Moral Hazard, Aggregate Risk and Linear Financial Contracts

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Abstract

We study competitive equilibria with moral hazard in economies with aggregate risk and where trading occurs with an incomplete set of financial assets. The main conclusion of the paper is that, contrary to the individual risk economies, moral hazard is compatible with trading in competitive linear financial contracts, and gives rise to no manipulation problem. We establish existence of nonmanipulable equilibria provided that there are no relative price effects (e.g. a one-commodity economy), and that financial markets display nonlinearly homogeneous payoffs (e.g., nominal), and are sufficiently incomplete. Finally, we justify the linear contract as the optimal pricing schedule in a specific trading game with an auctioneer. Journal of Economic Literature Classification Numbers: D50, D82.

1. Introduction

When economic agents are asymmetrically and privately informed, optimal exclusive contracts generally entail unavoidable nonlinearities.[1] If contracts are nonexclusive, Helpman and La¤ont [9], Cresta [7] and more recently Bisin and Gottardi [3] suggest that nonlinearities are necessary even if the contract itself is not optimally designed. As a result, principals (…rms) do not have access to linear hedging instruments in these models, and …nancial markets display nonlinearities to overcome the manipulation problem.[2] This is a strong negative conclusion on the viability of linear …nancial hedging instruments in the presence of moral hazard, and the goal of this paper is to …nd out conditions under which such a conclusion is reversed.

All this literature on adverse selection and moral hazard within a general equilibrium framework (see also Prescott and Townsend [19]) models the uncertainty a¤ected by the private information as individual risk.[3] We claim here that, if risk is not individual, nonlinearity is not necessary for the absence of manipulation on …nancial markets. With e¤ort decisions a¤ecting aggregate risk, it may happen that informed traders do not want to trade upon their information in equilibrium. We introduce a model of competitive …nancial market interaction where there is moral hazard, and where the risk a¤ected by the private action is not individual, but aggregate. E¤ort decisions may a¤ect aggregate risk in economies where entrepreneurs sell claims on the pro…ts they make to the market, or trade in …nancial assets whose payo¤s are correlated with their pro…ts (such as derivative

[1] By exclusive we mean a contract that cannot be transferred, or such that no other contract with third parties can be written by the parties involved (in particular, by the informed party), or that otherwise has terms depending on the parties' activities outside the contract. An optimal exclusive contract is objective-maximizing for the uninformed trader (the contract-designer). By linear we mean a contract where its terms (prices and payo¤s) do not depend on the quantities exchanged (the number of contracts purchased or sold). Note that the contract dependence on private information (separating contract) usually implies nonlinearities in prices.

[2] Indeed, a more direct way of incorporating borrowing and lending with moral hazard and competitive commodity markets is to model the …nancial contract as nonlinear from the start. This is essentially the approach taken by Prescott and Townsend [19] and by Bennardo [2] and Lisboa [14] for the case of zero-pro…t contracts, and by Citanna and Villanacci [5] when contracts may yield positive pro…t or utility to the principals.

[3] An important exception is La¤ont [13], which shows existence of fully revealing incentive compatible rational expectations equilibrium. Hence it can be seen as a model of adverse selection and aggregate risk. However, La¤ont assumes the existence of a continuum of agents identical in type, whereas here we will not.

instruments), or else, in economies where individual risks are correlated. Such economies have been previously studied in a partial equilibrium framework by Jensen and Meckling [11], among others, and by Kihlstrom and Matthews [12] in a simpli…ed general equilibrium setup, lately extended by Magill and Quinzii [15]. All these models, however, assume nonlinear pricing or nonlinear price conjectures. Instead, our simple but key observation is that with aggregate risk and incomplete …nancial assets, the asset real payo¤s are generally determined in equilibrium, since generally budget constraints are not linearly homogeneous in commodity prices. In equilibrium, asset payo¤s may adjust to make …nancial contracts manipulation-free even if asset prices are linear and so are conjectures.

For simplicity of the analysis, we will focus on the case where assets are nominal. In these economies and with incomplete …nancial markets there is endogenous uncertainty, that is, uncertainty regarding the future equilibrium spot price level, a phenomenon also known as indeterminacy, using a more structural language.[4] Because of this additional uncertainty, common to both the uninformed and the informed trader, when …nancial markets are su¢ ciently incomplete (and assets have nominal returns) we show that with only one commodity there is enough noise in the equilibrium price system to conceal the information (based on trade or price observation, the uninformed cannot tell what action the informed trader has chosen). At the same time, the informed trader is also uncertain regarding the future and values the asset identically no matter what e¤ort has been exerted. In other words, while the information remains concealed, it is depleted of value in trading the …nancial assets, so that trade occurs without manipulation. Of course, other equilibria may arise where prices di¤er across e¤ort realizations, assuming that these prices have the property of being incentive compatible from the informed trader's viewpoint. However, this property is not general in these models (see Blume and Easley, [4]), and this is why we concentrate on equilibria where prices are constant with respect to e¤ort. Moreover, equilibria where prices reveal e¤ort choices can be also modeled as equilibria with nonlinear price conjectures as in Kihlstrom and Matthews [12], partly losing their appeal if our goal is to study linear pricing with moral hazard.

We conclude that there cannot be any manipulation, since either information is truthfully revealed, so that the informational asymmetry disappears in equilibrium

[4] Moreover, if one eliminated the indeterminacy in the model via the introduction of money (see Magill and Quinzii [16]), one could think that these equilibria can be the result of a monetary policy constrained by the linearity of contracts (no interference with the pricing mechanism) and the incentive compatibility of the outcome, rather than the result of price level uncertainty.

and, with it, the moral hazard problem, or it is not revealed, but then informed traders cannot use the information to manipulate the …nancial contract.

Our view of this solution goes through an existence proof for strong fully nonrevealing equilibria, that is, for equilibria which are fully nonrevealing in the usual sense, and where additionally tomorrow commodity prices do not reveal signals chosen by informed traders.[5] As an additional feature of these equilibria, informed traders choose the signal, as opposed to passively receiving it. These equilibria are technically an extension of a result by Polemarchakis and Siconol… [18].

In the existing literature on informed trading, our paper mostly resembles Dow and Gorton [8], Kihlstrom and Matthews [12], and Magill and Quinzii [15]. With the …rst, it shares the goal of obtaining nonrevelation of private and ex post unveri…able information without making use of exogenous noise. It di¤ers in the results, in particular since we get informed trading which nevertheless is not profitable. With the second two papers, ours shares the interest in modelling moral hazard in competitive equilibrium with aggregate risk. The issue of price-taking behavior is usually seen as a major hurdle for applying competitive models à la Arrow-Debreu to the analysis of moral hazard.[6] Kihlstrom and Matthews solve the conceptual issue by assuming that the uninformed trader can infer from the quantity traded the e¤ort exerted, and that the assets value depends monotonically on the quantity sold by the informed agent. Magill and Quinzii develop this idea into a fully-‡edged general equilibrium model, and derive some other conclusions along the lines of real payo¤ determination, highlightling the role of options in creating the appropriate managerial incentives. In our model, because of nonrevelation, the link between quantity traded, e¤ort and price is broken, and we argue that price may be correctly conjectured not to depend on the quantity exchanged by the informed trader, modulo the usual 'irrationality' of price-taking behavior as in standard models.

Sections 2, 3 and 4 introduce the model setup, the existence proof and its extension to the case when the informed trader is himself a principal hiring a worker who exerts unobservable e¤ort. This version of the paper considers economies with one informed trader (possibly with his agent/worker) and one uninformed

[5] This is required to embed the standard principal-agent relation in a general equilibrium economy, since if prices tomorrow were e¤ort-dependent, the compensation scheme could also depend on e¤ort, a situation ruled out by assumption in the standard model.

[6] For example, Radner ([20], [21]), introducing his notion of competitive equilibrium with sequential markets, limits its applicability to situations in which states of the world can always be veri…ed ex post, ruling out by …at embedding the moral hazard problem into his model.

trader, but results can easily be extended to the general case of many principal-agent pairs, and many uninformed traders. The extension in Section 4 is key to show the link with the principal-agent literature. In a principal-agent model, can a …rm/principal hedge against risks on …nancial markets, when investors cannot observe the worker/agent's compensation scheme? The canonical answer is yes, (a) through a riskfree asset (but hedging is very limited, of course; see implicitly or explicitly most of the partial equilibrium literature on contracts where no limit is imposed on wages o¤ered by a principal) ; or (b) provided the contract is non-linear (another principal-agent contract, where the principal is the bank lending money) ; or (c) provided the investor understands the relation between choice of e¤ort and …nancial structure of the …rm (in particular, retained equity à la Jensen and Meckling [11]). In this case, a nonlinear 'price conjecture' is formed. Our answer is yes, even if the contract is linear (i.e., borrowing on …nancial markets and not through a bank, say ; this is what we have in mind here), provided the contract has payo¤s nonlinearly homogenous in prices (e.g., nominal, like a bond), and markets are su¢ ciently incomplete. Nevertheless, it should be noticed that our equilibria are not necessarily low-e¤ort equilibria, and that contrary to solution (a), the asset is not risk-free.

In our de…nition of equilibrium the informed trader does not control the amount of information revelation through prices, but takes it as given, and we intentionally limit the uninformed agent's knowledge of the structure of the economy, and in particular, of the informed traders' payo¤s. To justify the price-taking assumption, we complete the paper by discussing the informed traders' strategic use of their private information. In order to address this issue, we assume common knowledge of the structure of the economy, and formalize the uninformed traders' ignorance of the informed traders' payo¤ as incomplete information regarding the cost of e¤ort. In particular, we allow the uninformed traders to have complete ignorance of the e¤ect of the private action on the trader's utility: the uninformed traders do not even know whether it is relatively costlier to exert one action over the other. The model has then hidden-action and hidden-information moral hazard. This also formalizes the fact that in Section 2, 3 and 4 it is assumed that the uninformed traders cannot invert the informed trader's best response. In Section 5 they can invert, but the best response function is not one-to-one: knowing prices and the informed trader's optimal quantities the uninformed trader cannot …nd out what e¤ort has been chosen. We …rst extend our proof of existence of equilibria in these incomplete information economies (Section 5), and then show that our equilibrium allocations are weakly implementable (Section 6).

Finally, in Section 6 we also present an alternative trading game where the mechanism designer is a pure clearing house, and show that our equilibrium allocation is also one of this game (pure strategy) Bayesian-Nash equilibrium outcomes. Although the main point of the paper is not to investigate the optimality of these linear contracts, the conclusion on the consistency of linear financial contracts and moral hazard is shown to be sustainable in some strategic environment, and we explain linearities as the result of optimal equilibrium behavior, although not in the usual sense of maximizing the utility of the uninformed trader(s).

2. The model

The following setup is used in the paper.

Time, uncertainty and information structure

In a finite horizon economy, with two periods denoted by $t = 0, 1$ let $\infty > S > 1$ be the states of the world in $t = 1$. There are $H = 2$ traders, and trader 1 has private information about an action he takes and that affects the probability distribution of future states.[7] We assume that trader 1's private action is discrete, and denote it by n, with $n \in N = \{1, 2\}$. In general, N could be any finite set. We interpret $n = 1$ as 'low', and $n = 2$ as 'high'. Trader 1's action affects the probability distribution over the future states of the world in a first order stochastic dominance sense with respect to the natural order of the states, which may coincide with the endowment levels of individuals. Let $\pi^{s,n} = \pi(s|n)$ be the conditional probability of state s given action n. Then $\sum_{s=1}^{s'} \pi^{s,1} \geq \sum_{s=1}^{s'} \pi^{s,2}$; for all $s' \in S$; with a strict inequality for some s', and we assume that $\pi^{s,n} > 0$; all s, n. This is common knowledge.

Trader 2 has no private information, and instead has a prior distribution over trader 1's actions N, denoted by π_2^n. At this point, we do not assume that trader 2, the uninformed trader, knows the informed trader's preferences. We take N as a primitive from the uninformed trader's perspective. All the information $h = 2$

[7] The restriction on H is not essential, in the sense that we could easily deal with the case of many uninformed agents. Removing the assumption of a unique informed trader entails slightly changing the existence proof and it is not done in this paper. Interpreting H as types is also possible, in the sense of having many identical individuals of type 2. The type interpretation is indeed used as a justification for the price-taking behavior assumption adopted in the usual sense. However, having many identical individuals of type 1 is not a straightforward extension. To have many identical type-1 individuals is more problematic, since one has to impose restrictions on the way each individual's effort affects the outcome s relative to other individuals'.

has is market-related: prices, the price function, and possibly information deriving from understanding that markets clear. Further in the paper, when addressing issues of incentive compatibility, we will let the uninformed trader understand the structure of the economy, making a common knowledge assumption on the structure of the economy and specifying his knowledge of preferences and a prior on them, as opposed to over the action taken by the informed trader. Our notion of equilibrium at this point does not depend on this common knowledge assumption, and it is based on the tradition of competitive analysis of restricting to a minimum the traders' knowledge.

We therefore assume that $\pi_2^n > 0$; for all n; to avoid arbitrage problems. To denote the traders' information via partitions of N, we use the symbol L_h: Therefore we have $L_1 = (\{1\}; \{2\})$; while $L_2 = \{N\}$: We also let $\Pi_h = \{\pi_h \in \mathbb{R}^2 | \pi_h^1 + \pi_h^2 = 1; \pi_h^n \geq 0; n = 1; 2\}$ to denote the space of mixed strategies over actions for $h = 1$; and the space of prior beliefs for $h = 2$ (where we restrict our attention to strictly positive beliefs). Finally, let Π denote the set of conditional probabilities $\pi^{s;n}$:

Financial markets

Traders can exchange I financial assets, with $S > I \geq 2$, making trade plans at time $t = 0$ contingent upon their information. Then $b_h^n \in \mathbb{R}^I$ represents portfolio holdings if action n occurs, and $b_h = (b_h^1; \ldots; b_h^N)$: Let $q^n \in \mathbb{R}^I$ be the price vector of these assets. Also, let $q = (q^1; \ldots; q^N)$; and let

$$J(q) = \# \operatorname{Im} q; \text{ and } N_j = \{n \in N \mid q^n = q_j\}; \text{ all } j = 1; 2; \ldots; J(q):$$

Finally, let $L_h(q) = L_h \vee \{N_j\}_{j=1}^{J(q)}$ be the join of the private information partition and the price-induced partition. We will assume that traders use this information to make decisions. Hence traders' demand for assets must be $L_h(q)-$ measurable. To simplify notation, we denote by π_2^n also the posterior of trader 2 after observation of prices.

The assets are identified by an action-independent $S \times I$ payoff matrix Y; expressed in units of account: We assume that $Y > 0$ and that Y is in general position, so that in particular rank $Y = I$. To allow a straightforward comparison with the insurance contracts of the individual risk models, we could assume that the price q is paid in the future upon realization of the state s. This would make the asset spanning endogenous, and the analysis would be complicated by the need of establishing generic existence through the application of an otherwise well-known technique. To focus instead on existence issues related to the moral hazard problem only, we assume that q is paid today.[8]

Preferences and endowments

Each trader has preferences over future consumption of C commodities in each state-action pair. We assume that the commodity space is $\mathbb{R}^C_{++}$. Trader 1 incurs a cost a_1^n for choosing action n, and we assume that this cost is increasing with n. Preferences are represented by a von Neumann-Morgenstern utility

$$v_h : \mathbb{R}^{CSN}_{++} \times \Pi_h \times \Pi \to \mathbb{R}$$

equal to $v_h(x_h, \pi_h, \pi) = \sum_n \pi_h^n [\sum_s \pi^{s,n} u_h(x_h^{s,n}) - a_h^n]$, [let $a_2^n \equiv 0$, all n] where $u_h : \mathbb{R}^C_{++} \to \mathbb{R}$ is a smooth, differentially strictly increasing, differentially strictly concave function with closure of its indifference curves contained in the positive orthant. This amounts to risk aversion of traders, and to preferences (u_h) which are state-action independent, a standard assumption which will be key for the results of Section 5

As for notation, we will write $v_h(x_h, \pi_h, \pi, L_h(q))$ to stress the fact that probabilities π_h^n depend on $L_h(q)$ (although only for $h = 2$ and when prices reveal information), and v_h^n for $\sum_s \pi^{s,n} u_h(x_h^{s,n}) - a_h^n$. Also, $x_h^{s,n} \in \mathbb{R}^C_{++}$ has typical element $x_h^{s,n,c}$. Let $x_h^n = (x_h^{1,n}, ..., x_h^{S,n})$, and $x_h = (x_h^1, .., x_h^N)$. Notice that we posit that there is no consumption at time $t = 0$. Endowments are assumed to be action-independent, i.e. $e_h \in \mathbb{R}^{CS}_{++}$. Let $E = \mathbb{R}^{CSH}_{++}$ be the endowment space.

Hereafter, we will assume that $C = 1$. This assumption is similar to the dimensionality condition in rational expectations equilibrium models, and it plays a fundamental role in the existence proof.

Equilibrium

Let $p \in \mathbb{R}^{CSN}_{++}$ be the time $t = 1$ commodity price vector (keeping in mind that $p \in \mathbb{R}^{2S}_{++}$, since $C = 1$ and $N = 2$). Let $z_h = x_h - e_h$. An economy will be a point

$$(e, a, u, \pi, \pi_2, Y) \in E \times \mathbb{R}^2 \times U \times \Pi \times \Pi_2 \times \mathbb{R}^{SI}_+$$

where U is the space of utilities $u = (u_1, ..., u_H)$. In what follows (a, u, π, π_2, Y) are kept fixed. An economy is parametrized only by endowments. Let $\Theta = E$ be the parameter space, with θ an economy. Let Ψ^n be a standard $S \times SC$ matrix of prices, if action n occurs.

A vector $(\pi_1, (x_h, b_h)_{h=1}^H, p, q)$ is a private action equilibrium for an economy if:

[8] When insurance contracts are considered, we can have $I = 1$. Indeed, this was the case analyzed in a previous version of the paper.

(I) given p and q; trader 1 solves

$$\begin{aligned} &\max_{\pi_1^n; x_1; b_1} && v_1(x_1; \pi_1; L_1(q)) \\ &\text{s.t.} && q^n b_1^n = 0 \\ & && \alpha^n z_1^n = Y b_1^n \quad \text{for all } n \end{aligned}$$

and b_1 is $L_1(q) -$ measurable;

(U) given p and q; and prior (or revised) beliefs π_2^n; trader 2 solves

$$\begin{aligned} &\max_{x_2; b_2} && v_2(x_2; \pi_2; L_2(q)) \\ &\text{s.t.} && q^n b_2^n = 0 \\ & && \alpha^n z_2^n = Y b_2^n \quad \text{for all } n \end{aligned}$$

and b_2 is $L_2(q) -$ measurable;

(M) markets clear, that is,

$$\sum_h z_h^n = 0$$
$$\sum_h b_h^n = 0$$

(NR) $J(q) = 1$ and $\alpha^n = \alpha^{n'}$; all $n; n'$;

Remarks

a) The timing of the model is the following: first, for given asset prices and expected commodity prices traders exchange financial assets; then trader 1 chooses an action n; and finally, endowment (i.e., output) uncertainty is resolved, and commodity trades are carried through. The important feature of this equilibrium is that information is private even ex post, in the sense that the uninformed trader does not get to observe trader 1's action even after output uncertainty is resolved. This is an equilibrium despite the fact that trader 2 may place a positive probability on a zero probability event. Since there is no revelation of information through prices and no information extraction through direct signals, whatever prior trader 2 had, it is not revised. This depends on not having assumed knowledge of preferences or common knowledge of the structure of the economy. Below, we will remove this radical assumption and model ignorance of preferences in a common knowledge environment.

b) The moral hazard interpretation arises for assets whose payoffs y^s are increasing with s; and so are the endowments e_h^s. In this case, trader 1 can put in 'effort' ($n = 2$) to increase the likelihood of high output (that is, high endowment and high payoff for the investor), but he suffers the cost $a_1^2 > 0$ of exerting high effort. If the asset price depended on the effort level, trader 1 could manipulate the financial contract by claiming to exert high effort, selling the asset at a higher (by no arbitrage) price, and then use low effort, and use the extra cash to buy a position to hedge the endowment risk. Without nominal payoffs, price equality across n cannot in general be obtained.

c) In equilibrium the informed trader is not going to use his information, although he is not forced to do so. Instead, this follows from asset market clearing ($b_1^n + b_2^n = 0$) and from the measurability restrictions on prices and on trader 2's portfolio.

d) In the definition, we are assuming that trader 1 takes prices as given. Since his trade may reveal his information (his effort choice) through prices, the plausibility of this assumption will be further discussed at the end of the paper.

e) Since the financial contract is generally not optimal for the uninformed, as in Helpman and Laffont, no incentive compatibility constraint is embedded in the definition of equilibrium, except for implicitly requiring that the action n exerted in equilibrium be optimal for the informed trader. Only when discussing strategic behavior we will explicitly consider such a constraint. Condition (NR) singles out equilibria where prices are effort-independent. As discussed in the Introduction, and further in Section 6, other equilibria may arise, provided they satisfy an incentive compatibility condition.

f) As we will show later, we can see trader 1 as an entrepreneur himself facing a moral hazard problem with a worker, and trying to hedge his cash-flows through financial markets (see Section 4). One important consequence of existence is that the entrepreneur has access to competitive hedging instruments. As we claimed in the Introduction, these are not simply risk-free assets, but truly state-contingent claims. Normally, the principal-agent models assume that the principal has access to competitive credit markets to borrow at the risk-free rate without being bound by his current wealth in the amount of payments to the agent. Here we extend the access to guarantee that the principal can partially insulate his income from output uncertainty. The degree of incompleteness limits the principal's hedging opportunities; however, hedging occurs more substantially than with a risk-free asset. Note that in our equilibrium rank $Y = I \geq 2$; and one can show through a standard transversality argument that even in real terms $y^{s,i}/p^s \neq y^{s',i}/p^{s'}$ for all s, s' with $s \neq s'$; all i, generically in the parameters of the model. So moral hazard is not eliminated by trivially making the assets risk-free in real terms. Similarly, one can show that equilibria are not always 'low effort'. Of course, the issue remains of whether at least the principal can be made better off when offered a nonlinear hedging contract. In this paper we only address feasibility, leaving the optimality to further research, other than what mentioned in Section 6.

The issue of existence amounts to asking whether 'two economies' (different at the interim stage only through the informed trader's action n, and hence through the probabilities π_1^n) have the same equilibrium prices. This is possible if the Arrow security prices are identical in the 'two economies', a condition that we guarantee when there are enough degrees of freedom, i.e., when there is enough indeterminacy in the system of equations giving rise to an equilibrium, which in turn is derived from the degree of market incompleteness. This is formally done in the next section.

3. Existence of equilibrium

3.1. Modified equilibrium

The above-defined equilibrium can be expressed as the equilibrium of an economy where $h = 2$ is constrained in his portfolio choices, while $h = 1$ is unconstrained. We use this observation in this section, where we slightly modify the definition of equilibrium to an equivalent one, following the work of Balasko, Cass and Siconolfi [1], and similarly to what is done in Citanna and Villanacci [6].

To start, we look at an equilibrium by rewriting it as:

$(I; U)$ given p and q; traders solve

$$\begin{aligned} \max \quad & v_h(x_h; \pi_h; L_h(q)) \\ \text{s.t.} \quad & \textstyle\sum_n q^n b_h^n = 0 \\ & a^n z_h^n = Y b_h^n \qquad \text{for all } n \\ & B_h b_h = 0 \qquad \text{if } h > 1 \end{aligned}$$

(M) markets clear, that is,

$$\sum z_h(n) = 0$$
$$\sum b_h(n) = 0$$

and (NR) $Rq = 0$; and $p^{s1} = p^{s2}$; all s.

Here B_h is the $(N - 1)I \times NI$ matrix representing the measurability restrictions on b_h; and R is the $(N - 1)I \times NI$ matrix of price restrictions corresponding to $q^1 = q^2$ ($B_2 = R$; while $B_1 \equiv 0$).

We now describe the modified equilibrium as a solution to a system of equations after transforming the informed agent using the 'Cass' trick'. A transformed equilibrium will be shown to be a modified equilibrium, and this in turn a private action equilibrium.

After transforming trader 1 in walrasian, the first order conditions for problem (I), given prices and n, are expressed as

$$\begin{array}{ll} Dv_1^n - \omega \lambda^n a^n = 0 & (1) \\ \sum \lambda^n a^n z_1^n = 0 & (2) \end{array} \qquad (3.1)$$

with ω the appropriate Lagrange multiplier, and $\lambda^n \in R^S_{++}$, all n. For all other individuals (U), the first order conditions are

$$\begin{array}{ll} \pi_2^n Dv_2^n - \lambda_2^n a^n = 0 & (3) \\ -\sum \lambda_2^0 q^n + \lambda_2^n Y + \mu_2 B^n = 0 & (4) \\ q^n b_2^n = 0 & (5a) \\ -a^n z_2^n + Y b_2^n = 0 & (5b) \\ B_2 b_2 = 0 & (6) \end{array} \qquad (3.2)$$

where λ_2^0 is the multiplier associated with the first period budget constraint, and λ_2^n is the vector of Lagrange multipliers attached to the period 1 budget constraints.[9] Using Walras' law, market clearing is expressed as

$$\begin{array}{ll} \sum_h z_h^{n1,1} = 0 & (7) \\ \sum_h b_h^n = 0 & (8) \end{array} \qquad (3.3)$$

The informed trader chooses the unobservable action n comparing the indirect utility at each n. This can be expressed in equations as

$$\begin{array}{ll} v_1^2 - v_1^1 + \underline{\varepsilon} - \bar{\varepsilon} = 0 & (15) \\ \min(\underline{\varepsilon}, \pi_1^2) = 0 & (16) \\ \min(\bar{\varepsilon}, 1 - \pi_1^2) = 0 & (17) \end{array} \qquad (3.4)$$

Note that this is artificial timing, and does not imply that the informed trader first chooses n and then chooses b_h. This choice is simultaneous, as in the standard principal-agent model. System (3.1) through (3.4) with condition (NR) has

[9] Also, $\mu_h \in \Re^{(N-1)I}$ is the vector of multipliers corresponding to equations $B_2 b_2 = 0$. We set

$$\lambda_h = (\lambda_h^1, \ldots, \lambda_h^N)$$

for h = 2. B^n is the n-th supercolumn of B_2, corresponding to portfolio restrictions relative to action n.

$NS + NI + 1 - (N-1)(S+I)$ too many unknowns, namely among p, q and λ, which nevertheless have to satisfy the arbitrage equation $\lambda^n Y = q^n$; all n; further reducing the degrees of freedom in the system to $S - (N-1)I + 1$. A precondition for existence of a transformed equilibrium is that this number be nonnegative. Since not all of the q and λ can be exogenized, contrary to what happens in Balasko, Cass and Siconolfi [1], we have to choose appropriate normalizations. After setting $p^{1;1} = 1$; we use the following extra equations on q and λ,

$$
\begin{array}{ll}
\lambda^n Y - q^n = 0 & (9) \\
p^{1;1} = 1 & (10) \\
\sum_s \lambda^{s;1} - (3+I) = 0 & (11)
\end{array}
\qquad (3.5)
$$

and we construct a homotopy using the following equations:

$$
\begin{array}{ll}
(1-t)(q^{j;1} - q^{j;2}) + t(\lambda^{i;1} - 1) = 0; \text{ for } 3 \le i \le 3 + I - 1; j = i - 2 & (12) \\
(1-t)(\sum_s p^{s;1} - 2) + t(\lambda^{2;1} - 1) = 0 & (13) \\
(1-t)(p^{s;2} - p^{s;1}) + t(\lambda^{s;2} - \lambda^{s;1}) = 0 & (14)
\end{array}
\qquad (3.6)
$$

Let the homotopy be the function $H : \Xi \times T \times \Lambda \times \Theta \to \mathbb{R}^l$ defined by the left-hand side of the system of equations (3.1) to (3.6), where $t \in T = [0; 1]$ is the homotopy parameter and

$$\xi = \left(x_1; \omega; (x_h; b_h; \lambda_h; \mu_h)_{h=2}; p^{n1;1}; b_1; q; \lambda'; \pi_1^2; \ldots\right)$$

is a point in Ξ; and $l = \dim \Xi$: Here $\lambda = (\lambda'; \lambda'')$; where $\lambda' = (\lambda^2; (\lambda^{s;1})_{s=1}^{I+3})$; and $\Lambda \subset \mathbb{R}_{++}^{S-(I+3)}$ is the space of λ'' (the elements of λ^1 not in λ'). Also, $T = [0; 1]$; with generic element t. Then H is a homotopy such that $H_{\lambda'';\mu;t}(\xi)$ maps a boundaryless, smooth manifold into a (boundaryless, smooth, connected) manifold of the same dimension. If $H_t^{-1}(0)$ is a compact, boundaryless set, then $\deg_2(H_{\lambda'';\mu;t}; \{0\})$ is well defined and equal for all $t \in T$.

We will refer to the solutions to system of equations (3.1) to (3.6) at $t = 0$ as modified equilibria.

Lemma 3.1. *Modified equilibria are equilibria.*

Proof. First, observe that if (1), (2), (15), (16) and (17) are satisfied, then $h = 1$ solves $\max_{\pi_1; x_1} v_1(x_1; \pi_1; L_h(q))$ s.t. $\sum_n \lambda^n a^n z_1^n = 0$. The argument then is a standard application of the Cass trick (see Balasko, Cass and Siconolfi (1990)), and therefore is omitted. ■

Note that in system (3.1)-(3.6) we have to impose a restriction on S; that is: $S \geq I + 3$. This is not needed in order to establish Lemma 3.1, but it is required in order to get existence using this homotopy.

3.2. A homotopy argument

The key step in the existence proof is to show properness of the projection $pr : H^{-1}(0) \to \Lambda \times \Theta$; which we establish in the following Lemma.

Lemma 3.2. The projection $pr : H^{-1}(0) \to \Lambda \times \Theta$ is proper.

Proof. See the Appendix. ■

Now we can state the main result of the paper.

Theorem 3.3. For any $\theta \in \Theta$, if $S \geq I + 3$; an equilibrium exists.

Proof. For the proof, we need to show that $\deg_2(H_{\lambda'';\theta;1}; \{0\}) = 1$. We will do so by means of another, standard homotopy $\bar{H}$ between two points in the (path-connected) space of parameters $\Lambda \times \Theta$: This homotopy is defined by the left-hand side of the same system (3.1)-(3.6) for $t = 1$; where we substitute for any $(\lambda''; \theta)$ a convex combination through a $\bar{t} \in [0; 1]$ between two points in $\Lambda \times \Theta$: $\bar{t}(\lambda''; \theta) + (1 - \bar{t})(\lambda''^*; \theta^*)$; the first being an arbitrary point, and the second being our "test economy". For any given $\bar{t} \in [0; 1]$; this homotopy is a map between manifolds of the same dimension. It is proper, by an argument in all similar to the proof of Lemma 3.2 when $t = 1$, and degree modulo 2 at $\{0\}$ is well-defined. For an appropriate choice of λ''^* and θ^*; thought in the bigger space R^{CSN}_{++}; we can show that $\deg_2(\bar{H}_{\lambda'';\theta;\lambda''^*;\theta^*;t=1;\bar{t}=0}; \{0\}) = 1$:

A) (a test economy). Let $\lambda''^* = 1$; and choose a Pareto optimal allocation $e^{PO}_h \in R^{CSN}_{++}$; for all h: We can pick action-dependent endowments at this stage, if we can (as we do) homotope them to action-independent ones. This Pareto optimal allocation will have a corresponding unique (no trade) equilibrium, since $\lambda^1 = 1$; $\lambda^2 = \lambda^1$; and $q^1 = q^2$ is the unique solution to equation (9).

To be precise, let e^{PO}_h be the solution to this maximization problem: for given $\pi; \nabla^n$ and r^n; $n = 1; 2$,

$$\begin{array}{ll} \max_{\pi_1;x_1;x_2} \sum_n \pi_1^n [\sum_s \pi^{s;n} u_1(x_1^{s;n}) - a_1^n] & s{:}t{:} \\ V_2^n(x_2^n) \geq \nabla^n & (1) \\ \sum x_h^n = r^n & (2) \end{array} \quad (3.7)$$

with $x_h \gg 0$. This problem is equivalent to, for each n, maximizing $v_1^n(x_1^n) = \sum_s \pi^{s,n} u_1(x_1^{s,n}) - a_1^n$ subject to (1) and (2); and then choosing π_1^2 such that

$$\begin{array}{ll} v_1^2 - v_1^1 + \underline{\mu} - \bar{\mu} = 0 & (3) \\ \min(\bar{\mu}; 1 - \hat{\pi}_1^2) = 0 & (4) \\ \min(\underline{\mu}; \hat{\pi}_1^2) = 0 & (5) \end{array} \tag{3.8}$$

Note that for each n, the maximization problem is strictly concave, and has a unique solution, $\hat{x}_h^n$; given by the system of equations

$$\begin{array}{l} \pi^{s,n} Du_1(\hat{x}_1^{s,n}) - \hat{p}^{s,n} = 0 \\ \theta_2^n \pi^{s,n} Du_2(\hat{x}_2^{s,n}) - \hat{p}^{s,n} = 0 \\ v_2(\hat{x}_2) = \bar{v}^n \\ \sum \hat{x}_h^n = r^n \end{array} \tag{3.9}$$

and which does not depend on π_h^2; $h = 1; 2$. In general, in this solution it is not guaranteed that $\hat{x}_h^1 = \hat{x}_h^2$: However, generically in $(\bar{v}; r)$; $v_1^2 - v_1^1 \neq 0$; by a transversality argument, so without loss of generality we can pick a vector of parameters such that $v_1^2 - v_1^1 > 0$; so that $\hat{\pi}_1^2 = 1$, uniquely, from (3) $-$ (5); $\underline{\mu} = 0$ and $\bar{\mu} = v_1^2 - v_1^1$.

Now from (3.9) it is not difficult to go back uniquely to system (3.1)-(3.6), by setting $e_h^{n*} = (\hat{x}_h^n)$ as our choice of endowments (and recall, $\lambda''^{*} = 1$). Then $x_h^n = e_h^{n*}$; $\lambda^n = 1$; $\omega = \hat{p}^{1;1}$; $p^{1;1} = 1$, $p^{n1;1} = \hat{p}^{n1;1}/\hat{p}^{1;1}$; $\lambda_2^n = (\hat{p}^{1;1}\pi_2^n/\theta_2^n)1$; $b_h = 0$; $\mu_2 = 0$; $\pi_1^2 = \hat{\pi}_1^2$, $\underline{\varphi} = \underline{\mu}$; and $\bar{\varphi} = \bar{\mu}$ is a solution to (3.1)-(3.6), indeed the unique one.

B) (regularity). We need to show that $D_\xi F_{\lambda'';\theta;\lambda''^*;\theta^*;t=1;t=0}$; a square matrix of derivatives, has full rank I. Note that individual excess demands are all zero. We will show that

$$D_\xi F_{\lambda'';\theta;\lambda''^*;\theta^*;t=1;t=0}\,\Delta\xi = 0 \;\Rightarrow\; \Delta\xi = 0$$

The argument is essentially standard, and only needs to be adapted to the particular system of equations we are looking into. It is therefore deferred to the Appendix. ■

4. Extensions to a principal-agent setup

To make the model explicitly closer to the standard moral hazard setup, we should allow trader 1 (the principal) to buy the information off another individual (the agent), who is choosing the unobservable action n. Now the agent bears the cost of the action, a_1^n. The agent is assumed to have no access to financial markets, while the principal does. The agent has a v. Neumann-Morgenstern utility

$$\sum_s \pi^{s,n} u_1^A(x_{1A}^{s,n}) - a_1^n$$

for each level of action n; where u_1^A is a standard smooth utility function (with strictly convex preferences). The principal has utility $\sum_n \pi_1^n \sum_s \pi^{s,n} u_1^P(x_{1P}^{s,n})$; denoted by $v_1^P(x_{1P}; \pi_1; L_1(q))$. We assume that u_1^A is unbounded from below, so that the agent's limited liability restriction won't be binding. Now trader 1's problem becomes

$$\begin{array}{ll}
\max_{\pi_1^n; w_1; x_1; b_1} & v_1^P(x_{1P}; \pi_1; L_1(q)) \\
 & q^n b_1^n = 0 \\
s.t.: & \sum a^n (x_{1P}^n - [1 - \phi(w_1^n)] e_1) = Y b_1^n \quad \text{for all } n \\
(IC) & \sum_s (\pi^{s,n} - \pi^{s,n'}) w_1^{s,n} - a_1^n + a_1^{n'} \geq 0 \\
(PC) & \sum_s \pi^{s,n} w_1^{s,n} - a_1^n - \underline{u} \geq 0
\end{array}$$

where w_1^n is the wage-utility if action n is implemented, $0 \leq \phi(w_1^n) \leq 1$ (limited liability)[10] is the S-dimensional vector of wages, through the agent's indirect utility function ϕ; and $\underline{u}$ is the reservation utility level. Since the agent is forced to eat the wage in each spot, i.e. $x_{1A}^{s,n} = \phi(w_1^{s,n}) e_1^s$; his maximization problem is trivial, and will be omitted. In the modified equilibrium, system (3.1) is changed into

$$\begin{array}{lr}
Dv_1^n - \omega \lambda^n a^n = 0 & (1a) \\
-(\omega \lambda^{s,n} p^{s,n} e_1^s + \alpha^{s,n}) D\phi(w_1^{s,n}) + \beta^n \pi^{s,n} + \delta^n \left[\pi^{s,n} - \pi^{s,n'}\right] = 0 & (1b) \\
\min[\alpha^{s,n}; 1 - \phi(w_1^{s,n})] = 0 & (1c) \\
\sum \lambda^n a^n \sum z_P^n = 0 & (2a) \\
\min(\delta^n; \sum_s (\pi^{s,n} - \pi^{s,n'}) w_1^{s,n} - a_1^n + a_1^{n'}) = 0 & (2b) \\
\min(\beta^n; \sum_s \pi^{s,n} w_1^{s,n} - a_1^n - \underline{u}) = 0 & (2c)
\end{array} \qquad (4.1)$$

and the remaining equations are the same. Here $\alpha^{s,n} \in \mathbb{R}$, all s, n, and since $\beta^n > 0$; all n; (2c) holds as $\sum_s \pi^{s,n} w_1^{s,n} - a_1^n - \underline{u} = 0$; all n.

Then we have the following result, replicating Theorem 3.3.

[10] Limited liability for the principal could include his total wealth, both real and financial. Here we assume for simplicity that the principal cannot provide salaries above output (the endowment).

Theorem 4.1. For any $\theta \in \Theta$, if $S \geq I + 3$, an equilibrium exists.

Proof. To prove existence, we need to show properness of the projection and to find a test economy as we did before, in Theorem 3.3. While properness presents no additional difficulty, this time we need to specify parameters of the Pareto optimum in order to guarantee the differentiability of $\bar{H}_{\lambda'',\theta;\lambda''^*,\theta^*;t=1;t=0}$.

To construct the test economy, we pick endowments solving the following concave planning problem:

$$\begin{array}{ll} \max_{x_1^n, x_2^n} \sum_s \pi^{s,n} u_1^P(x_{1P}^{s,n}) & s.t.: \\ v_2^n(x_2^n) \geq \bar{v}^n & (1) \\ \sum_s \pi^{s,n} w^{s,n} - a_1^n - u \geq 0 & (2) \\ \sum_s (\pi^{s,n} - \pi^{s,n'}) w^{s,n} - a_1^n + a_1^{n'} \geq 0 & (3) \\ x_{1P}^n + x_{1A}^n + x_2^n = r^n & (4) \\ w^{s,n} \leq u_1^A(x_{1A}^{s,n}) & (5) \end{array} \tag{4.2}$$

and with the additional conditions $x_h \gg 0$. The Kuhn-Tucker conditions for this programming problem are

$$\begin{array}{ll} \pi^{s,n} Du_1^P(x_{1P}^{s,n}) - p^{s,n} = 0 & (1) \\ -p^{s,n} + \nu^{s,n} Du_1^A(x_{1A}^{s,n}) = 0 & (2) \\ \beta^n \pi^{s,n} + \delta^n(\pi^{s,n} - \pi^{s,n'}) - \nu^{s,n} = 0 & (3) \\ \min(\delta^n, \sum_s (\pi^{s,n} - \pi^{s,n'}) w^{s,n} - a_1^n + a_1^{n'}) = 0 & (4) \\ \sum_s \pi^{s,n} w^{s,n} - a_1^n - u = 0 & (5) \\ \theta_2^n \pi_2^{s,n} Du_2(x_2^{s,n}) - p^{s,n} = 0 & (6) \\ x_{1P}^n + x_{1A}^n + x_2^n = r^n & (7) \\ w^{s,n} = u_1^A(x_{1A}^{s,n}) & (8) \end{array}$$

and they map immediately to system (4.1), except for equation (1c). As before, one can show that generically in $(r, \bar{v}, u)$ we can select a test economy such that either $\delta^n = 0$ or $\sum_s (\pi^{s,n} - \pi^{s,n'}) w_1^{s,n} - a_1^n + a_1^{n'} = 0$, but not both: say $\delta^1 = 0$ and $\sum_s (\pi^{s,1} - \pi^{s,2}) w_1^{s,1} - a_1^1 + a_1^2 > 0$, while $\delta^2 > 0$. Also, generically the solution cannot imply $v_1^{P1} = v_1^{P2}$, and hence $\pi_1^2 = 1$. Note that in the test economy, $e_1 = x_{1P} + x_{1A}$, and the equilibrium is still no trade, with $x_{1A}^{s,n} = \phi(w_1^{s,n}) e_1^{s,n}$ and $x_{1P}^{s,n} = [1 - \phi(w_1^{s,n})] e_1^{s,n}$. Hence $1 > \phi(w_1^{s,n})$, and equation (1c) is $\alpha^{s,n} = 0$, all s, n.

We need to check that in this economy, rank of $D_\xi \bar{H}_{\lambda'',\theta;\lambda''^*,\theta^*;t=1;t=0}$ is full. This is done in the Appendix. ■

Adding more principal-agent pairs can be easily accommodated and is left to the reader.

5. Economies with common knowledge

As a first step to addressing the issue of implementability of our equilibrium, we need to characterize our economy as an incomplete information environment. The main element missing from the model in Section 2 and its extensions is an assumption of common knowledge of the structure of the economy, which allows to formally specify what exactly traders know and do not know in terms of priors over the parameters of the economy, as opposed to over the actions of other traders. Since complete knowledge of preferences would almost always lead the uninformed trader to correctly predict the best reply n for given otherwise uninformative equilibrium prices, we need to assume that uninformed traders do not know the preferences of the informed traders. More precisely, we assume that $h = 2$ does not know only the "cost" of the private action n. More formally, we assume that uninformed traders have a prior over a_1^2 ($\equiv a$; hereafter, fixing $a_1^1 = 0$). This prior is assumed to be a density function $f : A \equiv R \to R_+$. We take the support of this prior to always include the tails of R, implicitly considering that uninformed traders are not only uncertain about the cost of high effort, but also about whether it is a cost at all. Let F be the space of all such functions with the compact-open topology. It is path-connected, since the convex combination of two densities is still a density, and the condition on the tails is robust to convex combinations.

Still considering the case so far analyzed of $C = 1$ and $H = 2$; we first go back to the economy of Section 2. Trader $h = 2$ will now compute, for any admissible $\hat{p} \equiv (p^n; q) \in R^{SN}_{++} \times R^I$,[11] trader 1's optimal choice (as in (I)) of action n given the type a_1^2 ; a possibly stochastic choice denoted by $\pi_1^n(\hat{p}; a)$: Clearly, $\pi_1^n : R^{SN}_{++} \times R^I \times A \to [0; 1]$: To make π_1^n into a function, we select a number between zero and one whenever trader 1 is indifferent between actions. Then π_2^n is now derived, even in the case of uninformative prices $\hat{p}$, as a revised probability assessment over trader 1's action n;

$$\pi_2^n(\hat{p}) = \int \pi_1^n(\hat{p}; a) f(a) da$$

An incomplete information economy now is a tuple $(e; f; a; u; \pi; Y)$ and we consider it parametrized by endowments, the cost of the action $n = 2$ and the prior f only. Let $\theta = (e; a; f) \in \Theta$ be an economy. A private action equilibrium is a vector

[11] That is, such that problem (I) has a solution. We restrict asset prices to be equal across efforts n since we never relax this assumption, even in the homotopic economies.

$(\pi_1; (x_h; b_h)_{h=1}^H; p; q)$ such that (I); (M) and (NR) are given as before, and (U) is now:

given $\hat{p}$; and beliefs $\pi_2^n(\hat{p})$; trader 2 solves

$$\begin{array}{ll} \max_{x_2; b_2} & v_2(x_2; f; L_2(\hat{p})) \\ \text{s.t.} & q^n b_2^n = 0 \\ & a^n z_2^n = Y b_2^n \quad \text{for all } n \end{array}$$

and b_2 is $L_2(\hat{p}) -$ measurable.

Here L_2 represents the partition on n induced by prices and the prior f. Note that, for each admissible $\hat{p}$; there is a unique $a = a(\hat{p})$ such that $0 < \pi_1^n(\hat{p}; a(\hat{p})) < 1$: Moreover, it is immediate to check that given the smoothness of the optimal $(x_1; b_1)$ as a function of $\hat{p}$; $a(p)$ is a smooth function locally around each price $\hat{p}$. Hence $\pi_2^n(\hat{p})$ is a continuous function of $\hat{p}$: Also, in equilibrium, since the support of f contains the tails of R, $\pi_2^n(\hat{p}) > 0$; all n; consistently with the measurability restrictions.

Given these properties we can prove existence of equilibrium adapting the previous degree proof, as shown by the following theorem.

Theorem 5.1. For any $\theta \in \Theta$, if $S \geq I + 3$; an equilibrium exists.

Proof. For ease of notation, we provide the proof for the case of the model under Section 2. We can still use system (3.1)-(3.6) to characterize a modified equilibrium, provided that equation (3) be substituted by $\pi_2^n(\hat{p})Dv_2^n - \lambda_2^n a^n = 0$: Lemmas 3.1 and 3.2 still can be shown to hold true without substantial modification, and the proof is left to the reader. We can define $\deg_2(H_{\lambda'';\theta;1}; \{0\})$ and want to show that it is nonzero. To do this, we pick a test economy, which now involves a specification of $\lambda''; e; a; f$. Fixing $\lambda''^* = 1$; choose the endowment e^{n*} that solves system (3.9), then choose a^* such that system (3.8) yields $v_1^2 - v_1^1 > 0$; so that $\tilde{\pi}_1^2(\hat{p}; a^*) = 1$, uniquely, from (3) $-$ (5); $\underline{\wedge} = 0$ and $\overline{\wedge} = v_1^2 - v_1^1$: By construction, $a(\hat{p}) \neq a^*$. Now choose $f(a)$ to be zero on an open ball $B_{a(\hat{p})}$ centered around $a(\hat{p})$; and not containing a^*: Now $\pi_2^n(\hat{p})$ is uniquely determined, and we can choose f such that $\pi_2^n(\hat{p}) > 0$; all n. Moreover, the function $g(\hat{p}; a) = \tilde{\pi}_1^2(\hat{p}; a) f(a)$ is constant with respect to p around $\hat{p}$; therefore $\pi_2^n(\hat{p})$ also is, and its derivative $D_p \pi_2^n(\hat{p}) = 0$ at $\hat{p}$. The rest of the argument (uniqueness of the endogenous variables and regularity of the test economy) now follows as in Steps A and B of the proof of Theorem 3.3, concluding our proof that $\deg_2(H_{\lambda'';\theta;1}; \{0\}) = 1$. ■

It is now obvious that a private action equilibrium can be seen as a private information equilibrium for any incomplete information economy with properties as above. Note that it may well be the case that the uninformed trader's ignorance of the informed (dis)utility of the privately chosen action does not have to be dramatic: the uninformed may indeed restrict his prior to a that entail a cost for trader $h = 1$, especially in cases where the action chosen in equilibrium is not the most preferred by the uninformed trader. The extension to economies of Section 4 is straightforward, provided we assume that the principal observes a, but …nancial markets traders do not.

6. Fully strategic use of information: a discussion.

So far we have taken the rational expectations equilibrium perspective to focus on the leading factor yielding the result, the endogenous uncertainty derived from the presence of incomplete markets. This perspective implicitly assumes that the informed trader has no control over the amount of information revealed by prices. In other words, the informed trader is not exploiting the fact that, since prices may reveal the information he has, he could try to control the amount of revelation. In this scenario, the informed trader would not take prices as given, and could possibly manipulate the …nancial contract this way. While abstracting from the details of the price formation mechanisms may serve to highlight the general mechanism leading to the coexistence of moral hazard and linear …nancial contracts, conclusions would be severely limited if they were not sustainable by a model of strategic use of information by the informed trader. The purpose of this section is to illustrate a modelling alternative that would sustain our equilibrium allocations in a strategic environment, and to discuss a few more scenarios and the likelihood that they generate our competitive equilibrium as a strategic outcome.

For the sake of exposition, we once again use the setup of the economy of Section 2. In all that follows, it is assumed that the number of uninformed traders is large. The strategic models that we consider have embedded one or more additional players, which we call the mediators (following Myerson [17], Ch. 6, p.250). The mediator's objectives and strategies, or the timing of his move, obviously determine the kind of strategic model we look at. We examine two di¤erent setups.

A …rst modelling alternative consists in assuming that one mediator chooses allocations as a function of messages sent by the traders (in a direct mechanism), and that he moves …rst. This is the setup used in solving the (weak) implementation problem (see Myerson [17]). Let X be the feasible allocation space, and F be the Social Choice Correspondence (SCC) associated with (one of) our private action equilibrium, and mapping the space A of utility of action/effort into the feasible allocation space. Note that for each economy, F is constant in the allocation space. Hence it is immediate to see that F is weakly implementable. Morever, let G be the SCS corresponding to the fully revealing equilibrium, satisfying the ex post incentive compatibility condition. Then, from Blume and Easley [4], we know that there exists an open set of economies for which G is not weakly implementable, since the information structure of the economy does not satisfy the Non-Exclusivity condition. We conclude that there is at least an open set of economies where the strong fully nonrevealing equilibrium is the only weakly implementable outcome.

The trading game used to show weak implementability is quite unappealing (the constant game). Therefore in what follows we construct an alternative trading game, where the mediator essentially designs a mechanism where allocations are not directly assigned to traders from their messages.

First, assume that the mediator decides prices, as functions from messages $\tilde{A} = A$ to functions from $(R^I)^R \times R^I$ into $(R^S_{++} \times R^I)$, taking $\tilde{a} \mapsto p^*(b(a); \tilde{a}) = (p(b; \tilde{a}); q(b; \tilde{a}))$, where $b = (b_1(a); b_2)$: The mediator posts them for the traders as official terms of trade. These functions are chosen to maximize $e_0 - E_a||\Sigma_h b_h(a; p^*(:; \tilde{a}))||;$ where $b_h(:)$ is derived as follows. Let $b_1(n; a; p^*(:; \tilde{a}); b_2)$ solve:

$$\psi_1^n(a; p^*(:; \tilde{a}); b_2) \equiv \max_{b_1 \in R^I : q(b; \tilde{a}) b_1 = 0} \sum_s \pi^{s,n} u_1 f[1/p^s(b; \tilde{a})] y^s b_1 + e_1^s g - a_1^n$$

Further, let $\pi_1^n(a; p^*(:; \tilde{a}); b_2) = \arg\max_{\pi_1^n \geq 0; \sum \pi_1^n = 1} \sum_n \pi_1^n \psi_1^n(a; p^*(:; \tilde{a}); b_2)$, and let $b_1^{n^*}(a; p^*(:; \tilde{a}); b_2)$ be the correspondingly chosen asset portfolio. On the other hand, let $b_2(p^*(:; \tilde{a}); b_1(a)_a)$ solve:

$$\psi_2(p^*(:; \tilde{a}); b_1(a)_a) \equiv \max_{b_2 \in R^I : q(b; \tilde{a}) b_2 = 0} \sum_n \pi_2^n \sum_s \pi^{s,n} u_2 f[1/p^s(b; \tilde{a})] y^s b_2 + e_2^s g$$

and s.t. $b_2(:)$ is $L_2(p^*) -$ measurable (again, here π_2^n is the posterior probability on N, given that $p^*(:; \tilde{a})$ is the announced function). Let the (pure strategy, Bayesian) Nash equilibrium of this game be given by $B_h(a; p^*(:; \tilde{a}))$, for each h. Then $b_h(a; p^*(:; \tilde{a})) \in B_h(a; p^*(:; \tilde{a}))$; for all h: Note that $b_2(a; p^*(:; \tilde{a}))$ may not depend on a. Letting $\psi_1(a; p^*(:; \tilde{a}))$ be the value function for $h = 1$; while $\psi_2(p^*(:; \tilde{a}))$ be the value function for $h = 2$, the function $p^*(b; \tilde{a})$ must satisfy the additional incentive compatibility constraint

$$\psi_1(a; p^*(:; a)) \geq \psi_1(a; p^*(:; a')); \text{ all } a; a' \in A$$

The mechanism is the following: after $p^*()$ is posted, trader 1 submits a message $\hat{a}$ to the mediator, declaring his utility of the private action n, hence implictly his preferred action; the mediator announces the corresponding prices, trader 1 chooses his optimal action, and traders submit their demands at those prices. Traders will accept this mechanism since it o¤ers at least the level of utility derived from the expected value of the endowment (the reservation utility of traders).

This mediator's only concern is market clearing, so he will not try to extract information from traders. In this game, the mediator faces a cost in holding [selling] the asset (proportional to the excess supply [demand] of the asset), and gets no bene…t from trading. Therefore trading for the mediator is a private value exchange problem, since n does not a¤ect his pro…ts. We are assuming that mediators are similar to brokers, in that they are barred from trading on their account, and that competition prevents them from charging one-time commission fees.[12]

In this game, it is feasible for the mediator to guarantee for himself the initial wealth e_0; since the private action equilibrium is a feasible pricing schedule. In particular, the private action equilibrium is incentive compatible. Since e_0 is the maximum level of wealth the mediator can achieve, the private action equilibrium is a (pure strategy, Bayesian) Nash equilibrium of the game (between mediator and traders).[13]

The conclusions of this analysis are that: a) without restricting the class of mechanisms, we are guaranteed the existence of one sustaining our competitive

[12] Dow and Gorton [8] suggests studying a similar brokerage institution acting as a mediator between buyers and sellers. If instead we allowed the brokers to make pro…ts through trade, the absence of bid-ask spreads in equilibrium could not immediately be justi…ed by simple Bertrand competition among two exclusive brokers. That is, in this case it is not obvious that private action equilibria will also be a (pure strategy) Nash equilibrium of the trading game (deviations to a bid-ask spread may be pro…table). Further inquiry in this direction will be the object of future research. Also, observe that if the mediator dies before tomorrow, he has no independent interest in hedging. A mediator working as an agent for the uninformed trader generally will care about trading on his own account.

[13] The conclusions reached through these two games strongly depend on the timing of the mediator's move and his objective function. If the mediator moves after demands have been submitted, taking this into account, traders may choose to submit demand functions to the mediator, before he actually quotes a price (function). We can still assume (as in Jackson [10], e.g.) that the mediator chooses a price that clears markets as his only objective. In this setup, the mediator plays a more passive role and the informed agent now compares equilibria. It may still happen that, for an open set of economies, the no-revelation trading strategy gives rise to higher payo¤ to the informed trader when compared to the revelation strategy. This ultimately depends on comparing welfare of equilibria with di¤erent degrees of revelation.

equilibria as a strategic outcome; our private action equilibria are incentive compatible; b) further restricting the attention to trading games, we can …nd a reasonable game form whose equilibria sustain allocations (and prices) of a private action equilibrium, hence showing that the linear contracts are optimal in this sense.

A. Appendix

Proof of Lemma 3.2

As $\nu \to \infty$, we take a converging sequence $\{\lambda''^{\nu}, \theta^{\nu}\} \subset \Lambda \times \Theta$, and we will show that $\{\xi^{\nu}, t^{\nu}\}$ has a converging subsequence. From equations (2), (5), (7) and (8), $e_h \gg 0$ and the boundary condition, we have $\{x_h^{\nu}\}$ converges to $x_h \gg 0$. From equations (15-17), we get convergence of $\{\pi_1^{2\nu}\}$, and of the sequence $\{\varepsilon^{\nu}\}$. Since T is compact, $t^{\nu} \to t$ (or a subsequence does; we ignore the distinction hereafter). Then three cases are possible.

a) $t = 0$. In this case, from (1), (10) and $p^{1,1} = 1$, we get $\omega^{\nu} \to \omega$. From (11) and (13) we have that $\lambda^{1\nu} \to \lambda^{1}$ and $p^{1\nu} \to p^{1}$, and from (1), they must be both strictly positive. Hence from (14), $p^{2\nu} \to p^{2} \gg 0$, and from (1), $\lambda^{2\nu} \to \lambda^{2} \gg 0$. Now equation (9) implies that $q^{\nu} \to q$. Equation (3) will give convergence of $\{\lambda_2^{n\nu}\}$, while equation (5) implies now the convergence of $\{b_2^{\nu}\}$. It is immediate to get convergence of $\{\mu_2^{\nu}\}$ from (4) and of b_1 from (8).

b) $t = 1$. When $t = 1$, $H_{\lambda'',\theta,1}(\xi) = 0$ corresponds to a standard system for a fully nonrevealing equilibrium, and properness of the projection follows from a well-known argument.

c) $0 < t < 1$. In this case observe that we still have convergence of $\{\omega^{\nu}\}$. From (11) we get convergence of $\{\lambda^{1\nu}\}$, while from (13), that is,

$$(1 - t)\left(\sum_{s} p^{s,1} - 2\right) + t(\lambda^{2,1} - 1) = 0$$

we have that $\{p^{1\nu}\}$ converges. Since both λ^{1} and p^{1} do, (1) implies that they are both strictly positive. Now equation (9) for $n = 1$ implies that $\{q^{1\nu}\}$ converges, hence from (12) we get the convergence of $\{q^{2\nu}\}$. To conclude, from (14) we must have that $\lambda^{2\nu} \to \lambda^{2}$, with $\|\lambda^{2}\| < \infty$, and similarly $\{p^{2\nu}\} \to p^{2}$, with $\|p^{2}\| < \infty$. For suppose not. Then $\{t^{\nu}\}$, $\{\lambda^{1\nu}\}$ and $\{p^{1\nu}\}$ are all bounded sequences of positive numbers, and the left-hand side of the equation would go to infinity, a contradiction. The rest of the argument is now standard. ■

Proof of regularity in Theorem 3.3

The linear system of equations in $\Delta\xi$, $D_{\xi} H_{\lambda'',\theta,\lambda''^{*},\theta^{*},t=1,t=0}\,\Delta\xi = 0$, can be written as

$$
\begin{array}{ll}
D^2 v_1^n ¢ x_1^n \;¡\; (1\, ^{a\, n})^T ¢\, ! + ¤_1^n ¢ p^n = 0 & (1) \\
¡ \; \mathbf{P}_n (1\, ^{a\, n}) ¢ x_1^n = 0 & (2) \\
D^2 v_2^n ¢ x_2^n \;¡\; ^{a\, nT} ¢\, ¸_2^n + ¤_2^n ¢ p^n = 0 & (3) \\
¡ \mathbf{P} ¢\, ¸_2^0 q^{nT} + Y^T ¢\, ¸_2^n + B^{nT} ¢\, ^1{}_2 = 0 & (4) \\
\quad q^n ¢ b_2^n = 0 & (5a) \\
¡\; ^{a\, n} ¢ x_2^n + Y ¢ b_2^n = 0 & (5b) \\
B ¢ b_2 = 0 & (6) \\
¡ \mathbf{P}\; ^{a\, n} ¢ x_2^n + [Y \;¡\; Q^n] ¢ b_2^n = 0 & (6) \\
\mathbf{P}_h ¢ x_h^n = 0 & (7) \\
\mathbf{P}_h ¢ b_h^n = 0 & (8)
\end{array}
\qquad (A.1)
$$

and with $¢\, ¸^1 = ¢\, ¸^2 = 0$; using equations (10) through (14) ; with $¢ q^1 = ¢ q^2 = 0$ using (9) ; and with $¢ ¼_1^2 = ¢ \underline{''} = 0$; $¢ ^{\prime\prime} = ¢ v_1^2 \;¡\; ¢ v_1^1$. In system (A.1) we have

$$
¤_h = \begin{bmatrix} ¡\, ¸_h^{1s} \begin{bmatrix} I \\ 0 \end{bmatrix} & 0 \\ 0 & ¡\, ¸_h^{2s} I \end{bmatrix}
$$

for $h = 2$; and the same matrix, but multiplied by $!$, and with $¸^n$ replacing $¸_h^n$ for $h = 1$: Again, note that $¸_h^{ns} = ¸_h^{ns^0}$ for all $s; s^0$; all h:

Let $h = 2$: From (6) we have $¢\, ^1{}_2^T B ¢ b_2 = 0$; while from (4) we get

$$
¡\, ¢\, ¸_2^0 \sum_n ¢ b_2^{nT} q^{nT} + ¢ b_2^T \mathrm{diag} Y^T ¢\, ¸_2 + ¢ b_2^T B_2^T ¢\, ^1{}_2 = 0
$$

Combining these expressions and using (5a), we get $¢ b_2^T \mathrm{diag} Y^T ¢\, ¸_2 = 0$: From (5b) we have

$$
¢\, ¸_2^T\, ^a ¢ x_2 = ¢\, ¸_2^T \mathrm{diag} Y ¢ b_2 = 0
$$

This, combined with (3) after dividing this last by $¸_h^n$ and summing over n, leads to

$$
\sum_n ¢ x_2^{nT} \frac{D^2 v_2^n}{¸_2^n} ¢ x_2^n = \sum_n ¢ x_2^T I^n ¢ p^n
$$

with $I^n = I$ if $n > 1$; or equal to the same matrix after substituting its last row with zeros. Similar computations show that the same equation holds for $h = 1$; after replacing $¸_1^n$ by 1.

Summing over h, we get

$$\sum_h \sum_n \Delta x_h^{nT} \frac{D^2 v_h^n}{\lambda_h^n} \Delta x_h^n = \sum_n \sum_h \Delta x_h^T I^n \Delta p^n = 0$$

where last equality follows by multiplying equations (7) by Δp^n and summing over n. If $\Delta x_h \neq 0$, some h, this contradicts negative definiteness of Dv_h^{n2}, all h. Therefore $\Delta x_h = 0$ all h. System A.1 together with $\Delta x = 0$ leads to conclude that $\Delta \xi = 0$. ■

Proof of regularity in Theorem 4.1

The only difference from the previous argument revolves around equations (4.1.1) to (4.1.2c), and equations (4.1.7), and their derivative with respect to the relevant endogenous variables, which we rewrite below,

$$
\begin{aligned}
& D^2 v_1^n \Delta x_{1P}^n - (1 \otimes n)^T \Delta \omega + \Lambda_1^n \Delta p^n = 0 && \text{(1a)}\\
& -p^{s,n} e_1^{s,n} D\phi(w_1^{s,n}) \Delta \omega - p^{s,n} e_1^{s,n} D^2\phi(w_1^{s,n}) \Delta w_1^{s,n} - e_1^{s,n} D\phi(w_1^{s,n}) \Delta p^{s,n} \\
& \quad + \pi^{s,n} \Delta \beta^n + (\pi^{s,n} - \pi^{s,n'}) \Delta \delta^n = 0 && \text{(1b)}\\
& -\textstyle\sum_n (1 \otimes n) \Delta x_{1P}^n - \sum_{n,s} p^{s,n} e_1^{s,n} D\phi(w_1^{s,n}) \Delta w_1^{s,n} = 0 && \text{(2a)}\\
& \Delta \delta^n = 0 \text{ or } \textstyle\sum_s (\pi^{s,n} - \pi^{s,n'}) \Delta w_1^{s,n} = 0 && \text{(2b)}\\
& \textstyle\sum_s \pi^{s,n} \Delta w_1^{s,n} = 0 && \text{(2c)}\\
& \Delta x_{1P}^{s,n} + e_1^{s,n} D\phi(w_1^{s,n}) \Delta w_1^{s,n} + \Delta x_2^{s,n} = 0; \text{ all } (s,n) \neq (1,1) && \text{(7)}
\end{aligned}
$$

(A.2)

To show regularity, observe that from (1b), premultiplying by $\Delta w_1^{s,n}$, summing over s, n, and using (2b) and (2c), we get

$$\sum_{n,s} \Delta w_1^{s,n} p^{s,n} e_1^{s,n} D\phi(w_1^{s,n}) \Delta \omega = -\sum_{n,s} p^{s,n} e_1^{s,n} \Delta w_1^{s,n} D^2\phi(w_1^{s,n}) \Delta w_1^{s,n} - \sum_{n,s} \Delta w_1^{s,n} e_1^{s,n} D\phi(w_1^{s,n}) \Delta p^{s,n}$$

while from (2a) we have

$$\sum_{n,s} \Delta \omega\, p^{s,n} e_1^{s,n} D\phi(w_1^{s,n}) \Delta w_1^{s,n} = -\sum_n \Delta \omega\, (1 \otimes n) \Delta x_{1P}^n$$

Using (1a), premultiplying by $(\Delta x_{1P}^n)^T$ and summing over n, we have

$$\sum_n \Delta x_{1P}^{n\,T} D^2 v_1^n \Delta x_{1P}^n - \sum_n \Delta x_{1P}^{n\,T} (1 \otimes n)^T \Delta \omega - \sum_n \Delta x_{1P}^{n\,T} I^n \Delta p^n = 0$$

which combined with the previous expressions gives

$$\sum_n \Delta x_1^{n\,T} D^2 v_1^n \Delta x_1^n - \sum_{n;s} p^{s;n} e_1^{s;n} \Delta w_1^{s;n} D^2 \phi(w_1^{s;n}) \Delta w_1^{s;n} = \sum_{n;s} \Delta w_1^{s;n} e_1^{s;n} D\phi(w_1^{s;n}) \Delta p^{s;n} + \sum_n \Delta x_1^{n\,T} I^n \Delta p^n$$

so that, together with the equations corresponding to $h = 2$, we get

$$\sum_h \sum_n \Delta x_h^{nT} D^2 v_h^n \Delta x_h^n - \sum_{n;s} p^{s;n} e_1^{s;n} \Delta w_1^{s;n} D^2 \phi(w_1^{s;n}) \Delta w_1^{s;n} = 0 \qquad \text{(A.3)}$$

since, from (7);

$$\sum_n \Delta x_1^{n\,T} I^n \Delta p^n + \sum_{n;s} \Delta w_1^{s;n} e_1^{s;n} D\phi(w_1^{s;n}) \Delta p^{s;n} + \sum_n \Delta x_2^{nT} I^n \Delta p^n = 0$$

Now, suppose $\Delta x_h^n \neq 0$ or $\Delta w_1^{s;n} \neq 0$: Then by assumption on utilities, $\Delta x_h^{nT} D^2 v_h^n \Delta x_h^n < 0$ or $-\Delta w_1^{s;n} D^2 \phi(w_1^{s;n}) \Delta w_1^{s;n} < 0$; contradicting (A.3). Then $\Delta x_h^n = 0$ and $\Delta w_1^{s;n} = 0$; all h and n.

As a consequence, using (1a) one can show that $\Delta \omega = 0$; so that $\Delta p = 0$: From this we obtain that $\Delta \lambda = 0$; coming to the desired conclusion. ■

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