Indulgent Angels or Stingy Venture Capitalists?
The Entrepreneurs' Choice

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Abstract

This paper studies entrepreneurs' choice of investors, who must provide financial capital and effort for projects with externalities. Venture capitalists (VCs) and individual investors (angels) compete to finance the projects. VCs seek to invest into a portfolio of projects, while angels have more slack in how much they invest into one project. In the presence of externalities between projects, VCs can potentially increase the total value of their investment portfolio through better coordination of investment, while some angels behave indulgently and give more financial investment than necessary, earning zero profits in equilibrium. Surprisingly, externalities do not give VCs as much of an advantage as one would expect. Quite often VCs lose out to angels even when this means that some projects will not receive an optimal amount of effort. In the projects they invest in, VCs always make strictly positive profits despite the competition.

JEL classification: G24, G32

A recent trend in entrepreneurial finance has been the emergence of networks of related start-up companies funded by one investor. In spite of the abundance of anecdotal evidence, to our knowledge no comprehensive theory of how network externalities affect the financing decision and future values of firms has yet been put forward.

Empirical literature, from Sahlman (1990) to Kaplan and Stromberg (2001) describes the activities and compares the characteristics of two main groups of investors into innovative projects — business angels and venture capitalists (VCs). However, some questions remain unanswered: for instance, what determines entrepreneurs' choice of investors, and what role do the externality effects play in this choice? Is VCs' reputation as vultures who earn abnormal profits at the expense of entrepreneurs really justified? And if it is, why do these profits not fall with competition? When and why would entrepreneurs go to VCs instead of angels?

We provide some answers to these questions. In this paper we develop a theoretical framework to study how externalities between portfolio companies affect the investment decisions of different groups of investors. Using this framework, we analyze entrepreneurs' choice of investor for their innovative projects.

Due to the self interests of investors and entrepreneurs, externalities between the projects' outcomes can potentially lead to suboptimal solutions, even if contracts are publicly observable (see, e.g., Segal (1999)). Suboptimality can be exemplified as underinvestment or no investment at all into a worthy project.

The situation is only aggravated by the moral hazard problem caused by the noncontractibility of an investment's non-monetary element. In order to commit to higher investment, investors need higher stakes in the companies they invest in, and this results in strictly positive profits for these investors — a condition that entrepreneurs are not happy to agree to, if they have any alternative.

Angel investors and venture capitalists (VCs) use two different mechanisms to alleviate this problem. Because VCs invest in portfolios of firms, by internalizing externalities and coordinating investments in order to maximize the overall value of their investment portfolio, VCs are in a better position to commit to providing investment that will benefit all their portfolio firms. Rich angel investors, meanwhile, use the fact that the financial component of an investment is contractible. They ask for higher stakes in return for giving more money and indulgently allowing entrepreneurs to divert part of the invested money to the entrepreneurs' personal needs, thus potentially changing the projects' outcome. Since angels are investing their own money at their own discretion, there is nothing to stop such indulgent behavior, which can relieve the moral hazard problem and does tend to shift entrepreneurs' preferences to angel investment.

VCs often lose out to their rivals, the indulgent angels, who come out as surprise winners, earning zero profit not as a fair return on necessary investment, but by getting a higher stake in return for overinvestment in projects. When VCs are chosen as project investors, they earn strictly positive profit despite the competition with other VCs.

Our paper builds upon a growing body of literature studying investors in risky innovative projects and their contracts with the entrepreneurs. As stated earlier, these studies have identified two major groups emerge as the main suppliers of capital to this type of projects — business angels

and VCs. Angels are rich individuals who invest their own money. VCs are professionals, who raise money for VC funds from individuals and institutional investors and act as the general partners of VC funds managing the capital raised.

From the literature we know that both angels and VCs not only provide financial capital, but are also actively involved in monitoring, advising and formulating business strategy, (see, e.g., Kaplan and Stromberg (2001), Prowse (1998), Ehrlich et al. (1994), Gorman and Sahlman (1989)). Ehrlich et al. (1994) find that in comparison with business angels, VCs are more involved in the management of portfolio companies. Probably this is due to the fact that angels' resources, such as personal time, are more limited than VCs'. In this paper we assume that VCs can provide both financing and advising for a portfolio of companies, while angels can invest human capital into only one company, even if they are wealthy enough to provide financing for several companies. This assumption is consistent with the existent literature. For example, Gorman and Sahlman (1989) report that VC firms in their sample have an average portfolio of nine entrepreneurial companies. Kanniainen and Keuschnigg (2000) and Kanniainen and Keuschnigg (2001) model portfolio investment and optimal portfolio size. In their models, the VC faces a trade-off between rents from a bigger number of companies and the correspondingly diminishing quality of advice.

To summarize, for the purposes of this paper we assume two major differences between angels and VCs: 1) angels invest their own money, while VCs invest other partners' money and are more accountable for their investment and 2) although some extremely rich angels can provide financing for several companies, even they cannot be actively involved with more than one company.

In our model, portfolio investment does not diminish the quality of the advising effort, if this effort is exerted. The problem arises from the fact that unlike the amount of financial investment, the effort exerted by an investor is not contractible. This leads to the moral hazard problem, which can be resolved by a specific contract design. For example, Repullo and Suarez (1998) find that in a double-sided moral hazard problem the optimal contract between VCs and an entrepreneur has the characteristics of convertible preferred stock. Hellmann, in a series of papers (Hellmann (1994), Hellmann (1998a) and Hellmann (2002)), studies why and under what circumstances entrepreneurs would voluntarily relinquish control to VCs. This happens, for example, when VCs have better expertise in decisions affecting the value of the firm. Casamatta (2000) shows that when a VC's investment (in terms of both cash and effort) is high, it is optimal to give him convertible bonds, and when it is low, he should receive common stock.

Our model is close to those of Bhattacharya and Chiesa (1995) and Cabral (1998). In Cabral (1998) different parties commit together to a potentially rewarding joint venture. However, the free-rider problem hinders innovation — if the project is successful, the discovery (technological innovation) becomes a public good, and so the parties have an incentive to deviate from jointly-optimal behavior (to underinvest) at the outset, in order to free-ride. Bhattacharya and Chiesa (1995) have a similar model with implications closer to those of this paper, although they do not consider a moral hazard problem — in their model all investment is contractible. They study the interaction between financial decisions and the disclosure of interim research results to competing firms. Technological knowledge revealed to a firm's financier(s) need not also flow to its R&D and

product market competitors. The authors show that the choice of financing source can serve as a precommitment device for pursuing ex-ante efficient strategies in knowledge-intensive environments.

Hellmann (1998b) and Ueda (2000) have models where the entrepreneur's choice of investors takes into account the possibility that investors can steal the entrepreneur's idea. Unlike Ueda's paper, where stealing by a VC is seen necessarily as a bad thing, in our model information spillover is two-directional and can bring more value to the entrepreneur, because he could be the one who benefits from using others' ideas.

Our model studies entrepreneurs' choice of investors in the presence of externalities between their projects and unobservable effort by investors. It features two entrepreneurs, each with a risky two-stage project that requires investment from outside investors. The first stage develops a new technology and requires a small, but contractible financial investment, plus an expert human capital investment, which is not contractible. Only angels and VCs possess the necessary financial and human capital. In return for their investment, investors receive a share in a project. The second stage commercializes this technology.

The externalities in the model are due to the fact that the results of the R&D stage of one project affect the success/failure of the second stage of both projects.

Our objective is to study what kind of investors will be chosen by entrepreneurs for their projects. As in real life, the angel investors form a heterogeneous group. They have varying levels of wealth and some of them are rich enough to provide financing to both projects. However, even they cannot put the non-financial effort into both projects. Since angel investors invest their own money, they can be more lenient (indulgent) than VCs over what entrepreneurs do with their money. VCs invest other people's money, and accordingly seek to limit their investment to what is necessary and no more. On the other hand, each VC has enough human capital resources to support two projects at the same. Both angels and VCs want to maximize their profits and compete with each other.

In our paper we show that coordinated investment by VCs guarantees profitable investment in some projects for which angel investment would be suboptimal in terms of exerted effort. In this case the VCs' profits are strictly positive, although they do not depend on the value of the projects' payoff.

Surprisingly, the effect of information spillover between two projects does not give VCs as much of an advantage as one would expect. VCs do not usually provide better terms for entrepreneurs than the angel investors do, and underinvestment remains in a disappointingly large area. What is even more surprising is that in the regions where indulgent angels achieve the first best outcome, a VC can never match them.

Our results imply that in innovation driven industries, we should observe angel investment into relatively safe projects on generous financial investment terms, whereas VC investors play a more dominant role in financing more risky projects.

The rest of the paper is organized as follows. Section I describes the model. Section II derives the results, which are discussed in Section III. Finally, Section IV concludes.

I. Model

The model has three dates $t = 0$, 1 and 2. Two entrepreneurs, E_1 and E_2, are endowed with their own innovative projects. E_1 has project 1 and E_2 has project 2. Each project comprises two stages. The first stage, which we call the R&D stage, investigates the feasibility of a potentially promising technology for a given project. Each project can test only one technology at this stage. Technology is very broadly defined here. It includes, but is not limited to, new business models, distribution channels, markets, products and services. Amazon's "one-click" shopping, Dell's direct sales and FedEx' hub system are just few examples of such new technologies.

The R&D stage requires investment into a project of financial capital, K, and expert effort (human capital), e, in addition to the entrepreneur's own effort, which is here normalized to zero.[1] Unlike K, which is observable and verifiable, e can be interpreted as the advising and monitoring process of investors, which is observable, but not verifiable, and therefore cannot be contracted upon. Since the object of our attention is the effort provided by investors, to simplify our analysis we assume that K is very small, but still positive.

For a project that receives both K and e, the R&D stage has a probability of success, β. If the effort is zero, $e = 0$, then the R&D stage is unsuccessful. We interpret the result of the first stage as a costly answer to the question: "Will this technology work?" Investing K and e gives an entrepreneur and investor the right to put the question and obtain the answer, while investing only K simply entitles them to observe the other project's research result. If the project does not even receive K, then it goes out of business and cannot proceed to the next stage.

The success and failure of the R&D stage of individual projects are independent of the other project's outcome. However, as in Bhattacharya and Chiesa (1995), the final payoff of each project is affected by the other project. If at least one project is successful at the R&D stage, this technology can then be freely adopted by both projects provided that they have funding for the second stage. In other words, only failure of the R&D stage for both projects renders it impossible to go to the second stage, which we call the market stage.

At the market stage, the technology is actually implemented and production takes place. Projects 1 and 2 have payoffs V and RV, net of the second stage capital investment. R characterizes the degree of asymmetry between the projects . For clarity, we assume that $R \geq 1$.

Investors and contracts

Entrepreneurs need both financial and human external capital. As discussed in the introduction, only two categories of investors can provide both money and expertise. In comparison with VCs, the angels' human capital resources are more limited, therefore we assume that one angel investor can invest e into only one project, while a VC investor has enough resources to invest K and e into both projects. This is consistent with empirical evidence, e.g., Prowse (1998) and Ehrlich et al. (1994). Since K is small, in general we assume that rich angel investors can provide financing for both projects. Since angel investors invest their own money, we also allow them to invest more

than K, if they deem it necessary. We denote financial investment in project i as I_i.

Entrepreneurs choose their investors from a pool of angel investors and VCs. We assume that there are more than two angel investors and more than two VCs. All investors in the pool have financial wealth of no less than K. Expressed formally, investors offer financial investment I_i in project i in return for α_i share of the future cash flow.[2] An investor can also make a conditional offer to both entrepreneurs proposing I_1 in return for α_1 to E_1 and I_2 in return for α_2 to E_2 conditional on this offer's acceptance by both entrepreneurs.

As mentioned, we also allow wealthy individual investors to offer I_i that is greater than K. If $I_i > K$, then $(I_i - K)$ is appropriated by E_i for his personal use. Each entrepreneur's objective is to maximize his final cash flow, that is the sum of his share in the project plus the diverted cash flow $(I_i - K)$.

Entrepreneurs observe investors' offers and choose one investor for one project. In other words, E_i observes all α_i's offered and chooses the investor offering the most attractive one. A joint offer is considered accepted only if both entrepreneurs accept their respective parts of the offer. We assume that entrepreneurs cannot make transfer payments to each other. No investment is made until all the parties are satisfied with the share allocations and capital offered.

Only financial investment I_i is verifiable, but although the effort levels are not verifiable, entrepreneurs correctly anticipate them in equilibrium. To simplify our analysis we also assume that the effort levels are observed by entrepreneurs and investors and that investor 2 observes the degree of effort made by investor 1 before exerting his own effort.[3]

Since investors act competitively, the competition drives their profits down, although not necessarily to zero. Investors have zero reservation utility, so they prefer to participate in projects even if they have zero expected return.

We assume that everybody is risk-neutral.

As several authors have pointed out, e.g., Hellmann (1994), the VC-entrepreneur relationship should be analyzed as a two-sided incentive problem. Our model is simpler than the double-sided moral hazard problem in Repullo and Suarez (1998), because K is very small and this means that, when entrepreneurs act as agents, they will never divert the total capital investment I_i to their own benefit. We focus on a single-aspect moral hazard problem, in which entrepreneurs act as principals and investors act as agents.

Information structure

There is no information asymmetry at $t = 0$. The project characteristics, such as β, R, V and required investments K and e are common knowledge. α_1, I_1, α_2 and I_2 are publicly observable. Investor 2 observes investor 1's effort before exerting his own effort, although the effort is not verifiable. At $t = 1$ the R&D results become known and if at least one project is successful, all projects still in business (meaning those that received financing $I \geq K$ at $t = 0$) can freely use its result, provided that they have financing for the second stage.

The timeline

The timeline can be summarized as follows.

At $t = 0$, the entrepreneurs announce their projects. Investors decide whether or not to participate in projects and offer financial investment I_i in return for share α_i in project i. E_1 and E_2 choose their investors. If these investors are different, we call them investor 1 and investor 2. Once the choice of investors becomes final, i.e., when both E_1 and E_2 are satisfied with α_1 and α_2, investment of financial and human capital takes place. Investor 1 makes his investment first, investor 2 observes his exerted effort and makes his own investment.

Projects that receive financial investment of less than K do not get off the ground.

At $t = 1$ the success or failure of the R&D stage is observed by all parties that are still in business. The failed projects can potentially use the technology of the successful project, if there is one.

At $t = 2$, the net payoffs are realized.

Externality

The R&D externality is created by the transferability of R&D results of one project to the other project. It is characterized by the probability of success of an individual project, which depends on the level of effort put into the project. If no effort is put in any project, then the payoff to each project is zero. Investing e only in project 2 creates the externality for project 1, because its payoff becomes βV, which is the measure of externality in this case and it reaches its maximum at $\beta = 1$. The payoff to project 2 is $\beta R V$. If e is put in both projects, the expected payoff to project 2 becomes $\beta (2 - \beta) R V$. The difference between the two payoffs, $\beta (1 - \beta) R V$, is the externality that project 1 investment creates for project 2. This externality reaches its maximum at $\beta = \frac{1}{2}$.

II. Results

First we consider the first best outcome, defined as the result which maximizes the joint surplus (NPV) of both projects. Since $R > 1$ and K is very small,[4] it is never optimal to have only one project running at the second stage; therefore, there are three possible candidates for the first best outcome:

1. None of the projects receives financing. The NPV is zero.

2. One project receives both K and e, while the other project has only K. In the case of success, both projects continue at the second stage. The expected NPV is $\beta (1 + R) V - (K + e) - K$.

3. Both projects receive both K and e. In the case of success, both projects continue at the second stage. The expected NPV is $\beta(2 - \beta) (1 + R) V - 2 (K + e)$.

We can easily see that investing e into both projects is the first best outcome whenever

$$\begin{cases} \beta(2-\beta)(1+R)V - 2(K+e) > 0, \\ \beta(2-\beta)(1+R)V - 2(K+e) > \beta(1+R)V - (K+e) - K. \end{cases}$$

Since K is small, $K < \frac{\beta^2}{2}(1+R)V$, this is equivalent to

$$e < \beta(1-\beta)(1+R)V. \tag{1}$$

Similarly, we see that investing e into only one project is optimal for

$$\begin{cases} e > \beta(1-\beta)(1+R)V, \\ e \le \beta(1+R)V - 2K. \end{cases}$$

Finally, the projects are not worth investing in, if

$$\begin{cases} e > \beta(1-\frac{\beta}{2})(1+R)V - K, \\ e > \beta(1+R)V - 2K. \end{cases}$$

Again, for small K, $K < \frac{\beta^2}{2}(1+R)V$, this is equivalent to

$$e > \beta(1+R)V - 2K. \tag{2}$$

All other things being equal, the zone of investment into both projects grows with R. It is also more advantageous for β to be close to $\frac{1}{2}$. If β gets any bigger, then it becomes more advantageous to invest only into one project, because of the externality effect.

Interestingly, due to the externality effect, investment into project 1 can be optimal, even if the expected net payoff to this project is negative, i.e., if $\beta(2-\beta)V < K+e$ and $\beta(1-\beta)(1+R)V > e$.

Two sources of inefficiency might preclude from achieving the first best result: 1) the selfish interests of participants, both entrepreneurs and investors, and 2) the coordination problem for the investors. In the remaining part of this section we describe these inefficiencies and show how investors can restore the first best result.

A. Angel investment

We will first examine the possible outcomes with angel investment, if VC investment is not available and angel investors are only competing with each other. This competition drives their profits down to the level determined by the incentive compatibility (IC) and participation constraints (PC) of investors and entrepreneurs.

In order to understand what kind of contracts between entrepreneurs and angel investors can be observed in equilibrium, we start our analysis from the effort choice by two angel investors, who

become involved in projects 1 and 2. The angel investing in project i selects his level of effort (zero or e) so as to maximize his expected profit given his share α_i in project i and the share α_j in project j attributable to investor j.

The outcome should be the Nash equilibrium of the game described the tree in Figure 1 and by the following matrix

effort	0	e
0	$(0;0)$	$(\alpha_1\beta V; \alpha_2\beta RV - e)$
e	$(\alpha_1\beta V - e; \alpha_2\beta RV)$	$(\alpha_1\beta(2-\beta)V - e; \alpha_2\beta(2-\beta)RV - e)$

where the elements of the matrix are the investors' expected payoffs net of the effort exerted. Rows correspond to the level of effort exerted by investor 1 and columns correspond to the level of effort exerted by investor 2.

Possible equilibria are described in Appendix A. There are three subgame perfect equilibria: $(e;e)$, $(0;e)$ and $(0;0)$.

The minimum values of $\alpha_{A,1}^e$ and $\alpha_{A,2}^e$ that can provide the $(e;e)$ outcome are:

$$\alpha_{A,1}^e = \frac{e}{\beta(1-\beta)V}, \qquad \alpha_{A,2}^e = \frac{e}{\beta(1-\beta)RV}. \tag{3}$$

Despite the competition, the net profits of the investors involved in each can remain strictly positive even at the minimum values $\alpha_{A,1}^e$ and $\alpha_{A,2}^e$ and are equal to

$$\Pi_{A,i}^e = \frac{1}{1-\beta}e - I_i, \tag{4}$$

where I_i is the financial investment into project i provided by the angel investor. For example, if $I_1 = I_2 = K$, then both investors make positive profits.

The profits of entrepreneurs are

$$\begin{cases} \Pi_{E_1,A}^e = \left(1 - \frac{e}{\beta(1-\beta)V}\right)\beta(2-\beta)V + (I_1 - K) = \beta(2-\beta)V - \frac{2-\beta}{1-\beta}e + (I_1 - K), \\ \Pi_{E_2,A}^e = \left(1 - \frac{e}{\beta(1-\beta)RV}\right)\beta(2-\beta)RV + (I_2 - K) = \beta(2-\beta)RV - \frac{2-\beta}{1-\beta}e + (I_2 - K). \end{cases}$$

The minimum values of $\alpha_{A,1}^{ne}$ and $\alpha_{A,2}^{ne}$ that can provide the $(0;e)$ outcome are:

$$\alpha_{A,1}^{ne} = \frac{K}{\beta V}, \qquad \alpha_{A,2}^{ne} = \frac{K+e}{\beta RV}, \tag{5}$$

where the superscript "ne" refers to "no effort" by investor 1. Investor 2 still exerts e. The necessary

condition for this equilibrium to exist is

$$K + e \leq \beta RV. \tag{6}$$

If investors receive $\alpha_{A,1}^{ne}$ and $\alpha_{A,2}^{ne}$, their profits are zero. The entrepreneurs' profits are equal to the NPVs of their projects:

$$\Pi_{E_1,A}^{ne} = \beta V - K, \qquad \Pi_{E_2,A}^{e} = \beta RV - K - e.$$

In Appendix A we show that for fixed α_1 and α_2 these two equilibria do not coexist, and from observing α_1 and α_2 we can accurately infer the effort level exerted by investor 1.

If $(0;0)$ is the equilibrium both projects have zero gross payoff; hence, none of the investors is interested in investing in them. We can infer that if

$$e > \beta RV, \tag{7}$$

then no investment is possible, although for $e < \beta \left(1 - \beta\right)\left(1 + R\right) V$ the first best outcome might be to invest money and effort into **both** projects!

Possible outcomes when angel investment is financially constrained

Investors efforts are not observable. However, as we have mentioned, in equilibrium entrepreneurs correctly anticipate them by observing α_1 and α_2. If $\beta \left(1 - \beta\right) V < e < \beta RV$, then $(e; e)$ equilibrium in the effort choice is impossible and $\left(\alpha_{A,1}^{ne}, \alpha_{A,2}^{ne}\right)$ is the equilibrium allocation of shares received by investors. On the other hand, for $e < \beta \left(1 - \beta\right) V$, both equilibria are possible depending on the choice of $\alpha_{A,i}$ by entrepreneurs.

1. For small e, $e \leq \beta \left(1 - \beta\right), V$, angel investors offer E_1 and E_2 financing K in return for investors' shares $\alpha_{A,1}$ and $\alpha_{A,2}$, respectively, such that $\alpha_{A,1} \geq \alpha_{A,1}^{e}$ and $\alpha_{A,2} \geq \alpha_{A,2}^{e}$. If their offers are accepted, both investors provide money and effort e.

2. If e satisfies the inequality $e \leq \beta RV$, E_1 can be offered financing in return for investor's share $\alpha_{A,1}'$, in project 1, $\alpha_{A,1}^{ne} \leq \alpha_{A,1}' < \alpha_{A,1}^{e}$, and E_2 can be offered financing in return for $\alpha_{A,2}'$ share in project 2, $\alpha_{A,2}^{ne} \leq \alpha_{A,2}' < \alpha_{A,2}^{e}$. Only the project 2 investor provides money and effort e, while the project 1 investor provides money and zero effort.

3. For $e > \beta RV$ no angel financing is possible.

We will now turn to the contracts that can be offered in the equilibrium. Suppose that angel investors are financially constrained and cannot invest more than K — an assumption that we will relax later. Since they cannot invest more than K, they compete with each other by asking entrepreneurs for a smaller α_i share that would still elicit investors' choice of effort most preferred

by entrepreneurs. Since in both equilibria investor 2 provides e, the situation is not symmetrical. It is the preference by E_1 that plays the crucial role in determining the outcome.

For example, if both entrepreneurs prefer an outcome in which investors both provide e, then the smallest α_i share asked for by investor i will be $\alpha_{A,i}^e$. In this case an investor cannot win a contract by asking for a slightly smaller share, $\alpha_{A,1} = \alpha_{A,i}^e - \varepsilon$, because he cannot commit to provide e at the later stage. However, if $(e;e)$ is not an outcome preferred by E_1, then investor 1 cannot enforce it by asking for $\alpha_{A,1}^e$. Any investor who concedes ε of this share, thus signalling that he will not exert e, will leave investors asking for $\alpha_{A,1}^e$ without a chance of winning the contract. As in Bertrand competition, the decision to ask for the share $\left(\alpha_{A,i}^e - \varepsilon \right)$ is not sustainable in this case and $\alpha_{A,1}$ will go down to $\alpha_{A,i}^{ne}$, generating zero profit for investor 1.

When $e \leq \beta \left(1 - \beta \right) V$ angel investors are prepared to provide effort for both projects, which would lead to the first best outcome. In this case, the profits of participating investors are strictly positive. However, due to the incentive compatibility constraints of entrepreneurs, the condition $e \leq \beta \left(1 - \beta \right) V$ is not sufficient to achieve the first best outcome. We have to check whether the IC constraints for entrepreneurs are satisfied when $e \leq \beta \left(1 - \beta \right) V$ and investors are receiving $\alpha_{A,1}^e$ and $\alpha_{A,2}^e$. Suppose that alternatively E_1 could choose an investor who would provide investment K and zero effort in return for $\alpha_{A,1}^{ne}$. He would still prefer to choose an investor offering investment for $\alpha_{A,1}^e$, i.e., to give the investor a bigger share of project 1 iff the value of his share $\left(1 - \alpha_{A,1}^e \right)$ in the project were higher in this case than the value of his share $\left(1 - \alpha_{A,1}^{ne} \right)$ in the project with zero effort:

$$\left(1 - \frac{e}{\beta \left(1 - \beta \right) V} \right) \beta \left(2 - \beta \right) V \geq \left(1 - \frac{K}{\beta V} \right) \beta V,$$

which gives us

$$e \leq \frac{\beta \left(1 - \beta \right)^2 V + K \left(1 - \beta \right)}{2 - \beta}. \tag{8}$$

As we have mentioned, because investor 2 always provides effort, the preferences of E_2 do not really matter. In fact it turns out that E_2 prefers to receive $\left(1 - \alpha_{A,2}^e \right)$ if

$$e \leq \beta \left(1 - \beta \right)^2 RV + K \left(1 - \beta \right),$$

which holds, if inequality (8) holds.

It is easy to see that if inequality (8) does not hold, E_1 will prefer an investor who asks for $\alpha_{A,1}^{ne}$ in return for his investment. We have already seen why such investors exist despite the fact that the profit will be zero, while with $\alpha_{A,1}^e$ the investor's profit would be strictly positive.

We summarize the results in the following proposition:

Proposition 1. *If no angel investor can invest more than K, then angel investment leads to the following outcomes:*

1. For small e satisfying inequality (8), angel investors offer E_1 and E_2 financing K in return for investors' shares $\alpha_{A,1}^e = \frac{e}{\beta(1-\beta)V}$ and $\alpha_{A,2}^e = \frac{e}{\beta(1-\beta)RV}$ respectively. Both investors provide money K and effort e. Each investor makes strictly positive profit $\frac{1}{1-\beta}e - K$. E_1 has positive return $\beta(2-\beta)V - \frac{2-\beta}{1-\beta}e - K$ and E_2 has positive return $\beta(2-\beta)RV - \frac{2-\beta}{1-\beta}e - K$.

2. For e satisfying the double inequality

$$\frac{\beta(1-\beta)^2 V + K(1-\beta)}{2-\beta} < e \leq \beta RV - K \tag{9}$$

   investors 1 and 2 receive, respectively, shares $\alpha_{A,2}^{ne}$ and $\alpha_{A,2}^{ne}$ from (5) in return for financial investment K. Investor 2 exerts e, while investor 1 does not exert any effort.

3. For $e + K > \beta RV$ no angel financing is possible.

As we see from Proposition 1, quite often angel investment leads to a suboptimal outcome. For example, if $e < \beta(1-\beta)(1+R)V$, then exerting e for both projects is the first best action, while the angel investment achieves this result only under a much more restrictive condition of inequality (8). Similarly, no projects should receive funding only if $e + 2K > \beta(1+R)V$, while angel investors refuse financing whenever $e + K > \beta RV$.

The inefficiency of angel investment stems mainly from the angels' self interest, because they make their investment decisions without regard for the impact on the other project's outcome. However, it is also partly attributable to entrepreneurs' selfish interests — even when angels are ready to provide investment and effort to achieve the first best outcome, entrepreneurs might choose a solution that is in fact suboptimal in terms of the total value created.

Also, although investors act competitively and do not intentionally coordinate their investments, they do still take the existence of other projects into account.

If we ignored the externality effect of other existing projects, assumed the expected value of project 1 to be βV and tried to analyze the returns to angel investors on an isolated basis, then for e satisfying inequality (8) we would encounter the following "paradox": angel investors receive a smaller than "fair" share in the projects but nevertheless obtain strictly positive returns in the competitive world.

Of course, in reality there is no paradox once the externality effect is properly factored in.

So far in our analysis we have not allowed investors to invest money in more than one project, or to invest more than strictly necessary for the project's success. Therefore, even despite keeping this seemingly "smaller than fair" share $\alpha_{A,i}$, investors cannot be called indulgent — they do not give entrepreneurs more than the necessary investment K.

In the coming part of our paper, we will relax these two constraints by allowing angel investors to make financial, but not the human capital, investment, into two projects and by allowing them to invest more than K into one project. Of course, all players still remain rational and are not prepared to accept negative profits.

Possible outcomes with financially unconstrained angel investment

If angel investors do not have monetary constraints, then two separate factors can now affect the outcome: 1) the same angel investor can finance both projects, while exerting e for only one of them and 2) the amount of money invested into one project can exceed K. We have to emphasize that the same angel investor cannot exert effort e for both projects, because his human capital resources are limited.

Let us consider first that our angel investors are rich and the same investor can invest K into both projects. Investing into two projects makes sense either if it leads to a different outcome or if it gives the investor a higher profit without lowering that of the entrepreneurs.

If inequality (8) holds, then a rich angel investor cannot provide a viable alternative, for an obvious reason — he cannot exert e for both projects. If $(0; e)$ is an outcome in Proposition 1, then the rich investor cannot change the outcome, as he cannot increase his profit (making it positive) without asking for a higher share in at least one project and the entrepreneur who own the project concerned will not agree to that.

The only situation in which the two-project investment by the same angel investor might make a difference is where $e + K > \beta RV$ and $(0; 0)$ is the outcome in Proposition 1. This gives the following result

Proposition 2 (Projects' resuscitation by rich angels). *If the system of inequalities*

$$\begin{cases} K + e > \beta RV, \\ 2K + e \leq \beta \left(1 + R\right) V, \end{cases} \tag{10}$$

holds, then one rich angel investor finances both projects in return for shares $\alpha_{RA,1}$ and $\alpha_{RA,2}$ in projects 1 and 2 respectively, such that

$$(\alpha_{RA,1} + \alpha_{RA,2}R)\beta V = 2K + e. \tag{11}$$

Proof. *The angel investor will demand α_1 and α_2 such that*

$$(\alpha_1 + \alpha_2 R)\beta V \geq 2K + e$$

with competition driving it down to equality. Such an allocation with $\alpha_{RA,i} < 1$ always exists. For example, $\alpha_{RA,1} = \alpha_{RA,2} = \frac{2K+e}{\beta V(1+R)} < 1$ ∎

The exact values of $\alpha_{RA,1}$ and $\alpha_{RA,2}$ are the result of bargaining between entrepreneurs and the investor and are outside the scope of this paper.

The second and, we believe, more interesting situation is the one in which individual investment can exceed K. This would appear to be an unusual situation, because everybody knows that the required financial investment is K. So for institutional investors, like VCs, investing more than necessary is highly unlikely. Since angel investors do not have such restrictions, they can invest

more than K if they deem it necessary. Obviously, this can happen only if investors make non-negative profit as a result. Since in the $(0, e)$ equilibrium investors have zero profits, they will have an incentive to give entrepreneurs more than K only if this leads to an (e, e) equilibrium outcome.

In Proposition 1, for any e satisfying the double inequality

$$\frac{\beta (1 - \beta)^2 V + K (1 - \beta)}{2 - \beta} < e \leq \beta (1 - \beta) V$$

both investors were prepared to exert e, but the IC constraints for entrepreneurs, especially for E_1, precluded this outcome. By offering entrepreneurs a choice between $\alpha_{A,i}^{ne}$ in return for investment K and $\alpha_{A,i}^{e}$ in return for investment I_i, with $I_i > K$, investors can shift entrepreneurs' preferences to the second option if the following system of inequalities holds[5]

$$\begin{cases} \left(1 - \frac{e}{\beta(1-\beta)V}\right) \beta (2 - \beta) V + (I_1 - K) \geq \left(1 - \frac{K}{\beta V}\right) \beta V - K, \\ \left(1 - \frac{e}{\beta(1-\beta)RV}\right) \beta (2 - \beta) RV + (I_2 - K) \geq \left(1 - \frac{K+e}{\beta RV}\right) \beta RV - K; \end{cases}$$

or

$$\begin{cases} I_1 \geq \frac{(2-\beta)}{1-\beta} e - K - (1 - \beta) \beta V, \\ I_2 \geq \frac{1}{1-\beta} e - K - (1 - \beta) \beta RV; \end{cases} \tag{12}$$

From (4) we derive that the investors' participation constraints (non-negative profit condition) are

$$I_i \leq \frac{e}{1 - \beta}.$$

Combined with (12) this gives us an inequality

$$e \leq \beta (1 - \beta) V + K, \tag{13}$$

which always holds whenever angels' IC constraints for the $(e; e)$ outcome hold. Of course, in equilibrium, competition between angel investors will increase I_i until $I_i = \frac{e}{1-\beta}$ unless **all** investors' disposable financial resources investment are strictly smaller than $\frac{e}{1-\beta}$, in which case the outcome depends on whether or not there are at least two investors who can invest I_1 and I_2 that satisfy (12). We thus come to the following proposition.

Proposition 3 (Indulgent angel investors). *For $1 + R > 1$, if inequality 13 holds and there are at least two investors capable of and willing to provide financial investment which satisfies the system of inequalities (12), then in return for shares $\alpha_{A,1}^{e}$ and $\alpha_{A,2}^{e}$ satisfying condition (3), both projects will receive human capital investment e and financial investment I_1 and I_2, each in excess of K, such that*

1. $I_1 = I_2 = \frac{(2-\beta)}{1-\beta} e - K - (1 - \beta) \beta V$, if there are only two such investors. Both investors earn

positive profits

$$(1 - \beta) \beta V - e + K;$$

2. $I_1 = I_2 = I_{third}$, *if all but two investors can invest only strictly less than* $\frac{e}{1-\beta}$. I_{third} *is the third biggest amount that can be invested. The two richest investors earn positive profits*

$$\frac{e}{1 - \beta} - I_{third}$$

3. $I_1 = I_2 = \frac{e}{1-\beta}$ *if at least three investors can invest* $\frac{e}{1-\beta}$. *Investors' profits are equal to zero.*

We call these angel investors "indulgent", because they know that the entrepreneurs will appropriate for personal use part $(I_i - K)$ of their financial investment, and nevertheless allow it to happen.

The good news is that e is now put into both projects. The bad news for investors is that if there are enough rich investors, they lose all their profits, even in the area satisfying inequality (8) where angel investors would have positive profit, if they were not rich and indulgent. Of course, this is good news for entrepreneurs, who make higher profit. Since the combined net value of investors and entrepreneurs coincides with the first best outcome, we may call this result the "quasi" first best outcome.

B. VC investment

Let us mow suppose that VC investment is possible as well. If VC investors enter the market, then the only time they create higher value for the projects than angel investors would be in a situation where putting e into both projects is the first best outcome, while angel investors would put e only into one project. On the other hand, we assume that as institutional investors, VCs cannot make a financial investment in excess of the strictly required level, i.e., they cannot invest more than K in each project. Without this assumption, VCs could always at least match the angels' investments. We also assume that if VCs and angels create identical value for entrepreneurs, the entrepreneurs will prefer angel investors, due to the less formal nature of angel investment and the higher search costs involved in obtaining VC investment.

If K is small, then angel financing is always possible outside the "no investment" area determined by (2).[6] In order to win a contract, VCs must offer better deals to both entrepreneurs. The VC who wins the contract, must credibly commit to put e into both projects and continue both projects at $t = 1$, while both entrepreneurs obtain higher value for their stakes thanks to the VC's investment. The next proposition establishes, when these conditions hold

Proposition 4. *If he system of inequalities*

$$\begin{cases} e < \beta\left(1-\beta\right)\left(1+R\right)V, \\ e > \beta\left(1-\beta\right)V, \\ e \leq \beta RV - K, \\ e \leq \left(1-\beta\right)^2 \beta\left(1+R\right)V + 2K\left(1-\beta\right) \end{cases} \tag{14}$$

holds, then VC investment provides a more attractive alternative to angel investment. The VC investor provides financial capital K and human capital e for each project in return for shares $\alpha^*_{VC,1}$ and $\alpha^*_{VC,2}$, such that

$$\alpha^*_{VC,1} + \alpha^*_{VC,2}R = \frac{e}{\beta\left(1-\beta\right)V},$$

and

$$\begin{cases} \alpha^*_{VC,1} \leq \frac{1-\beta}{2-\beta} + \frac{K}{\beta(2-\beta)V}, \\ \alpha^*_{VC,2} \leq \frac{1-\beta}{2-\beta} - \frac{e-K}{\beta(2-\beta)RV}. \end{cases}$$

The VC makes a strictly positive profit

$$\Pi_{VC} = \frac{\beta}{1-\beta}e - 2K$$

Proof. *The proof is in Appendix B. The proof follows from the IC constraints for the VC and for the entrepreneurs.* ∎

The VC receives a strictly positive profit, because only then is his commitment to exert e for both projects credible. By assumption, VCs cannot "waste" money by overinvesting, therefore the profit remains strictly positive. If we relaxed this assumption, then the situation would resemble that described in Proposition 3 and instead of K, the VC investor would provide I_1 and I_2 such that $I_1 + I_2 = \frac{\beta}{1-\beta}e$, $I_i \geq K$.

The ability of VC investors to provide the first best result is not sufficient, because it can be obstructed by entrepreneurs' lack of interest in such a result. To summarize our results, we can write that the first best outcome which maximizes the combined value of the two projects is attainable in the following regions:

1. If either system of inequalities (14) or (**??**) holds, then entrepreneurs will choose a VC who exerts e for both projects.

2. If inequality (13) holds, then indulgent angel investors are the entrepreneurs' best choice. Both projects receive e.

3. If inequality (10) holds, then the same rich angel investor provides financing for both projects and exerts e for one of them.

4. For $\beta\left(1-\beta\right)\left(1+R\right)V \le e \le \beta RV - K$ two angel investors finance both projects, with only one project getting e.

Corollary 5. *If the system of inequalities*

$$\begin{cases} e < \beta\left(1-\beta\right)\left(1+R\right)V, \\ e > \beta\left(1-\beta\right)V, \\ e \le \beta RV - K, \\ e > \left(1-\beta\right)^2 \beta\left(1+R\right)V + 2K\left(1-\beta\right) \end{cases}$$

holds, then the first best outcome is never achieved by investors.
Proof. *The proof follows from Proposition 4.*

Corollary 5 shows that the despite strictly positive expected profit of a participating VC investor, when competing for contracts the VCs cannot lure entrepreneurs by conceding part of it (without investing more than K) due to the moral hazard problem. On the other hand, E_1 is adherent to the suboptimal outcome, because of his higher profits from it. If transfer payment from E_2 to E_1 were possible, it would make interests of E_1 and E_2 congruent, but it would not eliminate the VC's moral hazard problem.

The fact that in order to guarantee an input of e into both projects, VCs must earn strictly positive profit, means that when compared with indulgent investment from Proposition 3, VC investment will always be less attractive. In this case VCs cannot offer entrepreneurs terms that are comparable to the angels' contracts. We express this formally in the following proposition.

Proposition 6. *If $e \le \beta\left(1-\beta\right)V$, then VCs' offers are always strictly dominated by indulgent investors' offers.*
Proof. *The proof is in Appendix B.* ∎

III. Discussion and Empirical Implications

Figures 1-5 provide an illustration of our results. Numbers 1, 2 and 3 correspond to the areas of "poor", "rich" and "indulgent" angel investment, respectively, 4 corresponds to the region of VC investment, and in area 5 "no investment" is the first best outcome. Tildes denote underinvestment in the corresponding area. For computational simplicity we assume that $K = 0$.

Place Figure 1 here

Figure 1 shows how areas of angel and VC investment depend on e/V and β if there is no asymmetry between projects, i.e., $R = 1$. We see that the region of VC investment (area 4) is quite small in comparison with angels' investment. VCs invest in relatively risky but profitable projects.

Place Figure 2 here

As the asymmetry between projects grows (on Figure 2 $R = 2$), the area of the "two-projects effort" first best outcome also increases, and VCs start investing in a bigger number of projects, now including projects which are less risky and less profitable conditional on success, i.e., project 1 has a lower profitability index, $\frac{V}{e}$ of the realized payoff. The growth of VC investment area 4 is outpaced by the overall growth of the area where "two-projects effort" is optimal, thus creating inefficiency due to the angel underinvestment (areas $1'$ and $2'$).

The way the area of VC investment grows as the asymmetry between the projects increases is illustrated more clearly in Figures 3 and 4. For projects with a high $\frac{V}{e}$ ratio (on Figure 3 $\frac{V}{e} = 4.1$) VC investment into less risky projects is the contributor to this growth, while for projects with a more moderate $\frac{V}{e}$ ratio (in Figure 4 $\frac{V}{e} = 2$) VC investment expands both into higher and lower risk projects as the asymmetry grows.

Place Figure 3 here

Place Figure 4 here

In Subsection II. A we were considering equilibrium outcomes in an economy where angel investors are cash constrained. Figure 5 illustrates what happens in an economy which does not have rich individual investors. We see that VC investment takes over entire areas of the former portfolio investments of rich angels, creating additional efficiency in area 4* and encroaching on the former area of indulgent angel investment, but not completely — low risk projects are taken over by separate angel investors whose investment is suboptimal (area $1''$). In area 4* the VC investor still makes a positive profit, while in the remaining areas, formerly the reserve rich angel investment, his expected profit is zero.

Place Figure 5 here

Therefore, from Figure 5 we can infer that in comparison with rich economies, in poor economies one should observe more VC investment than angel investment, with angels holding the ground in investment into relatively safe and moderately profitable projects and VCs overtaking them in less profitable projects and highly profitable, but risky projects. In addition, unlike rich economies, where VC investment always makes positive profit despite competition, in poor economies VCs also make investments that generate zero profit.

The existence of areas of underinvestment, the strictly positive profit obtained the participating VC, and the cash overspending by indulgent angels all raise legitimate questions as to whether or

not the outcome can be further improved. One possible way of achieving the first best outcome could be to create a coalition of indulgent angels, with each investor holding stakes in both projects, while investing effort and (overs)investing cash in only one project. Such a coalition would make the "two-project effort" first best outcome always attainable; both investors would make zero profit, essentially driving VCs out of business. A quick analysis shows that capital investment I_1 and I_2 and allocations of investors' stakes in both projects is a situation that is possible, although the exact solution involves bargaining between four parties and is beyond the scope of this paper.

In fact, such coalitions of angel investors are, in all but name, VC funds without limited partners. Therefore, another alternative to avoid underinvestment would be to allow VCs to invest excess cash in the project they support. Probably, this is what some VCs did in the late nineties. The difficulty is that this solution leads to another agency conflict between VCs, who act as agents for the true owners of this money — the limited partners in venture capital funds.

A. Empirical Implications

Our results suggest the empirical implications of several types. First, there are implications concerning the VCs' returns in their portfolio companies. Second, we make inferences about investment patterns of angels' investment and VCs' investment. Third, we expect to observe some differences in start-up financing in countries with different per capita wealth of the richest angel investors.

To be more specific, when comparing VCs' investment portfolios to angels' portfolios, we should observe that VC-backed companies are relatively more heterogeneous in terms of their profitability or market share, but they nevertheless use compatible technology, i.e., belong to the same industry or related industries, while companies in the angel investment portfolio are more closely related. This difference can be tested by constructing a "heterogeneity index" for VCs' and angels' investment portfolios. For both portfolios, we should observe more information spillover (e.g., adoption of the same technology, use of similar business models) between portfolio companies than between companies financed independently.

Second, regarding the *ex-ante* probability of success for each company on an individual basis, we predict that VCs invest in more risky companies than angel investors. Ex-ante valuation of some of these companies might not even justify the investment made, showing the negative NPV. This prediction can be tested by comparing the business plans of entrepreneurial companies.

Third, the *ex-post* profitability of VC backed companies is *higher* than for those financed by rich angels who also invest into a portfolio of companies, but *lower* than for companies backed by angels who are single-company investors.

Despite these differences in the profitability of portfolio companies,VC returns remain positive and are higher than angel returns. This is consistent with the previous prediction, because VC investors apparently consistently receive a higher project share than angel investors in return for investment of the same size.

Finally, in the economies with lower per capita wealth of rich angel investors, we should observe a higher proportion of VC-backed start-ups. At the same time, VC would receive on average

lower profits from these start-ups, then from their investments in fewer start-up in rich economies. The reason is the following: in rich economies VCs lose out to angel investors the projects, in which investors earn zero expected profits, and are able to finance only the projects that generate positive profits to VCs. As we have indicated before, these positive profits serve as a commitment mechanism in the investor's moral hazard problem.

IV. Conclusion

In this article we study entrepreneurs' choice of investors, who must provide financial capital and effort for projects with externalities.

Each angel investor can exert effort for only one project, while VCs can invest in a portfolio of projects. When coordinated investment is necessary because of a strong externality effect, the VCs' portfolio approach can potentially lead to the first best outcome, whereas angels investing in separate projects may generate a suboptimal outcome, because they tend to undersupply effort for their projects and free-ride on competitors' results.

We have shown that angels can demonstrate their commitment to provide effort by supplying entrepreneurs with more cash than necessary for the projects' success. Because of the overinvestment, angels are then obliged to exert a high degree of effort in order to earn zero profit. This result offers an explanation for the lavish supply of cash attracted by certain entrepreneurial projects.

Due to the lack of coordination between angel investors, regions still remain where they either underinvest or fail to finance a group of projects, because they have negative NPV, if considered as stand-alone projects. Although portfolio investors, like angels and VCs, always manage to finance these projects, they are less successful in winning contracts from entrepreneurs running the underinvested projects.

The underinvestment problem can potentially be resolved by coalitions of angel investors who hold stakes in the portfolio of projects ("synthetic VCs"), or by allowing VCs to overinvest cash in their projects, which could be dangerous due to the principal-agency problem regarding relations between VCs (agents) and limited partners in venture capital funds (principals).

The results of this paper allow us to make the following conjectures.

First, a VC investment portfolio should include companies that are relatively heterogeneous in terms of their profitability or market share, but which nevertheless use compatible technology, i.e., belong to the same industry or related industries with high inter-industry externalities. Companies in the angel investment portfolio should be more closely related. For both portfolios, we should observe more information spillover (e.g., adoption of the same technology, use of similar business models) between portfolio companies than between companies financed independently.

Second, looking at the *ex-ante* probability of success for each company on an individual basis, we will find that VCs invest in more risky companies than angel investors. Ex-ante valuation of some of these companies might not even justify the investment made, showing the negative NPV.

Third, the *ex-post* profitability of VC backed companies is *higher* than for those financed by rich angels who also invest into a portfolio of companies, but *lower* than for companies backed by

angels who are single-company investors.

Finally, despite these differences in the profitability of portfolio companies, VC returns remain positive and are higher than angel returns. VC investors apparently consistently receive a higher project share than angel investors in return for investment of the same size. This result follows directly from our model, which made **no assumptions** about VCs' bargaining power or the scarceness of angel investment. On the contrary, if angel investors are poor and cannot invest cash into a portfolio of projects, VCs will invest in some zero profit projects. As individual investors become richer, VCs finance a smaller number of projects, but make higher profits on their investment!

References

Bhattacharya, Sudipto and Gabriela Chiesa, "Proprietary Information, Financial Intermediation and Research Incentives," *Journal of Financial Intermediation*, 1995, *4*, 328–357.

Cabral, Luis M.B., "R & D Cooperation and Product Market Competition," April 1998. LBS Working Paper.

Casamatta, Catherine, "Financing and Advising: Optimal Financial Contracts with Venture Capitalists," June 2000.

Ehrlich, S.B. et al., "After the Cash Arrives: A Comparative Study of Venture Capital and Private Investor Involvement in Entrepreneurial Firms," *Journal of Business Venturing*, 1994, *9*, 67–82.

Gorman, M. and W. Sahlman, "What Do Venture Capitalists Do," *Journal of Business Venturing*, 1989, *4*, 231–248.

Hellmann, Thomas, "Financial Structure and Control in Venture Capital," 1994. Stanford Working Paper.

——, "The Allocation of Control Rights in Venture Capital Contracts," *The Rand Journal of Economics*, Spring 1998, *29* (1), 57–76.

——, "A Theory of Corporate Venture Investing," July 1998. Stanford Working Paper.

——, "A Theory of Strategic Venture Investing," *Journal of Financial Economics*, May 2002, *64* (2), 285–314.

Kanniainen, Vesa and Christian Keuschnigg, "The Optimal Portfolio of Start-Up Firms in Venture Capital Finance," December 2000. CESifo Working Paper No. 381.

—— **and** ——, "Start-Up Investment with Scarce Venture Capital Support," March 2001. CESifo Working Paper No. 439.

Kaplan, Steven and Per Stromberg, "Venture Capitalists as Principals: Contracting, Screening, and Monitoring," *American Economic Review*, May 2001, *91* (2), 426–431.

Prowse, S., "Angel Investors and the Market for Angel Investment," *Journal of Banking and Finance*, 1998, *22*, 785–792.

Repullo, Rafael and Javier Suarez, "Venture Capital Finance: A Security Design Approach," September 1998. Working Paper.

Sahlman, W., "The Structure and Governance of Venture Capital Organization," *Journal of Financial Economics*, 1990, *27*, 473–521.

Segal, Ilya, "Contracting with Externalities," *Quarterly Journal of Economics*, May 1999, *114* (2), 337–388.

Ueda, Masako, "Bank versus Venture Capital," November 2000.

Notes

[1] Casamatta (2000) looks for contracts soliciting effort from both entrepreneur and investor, because she studies relations between the entrepreneur and investors, modeling them as a double-sided moral hazard problem. Here we focus on the entrepreneur's choice of investor, thus only the investor's effort is important.

[2] In Casamatta (2000) convertible debt should be used to elicit effort from the investor for a single-project model. Using convertible bonds does not change the results of this paper. Probably, this is due to the fact that we focus on a portfolio coordination problem.

[3] The model works without this assumption, but analysis of the observed equilibrium becomes unnecessarily over-complicated.

[4] Although the value of K is negligible, we keep K in the formulas in order to make a distinction from the situation in which the project gets no investment at all.

[5] Entrepreneurs are never interested in diverted K, because of the assumption that K is very small. Therefore, entrepreneurs always prefer to invest K rather than divert K and make the project fail.

[6] For bigger K, there exists a region where angel financing is impossible, while investing into both projects is the first best result

$$\begin{cases} e < \beta \left(1 - \beta\right) \left(1 + R\right) V, \\ e > \beta \left(1 + R\right) V - 2K. \end{cases}$$

In this case the VC investment is the entrepreneurs' only choice. The VC investor provides financial capital K and human capital e for each project in return for shares $\alpha_{VC,1}$ and $\alpha_{VC,2}$, such that

$$\alpha_{VC,1} + \alpha_{VC,2} R = \frac{e}{\beta \left(1 - \beta\right) V}.$$

The VC makes a strictly positive profit

$$\Pi_{VC} = \frac{\beta}{1 - \beta} e - 2K.$$

A. Appendix A.

In order to understand what kind of contracts between entrepreneurs and angel investors can be observed in equilibrium, we start our analysis from the effort choice by two angel investors, who become involved in projects 1 and 2. The angel investing in project i selects his level of effort (zero or e) so as to maximize his expected profit given his share α_i in project i and the share α_j in project j attributable to investor j.

The outcome should be the Nash equilibrium of the game described by the following matrix

effort	0	e
0	$(0; 0)$	$(\alpha_1 \beta V; \alpha_2 \beta RV - e)$
e	$(\alpha_1 \beta V - e; \alpha_2 \beta RV)$	$(\alpha_1 \beta (2 - \beta) V - e; \alpha_2 \beta (2 - \beta) RV - e)$

where the elements of the matrix are the investors' expected payoffs net of the effort exerted. Rows correspond to the level of effort exerted by investor 1 and columns correspond to the level of effort exerted by investor 2. Figure 1 shows the extensive form of the game

For $R > 1$ four equilibria are possible. They include three pure strategy equilibria $(e; e)$, $(0; e)$ and $(0; 0)$ and a mixed strategy equilibrium.

Equilibrium $(e; e)$

$(e; e)$ is the equilibrium if and only if

$$\begin{cases} e \leq \alpha_1 \beta (1 - \beta) V, \\ e \leq \alpha_2 \beta (1 - \beta) RV. \end{cases}$$

Assuming that when indifferent between making effort e or zero effort, the investors will exert effort e, we obtain the minimum values of $\alpha_{A,1}^e$ and $\alpha_{A,2}^e$ that can provide the $(e; e)$ outcome:

$$\begin{cases} \alpha_{A,1}^e = \frac{e}{\beta(1-\beta)V}, \\ \alpha_{A,2}^e = \frac{e}{\beta(1-\beta)RV}. \end{cases}$$

Despite the competition, the net profits of both investors can remain **strictly positive** even at the minimum values $\alpha_{A,1}^e$ and $\alpha_{A,2}^e$ and are equal to

$$\Pi_{A,i}^e = \frac{1}{1 - \beta} e - I_i,$$

where I_i is the financial investment into project i provided by the angel investor. For example, if $I_1 = I_2 = K$, then both investors make positive profits.

We are not saying that $\alpha_{A,1}^e$ and $\alpha_{A,2}^e$ will necessarily be the allocations of shares received by investors in equilibrium. This outcome is possible if both E_1 and E_2 agree to receive $(1 - \alpha_1)$ and
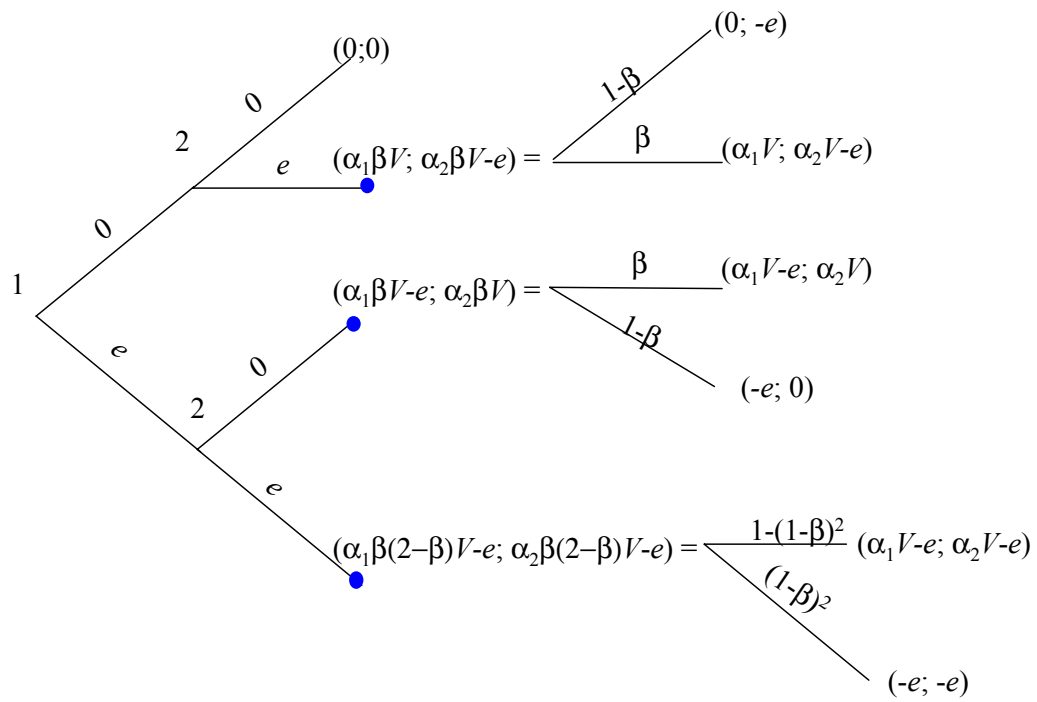
Figure 1: The tree of the effort choice by investors

$(1 - \alpha_2)$ shares, respectively. The values obtained by entrepreneurs in this case are

$$\begin{cases} \Pi^e_{E_1,A} = \left(1 - \frac{e}{\beta(1-\beta)V}\right)\beta(2-\beta)V + (I_1 - K) = \beta(2-\beta)V - \frac{2-\beta}{1-\beta}e + (I_1 - K), \\ \Pi^e_{E_2,A} = \left(1 - \frac{e}{\beta(1-\beta)RV}\right)\beta(2-\beta)RV + (I_2 - K) = \beta(2-\beta)RV - \frac{2-\beta}{1-\beta}e + (I_2 - K). \end{cases}$$

From $\alpha_1 < 1$ and $\alpha_2 < 1$ it follows that the necessary condition for this equilibrium to exist is $e < \beta(1-\beta)V$. Due to the moral hazard problem, angel investor 1 will not provide a positive effort, e, if e lies in the area $\beta(1-\beta)V < e < \beta(1-\beta)(1+R)V$ despite the strictly positive profit that this investor would make, if he could commit to e.

Equilibrium $(0; e)$

The outcome in which effort e is invested in project 2 only, is the Nash equilibrium iff

$$\begin{cases} e > \alpha_1 \beta(1-\beta)V, \\ e \leq \alpha_2 \beta RV. \end{cases}$$

Given the investors' participation constraints, i.e., that investors' profits cannot be negative, we obtain the minimum values of $\alpha^{ne}_{A,1}$ and $\alpha^{ne}_{A,2}$ that can provide the $(0; e)$ outcome:

$$\begin{cases} \alpha^{ne}_{A,1} = \frac{K}{\beta V}, \\ \alpha^{ne}_{A,2} = \frac{K+e}{\beta RV}, \end{cases}$$

where the superscript "ne" refers to "no effort" by investor 1 and only by investor 1, because investor 2 still exerts e. From $\alpha_1 < 1$ and $\alpha_2 < 1$ it follows that the necessary condition for this equilibrium to exist is

$$K + e \leq \beta RV.$$

If investors receive $\alpha^{ne}_{A,1}$ and $\alpha^{ne}_{A,2}$, their profits are zero. The entrepreneurs' profits are equal to the NPVs of the projects:

$$\begin{cases} \Pi^{ne}_{E_1,A} = \beta V - K, \\ \Pi^e_{E_2,A} = \beta RV - K - e. \end{cases}$$

Notice that for fixed α_1 and α_2 this equilibrium is not compatible with the $(e; e)$ equilibrium, because $\alpha_1 < \alpha^e_{A,1}$.

Equilibrium $(0; 0)$

$(0; 0)$ is the equilibrium iff

$$\begin{cases} e > \alpha_1 \beta V, \\ e > \alpha_2 \beta RV. \end{cases}$$

In this case both projects have zero gross payoff; hence, none of the investors is interested in investing in them. We can infer that if

$$e > \beta RV,$$

then no investment is possible, although for $e < \beta(1 - \beta)(1 + R)V$ the first best outcome might be to invest money and effort into **both** projects!

Mixed strategy equilibrium

If the following system of inequalities holds

$$\begin{cases} e < \alpha_1 \beta V, \\ e \geq \alpha_2 \beta(1 - \beta)RV, \end{cases}$$

then two Nash equilibria exist at the same time: $(0; e)$ and $(e; 0)$. Each angel investor would prefer to provide zero effort if he knew that the other was investing e at the R&D stage.

A mixed equilibrium exists in which investor i decides to provide zero effort at the R&D stage with probability p_i and decides to provide e with probability $(1 - p_i)$, where p_i is the solution to

$$\begin{cases} p_1 0 + (1 - p_1)\alpha_2 \beta RV = p_1 \alpha_2 \beta RV + (1 - p_1)\alpha_2 \beta(2 - \beta)RV - e, \\ p_2 0 + (1 - p_2)\alpha_1 \beta V = p_2 \alpha_1 \beta V + (1 - p_2)\alpha_1 \beta(2 - \beta)V - e. \end{cases}$$

The solution is: $\left\{ p_1 = \frac{-\alpha_2 \beta RV + \alpha_2 \beta^2 RV + e}{\alpha_2 \beta^2 RV}, p_2 = \frac{-\alpha_1 \beta V + \alpha_1 \beta^2 V + e}{\alpha_1 \beta^2 V} \right\}$. We thus have

$$\rho_1 = \frac{e - \alpha_2 \beta(1 - \beta)RV}{\alpha_2 \beta^2 RV} \qquad \text{and} \qquad \rho_2 = \frac{e - \alpha_1 \beta(1 - \beta)V}{\alpha_1 \beta^2 V},$$

The expected payoffs to investors 1 and 2 are

$$\Pi_{A,1} = \alpha_1 V - \frac{e}{\beta} - K, \qquad \Pi_{A,2} = \alpha_2 RV - \frac{e}{\beta} - K.$$

$\alpha_{A,i}$ for this equilibrium cannot be less than

$$\alpha_{A,1}^m = \frac{e + \beta K}{\beta V}, \qquad \alpha_{A,2}^m = \frac{e + \beta K}{\beta RV}$$

for which the investors' profits are zero. Since

$$\begin{cases} e < \alpha_1 \beta V, \\ e \geq \alpha_2 \beta(1 - \beta)RV, \end{cases}$$

must hold as well as $\alpha^m_{A,i} < 1$, we have

$$\begin{cases} e < e + \beta K, \\ e \geq (e + \beta K)(1 - \beta), \\ e + \beta K < \beta V. \end{cases}$$

So, the necessary condition for the mixed equilibrium to exist is $e + \beta K < \beta V$.

For $\alpha_{A,i} = \alpha^m_{A,i}$ entrepreneurs' profits are

$$\Pi^m_{E_1,A} = (\beta V - (e + \beta K)) \left(\frac{1}{\beta} - \frac{1}{\left(1 + \frac{\beta K}{e}\right)^2} \right)$$

$$\Pi^m_{E_2,A} = (\beta RV - (e + \beta K)) \left(\frac{1}{\beta} - \frac{1}{\left(1 + \frac{\beta K}{e}\right)^2} \right)$$

which are always positive

Since investor 2 observes the other investor's effort before he makes his own effort, the mixed strategy equilibrium is not subgame perfect and it is never realized in our model.

B. Appendix B.

Proof. Proposition 4. The incentive compatibility constraint for the VC is

$$\left(\alpha_{VC,1} + \alpha_{VC,2}R\right)\beta V - (2K + e) \leq \left(\alpha_{VC,1} + \alpha_{VC,2}R\right)\beta\left(2 - \beta\right)V - 2\left(K + e\right)$$

or

$$e \leq \left(\alpha_{VC,1} + \alpha_{VC,2}R\right)\beta\left(1 - \beta\right)V,$$

with competition driving it down to equality

$$e = \left(\alpha_{VC,1} + \alpha_{VC,2}R\right)\beta\left(1 - \beta\right)V.$$

Entrepreneurs are interested in VC investor, only if their profit sill be higher than with angel investment. Participation constraints for entrepreneurs E_i and E_2 are, respectively

$$\begin{cases} \left(1 - \alpha_{VC,1}\right)\beta\left(2 - \beta\right)V > \beta V - K, \\ \left(1 - \alpha_{VC,2}\right)\beta\left(2 - \beta\right)RV > \beta RV - \left(K + e\right), \end{cases}$$

or

$$\begin{cases} \alpha_{VC,1} \leq \frac{1-\beta}{2-\beta} + \frac{K}{\beta(2-\beta)V}, \\ \alpha_{VC,2} \leq \frac{1-\beta}{2-\beta} + \frac{(K+e)}{\beta(2-\beta)RV}. \end{cases}$$

Both incentive compatibility and participation constraints hold only if the following inequality is satisfied:

$$e \leq \left[\frac{1-\beta}{2-\beta}\left(1 + R\right) + \frac{\left(K + e\right) + K}{\beta\left(2 - \beta\right)V}\right]\beta\left(1 - \beta\right)V,$$

which gives

$$e \leq \left(1 - \beta\right)^2\beta\left(1 + R\right)V + 2\left(1 - \beta\right)K.$$

∎4

Proof. Proposition 6. IC constraints for VC and competition lead to

$$\alpha_{VC,1} + \alpha_{VC,2}R = \frac{e}{\beta\left(1 - \beta\right)V}$$

Indulgent angels hold $\alpha_{A,1}^e = \frac{e}{\beta(1-\beta)V}$ and $\alpha_{A,2}^e = \frac{e}{\beta(1-\beta)RV}$ in return for $\frac{e}{1-\beta}$

In order for VCs to win entrepreneurs contracts, the following IC constraints for entrepreneurs must hold:

$$\begin{cases} \left(1 - \alpha_{VC,1}\right)\beta\left(2 - \beta\right)V \geq \left(1 - \frac{e}{\beta(1-\beta)V}\right)\beta\left(2 - \beta\right)V + \frac{e}{1-\beta} - K \\ \left(1 - \alpha_{VC,2}\right)\beta\left(2 - \beta\right)RV \geq \left(1 - \frac{e}{\beta(1-\beta)RV}\right)\beta\left(2 - \beta\right)RV + \frac{e}{1-\beta} - K \end{cases}$$

which gives

$$\begin{cases} \alpha_{VC,1}\beta\left(2-\beta\right)V \leq e+K \\ \alpha_{VC,2}\beta\left(2-\beta\right)RV \leq e+K \end{cases}$$

and since the VC's profit cannot be negative, we arrive at an equality

$$(\alpha_{VC,1}+\alpha_{VC,2}R) = 2\frac{e+K}{\beta\left(2-\beta\right)V}$$

VC would in such a case make a zero profit, which contradicts VC's IC constraint.
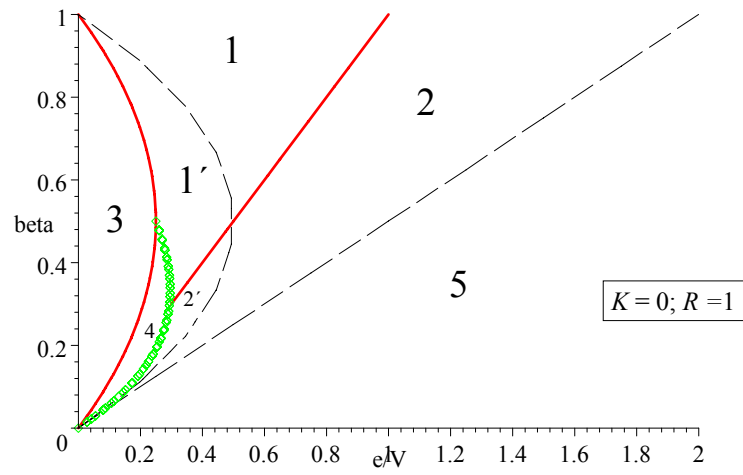
∎6

C. Appendix C.

Figure 1: Angel and VC investment as functions of e/V and β for $R = 1$, $K = 0$. Here and in the following figures numbers 1-3 denote areas of "poor", "rich" and "indulgent" angel investment, respectively, and 4 denotes VC investment. In 5 "no-investment" is the first best outcome. Tildes denote underinvestment.
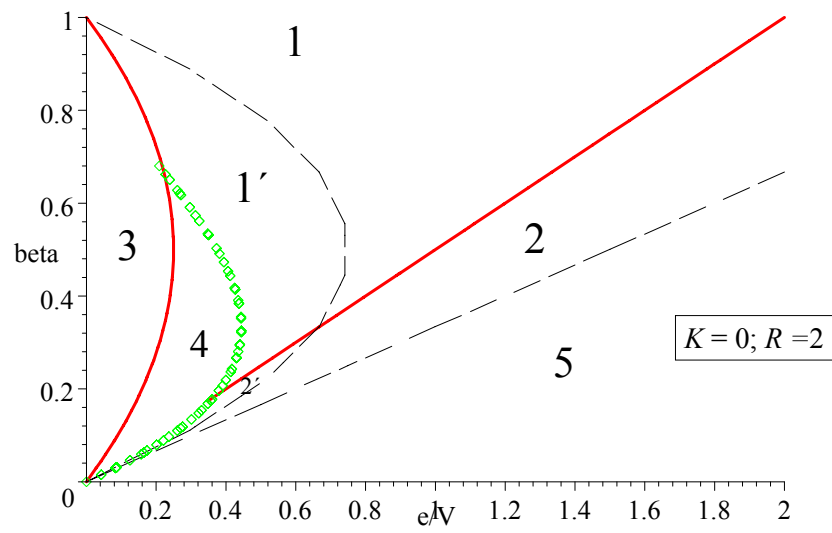
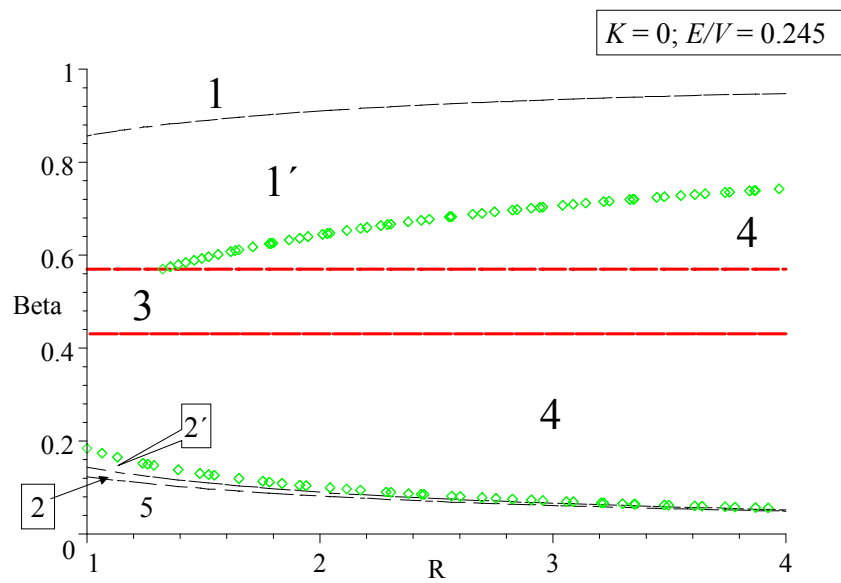Figure 2: Regions of angel and VC investment as functions of β and e/V for $R = 2$, $K = 0$.

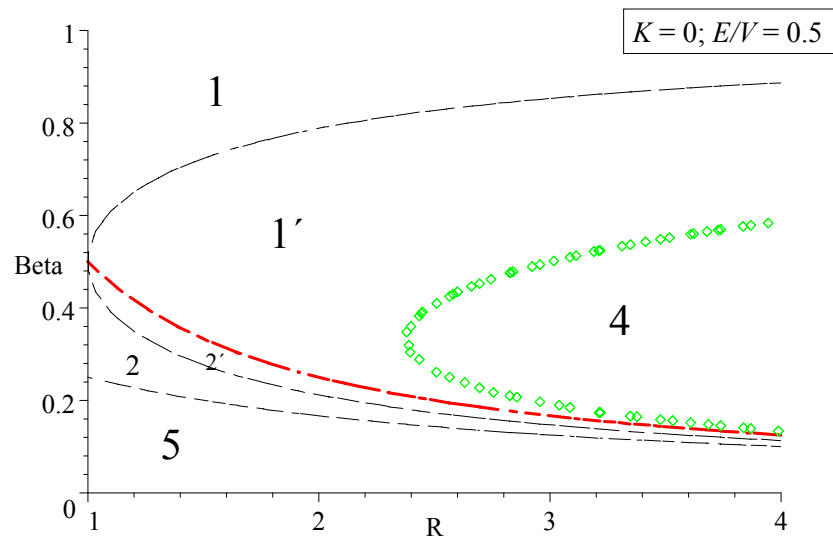Figure 3: Regions of angel and VC investment as functions of β and R for $e/V = 0.245$, $K = 0$.

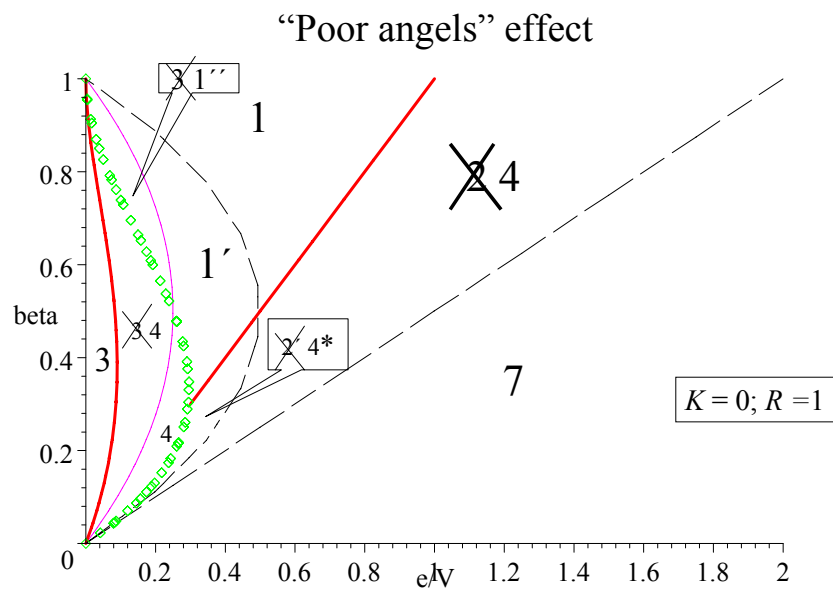Figure 4: Regions of angel and VC investment as functions of β and R for $e/V = 0.245$, $K = 0$.

Figure 5: Angel and VC investment as functions of e/V and β for $R = 1$, $K = 0$, if angels are wealth constrained. In 4* the first best outcome is now achieved by VCs. In 1″ efficiency is lost — angels cannot make excessive investment of cash, which leads to unerinvestment of human capital.