T}>k^{N}$. In this case,
the fall in hours worked produces a fall in non-traded output, due to the Rybczynski effect, while consumption increases. When $k^{N}>k^{T}$, the cumulative response is negative only when the tax cut is anticipated ten quarters before its implementation. Only in this case is the initial depreciation in the relative price of non-tradables large enough to moderate the non-traded output growth (see the sixth line of Panel D of Table 2) and thereby to produce a decumulation of physical capital.

In all cases, the current account adjustment is the mirror image of the dynamics of investment. Following an unexpected tax reform, the open economy experiences a current account deficit which shrinks over time, as shown in the second line of Figure 2. When the tax cut is anticipated, if $k^{T}>k^{N}$, a current account surplus shows up as investment falls sharply, though savings decline at the same time. By contrast, if $k^{N}>k^{T}$, the open economy experiences a large current account deficit as investment recovers quickly.

The fourth line of Figure 2 depicts the dynamics for output. Following an unexpected tax reform, GDP increases along the transitional path as a result of capital accumulation, regardless of sectoral capital intensities. When the tax cut is anticipated, the cumulative response of aggregate output over a six-quarter horizon summarized in the fourth line of Panel D of Table 2 is negative, ranging from $-2.5 \%$ if $k^{T}>k^{N}$ to $-1.8 \%$ if $k^{N}>k^{T}$, for the baseline scenario. In the former case, the large decline in investment drives down further GDP. When the tax cut is implemented, aggregate output rises sharply as shown in the fourth line of Figure 2. However, GDP rises above its original level only if $k^{N}>k^{T}$ at time $t=\mathcal{T}$.

Panel E of Tables 1 and 2 gives the cumulative response of GDP over a three-year horizon. Following an unanticipated 1 percent labor tax cut (relative to GDP), we find numerically that GDP increases by about 1.6 (1.5) percentage points after three years if $k^{T}>k^{N}\left(k^{T}>k^{N}\right)$. Note that our numerical results are close to the estimates provided by Perotti [2011] who find that a 1 percentage point of GDP unanticipated decrease in taxes leads to an increase in GDP by about 1.5 percentage points after three years. When the tax reform is anticipated, sectoral capital intensities play a key role in driving the response of aggregate output; more precisely, GDP falls by 2.8 percentage points if $k^{T}>k^{N}$ and declines by only 1 percentage point when the sectoral capital intensities are reversed. The reason for such a discrepancy is that the crowding-out of investment is more pronounced if $k^{T}>k^{N}$ since non-traded output falls in this case.
< Please insert Table 1 about here >
$<$ Please insert Table 2 about here $>$

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< Please insert Figure 2 about here >
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### 5.6 Short-Run Sectoral and Distribution Effects

The sectoral decomposition of the effects of fiscal shocks sheds light on the propagation mechanism in an open economy. The impact and cumulative responses (over a six-quarter horizon) of sectoral outputs are summarized in the eight and ninth lines of panel C and the fifth and sixth lines of Panel D of Tables 1 and 2 while Panel E gives the cumulative responses over a three-year horizon.

When the tax reform is unanticipated, traded output always falls while non-traded output expands, regardless of sectoral capital intensities. The reason is that output must increase to meet greater demand. Since non-tradables cannot be imported, the output of that sector must rise, regardless of sectoral capital intensities. If $k^{T}>k^{N}$, the Rybczynski effect produces a shift of labor towards the more labor intensive sector. If the sectoral capital intensities are reversed, the relative price of non-tradables appreciates as a result of the excess of demand which shifts resources towards the non-traded sector. As shown in the fifth line of Figures 2, traded output increases along the transitional path and recovers its original level only after twenty quarters. As shown in Panels C and D of Tables 1 and 2 , raising $\sigma_{L}$ amplifies the dispersion of sectoral output responses.

When the tax reform is anticipated, sectoral capital intensities now play a key role in driving sectoral output responses. If $k^{T}>k^{N}$, as agents supply less labor, the Rybczynski effect yields a drop in non-traded output while traded output expands. If $k^{N}>k^{T}$, the shift of labor towards the more capital intensive sector is large enough to depreciate the relative price of non-tradables (see the second line of Panel C of Table 2). While $Y^{N}$ falls on impact under certain circumstances, its cumulative response over a six-quarter horizon is positive, as shown in the sixth line of Panel D of Table 2. Along the transitional path, sectoral outputs vary in opposite directions as a result of the reallocation of inputs across sectors (see the fifth line of Figure 2). When $k^{T}>k^{N}$, capital decumulation produces a fall in traded output while non-traded output expands. Whereas sectoral outputs converge in this configuration, $Y^{T}$ and $Y^{N}$ diverge when $k^{N}>k^{T}$ as a result of the appreciation in $P$.

Importantly, as shown in the two last lines of Panel E of Tables 1 and 2, two striking results emerge from the numerical analysis. First, Panel E of Table 1 shows that the GDP response masks a large dispersion in sectoral output responses. More precisely, the sectoral decomposition of GDP growth over a three-year horizon shows that traded output declines
by 1.3 (1.5) percentage points of initial GDP while non-traded output rises by 2.9 (3.0) if $k^{T}>k^{N}\left(k^{N}>k^{T}\right)$. When considering an anticipated tax cut, the dispersion in sectoral output responses become between two and three times larger, as shown in Panel E of Table 2. Second, we find that the relative size of the non-traded sector increases substantially after three years, regardless of sectoral capital intensities and whether or not the tax cut is anticipated.

When investigating the distribution effects, the numerical results given in the last line of Panels C and D of Tables 1 and 2 show that the impact and cumulative effects are substantial only if the tax cut is anticipated. Starting with $k^{T}>k^{N}$, since the relative price dynamics degenerate, the labor share movement is only driven by the capital-labor ratio adjustment. For the baseline scenario, the last line of panel D of Tables 1 and 2 shows that the labor share increases by $0.12 \%$ of GDP if the tax reform is unanticipated and falls by $0.35 \%$ if the tax reform is anticipated six quarters before its implementation. In the former case, $K / L$ falls while in the latter case, the drop in worked hours pushes up the capital-labor ratio. Interestingly, if $k^{N}>k^{T}$, the labor share always drops. An unexpected tax reform leads to a cumulative response over a six-quarter horizon of about $-0.1 \%$ across all the scenarios, as the Stolper-Samuelson effect pushes up the return on capital. When the tax cut is anticipated, the cumulative response of the labor share is more negative. Its cumulative response varies between about $-0.2 \%$ if $\sigma_{L}=0.2$ and $-0.8 \%$ if $\sigma_{L}=1$. The reason is that investment recovers quickly which pushes up $K / L$, and more so if $\sigma_{L}$ is higher.

### 5.7 Anticipation Horizon

We now assess briefly to what extent our results depend on the assumption regarding the anticipation horizon of the labor tax cut. To begin with, we note that Mertens and Ravn [2009] stress that the implementation lag of the shock is an important determinant for the transmission of tax policy measures. More precisely, the authors find empirically that the longer the anticipation horizon, the deeper the pre-implementation downturn and the more muted the post-implementation expansion.

In deriving our results in the baseline scenario, we have assumed that labor tax cuts are anticipated six quarters before their implementation. The last column of Table 2 summarizes the numerical results for a longer anticipation horizon, i.e. setting $\mathcal{T}=10$. In the case of an announced future policy, the positive wealth effect is smaller, since the equilibrium change of $\bar{\lambda}$ is scaled down by $e^{-r^{\star} \mathcal{T}}$. As a result, labor, and thereby GDP decline less on impact, regardless of sectoral capital intensities, as shown in the fourth and seventh lines of Panel C of Table 2. An examination of the fourth line of panel D of Table 2 shows that the cumulative response of GDP over a six-quarter horizon is more negative as the
anticipation horizon increases, in line with the evidence from Mertens and Ravn [2009], only if $k^{N}>k^{T}$. The reason is that $P$ depreciates more over a six-quarter horizon which moderates substantially the traded output expansion (see the sixth line of Panel D of Table 2).

Figure 3 illustrates the impact of an anticipated tax reform when we vary the anticipation horizon (i.e. $\mathcal{T}$ ) between six and ten quarters. Regardless of the anticipation horizon, there is always an output decline prior to implementation and an output expansion after implementation of the labor tax cut. As stressed above, the model predicts a larger decline of GDP over a six-quarter horizon as the anticipation horizon $\mathcal{T}$ increases from six to ten, but only if $k^{N}>k^{T}$. Moreover, we find that the rise in GDP when the tax cut is implemented is smaller as $\mathcal{T}$ is raised, regardless of sectoral capital intensities. ${ }^{46}$ As shown in Figure 3(a), while GDP increases at time $\mathcal{T}$, aggregate output remains below its original level over two to three quarters if $k^{T}>k^{N}$. By contrast, if $k^{N}>k^{T}$, Figure 3(b) shows that GDP rises above its original level and then declines monotonically, regardless of the anticipation horizon.

## 6 Endogenous Markups and Sectoral Effects of Tax Reforms

Several papers have stressed that the variation in the number of competitors and the consequent change in the markup provide an important magnification mechanism, see e.g. Jaimovich and, Floetotto [2008], Wu and Zhang [2000], Zhang [2007], all of whom consider one-sector models. We therefore decided to revisit quantitatively the effects of unanticipated and anticipated tax reforms by allowing for the markup to be endogenous. Since the long-run effects remain almost unchanged compared to those in the case of a fixed markup, we will discuss them very briefly. Instead, we will concentrate on how an endogenous markup modifies the short-run adjustment of key variables and influences both the sectoral composition of GDP and the movement in the labor share. To save space, we concentrate on a revenue-neutral tax reform involving a fall in the employer's part of labor $\operatorname{tax}$ (i.e. a decrease in $\tau^{F}$ ) and a rise in consumption tax (i.e. an increase in $\tau^{C}$ ) so that the government budget is balanced.

[^20]
### 6.1 Extending the Model to Endogenous Markup

Following Jaimovich and Floetotto [2008], we depart from the usual practice by assuming that the number of firms is large enough so that the strategic effects can be ignored, but not so large that the effect of entry on the firm's demand curve is minuscule. Consequently, the price elasticity of demand faced by a single firm is no longer constant and equal to the elasticity of substitution between any two varieties, but rather a function of the number of firms $N$. Taking into account that output of one variety does not affect the general price index $P$, but does influence the sectoral price level, in a symmetric equilibrium the resulting price elasticity of demand is given by: ${ }^{47}$

$$
\begin{equation*}
e(N)=\epsilon-\frac{(\epsilon-\omega)}{N}, \quad N \in(1, \infty) . \tag{38}
\end{equation*}
$$

Assuming that $\epsilon>\omega$, the price elasticity of demand faced by any single firm is an increasing function of the number of firms $N$ within a sector. Henceforth, the markup $\mu=\frac{e}{e-1}$ decreases as the number of competitors increases.

In the interests of space, we restrict our attention to the major changes in deriving the macroeconomic equilibrium. First, the zero-profit condition in the intermediate good sector can be solved for the number of firms, i.e. $N=N(K, L, P)$. Bearing in mind that $\mu=\mu(N)$, the equalities of marginal products between sectors (i.e. eqs. (6a)-(6b)) imply that capital-labor ratios $k^{j}(j=T, N)$ are affected by the markup, i.e. $k^{j}=k^{j}(P, \mu)$, and so by the number of firms. Substituting the capital-labor ratios into $\theta^{T}\left(k^{T}\right)^{\theta^{T}-1} \equiv W^{F}$ to solve for the wage rate, and into the resource constraints (i.e. eqs. (7)) and the production functions to solve for the sectoral outputs, short-run static solutions become:

$$
\begin{equation*}
W=W\left(P, \tau^{F}, \mu\right), \quad Y^{T}=Y^{T}(K, P, L, \mu), \quad Y^{N}=Y^{N}(K, P, L, \mu), \tag{39}
\end{equation*}
$$

where $W_{\mu} \lessgtr 0$ depending on whether $k^{T} \gtrless k^{N}, Y_{\mu}^{T}>0$ and $Y_{\mu}^{N}<0$. To understand this result intuitively, i.e. the impact of markup variations, let us consider that the number of competitors increases so that $\mu$ falls. All things being equal, since the ratio $P / \mu$ rises, non-traded output $Y^{N}$ increases while traded output $Y^{T}$ falls. Additionally, if $k^{T}>k^{N}$, a fall in the markup $\mu$ raises the sectoral capital-labor ratios $k^{j}$ and thereby the wage rate. The same logic applies in the case of $k^{N}>k^{T}$ but $W$ falls.

### 6.2 Steady-state Effects

We first discuss very briefly the steady-state effects when the markups are endogenous. In the long run, the expansion of non-traded output triggers an entry of firms which lowers the markup. A lower $\mu$ leads to a long-run fall in the relative price of non-tradables $P$, regardless of sectoral capital intensities, to equalize the rates of return on domestic and

[^21]foreign assets, i.e. $\theta^{N}\left(\tilde{k}^{N}\right)^{\theta^{N}-1} / \mu(\tilde{N})-\delta_{K}=r^{\star}$. As shown in Panel A of Tables 1-2, the steady-state effects are similar, if not identical, to those obtained in the case of a fixed markup. The reason is that in the long run, the fall in the markup and the consequent adjustment in the relative price $P$ exert offsetting effects on all variables.

### 6.3 Short-Run Effects

We now investigate the short-term effects of a tax reform when the markup is endogenous, focusing on the GDP response, the reaction of investment, the adjustment of the real wage, and the movement in the labor share. Numerical results for impact and cumulative (over a six-quarter horizon) effects are summarized in panels C and D of Tables 1 and 2. Panel E gives the cumulative responses over a three-year horizon. The baseline calibration is identical to that described in section 5.1. The dashed line in Figure 2 shows the transitional paths for an endogenous markup.

To begin with, we note that the dynamics for the relative price of non-tradables are restored when $k^{T}>k^{N}$. Regardless of sectoral capital intensities, $P$ must adjust to equalize the rates of return on domestic capital and foreign bonds:

$$
\begin{equation*}
\frac{h_{k}\left\{k^{N}[P, \mu(N)]\right\}}{\mu(N)}+\frac{\dot{P}}{P}-\delta_{K}=r^{\star} . \tag{40}
\end{equation*}
$$

The markup $\mu$ depends on the number of firms $N$ which drives profits down towards zero in the non-traded sector at each instant of time. Depending on whether non-traded output is expected to increase or decrease, the number of firms rises or declines.

When the tax reform is unanticipated, non-traded output expands which lowers the markup, regardless of sectoral capital intensities. As producers perceive a more elastic demand, they are induced to produce more. As the marginal products of capital and labor increase in the non-traded sector, resources shift towards that sector. Hence, as shown in the two last lines of Panel E of Table 1, non-traded output increases by 3.4 (3.4) percentage points rather than 3.1 (3.3) while traded output declines by 1.6 (1.8) percentage points rather than $1.4(1.7)$ if $k^{T}>k^{N}\left(k^{N}>k^{T}\right)$. Overall, the rise in GDP after three years is larger when the markup is endogenous. Our numerical results show that abstracting from the competition channel would underestimate the three-year cumulative response of GDP by $5 \%$.

When the shock is anticipated, sectoral capital intensities and endogenous markup interact in determining the size of the effects. If the traded sector is more capital intensive, a model with a fixed markup would underestimate (overestimate) the drop of GDP after three years by almost $15 \%(10 \%)$ as the rise in the markup during the pre-implementation period triggers a recessionary (expansionary) effect on non-traded output if $k^{T}>k^{N}\left(k^{N}>k^{T}\right)$.

As shown in the seventh line of Panel C of Table 2, a model with an endogenous markup
produces a larger drop in investment after an anticipated tax reform, regardless of sectoral capital intensities. If $k^{T}>k^{N}$, investment falls more as a result of a higher markup which triggers a recessionary effect on non-traded output. When sectoral capital intensities are reversed, the relative price of non-tradables depreciates more (see the sixth line of Figure 2) which lowers non-traded output, and thereby investment. While these results on impact are in line with the evidence provided by Mertens and Ravn [2009] who find a fall in investment after an anticipated labor tax cut, the second line of Panel D of Table 2 shows that the cumulative response of investment is positive if $k^{N}>k^{T}$, and more so when the markup is endogenous.

By restoring the dynamics for the relative price $P$, an anticipated tax cut now produces a change in the real wage when $k^{T}>k^{N}$. In this case, the change in the real wage is the result of two opposite effects. On the one hand, the appreciation of $P$ on impact raises the real wage due to the Stolper-Samuelson effect. On the other hand, the higher markup exerts a negative impact on $W$. As shown in the third line of Panel C of Table 2, the latter effect predominates so that the real wage falls by about $0.1 \%$. As a result, the labor share falls further. In conclusion, the competition channel cannot produce the increase in the real wage documented by Mertens and Ravn over the pre-implementation period after a pre-announced labor tax cut when $k^{T}>k^{N}$. We find that the real wage increases after an anticipated tax reform only if $k^{N}>k^{T}$.

## 7 Conclusion

Low rates of employment and large public deficits accumulated by the current crisis remain a key policy concern across many industrialized countries. Policy makers have therefore shifted their attention away from resource-consuming public subsidies to resourceconserving reforms of the tax structure as ways of addressing the problems of growth and employment. While the theoretical literature analyzing the effects of revenue-neutral tax reforms emphasizes the beneficial effects of shifting the tax burden from labor to consumption, the empirical literature analyzing the effects of tax cuts finds evidence of a large decline in GDP during the pre-implementation period as a result of the fiscal foresight. Our paper revisits the macroeconomic effects of tax reforms in a two-sector open economy version of the neoclassical model, with traded and non-traded goods, by considering both unanticipated and anticipated tax cuts. Specifically, we analyze the effects of two revenue-neutral tax reforms that shift the tax burden from labor to consumption and a tax reform reducing a payroll tax while increasing a progressive wage tax that keeps the marginal tax wedge unchanged.

When considering an unanticipated tax reform, three main results emerge. First, we find that a shift from payroll taxes to consumption taxes produces an increase in GDP
which is about half-way between the large effects after a shift from progressive wage taxes to consumption taxes and the much smaller effects following a shift from payroll taxes to progressive wage taxes. Second, while at an aggregate level, our conclusions confirm the findings by Mendoza and Tesar [1998], numerical results reveal that the GDP response masks a large dispersion in sectoral output responses. In particular, we find that traded output falls while non-traded output expands, regardless of sectoral capital intensities, due to the shift of resources towards the non-traded sector. Third, as captured by the movement in the labor share, the workers reap the benefits of a labor tax cut only if the traded sector is more capital intensive.

In contrast to most papers analyzing the effects of tax reforms, we investigate the impact of anticipated tax cuts. Our model predicts that, regardless of sectoral capital intensities, GDP, hours worked and investment decline substantially during the pre-implementation period. Importantly, sectoral capital intensities play a major role in determining the GDP response after three years. If the traded sector is more capital intensive, GDP falls by about 2.8 percentage points after three years while it declines by only 1 percentage point when sectoral capital intensities are reversed. In the former case, the shift of labor towards the more capital intensive sector amplifies the crowding out of investment and thereby the contraction of GDP. Regarding the distribution effects, after an anticipated labor tax cut, the labor share always falls during the pre-implementation period, regardless of sectoral capital intensities, due to the rise in the capital-labor ratio.

Besides estimating the size of the aggregate effects of tax reforms, one key added value of our work here is to shed light on the sectoral decomposition of GDP response. More specifically, numerical results reveal that, in the baseline scenario, the relative size of the non-traded sector increases substantially after three years, regardless of sectoral capital intensities and whether or not the tax cut is anticipated.

As expected, we find that the elasticity of labor supply plays a key role in determining the magnitude of the GDP response. In contrast, raising traded openness does not alter our findings. Moreover, when conducting a sensitivity analysis of the cumulative GDP response over a six-quarter horizon with respect to the anticipation horizon, we find that the longer the anticipation horizon, the deeper the pre-implementation downturn, but only if the non-traded sector is more capital intensive. The reason is that the real exchange rate depreciates more as the anticipation horizon increases which results in a larger decline in investment. Finally, allowing for the markup to be endogenous, we find a model with fixed markup underestimates the positive GDP response after an unanticipated tax cut whereas it underestimates (overestimates) the decline in GDP if the traded (non-traded) sector is more capital intensive when the tax cut is pre-announced.

In conclusion, we must stress a number of caveats. When considering an endogenous
markup, we assume that entry of firms drives the profit down to zero at each instant of time. While this assumption simplifies the dynamics, we believe that considering a model with entry and exit of firms, in the lines of Bilbiie, Ghironi and Melitz [2010], could enrich the analysis. In this framework, the number of firms becomes a sluggish variable so that the markup will be unaffected on impact. Hence, after an anticipated tax cut, the real wage could rise rather than decrease as a result of the Stolper-Samuelson effect, when the traded sector is more capital intensive than the non-traded sector. Additionally, we consider time separable preferences which in turn produce a significant increase in consumption after an anticipated tax reform, whereas Mertens and Ravn [2009] find that the consumption response is quite muted in the short run. Considering habit formation in consumption would lower the intertemporal elasticity of substitution at a short-run horizon and thereby moderate the reaction of consumption on impact. Finally, due to our assumption of perfect labor mobility across sectors, traded and non-traded output vary in opposite directions while evidence from Benetrix and Lane [2010] mostly predicts that sectoral outputs covary. Further analysis of these issues has to be left for future research.

Unanticipated


Figure 2: Effect of Tax Reforms. Notes: variables are measured in percentage points of output, with the exception of labor and relative price of non tradables which are scaled by their initial steady-state values.

Table 1: Quantitative Effects of Unanticipated Tax Reforms (in \%)

| Variables | $k^{T}>k^{N}$ |  |  |  |  |  | $k^{N}>k^{T}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | bench $\tau^{F}-\tau^{C}$ |  |  |  | $\tau^{H}-\tau^{C}$ | $\tau^{F}-\tau^{H}$ | bench $\tau^{F}-\tau^{C}$ |  |  |  |
|  | $\left(\sigma_{L}=0.5\right)$ | ( $\sigma_{L}=0.2$ ) | $\left(\sigma_{L}=1\right)$ | ( $\mu$ end.) | $\left(\sigma_{L}=0.5\right)$ | $\left(\sigma_{L}=0.5\right)$ | $\left(\sigma_{L}=0.5\right)$ | ( $\sigma_{L}=0.2$ ) | $\left(\sigma_{L}=1\right)$ | ( $\mu$ end.) |
| A.Long-Term Consumption, $d \tilde{C}$ | 0.10 | 0.05 | 0.14 | 0.10 | 0.17 | 0.04 | 0.10 | 0.05 | 0.14 | 0.10 |
| Labor, $d \tilde{L}$ | 0.15 | 0.08 | 0.21 | 0.15 | 0.25 | 0.07 | 0.16 | 0.08 | 0.23 | 0.16 |
| Capital, $d \tilde{K}$ | 0.16 | 0.08 | 0.23 | 0.16 | 0.27 | 0.07 | 0.15 | 0.08 | 0.21 | 0.15 |
| GDP, $d \tilde{Y}$ | 0.15 | 0.08 | 0.22 | 0.15 | 0.26 | 0.07 | 0.15 | 0.08 | 0.22 | 0.15 |
| Traded output, $d \tilde{Y T}$ | 0.07 | 0.04 | 0.10 | 0.07 | 0.12 | 0.03 | 0.07 | 0.03 | 0.10 | 0.07 |
| Non traded output, $d \tilde{Y N}$ | 0.08 | 0.04 | 0.11 | 0.08 | 0.13 | 0.03 | 0.09 | 0.05 | 0.13 | 0.09 |
| B.Tax Change Tax change, $d \tau^{j}$ | 0.85 | 0.91 | 0.79 | 0.85 | 1.47 | 0.99 | 0.88 | 0.94 | 0.82 | 0.88 |
| C.Impact | 0.10 | 0.05 | 0.14 | 0.10 | 0.17 | 0.04 | 0.10 | 0.05 | 0.14 | 0.10 |
| Consumption, $d C(0)$ | 0.10 | 0.05 |  | 0.10 | 0.17 |  | 0.10 | 0.05 | 0.14 | 0.10 |
| RER, $d P(0)$ | 0.00 | 0.00 | 0.00 | -0.00 | 0.00 | 0.00 | 0.02 | 0.01 | 0.03 | 0.02 |
| Real wage, $d W(0)$ | 1.51 | 1.51 | 1.50 | 1.53 | 0.00 | 1.51 | 1.50 | 1.54 | 1.48 | 1.50 |
| Labor, $d L(0)$ | 0.15 | 0.08 | 0.21 | 0.16 | 0.25 | 0.07 | 0.12 | 0.07 | 0.14 | 0.12 |
| Investment, $d I(0)$ | 0.43 | 0.22 | 0.62 | 0.65 | 0.73 | 0.19 | 0.35 | 0.18 | 0.52 | 0.43 |
| Current Account, $d C A(0)$ | -0.43 | -0.22 | -0.62 | -0.63 | -0.73 | -0.19 | -0.43 | -0.22 | -0.65 | -0.51 |
| $\mathrm{GDP}, d Y(0)$ | 0.10 | 0.05 | 0.14 | 0.11 | 0.17 | 0.04 | 0.08 | 0.05 | 0.09 | 0.08 |
| Traded output, $d Y T(0)$ | -0.37 | -0.20 | -0.54 | -0.58 | -0.64 | -0.17 | -0.38 | -0.19 | -0.58 | -0.46 |
| Non traded output, $d Y N(0)$ | 0.47 | 0.25 | 0.68 | 0.68 | 0.81 | 0.21 | 0.46 | 0.24 | 0.67 | 0.54 |
| Labor Share, $d \beta_{L}(0)$ | 0.03 | 0.02 | 0.16 | 0.05 | 0.06 | 0.01 | -0.02 | -0.01 | -0.04 | -0.03 |
| D.Cumulative Response ( 6 qtrs ) |  |  |  |  |  |  |  |  |  |  |
| Real Wage | 9.04 | 9.05 | 9.03 | 9.11 | 0.00 | 9.04 | 9.19 | 9.31 | 9.10 | 9.18 |
| Investment, $d I$ | 1.62 | 0.85 | 2.33 | 2.07 | 2.76 | 0.71 | 1.38 | 0.71 | 2.03 | 1.55 |
| Current account, $d C A$ | -1.62 | -0.85 | -2.33 | -1.99 | -2.76 | -0.71 | -1.68 | -0.85 | -2.54 | -1.86 |
| $\mathrm{GDP}, d Y$ | 0.71 | 0.37 | 1.03 | 0.77 | 1.21 | 0.31 | 0.63 | 0.35 | 0.81 | 0.64 |
| Traded output, $d Y T$ | -1.26 | -0.66 | -1.82 | -1.62 | -2.15 | -0.56 | -1.35 | -0.68 | -2.06 | -1.52 |
| Non traded output, $d Y N$ | 1.97 | 1.04 | 2.85 | 2.39 | 3.36 | 0.87 | 1.98 | 1.03 | 2.87 | 2.17 |
| Labor Share, $d \beta_{L}$ | 0.12 | 0.06 | 0.86 | 0.16 | 0.20 | 0.05 | -0.08 | -0.04 | -0.13 | -0.10 |
| E.Cumulative Response (3 yrs) |  |  |  |  |  |  |  |  |  |  |
| GDP, $d Y$ | 1.56 | 0.82 | 2.26 | 1.66 | 2.66 | 0.69 | 1.46 | 0.79 | 1.98 | 1.50 |
| Traded output, $d Y T$ | -1.31 | -0.69 | -1.89 | -1.50 | -2.23 | -0.58 | -1.49 | -0.75 | -2.30 | -1.61 |
| Non traded output, $d Y N$ | 2.87 | 1.51 | 4.15 | 3.17 | 4.90 | 1.27 | 2.95 | 1.54 | 4.28 | 3.10 |

Notes: Effects of Unanticipated Tax reforms. We consider an unexpected permanent labor tax cut by $\tau^{j}(j=F, H)$ which lowers tax revenues by 1 percentage point of GDP; $\tau^{j}-\tau^{C}$ : revenue-neutral tax reform involving simultaneously cutting labor tax and raising consumption tax so that the government budget is balanced; in this case, Panel B gives the change in $\tau^{C} ; \tau^{F}-\tau^{H}$ : tax restructuring involving simultaneously cutting payroll taxes and raising the wage tax so that the marginal tax wedge is unchanged; in this case, Panel B gives the change in $\tau^{H}$. Responses are scaled by initial GDP, except for initial and long-run changes of real exchange rate, real wage, labor, and capital (percent of steady state).

Table 2: Quantitative Effects of Anticipated Tax Reforms (in \%)

| Variables | $k^{T}>k^{N}, \tau^{F}-\tau^{C}$ |  |  |  |  |  | $k^{N}>k^{T}, \tau^{F}-\tau^{C}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | bench $\mathcal{T}=6$ | low $\sigma_{L}$ | high $\sigma_{L}$ | open | markup | $\mathcal{T}=10$ | bench $\mathcal{T}=6$ | low $\sigma_{L}$ | high $\sigma_{L}$ | open | markup | $\mathcal{T}=10$ |
|  | $\left(\sigma_{L}=0.5\right)$ | ( $\left.\sigma_{L}=0.2\right)$ | $\left(\sigma_{L}=1\right)$ | ( $\varphi=0.8$ ) | ( $\mu$ end.) | $\left(\sigma_{L}=0.5\right)$ | $\left(\sigma_{L}=0.5\right)$ | ( $\left.\sigma_{L}=0.2\right)$ | $\left(\sigma_{L}=1\right)$ | ( $\varphi=0.8$ ) | ( $\mu$ end.) | $\left(\sigma_{L}=0.5\right)$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| Labor, $d \tilde{L}$ | 0.18 | 0.09 | 0.29 | 0.19 | 0.18 | 0.20 | 0.19 | 0.09 | 0.29 | 0.19 | 0.19 | 0.21 |
| Capital, $d \tilde{K}$ | 0.20 | 0.10 | 0.32 | 0.20 | 0.20 | 0.22 | 0.17 | 0.08 | 0.27 | 0.17 | 0.17 | 0.19 |
| $\mathrm{GDP}, d \tilde{Y}$ | 0.19 | 0.09 | 0.30 | 0.19 | 0.19 | 0.21 | 0.18 | 0.09 | 0.28 | 0.18 | 0.18 | 0.20 |
| Traded output, $d \tilde{Y T}$ | 0.11 | 0.06 | 0.17 | 0.14 | 0.11 | 0.13 | 0.10 | 0.05 | 0.16 | 0.13 | 0.10 | 0.12 |
| Non traded output, $d \tilde{Y N}$ | 0.08 | 0.04 | 0.12 | 0.05 | 0.08 | 0.08 | 0.08 | 0.04 | 0.13 | 0.05 | 0.08 | 0.08 |
| B.Tax Change Consumption tax, $d \tau^{C}$ | 0.84 | 0.91 | 0.77 | 0.86 | 0.84 | 0.84 | 0.87 | 0.94 | 0.80 | 0.84 | 0.87 | 0.87 |
| C.Impact |  |  |  |  |  |  |  |  |  |  |  |  |
| Consumption, $d C(0)$ | 0.31 | 0.28 | 0.34 | 0.31 | 0.31 | 0.30 | 0.32 | 0.28 | 0.36 | 0.32 | 0.32 | 0.31 |
| RER, $d P(0)$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | -0.03 | -0.01 | -0.07 | -0.03 | -0.03 | -0.04 |
| Real Wage, $d W(0)$ | 0.00 | 0.00 | 0.00 | 0.00 | -0.12 | 0.00 | 0.11 | 0.03 | 0.25 | 0.13 | 0.12 | 0.15 |
| Labor, $d L(0)$ | -0.50 | -0.18 | -1.07 | -0.51 | -0.55 | -0.48 | -0.47 | -0.18 | -0.89 | -0.44 | -0.46 | -0.43 |
| Investment, $d I(0)$ | -1.74 | -0.72 | -3.58 | -1.64 | -2.84 | -1.67 | -0.03 | 0.05 | -0.31 | -0.08 | -0.16 | -0.47 |
| Current Account, $d C A(0)$ | 1.10 | 0.32 | 2.52 | 0.99 | 2.08 | 1.05 | -0.50 | -0.42 | -0.44 | -0.44 | -0.36 | -0.01 |
| $\mathrm{GDP}, d Y(0)$ | -0.34 | -0.12 | -0.72 | -0.34 | -0.37 | -0.32 | -0.30 | -0.12 | -0.57 | -0.29 | -0.30 | -0.27 |
| Traded output, $d Y T(0)$ | 1.26 | 0.46 | 2.70 | 1.26 | 2.24 | 1.21 | -0.37 | -0.30 | -0.29 | -0.19 | -0.23 | 0.12 |
| Non traded output, $d Y N(0)$ | -1.60 | -0.59 | -3.42 | -1.59 | -2.61 | -1.53 | 0.07 | 0.18 | -0.28 | -0.10 | -0.07 | -0.40 |
| Labor Share, $d \beta_{L}(0)$ | -0.11 | -0.04 | -0.06 | -0.11 | -0.20 | -0.10 | -0.04 | -0.02 | -0.04 | -0.01 | -0.03 | -0.00 |
| D.Cumulative Response ( 6 qtrs ) |  |  |  |  |  |  |  |  |  |  |  |  |
| Real Wage | 0.00 | 0.00 | 0.00 | 0.00 | -0.62 | 0.00 | 0.43 | 0.11 | 0.99 | 0.50 | 0.39 | 0.58 |
| Investment, $d I$ | -6.61 | -2.72 | -13.59 | -6.22 | -10.44 | -6.34 | 2.96 | 1.27 | 5.24 | 2.93 | 3.38 | -0.45 |
| Current account, $d C A$ | 2.63 | 0.25 | 7.04 | 2.24 | 5.84 | 2.52 | -6.50 | -3.63 | -10.51 | -6.42 | -6.95 | -2.78 |
| $\mathrm{GDP}, d Y$ | -2.48 | -0.93 | -5.27 | -2.44 | -2.89 | -2.38 | -1.76 | -0.66 | -3.57 | -1.73 | -1.76 | -1.83 |
| Traded output, $d Y T$ | 3.52 | 1.11 | 7.90 | 3.74 | 6.65 | 3.37 | -5.53 | -2.80 | -9.40 | -4.79 | -5.98 | -1.94 |
| Non traded output, $d Y N$ | -6.00 | -2.04 | -13.17 | -6.19 | -9.54 | -5.75 | 3.78 | 2.14 | 5.84 | 3.06 | 4.22 | 0.10 |
| Labor Share, $d \beta_{L}$ | -0.35 | -0.11 | 0.17 | -0.40 | -0.67 | -0.33 | -0.46 | -0.22 | -0.80 | -0.36 | -0.49 | -0.19 |
| E.Cumulative Response (3 yrs) |  |  |  |  |  |  |  |  |  |  |  |  |
| GDP, $d Y$ | -2.77 | -0.97 | -6.22 | -2.68 | -3.31 | -4.83 | -0.96 | -0.27 | -2.30 | -0.92 | -0.87 | -3.00 |
| Traded output, $d Y T$ | -2.42 | -1.50 | -3.53 | -1.82 | -3.28 | 0.33 | -6.19 | -3.14 | -10.43 | -5.14 | -6.43 | -8.59 |
| Non traded output, $d Y N$ | -0.35 | 0.53 | -2.69 | -0.86 | -0.03 | -5.16 | 5.22 | 2.87 | 8.13 | 4.22 | 5.56 | 5.59 |


 GDP, exception with the real exchange rate, real wage, labor, and capital which are scaled by their initial steady-state values.


Figure 3: Impact on Output of a 1 Percent Anticipated Tax Cut for Alternative Implementation Lag. Notes: we consider an unanticipated tax reform (labelled "surprise") and anticipated tax reforms where the anticipation horizon $\mathcal{T}$ varies between 6 and 10 .

## A Data

## A. 1 Data for Figure 1

Figure 1 plots labor and consumption tax rates over the period 1990-2007 for six OECD countries: Belgium, Finland, France, Italy, Spain and Sweden. Both labor and consumption tax are taken from Mc Daniel [2007] who provides effective tax rates for a sample of fifteen OECD countries covering the period 1950-2007. Note that the data are taken from Mc Daniel as she estimates the aggregate labor tax as equal to the payroll tax rate (paid by employer and employee) plus the tax rate on household income. Further details of calculation can be retrieved in Mc Daniel [2007]. The procedure for choosing the six countries is as follows. Over the period 1990-1994, the labor tax rate averages 0.345 . Nine countries, including the six above plus Austria, Germany, and the Netherlands, feature labor tax rates that are higher than average. Among these nine countries, only six have unemployment rates that are higher than average. Note than we took the average of unemployment over 1993-1994 rather than 1990-1994 since the unemployment rate is not available for Austria before 1993. Source: OECD Main Economic Indicators.

## A. 2 Data for Calibration

Table 3 shows the non-tradable content of GDP, employment, consumption, gross fixed capital formation and government spending, and gives the share of government spending on the traded and non-traded good in the sectoral output, the shares of capital income in output in both sectors, and the markup charged by the non-traded sector for 14 OECD countries (Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, United Kingdom, United States). The choice of these countries has been dictated by data availability. For the countries of our sample, the period runs from 1990 to 2004. Details of construction of the data can be retrieved in Cardi and Restout [2011].

Table 3 also summarizes effective tax rates for the countries of our sample. The employers' part of labor taxes denoted by $\tau^{F}$ in the text is calculated as $E S S /(I E-E S S)$, where $E S S$ equal to employers' social security contributions and $I E$ equal to total compensation for employees; ESS comprise taxes paid by employers (2200) and taxes on payroll and workforce (3000). Source: OECD National Accounts.

The employees' part of labor taxes denoted by $\tau^{H}$ in the text is the labor income tax rate plus the rate of contribution to social security to be paid by households. It is calculated as $D T / H C R$, with $D T$ equal to income tax (1110) plus employees' social security contributions (2100) and $H C R$ equal to compensation of employees less labor taxes paid by employers. Source: OECD National Accounts.

We have computed the ratio $T I / W$, with $T I$ taxable income and $W$ the gross wage earnings before taxes. To calibrate the model, we set tax allowances $\kappa$ to target this ratio. Source: OECD National Accounts.

The consumption tax denoted by $\tau^{C}$ in the text is $T G S / C C$ with $T G S$ corresponding to taxes on goods and services $(5110+5121)$, and $C C$ represents the final consumption expenditure of households and of general government. Source: OECD National Accounts.

Table 3: Data to Calibrate the Two-Sector Model: Non Tradable Share and Tax Rates (1990-2004)

| Countries | Non tradable Share |  |  |  |  | $G^{3} / Y^{3}$ |  | Capital Share |  | $\begin{gathered} \text { Markup } \\ \mu \end{gathered}$ | Tax |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Output | Labor | Consumption | Investment | Gov. spending | $G^{N} / Y^{N}$ | $G^{T} / Y^{T}$ | $\theta^{T}$ | $\theta^{N}$ |  | $\tau^{C}$ | $\tau^{H}$ | $\tau^{F}$ | $\frac{W-\kappa}{W}$ |
| AUT | 0.70 | 0.66 | 0.47 | 0.60 | 0.91 | 0.28 | 0.06 | 0.33 | 0.35 | 1.37 | 0.17 | 0.36 | 0.23 | 0.77 |
| BEL | 0.71 | 0.71 | 0.44 | n.a. | 0.84 | 0.29 | 0.08 | 0.35 | 0.34 | 1.31 | 0.14 | 0.42 | 0.20 | 0.80 |
| CAN | 0.68 | 0.72 | 0.48 | 0.62 | 0.91 | 0.28 | 0.06 | 0.47 | 0.38 | 1.43 | 0.10 | 0.32 | 0.08 | n.a. |
| DEU | 0.69 | 0.67 | 0.44 | 0.61 | 0.90 | 0.28 | 0.07 | 0.23 | 0.37 | 1.45 | 0.14 | 0.33 | 0.15 | 0.92 |
| DNK | 0.71 | 0.71 | 0.47 | 0.53 | 0.93 | 0.39 | 0.07 | 0.35 | 0.32 | 1.33 | 0.24 | 0.50 | 0.01 | 0.88 |
| FIN | 0.61 | 0.64 | 0.48 | 0.63 | 0.89 | 0.37 | 0.08 | 0.40 | 0.28 | 1.33 | 0.21 | 0.40 | 0.23 | 0.93 |
| FRA | 0.74 | 0.70 | 0.44 | 0.61 | 0.94 | 0.33 | 0.06 | 0.29 | 0.36 | 1.41 | 0.15 | 0.28 | 0.31 | 0.60 |
| GBR | 0.69 | 0.73 | 0.58 | 0.51 | 0.93 | 0.30 | 0.05 | 0.29 | 0.29 | 1.28 | 0.14 | 0.24 | 0.07 | 0.83 |
| ITA | 0.68 | 0.64 | 0.41 | 0.52 | 0.91 | 0.28 | 0.06 | 0.28 | 0.37 | 1.57 | 0.12 | 0.39 | 0.27 | 0.82 |
| JPN | 0.69 | 0.66 | 0.45 | 0.59 | n.a. | n.a | n.a. | 0.42 | 0.38 | 1.46 | 0.06 | 0.20 | 0.09 | 0.50 |
| NLD | 0.71 | 0.73 | 0.45 | 0.62 | 0.92 | 0.33 | 0.07 | 0.39 | 0.31 | 1.31 | 0.16 | 0.35 | 0.08 | 0.96 |
| SPA | 0.68 | 0.67 | 0.50 | 0.65 | 0.91 | 0.26 | 0.05 | 0.41 | 0.34 | 1.33 | 0.12 | 0.22 | 0.21 | 0.66 |
| SWE | 0.68 | 0.71 | 0.51 | 0.47 | 0.91 | 0.42 | 0.09 | 0.34 | 0.32 | 1.32 | 0.19 | 0.44 | 0.32 | 0.94 |
| USA | 0.73 | 0.76 | 0.53 | 0.58 | 0.90 | 0.20 | 0.06 | 0.38 | 0.32 | 1.35 | 0.04 | 0.23 | 0.06 | 0.78 |
| Average | 0.69 | 0.69 | 0.48 | 0.58 | 0.91 | 0.31 | 0.07 | 0.35 | 0.34 | 1.38 | 0.14 | 0.33 | 0.17 | 0.80 |

Notes: $G^{j} / Y^{j}$ is the share of government spending on good $j$ in output of sector $j ; \theta^{j}$ is the share of capital income in output of sector $j=T, N ; \mu$ is the markup charged by the non-traded sector; $\tau^{C}$ is the consumption tax rate, $\tau^{H}$ : the employee' part of labor taxes, $\tau^{F}$ the employer' part of labor taxes.

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# UNANTICIPATED VS. ANTICIPATED TAX REFORMS IN A TWO SECTOR OPEN ECONOMY 

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## A Short-Run Static Solutions

In this section, we compute short-run static solutions. It is worthwhile noting that in this paper, we assume that the non-traded sector is imperfectly competitive and charges a markup denoted by $\mu$. We also allow for the markup to be endogenous in section 6 in the text. In order to isolate the influence of markup variations on variables, i.e. the competition channel, we express variables in terms of the markup; hence, we treat $\mu$ as an exogenous variable in computing short-run static solutions. For example, if a short-run static solution is given by $x=x(\bar{\lambda}, P, \mu)$ with $\bar{\lambda}$ the shadow value of wealth, $P$ the relative price of non tradables and $\mu$ the markup, the variable $x$ is only affected by $\bar{\lambda}$ and $P$ in the case of fixed markup while $x$ is influenced also by the competition channel when we allow for the markup to be endogenous. In section N , we set out the model with an imperfectly competitive non-traded sector, assuming that a limited number of competitors operate within each sector. When the number of competitors is large, the imperfectly competitive non-traded sector charges a fixed markup.

## A. 1 Short-Run Static Solutions for Consumption-Side

Static efficiency conditions (5b) and (5c) can be solved for real consumption and labor which of course must hold at any point of time:

$$
\begin{equation*}
c=C\left(\bar{\lambda}, P, \tau^{C}\right), \quad L=L\left(\bar{\lambda}, P, \tau^{F}, \tau^{H}, \mu\right) \tag{41}
\end{equation*}
$$

with

$$
\begin{align*}
C_{\bar{\lambda}} & =\frac{\partial C}{\partial \bar{\lambda}}=-\sigma_{C} \frac{C}{\bar{\lambda}}<0,  \tag{42a}\\
C_{P} & =\frac{\partial C}{\partial P}=-\alpha_{C} \sigma_{C} \frac{C}{p}<0,  \tag{42b}\\
C_{\tau^{C}} & =\frac{\partial C}{\partial \tau^{C}}=-\sigma_{C} \frac{C}{\left(1+\tau^{C}\right)}<0,  \tag{42c}\\
L_{\bar{\lambda}} & =\frac{\partial L}{\partial \bar{\lambda}}=\sigma_{L} \frac{L}{\bar{\lambda}}>0,  \tag{42d}\\
L_{P} & =\frac{\partial L}{\partial P}=\sigma_{L} L \frac{W_{P}\left(1-\tau^{H}\right)}{W^{A}}=-\sigma_{L} L \frac{\Lambda}{W^{F}} \frac{k^{T} h}{\mu\left(k^{N}-k^{T}\right)} \lessgtr 0,  \tag{42e}\\
L_{\tau^{F}} & =\frac{\partial L}{\partial \tau^{F}}=-\sigma_{L} L \frac{W_{\tau^{F}}\left(1-\tau^{H}\right)}{W^{A}}=-\sigma_{L} L \frac{\Lambda}{\left(1+\tau^{F}\right)}<0,  \tag{42f}\\
L_{\tau^{H}} & =\frac{\partial L}{\partial \tau^{H}}=-\sigma_{L} L \frac{(W-\kappa)}{W^{A}}<0,  \tag{42~g}\\
L_{\mu} & =\frac{\partial L}{\partial \mu}=\sigma_{L} L \frac{W_{\mu}\left(1-\tau^{H}\right)}{W^{A}}=\sigma_{L} L \frac{\Lambda}{W^{F}} \frac{k^{T} P h}{(\mu)^{2}\left(k^{N}-k^{T}\right)} \gtrless 0, \tag{42h}
\end{align*}
$$

where $\sigma_{C}=-\frac{u_{C}}{u_{C C}}>0$ corresponds to the intertemporal elasticity of substitution for consumption, $\sigma_{L}=\frac{v_{L}}{v_{L L} L}>0$ represents the intertemporal elasticity of substitution for labor. We denoted by $0<\Lambda \equiv \frac{\left(1-\tau^{H}\right)}{\left[\left(1-\tau^{H}\right)+\frac{\tau^{H} \kappa}{W}\right]}<1$ as long as $\kappa>0$; if $\kappa=0$, then $\Lambda=1$.

Denoting by $\phi$ the intratemporal elasticity of substitution between the tradable and the non tradable good and inserting short-run solution for consumption (41) into intra-temporal allocations between non tradable and tradable goods, we solve for $C^{T}$ and $C^{N}$ :

$$
\begin{equation*}
C^{T}=C^{T}\left(\bar{\lambda}, P, \tau^{C}\right), \quad C^{N}=C^{N}\left(\bar{\lambda}, P, \tau^{C}\right) \tag{43}
\end{equation*}
$$

with

$$
\begin{align*}
C_{\bar{\lambda}}^{T} & =-\sigma_{C} \frac{C^{T}}{\bar{\lambda}}<0  \tag{44a}\\
C_{P}^{T} & =\alpha_{C} \frac{C^{T}}{p}\left(\phi-\sigma_{C}\right) \lessgtr 0  \tag{44b}\\
C_{\tau^{C}}^{T} & =-\sigma_{C} \frac{C^{T}}{\left(1+\tau^{C}\right)}<0  \tag{44c}\\
C_{\bar{\lambda}}^{N} & =-\sigma_{C} \frac{C^{N}}{\bar{\lambda}}<0  \tag{44~d}\\
C_{P}^{N} & =-\frac{C^{N}}{p}\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right]<0  \tag{44e}\\
C_{\tau^{C}}^{N} & =-\sigma_{C} \frac{C^{N}}{\left(1+\tau^{C}\right)}<0, \tag{44f}
\end{align*}
$$

where we used the fact that $-\frac{P_{C}^{\prime \prime} P}{P_{C}^{\prime}}=\phi\left(1-\alpha_{C}\right)>0$ and $P_{C}^{\prime} C=C^{N}$.

## A. 2 Short-Run Static Solutions for Production-Side

## Sectoral Capital-Labor Ratios

First-order conditions (6) can be solved for the sectoral capital intensities:

$$
\begin{equation*}
k^{T}=k^{T}(P, \mu), \quad k^{N}=k^{N}(P, \mu), \tag{45}
\end{equation*}
$$

with

$$
\begin{align*}
k_{P}^{T} & =\frac{\partial k^{T}}{\partial P}=\frac{h}{\mu f_{k k}\left(k^{N}-k^{T}\right)},  \tag{46a}\\
k_{\mu}^{T} & =\frac{\partial k^{T}}{\partial \mu}=-\frac{P h}{(\mu)^{2} f_{k k}\left(k^{N}-k^{T}\right)},  \tag{46b}\\
k_{P}^{N} & =\frac{\partial k^{N}}{\partial P}=\frac{\mu f}{P^{2} h_{k k}\left(k^{N}-k^{T}\right)} .  \tag{46c}\\
k_{\mu}^{N} & =\frac{\partial k^{N}}{\partial \mu}=-\frac{f}{P h_{k k}\left(k^{N}-k^{T}\right)} . \tag{46~d}
\end{align*}
$$

## Real Wage

Equality $\left[f\left(k^{T}\right)-k^{T} f_{k}\left(k^{T}\right)\right] \equiv W^{F}$ can be solved for the wage rate:

$$
\begin{equation*}
W=W\left(P, \tau^{F}, \mu\right) \tag{47}
\end{equation*}
$$

with

$$
\begin{align*}
W_{P} & =\frac{\partial W}{\partial P}=-\frac{k^{T} f_{k k} k_{P}^{T}}{\left(1+\tau^{F}\right)}=-\frac{k^{T}}{\left(1+\tau^{F}\right)} \frac{h}{\mu\left(k^{N}-k^{T}\right)} \lessgtr 0,  \tag{48a}\\
W_{\tau^{F}} & =\frac{\partial W}{\partial \tau^{F}}=-\frac{w}{\left(1+\tau^{F}\right)}<0,  \tag{48b}\\
W_{\mu} & =-\frac{\partial W}{\partial \mu}=-\frac{k^{T} f_{k k} k_{\mu}^{T}}{\left(1+\tau^{F}\right)}=\frac{k^{T}}{\left(1+\tau^{F}\right)} \frac{P h}{(\mu)^{2}\left(k^{N}-k^{T}\right)} \gtrless 0 . \tag{48c}
\end{align*}
$$

## Sectoral Labor

Substituting short-run static solutions for labor (41) and capital-labor ratios (45) into the resource constraints for capital and labor (16), we can solve for traded and non-traded labor as follows:

$$
\begin{equation*}
L^{T}=L^{T}\left(K, P, \bar{\lambda}, \tau^{F}, \tau^{H}, \mu\right), \quad L^{N}=L^{N}\left(K, P, \bar{\lambda}, \tau^{F}, \tau^{H}, \mu\right) \tag{49}
\end{equation*}
$$

$$
\begin{align*}
L_{K}^{T} & =\frac{\partial L^{T}}{\partial K}=\frac{1}{k^{T}-k^{N}} \lessgtr 0,  \tag{50a}\\
L_{P}^{T} & =\frac{\partial L^{T}}{\partial P}=\frac{1}{\mu\left(k^{N}-k^{T}\right)^{2}}\left[\frac{L^{T} h}{f_{k k}}+\frac{\mu^{2} L^{N} f}{P^{2} h_{k k}}-\sigma_{L} L \frac{\Lambda}{W^{F}} k^{T} k^{N} h\right]<0,  \tag{50b}\\
L_{\mu}^{T} & =\frac{\partial L^{T}}{\partial \mu}=-\frac{1}{\left[\mu\left(k^{N}-k^{T}\right)\right]^{2}}\left[\frac{L^{T} P h}{f_{k k}}+\frac{\mu^{2} L^{N} f}{P h_{k k}}-\sigma_{L} L \frac{\Lambda}{W^{F}} k^{T} k^{N} P h\right]>0,  \tag{50c}\\
L_{\bar{\lambda}}^{T} & =\frac{\partial L^{T}}{\partial \bar{\lambda}}=\sigma_{L} \frac{L}{\bar{\lambda}} \frac{k^{N}}{k^{N}-k^{T}} \gtrless 0,  \tag{50d}\\
L_{\tau^{F}}^{T} & =\frac{\partial L^{T}}{\partial \tau^{F}}=-\frac{k^{N}}{k^{N}-k^{T}} \sigma_{L} L \frac{\Lambda}{\left(1+\tau^{F}\right)} \lessgtr 0,  \tag{50e}\\
L_{\tau^{H}}^{T} & =\frac{\partial L^{T}}{\partial \tau^{H}}=-\frac{k^{N}}{k^{N}-k^{T}} \sigma_{L} L \frac{(W-\kappa)}{W^{A}} \lessgtr 0,  \tag{50f}\\
L_{K}^{N} & =\frac{\partial L^{N}}{\partial K}=\frac{1}{k^{N}-k^{T}} \gtrless 0,  \tag{50~g}\\
L_{P}^{N} & =\frac{\partial L^{N}}{\partial P}=-\frac{1}{\mu\left(k^{N}-k^{T}\right)^{2}}\left[\frac{L^{T} h}{f_{k k}}+\frac{\mu^{2} L^{N} f}{P^{2} h_{k k}}-\sigma_{L} L \frac{\Lambda}{W^{F}}\left(k^{T}\right)^{2} h\right]>0,  \tag{50h}\\
L_{\mu}^{N} & =\frac{\partial L^{N}}{\partial \mu}=\frac{1}{\left[\mu\left(k^{N}-k^{T}\right)\right]^{2}}\left[\frac{L^{T} P h}{f_{k k}}+\frac{\mu^{2} L^{N} f}{P h_{k k}}-\sigma_{L} L \frac{\Lambda}{W^{F}}\left(k^{T}\right)^{2} P h\right]<0,  \tag{50i}\\
L_{\bar{\lambda}}^{N} & =\frac{\partial L^{N}}{\partial \bar{\lambda}}=-\sigma_{L} \frac{L}{\bar{\lambda}} \frac{k^{T}}{k^{N}-k^{T}} \lessgtr 0,  \tag{50j}\\
L_{\tau^{F}}^{N} & =\frac{\partial L^{T}}{\partial \tau^{F}}=\frac{k^{T}}{k^{N}-k^{T}} \sigma_{L} L \frac{\Lambda}{\left(1+\tau^{F}\right)} \gtrless 0,  \tag{50k}\\
L_{\tau^{H}}^{N} & =\frac{\partial L^{N}}{\partial \tau^{H}}=\frac{k^{T}}{k^{N}-k^{T}} \sigma_{L} L \frac{(W-k)}{W^{A}} \gtrless 0, \tag{501}
\end{align*}
$$

where $W^{F}=W\left(1+\tau^{F}\right)$.

## Sectoral Output

Inserting short-run static solutions for capital-labor ratios (45) and for labor (50) into the production functions, we can solve for the traded, $Y^{T}=L^{T} k^{T}$, and the non traded output, $Y^{N}=$ $L^{N} h^{N}$ :

$$
\begin{equation*}
Y^{T}=Y^{T}\left(K, P, \bar{\lambda}, \tau^{F}, \tau^{H}, \mu\right), \quad Y^{N}=Y^{N}\left(K, P, \bar{\lambda}, \tau^{F}, \tau^{H}, \mu\right) \tag{51}
\end{equation*}
$$

$$
\begin{align*}
Y_{K}^{T} & =\frac{\partial Y^{T}}{\partial K}=-\frac{f}{k^{N}-k^{T}} \lessgtr 0,  \tag{52a}\\
Y_{P}^{T} & =\frac{\partial Y^{T}}{\partial P}=\frac{1}{\mu\left(k^{N}-k^{T}\right)^{2}}\left[\frac{P L^{T}(h)^{2}}{\mu f_{k k}}+\frac{L^{N}(\mu f)^{2}}{(P)^{2} h_{k k}}-\sigma_{L} L \frac{\Lambda}{W^{F}} k^{T} k^{N} h f\right]<0,  \tag{52b}\\
Y_{\mu}^{T} & =\frac{\partial Y^{T}}{\partial \mu}=-\frac{1}{\left[\mu\left(k^{N}-k^{T}\right)\right]^{2}}\left[\frac{L^{T}(P h)^{2}}{\mu f_{k k}}+\frac{L^{N}(\mu f)^{2}}{P h_{k k}}-\sigma_{L} L \frac{\Lambda}{W^{F}} k^{T} k^{N} P h f\right]>0,  \tag{52c}\\
Y_{\bar{\lambda}}^{T} & =\frac{\partial Y^{T}}{\partial \bar{\lambda}}=\sigma_{L} \frac{L}{\bar{\lambda}} \frac{k^{N} f}{k^{N}-k^{T}} \gtrless 0,  \tag{52~d}\\
Y_{\tau^{F}}^{T} & =\frac{\partial Y^{T}}{\partial \tau^{F}}=-\frac{k^{N} f}{k^{N}-k^{T}} \sigma_{L} L \frac{\Lambda}{\left(1+\tau^{F}\right)} \lessgtr 0,  \tag{52e}\\
Y_{\tau^{H}}^{T} & =\frac{\partial Y^{T}}{\partial \tau^{H}}=-\frac{k^{N} f}{k^{N}-k^{T}} \sigma_{L} L \frac{(W-\kappa)}{W^{A}} \lessgtr 0,  \tag{52f}\\
Y_{K}^{N} & =\frac{\partial Y^{N}}{\partial K}=\frac{h}{k^{N}-k^{T}} \gtrless 0,  \tag{52~g}\\
Y_{P}^{N} & =\frac{\partial Y^{N}}{\partial P}=-\frac{1}{P\left(k^{N}-k^{T}\right)^{2}}\left[\frac{P L^{T}(h)^{2}}{\mu f_{k k}}+\frac{L^{N}(\mu f)^{2}}{P^{2} h_{k k}}-\frac{P}{\mu} \sigma_{L} L \frac{\Lambda}{W^{F}}\left(k^{T} h\right)^{2}\right]>0 .  \tag{52h}\\
Y_{\mu}^{N} & =\frac{\partial Y^{N}}{\partial \mu}=\frac{1}{\mu\left(k^{N}-k^{T}\right)^{2}}\left[\frac{P L^{T}(h)^{2}}{\mu f_{k k}}+\frac{L^{N}(\mu f)^{2}}{P^{2} h_{k k}}-\frac{P}{\mu} \sigma_{L} L \frac{\Lambda}{W^{F}}\left(k^{T} h\right)^{2}\right]<0,  \tag{52i}\\
Y_{\bar{\lambda}}^{N} & =\frac{\partial Y^{N}}{\partial \bar{\lambda}}=-\sigma_{L} \frac{L}{\bar{\lambda}} \frac{k^{T} h}{k^{N}-k^{T}} \lessgtr 0,  \tag{52j}\\
Y_{\tau^{F}}^{N} & =\frac{\partial Y^{N}}{\partial \tau^{F}}=\frac{k^{T} h}{k^{N}-k^{T} \sigma_{L} L \frac{\Lambda}{\left(1+\tau^{F}\right)} \gtrless 0,}  \tag{52k}\\
Y_{\tau^{H}}^{N} & =\frac{\partial Y^{N}}{\partial \tau^{H}}=\frac{k^{T} h}{k^{N}-k^{T}} \sigma_{L} L \frac{(W-\kappa)}{W^{A}} \gtrless 0 . \tag{52l}
\end{align*}
$$

As it will be useful to calculate tax multipliers for output, we give the partial derivatives of output in the traded and the non traded sector w. r. t. total employment:

$$
\begin{equation*}
Y_{L}^{T}=\frac{\partial Y^{T}}{\partial L}=\frac{k^{N} f}{k^{N}-k^{T}} \gtrless 0, \quad Y_{L}^{N}=\frac{\partial Y^{N}}{\partial L}=-\frac{k^{T} h}{k^{N}-k^{T}} \lessgtr 0 . \tag{53}
\end{equation*}
$$

## Useful Properties

Making use of (52b) and (52h), (52a) and (52g), we deduce the following useful properties:

$$
\begin{align*}
Y_{P}^{T}+P \frac{Y_{P}^{N}}{\mu} & =-\sigma_{L} L \Lambda \frac{k^{T} h}{\mu\left(k^{N}-k^{T}\right)} \lessgtr 0  \tag{54a}\\
\mu Y_{K}^{T}+p Y_{K}^{N} & =\frac{\mu f-P h}{k^{T}-k^{N}}=P h_{k}=\mu f_{k},  \tag{54b}\\
Y_{L}^{T}+P \frac{Y_{L}^{N}}{\mu} & =W^{F},  \tag{54c}\\
Y_{\mu}^{T}+P \frac{Y_{\mu}^{N}}{\mu} & =\sigma_{L} L \Lambda k^{T} \frac{P h}{\mu^{2}\left(k^{N}-k^{T}\right)} \gtrless 0,  \tag{54d}\\
Y_{\bar{\lambda}}^{T}+P \frac{Y_{\bar{\lambda}}^{N}}{\mu} & =\sigma_{L} \frac{L}{\bar{\lambda}} \frac{\left(k^{N} \mu f-k^{T} P h\right)}{\mu\left(k^{N}-k^{T}\right)}=\sigma_{L} \frac{L}{\bar{\lambda}} W^{F}>0,  \tag{54e}\\
Y_{\tau^{F}}^{T}+P \frac{Y_{\tau^{F}}^{N}}{\mu} & =-\sigma_{L} L w \Lambda<0,  \tag{54f}\\
Y_{\tau^{H}}^{T}+P \frac{Y_{\tau^{H}}^{N}}{\mu} & =-\sigma_{L} L \frac{(W-\kappa)}{W^{A}} W^{F}<0, \tag{54~g}
\end{align*}
$$

where we used the fact that $\mu f \equiv P\left[h-h_{k}\left(k^{N}-k^{T}\right)\right]$ and $k^{N} \mu f-k^{T} P h=P\left(h-h^{K} k^{N}\right)\left(k^{N}-k^{T}\right)=$ $\mu W^{F}\left(k^{N}-k^{T}\right)$.

In addition, using the fact that $R^{K}=f_{k}\left[k^{T}(P, \mu)\right]$, the rental rate of capital denoted by $R^{K}$ can be expressed as a function of the real exchange rate $P$ and the mark-up $\mu$ :

$$
\begin{equation*}
R^{K}=R^{K}(P, \mu), \tag{55}
\end{equation*}
$$

with partial derivatives given by:

$$
\begin{align*}
R_{P}^{K} & \equiv \frac{\partial R^{K}}{\partial P}=\frac{h}{\mu\left(k^{N}-k^{T}\right)} \gtrless 0  \tag{56a}\\
R_{\mu}^{K} & \equiv \frac{\partial R^{K}}{\partial \mu}=-\frac{P h}{\mu^{2}\left(k^{N}-k^{T}\right)} \lessgtr 0 . \tag{56b}
\end{align*}
$$

## B Equilibrium Dynamics and Formal Solutions

Inserting short-run static solutions (41), (43) and (51) into (10) and (5d), we obtain:

$$
\begin{align*}
\dot{K} & =\frac{1}{\mu} Y^{N}\left(K, P, \bar{\lambda}, \tau^{F}, \tau^{H}\right)-C^{N}\left(\bar{\lambda}, P, \tau^{C}\right)-\delta_{K} K-G^{N},  \tag{57a}\\
\dot{P} & =P\left[r^{\star}+\delta_{K}-\frac{h_{k}(P)}{\mu}\right] . \tag{57b}
\end{align*}
$$

Linearizing these two equations around the steady-state, and denoting $\tilde{X}=\tilde{K}, \tilde{P}$ the long-term values of $X=K, P$, we obtain in a matrix form:

$$
\begin{equation*}
(\dot{K}, \dot{P})^{T}=J(K(t)-\tilde{K}, P(t)-\tilde{P})^{T} \tag{58}
\end{equation*}
$$

where $J$ is given by

$$
J \equiv\left(\begin{array}{ll}
b_{11} & b_{12}  \tag{59}\\
b_{21} & b_{22}
\end{array}\right)
$$

with

$$
\begin{align*}
& b_{11}=\frac{Y_{K}^{N}}{\mu}-\delta_{K}=\frac{\tilde{h}}{\mu\left(\tilde{k}^{N}-\tilde{k}_{T}\right)}-\delta_{K} \gtrless 0, \quad b_{12}=\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}>0,  \tag{60a}\\
& b_{21}=0, \quad b_{22}=-\tilde{P} \frac{h_{k k} k_{P}^{N}}{\mu}=-\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}=\frac{Y_{K}^{T}}{\tilde{P}} \lessgtr 0 . \tag{60b}
\end{align*}
$$

## Equilibrium Dynamics

By denoting $\nu$ the eigenvalue of matrix $J$, the characteristic equation for the matrix of the linearized system (58) can be written as follows:

$$
\begin{equation*}
(\nu)^{2}-\frac{1}{\tilde{P}}\left(Y_{K}^{T}+\frac{\tilde{P}}{\tilde{\mu}} Y_{K}^{N}-\delta_{K} \tilde{P}\right) \nu+\frac{Y_{K}^{T}}{\tilde{P}}\left(\frac{Y_{K}^{N}}{\mu}-\delta_{K}\right)=0 . \tag{61}
\end{equation*}
$$

The determinant denoted by Det of the linearized $2 \times 2$ matrix (58) is unambiguously negative: ${ }^{48}$

$$
\begin{equation*}
\text { Det } \mathrm{J}=b_{11} b_{22}=\frac{Y_{K}^{T}}{\tilde{P}}\left(\frac{Y_{K}^{N}}{\mu}-\delta_{K}\right)=-\frac{\tilde{f} \tilde{h}}{\mu \tilde{P}\left(\tilde{k}^{N}-\tilde{k}^{T}\right)^{2}}-\delta_{K} \frac{Y_{K}^{T}}{\tilde{P}}<0, \tag{62}
\end{equation*}
$$

and the trace denoted by Tr given by

$$
\begin{equation*}
\operatorname{Tr} \mathrm{J}=b_{11}+b_{22}=\frac{1}{\tilde{P}}\left(Y_{K}^{T}+\frac{\tilde{P}}{\tilde{\mu}} Y_{K}^{N}\right)--\delta_{K}=\frac{h_{k}}{\mu}-\delta_{K}=r^{\star}>0, \tag{63}
\end{equation*}
$$

where we used the fact that at the long-run equilibrium $\frac{h_{k}}{\mu}=r^{\star}+\delta_{K}$.
From (58), the characteristic root obtained from $J$ writes as follows:

$$
\begin{equation*}
\nu_{i} \equiv \frac{1}{2}\left\{r^{\star} \pm \sqrt{\left(r^{\star}\right)^{2}-4 \frac{Y_{K}^{T}}{\tilde{P}}\left(\frac{Y_{K}^{N}}{\mu}-\delta_{K}\right)}\right\} \gtrless 0, \quad i=1,2 . \tag{64}
\end{equation*}
$$

Using (63), then (64) can be rewritten as follows:

$$
\begin{equation*}
\nu_{i} \equiv \frac{1}{2}\left\{r^{\star} \pm\left[\frac{Y_{K}^{T}}{\tilde{P}}-\left(\frac{Y_{K}^{N}}{\mu}-\delta_{K}\right)\right]\right\} \gtrless 0, \quad i=1,2 . \tag{65}
\end{equation*}
$$

[^22]We denote by $\nu_{1}<0$ and $\nu_{2}>0$ the stable and unstable real eigenvalues, satisfying

$$
\begin{equation*}
\nu_{1}<0<r^{\star}<\nu_{2} . \tag{66}
\end{equation*}
$$

Since the system features one state variable, $K$, and one jump variable, $P$, the equilibrium yields a unique one-dimensional stable saddle-path.

Formal Solutions for $K$ and $P$
General solutions paths are given by :

$$
\begin{align*}
K(t)-\tilde{K} & =B_{1} e^{\nu_{1} t}+B_{2} e^{\nu_{2} t}  \tag{67a}\\
P(t)-\tilde{P} & =\omega_{2}^{1} B_{1} e^{\nu_{1} t}+\omega_{2}^{2} B_{2} e^{\nu_{2} t} \tag{67b}
\end{align*}
$$

where we normalized $\omega_{1}^{i}$ to unity. The eigenvector $\omega_{2}^{i}$ associated with eigenvalue $\mu_{i}$ is given by

$$
\begin{equation*}
\omega_{2}^{i}=\frac{\nu_{i}-b_{11}}{b_{12}} \tag{68}
\end{equation*}
$$

with

$$
\begin{align*}
b_{11} & =\frac{Y_{K}^{N}}{\mu}-\delta_{K}=\frac{\tilde{h}}{\mu\left(\tilde{k}^{N}-\tilde{k}_{T}\right)}-\delta_{K} \gtrless 0  \tag{69a}\\
b_{12} & =\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}>0 \tag{69b}
\end{align*}
$$

where $C_{P}^{N}$ is given by (44e).
Case $k^{N}>k^{T}$
This assumption reflects the fact that the capital-labor ratio of the non-traded good sector exceeds the capital-labor of the traded sector. From (65), the stable and unstable eigenvalues can be rewritten as follows:

$$
\begin{align*}
\nu_{1} & =-\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}<0  \tag{70a}\\
\nu_{2} & =\frac{\tilde{h}}{\mu\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}-\delta_{K}>0 \tag{70b}
\end{align*}
$$

since we suppose that $k^{N}>k^{T}$.
We can deduce the signs of several useful expressions:

$$
\begin{align*}
Y_{K}^{N} & =\mu\left(\nu_{2}+\delta_{K}\right)>0  \tag{71a}\\
Y_{K}^{T} & =\tilde{P} \nu_{1}<0  \tag{71b}\\
\frac{\tilde{P} h_{k k} k_{P}^{N}}{\mu} & =-\nu_{1}>0  \tag{71c}\\
Y_{\tau^{F}}^{N} & =\tilde{k}^{T}\left(\nu_{2}+\delta_{K}\right) \sigma_{L} \tilde{L} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}>0,  \tag{71d}\\
Y_{\tau^{F}}^{T} & =\tilde{P} \tilde{k}^{N} \nu_{1} \sigma_{L} \tilde{L} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}<0  \tag{71e}\\
Y_{\bar{\lambda}}^{N} & =-\frac{1}{\bar{\lambda}} \sigma_{L} \tilde{L} \tilde{k}^{T} \mu\left(\nu_{2}+\delta_{K}\right)<0  \tag{71f}\\
Y_{\bar{\lambda}}^{T} & =-\frac{1}{\bar{\lambda}} \sigma_{L} \tilde{L} \tilde{P} \tilde{k}^{N} \nu_{1}>0 \tag{71g}
\end{align*}
$$

We write out eigenvector $\omega^{i}$, corresponding with stable eigenvalue $\nu_{1}$ with $i=1,2$, to determine their signs:

$$
\omega^{1}=\left(\begin{array}{cc}
1 & (+)  \tag{72}\\
\frac{\nu_{1}-\nu_{2}}{\left(\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}\right)} & (-)
\end{array}\right), \quad \omega^{2}=\left(\begin{array}{ll}
1 & (+) \\
0 &
\end{array}\right)
$$

Case $k^{T}>k^{N}$
This assumption reflects the fact that the capital-labor ratio of the traded good sector exceeds the capital-labor ratio of the non traded sector. From (65), the stable and unstable eigenvalues can be rewritten as follows:

$$
\begin{align*}
& \nu_{1}=\frac{\tilde{h}}{\mu\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}-\delta_{K}<0  \tag{73a}\\
& \nu_{2}=-\frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}>0 \tag{73b}
\end{align*}
$$

since we suppose that $k^{T}>k^{N}$.
We can deduce the signs of several useful expressions:

$$
\begin{align*}
Y_{K}^{N} & =\mu\left(\nu_{1}+\delta_{K}\right)<0,  \tag{74a}\\
Y_{K}^{T} & =\tilde{P} \nu_{2}>0  \tag{74b}\\
\frac{\tilde{P} h_{k k} k_{P}^{N}}{\mu} & =-\nu_{2}<0,  \tag{74c}\\
Y_{\tau^{F}}^{N} & =\tilde{k}^{T} \mu\left(\nu_{1}+\delta_{K}\right) \sigma_{L} \tilde{L} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}<0,  \tag{74d}\\
Y_{\tau^{F}}^{T} & =\tilde{P} \tilde{k}^{N} \nu_{2} \sigma_{L} \tilde{L} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}>0,  \tag{74e}\\
Y_{\bar{\lambda}}^{N} & =-\frac{1}{\bar{\lambda}} \sigma_{L} \tilde{L} \tilde{k}^{T} \mu\left(\nu_{1}+\delta_{K}\right)>0,  \tag{74f}\\
Y_{\bar{\lambda}}^{T} & =-\frac{1}{\bar{\lambda}} \sigma_{L} \tilde{L} \tilde{P} \tilde{k}^{N} \nu_{2}<0 . \tag{74~g}
\end{align*}
$$

We write out the four eigenvectors $\omega^{i}$, corresponding with stable eigenvalues $\nu_{i}$ with $i=1,2$, to determine their signs:

$$
\omega^{1}=\left(\begin{array}{cc}
1 & (+)  \tag{75}\\
0 &
\end{array}\right), \quad \omega^{2}=\left(\frac{0}{\frac{\nu_{2}-\nu_{1}}{\left(\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}\right)}} \quad(+)\right) .
$$

## C Current Account

In this section, we first derive the current account equation and then determine the formal solution for the stock of foreign assets.

## Derivation of the Current Account Equation

Using the definition of lump-sum transfer $Z$ given by (8), and substituting the market clearing condition for non traded goods (10) into (3) yields:

$$
\begin{aligned}
\dot{B} & =r^{\star} B(t)+R^{K} K(t)+W^{A} L(t)-P_{C}\left(1+\tau^{C}\right) C(t)-P(t) I(t)+Z \\
& =r^{\star} B+\left(R^{K} K+W^{F} L\right)-P_{C} C-P\left(\frac{Y^{N}}{\mu}-C^{N}-G^{N}\right)
\end{aligned}
$$

Using the fact that $L^{T}+L^{N}=L, K^{T}+K^{N}=K$, the dynamic equation for the current account can be rewritten as follows:

$$
\begin{aligned}
\dot{B} & =r^{\star} B+\left[W^{F} L^{T}+\left(R^{K}\right) K^{T}\right]+\left[W^{F} L^{N}+\left(R^{K}+\delta_{K}\right) K^{N}\right]-P \frac{Y^{N}}{\mu}-C^{T}-G^{T}, \\
& =r^{\star} B+Y^{T}-C^{T}-G^{T},
\end{aligned}
$$

where the overall variable cost $W^{F} L^{N}+R^{K} K^{N}$ in the non traded sector and output net of fixed cost in that sector, i. e. $P \frac{Y^{N}}{\mu}=P Z^{N}$, cancel each other. ${ }^{49}$

## Formal Solution for the Stock of Foreign Assets

We now derive the formal solution for the stock of foreign assets. Inserting first the short-run static solutions for consumption in tradables given by (43) and output in the traded sector given by (51) into the current account dynamic equation (13) and linearizing around the steady-state yields:

$$
\begin{equation*}
\dot{B}(t)=r^{\star}(B(t)-\tilde{B})+Y_{K}^{T}(K(t)-\tilde{K})+\left[Y_{P}^{T}-C_{P}^{T}\right](P(t)-\tilde{P}) \tag{76}
\end{equation*}
$$

where $C_{P}^{T}$ is given by (44b).
Inserting general solutions for $K(t)$ and $P(t)$ given by eqs. (67), the solution for the stock of foreign assets is:

$$
\begin{equation*}
\dot{B}(t)=r^{\star}(B(t)-\tilde{B})+Y_{K}^{T} \sum_{i=1}^{2} B_{i} e^{\nu_{i} t}+\left[Y_{P}^{T}-C_{P}^{T}\right] \sum_{i=1}^{2} B_{i} \omega_{2}^{i} e^{\nu_{i} t} \tag{77}
\end{equation*}
$$

Solving the differential equation leads to:

$$
\begin{equation*}
B(t)-\tilde{B}=\left[\left(B_{0}-\tilde{B}\right)-\Phi_{1} B_{1}-\Phi_{2} B_{2}\right] e^{r^{\star} t}+\Phi_{1} B_{1} e^{\nu_{1} t}+\Phi_{2} B_{2} e^{\nu_{2} t} \tag{78}
\end{equation*}
$$

[^23]where
\[

$$
\begin{equation*}
\Phi_{i}=\frac{N_{i}}{\nu_{i}-r^{\star}}=\frac{Y_{K}^{T}+\left[Y_{P}^{T}-C_{P}^{T}\right] \omega_{2}^{i}}{\nu_{i}-r^{\star}}, \quad i=1,2 . \tag{79}
\end{equation*}
$$

\]

Invoking the transversality condition for intertemporal solvency, the terms in brackets of equation (78) must be null and which implies $B_{2}=0$. We get the linearized version of the nation's intertemporal budget constraint:

$$
\begin{equation*}
B_{0}-\tilde{B}=\Phi_{1}\left(K_{0}-\tilde{K}\right) \tag{80}
\end{equation*}
$$

The stable solution for net foreign assets finally reduces to:

$$
\begin{equation*}
B(t)-\tilde{B}=\Phi_{1}(K(t)-\tilde{K}) \tag{81}
\end{equation*}
$$

Case $k^{N}>k^{T}$

$$
\begin{align*}
N_{1} & =Y_{K}^{T}+\left(Y_{P}^{T}-C_{P}^{T}\right) \omega_{2}^{1}, \\
& =\tilde{P} \nu_{2}\left\{1+\frac{\omega_{2}^{1}}{\tilde{P} \nu_{2}}\left[\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T}\left(\nu_{2}+\delta_{K}\right) \tilde{\Lambda}\right]\right\} \gtrless 0,  \tag{82a}\\
N_{2} & =Y_{K}^{T}+\left(Y_{P}^{T}-C_{P}^{T}\right) \omega_{2}^{2},  \tag{82b}\\
& =Y_{K}^{T}=\tilde{P} \nu_{1}<0, \tag{82c}
\end{align*}
$$

where (82c) follows from the fact that $\omega_{2}^{2}=0$. We made use of property (54a) together with the fact that $C_{P}^{T}=P_{C} C_{P}-p C_{P}^{N}$ to compute $Y_{P}^{T}-C_{P}^{T}=-\tilde{P}\left(\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}\right)-P_{C} C_{P}-\sigma_{L} \tilde{L} \tilde{k}^{T}\left(\nu_{2}+\delta_{K}\right) \tilde{\Lambda} \gtrless 0$.

Since it is equal to $\frac{N_{1}}{\nu_{1}-r^{\star}}, \Phi_{1}$ is given by:

$$
\begin{equation*}
\Phi_{1}=-\tilde{P}\left\{1+\frac{\omega_{2}^{1}}{\tilde{P} \nu_{2}}\left[\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T}\left(\nu_{2}+\delta_{K}\right) \tilde{\Lambda}\right]\right\} \tag{83}
\end{equation*}
$$

and $\Phi_{2}=\frac{N_{2}}{\nu_{2}-r^{\star}}=-\tilde{P}$.
The sign of $\Phi_{1}$ is ambiguous and reflects the impact of the capital accumulation on the net foreign assets accumulation along a stable transitional path:

$$
\dot{B}(t)=\Phi_{1} \dot{K}(t) .
$$

where $\dot{K}(t)=\nu_{1} B_{1} e^{\nu_{1} t}$. Following empirical evidence suggesting that the current account and investment are negatively correlated (see e. g. Glick and Rogoff [1995]), we will impose thereafter:

Assumption $1 \Phi_{1}<0$ which implies that $N_{1}>0$.
Hence, we impose the following condition when $k^{N}>k^{T}$ :

$$
\begin{equation*}
\nu_{2}>-\frac{\omega_{2}^{1}}{\tilde{P}}\left[\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T}\left(\nu_{2}+\delta_{K}\right) \tilde{\Lambda}\right] \tag{84}
\end{equation*}
$$

For all parametrization, the inequality above holds.
Case $k^{T}>k^{N}$

$$
\begin{align*}
N_{1} & =Y_{K}^{T}+\left(Y_{P}^{T}-C_{P}^{T}\right) \omega_{2}^{1}, \\
& =Y_{K}^{T}=\tilde{P} \nu_{2}>0  \tag{85a}\\
N_{2} & =Y_{K}^{T}+\left(Y_{P}^{T}-C_{P}^{T}\right) \omega_{2}^{2}, \\
& =\tilde{P} \nu_{1}\left\{1+\frac{\omega_{2}^{2}}{\tilde{P} \nu_{1}}\left[\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T}\left(\nu_{1}+\delta_{k}\right) \tilde{\Lambda}\right]\right\}, \lessgtr 0, \tag{85b}
\end{align*}
$$

where (86) follows from the fact that $\omega_{2}^{1}=0$. We made use of property (54a) together with $C_{P}^{T}=P_{C} C_{P}-P C_{P}^{N}$ to compute $Y_{P}^{T}-C_{P}^{T}=-\tilde{P}\left(\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}\right)-P_{C} C_{P}-\sigma_{L} \tilde{L} \tilde{k}^{T}\left(\nu_{1}+\delta_{K}\right) \gtrless 0$.

Hence, $\Phi_{1}=\frac{N_{1}}{\nu_{1}-r^{\star}}=-\tilde{P}$. Furthermore, since it is equal to $\frac{N_{2}}{\nu_{2}-r^{\star}}, \Phi_{2}$ is given by:

$$
\begin{equation*}
\Phi_{2}=-\tilde{P}\left\{1+\frac{\omega_{2}^{2}}{\tilde{P} \nu_{1}}\left[\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T}\left(\nu_{1}+\delta_{k}\right) \tilde{\Lambda}\right]\right\} \tag{86}
\end{equation*}
$$

## D Savings

## D. 1 Formal Solution for Financial Wealth

The law of motion for financial wealth $(S(t)=\dot{A}(t))$ is given by:
$\dot{A}(t)=r^{\star} A(t)+\left[W\left(P, \tau^{F}, \eta\right)\left(1-\tau^{H}\right)+\tau^{H} \kappa\right] L\left(\bar{\lambda}, P, \tau^{F}, \tau^{H}\right)-P_{C}(P)\left(1+\tau^{C}\right) C\left(\bar{\lambda}, P, \tau^{C}\right)+Z$,
with $Z=\tau^{C} P_{C} C+\left[\left(\tau^{H}+\tau^{F}\right) w-\tau^{H} \kappa\right] L-G^{T}-\tilde{P} G^{N}$.
The linearized version of (87) is:

$$
\begin{equation*}
\dot{A}(t)=r^{\star}(A(t)-\tilde{A})+M_{1}(P(t)-\tilde{P}) \tag{88}
\end{equation*}
$$

with $M_{1}$ given by

$$
\begin{align*}
M_{1} & =\left(W_{P} \tilde{L}+\tilde{W} L_{P}\right)\left(1+\tau^{F}\right)-\left(\tilde{C}^{N}+P_{C} C_{P}+G^{N}\right) \\
& =\left(1+\tau^{F}\right) \tilde{L} W_{P}\left(1+\tilde{\Lambda} \sigma_{L}\right)-\left[\tilde{C}^{N}\left(1-\sigma_{C}\right)+G^{N}\right] \\
& =-\left\{\tilde{K}\left(\nu_{2}+\delta_{K}\right)+\left[\sigma_{L} \tilde{L} \tilde{\Lambda} \tilde{k}^{T}\left(\nu_{2}+\delta_{K}\right)-\sigma_{C} \tilde{C}^{N}\right]\right\}<0 \tag{89}
\end{align*}
$$

From the second line of (89), if $\sigma_{C}<1$ as empirical studies suggest, then the term in square brackets is positive and $M_{1}$ is negative. The last line has been computed by using the fact that $\tilde{L}=\tilde{L}^{N}+\tilde{L}^{T}$ and $\tilde{K}=\tilde{k}^{T} \tilde{L}^{T}+\tilde{k}^{N} \tilde{L}^{N}$ which allows to simplify $\frac{1}{\mu}\left[\tilde{Y}^{N}+\tilde{L} \tilde{k}^{T}\left(\nu_{2}+\delta_{K}\right) \mu\right]$ to $\tilde{K}\left(\nu_{2}+\delta_{K}\right)$.

The general solution for the stock of financial wealth is:

$$
\begin{equation*}
A(t)=\tilde{A}+\left[\left(A_{0}-\tilde{A}\right)-\frac{M_{1} \omega_{2}^{1}}{\nu_{1}-r^{\star}} B_{1}\right] e^{r^{\star} t}+\frac{M_{1} \omega_{2}^{1}}{\nu_{1}-r^{\star}} B_{1} e^{\nu_{1} t} \tag{90}
\end{equation*}
$$

where we used the fact that $\omega_{2}^{2}=0$.
Invoking the transversality condition, we obtain the stable solution for financial wealth:

$$
\begin{equation*}
A(t)=\tilde{A}+\frac{M_{1} \omega_{2}^{1}}{\nu_{1}-r^{\star}} B_{1} e^{\nu_{1} t} \tag{91}
\end{equation*}
$$

and the intertemporal solvency condition

$$
\begin{equation*}
\tilde{A}-A_{0}=\frac{M_{1} \omega_{2}^{1}}{\nu_{1}-r^{\star}}\left(\tilde{K}-K_{0}\right) . \tag{92}
\end{equation*}
$$

## D. 2 Steady-State and Dynamic Effects of Tax Changes

Differentiating (92) w. r. t. $\tau^{j}(j=F, H)$, long-term changes of financial wealth are given by:

$$
\begin{equation*}
\frac{\mathrm{d} \tilde{A}}{\mathrm{~d} \tau^{j}}=\frac{\omega_{2}^{1}}{\nu_{2}}\left(\tilde{K} \nu_{2}+\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{2}-\sigma_{C} \tilde{C}^{N}\right) \frac{\mathrm{d} \tilde{K}}{\mathrm{~d} \tau^{j}} \tag{93}
\end{equation*}
$$

Differentiating (91) w. r. t. $\tau^{C}$ and $\tau^{j}(j=F, H)$, gives the dynamics for savings:

$$
\begin{align*}
& S(t)=\dot{A}(t)=\nu_{1} \frac{M_{1} \omega_{2}^{1}}{\nu_{1}-r^{\star}} \frac{B_{1}}{\mathrm{~d} \tau^{C}} \mathrm{~d} \tau^{C} e^{\nu_{1} t}>0,  \tag{94a}\\
& S(t)=\dot{A}(t)=\nu_{1} \frac{M_{1} \omega_{2}^{1}}{\nu_{1}-r^{\star}} \frac{B_{1}}{\mathrm{~d} \tau^{j}} \mathrm{~d} \tau^{j} e^{\nu_{1} t}<0, \quad j=F, H, \tag{94b}
\end{align*}
$$

where $\frac{B_{1}}{\mathrm{~d} \tau^{C}}=-\frac{\mathrm{d} \tilde{K}}{\mathrm{~d} \tau^{C}}<0$ and $\frac{B_{1}}{\mathrm{~d} \tau^{j}}=-\frac{\mathrm{d} \tilde{K}}{\mathrm{~d} \tau^{j}}>0$ as it is shown in the next section.

## E Long-Run Effects of Labor and Consumption Tax Changes

In this section, we calculate formal expressions of steady-state changes. For clarity purpose, we assume that $\delta_{K}=0$ since it does not modify qualitatively the long-run effects of tax policies. This assumption will be relaxed in numerical analysis. We totally differentiate the steady-state which yields in a matrix form:

$$
\left(\begin{array}{cccc}
\frac{h_{k k} k_{P}^{N}}{\mu} & 0 & 0 & 0  \tag{95}\\
\left(\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}\right) & \frac{Y_{K}^{N}}{\mu} & \left(\frac{Y_{\bar{\lambda}}^{N}}{\mu}-C_{\bar{\lambda}}^{N}\right) & 0 \\
\left(Y_{P}^{T}-C_{P}^{T}\right) & Y_{K}^{T} & \left(Y_{\bar{\lambda}}^{T}-C_{\bar{\lambda}}^{T}\right) & r^{\star} \\
0 & -\Phi_{1} & 0 & 1
\end{array}\right)\left(\begin{array}{c}
0 \\
\mathrm{~d} \tilde{P} \\
\mathrm{~d} \tilde{K} \\
\mathrm{~d} \bar{\lambda} \\
\mathrm{~d} \tilde{B}
\end{array}\right)=\left(\begin{array}{c}
0 \\
-\frac{Y_{\tau F}^{N}}{\mu} \mathrm{~d} \tau^{F}-\frac{Y_{\tau H}^{N}}{\mu} \mathrm{~d} \tau^{H}+C_{\tau_{C}}^{N} \mathrm{~d} \tau^{C} \\
-Y_{\tau^{F}}^{T} \mathrm{~d} \tau^{F}-Y_{\tau_{H}^{H}}^{T} \mathrm{~d} \tau^{H}+C_{\tau^{C}}^{T} \mathrm{~d} \tau^{C} \\
0
\end{array}\right)
$$

The determinant denoted by $D$ of the matrix of coefficients is given by:

$$
\begin{equation*}
D \equiv \frac{h_{k k} k_{P}^{N}}{\mu}\left\{\frac{Y_{K}^{N}}{\mu}\left(Y_{\bar{\lambda}}^{T}-C_{\bar{\lambda}}^{T}\right)-\left(\frac{Y_{\bar{\lambda}}^{N}}{\mu}-C_{\bar{\lambda}}^{N}\right)\left[Y_{K}^{T}+r^{\star} \Phi_{1}\right]\right\} \tag{96}
\end{equation*}
$$

We have to consider two cases, depending on wether the non traded sector is more or less capital intensive than the traded sector:

$$
\begin{align*}
D= & -\frac{\nu_{1} \nu_{2}}{\tilde{P} \bar{\lambda}}\left(\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}\right)>0, \text { case } k^{T}>k^{N},  \tag{97a}\\
D= & -\frac{\nu_{1} \nu_{2}}{\tilde{P} \bar{\lambda}}\left\{\left(\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}\right)+\frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{2}\right)\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{2} \tilde{\Lambda}\right)\right\}  \tag{970}\\
& \text { case } k^{N}>k^{T},
\end{align*}
$$

where we used the fact that and $\mu f k^{N}-P h k^{T}=\mu W^{F}\left(k^{N}-k^{T}\right)$ together with $-P\left(k^{N} \nu_{2}+k^{T} \nu_{1}\right) \equiv$ $W^{F}$ if $k^{T}>k^{N}$ or $-P\left(k^{N} \nu_{1}+k^{T} \nu_{2}\right) \equiv W^{F}$ if $k^{N}>k^{T}$.

Useful Expressions
We have computed these useful expressions:

$$
\begin{align*}
\frac{Y_{K}^{N}}{\mu} Y_{\bar{\lambda}}^{T}-Y_{K}^{T} \frac{Y_{\bar{\lambda}}^{N}}{\mu} & =\sigma_{L} \frac{\tilde{L}}{\bar{\lambda}} \frac{\tilde{h} \tilde{f}}{\left(\tilde{k}^{N}-\tilde{k}^{T}\right)},  \tag{98a}\\
P_{C}^{\prime} Y_{K}^{T}-\left(1-\alpha_{C}\right) P_{C} \frac{Y_{K}^{N}}{\mu} & =-\frac{P_{C}}{\tilde{P}}\left[\frac{\alpha_{C} \tilde{f}+\left(1-\alpha_{C}\right) \tilde{P} \tilde{h}}{\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}\right]  \tag{98b}\\
\frac{Y_{\bar{\lambda}}^{N}}{\mu}-C_{\bar{\lambda}}^{N} & =\frac{1}{\bar{\lambda}}\left[-\sigma_{L} \tilde{L}^{2} \tilde{k}^{T} \frac{\tilde{h}}{\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}+\sigma_{C} \tilde{C}^{N}\right]  \tag{98c}\\
Y_{K}^{T}-C_{\bar{\lambda}}^{T} & =\frac{\tilde{P}}{\bar{\lambda}}\left[\sigma_{L} \tilde{L} \tilde{k}^{N} \frac{\tilde{f}}{\tilde{P}\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}+\sigma_{C} \frac{\tilde{C}^{T}}{\tilde{P}}\right] . \tag{98d}
\end{align*}
$$

If $\underline{k}^{N}>k^{T}$, useful expressions (105) become:

$$
\begin{align*}
& \frac{Y_{K}^{N}}{\mu} Y_{\bar{\lambda}}^{T}-Y_{K}^{T} \frac{Y_{\bar{\lambda}}^{N}}{\mu}=-\tilde{P} \nu_{1} \nu_{2} \frac{\sigma_{L} \tilde{L}}{\bar{\lambda}}\left(\tilde{k}^{N}-\tilde{k}^{T}\right)>0  \tag{99a}\\
& P_{C}^{\prime} Y_{K}^{T}-\left(1-\alpha_{C}\right) P_{C} \frac{Y_{K}^{N}}{\mu}=-P_{C}\left[\nu_{2}-\alpha_{C} r^{\star}\right]<0  \tag{99b}\\
& \frac{Y_{\bar{\lambda}}^{N}}{\mu}-C_{\bar{\lambda}}^{N}=-\frac{1}{\bar{\lambda}}\left[\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{2}-\sigma_{C} \tilde{C}^{N}\right]<0  \tag{99c}\\
& Y_{\bar{\lambda}}^{T}-C_{\bar{\lambda}}^{T}=-\frac{1}{\bar{\lambda}}\left[\sigma_{L} \tilde{L} \tilde{P} \tilde{k}^{N} \nu_{1}-\sigma_{C} \tilde{C}^{T}\right]>0  \tag{99d}\\
& \frac{Y_{K}^{N}}{\mu} C_{\tau^{C}}^{T}-Y_{K}^{T} C_{\tau^{C}}^{N}=\frac{\sigma_{C} P_{C} \tilde{C}}{\left(1+\tau^{C}\right)}\left[-\nu_{2}\left(1-\alpha_{C}\right)+\nu_{1} \alpha_{C}\right]<0  \tag{99e}\\
& Y_{K}^{T} \frac{Y_{\tau^{F}}^{N}}{\mu}-\frac{Y_{K}^{N}}{\mu} Y_{\tau^{F}}^{T}=-\tilde{P} \nu_{1} \nu_{2} \sigma_{L} \tilde{L} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}\left(\tilde{k}^{N}-\tilde{k}^{T}\right)>0  \tag{99f}\\
& Y_{K}^{T} \frac{Y_{\tau^{H}}^{N}}{\mu}-\frac{Y_{K}^{N}}{\mu} Y_{\tau^{H}}^{T}=-\tilde{P}_{\nu_{1} \nu_{2} \sigma_{L} \tilde{L} \frac{(\tilde{W}-\kappa)}{\tilde{W} \tilde{N}^{A}}\left(\tilde{k}^{N}-\tilde{k}^{T}\right)>0}^{Y_{P}^{T}-C_{P}^{T}}  \tag{99~g}\\
&=\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{\Lambda} \tilde{k}^{T} \nu_{2}\right)-\tilde{P}\left(\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}\right) . \tag{99h}
\end{align*}
$$

where we used the fact that $Y_{P}^{T}=-\tilde{P} \frac{Y_{P}^{N}}{\mu}-\sigma_{L} \tilde{L} \tilde{\Lambda} \tilde{k}^{T} \nu_{2}, C_{P}^{T}=P_{C} C_{P}-\tilde{P} C_{P}^{N}$ and $P_{C} C_{P}=-\sigma_{C} \tilde{C}^{N}$ to rewrite $Y_{P}^{T}-C_{P}^{T}($ see $(99 \mathrm{~h}))$.

If $\underline{k^{T}>k^{N}}$, useful expressions (105) become:

$$
\begin{align*}
\frac{Y_{K}^{N}}{\mu} Y_{\bar{\lambda}}^{T}-Y_{K}^{T} \frac{Y_{\bar{\lambda}}^{N}}{\mu} & =\tilde{P} \nu_{1} \nu_{2} \frac{\sigma_{L} \tilde{L}}{\bar{\lambda}}\left(\tilde{k}^{T}-\tilde{k}^{N}\right)<0,  \tag{100a}\\
P_{C}^{\prime} Y_{K}^{T}-\left(1-\alpha_{C}\right) P_{C} Y_{K}^{N} & =-P_{C}\left[\nu_{1}-\alpha_{C} r^{\star}\right]>0,  \tag{100b}\\
\frac{Y_{\bar{\lambda}}^{N}}{\mu}-C_{\bar{\lambda}}^{N} & =-\frac{1}{\bar{\lambda}}\left(\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}-\sigma_{C} \tilde{C}^{N}\right)>0,  \tag{100c}\\
Y_{\bar{\lambda}}^{T}-C_{\bar{\lambda}}^{T} & =-\frac{1}{\bar{\lambda}}\left[\sigma_{L} \tilde{L} \tilde{P} \tilde{k}^{N} \nu_{2}-\sigma_{C} \tilde{C}^{T}\right] \gtrless 0,  \tag{100d}\\
\frac{Y_{K}^{N}}{\mu} C_{\tau^{C}}^{T}-Y_{K}^{T} C_{\tau^{C}}^{N} & =\frac{\sigma_{C} P_{C} \tilde{C}}{\left(1+\tau^{C}\right)}\left[-\nu_{1}\left(1-\alpha_{C}\right)+\nu_{2} \alpha_{C}\right]>0,  \tag{100e}\\
Y_{K}^{T} \frac{Y_{\tau^{F}}^{N}}{\mu}-\frac{Y_{K}^{N}}{\mu} Y_{\tau^{F}}^{T} & =\tilde{P} \nu_{1} \nu_{2} \sigma_{L} \tilde{L} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}\left(\tilde{k}^{T}-\tilde{k}^{N}\right)<0,  \tag{100f}\\
Y_{P}^{T}-C_{P}^{T} & =\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{\Lambda} \tilde{k}^{T} \nu_{1}\right)-\tilde{P}\left(\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}\right),  \tag{100g}\\
Y_{K}^{T}+r^{\star} \Phi_{1} & =-\tilde{P} \nu_{1} . \tag{100h}
\end{align*}
$$

## E. 1 Long-Run Effects of an Unanticipated Permanent Consumption Tax Change

Case $k^{N}>k^{T}$

$$
\begin{align*}
\frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{C}} & =-\left(\frac{\sigma_{C} \tilde{C} \sigma_{L} \tilde{L}}{\Delta\left(1+\tau^{C}\right)}\right)\left[W^{F}-\tilde{k}^{T} \frac{r^{\star}}{\nu_{2}} \omega_{2}^{1}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right)\right] \lessgtr 0  \tag{101a}\\
\frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{C}} & =-\left(\frac{\sigma_{C} P_{C} \tilde{C} \sigma_{L} \tilde{L}}{1+\tau^{C}}\right) \frac{1}{\Delta}\left[1+\alpha_{C} \frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\tilde{P} \nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right)\right]<0  \tag{101b}\\
\frac{\mathrm{~d} \bar{\lambda}}{\mathrm{~d} \tau^{C}} & =-\left(\frac{\sigma_{C} \bar{\lambda} P_{C} \tilde{C}}{1+\tau^{C}}\right) \frac{1}{\Delta}\left[1+\alpha_{C} \frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\tilde{P} \nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right)\right]<0  \tag{101c}\\
\frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{C}} & =\frac{1}{\nu_{2}}\left(\frac{1}{1+\tau^{C}}\right)\left(\frac{\sigma_{C} P_{C} \tilde{C} \sigma_{L} \tilde{L}}{\Delta}\right)\left[\alpha_{C} \tilde{k}^{N} \nu_{1}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{2}\right]<0  \tag{101d}\\
\frac{\mathrm{~d} \tilde{B}}{\mathrm{~d} \tau^{C}} & =\Phi_{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{C}}>0 \tag{101e}
\end{align*}
$$

where $\Delta=\left[\left(\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}\right)+\frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{2}\right)\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L}^{T} \tilde{k}^{T} \nu_{2} \tilde{\Lambda}\right)\right]$ is assumed to be positive.

Case $k^{T}>k^{N}$

$$
\begin{align*}
\frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{C}} & =-\left(\frac{1}{1+\tau^{C}}\right)\left(\frac{\sigma_{C} \tilde{C} \sigma_{L} \tilde{W}^{F} \tilde{L}}{\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}}\right)<0  \tag{102a}\\
\frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{C}} & =-\left(\frac{1}{1+\tau^{C}}\right)\left(\frac{\sigma_{C} P_{C} \tilde{C} \sigma_{L} \tilde{L}}{\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}}\right)<0  \tag{102b}\\
\frac{\mathrm{~d} \bar{\lambda}}{\mathrm{~d} \tau^{C}} & =-\bar{\lambda}\left(\frac{1}{1+\tau^{C}}\right)\left(\frac{\sigma_{C} P_{C} \tilde{C}}{\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}}\right)<0  \tag{102c}\\
\frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{C}} & =\frac{1}{\nu_{1}}\left(\frac{1}{1+\tau^{C}}\right)\left(\frac{\sigma_{C} P_{C} \tilde{C} \sigma_{L} \tilde{L}}{\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}}\right)\left(\alpha_{C} \tilde{k}^{N} \nu_{2}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{1}\right)<0  \tag{102d}\\
\frac{\mathrm{~d} \tilde{B}}{\mathrm{~d} \tau^{C}} & =-\tilde{P} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{C}}>0 \tag{102e}
\end{align*}
$$

## E. 2 Long-Run Effects of an Unanticipated Permanent Change in Payroll Taxes

$$
\begin{align*}
\frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{F}} & =-\sigma_{C} \tilde{C} \frac{\sigma_{L} \tilde{L}}{\Delta} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}\left[\tilde{W}^{F}-\tilde{k}^{T} r^{\star} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right)\right] \lessgtr 0,  \tag{103a}\\
\frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{F}} & =-\frac{\sigma_{L} \tilde{L}}{\Delta} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}\left\{\sigma_{C} P_{C} \tilde{C}+\frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right) \sigma_{C} \tilde{C}^{N}\right\}<0,  \tag{103b}\\
\frac{\mathrm{~d} \tilde{\lambda}}{\mathrm{~d} \tau^{F}} & =\bar{\lambda} \frac{\sigma_{L} \tilde{L}}{\Delta} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}\left[\tilde{W}^{F}-\tilde{k}^{T} r^{\star} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right)\right] \lessgtr 0,  \tag{103c}\\
\frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{F}} & =\frac{\sigma_{L} \tilde{L}}{\Delta \nu_{2}} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)} \sigma_{C} P_{C} \tilde{C}\left[\alpha_{C} \tilde{k}^{N} \nu_{1}-\left(1-\alpha_{C} \tilde{k}^{T} \nu_{2}\right]<0,\right.  \tag{103d}\\
\frac{\mathrm{d} \tilde{B}}{\mathrm{~d} \tau^{F}} & =\Phi_{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{F}}>0, \tag{103e}
\end{align*}
$$

where we used the fact that $\tilde{P}\left(\nu_{2} \tilde{k}^{T}+\nu_{1} \tilde{k}^{N}\right)=-\tilde{P}\left(\tilde{h}-h_{k} \tilde{k}^{N}\right) \equiv-\tilde{W}^{F}$.
Case $k^{T}>k^{N}$

$$
\begin{align*}
\frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{F}} & =-\sigma_{C} \tilde{C} \frac{\sigma_{L} \tilde{L}}{\Delta} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)} \tilde{W}^{F}  \tag{104a}\\
\frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{F}} & =-\frac{\sigma_{L} \tilde{L}}{\Delta} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)} \sigma_{C} P_{C} \tilde{C},  \tag{104b}\\
\frac{\mathrm{~d} \bar{\lambda}}{\mathrm{~d} \tau^{F}} & =\bar{\lambda} \frac{\sigma_{L} \tilde{L}}{\Delta} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)} \tilde{W}^{F}>0,  \tag{104c}\\
\frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{F}} & =\frac{\sigma_{L} \tilde{L}}{\nu_{1} \Delta} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)} \sigma_{C} P_{C} \tilde{C}\left[\alpha_{C} \tilde{k}^{N} \nu_{2}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{1}\right]<0,  \tag{104d}\\
\frac{\mathrm{~d} \tilde{B}}{\mathrm{~d} \tau^{F}} & =-\tilde{P} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{F}}>0, \tag{104e}
\end{align*}
$$

where we let $\Delta \equiv \sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}$ and we used the fact that $-\tilde{P}\left(\nu_{1} \tilde{k}^{T}+\nu_{2} \tilde{k}^{N}\right)=\tilde{P}\left(\tilde{h}-h_{k} \tilde{k}^{N}\right) \equiv$ $\tilde{W}^{F}$.

## E. 3 Long-Run Effects of an Unanticipated Permanent Progressive Wage Tax Change

Case $k^{N}>k^{T}$

$$
\begin{align*}
\frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{H}} & =-\sigma_{C} \tilde{C} \frac{\sigma_{L} \tilde{L}}{\Delta} \frac{(\tilde{W}-\kappa)}{\tilde{W}^{A}}\left[\tilde{W}^{F}-\tilde{k}^{T} r^{\star} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right)\right] \lessgtr 0  \tag{105a}\\
\frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{H}} & =-\frac{\sigma_{L} \tilde{L}}{\Delta} \frac{\tilde{W}-\kappa)}{\tilde{W}^{A}}\left\{\sigma_{C} P_{C} \tilde{C}+\frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right) \sigma_{C} \tilde{C}^{N}\right\}<0,  \tag{105b}\\
\frac{\mathrm{~d} \bar{\lambda}}{\mathrm{~d} \tau^{H}} & =\bar{\lambda} \frac{\sigma_{L} \tilde{L}}{\Delta} \frac{(\tilde{W}-\kappa)}{\tilde{W}^{A}}\left[\tilde{W}^{F}-\tilde{k}^{T} r^{\star} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right)\right] \lessgtr 0,  \tag{105c}\\
\frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{H}} & \left.=\frac{\sigma_{L} \tilde{L}}{\nu_{2} \Delta} \sigma_{C} P_{C} \tilde{C} \frac{(\tilde{W}-\kappa)}{\tilde{W}^{A}}\right)\left[\alpha_{C} \tilde{k}^{N} \nu_{1}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{2}\right]<0,  \tag{105d}\\
\frac{\mathrm{~d} \tilde{B}}{\mathrm{~d} \tau^{H}} & =-\Phi_{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{H}}>0, \tag{105e}
\end{align*}
$$

where we used the fact that $\frac{\tilde{W}\left(1-\tau^{H}\right)}{\tilde{W}^{A}}=\tilde{\Lambda}$.
case $k^{T}>k^{N}$

$$
\begin{align*}
\frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{H}} & =-\sigma_{C} \tilde{C} \frac{\sigma_{L} \tilde{L}}{\Delta} \frac{\left(\tilde{W}^{\prime}-\kappa\right)}{\tilde{W}^{A}} \tilde{W}^{F}<0  \tag{106a}\\
\frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{H}} & =-\frac{\sigma_{L} \tilde{L}}{\Delta} \frac{\tilde{W}-\kappa)}{\tilde{W}^{A}} \sigma_{C} P_{C} \tilde{C}<0  \tag{106b}\\
\frac{\mathrm{~d} \bar{\lambda}}{\mathrm{~d} \tau^{H}} & =\bar{\lambda} \frac{\sigma_{L} \tilde{L}}{\Delta} \frac{(\tilde{W}-\kappa)}{\tilde{W}^{A}} \tilde{W}^{F}>0  \tag{106c}\\
\frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{H}} & =\frac{\sigma_{L} \tilde{L}}{\nu_{1} \Delta} \frac{(\tilde{W}-\kappa)}{\tilde{W}^{A}} \sigma_{C} P_{C} \tilde{C}\left[\alpha_{C} \tilde{k}^{N} \nu_{2}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{1}\right]<0  \tag{106d}\\
\frac{\mathrm{~d} \tilde{B}}{\mathrm{~d} \tau^{H}} & =-\tilde{P} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{H}}>0 \tag{106e}
\end{align*}
$$

where we set $\Delta=\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}$.

## E. 4 Useful Properties

We denote by $X$ the macroeconomic aggregates $C, L, K, N X$. Inspection of long-run changes shows that, regardless of sectoral capital intensities, we have the following useful properties;

$$
\begin{equation*}
\frac{\mathrm{d} \tilde{X}}{\mathrm{~d} \tau^{C}}=\frac{1}{1+\tau^{C}} \frac{1+\tau^{F}}{\tilde{\Lambda}} \frac{\mathrm{~d} \tilde{X}}{\mathrm{~d} \tau^{F}}, \quad \frac{\mathrm{~d} \tilde{X}}{\mathrm{~d} \tau^{H}}=\frac{1+\tau^{F}}{\tilde{\Lambda}} \frac{\tilde{W}-\kappa}{\tilde{W}^{A}} \frac{\mathrm{~d} \tilde{X}}{\mathrm{~d} \tau^{F}} \tag{107a}
\end{equation*}
$$

## E. 5 Rewriting the Long-Run Effects

In this subsection, we rewrite expressions of steady-state changes following a labor tax cut, i.e. after a drop in $\tau^{j}(j=F, H)$ when the traded sector is more capital intensive than the non traded sector. It is useful to introduce some notations:

$$
\begin{align*}
& \hat{\tau}^{F}=\frac{\mathrm{d} \tau^{F}}{1+\tau^{F}}, \hat{\tau}^{H}=\frac{\mathrm{d} \tau^{H}}{1-\tau^{H}}  \tag{108a}\\
& 0<\Lambda^{F} \equiv \frac{\left(1-\tau^{H}\right) \tilde{W}}{\tilde{W}^{A}}<1, \quad 0<\Lambda^{H} \equiv \frac{(\tilde{W}-\kappa)\left(1-\tau^{H}\right)}{\tilde{W}^{A}}=1-\frac{\kappa}{\tilde{W}^{A}}<1  \tag{108b}\\
& 0<\tilde{\xi} \equiv \frac{\sigma_{L} \tilde{W}^{F} \tilde{L}}{\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}}<1 \tag{108c}
\end{align*}
$$

where we used the fact that $\tilde{W}\left(1-\tau^{H}\right)=\tilde{W}^{A}-\tau^{H} \kappa$ to determine (108b).
Case $k^{T}>k^{N}$
Denoting by a hat the percentage deviation relative to initial steady-state, the change in the shadow value of wealth following a labor tax cut is:

$$
\begin{equation*}
\hat{\bar{\lambda}}=\Lambda^{j} \tilde{\xi} \hat{\tau}^{j}<0, \quad j=F, H \tag{109}
\end{equation*}
$$

where $\Lambda^{j} \tilde{\xi}<1$.
The change in labor following a labor tax cut is:

$$
\begin{equation*}
\hat{\tilde{L}}=-\sigma_{L} \Lambda^{j}(1-\tilde{\xi}) \hat{\tau}^{j}>0, \quad j=F, H, \tag{110}
\end{equation*}
$$

where $\Lambda^{j}(1-\tilde{\xi})<1$.
The change in the capital stock following a labor tax cut is:

$$
\begin{equation*}
\mathrm{d} \tilde{K}=\frac{\sigma_{L} \tilde{L}}{\nu_{1}} \Lambda^{j}(1-\tilde{\xi})\left[\alpha_{C} \tilde{k}^{N} \nu_{2}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{1}\right] \hat{\tau}^{j}>0, \quad j=F, H, \tag{111}
\end{equation*}
$$

where $\Lambda^{j}(1-\tilde{\xi})<1$ and $\left[\alpha_{C} \tilde{k}^{N} \nu_{2}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{1}\right]>0$.
Remembering that the relative price of non tradables remains unaffected in the long-run, and using the fact that $\left.\mathrm{d} \tilde{Y}\right|^{j, C}=\left.\left(Y_{K}^{T}+\frac{\tilde{P}}{\mu} Y_{K}^{N}\right) \mathrm{d} \tilde{K}\right|^{j, C}+\left.\left(Y_{L}^{T}+\frac{\tilde{P}}{\mu} Y_{L}^{N}\right) \mathrm{d} \tilde{L}\right|^{j, C}$ together with $Y_{K}^{T}+$ $\frac{\tilde{P}}{\mu} Y_{K}^{N}=\tilde{P} r^{\star}$ and $Y_{L}^{T}+\frac{\tilde{P}}{\mu} Y_{L}^{N}=\tilde{W}^{F}$, the change in aggregate output following a labor tax cut is:

$$
\begin{equation*}
\hat{\tilde{Y}}=\left(1-\tilde{\beta}_{L}\right) \hat{\tilde{K}}+\tilde{\beta}_{L} \hat{\tilde{L}}>0 \tag{112}
\end{equation*}
$$

where $\left(1-\tilde{\beta}_{L}\right)=\frac{\tilde{P}^{\star} \tilde{K}}{\tilde{Y}}$ and $\tilde{\beta}_{L}=\frac{\tilde{W}^{F} \tilde{L}}{\tilde{Y}}$ are the shares of capital and labor income in GDP, respectively.

## E. 6 Inelastic Labor Supply Case: $\sigma_{L}=0$

To get further insight about the transmission mechanism, we derive the long-run effects when labor supply is inelastic, i.e. we set $\sigma_{L}=0$.

Long-Run Effects of an Unanticipated Permanent Consumption Tax Change Set $\sigma_{L}=0$ into (101) or (102), we get:

$$
\begin{align*}
\frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{C}} & =\frac{\mathrm{d} \tilde{L}}{\mathrm{~d} \tau^{C}}=\frac{\mathrm{d} \tilde{P}}{\mathrm{~d} \tau^{C}}=\frac{\mathrm{d} \tilde{K}}{\mathrm{~d} \tau^{C}}=\frac{\mathrm{d} \tilde{B}}{\mathrm{~d} \tau^{C}}=0  \tag{113a}\\
\frac{\mathrm{~d} \bar{\lambda}}{\mathrm{~d} \tau^{C}} & =-\frac{\bar{\lambda}}{\left(1+\tau^{C}\right)} \tag{113b}
\end{align*}
$$

From (113a)-(113b), the elasticity of the marginal utility of wealth is equal to unity in absolute terms and the long-run levels of variables remain unaffected. A rise in consumption tax raises the marginal cost of current consumption. Since the trade-off between labor and leisure turns out to be irrelevant, total employment remains fixed such that $\bar{\lambda}$ must fall by the same proportion than the rise in $\tau^{C}$ thus leaving unaffected real consumption as the tax effect and the wealth effect cancel each other. Since demand for non tradables and tradables remain unaffected, capital stock and net foreign assets must not change for investment and the current account to be zero in the long-run. As the capital stock remains unchanged in the long-run, dynamics degenerate.

Long-Run Effects of an Unanticipated Permanent Change in Payroll Taxes
Set $\sigma_{L}=0$ into (103) or (104), we get:

$$
\begin{align*}
\frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{F}} & =\frac{\mathrm{d} \tilde{L}}{\mathrm{~d} \tau^{F}}=\frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{F}}=\frac{\mathrm{d} \tilde{P}}{\mathrm{~d} \tau^{F}}=\frac{\mathrm{d} \tilde{K}}{\mathrm{~d} \tau^{F}}=\frac{\mathrm{d} \tilde{B}}{\mathrm{~d} \tau^{F}}=0  \tag{114a}\\
\frac{\mathrm{~d} \tilde{W}}{\mathrm{~d} \tau^{F}} & =-\frac{\tilde{W}}{\left(1+\tau^{F}\right)}<0 \tag{114b}
\end{align*}
$$

From (114a)-(114b), a fall in $\tau^{F}$ leaves unchanged the steady-state levels of variables, and more importantly does no longer induce a wealth effect. The explanation is that whenever the trade-off between labor and leisure turns out to be irrelevant, total employment remains fixed. To insure that equality of sectoral labor marginal products holds, the wage must rise by the same proportion than the fall in the payroll tax. As the capital stock remains unchanged in the long-run, dynamics degenerate. In words, if labor is fixed, a change in the tax on wage paid by producers induces solely a tax effect on the wage rate.

## F The Two-Step Procedure: Wealth Effect and Tax Effects

By analytical convenience, we rewrite the system of steady-state equations, assuming that $\delta_{K}=0$ :

$$
\begin{gather*}
\frac{h_{k}\left[k^{N}(\tilde{P})\right]}{\mu}=r^{\star}  \tag{115a}\\
\frac{1}{\mu} Y^{N}\left(\tilde{K}, \tilde{P}, \bar{\lambda}, \tau^{F}, \tau^{H}\right)-C^{N}\left(\bar{\lambda}, \tilde{P}, \tau^{C}\right)-G^{N}=0  \tag{115b}\\
r^{\star} \tilde{B}+Y^{T}\left(\tilde{K}, \tilde{P}, \bar{\lambda}, \tau^{F}, \tau^{H}\right)-C^{T}\left(\bar{\lambda}, \tilde{P}, \tau^{C}\right)-G^{T}=0  \tag{115c}\\
\text { together with the intertemporal solvency condition }
\end{gather*}
$$

$$
\begin{equation*}
\left(\tilde{B}-B_{\mathcal{T}}\right)=\Phi_{1}\left(\tilde{K}-K_{\mathcal{T}}\right) \tag{115d}
\end{equation*}
$$

where $K_{0}$ and $B_{0}$ correspond to the initially predetermined stocks of physical capital and foreign assets, the open economy starting from an initial steady-state at time $\mathcal{T}$. If the fiscal shock is permanent, then $\mathcal{T}=0$.

## Derivation of Steady-State Functions

In a first step, we solve the system (115a)-(115c) for $\tilde{P}, \tilde{K}$ and $\tilde{B}$ as functions of the marginal utility of wealth, $\bar{\lambda}$, the tax rates on consumption and labor together with the mark-up. Totally differentiating equations (115a)-(115c) yields in matrix form:

$$
\begin{align*}
& \left(\begin{array}{ccc}
h_{k k} k_{P}^{N} & 0 & 0 \\
\left(\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}\right) & \frac{Y_{K}^{N}}{\mu} & 0 \\
\left(Y_{P}^{T}-C_{P}^{T}\right) & Y_{K}^{T} & r^{\star}
\end{array}\right)\left(\begin{array}{c}
\mathrm{d} \tilde{P} \\
\mathrm{~d} \tilde{K} \\
\mathrm{~d} \tilde{B}
\end{array}\right) \\
= & \left(\begin{array}{c}
\frac{Y_{K}^{N}}{\mu} \mathrm{~d} \mu \\
-\left(\frac{Y_{\bar{\lambda}}^{N}}{\mu}-C_{\bar{\lambda}}^{N}\right) \mathrm{d} \bar{\lambda}+C_{\tau^{C}}^{N} \mathrm{~d} \tau^{C}-\frac{Y_{\tau F}^{N}}{\mu} \mathrm{~d} \tau^{F}-\frac{Y_{\tau H}^{N}}{\mu} \mathrm{~d} \tau^{H}-\left(\frac{Y_{\mu}^{N}}{\mu}-\frac{Y^{N}}{\mu^{2}}\right) \mathrm{d} \mu \\
-\left(Y_{\bar{\lambda}}^{T}-C_{\bar{\lambda}}^{T}\right) \mathrm{d} \bar{\lambda}+C_{\tau_{C}^{C}}^{T} \mathrm{~d} \tau^{C}-Y_{\tau_{F}}^{T} \mathrm{~d} \tau^{F}-Y_{\tau^{H}}^{T} \mathrm{~d} \tau^{H}-Y_{\mu}^{T} \mathrm{~d} \mu
\end{array}\right), \tag{116}
\end{align*}
$$

where we used the fact that $\mu f=P\left[h-h_{k}\left(k^{N}-k^{T}\right)\right]$ and $\frac{h_{k}}{\mu}=r^{\star}$ at the steady-state to rewrite $r^{\star}-h_{k k} k_{\mu}^{N}$ as $\frac{\tilde{h}}{\mu\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}=\frac{Y_{K}^{N}}{\mu}$.

The equilibrium value of the marginal utility of wealth $\bar{\lambda}$, tax rates (i.e., $\tau^{C}, \tau^{F}, \tau^{H}$ ) and the markup $\mu$ determine the following steady-state values:

$$
\begin{align*}
\tilde{P} & =P(\mu),  \tag{117a}\\
\tilde{K} & =K\left(\bar{\lambda}, \tau^{C}, \tau^{F}, \tau^{H}, \mu\right),  \tag{117b}\\
\tilde{B} & =B\left(\bar{\lambda}, \tau^{C}, \tau^{F}, \tau^{H}, \mu\right), \tag{117c}
\end{align*}
$$

with partial derivatives given by:

$$
\begin{align*}
K_{\bar{\lambda}} \equiv \frac{\partial \tilde{K}}{\partial \bar{\lambda}} & =-\frac{1}{\bar{\lambda}} \frac{1}{\nu_{1}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)>0 \quad \text { if } \quad k^{T}>k^{N}  \tag{118a}\\
& =-\frac{1}{\bar{\lambda}} \frac{1}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{2}\right)>0 \quad \text { if } \quad k^{N}>k^{T},  \tag{118b}\\
B_{\bar{\lambda}} \equiv \frac{\partial \tilde{B}}{\partial \bar{\lambda}} & =-\frac{1}{\bar{\lambda}} \frac{1}{r^{\star} \tilde{h}}\left[\sigma_{C}\left(\tilde{f} \tilde{C}^{N}+\tilde{h} \tilde{C}^{T}\right)+\sigma_{L} \tilde{L} \tilde{h} \tilde{f}\right]<0, \tag{118c}
\end{align*}
$$

and

$$
\begin{align*}
K_{\tau^{C}} \equiv \frac{\partial \tilde{K}}{\partial \tau^{C}} & =-\frac{1}{\nu_{1}}\left(\frac{\sigma_{C} \tilde{C}^{N}}{1+\tau^{C}}\right)>0 \quad \text { if } \quad k^{T}>k^{N},  \tag{119a}\\
& =-\frac{1}{\nu_{2}}\left(\frac{\sigma_{C} \tilde{C}^{N}}{1+\tau^{C}}\right)<0 \quad \text { if } \quad k^{N}>k^{T},  \tag{119b}\\
B_{\tau^{C}} \equiv \frac{\partial \tilde{B}}{\partial \tau^{C}} & =\frac{\sigma_{C}\left(\tilde{P} \tilde{C}^{N} \nu_{2}-\tilde{C}^{T} \nu_{1}\right)}{\nu_{1} r^{\star}\left(1+\tau^{C}\right)}<0, \quad \text { if } \quad k^{T}>k^{N},  \tag{119c}\\
& =\frac{\sigma_{C}\left(\tilde{P} \tilde{C}^{N} \nu_{1}-\tilde{C}^{T} \nu_{2}\right)}{\nu_{2} r^{\star}\left(1+\tau^{C}\right)}<0, \quad \text { if } \quad k^{T}>k^{N} \tag{119d}
\end{align*}
$$

and

$$
\begin{align*}
& K_{\tau^{F}} \equiv \frac{\partial \tilde{K}}{\partial \tau^{F}}=-\frac{\sigma_{L} \tilde{L}}{1+\tau^{F}} \tilde{k}^{T}<0,  \tag{120a}\\
& B_{\tau^{F}} \equiv \frac{\partial \tilde{B}}{\partial \tau^{F}}=\frac{\tilde{f}}{r^{\star}} \frac{\sigma_{L} \tilde{L}}{1+\tau^{F}}>0, \tag{120b}
\end{align*}
$$

and

$$
\begin{align*}
& K_{\tau^{H}} \equiv \frac{\partial \tilde{K}}{\partial \tau^{H}}=-\frac{\sigma_{L} \tilde{L}}{1-\tau^{H}} \tilde{k}^{T}<0,  \tag{121a}\\
& B_{\tau^{H}} \equiv \frac{\partial \tilde{B}}{\partial \tau^{H}}=\frac{\tilde{f}}{r^{\star}} \frac{\sigma_{L} \tilde{L}}{1-\tau^{H}}>0 . \tag{121b}
\end{align*}
$$

and

$$
\begin{align*}
& P_{\mu} \equiv \frac{\partial \tilde{P}}{\partial \mu}=-\frac{\tilde{P}}{\mu} \frac{\tilde{P} Y_{K}^{N}}{\mu Y_{K}^{T}}=-\frac{\tilde{P} \nu_{1}}{\mu \nu_{2}}>0, \quad \text { if } k^{T}>k^{N},  \tag{122a}\\
&=-\frac{\tilde{P} \nu_{2}}{\mu \nu_{1}}>0, \quad \text { if } k^{N}>k^{T},  \tag{122b}\\
& K_{\mu} \equiv \frac{\partial \tilde{K}}{\partial \mu}=\frac{\tilde{P}}{\mu \nu_{1} \nu_{2}}\left[\frac{Y_{P}^{N}}{\mu}-\nu_{1} C_{P}^{N}\right]+\frac{Y^{N}}{\mu^{2} \nu_{1}} \lessgtr 0, \quad \text { if } k^{T}>k^{N},  \tag{122c}\\
&=\frac{\tilde{P}}{\mu \nu_{1} \nu_{2}}\left[\frac{Y_{P}^{N}}{\mu}-\nu_{2} C_{P}^{N}\right]+\frac{Y^{N}}{\mu^{2} \nu_{2}} \lessgtr 0, \quad \text { if } k^{N}>k^{T},  \tag{122d}\\
& B_{\mu} \equiv \frac{\partial \tilde{B}}{\partial \mu}=-\frac{\tilde{P}}{\mu \nu_{2}}\left[\tilde{P}\left(\frac{Y_{P}^{N}}{\mu} \frac{r^{\star}}{\nu_{1}}-C_{P}^{N}\right)+\left(\sigma_{L} \tilde{L} \tilde{\Lambda} \tilde{k}^{T} \nu_{1}-\frac{\nu_{1}}{r^{\star}} \sigma_{C} \tilde{C}^{N}\right)\right]+\frac{\tilde{L}^{N} \tilde{f}}{\mu r^{\star}} \gtrless 0, \\
& \quad \text { if } k^{T}>k^{N}  \tag{122e}\\
&=-\frac{\tilde{P}}{\mu \nu_{1}}\left[\tilde{P}\left(\frac{Y_{P}^{N}}{\mu} \frac{r^{\star}}{\nu_{2}}-C_{P}^{N}\right)+\left(\sigma_{L} \tilde{L} \tilde{\Lambda} \tilde{k}^{T} \nu_{2}-\frac{\nu_{2}}{r^{\star}} \sigma_{C} \tilde{C}^{N}\right)\right]+\frac{\tilde{L}^{N} \tilde{f}}{\mu r^{\star}} \gtrless 0, \\
& \text { if } k^{N}>k^{T} \tag{122f}
\end{align*}
$$

where we used the fact that $h_{k k} k_{P}^{N}=-\frac{\mu}{P} \frac{Y_{K}^{T}}{P}$ to derive the first equality of (122a). In addition, we made use of the following property $Y_{\mu}^{N}=-\frac{P}{\mu} Y_{P}^{N}$ and $Y_{\mu}^{T}=-\frac{P}{\mu} Y_{P}^{T}$ to determine (122c)-(122d) and (122e)-(122f). Finally, use has been made of property (54a) to rewrite $Y_{P}^{T}-C_{P}^{T}$ and property (54b) to simplify $\mu Y_{K}^{T}+\mu Y_{K}^{N}$ which is equal to $\tilde{P} \mu r^{\star}$ in the long-run.

Since the change in the markup modifies the long-run levels of real consumption and labor supply through the steady-state change in the relative price of non tradables, it is convenient to write their steady-state functions by substituting (117a) into their static solutions (41) that hold in the long-run:

$$
\begin{equation*}
C=m\left(\bar{\lambda}, \tau^{C}, \mu\right), \quad L=n\left(\bar{\lambda}, \tau^{F}, \tau^{H}, \mu\right) \tag{123}
\end{equation*}
$$

where partial derivatives are given by (42) evaluated at the steady-state (that's why we substitute respectively the notations $m$ and $n$ for $c$ and $L$ ) and

$$
\begin{align*}
m_{\mu} \equiv \frac{\partial \tilde{C}}{\partial \mu} & =\alpha_{C} \sigma_{C} \tilde{C} \frac{\nu_{1}}{\nu_{2}}<0, \quad \text { if } \quad k^{T}>k^{N}  \tag{124a}\\
& =\alpha_{C} \sigma_{C} \tilde{C} \frac{\nu_{2}}{\nu_{1}}<0, \quad \text { if } k^{N}>k^{T},  \tag{124b}\\
n_{\mu} \equiv \frac{\partial \tilde{L}}{\partial \mu} & =-\frac{\sigma_{L} \tilde{L} \tilde{\Lambda} \tilde{k}^{T}}{\tilde{W}^{F}} \frac{\tilde{P} \tilde{h}}{\tilde{f}} \frac{\tilde{P} r^{\star}}{\mu^{2}}<0, \tag{124c}
\end{align*}
$$

where partial derivatives w. r. t. to $\bar{\lambda}, \tau^{C}, \tau^{F}$, and $\tau^{H}$ are given by (42); we computed (124c) as follows: $n_{\mu}=\frac{\sigma_{L} \tilde{L} \tilde{\Lambda} \tilde{k}^{T}}{\tilde{W}^{F}} \frac{\tilde{P} Y_{K}^{N}}{\mu Y_{K}^{T}} \frac{\tilde{P} r^{\star}}{\mu}$.

Following the same procedure, i. e. substituting the steady-state function for the real exchange rate into the static solution for wage evaluated at the steady-state, the steady-state function for the wage rate is:

$$
\begin{equation*}
W=W\left(\tau^{F}, \mu\right) \tag{125}
\end{equation*}
$$

where the partial derivative w. r. t. $\mu$ is given by:

$$
\begin{equation*}
W_{\mu} \equiv \frac{\partial \tilde{W}}{\partial \mu}=-\frac{\tilde{k}^{T}}{1+\tau^{F}} \frac{\tilde{P} \tilde{h}}{\tilde{f}} \frac{\tilde{P} r^{\star}}{\mu^{2}}<0 \tag{126}
\end{equation*}
$$

where $W_{\mu}=\frac{\tilde{k}^{T}}{1+\tau^{F}} \frac{\tilde{P} Y_{K}^{N}}{\mu Y_{K}^{T}} \frac{\tilde{P} r^{\star}}{\mu}$ with $\frac{Y_{K}^{N}}{Y_{K}^{T}}=-\frac{\tilde{h}}{\tilde{f}}<0$.
Finally, following a similar procedure, we can express the rental rate of physical capital as a function of $\tau^{F}$ and $\mu$ :

$$
\begin{equation*}
R^{K}=R^{K}(\mu), \tag{127}
\end{equation*}
$$

where the partial derivative w. r. t. $\mu$ is given by:

$$
\begin{align*}
R_{\mu}^{K} & \equiv \frac{\partial \tilde{r}^{K}}{\partial \mu}=-r^{\star} \frac{\tilde{P}}{\mu} \frac{\nu_{1}}{\nu_{2}}>0, \quad \text { if } \quad k^{T}>k^{N}  \tag{128}\\
R_{\mu}^{K} & \equiv \frac{\partial \tilde{r}^{K}}{\partial \mu}=-r^{\star} \frac{\tilde{P}}{\mu} \frac{\nu_{2}}{\nu_{1}}>0, \quad \text { if } \quad k^{N}>k^{T} \tag{129}
\end{align*}
$$

and the partial derivative w. r. t. $\tau^{F}$ is given by (48b).

## G Long-Term Effects of Revenue-Neutral Tax Reforms

In this section, we derive the steady-state effects of a shift of the tax burden from labor to consumption. Since we consider a revenue-neutral tax reform, the consumption tax must change accordingly to balance the budget after a labor tax cut. To derive the direction and the size of the change in the consumption tax, we first substitute short-run static solutions for consumption, wage and labor given by (41) and (47), into the balanced government budget constraint (8) evaluated at the steady-state:

$$
\begin{equation*}
\tau^{C} P_{C}(\tilde{P}) C\left(\bar{\lambda}, \tilde{P}, \tau^{C}\right)+\left[\left(\tau^{F}+\tau^{H}\right) W\left(\tilde{P}, \tau^{F}\right)-\tau^{H} \kappa\right] L\left(\bar{\lambda}, \tilde{P}, \tau^{F}, \tau^{H}\right)=Z \tag{130}
\end{equation*}
$$

keeping in mind that the long-run value of the real exchange rate is unaffected by fiscal tax changes and $\bar{\lambda}=\lambda\left(\tau^{C}, \tau^{F}, \tau^{H}\right)$.

In deriving the long-run effects of revenue-neutral tax reforms, we concentrate on the case $k^{T}>k^{N}$ since we cannot determine the sign of formal expressions when $k^{N}>k^{T}$. However, as shown by numerical results, the long-run effects are both qualitatively and quantitatively similar whether $k^{T} \gtrless k^{N}$.

## G. 1 Steady-State Changes: A Shift from Payroll Taxes to Consumption Taxes

In this section, we estimate the long-run effects of a shift from a payroll tax $\tau^{F}$ to a consumption tax $\tau^{C}$, which is adjusted accordingly to balance the government budget. Additionally, we assume that taxes on labor income are progressive so that $\kappa>0$ and $\Lambda<1$. To avoid confusion, we denote by $\left.\right|^{j, C}$ the effects of the tax reform which involves simultaneously cutting the tax $j=F$ and increasing the $\operatorname{tax} k=c$ so as to keep the government budget balanced. In brief, the tax reform strategy involves simultaneously cutting the payroll tax by $\mathrm{d} \tau^{F}<0$ and increasing the consumption tax $\left.\mathrm{d} \tau^{C}\right|^{F, C}>0$.

Holding $\tau^{H}$ constant, we differentiate (130)

$$
\begin{equation*}
\left.P_{C} \tilde{C} \mathrm{~d} \tau^{C}\right|^{F, C}+\left.\tau^{C} P_{C} \mathrm{~d} \tilde{C}\right|^{F, C}+\left[\left(\tau^{F}+\tau^{H}\right) W_{\tau^{F}}+\tilde{W}\right] \mathrm{d} \tau^{F}+\left.\left(\tilde{W}^{F}-\tilde{W}^{A}\right) \mathrm{d} \tilde{L}\right|^{F, C}=0, \tag{131}
\end{equation*}
$$

with $\left[\left(\tau^{F}+\tau^{H}\right) W_{\tau^{F}}+\tilde{W}\right]=\tilde{W}\left(\frac{1-\tau^{H}}{1+\tau^{F}}\right)>0$.
We denote by $X$ the aggregate $C, L, K, N X$. By using the fact that $\left.\mathrm{d} \tilde{X}\right|^{F, C}=\frac{\mathrm{d} \tilde{X}}{\mathrm{~d} \tau^{F}} \mathrm{~d} \tau^{F}+$ $\left.\frac{\mathrm{d} \tilde{X}}{\mathrm{~d} \tau^{C}} \mathrm{~d} \tau^{C}\right|^{F, C}$, and by rearranging terms, we can determine the size of the rise in the consumption tax rate $\left.\tau^{C}\right|^{F, C}$ after a fall in the payroll tax $\tau^{F}$ such that the government budget constraint (130) remains balanced:

$$
\begin{equation*}
\left.\mathrm{d} \tau^{C}\right|^{F, C}=-\frac{\chi_{F}}{\chi_{C}} \mathrm{~d} \tau^{F}=-\left\{\frac{\tau^{C} P_{C} \frac{\mathrm{~d} \tilde{C}}{\mathrm{~d} \tau^{F}}+\left(\tilde{W}^{F}-\tilde{W}^{A}\right) \frac{\mathrm{d} \tilde{L}}{\mathrm{~d} \tau^{F}}+\left(\frac{1-\tau^{H}}{1+\tau^{F}}\right) \tilde{W} \tilde{L}}{\tau^{C} P_{C} \frac{\mathrm{~d} \tilde{C}}{\mathrm{~d} \tau^{C}}+\left(\tilde{W}^{F}-\tilde{W}^{A}\right) \frac{\mathrm{d} \tilde{L}}{\mathrm{~d} \tau^{C}}+P_{C} \tilde{C}}\right\} \mathrm{d} \tau^{F}, \tag{132}
\end{equation*}
$$

where analytical expressions of $\chi_{F}$ and $\chi_{C}$ are shown below.
We first compute $\chi^{F}$ which reflects two opposite effects on tax revenues. Whereas a labor tax cut lowers tax revenue, keeping unchanged consumption and employment, a labor tax cut raises employment and consumption, and thereby tax revenues. By noting that $\left(\frac{1-\tau^{H}}{1+\tau^{F}}\right) \tilde{W} \tilde{L}=\frac{\tilde{\Lambda}}{1+\tau^{F}} \tilde{W}^{A} \tilde{L}$, and by substituting the long-run changes of $L$ and $C$ (see eqs (104a) and (104b)), $\chi_{F}$ is given by:

$$
\begin{equation*}
\chi^{F}=\frac{\tilde{\Lambda}}{1+\tau^{F}} \tilde{W}^{A} \tilde{L}\left\{1-\sigma_{L} \frac{\tilde{W}^{F}}{\tilde{W}^{A}}(1-\tilde{\xi})\left[\tau^{C}+\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right)\right]\right\} \tag{133}
\end{equation*}
$$

where $0<\tilde{\xi}<1$ is given by (108c). As long as tax rates take reasonable values and the elasticity of labor supply (i.e., $\sigma_{L}$ ) is not too large, $\chi^{F}$ is positive. Hence, a labor tax cut leads to a fall in tax revenue, and more so the smaller $\sigma_{L}$ and the lower the tax rates.

The same logic applies to a change in the consumption tax. Substituting long-run changes of $C$ and $L$ given by eqs. (102a) and (102b), $\chi^{C}$ is given by:

$$
\begin{equation*}
\chi^{C}=P_{C} \tilde{C}\left\{1-\frac{\sigma_{C}}{\left(1+\tau^{C}\right)} \tilde{\xi}\left[\tau^{C}+\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right)\right]\right\} \tag{134}
\end{equation*}
$$

where $0<\tilde{\xi}<1$ is given by (108c). As long as tax rates take reasonable values and the intertemporal elasticity of substitution for consumption (i.e., $\sigma_{C}$ ) is smaller than one, $\chi^{C}$ is positive. Hence, a rise in consumption tax leads to a rise in tax revenue, and more so the smaller $\sigma_{C}$ and the lower the tax rates.

As long as $\sigma_{L}, \sigma_{C}, \tau^{H}, \tau^{F}, \tau^{C}$ take reasonable values, $\chi^{F}$ and $\chi^{C}$ are positive. Hence, according to (132), a labor tax cut leads to a rise in the consumption tax. As shown in Panel B of Table 1, a labor tax cut induces a rise in the consumption tax for all scenarios.

Since the consumption tax increases, we cannot exclude that macroeconomic variables decline after a shift of the tax burden from labor to consumption. We show below that changes in aggregates following a tax reform are simply a scaled-down version of the changes after a labor tax cut associated with a fall in lump-sum transfer. The long-run change of a shift of the tax burden from labor to consumption is equal to the sum of the impact of the labor tax cut by $\mathrm{d} \tau^{F}<0$ financed by a lump-sum transfer (i.e., $\frac{\partial \tilde{X}}{\partial \tau^{F}} \mathrm{~d} \tau^{F}$ ) and the effect triggered by a change in the consumption tax by
$\left.\mathrm{d} \tau^{C}\right|^{j, C}$. If $k^{T}>k^{N}$, the long-run effects are:

$$
\begin{align*}
\left.\mathrm{d} \bar{\lambda}\right|^{F, C} & =\lambda_{\tau^{F}} \mathrm{~d} \tau^{F}+\left.\lambda_{\tau^{C}} \mathrm{~d} \tau^{C}\right|^{F, C} \\
& =\bar{\lambda}\left[\tilde{\xi} \frac{\tilde{\Lambda}}{\left(1+\tau^{F}\right)}+\frac{(1-\tilde{\xi})}{\left(1+\tau^{C}\right)} \frac{\chi^{F}}{\chi^{C}}\right] \mathrm{d} \tau^{F}<0  \tag{135a}\\
\left.\mathrm{~d} \tilde{X}\right|^{F, C} & =\frac{\partial \tilde{X}}{\partial \tau^{F}} \mathrm{~d} \tau^{F}+\left.\frac{\partial \tilde{X}}{\partial \tau^{C}} \mathrm{~d} \tau^{C} \lambda_{\tau^{C}} \mathrm{~d} \tau^{C}\right|^{F, C} \\
& =\frac{\partial \tilde{X}}{\partial \tau^{F}} \frac{P_{C} \tilde{C}}{\chi_{C}}\left[1-\frac{\tilde{W}^{A} \tilde{L}}{P_{C} \tilde{C}\left(1+\tau^{C}\right)}\right] \mathrm{d} \tau^{F}>0 \tag{135b}
\end{align*}
$$

where $\mathrm{d} \tau^{F}<0$ and $\chi^{F}>0, \chi^{C}>0$. To derive (135a), we substituted eqs. (102c) and (104c). To determine (135b), we used the fact that $\frac{\partial \tilde{X}}{\partial \tau^{C}}=\frac{\partial \tilde{X}}{\partial \tau^{F}} \frac{1+\tau^{F}}{1+\tau^{C}} \frac{1}{\bar{\Lambda}}$ and substituted (132), by remembering that $\chi_{C}=\frac{1+\tau^{F}}{1+\tau^{C}} \frac{1}{\tilde{\Lambda}}\left[\chi_{F}-\frac{\tilde{\Lambda}}{1+\tau^{F}} \tilde{W}^{A} \tilde{L}\right]+P_{C} \tilde{C}$.

According to (135a), a labor tax cut induces unambiguously a decline in the marginal utility of wealth since both a fall in $\tau^{F}$ and a rise in $\tau^{C}$ produce a drop in $\bar{\lambda}$. According to eq. (135b), a shift of the tax burden from labor to consumption raises unambiguously macroeconomic aggregates (i.e., $C, L, K, N X)$ as long as $\chi^{C}>0$ and $0<\frac{\tilde{W}^{A} \tilde{L}}{P_{C} \tilde{C}\left(1+\tau^{C}\right)}<1$. The former condition is easily fulfilled if $\sigma_{C}<1$ and tax rates take initially reasonable values. The latter condition is fulfilled as long as $r^{\star} \tilde{A}+Z>0$ since it $\underset{\tilde{A}}{ }$ implies that $\tilde{W}^{A} \tilde{L}<P_{C} \tilde{C}\left(1+\tau^{C}\right)$. Hence, if the representative household is a net creditor (i.e., $\tilde{A}>0$ ), the consumption tax base is higher than the labor tax base that the rise in $\tau^{C}$ necessary to balance the government budget is smaller than the decline in the labor tax cut. Finally, while $\tau^{C}$ increases to balance the government budget following the labor tax cut, the tax reform produces an expansion of macroeconomic aggregates. Hence, the long-run change of $X=C, L, K, N X$ following a tax reform is simply a scaled-down version of the long-term change of the aggregate $X$ after a labor tax cut financed by a fall in lump-sum transfer. More formally, we have:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{X}\right|^{F, C}=\Phi^{F, C} \frac{\partial \tilde{X}}{\partial \tau^{F}} \mathrm{~d} \tau^{F} \tag{136}
\end{equation*}
$$

where $0<\Phi^{F, C}=\frac{P_{C} \tilde{C}}{\chi_{C}}\left[1-\frac{\tilde{W}^{A} \tilde{L}}{P_{C} \tilde{C}\left(1+\tau^{C}\right)}\right]<1$ as long as $\chi^{F}>0, \chi^{C}>0$, and $\tilde{A}>0$. Eq. (136) corresponds to eq. (21) in the text.

## G. 2 Steady-State Changes: A Shift from Progressive Wage Taxes to Consumption Taxes

In this section, we now estimate the long-run effects of a shift from a progressive wage tax $\tau^{H}$ to a consumption tax $\tau^{C}$, which is adjusted accordingly to balance the government budget. Note that the tax scheme is progressive as long as $\kappa>0$ so that $\Lambda<1$. To avoid confusion, we denote by the superscript $\left.\right|^{H, C}$ the effects of a tax reform which involves simultaneously cutting the progressive wage $\operatorname{tax} \tau^{H}$ and increasing the consumption $\operatorname{tax} \tau^{C}$ so as to balance the government budget.

Differentiating (130), keeping $\tau^{F}$ unchanged, yields the change in the consumption tax:

$$
\begin{equation*}
\left.\mathrm{d} \tau^{C}\right|^{H, C}=-\frac{\chi_{H}}{\chi_{C}} \mathrm{~d} \tau^{H}=-\left\{\frac{\tau^{C} P_{C} \frac{\mathrm{~d} \tilde{C}}{\mathrm{~d} \tau^{H}}+\left(\tilde{W}^{F}-\tilde{W}^{A}\right) \frac{\mathrm{d} \tilde{L}}{\mathrm{~d} \tau^{H}}+(\tilde{W}-\kappa) \tilde{L}}{\tau^{C} P_{C} \frac{\mathrm{~d} \tilde{C}}{\mathrm{~d} \tau^{C}}+\left(\tilde{W}^{F}-\tilde{W}^{A}\right) \frac{\mathrm{d} \tilde{L}}{\mathrm{~d} \tau^{C}}+P_{C} \tilde{C}}\right\} \mathrm{d} \tau^{H} . \tag{137}
\end{equation*}
$$

where $\chi^{C}$ is given by (134) and $\chi_{H}$ can be written as follows:

$$
\begin{equation*}
\chi^{H}=(\tilde{W}-\kappa)\left\{1-\sigma_{L} \frac{\tilde{W}^{F}}{\tilde{W}^{A}}(1-\tilde{\xi})\left[\tau^{C}+\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right)\right]\right\} \tag{138}
\end{equation*}
$$

where we have substituted the long-run changes of $L$ and $C$ (see eqs (106a) and (106b)) to derive (138); $0<\tilde{\xi}<1$ is given by (108c); $\chi^{H}$ reflects two conflictory effects on tax revenues. Whereas a labor tax cut lowers tax revenue, keeping unchanged consumption and employment, a labor tax cut raises employment and consumption, and thereby tax revenues. As for $\chi^{F}$, as long as tax rates take reasonable values and the elasticity of labor supply (i.e., $\sigma_{L}$ ) is not too large, $\chi^{H}$ is positive. Hence, a cut in progressive wage taxes leads to a fall in tax revenue, and more so the smaller $\sigma_{L}$ and the lower the tax rates. Since $\chi^{C}$ and $\chi^{H}$ are both positive for reasonable values of parameters (i.e., $\sigma_{C}$ and $\sigma_{L}$ ) and tax rates, the consumption tax must increase to balance the budget following a cut in progressive wage taxes.

Since the consumption tax increases, we cannot exclude that macroeconomic variables decline after a shift of the tax burden from labor to consumption. We show below that changes in aggregates following a tax reform are simply a scaled-down version of the changes after a labor tax cut associated with a fall in lump-sum transfer. We denote by $X$ the aggregates $C, L, K, N X$. The long-run effect of a shift of the tax burden from labor to consumption is equal to the sum of the impact of the labor tax cut by $\mathrm{d} \tau^{H}<0$ financed by a lump-sum transfer (i.e., $\frac{\partial \tilde{X}}{\partial \tau^{H}} \mathrm{~d} \tau^{H}$ ) and the effect triggered by a change in the consumption tax by $\left.\mathrm{d} \tau^{C}\right|^{j, C}$. If $k^{T}>k^{N}$, the long-run effects are:

$$
\begin{align*}
\left.\mathrm{d} \bar{\lambda}\right|^{H, C} & =\lambda_{\tau^{H}} \mathrm{~d} \tau^{H}+\left.\lambda_{\tau^{C}} \mathrm{~d} \tau^{C}\right|^{H, c} \\
& =\bar{\lambda}\left[\tilde{\xi}\left(\frac{\tilde{W}-\kappa}{\tilde{W}^{A}}\right)+\frac{(1-\tilde{\xi})}{\left(1+\tau^{C}\right)} \frac{\chi^{H}}{\chi^{C}}\right] \mathrm{d} \tau^{H}<0  \tag{139a}\\
\left.\mathrm{~d} \tilde{X}\right|^{H, C} & =\frac{\partial \tilde{X}}{\partial \tau^{H}} \mathrm{~d} \tau^{H}+\left.\frac{\partial \tilde{X}}{\partial \tau^{C}} \mathrm{~d} \tau^{C}\right|^{H, c} \\
& =\frac{\partial \tilde{X}}{\partial \tau^{H}} \frac{P_{C} \tilde{C}}{\chi_{C}}\left[1-\frac{\tilde{W}^{A} \tilde{L}}{P_{C} \tilde{C}\left(1+\tau^{C}\right)}\right] \mathrm{d} \tau^{H} \tag{139b}
\end{align*}
$$

where $\mathrm{d} \tau^{H}<0$ and $\chi^{H}>0, \chi^{C}>0$. To derive (139a), we substituted eqs. (102c) and (106c). To determine (139b), we used the fact that $\frac{\partial \tilde{X}}{\partial \tau^{C}}=\frac{\partial \tilde{X}}{\partial \tau^{H}} \frac{1}{1+\tau^{C}}\left(\frac{\tilde{W}^{A}}{\tilde{W}-\kappa}\right)$ and substituted (137), by remembering that $\chi_{C}=\frac{1}{1+\tau^{C}}\left(\frac{\tilde{W}^{A}}{\tilde{W}-\kappa}\right)\left[\chi_{H}-(\tilde{W}-\kappa) \tilde{L}\right]+P_{C} \tilde{C}$.

According to (139a), a cut in progressive wage taxes induces unambiguously a decline in the marginal utility of wealth since both a fall in $\tau^{H}$ and a rise in $\tau^{C}$ produce a drop in $\bar{\lambda}$. According to eq. (139b), a shift of the tax burden from labor to consumption raises unambiguously macroeconomic aggregates (i.e., $C, L, K, N X)$ as long as $\chi^{C}>0$ and $0<\frac{\tilde{W}^{A} \tilde{L}}{P_{C} \tilde{C}\left(1+\tau^{C}\right)}<1$. As stressed previously, the former condition is fulfilled if $\sigma_{C}<1$ and tax rates take initially reasonable values. The latter condition is fulfilled as long as $r^{\star} \tilde{A}+Z>0$ since it implies that $\tilde{W}^{A} \tilde{L}<P_{C} \tilde{C}\left(1+\tau^{C}\right)$. Hence, if the representative household is a net creditor (i.e., $\tilde{A}>0$ ), the consumption tax base is higher than the labor tax base so that the rise in $\tau^{C}$ necessary to balance the government budget is smaller than the decline in the labor tax cut. Finally, while $\tau^{C}$ increases to balance the government budget following the labor tax cut, the tax reform produces an expansion of macroeconomic aggregates. Hence, the long-run change of $X=C, L, K, N X$ following a tax reform is simply a scaled-down version of the long-term change of the aggregate $X$ after a labor tax cut financed by a fall in lump-sum transfer. More formally, we have:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{X}\right|^{H, C}=\Phi^{H, C} \frac{\partial \tilde{X}}{\partial \tau^{H}} \mathrm{~d} \tau^{H} \tag{140}
\end{equation*}
$$

where $\Phi^{H, C}<1$ as long as $\chi^{H}>0, \chi^{C}>0$, and $\tilde{A}>0$.

## G. 3 Derivation of Formal Expressions in the Text

Having determined the long-run effects of a revenue-neutral tax reform which involves a tax cut in payroll taxes or in progressive wage taxes coordinated with a rise in the consumption tax so as to balance the government budget, we present below the main steps to derive expressions in the text.

We denote by the superscript $\left.\right|^{j, C}$ the effects of a fall in the labor tax by $\mathrm{d} \tau^{j}<0(j=F, H)$ coordinated with a rise in the consumption tax rate by $\left.\mathrm{d} \tau^{C}\right|^{j, C}$ which is endogenously determined so as the government budget constraint is met. Assuming that the stock of financial wealth plus transfers is positive, the labor tax base is smaller than the consumption tax base. Hence, $\tau^{C}$ must increase less than the drop in labor tax to balance the budget. As a result, denoting by $X$ the macroeconomic aggregates $C, L, K, N X$, the long-term effect of a tax reform is simply a scaleddown version of the long-term change in the aggregate $X$ after a labor tax cut financed by a fall in lump-sum transfer. This point is formalized below.

To start with, we rewrite the change in the consumption tax by denoting $\Gamma^{C}=\chi^{C}\left(1+\tau^{C}\right)$ and $\Gamma^{j}=\left(1 \pm \tau^{j}\right)$ :

$$
\begin{equation*}
\left.\hat{\tau}^{C}\right|^{j, C}=-\frac{\Gamma^{j}}{\Gamma^{C}}=-\frac{\Lambda^{j} \tilde{W}^{A} \tilde{L}\left\{1-\sigma_{L} \frac{\tilde{W}^{F}}{\tilde{W}^{A}}(1-\tilde{\xi})\left[\tau^{C}+\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right)\right]\right\}}{P_{C} \tilde{C}\left(1+\tau^{C}\right)\left\{1-\frac{\sigma_{C}}{\left(1+\tau^{C}\right)} \tilde{\xi}\left[\tau^{C}+\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right)\right]\right\}} \hat{\tau}^{j}>0 \tag{141}
\end{equation*}
$$

where $\hat{\tau}^{F}=\frac{\mathrm{d} \tau^{F}}{1+\tau^{F}}, \hat{\tau}^{H}=\frac{\mathrm{d} \tau^{H}}{1-\tau^{H}}$, and we set $0<\Lambda^{F} \equiv \frac{\left(1-\tau^{H}\right) \tilde{W}}{\tilde{W}^{A}}<1$, and $0<\Lambda^{H} \equiv 1-\frac{\kappa}{\tilde{W}^{A}}<1$. Eq. (141) corresponds to eq. (19) in the text.

The long-term change in the aggregate $X=C, L, K, N X$ following a shift of the tax burden from labor to consumption is equal to the sum of the expansionary impact of the labor tax cut by $\mathrm{d} \tau^{j}<0(j=F, H)$ financed by a lump-sum transfer (i.e., $\left.\frac{\partial \tilde{X}}{\partial \tau^{j}} \mathrm{~d} \tau^{j}>0\right)$ and the recessionary effect triggered by the rise in the consumption tax by $\left.\mathrm{d} \tau^{C}\right|^{j, C}$ (i.e., $\left.\frac{\partial \tilde{X}}{\partial \tau^{C}} \mathrm{~d} \tau^{C}\right|^{j, C}<0$ ). Hence, we have:

$$
\left.\hat{\tilde{X}}\right|^{j, C}=\frac{\hat{\tilde{X}}}{\hat{\tau}^{j}} \hat{\tau}^{j}+\left.\frac{\hat{\tilde{X}}}{\hat{\tau}^{C}} \hat{\tau}^{C}\right|^{j, C},
$$

Using the fact that $\frac{\hat{\hat{X}}}{\hat{\tau}^{C}}=\frac{1}{\Lambda^{j}} \hat{\tilde{X}} \hat{\tau}^{j}$ and substituting (141), keeping in mind that $\Gamma^{C}=\left[\frac{\Gamma^{j}}{\Lambda^{j}}-\tilde{W}^{A} \tilde{L}\right]+$ $P_{C} \tilde{C}\left(1+\tau^{C}\right)$, yields:

$$
\begin{equation*}
\left.\hat{\tilde{X}}\right|^{j, C}=\Phi^{j, C} \frac{\hat{\tilde{X}}}{\hat{\tau}^{j}} \hat{\tau}^{j}>0, \quad j=F, H \tag{142}
\end{equation*}
$$

where $0<\Phi^{j, C}=\frac{P_{C} \tilde{C}}{\chi_{C}}\left[1-\frac{\tilde{W}^{A} \tilde{L}}{P_{C} \tilde{C}\left(1+\tau^{C}\right)}\right]<1$. Eq. (142) corresponds to eq. (21) in the text.
Denoting by $0<\tilde{\xi} \equiv \frac{\sigma_{L} \tilde{W}^{F} \tilde{L}}{\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}}<1$, and denoting by a hat the percentage deviation of the aggregate from its initial steady-state value, (135a) and (139a) can be reduced to:

$$
\begin{equation*}
\left.\hat{\bar{\lambda}}\right|^{j, C}=\tilde{\xi} \Lambda^{j} \hat{\tau}^{j}-\left.(1-\tilde{\xi}) \hat{\tau}^{C}\right|^{j, C}<0, \quad j=F, H . \tag{143}
\end{equation*}
$$

where $\left.\hat{\tau}^{C}\right|^{j, C}=-\frac{\chi^{j}}{\chi^{C}} \hat{\tau}^{j}>0$. Eq. (143) corresponds to eq. (20) in the text.
Combining (110) and (142) yields the long-run change in labor following a revenue-neutral tax reform:

$$
\begin{equation*}
\left.\hat{\tilde{L}}\right|^{j, C}=-\Phi^{j, C} \sigma_{L} \Lambda^{j}(1-\tilde{\xi}) \hat{\tau}^{j}>0, \quad j=F, H \tag{144}
\end{equation*}
$$

Eq. (144) corresponds to eq. (22) in the text.
Combining (111) and (142) yields the long-run change in the capital stock following a revenueneutral tax reform:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{K}\right|^{j, C}=\Phi^{j, C} \frac{\sigma_{L} \tilde{L}}{\nu_{1}} \Lambda^{j}(1-\tilde{\xi})\left[\alpha_{C} \tilde{k}^{N} \nu_{2}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{1}\right] \hat{\tau}^{j}>0, \quad j=F, H \tag{145}
\end{equation*}
$$

Eq. (145) corresponds to eq. (23) in the text.
Combining (112) and (142) yields the long-run change in GDP following a revenue-neutral tax reform:

$$
\begin{equation*}
\left.\hat{\tilde{Y}}\right|^{j, C}=\left.\left(1-\tilde{\beta}_{L}\right) \hat{\tilde{K}}\right|^{j, C}+\left.\tilde{\beta}_{L} \hat{\tilde{L}}\right|^{j, C}>0 \tag{146}
\end{equation*}
$$

Eq. (146) corresponds to eq. (24) in the text. To derive (146), we used the fact that $Y^{T} \equiv$ $Y^{T}(K, L, P)$ and $Y^{N} \equiv Y^{N}(K, L, P)$; differentiating $Y=Y^{T}+(P / \mu) Y^{N}$ yields: d $\left.\tilde{Y}\right|^{j, C}=$ $\left.\left(Y_{K}^{T}+\frac{\tilde{P}}{\mu} Y_{K}^{N}\right) \mathrm{d} \tilde{K}\right|^{j, C}+\left.\left(Y_{L}^{T}+\frac{\tilde{P}}{\mu} Y_{L}^{N}\right) \mathrm{d} \tilde{L}\right|^{j, C}$ where $Y_{K}^{T}+\frac{\tilde{P}}{\mu} Y_{K}^{N}=\tilde{P} r^{\star}$ and $Y_{L}^{T}+\frac{\tilde{P}}{\mu} Y_{L}^{N}=\tilde{W}^{F}$.

## H Tax Wedge: Some Definitions

In line with general practice, payroll taxes are assumed to be proportional and wage income taxes are taken to be progressive. Following Heijdra and Lightart [2009], we define the average tax wedge as the difference between the producer wage (paid by the firm) and the purchasing power on consumption goods of after-tax average wage expressed as a percentage of the wage including payroll taxes:

$$
\begin{align*}
\tau^{A} & \equiv \frac{W L\left(1+\tau^{F}\right)-\left[\left(1-\tau^{H}\right) W+\tau^{H} \kappa\right] L}{W^{F} L} \\
& \equiv 1-\frac{\left[\left(1-\tau^{H}\right)+\frac{\tau^{H} \kappa}{W}\right]}{\left(1+\tau^{F}\right)} \tag{147}
\end{align*}
$$

where $W^{F}=W\left(1+\tau^{F}\right)$. In addition, we denote by $\tau^{M}$ the marginal tax wedge as the difference between the producer wage (paid by the firm) and the after-tax marginal wage expressed as a percentage of the producer cost (i. e. including payroll taxes):

$$
\begin{align*}
\tau^{M} & \equiv \frac{W L\left(1+\tau^{F}\right)-W L\left(1-\tau^{H}\right)}{W^{F} L} \\
& \equiv 1-\frac{\left(1-\tau^{H}\right)}{\left(1+\tau^{F}\right)} \tag{148}
\end{align*}
$$

The closer to unity $\tau^{M}$, the larger the gap between the wage paid by firms and the real wage received by households.

Using the definition of $\tau^{M}$ given by (148), we can rewrite the average tax wedge as follows:

$$
\begin{equation*}
\tau^{A} \equiv \tau^{M}-\frac{\tau^{H} \kappa}{W^{F}} \tag{149}
\end{equation*}
$$

Finally, we provide a measure of the degree of tax progressiveness by the means of the coefficient of average tax progression:

$$
\begin{equation*}
\Gamma\left(\tau^{F}, \tau^{H}, \kappa, P\right) \equiv \tau^{M}-\tau^{A}=\frac{\tau^{H} \kappa}{X^{F}} \tag{150}
\end{equation*}
$$

where $W^{F}=W\left(1+\tau^{F}\right)$ with $W=w\left(\tau^{F}, P\right)$.
As the average tax burden $\tau^{A}$ rises with the wage rate, the system tax is progressive such that $\Gamma()>$.0 which holds as long as $\kappa>0$. It is worth emphasizing that our approach which defines the average tax together with the marginal tax wedge by taking into account the wage paid by the firm allows for "scaling" the tax burden faced by households in terms of firms' labor cost, the index of average tax progression being expressed in terms of consumption goods; that's why we use the "wedge" label. By abstracting from this "scaling" approach, we would define the marginal and average tax wedges together with the coefficient of average tax progression as follows : $\tau^{M} \equiv \tau^{H} w$, $\tau^{A} \equiv \tau^{H}(W-\kappa)$ and $\Gamma \equiv \tau^{H} \kappa>0$ (as long as $\kappa>0$ ).

## I A Tax reform Keeping Unchanged the Marginal Tax Wedge

In this section, we consider a labor tax strategy which involves simultaneously cutting a payroll tax by $\mathrm{d} \tau^{F}<0$ and increasing a progressive wage tax by $\mathrm{d} \tau^{H}>0$ so as to leave unchanged the marginal tax wedge, i. e. $\mathrm{d} \tau^{M}=0$. By making use of (148), the labor tax reform strategy requires a rise in the wage income tax by the following amount:

$$
\begin{equation*}
\left.\mathrm{d} \tau^{H}\right|^{F, H} \equiv-\theta \mathrm{d} \tau^{F}, \quad \theta \equiv \frac{1-\tau^{H}}{1+\tau^{F}}<1 . \tag{151}
\end{equation*}
$$

Eq. (151) corresponds to eq. (27) in the text. According to eq. (151), the progressive wage tax must be increased by a smaller amount than the fall in $\tau^{F}$ so as to leave unchanged the marginal tax wedge.

Substituting the short-run static solution for the wage rate (47) that holds in the long-run, and differentiating the coefficient of average tax progression (150) w. r. t. $\tau^{H}$ and $\tau^{F}$, and then using (151), we find that the tax reform raises the degree of average tax progression:

$$
\begin{equation*}
\mathrm{d} \Gamma=-\frac{\kappa}{W^{F}} \theta \mathrm{~d} \tau^{F}>0 \tag{152}
\end{equation*}
$$

where $\mathrm{d} \tau^{F}<0$ since we considered a fall in payroll taxes. The explanation comes from the fact that the wage rate is raised by the same proportion than the fall in $\tau^{F}$. Consequently, as long as $\kappa>0$, the rise in $\tau^{H}$ leads to an increase in $\Gamma$.

Making use of long-term effects of permanent changes in $\tau^{F}$ and $\tau^{H}$ and substituting $\left.\mathrm{d} \tau^{H}\right|^{F, H}$ given by (151), we are able to estimate the directions and the sizes of the long-run changes of main economic variables after a fall in $\tau^{F}$ associated with a rise in $\tau^{H}$ by an amount that leaves unaffected $\tau^{M}$. We derive below the steady-state effects of a tax reform keeping unchanged the marginal tax wedge by assuming that the traded sector is more capital intensive than the non traded sector.

If $k^{T}>k^{N}$, denoting macroeconomic aggregates $C, L, K, N X$ by $X$, and denoting by a hat the percentage deviation from initial steady-state, steady-state changes are given by:

$$
\begin{align*}
\left.\hat{\bar{\lambda}}\right|^{F, H} & =\frac{\hat{\bar{\lambda}}}{\hat{\tau}^{F}} \hat{\tau}^{F}+\left.\frac{\hat{\bar{\lambda}}}{\hat{\tau}^{H}} \hat{\tau}^{H}\right|^{F, H}, \\
& =\tilde{\xi}\left(\Lambda^{F}-\Lambda^{H}\right) \hat{\tau}^{F} \lessgtr 0, \\
& =\tilde{\xi} \Lambda^{F} \frac{\kappa}{\tilde{W}} \hat{\tau}^{F}<0,  \tag{153a}\\
\left.\hat{\tilde{X}}\right|^{F, H} & =\frac{\hat{\tilde{X}}}{\hat{\tau}^{F}} \hat{\tau}^{F}+\left.\frac{\hat{\tilde{X}}}{\hat{\tau}^{H}} \hat{\tau}^{H}\right|^{F, H}, \\
& =\frac{\hat{\tilde{X}}}{\hat{\tau}^{F}}\left(\frac{\Lambda^{F}-\Lambda^{H}}{\Lambda^{F}}\right) \hat{\tau}^{F},  \tag{153b}\\
& =\frac{\hat{\tilde{X}}}{\hat{\tau}^{F}} \frac{\kappa}{\tilde{W}} \hat{\tau}^{F}, \tag{153c}
\end{align*}
$$

where $\hat{\tau}^{F}<0$ and $\tilde{\xi}, \Lambda^{F}$ and $\Lambda^{H}$ are given by (108); to get (153b), we used the fact that $\frac{\hat{X}}{\hat{\tau}^{H}}=\frac{\Lambda^{H}}{\Lambda^{F}} \frac{\hat{\hat{X}}}{\hat{\tau}^{F}}$ and $\left.\hat{\tau}^{H}\right|^{F, H}=-\hat{\tau}^{F}$; we used the fact that $\frac{\Lambda^{F}-\Lambda^{H}}{\Lambda^{F}}=\frac{\kappa}{\tilde{W}}$ to get (153c). Eq. (153c) corresponds to eq. (28) in the text.

## J Dynamic Effects of a Tax Reform

This section estimates the dynamic effects of a tax restructuring. Steady-state changes are those derived in the previous section where we estimated the long-run variations such that the rise in $\tau^{C}$ guarantees that the balanced condition for the government holds. Note that the change of the tax scheme can be viewed as an unanticipated permanent tax shock.

The stable adjustment of the economy is described by a saddle-path in $(K, P)$-space. The capital stock, the real exchange rate, and the stock of traded bonds evolve according to:

$$
\begin{align*}
K(t) & =\tilde{K}+B_{1} e^{\nu_{1} t}  \tag{154a}\\
P(t) & =\tilde{P}+\omega_{2}^{1} B_{1} e^{\nu_{1} t}  \tag{154b}\\
B(t) & =\tilde{B}+\Phi_{1} B_{1} e^{\nu_{1} t} \tag{154c}
\end{align*}
$$

where $\omega_{2}^{1}=0, \Phi_{1}=-\tilde{P}$ if $k^{T}>k^{N}$ and with

$$
B_{1}=K_{0}-\tilde{K}=-\left.\mathrm{d} \tilde{K}\right|^{j, k}=-\Phi^{j, k} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}} \mathrm{~d} \tau^{j}
$$

where we made use of the constancy of $K$ at time $t=0$ (i. e. $K_{0}$ is predetermined).
In section $F$, we show that steady-state values of macroeconomic aggregates can be expressed as function of the marginal utility of wealth and tax rates. Using (142) and totally differentiating eq. (117b) yields the steady-state change of the capital stock after a tax shock:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{K}\right|^{j, k}=\Phi^{j, k}\left(K_{\bar{\lambda}} \lambda_{\tau^{j}}+K_{\tau^{j}}\right) \mathrm{d} \tau^{j}, \tag{155}
\end{equation*}
$$

where the first term on the RHS reflects the impact of the wealth effect on steady-state capital stock while the second term represents the influence of the tax effect.

## J. 1 Investment and Current Account

We derive the initial reactions of investment and the current account by abstracting from depreciation of physical capital to avoid uninteresting complications. As previously, we consider that $k^{T}>k^{N}$ to avoid uninteresting complications.

Differentiating (154a) w.r.t. time, evaluating at time $t=0$, and substituting (155), the initial response of investment is:

$$
\begin{align*}
\left.I(0)\right|^{j, k} & =-\nu_{1} \Phi^{j, k}\left(K_{\bar{\lambda}} \lambda_{\tau^{j}}+K_{\tau^{j}}\right) \mathrm{d} \tau^{j} \\
& =\Phi^{j, k}\left[\Lambda^{j} \tilde{\xi}\left(\sigma_{C} \tilde{C}^{N}-\nu_{1} \sigma_{L} \tilde{L} \tilde{k}^{T}\right)+\nu_{1} \sigma_{L} \tilde{L} \tilde{k}^{T}\right] \hat{\tau}^{j} \gtrless 0, \quad j=F, H \tag{156}
\end{align*}
$$

where $\hat{\tau}^{F}=\frac{\mathrm{d} \tau^{F}}{1+\tau^{F}}, \hat{\tau}^{H}=\frac{\mathrm{d} \tau^{H}}{1+\tau^{H}}, 0<\Lambda^{F} \equiv \frac{\left(1-\tau^{H}\right) \tilde{W}}{\tilde{W}^{A}}<1,0<\Lambda^{H} \equiv 1-\frac{\kappa}{\tilde{W}^{A}}<1,0<\tilde{\xi} \equiv$ $\frac{\sigma_{L} \tilde{W}^{F} \tilde{L}}{\sigma_{L} \tilde{W}^{F} \tilde{L}+\sigma_{C} P_{C} \tilde{C}}<1$. Eq. (156) corresponds to eq. (25) in the text.

By using the steady-state change of the capital stock after a labor tax cut given by (104d), we are able to sign eq. (156):

$$
\begin{align*}
\left.I(0)\right|^{j, k} & =-\Phi^{j, k} \nu_{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}} \mathrm{~d} \tau^{j} \\
& =-\Phi^{j, k} \sigma_{L} \tilde{L}(1-\tilde{\xi}) \Lambda^{j}\left[\alpha_{C} \tilde{k}^{N} \nu_{2}-\left(1-\alpha_{C}\right) \tilde{k}^{T} \nu_{1}\right] \hat{\tau}^{j},>0, \quad j=F, H \tag{157}
\end{align*}
$$

where $0 \leq \Lambda^{j} \leq 1$ and $0<\tilde{\xi}<1$.
Differentiating the stable solution for foreign bonds (154c) w.r.t. time, evaluating at time $t=0$, the initial response of the current account is:

$$
\begin{equation*}
\left.\left.C A(0)\right|^{j, k} \equiv \dot{B}(0)\right|^{j, k}=-\left.\tilde{P} I(0)\right|^{j, k}<0, \quad j=F, H \tag{158}
\end{equation*}
$$

where we used the fact that $\Phi_{1}=-\tilde{P}$.

## J. 2 Labor Share

Before analyzing the short-run distribution effects of a labor tax cut, we investigate the steady-state change of the labor share. The share of labor income in GDP is defined as follows:

$$
\begin{equation*}
\tilde{\beta}_{L}=\frac{\tilde{W}^{F} \tilde{L}}{\tilde{W}^{F} \tilde{L}^{2}+\tilde{R}^{K} \tilde{K}}=\frac{\tilde{\omega}}{\tilde{\omega}+\tilde{k}} \tag{159}
\end{equation*}
$$

where $\tilde{R}^{K}=\tilde{P} r^{\star}$; we denoted by $\tilde{\omega}=\tilde{W}^{F} / \tilde{R}^{K}$ the wage-interest ratio and by $\tilde{k}=\tilde{K} / \tilde{L}$ the capitallabor ratio. As long as the relative price of non tradables $\tilde{P}$ is unaffected by the tax shock, the wage-interest ratio $\tilde{\omega}$ remains unaffected. Hence, the labor share movement is driven only by the capital-labor ratio. Eq. (159) corresponds to eq. (30) in the text.

Differentiating eq. (159), we have:

$$
\begin{equation*}
\mathrm{d} \tilde{\beta}_{L}=\tilde{\beta}_{L}\left(1-\tilde{\beta}_{L}\right)\left(\frac{\mathrm{d} \tilde{L}}{\tilde{L}}-\frac{\mathrm{d} \tilde{K}}{\tilde{K}}\right) . \tag{160}
\end{equation*}
$$

To analyze the distributions effects of a tax reform in the short-run, we have to linearize $\beta_{L}=$ $\frac{W(P)\left(1+\tau^{F}\right) L(P)}{Y(K, L)}$ in the neighborhood of the steady-state: $5^{50}$

$$
\begin{aligned}
\beta_{L}(t)-\tilde{\beta}_{L} & =\tilde{\beta}_{L}\left[\left(\frac{W_{P} \tilde{P}}{\tilde{W}}+\frac{L_{P} \tilde{P}}{\tilde{L}}-\frac{Y_{P} \tilde{P}}{\tilde{Y}}\right)-\frac{Y_{K} \tilde{K}}{\tilde{K}}\right] \\
& =-\tilde{\beta}_{L}\left\{\frac{\tilde{P} \tilde{h}^{T}}{\tilde{W}^{F} \mu\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}\left[1+\sigma_{L} \Lambda^{j}\left(1-\tilde{\beta}_{L}\right)\right] \frac{\omega_{2}^{1}}{\tilde{P}}+\left(1-\beta_{L}\right)\right\}\left(\frac{K(t)-\tilde{K}}{\tilde{K}}() 61\right)
\end{aligned}
$$

where we substituted $P(t)-\tilde{P}=\omega_{2}^{1}(K(t)-\tilde{K})$.
If $k^{T}>k^{N}$, then $\omega_{2}^{1}=0$. Hence, evaluating (161) at time $t=0$ and differentiating yields:

$$
\begin{equation*}
\mathrm{d} \beta_{L}(0)=\mathrm{d} \tilde{\beta}_{L}+\tilde{\beta}_{L}\left(1-\tilde{\beta}_{L}\right) \frac{\mathrm{d} \tilde{K}}{\tilde{K}}=\tilde{\beta}_{L}\left(1-\tilde{\beta}_{L}\right) \frac{\mathrm{d} \tilde{L}}{\tilde{L}}>0, \tag{162}
\end{equation*}
$$

where we used the fact that $K$ is initially predetermined. Hence, a labor tax cut unambiguously raises the labor share $\beta_{L}$ on impact. Eq. (162) corresponds to eq. (31a) in the text.

Differentiating (161) w.r.t. time, we find that capital accumulation lowers unambiguously the labor share along the transitional path:

$$
\begin{equation*}
\dot{\beta}_{L}(t)=-\tilde{\beta}_{L}\left(1-\tilde{\beta}_{L}\right) \frac{\dot{K}(t)}{\tilde{K}}<0 \tag{163}
\end{equation*}
$$

If $k^{N}>k^{T}$, the initial response of the labor share to a labor tax cut is given by:

$$
\begin{align*}
\mathrm{d} \beta_{L}(0) & =\mathrm{d} \tilde{\beta}_{L}+\tilde{\beta}_{L}\left\{\frac{\tilde{P} \tilde{h}^{T} \tilde{k}^{T}}{\tilde{W}^{F} \mu\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}\left[1+\sigma_{L} \Lambda^{j}\left(1-\tilde{\beta}_{L}\right)\right] \frac{\omega_{2}^{1}}{\tilde{P}}+\left(1-\beta_{L}\right)\right\} \frac{\mathrm{d} \tilde{K}}{\tilde{K}} \\
& =\tilde{\beta}_{L}\left(1-\tilde{\beta}_{L}\right) \frac{\mathrm{d} \tilde{L}}{\tilde{L}}+\left\{\frac{\tilde{h} \tilde{k}^{T}}{\tilde{W}^{F} \mu\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}\left[1+\sigma_{L} \Lambda^{j}\left(1-\tilde{\beta}_{L}\right)\right] \omega_{2}^{1}\right\} \frac{\mathrm{d} \tilde{K}}{\tilde{K}} \gtrless 0, \tag{164}
\end{align*}
$$

where the sign of (164) is ambiguous. If $k^{N}>k^{T}$, the first term on the RHS is positive, i.e., the rise in labor supply exerts a positive impact on the labor share, while the second term is negative, i.e., the initial appreciation in the relative price of non tradables exerts a negative impact on $\beta_{L}$. Eq. (164) corresponds to eq. (31b) in the text.

## K Tax Multipliers

In this section, we derive analytical expressions of tax multipliers for overall and sectoral output.

[^24]
## K. 1 Tax Multiplier for Overall Output

## Long-Run Tax Multiplier

Because overall output denoted by $Y$ is the sum of traded output $Y^{T}$ and non traded output measured in terms of the traded good $\frac{P}{\mu} Y^{N}$, using the fact that $Y^{T} \equiv Y^{T}(K, L, p)$ and $Y^{N} \equiv$ $Y^{N}(K, L, p)$, remembering that steady-state level of the real exchange rate is unaffected by a tax reform, the steady-state change of overall output is:

$$
\begin{align*}
\left.\mathrm{d} \tilde{Y}\right|^{j, k} & =\left.\left(Y_{K}^{T}+\frac{\tilde{P}}{\mu} Y_{K}^{N}\right) \mathrm{d} \tilde{K}\right|^{j, k}+\left.\left(Y_{L}^{T}+\frac{\tilde{P}}{\mu} Y_{L}^{N}\right) \mathrm{d} \tilde{L}\right|^{j, k} \\
& =\left.\tilde{P} r^{\star} \mathrm{d} \tilde{K}\right|^{j, k}+\left.W^{F} \mathrm{~d} \tilde{L}\right|^{j, k}>0 \tag{165}
\end{align*}
$$

where we use properties (54b) and (54c) to get (165).

## Initial Tax Multiplier

Adopting a similar procedure keeping in mind that the capital stock is initially predetermined, the short-run tax multiplier is:

$$
\begin{align*}
\left.\mathrm{d} Y(0)\right|^{j, k} & =\left.\left(Y_{L}^{T}+\frac{P}{\mu} Y_{L}^{N}\right) \mathrm{d} L(0)\right|^{j, k}+\left.\left(\hat{Y}_{P}^{T}+\frac{P}{\mu} \hat{Y}_{P}^{N}\right) \mathrm{d} P(0)\right|^{j, k} \\
& =\left.W^{F} \mathrm{~d} L(0)\right|^{j, k}>0 \tag{166}
\end{align*}
$$

where we use properties (54c) to get (166); according to property (54a), denoting by a hat the partial derivative of $Y$ w. r. t. $P$ for given labor, $\hat{Y}_{P}^{T}+\frac{P}{\mu} \hat{Y}_{P}^{N}=0$;

## K. 2 Tax Multipliers for Sectoral Outputs

## Long-Run Tax Multipliers

$k^{N}>k^{T}$
We calculate the tax multiplier for traded output by differentiating the short-run static solution for $Y^{T}$ evaluated at the steady-state:

$$
\begin{align*}
\left.\mathrm{d} \tilde{Y}^{T}\right|^{j, k} & =\left.Y_{K}^{T} \mathrm{~d} \tilde{K}\right|^{j, k}+\left.Y_{L}^{T} \mathrm{~d} \tilde{L}\right|^{j, k}=\Phi^{j, k}\left[Y_{K}^{T} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}+Y_{L}^{T} \frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}\right] \mathrm{d} \tau^{F} \\
& =\frac{\nu_{1}}{\nu_{2}} \frac{\sigma_{L} \tilde{L}}{\Delta} \sigma_{C} P_{C} \tilde{C} \Upsilon^{j}\left\{\left[\left(1-\alpha_{C}\right) \tilde{W}^{F}+r^{\star} \tilde{P} \tilde{k}^{N}\right]\right. \\
& \left.+\alpha_{C} \tilde{k}^{N} r^{\star} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right)\right\} \Phi \mathrm{d} \tau^{j}>0, \quad j=F, H \tag{167}
\end{align*}
$$

where $\mathrm{d} \tau^{j}<0$ since we consider a fall in labor $\operatorname{tax} \tau^{j}, \Phi^{j, c}=\frac{P_{C} \tilde{C}}{\chi C}\left[1-\frac{\tilde{W}^{A} \tilde{L}}{P_{C}\left(1+\tau^{C}\right) \tilde{C}}\right]>0$ (with $j=F, H)$ as long as $\tilde{a}>0$, and $\Phi^{F, H}=\frac{\tilde{W}}{\kappa}>0$. To derive (167), we used the fact that $Y_{L}^{T}=$ $\left.-\tilde{P} \nu_{1} \tilde{k}^{N}>0, Y_{K}^{T}=\tilde{P} \nu_{1}<0,\left(\nu_{2} \tilde{k}^{T}+\nu_{1} \tilde{k}^{N}\right)^{\kappa}\right)=-\frac{\tilde{W}^{F}}{\tilde{P}}<0$ and $\nu_{1}+\nu_{2}=r^{\star}$.

We calculate the tax multiplier for non-traded output by differentiating the short-run static solution for $Y^{N} / \mu$ evaluated at the steady-state:

$$
\begin{align*}
\left.\frac{\tilde{P}}{\mu} \mathrm{~d} \tilde{Y}^{N}\right|^{j, k} & =\left.\frac{\tilde{P}}{\mu} Y_{K}^{N} \mathrm{~d} \tilde{K}\right|^{j, k}+\left.\frac{\tilde{P}}{\mu} Y_{L}^{N} \mathrm{~d} \tilde{L}\right|^{j, k}=\Phi^{j, k}\left[Y_{K}^{N} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}+\frac{Y_{L}^{N}}{\mu} \frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}\right] \mathrm{d} \tau^{j} \\
& =-\frac{\sigma_{L} \tilde{L}}{\Delta} \sigma_{C} \tilde{C}^{N} \Upsilon^{j}\left\{\tilde{W}^{F}-\tilde{k}^{T} r^{\star} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} k^{T} \nu_{2} \tilde{\Lambda}\right)\right\} \Phi \mathrm{d} \tau^{j}>0 \tag{168}
\end{align*}
$$

where we used the fact that $Y_{K}^{N}=\mu \nu_{2}>0$ and $Y_{L}^{N}=-\tilde{k}^{T} \mu \nu_{2}<0$.
$k^{T}>k^{N}$
We calculate the tax multiplier for traded output by differentiating the short-run static solution for $Y^{T}$ evaluated at the steady-state:

$$
\begin{align*}
\left.\mathrm{d} \tilde{Y}^{T}\right|^{j, k} & =\left.Y_{K}^{T} \mathrm{~d} \tilde{K}\right|^{j, c}+\left.Y_{L}^{T} \mathrm{~d} \tilde{L}\right|^{j, c}=\Phi^{j, k}\left[Y_{K}^{T} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}+Y_{L}^{T} \frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}\right] \mathrm{d} \tau^{F} \\
& =\frac{\nu_{2}}{\nu_{1}} \frac{\sigma_{L} \tilde{L}}{\Delta} \Upsilon^{j} \sigma_{C} P_{C} \tilde{C}\left[\left(1-\alpha_{C}\right) \tilde{W}^{F}+r^{\star} \tilde{P} \tilde{k}^{N}\right] \Phi^{j, k} \mathrm{~d} \tau^{j}>0, \quad j=F, H \tag{169}
\end{align*}
$$

where $\Upsilon^{j}>0$ and $\Phi^{j, k}>0$ (as long as $\tilde{A}>0$. We used the fact that $Y_{L}^{T}=-\tilde{P} \nu_{2} \tilde{k}^{N}<0$, $Y_{K}^{T}=\tilde{P} \nu_{2}>0,\left(\nu_{1} \tilde{k}^{T}+\nu_{2} \tilde{k}^{N}\right)=-\frac{\tilde{W}^{F}}{\tilde{P}}<0$ and $\nu_{1}+\nu_{2}=r^{\star}$ to get (169).

We calculate the tax multiplier for non-traded output by differentiating the short-run static solution for $Y^{N} / \mu$ evaluated at the steady-state:

$$
\begin{align*}
\left.\frac{\tilde{P}}{\mu} \mathrm{~d} \tilde{Y}^{N}\right|^{j, k} & =\frac{\tilde{P}}{\mu} Y_{K}^{N} \mathrm{~d} \tilde{K}^{j, k}+\left.\frac{\tilde{P}}{\mu} Y_{L}^{N} \mathrm{~d} \tilde{L}\right|^{j, k}=\Phi^{j, k}\left[\frac{Y_{K}^{N}}{\mu} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}+\frac{Y_{L}^{N}}{\mu} \frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}\right] \mathrm{d} \tau^{j} \\
& =-\frac{\sigma_{L} \tilde{L}}{\Delta} \Upsilon^{j} \sigma_{C} \tilde{C}^{N} \tilde{W}^{F} \Phi^{j, k} \mathrm{~d} \tau^{j}>0, \quad j=F, H \tag{170}
\end{align*}
$$

where we used the fact that $Y_{K}^{N}=\mu \nu_{1}<0$ and $Y_{L}^{N}=-\tilde{k}^{T} \mu \nu_{1}>0$ to get (170).

## Short-Run Tax Multipliers

$k^{N}>k^{T}$
Remembering that the short-run solution $Y^{T} \equiv Y^{T}(K, L, P)$, using the fact that the capital stock is initially predetermined, $\left.\mathrm{d} L(0)\right|^{j, k}=\left.\mathrm{d} \tilde{L}\right|^{j, k}+\left.L_{P} \mathrm{~d} P(0)\right|^{j, k}$ and $\left.\mathrm{d} P(0)\right|^{j, k}=-\left.\omega_{2}^{1} \mathrm{~d} \tilde{K}\right|^{j, k}$, the short-run tax multiplier is given by:

$$
\begin{align*}
\left.\mathrm{d} Y^{T}(0)\right|^{j, k} & =\left.Y_{L}^{T} \mathrm{~d} L(0)\right|^{j, k}+\left.\hat{Y}_{P}^{T} \mathrm{~d} P(0)\right|^{j, k} \\
& =-\Phi^{j, k}\left[\tilde{P} \nu_{1} \tilde{k}^{N} \frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}+Y_{P}^{T} \omega_{2}^{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}\right] \mathrm{d} \tau^{j} \gtrless 0, \tag{171}
\end{align*}
$$

where we used the fact that $Y_{L}^{T}=-\tilde{P} \nu_{1} \tilde{k}^{N}>0$; we denoted by a hat the partial derivative of $Y^{T}$ w. r. t. $P$ for given labor, i. e. $\hat{Y}_{P}^{T}<0$, and we used the fact that $Y_{L}^{T} L_{P}+\hat{Y}_{P}^{T}=Y_{P}^{T}$. The short-run tax multiplier for traded output is the result of two opposite effects: while the initial stimulus of labor supply induces a labor inflow in the traded sector, the real exchange appreciation shifts away resources from the traded sector towards the non-traded sector.

Differentiating the short-run solution for $Y^{N} \equiv Y^{N}(K, L, P)$ and remembering that the capital stock is initially predetermined, the short-run tax multiplier is given by:

$$
\begin{align*}
\left.\frac{P}{\mu} \mathrm{~d} Y^{N}(0)\right|^{j, k} & =\left.\frac{P}{\mu} Y_{L}^{N} \mathrm{~d} L(0)\right|^{j, k}+\left.\frac{P}{\mu} \hat{Y}_{P}^{N} \mathrm{~d} P(0)\right|^{j, k} \\
& =-\Phi^{j, k}\left[\tilde{P} \nu_{1} \tilde{k}^{N} \frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}+\frac{Y_{P}^{N}}{\mu} \omega_{2}^{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}\right] \mathrm{d} \tau^{j} \gtrless 0, \tag{172}
\end{align*}
$$

where we used the fact that $Y_{L}^{N}=-\tilde{k}^{T} \mu \nu_{2}<0$, and we denoted by a hat the partial derivative of $Y^{N}$ w. r. t. $P$ for given labor, i. e. $\hat{Y}_{P}^{N}>0$. The short-run tax multiplier for non-traded output is the result of two opposite effects: while the initial stimulus of labor supply induces a labor outflow from the non-traded sector, the real exchange appreciation attracts resources in this sector.
$k^{T}>k^{N}$
Remembering that the short-run solution $Y^{T} \equiv Y^{T}(K, L, P)$, using the fact that the capital stock is initially predetermined and $P$ is unaffected by a tax reform, the short-run tax multiplier is given by:

$$
\begin{equation*}
\left.\mathrm{d} Y^{T}(0)\right|^{j, k}=\left.Y_{L}^{T} \mathrm{~d} L(0)\right|^{j, k}=\left.Y_{L}^{T} \mathrm{~d} \tilde{L}\right|^{j, k}<0, \tag{173}
\end{equation*}
$$

where we used the fact that $Y_{L}^{T}=-\tilde{P} \nu_{2} \tilde{k}^{N}<0$,
Differentiating the short-run solution for $Y^{N} \equiv Y^{N}(K, L, p)$ and remembering that the capital stock is initially predetermined, the short-run tax multiplier is given by:

$$
\begin{equation*}
\left.\frac{P}{\mu} \mathrm{~d} Y^{N}(0)\right|^{j, k}=\left.\frac{P}{\mu} Y_{L}^{N} \mathrm{~d} L(0)\right|^{j, k}=\left.\frac{P}{\mu} Y_{L}^{N} \mathrm{~d} \tilde{L}\right|^{j, k}>0, \tag{174}
\end{equation*}
$$

where we used the fact that $Y_{L}^{N}=-\tilde{k}^{T} \mu \nu_{1}>0$.

## L Derivation of Formal Solutions after Anticipated Tax Reforms

In this section, we provide the main steps to derive formal solutions for key variables after future anticipated permanent tax shocks, by applying the procedure developed by Schubert and Turnovsky [2002]. For simplicity purpose, we assume that $\delta_{K}=0$ to avoid uninteresting complications.

## L. 1 Steady-State

As in Schubert and Turnovsky [2002], we define a viable steady-state $i$ starting at time $\mathcal{T}_{i}$ to be one that is consistent with long run solvency, given the stocks of capital, $K_{\mathcal{T}_{i}}$ and foreign bonds, $B_{\mathcal{T}_{i}}$.

We rewrite the system of steady-state equations for an arbitrary period $i$ (with $i=0,1,2$ ):

$$
\begin{gather*}
h_{k}\left[\tilde{k}^{N}\left(\tilde{P}_{i}\right)\right]=\mu\left(r^{\star}+\delta_{K}\right),  \tag{175a}\\
\frac{1}{\mu} Y^{N}\left(\tilde{K}_{i}, \tilde{P}_{i}, \bar{\lambda}_{i}, \tau_{i}^{F}, \tau_{i}^{H}\right)-C^{N}\left(\bar{\lambda}_{i}, \tilde{P}_{i}, \tau_{i}^{C}\right)-G^{N}=0  \tag{175b}\\
r^{\star} \tilde{B}_{i}+Y^{T}\left(\tilde{K}_{i}, \tilde{P}_{i}, \bar{\lambda}_{i}, \tau_{i}^{F}, \tau_{i}^{H}\right)-C^{T}\left(\bar{\lambda}_{i}, \tilde{P}_{i}, \tau_{i}^{C}\right)-G^{T}=0 \tag{175c}
\end{gather*}
$$

together with the intertemporal solvency condition

$$
\begin{equation*}
\left(\tilde{B}_{i}-B_{\mathcal{T}_{i}}\right)=\Phi_{1}\left(\tilde{K}_{i}-K_{\mathcal{T}_{i}}\right) . \tag{175d}
\end{equation*}
$$

## L. 2 Steady-State Functions

The new consistent procedure consists in two steps. In a first step, we solve the system (175a)$(175 \mathrm{c})$ for $\tilde{P}_{i}, \tilde{K}_{i}$ and $\tilde{B}_{i}$ as functions of the marginal utility of wealth, $\bar{\lambda}_{i}$, and tax rates, i.e. $\tau_{i}^{F}$ and $\tau_{i}^{H}, \tau_{i}^{C}$, and the markup $\mu_{i}$.

The equilibrium value of the marginal utility of wealth $\bar{\lambda}_{i}$ and tax rates, $\tau^{C}, \tau^{F}, \tau^{H}$ and $\mu$ determine the following steady-state values:

$$
\begin{align*}
\tilde{P}_{i} & =P\left(\mu_{i}\right),  \tag{176a}\\
\tilde{K}_{i} & =K\left(\bar{\lambda}_{i}, \tau_{i}^{C}, \tau_{i}^{F}, \tau_{i}^{H}, \mu_{i}\right),  \tag{176b}\\
\tilde{B}_{i} & =B\left(\bar{\lambda}_{i}, \tau_{i}^{C}, \tau_{i}^{F}, \tau_{i}^{H}, \mu_{i}\right), \tag{176c}
\end{align*}
$$

where partial derivatives are given by (118), (119), (120), (121) and (122). Note that as long as the markup is fixed, i.e., the number of competitors is large, then the relative price of non-tradables remains unaffected after a tax change.

The second step consists in determining the equilibrium change of $\bar{\lambda}$ by taking the total differential of the intertemporal solvency condition (175d):

$$
\begin{align*}
{\left[B_{\bar{\lambda}}-\Phi_{1} K_{\lambda}\right] \mathrm{d} \bar{\lambda}_{i}=} & \mathrm{d} B_{\mathcal{T}_{i}}-\Phi_{1} \mathrm{~d} K_{\mathcal{T}_{i}}-\left[B_{\tau^{j}}-\Phi_{1} K_{\tau^{j}}\right] \mathrm{d} \tau^{j} \\
& -\left[B_{\tau^{C}}-\Phi_{1} K_{\tau^{C}}\right] \mathrm{d} \tau^{C} \tag{177}
\end{align*}
$$

from which may solve for the equilibrium value of $\bar{\lambda}$ as a function of tax rates:

$$
\begin{equation*}
\bar{\lambda}=\lambda\left(K_{\mathcal{T}_{i}}, B_{\mathcal{T}_{i}}, \tau^{j}, \tau^{C}\right), \tag{178}
\end{equation*}
$$

with

$$
\begin{align*}
\lambda_{K} \equiv \frac{\partial \bar{\lambda}_{i}}{\partial K_{\mathcal{T}_{i}}} & =-\frac{\Phi_{1}}{\left[B_{\bar{\lambda}}-\Phi_{1} K_{\bar{\lambda}}\right]}<0  \tag{179a}\\
\lambda_{B} \equiv \frac{\partial \bar{\lambda}_{i}}{\partial B_{\mathcal{T}_{i}}} & =\frac{1}{\left[B_{\bar{\lambda}}-\Phi_{1} K_{\bar{\lambda}}\right]}<0  \tag{179b}\\
\lambda_{\tau^{j}} \equiv \frac{\partial \bar{\lambda}}{\partial \tau^{j}} & =-\frac{\left[B_{\tau^{j}}-\Phi_{1} K_{\tau^{j}}\right]}{\left[B_{\bar{\lambda}}-\Phi_{1} K_{\bar{\lambda}}\right]}>0  \tag{179c}\\
\lambda_{\tau^{C}} \equiv \frac{\partial \bar{\lambda}}{\partial \tau^{C}} & =-\frac{\left[B_{\tau^{C}}-\Phi_{1} K_{\tau^{C}}\right]}{\left[B_{\bar{\lambda}}-\Phi_{1} K_{\bar{\lambda}}\right]}<0 \tag{179d}
\end{align*}
$$

From (179), we obtain the following properties:

$$
\begin{align*}
\lambda_{B}\left[B_{\bar{\lambda}}-\Phi_{1} K_{\bar{\lambda}}\right] & =1,  \tag{180a}\\
\lambda_{B}\left[B_{\tau^{j}}-\Phi_{1} K_{\tau^{j}}\right] & =-\lambda_{\tau^{j}},  \tag{180b}\\
\lambda_{B}\left[B_{\tau^{C}}-\Phi_{1} K_{\tau^{C}}\right] & =-\lambda_{\tau^{C}} . \tag{180c}
\end{align*}
$$

## L. 3 Procedure to Derive Solutions for Future Anticipated Tax Shocks

In this subsection, we derive formal solutions after a future anticipated permanent change in the tax rate. In the text, we consider a cut in $\tau^{F}$ coordinated with a rise in $\tau^{C}$ so as to balance the government budget. Hence, we restrict ourselves to such revenue-neutral tax reform; note that a fall in $\tau^{H}$ yields very similar qualitative results. Importantly, while after an unanticipated permanent tax reform, the long-run effects are simply a scaled-down version of the steady-state changes after a labor tax cut financed by a decline in lump-sum transfer $Z$, this result does no longer hold when the tax reform is anticipated since only the wealth effect is in effect over the pre-implementation period. Hence, we proceed as follows. We determine separately formal solutions after a labor tax cut and a rise in consumption tax and then we consider a tax reform. We are able to derive impact
and steady-state effects only if $k^{T}>k^{N}$, and thereby we present analytical results only in this case. With reversal capital intensities, the effects of an anticipated tax reform shifting the tax burden from labor (i.e. $\tau^{F}$ ) to consumption are estimated numerically and shown in Table 2.

We assume that the small open economy is initially in steady-state equilibrium, denoted by the subscript $i=0$ :

$$
\begin{align*}
K_{0} & =\tilde{K}_{0}=K\left(\bar{\lambda}_{0}, \tau_{0}^{j}, \tau_{0}^{C}\right)  \tag{181a}\\
B_{0} & =\tilde{B}_{0}=B\left(\bar{\lambda}_{0}, \tau_{0}^{j}, \tau_{0}^{C}\right)  \tag{181b}\\
\lambda_{0} & =\bar{\lambda}_{0}=\lambda\left(K_{0}, B_{0}, \tau_{0}^{j}, \tau_{0}^{C}\right) \tag{181c}
\end{align*}
$$

Period $1(0 \leq t<\mathcal{T})$
We assume that at time $t=0$ agents perfectly anticipate that the tax rate $\tau^{j}\left(\tau^{C}\right)$ falls (rises) permanently at time $\mathcal{T}$, from $\tau_{0}^{j}=\tau_{1}^{j}\left(\tau_{0}^{C}=\tau_{1}^{C}\right)$ to $\tau_{T}^{j}=\tau_{2}^{j}\left(\tau_{T}^{C}=\tau_{2}^{C}\right)$, with $\mathrm{d} \tau_{T}^{j} \equiv \tau_{2}^{j}-\tau_{1}^{j}=$ $\tau_{T}^{j}-\tau_{0}^{j}<0\left(\mathrm{~d} \tau_{T}^{C} \equiv \tau_{2}^{C}-\tau_{1}^{C}=\tau_{T}^{C}-\tau_{0}^{C}>0\right)$.

When the tax shock is in effect, the economy follows unstable transitional paths:

$$
\begin{align*}
K(t)= & \tilde{K}_{1}+B_{1} e^{\nu_{1} t}+B_{2} e^{\nu_{2} t}  \tag{182a}\\
P(t)= & \tilde{P}_{1}+\omega_{2}^{1} B_{1} e^{\nu_{1} t}+\omega_{2}^{2} B_{2} e^{\nu_{2} t},  \tag{182b}\\
B(t)= & \tilde{B}_{1}+\left[\left(B_{0}-\tilde{B}_{1}\right)-\Phi_{1} B_{1}-\Phi_{2} B_{2}\right] e^{r^{\star} t}+ \\
& +\Phi_{1} B_{1} e^{\nu_{1} t}+\Phi_{2} B_{2} e^{\nu_{2} t} \tag{182c}
\end{align*}
$$

with the steady-state values $\tilde{K}_{1}$ and $\tilde{B}_{1}$ given by the following functions (set $i=1$ into (176b)(176c)):

$$
\begin{align*}
\tilde{K}_{1} & =K\left(\bar{\lambda}, \tau_{1}^{j}, \tau_{1}^{C}\right)  \tag{183a}\\
\tilde{B}_{1} & =B\left(\bar{\lambda}, \tau_{1}^{j}, \tau_{1}^{C}\right) \tag{183b}
\end{align*}
$$

where the marginal utility of wealth remains constant over periods 1 and 2 at level $\bar{\lambda}_{1}=\bar{\lambda}_{\mathcal{T}}=\bar{\lambda}_{2}=\bar{\lambda}$ after its initial jump at time $t=0$.

Period $2(t \geq \mathcal{T})$
Once the tax rate reverts back to its initial level, the economy follows stable paths

$$
\begin{align*}
K(t) & =\tilde{K}_{2}+B_{1}^{\prime} e^{\nu_{1} t}  \tag{184a}\\
P(t) & =\tilde{P}_{2}+\omega_{2}^{1} B_{1}^{\prime} e^{\nu_{1} t}  \tag{184b}\\
B(t) & =\tilde{B}_{2}+\Phi_{1} B_{1}^{\prime} e^{\nu_{1} t} \tag{184c}
\end{align*}
$$

with the steady-state values $\tilde{K}_{2}$ and $\tilde{B}_{2}$ given by the following functions (set $i=2$ into (176b)(176c)):

$$
\begin{align*}
\tilde{K}_{2} & =K\left(\bar{\lambda}, \tau_{2}^{j}, \tau_{2}^{C}\right)  \tag{185a}\\
\tilde{B}_{2} & =B\left(\bar{\lambda}, \tau_{2}^{j}, \tau_{2}^{C}\right) \tag{185b}
\end{align*}
$$

During the transition period 1 , the economy accumulates (or decumulates) capital and foreign assets. Since this period is unstable, it would lead the nation to violate its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy's intertemporal budget constraint. At the same time, the zero-root problem requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hits the economy. So $\lambda$ remains constant over the periods 1 and 2 . The aim of the two-step method is to calculate the deviation of $\lambda$ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial conditions, $K_{\mathcal{T}}$ and $B_{\mathcal{T}}$, accumulated over the unstable period (before the shock in in effect). Therefore, for the country to remain intertemporally solvent, we require:

$$
\begin{equation*}
B_{\mathcal{T}}-\tilde{B}_{2}=\Phi_{1}\left(K_{\mathcal{T}}-\tilde{K}_{2}\right) \tag{186}
\end{equation*}
$$

In order to determine the three constants $B_{1}, B_{2}$, and $B_{1}^{\prime}$, and the equilibrium value of marginal utility of wealth, we impose three conditions:

1. Initial conditions $K(0)=K_{0}, B(0)=B_{0}$ must be met.
2. Economic aggregates $K$ and $P$ remain continuous at time $\mathcal{T}$.
3. The intertemporal solvency constraint (186) must hold implying that the net foreign assets remain continuous at time $\mathcal{T}$.

Set $t=0$ in solution (182a); evaluate at time $t=\mathcal{T}$ and equate (182a) and (184a), (182c) and (184c):

$$
\begin{gather*}
\tilde{K}_{1}+B_{1}+B_{2}=K_{0},  \tag{187a}\\
\tilde{K}_{1}+B_{1} e^{\nu_{1} \mathcal{T}}+B_{2} e^{\nu_{2} \mathcal{T}}=\tilde{K}_{2}+B_{1}^{\prime} e^{\nu_{1} \mathcal{T}},  \tag{187b}\\
\tilde{P}_{1}+\omega_{2}^{1} B_{1} e^{\nu_{1} \mathcal{T}}+\omega_{2}^{2} B_{2} e^{\nu_{2} \mathcal{T}}=\tilde{P}_{2}+\omega_{2}^{1} B_{1}^{\prime} e^{\nu_{1} \mathcal{T}}, \tag{187c}
\end{gather*}
$$

where we used the continuity condition.
Evaluating $K_{\mathcal{T}}$ and $B_{\mathcal{T}}$ by using (182a) and (182c), substituting into (182c) evaluated at time $t=\mathcal{T}$, and using functions of steady-state values $\tilde{K}_{i}$ and $\tilde{B}_{i}$ given by (176) for appropriate periods, the intertemporal solvency condition can be rewritten as

$$
\begin{align*}
& B\left(\bar{\lambda}, \tau_{1}^{j}, \tau_{1}^{C}\right)+\left\{\left[B\left(\lambda_{0}, \tau_{0}^{j}, \tau_{0}^{C}\right)-B\left(\bar{\lambda}, \tau_{1}^{j}, \tau_{1}^{C}\right)\right]-\Phi_{1} B_{1}-\Phi_{2} B_{2}\right\} e^{r^{\star} \mathcal{T}}+\Phi_{1} B_{1} e^{\nu_{1} \mathcal{T}} \\
& +\Phi_{2} B_{2} e^{\nu_{2} \mathcal{T}}-B\left(\bar{\lambda}, \tau_{2}^{j}, \tau_{2}^{C}\right)=\Phi_{1}\left[K\left(\bar{\lambda}, \tau_{1}^{j}, \tau_{1}^{C}\right)+B_{1} e^{\nu_{1} \mathcal{T}}+B_{2} e^{\nu_{2} \mathcal{T}}-K\left(\bar{\lambda}, \tau_{2}^{j}, \tau_{2}^{C}\right)\right](\overline{1} \tag{188}
\end{align*}
$$

Then, we approximate the steady-state changes with the differentials:

$$
\begin{align*}
& \tilde{K}_{1}-\tilde{K}_{0} \equiv K\left(\bar{\lambda}, \tau_{0}^{j}, \tau_{0}^{C}\right)-K\left(\lambda_{0}, \tau_{0}^{j}, \tau_{0}^{C}\right)=\left.K_{\bar{\lambda}} \frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{F}}\right|_{f u t}+\left.K_{\bar{\lambda}} \frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{C}}\right|_{f u t},  \tag{189a}\\
& \tilde{K}_{2}-\tilde{K}_{1} \equiv K\left(\bar{\lambda}, \tau_{2}^{j}, \tau_{2}^{C}\right)-K\left(\bar{\lambda}, \tau_{1}^{j}, \tau_{1}^{C}\right)=K_{\tau^{j}} \mathrm{~d} \tau^{j}+K_{\tau^{C}} \mathrm{~d} \tau^{C}  \tag{189b}\\
& \tilde{B}_{1}-\tilde{B}_{0} \equiv B\left(\bar{\lambda}, \tau_{0}^{j}, \tau_{0}^{C}\right)-B\left(\lambda_{0}, \tau_{0}^{j}, \tau_{0}^{C}\right)=\left.B_{\bar{\lambda}} \frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{F}}\right|_{f u t} \mathrm{~d} \tau^{F}+\left.B_{\bar{\lambda}} \frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{C}}\right|_{f u t} \mathrm{~d} \tau^{C},  \tag{189c}\\
& \tilde{B}_{2}-\tilde{B}_{1} \equiv B\left(\bar{\lambda}, \tau_{2}^{j}, \tau_{2}^{C}\right)-B\left(\bar{\lambda}, \tau_{1}^{j}, \tau_{1}^{C}\right)=B_{\tau^{j}} \mathrm{~d} \tau^{j}+B_{\tau^{C}} \mathrm{~d} \tau^{C} \tag{189d}
\end{align*}
$$

By substituting these expressions into (187) and (188), we obtain finally

$$
\begin{gather*}
B_{1}+B_{2}=-K_{\bar{\lambda}} \mathrm{d} \bar{\lambda}  \tag{190a}\\
B_{1} e^{\nu_{1} \mathcal{T}}+B_{2} e^{\nu_{2} \mathcal{T}}-B_{1}^{\prime} e^{\nu_{1} \mathcal{T}}=K_{\tau^{j}} \mathrm{~d} \tau^{j}+K_{\tau^{C}} \mathrm{~d} \tau^{C}  \tag{190b}\\
\omega_{2}^{1} B_{1} e^{\nu_{1} \mathcal{T}}+\omega_{2}^{2} B_{2} e^{\nu_{2} \mathcal{T}}-\omega_{2}^{1} B_{1}^{\prime} e^{\nu_{1} \mathcal{T}}=0 \tag{190c}
\end{gather*}
$$

where $\mathrm{d} \bar{\lambda} \equiv \bar{\lambda}-\lambda_{0}$ and

$$
\begin{equation*}
B_{1} \Upsilon_{1}+B_{2} \Upsilon_{2}+B_{\bar{\lambda}} \mathrm{d} \bar{\lambda}=-\Omega_{j}-\Omega_{C} \tag{191}
\end{equation*}
$$

where we set

$$
\begin{align*}
\Upsilon_{1} & \equiv \Phi_{1}  \tag{192a}\\
\Upsilon_{2} & \equiv \Phi_{2}+\left(\Phi_{1}-\Phi_{2}\right) e^{-\nu_{1} \mathcal{T}}  \tag{192b}\\
\Omega_{j} & \equiv\left(B_{\tau^{j}}-\Phi_{1} K_{\tau^{j}}\right) e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{j}  \tag{192c}\\
\Omega_{C} & \equiv\left(B_{\tau^{C}}-\Phi_{1} K_{\tau^{C}}\right) e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{C} . \tag{192d}
\end{align*}
$$

We solve the system written in a matrix form for the constants $B_{1}, B_{2}, B_{1}^{\prime}$, and the change in the equilibrium value of the marginal utility of wealth $d \bar{\lambda}$ :

$$
\left(\begin{array}{cccc}
\Phi_{1} & \Upsilon_{2} & 0 & B_{\bar{\lambda}}  \tag{193}\\
1 & 1 & 0 & K_{\bar{\lambda}} \\
e^{\nu_{1} T} & e^{\nu_{2} T} & -e^{\nu_{1} T} & 0 \\
\omega_{2}^{1} e^{\nu_{1} T} & \omega_{2}^{2} e^{\nu_{2} T} & -\omega_{2}^{1} e^{\nu_{1} T} & 0
\end{array}\right)\left(\begin{array}{c}
B_{1} \\
B_{2} \\
B_{1}^{\prime} \\
\mathrm{d} \bar{\lambda}
\end{array}\right)=\left(\begin{array}{c}
-\Omega_{1}-\Omega_{C} \\
0 \\
K_{\tau^{j}} \mathrm{~d} \tau^{j}+K_{\tau^{C}} \mathrm{~d} \tau^{C} \\
0
\end{array}\right)
$$

where the determinant $E$ is:

$$
\begin{equation*}
E \equiv-\left(B_{\bar{\lambda}}-\Phi_{1} K_{\bar{\lambda}}\right)\left(\omega_{2}^{2}-\omega_{2}^{1}\right) e^{r^{\star} T}>0, \tag{194}
\end{equation*}
$$

## L. 4 Formal Solutions for an Anticipated Change in $\tau^{F}$

In deriving the formal solutions for an anticipated labor tax cut at time $\mathcal{T}$, we keep unchanged $\tau^{C}$ so that $\mathrm{d} \tau^{C}=0$ and $\mathrm{d} \tau^{j}=\mathrm{d} \tau^{F}<0$ since we consider a decline in payroll taxes.

Case $k^{T}>k^{N}$

Remembering that $\omega_{2}^{1}=0$ in this case, solving (190)-(191) gives:

$$
\begin{align*}
B_{1} & =-K_{\bar{\lambda}} \lambda_{\tau^{F}} e^{-r^{\star} \mathcal{T}}=-\frac{1}{\bar{\lambda} \nu_{1}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right) \lambda_{\tau^{F}} e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{F}>0  \tag{195a}\\
B_{2} & =0  \tag{195b}\\
B_{1}^{\prime} & =B_{1}  \tag{195c}\\
\left.\frac{\mathrm{~d} \bar{\lambda}}{\mathrm{~d} \tau^{F}}\right|_{f u t} & =\lambda_{\tau^{F}} e^{-r^{\star} \mathcal{T}}>0 \tag{195d}
\end{align*}
$$

where $\lambda_{\tau^{F}}>0$ represents the change in the equilibrium of the marginal utility of wealth following an unexpected permanent labor tax cut; we used property (180b) to get (200c).

Case $k^{N}>k^{T}$
Remembering that $\omega_{2}^{2}=0$ in this case, solving (190)-(191) gives:

$$
\begin{align*}
B_{1} & =-\left.K_{\bar{\lambda}} \frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{F}}\right|_{f u t} \mathrm{~d} \tau^{F}-K_{\tau^{F}} e^{-\mu_{2} \mathcal{T}} \mathrm{~d} \tau^{F}>0  \tag{196a}\\
B_{2} & =K_{\tau^{F}} e^{-\mu_{2} \tau} \mathrm{~d} \tau^{F}>0  \tag{196b}\\
B_{1}^{\prime} & =B_{1}>0 \tag{196c}
\end{align*}
$$

where the change in the equilibrium value of the marginal utility of wealth is:

$$
\begin{equation*}
\left.\frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{F}}\right|_{f u t}=\lambda_{\tau^{F}} e^{-r^{\star} \mathcal{T}}-\lambda_{B}\left(\Phi_{1}-\Phi_{2}\right)\left(e^{-r^{\star} \mathcal{T}}-e^{-\nu_{2} \mathcal{T}}\right) K_{\tau^{F}}>0 \tag{197}
\end{equation*}
$$

with $\lambda_{B}<0$ and

$$
\begin{equation*}
\Phi_{1}-\Phi_{2}=\frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{2} \tilde{\Lambda}-\sigma_{C} \tilde{C}^{N}\right)<0 \tag{198}
\end{equation*}
$$

## Long-Run Effects

We determine the impacts of an anticipated labor tax cut by assuming that $k^{T}>k^{N}$ since when $k^{N}>k^{T}$, we are unable to determine the signs of expressions as the change in the marginal utility of wealth given by (197) complicates substantially algebra.

Note that numerical results reported in Table 2 show that the long-run effects are similar whether $k^{T} \gtrless k^{N}$. Denoting by $X$ the macroeconomic aggregate $C, L, K, N X$, and using the fact that $\tilde{X}=X\left(\bar{\lambda}, \tau^{j}, \tau^{C}\right)$ (see (176)), the steady-state change of $X$ following an anticipated labor tax cut is given by:

$$
\begin{equation*}
\left.\frac{\mathrm{d} \tilde{X}}{\mathrm{~d} \tau^{F}}\right|_{f u t}=\left.X_{\bar{\lambda}} \frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{F}}\right|_{f u t}+X_{\tau^{F}} \tag{199}
\end{equation*}
$$

Applying this formula to consumption, labor and capital stock, the long-run effects expressed in percentage deviations from initial steady-state (denoted by a hat) are:

$$
\begin{align*}
\left.\hat{\tilde{C}}\right|_{f u t} & =-\sigma_{C} \Lambda^{F} \tilde{\xi} e^{-r^{\star} \mathcal{T}} \hat{\tau}^{F}>0  \tag{200a}\\
\left.\tilde{\tilde{L}}\right|_{f u t} & =-\sigma_{L} \Lambda^{F}\left(1-\tilde{\xi} e^{-r^{\star} \mathcal{T}}\right) \hat{\tau}^{F}>0,  \tag{200b}\\
\left.\mathrm{~d} \tilde{K}\right|_{f u t} & =-\left[\sigma_{L} \tilde{L} \tilde{k}^{T}\left(1-\Lambda^{F} \tilde{\xi} e^{-r^{\star} \mathcal{T}}\right)+\frac{\Lambda^{F} \tilde{\xi} \sigma_{C} \tilde{C}^{N}}{\nu_{1}} e^{-r^{\star} \mathcal{T}}\right] \hat{\tau}^{F}>0, \tag{200c}
\end{align*}
$$

where $\hat{\tau}^{F}<0,0<\Lambda^{F}<1$ (see (108b)) $0<\tilde{\xi}<1$ (see (108c)). The sign of eq. (200c) comes from the fact that the wealth effect is smaller than after an unexpected labor tax cut (see eq. (111)).

## Impact Effects

Differentiating eq. (184a) w.r.t. time, evaluating at time $t=0$, we obtain the initial response of investment following an anticipated permanent labor tax cut:

$$
\left.I(0)\right|_{f u t}=\nu_{1} B_{1}+\nu_{2} B_{2} .
$$

Substituting (200) and using the fact that $B_{2}=0$, the initial reaction of investment becomes:

$$
\begin{align*}
\left.I(0)\right|_{f u t} & =-\nu_{1} K_{\bar{\lambda}} \lambda_{\tau^{F}} e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{F} \\
& =\left(\sigma_{C} \tilde{C}^{N} \sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right) \frac{\sigma_{L} \tilde{W} \tilde{L}}{\left(\sigma_{C} P_{C} \tilde{C}+\sigma_{L} \tilde{W}^{F} \tilde{L}\right)} \Lambda^{F} e^{-r^{\star} \mathcal{T}} \hat{\tau}^{F}<0 \\
& =\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right) \tilde{\xi} \Lambda^{F} e^{-r^{\star} \mathcal{T}} \hat{\tau}^{F}<0 \tag{201}
\end{align*}
$$

The general solution for the stock of foreign assets is given by:

$$
\begin{equation*}
B(t)=\tilde{B}+\left[\left(B_{0}-\tilde{B}\right)-\Phi_{1} B_{1}-\Phi_{2} B_{2}\right] e^{r^{\star} t}+\Phi_{1} B_{1} e^{\nu_{1} t}+\Phi_{2} B_{2} e^{\nu_{2} t} \tag{202}
\end{equation*}
$$

Differentiating eq. (202) w.r.t. time, evaluating at time $t=0$, and remembering that $B_{2}=0$, we obtain the initial response of the current account following a future anticipated permanent change in the tax rate:

$$
\left.C A(0)\right|_{f u t}=r^{\star}\left[\left(B_{0}-\tilde{B}_{1}\right)-\Phi_{1} B_{1}-\Phi_{2} B_{2}\right]+\nu_{1} \Phi_{1} B_{1}+\nu_{2} \Phi_{2} B_{2} .
$$

We compute:

$$
\begin{align*}
& -\left.\mathrm{d} \tilde{B}_{1}\right|_{f u t}-\Phi_{1} B_{1} \\
= & -\left.\left(B_{\bar{\lambda}}-\Phi_{1} K_{\bar{\lambda}}\right) \mathrm{d} \bar{\lambda}\right|_{f u t}, \\
= & \frac{\lambda_{\tau^{F}}}{\lambda_{B}} e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{F}=\left(B_{\tau^{F}}-\Phi_{1} K_{\tau^{F}}\right) e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{F}, \\
= & \frac{\sigma_{L} \tilde{W}^{F} \tilde{L}}{r^{\star}} e^{-r^{\star} \mathcal{T}} \hat{\tau}^{F}, \tag{203}
\end{align*}
$$

where we used the fact that $\tilde{f}-\tilde{P} r^{\star} f_{k}=\tilde{W}^{F}, B_{2}=0$ and property (180b). The initial reaction of the current account becomes:

$$
\begin{align*}
\left.C A(0)\right|_{f u t} & =\sigma_{L} \tilde{W} \tilde{L} e^{-r^{\star} \mathcal{T}} \hat{\tau}^{F}-\nu_{1} \tilde{P} B_{1} \\
& =\sigma_{L} \tilde{W}^{F} \tilde{L}\left[1-\frac{\Lambda^{F} \tilde{P}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)}{\left(\sigma_{C} P_{C} \tilde{C}+\sigma_{L} \tilde{W}^{F} \tilde{L}\right)}\right] e^{-r^{\star} \mathcal{T}} \hat{\tau}^{F} \gtrless 0, \tag{204}
\end{align*}
$$

where we used the fact that $\Phi_{1}=-\tilde{P}$.

## L. 5 Formal Solutions for an Anticipated Change in $\tau^{C}$

In deriving the formal solutions for an anticipated rise in the consumption tax at time $\mathcal{T}$, we keep unchanged $\tau^{F}$ so that $\mathrm{d} \tau^{F}=0$ and consider $\mathrm{d} \tau^{C}>0$.

Case $k^{T}>k^{N}$
Remembering that $\omega_{2}^{1}=0$ in this case, solving (190)-(191) gives:

$$
\begin{align*}
B_{1} & =-K_{\bar{\lambda}} \lambda_{\tau^{C}} e^{-r^{\star} \mathcal{T}}=-\frac{1}{\bar{\lambda} \nu_{1}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right) \lambda_{\tau^{C}} e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{C}<0  \tag{205a}\\
B_{2} & =0  \tag{205b}\\
B_{1}^{\prime} & =B_{1},  \tag{205c}\\
\left.\frac{\mathrm{~d} \bar{\lambda}}{\mathrm{~d} \tau^{C}}\right|_{f u t} & =\lambda_{\tau^{C}} e^{-r^{\star} \mathcal{T}}<0, \tag{205d}
\end{align*}
$$

where $\lambda_{\tau^{c}}<0$ represents the change in the equilibrium of the marginal utility of wealth following an unexpected permanent labor tax cut; we used property (180b) to get (205d).

Case $k^{N}>k^{T}$
Remembering that $\omega_{2}^{2}=0$ in this case, solving (190)-(191) gives:

$$
\begin{align*}
B_{1} & =-\left.K_{\bar{\lambda}} \frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{C}}\right|_{f u t} \mathrm{~d} \tau^{C}-K_{\tau^{C}} e^{-\mu_{2} \mathcal{T}} \mathrm{~d} \tau^{C}>0  \tag{206a}\\
B_{2} & =K_{\tau^{C}} e^{-\mu_{2} \mathcal{T}} \mathrm{~d} \tau^{C}>0  \tag{206b}\\
B_{1}^{\prime} & =B_{1}>0 \tag{206c}
\end{align*}
$$

where the change in the equilibrium value of the marginal utility of wealth:

$$
\begin{equation*}
\left.\frac{\mathrm{d} \bar{\lambda}}{\mathrm{~d} \tau^{C}}\right|_{f u t}=\lambda_{\tau^{C}} e^{-r^{\star} \mathcal{T}}-\lambda_{B}\left(\Phi_{1}-\Phi_{2}\right)\left(e^{-r^{\star} \mathcal{T}}-e^{-\nu_{2} \mathcal{T}}\right) K_{\tau^{C}}<0 \tag{207}
\end{equation*}
$$

with $\lambda_{B}<0$ and $\left(\Phi_{1}-\Phi_{2}\right)$ given by (201).

## Long-Run Effects

We determine the effects of an anticipated labor tax cut by assuming that $k^{T}>k^{N}$ since when $k^{N}>k^{T}$, we are unable to determine the signs of formal expressions as the change in the marginal utility of wealth given by (207) complicates substantially algebra.

Applying formula (199) to consumption, labor and capital stock, the long-run effects expressed in percentage deviations from initial steady-state (denoted by a hat) are:

$$
\begin{align*}
\left.\hat{\tilde{C}}\right|_{f u t} & =-\sigma_{C}\left[1-(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}}\right] \hat{\tau}^{C}<0,  \tag{208a}\\
\left.\hat{\tilde{L}}\right|_{f u t} & =-\sigma_{L}(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}} \hat{\tau}^{C}<0  \tag{208b}\\
\left.\mathrm{~d} \tilde{K}\right|_{f u t} & =-\left\{\frac{\sigma_{C} \tilde{C}^{N}}{\nu_{1}}\left[1-(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}}\right]+\sigma_{L} \tilde{L} \tilde{k}^{T}(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}}\right\} \hat{\tau}^{C}<0, \tag{208c}
\end{align*}
$$

where $\hat{\tau}^{C}>0,0<\Lambda^{F}<1$ (see (108b)) $0<\tilde{\xi}<1$ (see (108c)). The sign of eq. (208c) comes from the fact that the wealth effect is smaller than after an unexpected labor tax cut (see eq. (102d)).

## Impact Effects

Differentiating eq. (184a) w.r.t. time, evaluating at time $t=0$, we obtain the initial response of investment following an anticipated permanent increase in the consumption tax:

$$
\left.I(0)\right|_{f u t}=\nu_{1} B_{1}+\nu_{2} B_{2} .
$$

Substituting (205a) and using the fact that $B_{2}=0$, the initial reaction of investment becomes:

$$
\begin{align*}
\left.I(0)\right|_{f u t} & =-\nu_{1} K_{\bar{\lambda}} \lambda_{\tau^{C}} e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{C}, \\
& =-\left(\sigma_{C} \tilde{C}^{N} \sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right) \frac{\sigma_{C} P_{C} \tilde{C}}{\left(\sigma_{C} P_{C} \tilde{C}+\sigma_{L} \tilde{W}^{F} \tilde{L}\right)} e^{-r^{\star} \mathcal{T}} \hat{\tau}^{C}<0, \\
& =-\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}} \hat{\tau}^{C}<0 . \tag{209}
\end{align*}
$$

Differentiating eq. (202) w.r.t. time, evaluating at time $t=0$, and remembering that $B_{2}=0$, we obtain the initial response of the current account following a future anticipated permanent change in the tax rate:

$$
\left.C A(0)\right|_{f u t}=r^{\star}\left[\left(B_{0}-\tilde{B}_{1}\right)-\Phi_{1} B_{1}-\Phi_{2} B_{2}\right]+\nu_{1} B_{1} \Phi_{1}+\nu_{2} B_{2} \Phi_{2} .
$$

We compute:

$$
\begin{align*}
& -\left.\mathrm{d} \tilde{B}_{1}\right|_{f u t}-\Phi_{1} B_{1} \\
= & -\left.\left(B_{\bar{\lambda}}-\Phi_{1} K_{\bar{\lambda}}\right) \mathrm{d} \bar{\lambda}\right|_{f u t}, \\
= & \frac{\lambda_{\tau^{C}}}{\lambda_{B}} e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{C}=\left(B_{\tau^{C}}-\Phi_{1} K_{\tau^{C}}\right) e^{-r^{\star} \mathcal{T}} \mathrm{d} \tau^{C} \\
= & -\frac{\sigma_{C} P_{C} \tilde{C}}{r^{\star}} e^{-r^{\star} \mathcal{T}} \hat{\tau}^{C} \tag{210}
\end{align*}
$$

where we used the fact that $B_{2}=0$ and property (180b). The initial reaction of the current account becomes:

$$
\begin{align*}
\left.C A(0)\right|_{f u t} & =-\sigma_{C} P_{C} \tilde{C} e^{-r^{\star} \mathcal{T}} \hat{\tau}^{C}-\nu_{1} \tilde{P} B_{1} \\
& =-\sigma_{C} P_{C} \tilde{C}\left[1-\frac{\tilde{P}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)}{\left(\sigma_{C} P_{C} \tilde{C}+\sigma_{L} \tilde{W}^{F} \tilde{L}\right)}\right] e^{-r^{\star} \mathcal{T}} \hat{\tau}^{C} \gtrless 0, \tag{211}
\end{align*}
$$

where we used the fact that $\Phi_{1}=-\tilde{P}$.

## L. 6 Formal Solutions for an Anticipated Tax Reform

We now derive the steady-state and impact effects of a tax reform which involves cutting payroll taxes and raising consumption taxes so as to keep the government budget balanced. We denote the superscript $\left.\right|^{F, C}$ the effects of such a tax reform.

## Steady-State Changes

Using (200c) and (205d), the change in the marginal utility of wealth after an anticipated tax reform is given by:

$$
\begin{align*}
\left.\hat{\bar{\lambda}}\right|_{f u t} ^{F, C} & =\left.\frac{\hat{\bar{\lambda}}}{\hat{\tau}^{F}}\right|_{f u t} \hat{\tau}^{F}+\left.\left.\frac{\hat{\bar{\lambda}}}{\hat{\tau}^{C}}\right|_{f u t} \hat{\tau}^{C}\right|_{f u t} ^{F, C}, \\
& =\left[\tilde{\xi} \Lambda^{F} \hat{\tau}^{F}-\left.(1-\tilde{\xi}) \hat{\tau}^{C}\right|_{f u t} ^{F, C}\right] e^{-r^{\star} \mathcal{T}}<0 . \tag{212}
\end{align*}
$$

Eq. (212) corresponds to eq. (32) in the text.
The change in the consumption tax for a given decline in the labor tax cut is given by:

$$
\begin{equation*}
\left.\hat{\tau}^{C}\right|^{F, C}=-\frac{\left.\Gamma^{F}\right|_{f u t}}{\left.\Gamma^{C}\right|_{f u t}}>0, \tag{213}
\end{equation*}
$$

where

$$
\begin{equation*}
\left.\Gamma^{F}\right|_{f u t}=\Lambda^{F} \tilde{W}^{A} \tilde{L}\left\{1-\sigma_{L} \frac{\tilde{W}^{F}}{\tilde{W}^{A}}\left[\tau^{C}(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}}+\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right)\left(1-\tilde{\xi} e^{-r^{\star} \mathcal{T}}\right)\right]\right\} \tag{214}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\Gamma^{C}\right|_{f u t}=P_{C} \tilde{C}\left(1+\tau^{C}\right)\left\{1-\frac{\sigma_{C}}{\left(1+\tau^{C}\right)}\left[\tau^{C}\left[1-(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}}\right]+\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right) \tilde{\xi} e^{-r^{\star} \mathcal{T}}\right]\right\} . \tag{215}
\end{equation*}
$$

Comparing $\Gamma^{F}$ after an unanticipated and anticipated tax reforms, we find that:

$$
\begin{equation*}
\Gamma^{F}>\left.\Gamma^{F}\right|_{f u t}, \quad \text { if } \quad \tau^{C}(1-\tilde{\xi})<\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right) \tilde{\xi} \tag{216}
\end{equation*}
$$

Comparing $\Gamma^{C}$ after an unanticipated and anticipated tax reforms, we find that:

$$
\begin{equation*}
\Gamma^{C}>\left.\Gamma^{C}\right|_{f u t}, \quad \text { if } \quad \tau^{C}(1-\tilde{\xi})>\left(\frac{\tilde{W}^{F}-\tilde{W}^{A}}{\tilde{W}^{F}}\right) \tilde{\xi} \tag{217}
\end{equation*}
$$

We find numerically that the change in the consumption tax (see Panel B of Table 2) so as to balance the government budget after an anticipated tax reform is roughly equal to that after an unanticipated tax reform (see Panel B of Table 1). Hence, the consumption tax must increase for a given labor tax cut. However, we are no longer able to express steady-state changes after an anticipated tax reform as scaled-down versions of the steady-state effects after a labor tax cut financed by a decline in lump-sum transfer. We compute below steady-state effects after an anticipated tax reform.

Using (200) and (208), the long-run effects of an anticipated tax reform, expressed in percentage deviations from initial steady-state (denoted by a hat), are:

$$
\begin{align*}
\left.\hat{\tilde{C}}\right|_{f u t} ^{F, C} & =\frac{\left.\hat{\tilde{C}}\right|_{f u t}}{\hat{\tau}^{F}} \hat{\tau}^{F}+\left.\frac{\left.\hat{\tilde{C}}\right|_{f u t}}{\hat{\tau}^{C}} \hat{\tau}^{C}\right|_{f u t} ^{F, C}, \\
& =-\sigma_{C} \Lambda^{F} \tilde{\xi}^{-e^{*} \mathcal{T}} \hat{\tau}^{F}-\left.\sigma_{C}\left[1-(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}}\right] \hat{\tau}^{C}\right|_{f u t} ^{F, C}>0,  \tag{218a}\\
\left.\hat{\tilde{L}}\right|_{f u t} ^{F, C} & =\frac{\left.\hat{\tilde{L}}\right|_{f u t}}{\hat{\tau}^{F}} \hat{\tau}^{F}+\left.\frac{\left.\hat{\tilde{L}}\right|_{f u t}}{\hat{\tau}^{C}} \hat{\tau}^{C}\right|_{f u t} ^{F, C}, \\
& =-\sigma_{L} \Lambda^{F}\left(1-\tilde{\xi} e^{-r^{\star} \mathcal{T}}\right) \hat{\tau}^{F}-\left.\sigma_{L}(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}^{\prime}} \hat{\tau}^{C}\right|_{f u t} ^{F, C}>0,  \tag{218b}\\
\left.\mathrm{~d} \tilde{K}\right|_{f u t} ^{F, C} & =\left.K_{\bar{\lambda}} \mathrm{d} \bar{\lambda}\right|_{f u t} ^{F, C}+K_{\tau^{F}} \mathrm{~d} \tau^{F}+\left.K_{\tau^{C}} \mathrm{~d} \tau^{C}\right|_{f u t} ^{F, C}, \\
& =-\left.\frac{1}{\nu_{1}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right) \hat{\bar{\lambda}}\right|_{f u t} ^{F, C}-\sigma_{L} \tilde{L} \tilde{k}^{T} \hat{\tau}^{F}-\left.\frac{\sigma_{C} \tilde{C}^{N}}{\nu_{1}} \hat{\tau}^{C}\right|_{f u t} ^{F, C}>0, \tag{218c}
\end{align*}
$$

where $\hat{\tau}^{F}<0,\left.\hat{\tau}^{C}\right|_{f u t} ^{F, C}>0\left(\right.$ see (213)), $0<\Lambda^{F}<1$ (see (108b)) $0<\tilde{\xi}<1$ (see (108c)). To derive eq. (218c), we totally differentiated eq. (176b). Eq. (218b) corresponds to eq. (33) in the text. Eq. (218c) corresponds to eq. (34) in the text.

Combining (218b) and (218c) yields the long-run change in GDP following an anticipated revenue-neutral tax reform:

$$
\begin{equation*}
\left.\hat{\tilde{Y}}\right|_{f u t} ^{F, C}=\left.\left(1-\tilde{\beta}_{L}\right) \hat{\tilde{K}}\right|_{f u t} ^{F, C}+\left.\tilde{\beta}_{L} \hat{\tilde{L}}\right|_{f u t} ^{F, C}>0 \tag{219}
\end{equation*}
$$

Eq. (219) corresponds to eq. (35) in the text.

## Impact Effects

Combining (201) and (209), the initial response of investment after an anticipated revenueneutral tax reform is unambiguously negative. Formally, it is given by:

$$
\begin{equation*}
\left.I(0)\right|_{f u t} ^{F, C}=\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)\left[\tilde{\xi} \Lambda^{F} \hat{\tau}^{F}-\left.(1-\tilde{\xi}) e^{-r^{\star} \mathcal{T}} \hat{\tau}^{C}\right|_{f u t} ^{F, C}\right] e^{-r^{\star} \mathcal{T}}<0 \tag{220}
\end{equation*}
$$

$\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)>0, \hat{\tau}^{F}<0,\left.\hat{\tau}^{C}\right|_{f u t} ^{F, C}>0$. Eq. (220) corresponds to eq. (36) in the text.

Combining (204) and (211), the initial response of the current account after an anticipated revenue-neutral tax reform is unambiguously positive. Formally, it is given by:

$$
\begin{align*}
\left.C A(0)\right|_{f u t} ^{F, C}= & \left(\sigma_{L} \tilde{W}^{F} \tilde{L} \hat{\tau}^{F}-\left.\sigma_{C} P_{C} \tilde{C} \hat{\tau}^{C}\right|_{f u t} ^{F, C}\right) e^{-r^{\star} \mathcal{T}} \\
& -\tilde{P}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{1}\right)\left[\tilde{\xi} \Lambda^{F} \hat{\tau}^{F}-\left.(1-\tilde{\xi}) \tilde{C} \hat{\tau}^{C}\right|_{f u t} ^{F, C}\right] e^{-r^{\star} \mathcal{T}} \gtrless 0, \tag{221}
\end{align*}
$$

Eq. (221) corresponds to eq. (37) in the text.

## M Welfare Analysis

In this section, we investigate analytically the welfare effects of an unanticipated tax reform which involves simultaneously cutting the labor tax by $\mathrm{d} \tau^{j}<0(j=F, H)$ and raising the consumption tax or the progressive wage taxes by $\mathrm{d} \tau^{k}>0(k=c, H)$. We denote by $\phi(t)$ the instantaneous utility at time $t$ :

$$
\begin{equation*}
\phi(t)=u(C(t))+v(L(t)), \tag{222}
\end{equation*}
$$

and by $U$ its discounted value over an infinite horizon:

$$
\begin{equation*}
U=\int_{0}^{\infty} \phi(t) \exp (-\delta t) \mathrm{d} t \tag{223}
\end{equation*}
$$

## M. 1 Instantaneous Welfare

We first linearize the instantaneous utility function (222) in the neighborhood of the steady-state:

$$
\begin{equation*}
\phi(t)=\tilde{\phi}+u_{C}(\tilde{C})(C(t)-\tilde{C})+v_{L}(\tilde{L})(L(t)-\tilde{L}) \tag{224}
\end{equation*}
$$

with $\tilde{\phi}$ given by

$$
\begin{equation*}
\tilde{\phi}=u(\tilde{C})+v(\tilde{L}) \tag{225}
\end{equation*}
$$

By substituting solutions for $C(t)$ and $L(t)$, we obtain the stable solution for instantaneous welfare:

$$
\begin{equation*}
\phi(t)=\tilde{\phi}+\left[u_{C} C_{P}+v_{L} L_{P}\right] \omega_{2}^{1}\left(K_{0}-\tilde{K}\right) e^{\nu_{1} t} \tag{226}
\end{equation*}
$$

where partial derivatives are evaluated at the steady-state, i. e. $u_{C}=u_{C}(\tilde{C})$ and $v_{L}=v_{L}(\tilde{L})$. We estimate the expression in square brackets by making use of the first-order conditions for consumption and labor supply supply decisions evaluated at the steady-state, i. e. $u_{C}=P_{C} \bar{\lambda}\left(1+\tau^{C}\right)$ and $v_{L}=-\bar{\lambda} \tilde{W}^{A}$. We obtain:

$$
\begin{equation*}
u_{C} C_{P}+v_{L} L_{P}=\bar{\lambda}\left[-\sigma_{C} \tilde{C}^{N}\left(1+\tau^{C}\right)+\frac{\tilde{W}^{A}}{\tilde{W}^{F}} \sigma_{L} \tilde{L} \tilde{k}^{T} \tilde{\Lambda} \frac{\tilde{h}}{\mu\left(\tilde{k}^{N}-\tilde{k}^{T}\right)}\right] \gtrless 0 . \tag{227}
\end{equation*}
$$

Evaluate (222) at the steady-state and differentiate allows us to derive the long-run change of $\phi$ after a tax reform:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{\phi}\right|^{j, k}=\Phi^{j, k}\left[u_{C} \frac{\mathrm{~d} \tilde{C}}{\mathrm{~d} \tau^{j}} \mathrm{~d} \tau^{j}+v_{L} \frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}\right] \mathrm{d} \tau^{j} \gtrless 0 \tag{228}
\end{equation*}
$$

where $0<\Phi^{j, k}<1$.
Evaluate (226) at time $t=0$ and differentiate allows us to derive the initial reaction of $\phi$ after a tax reform:

$$
\begin{align*}
\left.\mathrm{d} \phi(0)\right|^{j, k} & =\mathrm{d} \tilde{\phi}^{j, k}-\left.\left(u_{C} C_{P}+v_{L} L_{P}\right) \omega_{2}^{1} \mathrm{~d} \tilde{K}\right|^{j, k} \\
& =\Phi^{j, k}\left\{u_{C}\left[\frac{\mathrm{~d} \tilde{C}}{\mathrm{~d} \tau^{j}}-C_{P} \omega_{2}^{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}\right]+v_{L}\left[\frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}-L_{P} \omega_{2}^{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}\right]\right\} \mathrm{d} \tau^{j}, \tag{229}
\end{align*}
$$

where we used (228) to get (229).
Case $k^{N}>k^{T}$
If the non-traded sector is more capital intensive than the traded sector, the long-run change of $\phi$ after a tax reform is:

$$
\begin{align*}
\left.\mathrm{d} \tilde{\phi}\right|^{j, k} & =\Phi^{j, k}\left\{P_{C} \bar{\lambda}\left(1+\tau^{C}\right)\left[1-\frac{\tilde{W}^{A}}{\tilde{W}^{F}} \frac{1}{P_{C}\left(1+\tau^{C}\right)}\right] \frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{j}} \mathrm{~d} \tau^{j}\right. \\
& \left.+\Gamma^{j} \bar{\lambda} \frac{\sigma_{L} \tilde{L}}{\Delta} \tilde{W}^{A} \frac{\sigma_{C} P_{C} \tilde{C}}{\tilde{P}} \frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{2} \tilde{\Lambda}\right)\left(\alpha_{C}+\frac{\tilde{k}^{T} \tilde{P} \nu_{2}}{\tilde{W}^{F}}\right)\right\} \mathrm{d} \tau^{j} \gtrless 0, \tag{230}
\end{align*}
$$

where $0<\Phi^{j, k}<1$ and $\Gamma^{j}>0$. We substituted $u_{C}=P_{C} \bar{\lambda}\left(1+\tau^{C}\right)$ and $v_{L}=-\bar{\lambda} \tilde{W}^{A}$, and we used the fact that $\frac{\mathrm{d} \tilde{L}}{\mathrm{~d} \tau^{j}}=\frac{P_{C}}{\tilde{W}^{F}} \frac{\mathrm{~d} \tilde{C}}{\mathrm{~d} \tau^{j}}-\Gamma^{j} \frac{\sigma_{L} \tilde{L}}{\Delta} \frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\nu_{2}}\left(\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T} \nu_{2} \tilde{\Lambda}\right) \sigma_{C} \tilde{C}^{N}$ to determine (230). Since the sign of the long-run change of instantaneous utility is ambiguous, we dot not calculate the initial reaction of $\phi$ which in particular depends on eq. (230).

Case $k^{T}>k^{N}$
If the traded sector is more capital intensive than the non-traded sector, the long-run change of $\phi$ after a tax reform is:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{\phi}\right|^{j, k}=\Phi^{j, k} P_{C} \bar{\lambda}\left(1+\tau^{C}\right)\left[1-\frac{\tilde{W}^{A}}{\tilde{W}^{F}} \frac{1}{1+\tau^{C}}\right] \frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{j}} \mathrm{~d} \tau^{j}>0, \tag{231}
\end{equation*}
$$

where $0<\Phi^{j, k}<1$. We substituted $u_{C}=P_{C} \bar{\lambda}\left(1+\tau^{C}\right)$ and $v_{L}=-\bar{\lambda} \tilde{W}^{A}$, and we used the fact that $\frac{\mathrm{d} \tilde{L}}{\mathrm{~d} \tau^{j}}=\frac{P_{C}}{\tilde{W}^{F}} \frac{\mathrm{~d} \tilde{C}}{\mathrm{~d} \tau^{j}}$ to determine (231).

## M. 2 Overall Welfare

Until now, we have analyzed the instantaneous welfare implications of an unanticipated permanent tax reform, say at different points of times. To address welfare effects in a convenient way within an intertemporal-maximizing framework, we have to evaluate the discounted value of (222) over the agent's infinite planning horizon. Whereas the change of overall welfare can be estimated numerically, we determine its measure along a transitional path after a tax restructuring.

In order to have a correct and comprehensive measure of welfare, we calculate first the discounted value of instantaneous welfare over the entire planning horizon:

$$
\begin{align*}
U & =\frac{\tilde{\phi}}{\delta}+\frac{\left[u_{C} C_{P}+v_{L} L_{P}\right] \omega_{2}^{1}}{r^{\star}-\nu_{1}} A_{1} \\
& =\frac{\tilde{\phi}}{\delta}+\frac{\phi(0)-\tilde{\phi}}{r^{\star}-\nu_{1}} . \tag{232}
\end{align*}
$$

The first term on the right hand-side of (232) represents the capitalized value of instantaneous welfare evaluated at the steady-state. The second term on the RHS of (232) vanishes whenever the traded sector is more capital intensive than the non traded sector since the dynamics of the real exchange degenerate. If consumption reacts strongly on impact and labor is not too much responsive, then $\phi(0)$ can overshoot its long-run level which exerts a positive influence on overall welfare.

Case $k^{N}>k^{T}$
If the non-traded sector is more capital intensive than the traded sector, the long-run change of $U$ after a tax reform is:

$$
\begin{align*}
\left.\mathrm{d} U\right|^{j, k} & =\left.\frac{1}{\delta} \mathrm{~d} \tilde{\phi}\right|^{j, k}-\left.\frac{\left[u_{C} C_{P}+v_{L} L_{P}\right] \omega_{2}^{1}}{r^{\star}-\nu_{1}} \mathrm{~d} \tilde{K}\right|^{j, k} \\
& =\frac{1}{r^{\star} \nu_{2}} \Phi^{j, k}\left\{u_{C}\left[\frac{\mathrm{~d} \tilde{C}}{\mathrm{~d} \tau^{j}}-r^{\star} C_{P} \omega_{2}^{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}\right]+v_{L}\left[\frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}-r^{\star} L_{P} \omega_{2}^{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}}\right]\right\} \mathrm{d} \tau^{j} \gtrless 0 \tag{233}
\end{align*}
$$

with

$$
\begin{align*}
\frac{\mathrm{d} \tilde{C}}{\mathrm{~d} \tau^{j}}-r^{\star} C_{P} \omega_{2}^{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}} & =\frac{\sigma_{L} \tilde{L}}{\Delta} \sigma_{C} \tilde{C} \Upsilon^{j}\left\{\tilde{P} \nu_{1} \tilde{k}^{N}\left[1+\alpha_{C} \sigma_{C} \frac{\tilde{C}^{N}}{\tilde{P}} \frac{r^{\star}}{\nu_{2}} \omega_{2}^{1}\right]\right. \\
& \left.+r^{\star} \tilde{k}^{T} \nu_{2} \omega_{2}^{1}\left[\sigma_{C} \tilde{C}^{N} \alpha_{C}-\sigma_{L} \tilde{L} \tilde{k}^{T} \tilde{\Lambda}\right]\right\}<0  \tag{234a}\\
\frac{\mathrm{~d} \tilde{L}}{\mathrm{~d} \tau^{j}}-r^{\star} L_{P} \omega_{2}^{1} \frac{\mathrm{~d} \tilde{K}}{\mathrm{~d} \tau^{j}} & =-\frac{\sigma_{L} \tilde{L}}{\Delta} \sigma_{C} \tilde{C} \Upsilon^{j}\left\{\nu_{2}\left[1+\alpha_{C} \sigma_{C} \frac{\tilde{C}^{N}}{\tilde{P}} \frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\nu_{2}}\right]\right. \\
& \left.+\frac{r^{\star}}{\nu_{2}} \frac{\omega_{2}^{1}}{\tilde{P}} \sigma_{L} \tilde{L} \tilde{k}^{T} \tilde{\Lambda}\left[\alpha_{C}\left(1-\sigma_{C}\right)+\frac{\tilde{P} \nu_{2} \tilde{k}^{T}}{\tilde{W}^{F}}\right]\right\} \lessgtr 0 \tag{234b}
\end{align*}
$$

Case $k^{T}>k^{N}$
If the traded sector is more capital intensive than the non-traded sector, the long-run change of $U$ after a tax reform is:

$$
\begin{equation*}
\left.\mathrm{d} U\right|^{j, k}=\left.\frac{1}{\delta} \mathrm{~d} \tilde{\phi}\right|^{j, k}>0, \tag{235}
\end{equation*}
$$

where $\left.\mathrm{d} \tilde{\phi}\right|^{j, k}>0$ is given by (231).

## N The Case of Endogenous Markup

The framework builds on Jaimovich and Floetotto [2008]. While we consider the case of an endogenous markup, it holds for an exogenous markup, though in the latter case the number of competitors is large enough so that the price-elasticity of demand is not affected by firm entry. There are two sectors in the economy: a perfectly competitive sector which produces a traded good denoted by the superscript $T$ and an imperfectly competitive sector which produces a non-traded good denoted by the superscript $N$. We assume that each producer of a unique variety of the non-traded good has the following technology $X_{j}^{N}=H\left(\mathcal{K}_{j}, \mathcal{L}_{j}\right)$ with $\mathcal{K}_{j}$ the capital stock and $\mathcal{L}_{j}$ labor.

## N. 1 Framework

The final non-traded output, $Y^{N}$, is produced in a competitive retail sector using a constant-returns-to-scale production function which aggregates a continuum measure one of sectoral non traded goods:

$$
\begin{equation*}
Y^{N}=\left[\int_{0}^{1}\left(\mathcal{Q}_{j}^{N}\right)^{\frac{\omega-1}{\omega}} \mathrm{~d} j\right]^{\frac{\omega}{\omega-1}} \tag{236}
\end{equation*}
$$

where $\omega>0$ represents the elasticity of substitution between any two different sectoral goods and $\mathcal{Q}_{j}^{N}$ stands for intermediate consumption of sector'j variety (with $j \in[0, N]$ ). The final good producers behave competitively, and the households use the final good for both consumption and investment.

In each of the $j$ sectors, there are $N>1$ firms producing differentiated goods that are aggregated into a sectoral non traded good by a CES aggregating function. The non traded output sectoral $\operatorname{good} j$ is: ${ }^{51}$

$$
\begin{equation*}
\mathcal{Q}_{j}^{N}=N^{-\frac{1}{\epsilon-1}}\left[\int_{0}^{N}\left(\mathcal{X}_{i, j}^{N}\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{~d} i\right]^{\frac{\epsilon}{\epsilon-1}} \tag{237}
\end{equation*}
$$

where $\mathcal{X}_{i, j}^{N}$ stands for output of firm $i$ in sector $j$ and $\epsilon$ is the elasticity of substitution between any two varieties.

Denoting by $P$ and $\mathcal{P}_{j}$ the relative price of the final good and of the jth variety of the intermediate good, respectively, the profit of the final good producer is:

$$
\begin{equation*}
\Pi^{N}=p\left[\int_{0}^{N}\left(\mathcal{Q}_{j}^{N}\right)^{\frac{\omega-1}{\omega}} \mathrm{~d} j\right]^{\frac{\omega}{\omega-1}}-\int_{0}^{1} \mathcal{P}_{j} \mathcal{Q}_{j}^{N} \mathrm{~d} j . \tag{238}
\end{equation*}
$$

Total cost minimizing for a given level of final output gives the (intratemporal) demand function for each input:

$$
\begin{equation*}
\mathcal{Q}_{j}^{N}=\left(\frac{\mathcal{P}_{j}}{p}\right)^{-\omega} Y^{N} \tag{239}
\end{equation*}
$$

and the price of the final output is given by:

$$
\begin{equation*}
P=\left(\int_{0}^{1} \mathcal{P}_{j}^{1-\omega} \mathrm{d} j\right)^{\frac{1}{1-\omega}}, \tag{240}
\end{equation*}
$$

where $\mathcal{P}_{j}$ is the price index of sector $j$ and $p$ is the price of the final good.
Within each sector, there is monopolistic competition; each firm that produces one variety $\mathcal{X}_{i, j}^{N}$ is a price setter. Intermediate output $\mathcal{X}_{i, j}^{N}$ is produced using capital $\mathcal{K}_{i, j}^{N}$ and labor $\mathcal{L}_{i, j}^{N}$ :

$$
\begin{equation*}
\mathcal{X}_{i, j}^{N}=H\left(\mathcal{K}_{i, j}^{N}, \mathcal{L}_{i, j}^{N}\right) \tag{241}
\end{equation*}
$$

Denoting by $\mathcal{P}_{i, j}$ the price of good $i$ in sector $j$, the profit function for the jth sector good producer denoted by $\pi_{j}^{N}$ is:

$$
\begin{equation*}
\pi_{j}^{N} \equiv \mathcal{P}_{j} N^{-\frac{1}{\epsilon-1}}\left(\int_{0}^{N}\left(\mathcal{X}_{i, j}^{N}\right)^{\frac{\epsilon-1}{\epsilon}} \mathrm{~d} i\right)^{\frac{\epsilon}{\epsilon-1}}-\int_{0}^{N} \mathcal{P}_{i, j} \mathcal{X}_{i, j}^{N} \mathrm{~d} i \tag{242}
\end{equation*}
$$

The demand faced by each producer $\mathcal{X}_{i, j}^{N}$ is defined as:

$$
\begin{equation*}
\mathcal{X}_{i, j}^{N}=\left(\frac{\mathcal{P}_{i, j}}{\mathcal{P}_{j}}\right)^{-\epsilon} \frac{\mathcal{Q}_{j}^{N}}{N} \tag{243}
\end{equation*}
$$

[^25]and the price index of sector $j$ is given by:
\[

$$
\begin{equation*}
\mathcal{P}_{j}=N^{-\frac{1}{1-\epsilon}}\left(\int_{0}^{N} \mathcal{P}_{i, j}^{1-\epsilon} \mathrm{d} i\right)^{\frac{1}{1-\epsilon}} \tag{244}
\end{equation*}
$$

\]

Combining (239) and (241), the demand for variety $\mathcal{X}_{i, j}^{N}$ can be expressed in terms of the relative price of the final non traded good:

$$
\begin{equation*}
\mathcal{X}_{i, j}^{N}=\left(\frac{\mathcal{P}_{i, j}}{\mathcal{P}_{j}}\right)^{-\epsilon}\left(\frac{\mathcal{P}_{j}}{p}\right)^{-\omega} \frac{Y^{N}}{N} \tag{245}
\end{equation*}
$$

In order to operate, each intermediate good producer must pay a fixed cost denoted by $F C$ measured in terms of the final good which is assumed to be symmetric across firms. Each firm $j$ chooses capital and labor to maximize profits. The profit function for the ith producer in sector $j$ denoted by $\pi_{i, j}^{N}$ is:

$$
\begin{equation*}
\pi_{i, j}^{N} \equiv \mathcal{P}_{j} H\left(\mathcal{K}_{j}^{N}, \mathcal{L}_{j}^{N}\right)-r^{K} \mathcal{K}_{j}^{N}-W^{F} \mathcal{L}_{j}^{N}-p F C \tag{246}
\end{equation*}
$$

The demands for capital and hours worked are given by the equalities of the markup-adjusted marginal revenues of capital $\frac{\mathcal{P}_{j} H_{K}}{\mu}$ and labor $\frac{\mathcal{P}_{j} H_{L}}{\mu}$, to the capital rental rate $R^{K}$ and the producer wage $W^{F}$, respectively.

## N. 2 First-Order Conditions

The current-value Hamiltonian for the j-th firm's optimization problem in the non traded sector writes as follows:

$$
\begin{equation*}
\mathcal{H}_{j}^{N}=\mathcal{P}_{i, j} H\left(\mathcal{K}_{j}^{N}, \mathcal{L}_{j}^{N}\right)-r^{K} \mathcal{K}_{i, j}^{N}-W^{F} \mathcal{L}_{i, j}^{N}-P F C+\eta\left[H\left(\mathcal{K}_{i, j}^{N}, \mathcal{L}_{i, j}^{N}\right)-\mathcal{X}_{i, j}^{N}\right] \tag{247}
\end{equation*}
$$

where $\mathcal{X}_{j}^{N}$ stands for the demand for variety $j$; firm $j$ chooses its price $\varrho_{j}$ to maximize profits treating the factor prices as given. First-order conditions for the non traded sector write as follows:

$$
\begin{align*}
\mathcal{P}_{j} H_{K}+\eta H_{K} & =R^{K},  \tag{248a}\\
\mathcal{P}_{j} H_{L}+\eta H_{L} & =W^{F}  \tag{248b}\\
\eta & =\mathcal{P}_{j}^{\prime} \mathcal{X}_{i, j}^{N}, \tag{248c}
\end{align*}
$$

Combining (248a)-(248b) with (248c) yields:

$$
\begin{align*}
\mathcal{P}_{i, j} H_{K}\left(1-\frac{1}{e}\right) & =R^{K}  \tag{249a}\\
\mathcal{P}_{i, j} H_{L}\left(1-\frac{1}{e}\right) & =W^{F} \tag{249b}
\end{align*}
$$

where we used the fact that $-\frac{\mathcal{P}_{i, j}^{\prime}}{\mathcal{P}_{i, j} \mathcal{X}_{i, j}^{\mathcal{N}^{N}}}=\frac{1}{e}$.
We consider a symmetric equilibrium where all firms in the intermediate good sector produce the output level $\mathcal{X}_{i, j}^{N}=\mathcal{X}^{N}$ with the same quantities of labor $\mathcal{L}_{i, j}^{N}=\mathcal{L}^{N}$ and capital $\mathcal{K}_{i, j}^{N}=\mathcal{K}^{N}$. Hence, the aggregate stock of physical capital and hours worked are $K^{N}=N \mathcal{K}^{N}$ and $L^{N}=N \mathcal{L}^{N}$, respectively. They also set the same price $\mathcal{P}_{i, j}=\mathcal{P}$. Hence, eq. (240) and eq. (244) imply that $\mathcal{P}=P$.

Defining the markup $\mu$ as $\frac{e}{e-1}$, first-order conditions can be rewritten as follows:

$$
\begin{equation*}
P \frac{H_{K}}{\mu}=R^{K}, \quad P \frac{H_{L}}{\mu}=W^{F} \tag{250}
\end{equation*}
$$

We follow Yang and Heijdra [1993] and Jaimovich and Floetotto [2008] by taking into account the influence of the individual price on the sectoral price index:

$$
\begin{equation*}
e(N)=\epsilon-\frac{(\epsilon-\omega)}{N}, \quad N \in(1, \infty) \tag{251}
\end{equation*}
$$

As will be useful later, we calculate expressions of the partial derivatives of the price-elasticity of demand and the markup with respect to the number of firms:

$$
\begin{equation*}
e_{N}=\frac{\partial e}{\partial N}=\frac{\epsilon-\omega}{N^{2}}>0, \quad \mu_{N}=\frac{\partial \mu}{\partial N}=-\frac{e_{N}}{(e-1)^{2}}=-\frac{e_{N}}{e-1} \frac{\mu}{e}<0 \tag{252}
\end{equation*}
$$

where we let $\mu=\frac{e}{e-1}$.

We further assume that free entry drives profits down to zero in each nindustry at each instant of time. Using constant returns to scale in production, i. e. $X=H(K, L)=H_{K} K+H_{L} L$, and the zero profit condition, in the aggregate, we have

$$
\begin{equation*}
P H\left(K^{N}, L^{N}\right)-R^{K} K^{N}-W^{F} L^{N}-P N F C=0 . \tag{253}
\end{equation*}
$$

Substituting the short-run static solution for non traded output (51), the zero-profit condition (253) can be rewritten as:

$$
\begin{equation*}
Y^{N}\left(K, P, \bar{\lambda}, \tau^{F}, \tau^{H}, \mu(N)\right)\left(1-\frac{1}{\mu(N)}\right)=N F C . \tag{254}
\end{equation*}
$$

## N. 3 Short-Run Static Solution for the Number of Firms

The zero profit condition (254) can be solved for the number of producers in the non traded sector:

$$
\begin{equation*}
N=N\left(K, P, \bar{\lambda}, \tau^{F}, \tau^{H}\right), \tag{255}
\end{equation*}
$$

with partial derivatives given by:

$$
\begin{equation*}
N_{X} \equiv \frac{\partial N}{\partial x}=-\frac{Y_{X}^{N} \omega_{F C}}{\chi} \gtrless 0 \tag{256}
\end{equation*}
$$

where $X=K, P, \bar{\lambda}, \tau^{F}, \tau^{H}, \omega_{F C} \equiv N F C / Y^{N}$ stands for the share of fixed costs in markup adjusted output and we set

$$
\begin{equation*}
\chi=\frac{Y^{N}}{N}\left\{\left[\eta_{Y^{N}, \mu}(\mu-1)+1\right] \frac{\eta_{\mu, N}}{\mu}-\omega_{F C}\right\}, \tag{257}
\end{equation*}
$$

Inspection of (257) shows that $\chi<0$ if $\eta_{\mu, N}$ is not too large. This implies that an input inflow in the non traded sector that raises $Y^{N}$ and thereby yields to profit opportunities stimulates entry of firms.

## N. 4 Equilibrium Dynamics and Formal Solutions

Inserting short-run static solutions for non traded output and consumption, given by (51) and (43) respectively, into the non traded good market-clearing condition (57a), and inserting the shortrun static solution for sectoral the capital-labor ratio in the non traded good sector (45) into the dynamic equation for the real exchange rate (57b), and substituting the short-run static solution for the number of firms (255) yields:

$$
\begin{align*}
\dot{K} & =\frac{Y^{N}\{K, P, \mu[N(K, P)]\}}{\mu[N(K, p)]}-C^{N}(P)-\delta_{K} K-G^{N}  \tag{258a}\\
\dot{P} & =P\left\{r^{\star}+\delta_{K}-\frac{h_{k}\left(k^{N}\{P, \mu[N(K, P)]\}\right)}{\mu[N(K, P)]}\right\} \tag{258b}
\end{align*}
$$

For clarity purposes, we dropped variables which are constant over time as arguments of short-run static solutions.

Linearizing these two equations around the steady-state, and denoting by $\tilde{x}=\tilde{K}, \tilde{P}$ the steadystate values of $x=K, P$, we obtain in a matrix form:

$$
\begin{equation*}
(\dot{K}, \dot{p})^{T}=J(K(t)-\tilde{K}, P(t)-\tilde{P})^{T} \tag{259}
\end{equation*}
$$

where $J$ is given by

$$
J \equiv\left(\begin{array}{ll}
b_{11} & b_{12}  \tag{260}\\
b_{21} & b_{22}
\end{array}\right)
$$

with

$$
\begin{align*}
b_{11} & =\frac{Y^{N}}{\mu}\left[\frac{Y_{K}^{N}}{Y^{N}}-\frac{\mu_{N}}{\mu} N_{K}\left(1-\frac{Y_{\mu}^{N} \mu}{Y^{N}}\right)\right]-\delta_{K},  \tag{261a}\\
b_{1} 2 & =\frac{Y^{N}}{\mu}\left[\frac{Y_{P}^{N}}{Y^{N}}-\frac{\mu_{N}}{\mu} N_{p}\left(1-\frac{Y_{\mu}^{N} \mu}{Y^{N}}\right)\right]-c_{p}^{N}  \tag{261b}\\
b_{21} & =\frac{P}{\mu} h_{k k} \frac{\mu_{N} N_{K}}{\mu} k^{N}\left(\frac{h_{k}}{h_{k k} k^{N}}-\frac{k_{\mu}^{N} \mu}{k^{N}}\right),  \tag{261c}\\
b_{22} & =-\frac{P}{\mu} h_{k k}\left[k_{p}^{N}-\frac{\mu_{N} N_{p}}{\mu} k^{N}\left(\frac{h_{k}}{h_{k k} k^{N}}-\frac{k_{\mu}^{N} \mu}{k^{N}}\right)\right] . \tag{261d}
\end{align*}
$$

## Equilibrium Dynamics

The determinant of the Jacobian matrix is unambiguously negative:
Det J $=b_{11} b_{22}-b_{12} b_{21}$

$$
\begin{align*}
= & \left(\frac{Y_{K}^{N}}{\mu}-\delta_{K}\right)\left[\frac{Y_{K}^{T}}{\tilde{P}}+\frac{P}{\mu} h_{k k} k^{N} \frac{\mu_{N} N_{p}}{\mu}\left(\frac{h_{k}}{h_{k k} k^{N}}-\frac{k_{\mu}^{N} \mu}{k^{N}}\right)\right] \\
& -\frac{\mu_{N}}{\mu} N_{K}\left[\frac{Y^{N}}{\mu}\left(1-\frac{Y_{\mu}^{N} \mu}{Y^{N}}\right) \frac{Y_{K}^{T}}{\tilde{P}}+\left(\frac{Y_{P}^{N}}{\mu}-C_{P}^{N}\right) \frac{P}{\mu} h_{k k} k^{N}\left(\frac{h_{k}}{h_{k k} k^{N}}-\frac{k_{\mu}^{N} \mu}{k^{N}}\right)\right] \tag{262}
\end{align*}
$$

and the trace is given by:

$$
\begin{align*}
\operatorname{Tr} \mathrm{J}= & b_{11}+b_{22}=\frac{Y_{K}^{T}}{\mu}+\frac{Y_{K}^{N}}{p}-\delta_{K} \\
& -\frac{\mu_{N}}{\mu}\left[N_{K} \frac{Y^{N}}{\mu}\left(1-\frac{Y_{\mu}^{N} \mu}{Y^{N}}\right)-N_{P} \frac{P}{\mu} h_{k k} k^{N}\left(\frac{h_{k}}{h_{k k} k^{N}}-\frac{k_{\mu}^{N} \mu}{k^{N}}\right)\right] \\
= & r^{\star}-\frac{\mu_{N}}{\mu} N_{K} \frac{Y^{N}}{\mu}>0 \tag{263}
\end{align*}
$$

where we used the fact that $\frac{Y_{K}^{T}}{\mu}+\frac{Y_{K}^{N}}{P}=\frac{h_{k}}{\mu}=r^{\star}+\delta_{K}$; the positive sign follows from $N_{K}>0$ and $\mu_{N}<0$.

Characteristic roots from $J$ are:

$$
\begin{equation*}
\nu_{i} \equiv \frac{1}{2}\left\{\operatorname{Tr} \mathrm{~J} \pm \sqrt{(\operatorname{Tr} \mathrm{J})^{2}-4 \operatorname{Det} \mathrm{~J}}\right\} \gtrless 0, \quad i=1,2 . \tag{264}
\end{equation*}
$$

We denote by $\nu_{1}<0$ and $\nu_{2}>0$ the stable and unstable real eigenvalues, satisfying

$$
\begin{equation*}
\nu_{1}<0<r^{\star}<\nu_{2} \tag{265}
\end{equation*}
$$

Since the system features one state variable, $K$, and one jump variable, $p$, the equilibrium yields a unique one-dimensional stable saddle-path.

General solutions are those described by (67) with eigenvector $\omega_{2}^{i}$ associated with eigenvalue $\mu_{i}$ given by:

$$
\begin{equation*}
\omega_{2}^{i}=\frac{\nu_{i}-b_{11}}{b_{12}} \tag{266}
\end{equation*}
$$

## Formal Solution for the Stock of Foreign Assets

Substituting first the short-run static solution for consumption in the traded good (43) and the short-run static solution for traded output (51) into the accumulation equation of traded bonds (13), and linearizing around the steady-state gives:

$$
\begin{equation*}
\dot{B}(t)=r^{\star}(B(t)-\tilde{B})+\left[Y_{K}^{T}+Y_{\mu}^{T} \mu_{N} N_{K}\right](K(t)-\tilde{K})+\left[\left(Y_{P}^{T}+Y_{\mu}^{T} \mu_{N} N_{P}\right)-C_{P}^{T}\right](P(t)-\tilde{P}) \tag{267}
\end{equation*}
$$

where $C_{P}^{T}$ is given by (44b).
Using the fact that $P(t)-\tilde{P}=\omega_{2}^{1}(K(t)-\tilde{K})$, setting

$$
\begin{equation*}
N_{1}=\left[Y_{K}^{T}+Y_{\mu}^{T} \mu_{N} N_{K}\right]+\left[\left(Y_{P}^{T}+Y_{\mu}^{T} \mu_{N} N_{P}\right)-C_{P}^{T}\right] \omega_{2}^{1} \tag{268}
\end{equation*}
$$

solving for the differential equation and invoking the transversality condition for intertemporal solvency, the stable solution for net foreign assets finally reduces to:

$$
\begin{equation*}
B(t)-\tilde{B}=\Phi_{1}(K(t)-\tilde{K}) \tag{269}
\end{equation*}
$$

and the linearized version of the nation's intertemporal budget constraint is:

$$
\begin{equation*}
\tilde{B}-B_{0}=\Phi_{1}\left(\tilde{K}-K_{0}\right) \tag{270}
\end{equation*}
$$

where we substituted $B_{1} \equiv K_{0}-\tilde{K}$.

## N. 5 Stable Solutions for $L, N$, and $W$

Linearizing the short-run static solution $N=N(K, P)$ gives the stable solution for the number of firms:

$$
\begin{align*}
N(t) & =\tilde{N}+N_{K}(K(t)-\tilde{K})+N_{P}(P(t)-\tilde{P}) \\
& =\tilde{N}+\left(N_{K}+N_{P} \omega_{2}^{1}\right)(K(t)-\tilde{K}) \tag{271}
\end{align*}
$$

Evaluating at time $t=0$ and differentiating gives the initial response of the number of firms :

$$
\begin{equation*}
\left.\mathrm{d} N(0)\right|^{j, k}=\left.\mathrm{d} \tilde{N}\right|^{j, k}-\left.\left(N_{K}+N_{P} \omega_{2}^{1}\right) \mathrm{d} \tilde{K}\right|^{j, k} . \tag{272}
\end{equation*}
$$

Linearizing the short-run static solution for labor $L=L(P, \mu)$, using the fact that $\mu=\mu(N)$, and substituting the appropriate solutions, the solution for $L(t)$ reads:

$$
\begin{align*}
L(t) & =\tilde{L}+L_{P}(P(t)-\tilde{P})+L_{\mu}(\mu(t)-\tilde{\mu})  \tag{273}\\
& =\tilde{L}+L_{P}\left[\omega_{2}^{1}-\frac{\tilde{P}}{\tilde{\mu}} \mu_{N}\left(N_{K}+N_{P} \omega_{2}^{1}\right)\right](K(t)-\tilde{K}), \tag{274}
\end{align*}
$$

where we used the fact that $L_{\mu}=-\frac{L_{P} p}{\mu}$. Evaluating at time $t=0$ and differentiating yields the initial response of employment:

$$
\begin{equation*}
\left.\mathrm{d} L(0)\right|^{j, k}=\mathrm{d} \tilde{L}^{j, k}-L_{P}\left[\omega_{2}^{1}-\frac{\tilde{P}}{\tilde{\mu}} \mu_{N}\left(N_{K}+N_{P} \omega_{2}^{1}\right)\right] \mathrm{d} \tilde{K}^{j, k} \tag{275}
\end{equation*}
$$

Linearizing the short-run static solution for the wage rate $W=W(P, \mu)$ and substituting appropriate solutions gives:

$$
\begin{align*}
W(t) & =\tilde{W}+W_{P} \omega_{2}^{1}(K(t)-\tilde{K})+w_{\mu} \mu_{N}(N(t)-\tilde{N}), \\
& =\tilde{W}+W_{P}\left[\omega_{2}^{1}-\frac{\tilde{P}}{\tilde{\mu}} \mu_{N}\left(N_{K}+N_{P} \omega_{1}^{1}\right)\right](K(t)-\tilde{K}), \tag{276}
\end{align*}
$$

where we used the fact that $W_{\mu}=-\frac{W_{P} P}{\mu}$. Evaluating at time $t=0$ and differentiating gives the initial response of the wage rate:

$$
\begin{equation*}
\left.\mathrm{d} W(0)\right|^{j, k}=\mathrm{d} \tilde{W}^{j, k}-W_{P}\left[\omega_{2}^{1}-\frac{\tilde{P}}{\tilde{\mu}} \mu_{N}\left(N_{K}+N_{P} \omega_{2}^{1}\right)\right] \mathrm{d} \tilde{K}^{j, k} \tag{277}
\end{equation*}
$$

## N. 6 Tax Multiplier for Overall Output

## Long-Run Tax Multiplier

Because overall output denoted by $Y$ is the sum of traded output $Y^{T}$ and non-traded output measured in terms of the traded good $\frac{P}{\mu} Y^{N}$, using the fact that $Y^{T} \equiv Y^{T}(K, L, P, \mu)$ and $Y^{N} \equiv$ $Y^{N}(K, L, P, \mu)$, remembering that a tax reform exerts a long-term effect on the relative price of non tradables, the steady-state change of overall output becomes:

$$
\begin{align*}
\left.\mathrm{d} \tilde{Y}\right|^{j, k} & =\left.\left(Y_{K}^{T}+\frac{\tilde{P}}{\mu} Y_{K}^{N}\right) \mathrm{d} \tilde{K}\right|^{j, k}+\left.\left(Y_{L}^{T}+\frac{\tilde{P}}{\mu} Y_{L}^{N}\right) \mathrm{d} \tilde{L}\right|^{j, k} \\
& =\left.\tilde{P} r^{\star} \mathrm{d} \tilde{K}\right|^{j, k}+\left.W^{F} \mathrm{~d} \tilde{L}\right|^{j, k}>0 . \tag{278}
\end{align*}
$$

where we used properties (54b) and (54c) to get (165); according to property (54a), denoting by a hat the partial derivative of $Y$ w. r.t. $P$ for given labor, $\hat{Y}_{P}^{T}+\frac{P}{\mu} \hat{Y}_{P}^{N}=\hat{Y}_{\mu}^{T}+\frac{P}{\mu} \hat{Y}_{\mu}^{N}=0$.

Using the fact that $Y^{T}(t)=C^{T}(t)+G^{T}-r^{\star} B(t)+C A(t)=C^{T}+G^{T}+N X(t)$ and $\frac{Y^{N}(t)}{\mu}=$ $C^{N}(t)+G^{N}+I(t)$, the overall output is equal to $Y(t)=P_{C}(P(t)) C(t)+G^{T}+P(t)+G^{N}+N X(t)+$ $I(t)$. The steady-state change of GDP following a tax reform is:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{Y}\right|^{j, k}=\left.\frac{\tilde{Y}^{N}}{\tilde{\mu}} \mathrm{~d} \tilde{P}\right|^{j, k}+\left.P_{C} \mathrm{~d} \tilde{C}\right|^{j, k}+\left.\mathrm{d} \tilde{N} X\right|^{j, k}+\left.\tilde{P} \mathrm{~d} \tilde{I}\right|^{j, k}, \tag{279}
\end{equation*}
$$

where $\left.\mathrm{d} \tilde{N X}\right|^{j, k}=-\left.r^{\star} \mathrm{d} \tilde{B}\right|^{j, k}$ and $\left.\mathrm{d} \tilde{I}\right|^{j, k}=\left.\delta_{K} \mathrm{~d} \tilde{K}\right|^{j, k}$.

## Initial Tax Multiplier

Keeping in mind that the capital stock is initially predetermined, the short-run tax multiplier writes as follows:

$$
\begin{align*}
\left.\mathrm{d} Y(0)\right|^{j, k} & =\left.\left(Y_{L}^{T}+\frac{P}{\mu} Y_{L}^{N}\right) \mathrm{d} L(0)\right|^{j, k}+\left.\left(\hat{Y}_{P}^{T}+\frac{P}{\mu} \hat{Y}_{P}^{N}\right) \mathrm{d} P(0)\right|^{j, k}+\left.\left(\hat{Y}_{\mu}^{T}+\frac{P}{\mu} \hat{Y}_{\mu}^{N}\right) \mu_{N} \mathrm{~d} N(0)\right|^{j, k} \\
& =\left.W^{F} \mathrm{~d} L(0)\right|^{j, k}>0 \tag{280}
\end{align*}
$$

where we used properties (54c) to get (166); according to property (54a), denoting by a hat the partial derivative of $Y$ w. r. t. $P$ for given labor, $\hat{Y}_{P}^{T}+\frac{P}{\mu} \hat{Y}_{P}^{N}=\hat{Y}_{\mu}^{T}+\frac{P}{\mu} \hat{Y}_{\mu}^{N}=0$.

Linearizing around the steady-state yields:

$$
Y(t)=\tilde{Y}+\frac{\tilde{Y}^{N}}{\tilde{\mu}}(P(t)-\tilde{P})+(N X(t)-\tilde{N X})+\tilde{P}(I(t)-\tilde{I})
$$

Evaluating at time $t=0$ and differentiating yields the initial reaction of GDP:

$$
\begin{equation*}
\left.\mathrm{d} Y(0)\right|^{j, k}=\left.\frac{\tilde{Y}^{N}}{\tilde{\mu}} \mathrm{~d} P(0)\right|^{j, k}+\left.P_{C} \mathrm{~d} C(0)\right|^{j, k}+\left.\mathrm{d} N X(0)\right|^{j, k}+\left.\mathrm{d} I(0)\right|^{j, k} \tag{281}
\end{equation*}
$$

where $\left.\mathrm{d} C(0)\right|^{j, k}=\left.\mathrm{d} \tilde{C}\right|^{j, k}-\left.C_{P} \omega_{2}^{1} \mathrm{~d} \tilde{K}\right|^{j, k},\left.\mathrm{~d} P(0)\right|^{j, k}=\left.\mathrm{d} \tilde{P}\right|^{j, k}-\left.\omega_{2}^{1} \mathrm{~d} \tilde{K}\right|^{j, k}, \mathrm{~d} I(0)=-\left.\mu_{1} \mathrm{~d} \tilde{K}\right|^{j, k}$ and $\left.\mathrm{d} N X(0)\right|^{j, k}=\left.\mathrm{d} C A(0)\right|^{j, k}=-\left.\mu_{1} \Phi_{1} \mathrm{~d} \tilde{K}\right|^{j, k}$.

## N. 7 Tax Multipliers for Sectoral Outputs

## Long-Run Sectoral Tax Multipliers

We calculate the tax multiplier for traded output by differentiating the short-run static solution for $Y^{T}$ evaluated at the steady-state:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{Y}^{T}\right|^{j, k}=\left.Y_{K}^{T} \mathrm{~d} \tilde{K}\right|^{j, k}+\left.Y_{L}^{T} \mathrm{~d} \tilde{L}\right|^{j, k}+\left.\hat{Y}_{P}^{T} \mathrm{~d} \tilde{P}\right|^{j, k}+\left.\hat{Y}_{\mu}^{T} \mu_{N} \mathrm{~d} \tilde{N}\right|^{j, k} \tag{282}
\end{equation*}
$$

where $\hat{Y}_{P}^{T}<0, \hat{Y}_{\mu}^{T}>0$ and $\mu_{N}<0$.
Using the fact that $Y^{T}(t)=C^{T}+G^{T}+N X(t)$ and totally differentiating yields the steady-state change of traded output following a tax reform:

$$
\begin{equation*}
\left.\mathrm{d} \tilde{Y}^{T}\right|^{j, k}=\left.\mathrm{d} \tilde{C}^{T}\right|^{j, k}+\left.\mathrm{d} \tilde{N X}\right|^{j, k} \tag{283}
\end{equation*}
$$

where $\left.\mathrm{d} \tilde{N X}\right|^{j, k}=-\left.r^{\star} \mathrm{d} \tilde{B}\right|^{j, k}$.
We calculate the tax multiplier for non-traded output by differentiating the short-run static solution for $Y^{N} / \mu$ evaluated at the steady-state:

$$
\begin{equation*}
\left.\frac{\tilde{P}}{\mu} \mathrm{~d} \tilde{Y}^{N}\right|^{j, k}=\left.\frac{\tilde{P}}{\mu} Y_{K}^{N} \mathrm{~d} \tilde{K}\right|^{j, k}+\left.\frac{\tilde{P}}{\mu} Y_{L}^{N} \mathrm{~d} \tilde{L}\right|^{j, k}+\left.\frac{\tilde{P}}{\mu} \hat{Y}_{P}^{T} \mathrm{~d} \tilde{P}\right|^{j, k}+\left.\frac{\tilde{P}}{\mu} \hat{Y}_{\mu}^{T} \mu_{N} \mathrm{~d} \tilde{N}\right|^{j, k} \tag{284}
\end{equation*}
$$

where $\hat{Y}_{P}^{N}>0, \hat{Y}_{\mu}^{T}<0$ and $\mu_{N}<0$.
Using the fact that $\frac{Y^{N}(t)}{\mu}=C^{N}(t)+G^{N}+I(t)$, and totally differentiating gives the steady-state change of non-traded output following a tax reform:

$$
\begin{equation*}
\left.\frac{1}{\tilde{\mu}} \mathrm{~d} \tilde{Y}^{N}\right|^{j, k}=\left.\mathrm{d} \tilde{C}^{N}\right|^{j, k}+\left.\mathrm{d} \tilde{I}\right|^{j, k}, \tag{285}
\end{equation*}
$$

where $\left.\mathrm{d} \tilde{I}\right|^{j, k}=\left.\delta_{K} \mathrm{~d} \tilde{K}\right|^{j, k}$.

## Short-Run Sectoral Tax Multipliers

$k^{N}>k^{T}$
Remembering that the short-run solution $Y^{T} \equiv Y^{T}(K, L, P, \mu)$, using the fact that the capital stock is initially predetermined, the short-run tax multiplier is given by:

$$
\begin{equation*}
\left.\mathrm{d} Y^{T}(0)\right|^{j, k}=\left.Y_{L}^{T} \mathrm{~d} L(0)\right|^{j, k}+\left.\hat{Y}_{P}^{T} \mathrm{~d} P(0)\right|^{j, k}+\left.\hat{Y}_{\mu}^{T} \mu_{N} \mathrm{~d} N(0)\right|^{j, k} \tag{286}
\end{equation*}
$$

where $\left.\mathrm{d} L(0)\right|^{j, k}$ and $\left.\mathrm{d} N(0)\right|^{j, k}$ are given by (272) and (275), respectively, and $\left.\mathrm{d} P(0)\right|^{j, k}=\left.\mathrm{d} \tilde{P}\right|^{j, k}-$ $\left.\omega_{2}^{1} \mathrm{~d} \tilde{K}\right|^{j, k}$.

Linearizing $Y^{T}(t)=C^{T}+G^{T}+N X(t)$ around the steady-state, evaluating at time $t=0$ and totally differentiating yields the initial change of traded output following a tax reform:

$$
\begin{equation*}
\left.\mathrm{d} Y^{T}(0)\right|^{j, k}=\left.\mathrm{d} C^{T}(0)\right|^{j, k}+\left.\mathrm{d} N X(0)\right|^{j, k} \tag{287}
\end{equation*}
$$

where $\left.\mathrm{d} N X(0)\right|^{j, k}=\left.\mathrm{d} C A(0)\right|^{j, k}=-\left.\mu_{1} \Phi_{1} \mathrm{~d} \tilde{K}\right|^{j, k}$ and $\left.\mathrm{d} C^{T}(0)\right|^{j, k}=-\left.C_{P}^{T} \omega_{2}^{1} \mathrm{~d} \tilde{K}\right|^{j, k}$.
Differentiating the short-run solution for $Y^{N} \equiv Y^{N}(K, L, P, \mu)$ and remembering that the capital stock is initially predetermined, the short-run tax multiplier for non-traded output is given by:

$$
\begin{equation*}
\left.\frac{P}{\mu} \mathrm{~d} Y^{N}(0)\right|^{j, k}=\left.\frac{P}{\mu} Y_{L}^{N} \mathrm{~d} L(0)\right|^{j, k}+\left.\frac{P}{\mu} \hat{Y}_{P}^{N} \mathrm{~d} P(0)\right|^{j, k}+\left.\frac{P}{\mu} \hat{Y}_{\mu}^{N} \mu_{N} \mathrm{~d} N(0)\right|^{j, k} \tag{288}
\end{equation*}
$$

where $\left.\mathrm{d} L(0)\right|^{j, k}$ and $\left.\mathrm{d} N(0)\right|^{j, k}$ are given by (272) and (275), respectively, and $\left.\mathrm{d} P(0)\right|^{j, k}=\left.\mathrm{d} \tilde{P}\right|^{j, k}-$ $\left.\omega_{2}^{1} \mathrm{~d} \tilde{K}\right|^{j, k}$.

Linearizing $\frac{Y^{N}(t)}{\mu}=C^{N}(t)+G^{N}+I(t)$, around the steady-state, evaluating at time $t=0$ and totally differentiating gives the initial change of non-traded output following a tax reform:

$$
\begin{equation*}
\left.\frac{1}{\tilde{\mu}} \mathrm{~d} \tilde{Y}^{N}\right|^{j, k}=\left.\mathrm{d} C^{N}(0)\right|^{j, k}+\left.\mathrm{d} I(0)\right|^{j, k} \tag{289}
\end{equation*}
$$

where $\left.\mathrm{d} C^{N}(0)\right|^{j, k}=-\left.C_{P}^{N} \omega_{2}^{1} \mathrm{~d} \tilde{K}\right|^{j, k}$ and $\mathrm{d} I(0)=-\left.\mu_{1} \mathrm{~d} \tilde{K}\right|^{j, k}$.

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Jaimovich, Nir and Joseph Floetotto (2008) Firm Dynamics, Markup Variations and the Business Cycle. Journal of Monetary Economics, 55, 1238-1252.

Schubert, Stefan F., and Stephen J. Turnovsky (2002) The Dynamics of Temporary Policies in a Small Open Economy. Review of International Economics 10(4), 604-622.

## Documents de travail du BETA

2012-01 Unanticipated vs. Anticipated Tax Reforms in a Two-Sector Open Economy Olivier CARDI, Romain RESTOUT, janvier 2012.

La présente liste ne comprend que les Documents de Travail publiés à partir du $1^{\text {er }}$ janvier 2012. La liste complète peut être donnée sur demande.
This list contains the Working Papers written after January 2012, 1rst. The complet list is available upon request.


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[^1]:    ${ }^{1}$ See Auerbach [2011] and IMF [2010].
    ${ }^{2}$ Coto-Martinez and Dixon [2003] use a similar framework to ours, but they analyze the effects of a permanent rise in government spending by assuming that the traded sector is more capital intensive than the non-traded sector.
    ${ }^{3}$ Non-tradable proportions are given in Table 3 (see Appendix A).

[^2]:    ${ }^{4}$ When studying the effects of revenue-neutral tax reforms, we consider alternatively a fall in payroll taxes or a drop in progressive wage taxes coordinated with a rise in consumption tax keeping the government budget balanced.

[^3]:    ${ }^{5}$ Note that the literature analyzing the effects of a tax reform that shifts the tax burden from labor to consumption commonly use a closed economy framework, see e.g. Auerbach [1996], Heer and Trede [2003], Lehmus [2011].
    ${ }^{6}$ Exploiting the distinction between anticipated and unanticipated tax shocks proposed by Mertens and Ravn [2009], Favero and Giavazzi [2011] also find that anticipated tax shocks have opposite effects on GDP in the pre-implementation and post-implementation periods.
    ${ }^{7}$ Yang [2005] and House and Shapiro [2006] analyze the effects of anticipated tax cuts by using a neoclassical framework. In contrast to these two studies, we consider an open economy model with tradables and non-tradables.
    ${ }^{8}$ According to Romer and Romer's [2010] estimates, an exogenous tax cut results in a trade balance deficit by driving down exports and raising imports. Cloyne [2011] finds similar results for the UK, although the response of exports is quite muted.

[^4]:    ${ }^{9}$ Our numerical results are close to the estimates provided by Perotti [2011] who finds that an unexpected 1 percentage point of GDP decrease in taxes leads to an increase in GDP by about 1.5 percentage points after three years.
    ${ }^{10}$ For a group of 14 OECD countries and 5 industries, Wu and Zhang [2000] find evidence of a significant impact of tax rates on markups. Specifically, the authors find that higher income tax raises the markups by reducing the number of firms.
    ${ }^{11}$ Note that our model is close to Coto-Martinez and Dixon's [2003] framework. In contrast to the authors, we allow for the markup to be endogenous and estimate numerically the effects of both unanticipated and anticipated tax reforms.

[^5]:    ${ }^{12}$ More details on the model as well as the derivations of the results which are stated below are provided in an Appendix which is available from the authors on request.

[^6]:    ${ }^{13}$ As stressed by Turnovsky and Sen [1995], allowing for traded capital investment would not affect the results (qualitatively). Furthermore, like Burstein et al. [2004] and Bems [2008], we find that the nontradable content of investment accounts for the lion's share of total investment expenditure (averaging to $60 \%$ ).
    ${ }^{14}$ The price of the traded good is determined on the world market and exogenously given for the small open economy.
    ${ }^{15}$ We abstract from capital income tax which is beyond the scope of this paper.

[^7]:    ${ }^{16}$ Specifically, we have $\alpha_{C}=\frac{(1-\varphi) P^{1-\phi}}{\varphi+(1-\varphi) P^{1-\phi}}$. Note that it depends negatively on the relative price $P$ as long as $\phi>1$.
    ${ }^{17}$ In the lines of De Gregorio, Giovannini and Wolf [1994], the non-traded sector is assumed to be monopolistically competitive. This assumption also relies upon observed empirical facts. According to our estimates for a sample of fourteen OECD economies, the markups in the traded sector average 1.2 with a

[^8]:    ${ }^{20}$ Government spending on traded $G^{T}$ and non traded goods $P G^{N}$ are considered for calibration purposes.

[^9]:    ${ }^{21}$ See the Appendix for further details.

[^10]:    ${ }^{22}$ If $k^{T}>k^{N}$, then $\Phi_{1}=-\tilde{P}<0$ and $\Phi_{2}=-\tilde{P}\left\{1+\frac{\omega_{2}^{2}}{\tilde{P} \nu_{1}}\left[\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L}^{T} \tilde{k}^{T}\left(\nu_{1}+\delta_{k}\right) \tilde{\Lambda}\right]\right\}$, with $0<\tilde{\Lambda} \equiv$ $\frac{\left(1-\tau^{H}\right)}{\left[\left(1-\tau^{H}\right)+\frac{\tau^{H}}{\tilde{W}}\right]}<1$. If $k^{N}>k^{T}$, then $\Phi_{1}=-\tilde{P}\left\{1+\frac{\omega_{2}^{1}}{\tilde{P} \nu_{2}}\left[\sigma_{C} \tilde{C}^{N}-\sigma_{L} \tilde{L} \tilde{k}^{T}\left(\nu_{2}+\delta_{K}\right) \tilde{\Lambda}\right]\right\}$ and $\Phi_{2}=-\tilde{P}$.

[^11]:    ${ }^{23}$ Following an unexpected tax reform, the economy moves along a stable path; hence, the trajectory for $B(t)$ is obtained by invoking the transversality condition $\lim _{t \rightarrow \infty} \bar{\lambda} B(t) e^{-r^{\star} t}=0$ which implies that the constant $B_{2}$ must be set to zero.
    ${ }^{24}$ Substituting first the short-run solutions, then linearizing the dynamic equation of the internationally traded bonds (13) in the neighborhood of the steady-state, substituting the solutions for $K(t)$ and $P(t)$ and finally invoking the transversality condition, we obtain the linearized version of the nation's intertemporal budget constraint (18).
    ${ }^{25}$ Since for all parameterizations, $\Phi_{1}$ is always negative, we assume $\Phi_{1}<0$ from now on. Hence, capital accumulation deteriorates the current account along the transitional path.

[^12]:    ${ }^{26}$ In deriving formal solutions, without loss of generality, we assume that the rate of depreciation of physical capital is zero. In the numerical analysis, we relax this assumption.
    ${ }^{27}$ Note that $\hat{\tau}^{F}=\frac{\mathrm{d} \tau^{F}}{1+\tau^{F}}$ and $\hat{\tau}^{H}=\frac{\mathrm{d} \tau^{H}}{1-\tau^{H}}$.

[^13]:    ${ }^{28}$ As will become clear when discussing numerical results, long-run effects are similar both qualitatively and quantitatively whether $k^{T}>k^{N}$ or $k^{N}>k^{T}$.
    ${ }^{29}$ The change in the marginal utility of wealth after a revenue-neutral tax reform is: $\left.\mathrm{d} \bar{\lambda}\right|^{j, C}=\frac{\partial \bar{\lambda}}{\partial \tau^{j}} \mathrm{~d} \tau^{j}+$ $\left.\frac{\partial \bar{\lambda}}{\partial \tau^{C}} \mathrm{~d} \tau^{C}\right|^{j, C}$. Substituting the steady-state change after a labor tax change, i.e. $\partial \bar{\lambda} / \partial \tau^{j}$, and the steady-state change after a consumption tax change, i.e. $\partial \bar{\lambda} / \partial \tau^{C}$, gives eq. (20).
    ${ }^{30}$ Denoting by $A \equiv B+P K$ the stock of financial wealth, at the steady-state, we have: $r^{\star} \tilde{A}+Z+\tilde{W}^{A} \tilde{L}=$ $P_{C}\left(1+\tau^{C}\right) \tilde{C}$. As long as $r^{\star} \tilde{A}+Z>0$, the consumption tax base is larger than the labor tax base.

[^14]:    ${ }^{31}$ Note that a similar conclusion is reached by Mendoza and Tesar [1998] who find that trade in world financial markets amplifies the rise in GDP after a tax reform as the open economy must service the debt accumulated during the transition by running a trade balance surplus in the long run.
    ${ }^{32}$ Note that in a one-sector model with perfectly competitive markets and linearly homogenous production function, the share of labor income in GDP is constant. Under the same assumptions, the aggregate labor share can be defined as a sectoral value added-weighted sum of the traded and non-traded labor shares. Because the share of sectoral output in GDP may vary, the aggregate labor share is no longer fixed.

[^15]:    ${ }^{33}$ To derive formal solutions after an anticipated future permanent tax cut, we applied the procedure developed by Schubert and Turnovsky [2002].
    ${ }^{34}$ Unfortunately, in contrast to an unanticipated tax reform, both long-run and short-run changes of macroeconomic aggregates cannot be expressed as scaled-down versions of changes after a labor tax cut associated with a drop in lump-sum transfer. Hence, the scaled-down term $\Phi^{j, k}$ vanishes.

[^16]:    ${ }^{35}$ Technically, the assumption $\beta=r^{\star}$ requires the joint determination of the transition and the steadystate.
    ${ }^{36}$ Table 3 shows the non-tradable content of GDP components for fourteen OECD countries.
    ${ }^{37}$ Note that consumption expenditure is $60 \%$ of GDP for our baseline calibration.
    ${ }^{38}$ Table 3 gives the values of $\theta^{j}(j=T, N)$ for fourteen OECD countries. The values of $\theta^{T}$ and $\theta^{N}$ we have chosen correspond roughly to the averages for countries with $k^{T}>k^{N}$. For these values, the non-tradable content of GDP and labor are $67 \%$ and $70 \%$, respectively. When $k^{N}>k^{T}$, we can use reverse but symmetric values for $\theta^{N}$ so that the difference between sectoral capital-labor ratios $k^{T}-k^{N}$ remains unchanged. For $\theta^{T}=0.3$ and $\theta^{N}=0.38$, the non-tradable content of GDP and labor are $73 \%$ and $70 \%$, respectively.
    ${ }^{39}$ Close to the average of the values reported in Table 3, the ratios $G^{T} / Y^{T}$ and $G^{N} / Y^{N}$ are $6 \%$ and $20 \%$ in the baseline calibration. The share of government spending in non-traded output is a bit small since we consider that the whole investment is non-traded.

[^17]:    ${ }^{40}$ When considering a revenue-neutral tax reform shifting payroll (wage) taxes to consumption taxes, $\tau^{F}$ $\left(\tau^{H}\right)$ must decrease by about $1.7(2.2)$ percentage points to lower the tax receipts by 1 percent of GDP.
    ${ }^{41}$ Results for alternative tax reforms when $k^{N}>k^{T}$ are available from the authors.
    ${ }^{42}$ Mertens and Ravn [2009] find a median for the implementation lag of tax changes of six quarters amongst tax shocks categorized as anticipated but stress that there is some variation in the anticipation lags.

[^18]:    ${ }^{43}$ The steady-state change of $X=C, L, K, N X$ after a change in progressive wage taxes can be related to the steady-state change of $X$ following a drop in payroll taxes as follows: $\frac{\hat{\tilde{x}}}{\hat{\tau}^{H}}=\frac{\Lambda^{H}}{\Lambda^{F}} \frac{\hat{\tilde{X}}^{F}}{\hat{\tau}^{F}}$ where $\Lambda^{F}$ and $\Lambda^{H}$ are terms which depend on tax rates, tax allowances and the wage rate. If $\Lambda^{H}>\Lambda^{F}$, then a cut in $\tau^{H}$ produces larger effects on $L$ and $K$ by raising further the after-tax wage rate.
    ${ }^{44}$ Following a tax reform keeping the marginal tax wedge constant, the after-tax wage increases by $\tilde{W} \frac{1-\tau^{H}}{1+\tau^{F}}$ as $\tau^{F}$ is decreased while the rise in $\tau^{H}$ lowers the after-tax wage by $(\tilde{W}-\kappa) \frac{1-\tau^{H}}{1+\tau^{F}}$. The lower $\kappa$ is, the smaller the effects of the tax reform keeping $\tau^{M}$ constant.

[^19]:    ${ }^{45}$ Since steady-state changes of $\beta_{L}$ are almost zero in all cases, we did not report the numerical results to economize space.

[^20]:    ${ }^{46}$ The numerical results are not reported in the paper but are available from the authors upon request.

[^21]:    ${ }^{47}$ Details of the derivation can be found in the Appendix.

[^22]:    ${ }^{48}$ Starting from the equality of labor marginal products between sectors, using the fact that $f_{k}=P h_{k}$ and $h_{k}=r^{\star}+\delta_{K}$, it is straightforward to prove that $b_{11}$ is positive in the case $k^{N}>k^{T}$.

[^23]:    ${ }^{49}$ In the traded sector which is perfectly competitive, we have : $Y^{T}=F_{L} L^{T}+R^{K} K^{T}=W^{F} L^{T}+R^{K} K^{T}$. Instead, in the non traded sector which is imperfectly competitive we have: $p Z^{N}=P \frac{H_{L}}{\mu} L^{N}+P \frac{H_{K}}{\mu} K^{N}$ or $P \mu Z^{N}=P Y^{N}=P h_{L} L^{N}+P h_{k} K^{N}=W^{F} L^{N}+R^{K} K^{N}$.

[^24]:    ${ }^{50}$ We have used the short-run static solutions for $W, L, Y$ by abstracting from exogenous variables for clarity purpose.

[^25]:    ${ }^{51}$ By having the term $N^{-\frac{1}{\epsilon-1}}$ in (237), the analysis abstracts from the variety effect and concentrates solely on the effects of markup variation.

