Majority Vote Following a Debate*

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Abstract

Voters determine their preferences over alternatives based on cases (or arguments) that are raised in the public debate. Each voter is characterized by a matrix, measuring how much support each case lends to each alternative, and her ranking is additive in cases. We show that the majority vote in such a society can be any function from sets of cases to binary relations over alternatives. A similar result holds for voting with quota in the case of two alternatives.

1 Introduction

Information that becomes available to the public prior to elections may have unpredictable effects. The fact that a presidential candidate has used drugs in his youth may be a fatal blow to his popularity among some voters. Among others, it may be taken as a minor misdemeanor or even a sign of an open mind. Having been a member of a Trotzkyist party three decades before the upcoming elections may well be viewed as a virtue by some voters, and as a vice by others. Even less anecdotal pieces of information, such as a successful military career, are open to various interpretations, and will typically have differential impact on voters.

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It follows that it is not always clear which facts, or cases will affect elections in favor of a given candidate. It is even less clear how such cases interact. Imagine, for instance, that the ex-Trotzkyist candidate has also used drugs in his youth. Assume that none of these cases can turn a majority of voters against the candidate. But if the voters who find that drugs are a sign of an open mind are not those who favor Trozkyism, it is possible that the combination of the two cases will generate a "coalition of minorities" (Downs (1957)) against the candidate.

In this paper we consider a very simple model, according to which each voter uses cases in an additive manner. Specifically, for each voter i, each case c, and each candidate x, there is a number $w_i(x,c)$ such that, given a set of cases M, voter i prefers candidate x to y iff[1]

$$\sum_{c\in M} w_i(x,c) > \sum_{c\in M} w_i(y,c).$$

or

$$\sum_{c\in M} [w_i(x,c) - w_i(y,c)] > 0.$$

While this model is rather restrictive, we will see that it allows for significant freedom in terms of patterns of majority votes. Consider, for instance, a society containing three individuals, who have to choose between candidate x and candidate y. Suppose that there are three conceivable cases. The following matrix provides the value of the vector $(w_i(x,c) - w_i(y,c))_c$ for each voter:

W1	c	d	e
1	3	−1	−1
2	−1	3	−1
3	−1	−1	3

[1] Gilboa and Schmeidler (1999) axiomatize this rule. They assume that the voter can rank the candidates given *any* conceivable memory that is composed of repetitions of past cases.

Rows correspond to individuals, and columns – to cases. Thus the entry in row i and column c is the number $w_i(x,c) - w_i(y,c)$, measuring the degree of support that case c lends to candidate x, as compared to candidate y, in the eyes of voter i. Given the set $M_1 = \{c\}$, a majority of candidates favor y to x: case c convinces voter 1 that x is preferred to y, but it convinces voters 2 and 3 of the opposite. Alternative x will also be voted down given the set $M_2 = \{d\}$: this time it is the coalition of voters 1 and 3 that oppose alternative x. But if the union of the two sets, $M_1 \cup M_2 = \{c,d\}$, is brought forth, voters 1 and 2 vote for x and only voter 3 prefers y. In fact, in this example majority vote ranks alternative y above x given *any* single case, but this ranking is reversed for any set that contains at least two cases. Moreover, if all cases are cited, x is unanimously chosen.

To consider another example, consider the following matrix

W2	c	d	e
1	5	−3	−3
2	−3	5	−3
3	−3	−3	5

In this example, y is preferred to x given any single case. Given any pair of cases, majority vote favors x to y. But, as opposed to the example W1, in W2 citing all cases together reverses the pattern again, and y is chosen over x. Finally, in the following matrix (with five voters)

W3	c	d	e
1	3	−1	−1
2	−1	3	−1
3	−1	−1	3
4	−1	−1	−1
5	−1	−1	−1

alternative y is preferred to x for all sets of cases apart from the entire set $C = \{c,d,e\}$.

How erratic can majority vote be? Are there any conclusions that can be drawn from the way a society votes given certain subsets of cases to the way it votes given other subsets? The negative answer is given below. Our main result is that practically any pattern of votes (as a function of subsets of cases) can emerge as a result of a democratic vote. We present the model and the main result in Section 2. In Section 3 we discuss the case of votes with quotas. We show that the choice between two alternatives is as unpredictable in this case as in the case of simple majority vote. Section 4 is devoted to the proofs, and Section 5 – to a brief discussion.

2 Model and Main Result

Let $N = \{1, ..., n\}$ $(n \geq 0)$ be a set of *voters*. They are asked to rank *alternatives* in a finite set X. The public debate preceding the vote mentions a subset M of *cases* from a finite set of conceivably relevant cases C. For each voter $i \in N$, each case $c \in C$, and each alternative $x \in X$, there is a *decision weight* $w_i(x, c)$, interpreted as in the Introduction. We refer to the set of voters, coupled with their decision weights $w = (w_i(x, c))_{i \in N, x \in X, c \in C}$, as a *population* (N, w).

A binary relation $\succsim \subset X \times X$ is *reflexive* if $x \succsim x$ for every $x \in X$. It is *complete* if $x \succsim y$ or $y \succsim x$ for every $x, y \in X$. Observe that a complete relation is also reflexive. The asymmetric and symmetric parts of $\succsim$ are denoted, as usual, as $\succ$ and $\sim$, respectively. A relation $\succsim$ is *trivial* if $\succsim = X \times X$, that is, if $x \sim y$ for every $x, y \in X$. Let the set of all complete binary relations on X be $\mathcal{R} = \mathcal{R}(X)$.

Given a set of cases $M \subset C$, we define society's preferences by a majority vote. Formally, $\succsim_{((N,w),M)} \subset X \times X$ is defined as follows: for every $x, y \in X$ and every $M \subset C$, $x \succsim_{((N,w),M)} y$ iff

$$\#\{i \in N \mid \textstyle\sum_{c \in M} w_i(x, c) > \sum_{c \in M} w_i(y, c)\} \geq$$
$$\#\{i \in N \mid \textstyle\sum_{c \in M} w_i(x, c) < \sum_{c \in M} w_i(y, c)\}.$$

Observe that $\succsim_{((N,w),M)}$ is complete for every M. Thus, given a population of voters (N,w), majority vote defines a function

$$V_{(N,w)} : 2^C \to \mathcal{R}$$

by

$$V_{(N,w)}(M) = \succsim_{((N,w),M)}.$$

The question we address in this paper is the following: given a function $U : 2^C \to \mathcal{R}$, can it be the majority vote of some population? That is, is there a population (N,w) such that $V_{(N,w)} = U$? An obvious necessary condition is that $U(\varnothing)$ be trivial. The following result states that this is also a sufficient condition.

Theorem 1 *Let there be given a function $U : 2^C \to \mathcal{R}$. There exists a population (N,w) such that $V_{(N,w)} = U$ iff $U(\varnothing)$ is trivial.*

In the very specific case where C contains only one element, the statement is a slight generalization of McGarvey (1953) Theorem.

3 Voting with a Quota

There are many decisions in which regular majority does not suffice. For instance, suppose that the set X does not represent candidates for a public position, but two choices: approve a proposed constitutional amendment or reject it. In many countries, an amendment would require more than 50% of the votes in order to be approved. Assume, then, that there is a quota $q \in [\frac{1}{2}, 1)$ such that an amendment is approved only if the proportion of voters supporting it is q or higher. Which sets of cases will induce a q-majority for the amendment?

Formally, define, for $q \in [\frac{1}{2}, 1)$, $\succsim_{((N,w),M,q)} \subset X \times X$ as follows: for every $x, y \in X$ and every $M \subset C$, $x \succsim_{((N,w),M,q)} y$ iff

$$\#\{i \in N \mid \textstyle\sum_{c \in M} w_i(x,c) > \sum_{c \in M} w_i(y,c)\} \geq q|N|.$$

When $q > \frac{1}{2}$, $\succsim_{((N,w),M,q)}$ is not expected to be complete. One may ask whether any function from subsets of cases to (not necessarily complete) binary relations can be the result of a q-majority vote of some population. The negative answer is given by Vieille (2002). He shows that, even if $|X| = 2$, for every $q > \frac{1}{2}$, there exists a set of cases C and a function from 2^C to $\mathcal{R}$ that cannot coincide with $\succsim_{((N,w),M,q)}$ for any (N,w) (that is, that cannot be the q-majority vote of any population).

In the absence of a q-majority for either alternative, society still has to make a choice. To this end, there should be a default alternative that is chosen unless there is a q-majority against it. For instance, in the vote on a constitutional amendment, the default is that the amendment is not approved, unless it is supported by at least q of the votes.

Assume, then, that $X = \{x,y\}$ and that y is the default alternative. Thus, we re-define $\succsim_{((N,w),M,q)}$ as follows: $x \succsim (\succ)_{((N,w),M,q)} y$ iff

$$\#\{i \in N \mid \textstyle\sum_{c \in M} w_i(x,c) > \sum_{c \in M} w_i(y,c)\} \geq (>) q|N|$$

and $y \succ_{((N,w),M,q)} x$ whenever $x \succsim_{((N,w),M,q)} y$ does not hold. Let $\mathcal{R}_s$ be the subset of $\mathcal{R}$ consisting of strict relations (i.e., for $\succsim \in \mathcal{R}_s$, either $x \succ y$ or $y \succ x$, but not both). Given a population (N,w) we define $V_{(N,w,q)}(M) = \succsim_{((N,w),M,q)}$ and ask, which functions $U : 2^C \to \mathcal{R}_s$ can be the q-majority vote of some population? That is, for which U is there a population (N,w) such that $V_{(N,w,q)} = U$?

Proposition 2 *Assume that $X = \{x,y\}$ as above and $q > \frac{1}{2}$. Let there be given a function $U : 2^C \to \mathcal{R}_s$. There exists a population (N,w) such that $V_{(N,w,q)} = U$ iff $U(\varnothing)$ is trivial.*

Thus, in the case of two alternatives our results extends to a majority vote with quota $q > \frac{1}{2}$. This result does not extend to the case $|X| > 2$.

Observe that with more than two alternatives the default choice defines a complete binary relation on X. In general, it is easy to see that not every pattern of choices may be the majority vote for any q. For instance, for $q > \frac{2}{3}$ one may set the default to be a cycle $x \succ y \succ z \succ x$, and require that, for a given case, preferences be the reverse cycle. It is easy to see that no population can exhibit such preferences, because no population can vote for a Condorcet cycle with $q > \frac{2}{3}$.[2]

4 Proofs

4.1 Proof of Theorem 1:

Step 1: The case $|X| = 2$

Assume that $X = \{x, y\}$. Without loss of generality, we will assume that all voters discussed will satisfy $w_i(y, c) = 0$ for all $c \in C$. Thus, a voter with decision weights $(w_i(x, c))_{x \in X, c \in C}$ will be characterized by a vector $w_i = (w_i(x, c))_{c \in C}$. She *prefers* x to y *given* $M \subset C$ if $\sum_{c \in M} w_i(x, c) > 0$.

Some preliminary definitions will prove useful. For a set of cases $D \subset C$ with $|D| = d$, a voter with decision weights w_i is said to be a D^+ *voter* if

$$w_i(c) = \begin{cases} \frac{1}{d+1} & \text{if } c \in D \\ -1 & \text{if } c \notin D \end{cases}.$$

Observe that a D^+ voter (strictly) prefers x to y given $M \neq \varnothing$ if and only if $M \subset D$, and she (strictly) prefers y to x otherwise. A voter is a D^- *voter* if $-w_i(c)$ defines a D^+ voter. Thus, a D^- voter (strictly) prefers y to x given $M \neq \varnothing$ if and only if $M \subset D$, and she (strictly) prefers x to y otherwise.

A population (N, w) is a k-D^+ *population* if N consists of $2k$ voters, where k are D^+ voters, and k are $\varnothing^-$ voters. If (N, w) is a k-D^+ population, then, given $M \neq \varnothing$, $x \succ_{((N,w),M)} y$ if $M \subset D$ and $x \sim_{((N,w),M)} y$ otherwise.

[2] However, it is not clear that this is the most natural definition of the problem when $|X| > 2$. Indeed, majority vote with $q > \frac{1}{2}$ may not be a very natural procedure for more than two alternatives.

Similarly, a population (N, w) is a k-D^- *population* if N consists of $2k$ voters, where k are D^- voters, and k are $\varnothing^+$ voters. Thus, if (N, w) is a k-D^- population, then, given $M \neq \varnothing$, $y \succ_{((N,w),M)} x$ if $M \subset D$ and $x \sim_{((N,w),M)} y$ otherwise.

We now turn to the proof. Let there be given a function $U : 2^C \to \mathcal{R}$ such that $U(\varnothing)$ is trivial. For $M \subset D$, denote $U(M)$ by $\succsim_M$ and let $\succ_M$, $\sim_M$ have their usual meaning. We wish to construct a population (N, w) such that $V_{(N,w)}(M) = \succsim_{((N,w),M)} = \succsim_M$. This population will be constructed inductively as the union of k-D^+ and k-D^- populations, for appropriately chosen sets D and numbers k.

Let $(D_1, ..., D_{2^{|C|}-1})$ be an enumeration of all non-empty subsets of C that is non-increasing with respect to set cardinality. That is, if $r > s$, then $|D_r| \leq |D_s|$. Thus, $D_1 = C$, whereas $D_{2^{|C|}-|C|}, ..., D_{2^{|C|}-1}$ are singletons. We will prove the following claim by induction:

Claim: For every $1 \leq r \leq 2^{|C|} - 1$, there exists a population $(N_r, w_{(r)})$ such that, for all D_s with $s \leq r$, $\succsim_{((N_r,w(r)),D_s)} = \succsim_{D_s}$.

Thus, the majority vote of population $(N_r, w_{(r)})$, $V_{(N_r,w_{(r)})}(M)$, will agree with the target relation $\succsim_M$ for the first r sets in $(D_1, ..., D_{2^{|C|}-1})$. Setting $r = 2^{|C|} - 1$ will complete the proof of Step 1.

Proof of Claim:

For $r = 1$, consider $\succsim_C$. If $x \sim_C y$, set N_r to be empty. Otherwise, if $x \succ_C y$, let $(N_r, w_{(r)})$ be a 1-C^+ population.

Assume that the claim is true for $r - 1 \geq 1$, and that $(N_{r-1}, w_{(r-1)})$ is the population provided by the induction hypothesis. We will construct $(N_r, w_{(r)})$ such that N_r is a superset of N_{r-1} and $w_{(r)}$ – an extension of $w_{(r-1)}$. Consider D_r. If $\succsim_{D_r}$ equals $\succsim_{((N_{r-1},w(r-1)),D_s)}$, $(N_r, w_{(r)})$ can be set equal to $(N_{r-1}, w_{(r-1)})$. Suppose, then, that the two differ. Assume, first, that $x \succsim_{((N_{r-1},w(r-1)),D_s)} y$ but that $y \succ_{D_r} x$. Define $(N_r, w_{(r)})$ to be the disjoint union of $(N_{r-1}, w_{(r-1)})$ and a k-D_r^- population for a large enough k. Observe that k can be chosen so that the majority for y in the k-D_r^-

population outweighs the majority that might exist for x in $(N_{r-1}, w_{(r-1)})$. Specifically, choose

$$k = \tfrac{1}{2}[\#\{i \in N_r \mid \textstyle\sum_{c\in M} w_{(r-1)i}(x,c) > \sum_{c\in M} w_{(r-1)i}(y,c)\} - \#\{i \in N_r \mid \sum_{c\in M} w_{(r-1)i}(x,c) < \sum_{c\in M} w_{(r-1)i}(y,c)\}] + 1.$$

Observe that the difference in square brackets is even, since our construction involves only the disjoint union of populations, within each of which either there is a tie between the two alternatives, or there is an even-size majority for one of them.

Thus $\succsim_{((N_r,w(r)),D_r)}$ equals $\succcurlyeq_{D_r}$. The main observation is, however, that $\succsim_{((N_{r-1},w(r-1)),D_s)}$ also equals $\succcurlyeq_{D_s}$ for $s < r$. To see this, let $s < r$ and consider D_s. D_s differs from D_r (since $s \neq r$), and it is not a subset of D_r (which is possible only if $s > r$). Hence the k-D_r^- population we add, $(N_r \backslash N_{r-1}, w_{(r)})$, consists of exactly k voters who prefer x to y given D_s, and k voters whose preferences are reversed. This implies that whatever was the majority vote in $(N_{r-1}, w_{(r-1)})$ given D_s, it is identical for $(N_r, w_{(r)})$ given D_s.

Next assume that $x \succ_{((N_{r-1},w(r-1)),D_s)} y$ but that $y \sim_{D_r} x$. The same construction applies with

$$k = \tfrac{1}{2}[\#\{i \in N_r \mid \textstyle\sum_{c\in M} w_{(r-1)i}(x,c) > \sum_{c\in M} w_{(r-1)i}(y,c)\} - \#\{i \in N_r \mid \sum_{c\in M} w_{(r-1)i}(x,c) < \sum_{c\in M} w_{(r-1)i}(y,c)\}].$$

Finally, the cases in which $y \succsim_{((N_{r-1},w(r-1)),D_s)} x$ are dealt symmetrically (by addition of an appropriate k-D_r^+ population). ¤

Step 2: The case $|X| > 2$

Assume that $X = \{x_1, ..., x_m\}$. Consider two alternatives, x_p and x_q. For every $M \subset C$, restrict the relation $U(M)$ to $\{x_p, x_q\}$. Use Step 1 to construct a population $(N_{p,q}, w_{(p,q)})$, defined for $\{x_p, x_q\}$, such that $V_{(N_{p,q},w_{(p,q)})}(M)$ equals $U(M)$ on $\{x_p, x_q\}$ for every M. We now extend the decision weights

$w_{(p,q)}$ of voters in $N_{p,q}$ from $\{x_p, x_q\}$ to all of X in two distinct ways, and we will eventually take the union of the two populations thus generated.

First, let $(N^t_{p,q}, w^t_{(p,q)})$ be a population of voters (with preferences defined over all of X), where, for each voter and given any M, $\{x_p, x_q\}$ are ranked above all other alternatives, where the rest are ranked by their index. This can be done, for instance, by setting $w_{(p,q)_i}(x_r, c) = -r - 1$ for all $r \notin \{p, q\}$, for all $c \in C$, and all $i \in N^t_{p,q}$. Recall that the construction in step 1 produced weights $w_i(x, c) \in [-1, 1]$. It follows that the new weights defined for x_r are lower than those for $\{x_p, x_q\}$.

Next, let $(N^b_{p,q}, w^b_{(p,q)})$ be a population of voters (again, with preferences defined over all of X), for which the opposite is true: given any M, $\{x_p, x_q\}$ are ranked below all other alternatives, where the rest are ranked in reverse index order. For instance, set $w_{(p,q)_i}(x_r, c) = +r + 1$ for all $r \notin \{p, q\}$, for all $c \in C$, and all $i \in N^b_{p,q}$.

Now consider the population generated by the union of $(N^t_{p,q}, w^t_{(p,q)})$ and $(N^b_{p,q}, w^b_{(p,q)})$. Let there be given a set $M \subset C$. Majority vote between x_p and x_q is identical in both sub-populations, and is identical to $U(M)$. Hence it is also the majority vote in the new population $(N^t_{p,q} \cup N^b_{p,q}, w^t_{(p,q)} \cup w^b_{(p,q)})$. For any pair of indices $\{r, s\} \neq \{p, q\}$, exactly half of the new population prefers x_r to x_s, and the other half has reverse preferences. Hence the new population is indifferent between any pair $\{x_r, x_s\}$ such that $\{r, s\} \neq \{p, q\}$.

Finally, consider a population that is the disjoint union of $\binom{m}{2}$ sub-population, one for each pair $\{x_p, x_q\}$, constructed as above. Majority vote in the entire population between $\{x_p, x_q\}$ is determined by the $\{x_p, x_q\}$ sub-population, and therefore coincides with $U(\cdot)$. ¤ ¤

4.2 Proof of Proposition 2:

Step 1: rational q

Assume that $q = \frac{t}{t+s}$ where $t > s > 0$ are natural numbers. The proof in this case mimics the Step 1 in the proof of Theorem 1, with the following modification.

A population (N, w) is a k-l-D^+ *population* if N consists of $k + l$ voters, where k are D^+ voters, and l are $\varnothing^-$ voters. If (N, w) is a s-t-D^+ population, then, given $M \neq \varnothing$, $x \succ_{((N,w),M,q)} y$ if $M \subset D$ and $x \sim_{((N,w),M,q)} y$ otherwise. Similarly, a population (N, w) is a k-l-D^- *population* if N consists of $k + l$ voters, where l are D^- voters, and k are $\varnothing^+$ voters. Thus, if (N, w) is a s-t-D^- population, then, given $M \neq \varnothing$, $y \succ_{((N,w),M,q)} x$ if $M \subset D$ and $x \sim_{((N,w),M,q)} y$ otherwise.

One continues to construct the population (N, w) inductively, as in the Claim in the proof of Theorem 1. The only difference is that, if there is a need to add a sub-population to $(N_{r-1}, w_{(r-1)})$ in order to obtain $(N_r, w_{(r)})$, one adds a ks-kt-D^+ population for a large enough natural k (in case $x \succ_{D_r} y$) and a ks-kt-D^- population for a large enough natural k (in case $y \succ_{D_r} x$). ¤

Step 2: irrational q

The proof relies on approximating q by rational numbers. As in the case of a rational q, the construction is based on successive additions of k-l-D^+ populations and k-l-D^- populations, as the need may be. Only in this construction one uses ks-kt-D^+ populations and ks-kt-D^- populations, where $t > s > 0$ are natural numbers such that $\frac{t}{t+s}$ approximates q. Specifically, consider stage r in the induction of the Claim. Assume, without loss of generality that, that $x \succ_{D_r} y$ but that $y \succ_{((N_{r-1},w(r-1)),D_r,q)} x$, hence we are about to add a ks-kt-D_r^+ population. We first choose k, then s and t. Choose $k > \frac{q}{1-q}|N_{r-1}|$. The population we add will have $k(s + t) > k$ voters, and will therefore outweigh the existing population N_{r-1} by a ratio of $\frac{q}{1-q}$ or more. That is, for any $s, t > 0$ adding a ks-kt-D_r^+ population will result in $x \succ_{((N_r,w(r)),D_r,q)} y$. It is left to choose $s, t > 0$ such that $\succ_{((N_r,w(r)),D_p,q)}$ agrees with $\succ_{((N_{r-1},w(r-1)),D_p,q)}$ for $p < r$. To this end, let

$$\varepsilon = \min_{p<r} \left| q - \frac{\#\{i \in N_{r-1} \mid \sum_{c \in D_p} w_{(r-1)i}(x,c) > \sum_{c \in D_p} w_{(r-1)i}(y,c)\}}{|N_{r-1}|} \right|$$

Observe that $\varepsilon > 0$. Choose $t > s > 0$ such that $\left|\frac{t}{t+s} - q\right| < \frac{|N_{r-1}|}{k(s+t)}\varepsilon$. The existence of such s, t can be derived from the theory of continued fractions. Indeed, the approximation by continued fractions of an irrational q yields a sequence (p_n, q_n) of integers such that $\lim_{n\to+\infty} q_n = +\infty$ and $\left|q - \frac{p_n}{q_n}\right| \leq \frac{1}{q_n^2}$. Thus, one may set $t = p_n$ and $s + t = q_n$ where n is large enough.

It follows that, for every $p < r$,

$$sign\left(q - \frac{\#\{i \in N_{r-1} \mid \sum_{c\in D_p} w_{(r-1)i}(x,c) > \sum_{c\in D_p} w_{(r-1)i}(y,c)\}}{|N_{r-1}|}\right) =$$
$$sign\left(q - \frac{\#\{i \in N_{r} \mid \sum_{c\in D_p} w_{(r)i}(x,c) > \sum_{c\in D_p} w_{(r)i}(y,c)\}}{|N_{r}|}\right)$$

that is, $x \succ_{((N_r,w(r)),D_p,q)} y$ iff $x \succ_{((N_{r-1},w(r-1)),D_p,q)} y$. This completes the proof. ¤ ¤

5 Discussion

Our result assumes that voter's preferences are additive in cases. There are many reasons for which this assumption may be unrealistic. For instance, imagine that voters prefer candidates who exhibited strong ideological convictions in their youth, irrespective of the ideology they subscribed to. A case in which a candidate supported a communist party, as well as a case in which the candidate supported a fascist party, will speak well of the candidate. But the combination of these cases will point to incoherence, lack of integrity, or opportunism.

Preferences may not be additive in cases also due to logical inferences that voters can make, based on strategic reasoning. As pointed out by Glazer and Rubinstein (2001), the very fact that one party brings forth a particular argument while it could have brought forth another may be informative in its own right. Whereas Grice (1975, 1989) may be viewed as suggesting a strategic analysis of conversations based on the assumption that speakers and listeners play a common interest game, Glazer and Rubinstein apply strategic reasoning to debates, in which interests are far from common. Athreya, Gilboa, and Schmeidler (2002) analyze Glazer and Rubinstein's example in the case-based model we use here. They show that even if preferences are additive in cases, inferences based on strategic reasoning may lead to non-additive functions, because mentioning one case in a debate is equivalent to bringing forth an entire set of cases.

Aragones, Gilboa, Postlewaite, and Schmeidler (2002) discuss situation in which cases are used to draw the listeners' attention to analogies or to certain regularities. These may change the way voters view cases they already know of, and may therefore be another reason for non-additivity in the way voters react to cases.

Our main thesis is that the impact of a set of cases on voters may be hard to predict based on the impact of other sets of cases. Our results show that even a simple preference structure suffices to render society choice rather complex. Introducing more realistic preferences will only strengthen our point.

Throughout the paper we refer to elements of C as "cases", which are to be thought of as facts or stories. But the formal model also allows other interpretations. In particular, members of C may be arguments that are being raised for or against certain alternatives. Again, one finds that a very simple rule for aggregation of arguments at the individual level already yields complex patterns of majority votes.

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