

# The (topo)logic of vagueness\*

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## Abstract

Zeno's "dichotomy" paradox of the runner and the sorites paradox exhibit certain interesting similarities. Both of them involve a long series of steps, each of which seems legitimate, but which, taken together, apparently lead to an unacceptable conclusion. In this article, a particular interpretation of a common reply to Zeno's paradox is presented, which recognises that to defuse the paradox, it is necessary to assert that the number of stages that the runner has completed on Zeno's infinite sequence of times is not an appropriate measure of whether he finishes the race or not. Applying this style of reply to the sorites argument, one would reject the argument on the grounds of the inappropriateness of the number of hairs for reasoning about baldness. Such an attitude to the sorites argument implies a certain conception of the problem posed by vague terms, according to which the problem is to understand such relationships between terms as the appropriateness of one for reasoning about the other. Consequently, it poses a certain set of challenges to prospective theories of vagueness.

**Keywords** Vagueness, theories of, problem of; sorites paradox; Zeno's paradox; scale.

Zeno's "dichotomy" paradox argues that a runner in a race will never finish: before reaching the finish, he must get to the half-way point  $p_1$ ; but once he has got to  $p_1$ , he must still reach the point half-way between  $p_1$  and the finish, say  $p_2$ ; but once he has reached  $p_2$  . . . So, one might conclude, he never really finishes the race.

The sorites paradox argues that a man with a full head of hair may count as bald: for a man with 0 hairs counts as bald; and if a man with 0 hairs counts as bald, a man with 1 hair counts as bald; and if a man with 1 hair counts as bald, a

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man with 2 hairs counts as bald; and . . . ; and if a man with 4999 hair counts as bald, a man with 5000 hairs counts as bald, so a man with 5000 counts as bald.

There is a certain structural resemblance between these arguments. Each involves a long series of apparently legitimate steps, which, taken together, yield an unacceptable conclusion. Despite this similarity, philosophers have tended to treat the two paradoxes separately and in different ways. One commonly accepts Zeno's infinite sequence as "compatible" with the race finishing in a finite number of seconds, whereas one often reacts to the sorites paradox by proposing a semantic, epistemic or pragmatic theory of vagueness which denies the sorites argument. In this paper, we shall employ the apparent *structural* similarity between the two paradoxes to transpose aspects of a common reply to Zeno's paradox onto the sorites case. More precisely, it shall be noted that compatibility of Zeno's argument with the race finishing in a finite time is not enough to defuse the paradox, for it is also necessary to assert that the number of seconds (not the number of stages of Zeno's sequence) is the *appropriate* measure for whether the runner has finished the race or not. If one tries to apply this aspect of the reply to Zeno's paradox in the sorites case, one ends up rejecting the argument on the grounds that the number of hairs is not appropriate for reasoning about baldness. This will have some interesting consequences for the conception of the *problem* which vagueness poses, and thus for what one should expect from a theory of vagueness. Questions regarding general relationships (such as compatibility and appropriateness) between terms (be they vague or precise) shall take precedent over the question of the truth of sentences featuring vague terms.

## 1 Zeno's paradox

Given that the interest of Zeno's paradox for the purposes of this paper lies in its structural similarities and differences with the sorites arguments, it shall be understood in a naïve sense, as an argument that the runner never reaches the finish.<sup>1</sup>

Common replies point out that although the times  $\tau_i$  it takes to run from points  $p_i$  to  $p_{i+1}$  are finite for each  $i$ , the sum of the infinite series of these finite times is finite, so it takes the runner a finite time to reach the finish (and thus he does finish the race).<sup>2</sup> There are two aspects of this response that should be distinguished:

- Firstly, there is the fact that the infinite series of the times  $\tau_i$  sums to a finite time,  $\tau$ ;

<sup>1</sup>Questions about the divisibility of space and of time shall thus be ignored.

<sup>2</sup>We shall reason here with times, although a similar argument can be run in terms of distances.

- Secondly, there is the implicit assertion that one judges whether or not the runner finishes the race in terms of the time  $\tau$ , which is finite, and not in terms of the number of steps of Zeno’s sequence, which are infinite.

Instead of presenting the discussion in terms of sequences of times, it is worth introducing the more general notion of a scale: a *scale*  $S$  on a collection of objects  $\mathcal{O}$  is a set of sets of objects of  $\mathcal{O}$ <sup>3</sup>. The idea is that these sets characterise the ‘fineness’ of the scale, that is to say what it can distinguish and what it cannot. A scale  $S$  can *distinguish* between two objects  $o_1$  and  $o_2$  of  $\mathcal{O}$  if there exists two sets of  $S$ , say  $E_1$  and  $E_2$ , such that each object is a member of only one of the sets: for example,  $o_1 \in E_1 \setminus E_2$  and  $o_2 \in E_2 \setminus E_1$ . So a scale for a time-line – a real line whose points, or ‘objects’, are time-points<sup>4</sup> – consists of a set of sets of time-points. Intuitively, these sets determine the points between which the scale does and does not distinguish.

For a given (increasing) sequence of time-points, one can construct a corresponding scale which distinguishes among the points of the sequence but does not distinguish between pairs of points which both lie between consecutive points of the sequence. For example, for a “constant” sequence of time points  $0, 1, 2 \dots$  at one second intervals, there is a constant scale  $S_C$ , whose sets are just intervals of constant unit length, that is, sets of the form  $[i, i + 1)$ , where  $i + 1$  is the time-point one second after  $i$  and the brackets indicate that the set contains the time-point  $i$  and all time-points after  $i$  but before  $i + 1$ . This scale only distinguishes between pairs of points which are at least a second apart: for example, it distinguishes between 2 and 3 but not between 2.25 and 2.75.

On the other hand, Zeno’s paradox uses a different sequence and thus a different scale on the time-line. Let  $t_i$  be the time-point at which the runner reaches the (space) point  $p_i$ . Zeno’s paradox uses the scale  $S_Z$  whose sets are intervals  $[t_i, t_{i+1})$ : after passing the time-point  $t_i$ , the paradox picks up on the moment it leaves the corresponding set of  $S_Z$ , that is, it picks up the time  $t_{i+1}$ . At small  $i$ , this scale cannot distinguish between many pairs of time points, but at large  $i$ , the scale becomes more refined, and can distinguish between points which are increasingly close. The two scales are shown, for a simple example, in Figure 1.

Zeno’s paradox occurs when one insists on “clocking” the run on the Zeno scale  $S_Z$ , that is, when one counts one stage in the progression of the runner for each transition between consecutive time-points in different sets of  $S_Z$ . We know very well that, since the Zeno scale becomes infinitely fine around the time at

<sup>3</sup>There are interesting questions pertaining to the constraints on this set of sets which are potentially pertinent to the notion of scale; however, we shall not need to concern ourselves with such details questions in this paper.

<sup>4</sup>Given that “physical” questions are ignored here (see note 1), we shall assume that time can be considered as a continuum of instants or time-points.

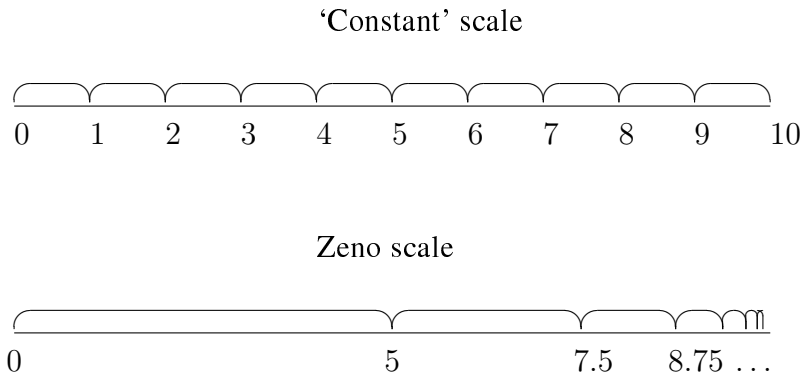


Figure 1: Scales on a time-line (Example of a runner running a 100m race at the constant speed of 10m/s; the values of time-points are in seconds)

which the runner should finish the race, it distinguishes between ever closer time-points. In fact, it distinguishes among infinitely many time-points in a finite time interval.

The first part of the reply to the paradox notes a *certain relationship between the two scales*, namely, that the union of the infinite number of sets of the Zeno scale  $S_Z$  is contained in the union of a finite number of sets of the ‘constant’ scale  $S_C$ . More generally, for *any* ‘constant’ scale (whether it distinguishes between points one second apart or one microsecond apart), there are a finite number of sets of that scale whose union contains the union of the infinite number of sets of the Zeno scale. This is just to say that the infinite number of time steps taken in the Zeno paradox sequence sums up to a finite number, on our ‘ordinary’ notion of time.

However, this first part of the reply is insufficient *on its own*. There are (at least) two scales supporting seemingly different conclusions: if one measures the progress of the runner in terms of the number of stages he covers on the constant scale  $S_C$  (the number of ticks of a watch), he finishes in a finite number of steps; if one measures his progress in terms of the number of stages he covers on the Zeno scale  $S_Z$ , he has not finished in an infinite number of steps. One of these scales rather than the other is relevant for deciding whether the runner finishes or not. Therefore, the *existence* of a scale does not guarantee its *appropriateness* for use in a given discussion. Whatever the relationship between the Zeno scale and the constant scale, a critic of the Zeno argument still has to assert or argue that *it is the ordinary scale of time which should be used in deciding whether the runner finishes the race*. This is the second part of his response.

The introduction of the notion of scale thus allows a clarification of the full brunt of the ordinary response to the Zeno argument. Two properties of this notion

allow it to bring out two important factors. Firstly, it is *innocent*: no *a priori* assumption is made about the priority or importance of one scale over another. For example, if one considers only its properties as a scale, there is nothing “constant” about the constant scale (thus the scare-quotes when it was introduced). The priority accorded to the constant scale in the rejection of the Zeno argument is thus highlighted, and identified as a second premise in the argument. Secondly, the notion of scale is *holist*: the differentiations between objects which are available are sensitive to properties of the whole scale as such; any fiddling with the scale may lead to very different differentiations being available, and possibly to the breakdown of the argument. In particular, any time interval may belong to scales which support a Zeno styled argument, as well as to scales which do not.

The notion of scale is also philosophically neutral in the sense that it can be given different sorts of interpretations: epistemic (our best way of dividing up the world), ontological (the way the world is actually divided up), or linguistic (the way language divides up the world) are but a few examples. It may help the reader’s understanding of the analysis proposed here to recast it in terms of languages, though it should be remembered that this is but one interpretation. In order to do this, let us consider languages which have *just* the resources to capture the differentiations permitted by the scale: given the holistic aspect of scales, it is important to keep close control on *everything* that can be said in the language adopted. There is a thus language  $L_Z$  containing terms “after  $n$  steps (on the Zeno scale)” for each  $n$  and a predicate “finishes $_Z$ ”, with a theory  $T_Z$  containing the proposition “ $X$  finishes $_Z$  if and only if there is a number  $n$  such that  $X$  has passed the finish line after  $n$  ticks”. The crux of the reply to the Zeno’s paradox is that “finishes $_Z$ ” is not the relevant notion of finishing the race. The relevant notion is rather the ordinary one, which is better captured by the predicate “finishes $_C$ ” of  $L_C$ , which is just like  $L_Z$  (and has a theory  $T_C$  just like  $T_Z$ ), except that it is the constant scale  $S_C$  rather than the Zeno scale  $S_Z$  which is involved.

Finally, it is worth noting that, although the first part of the response without the second will not suffice to neutralise Zeno’s argument, the second part without the first is more effective. It is not *necessary* to understand the relationship between two scales to assert that one should be used instead of the other. A response consisting of this second part only would claim that, whether or not there is a simple or illuminating relationship between the Zeno scale and the everyday constant scale, it is the everyday scale which is appropriate when deciding whether or not the runner has finished the race, and not the Zeno scale. Whatever is implied by the use of the Zeno scale is irrelevant, according to this reply, since it is quite simply the wrong scale for the job in hand.

## 2 The Sorites paradox

If one tries to apply the two pronged reply to Zeno's paradox in the case of the sorites, one finds that the first part seems not to work, but the second part poses a valid question.

For the sorites example concerning baldness, presented at the beginning of the article, one can take, as the collection of objects, the set of all heads,  $\mathcal{H}$ . The scale used by the argument distinguishes between heads only by the number of hairs; that is, the sorites scale  $S_S$  contains, for each  $i$ , the set of all heads with  $i$  hairs.

What happens if we try to run the reply to Zeno's paradox on the sorites case? The first part of the reply established a particular relationship between the Zeno scale and the everyday constant scales. As presented, this part of the reply does not apply in the sorites: no everyday scale pertinent for the term *bald* has been proposed yet, and no relationship with the sorites scale has yet been worked out. But recall that the second part of the reply is sufficient without the first. And this part of the reply, if applied to the sorites case, is not devoid of validity. Just as the Zeno scale is not appropriate for deciding whether the runner finishes the race, couldn't one assert that the sorites scale is *not appropriate* for reasoning about baldness?

The purpose of this paper is to draw attention to the possibility of this sort of reply to the sorites paradox and indicate some of its consequences for theories of vagueness. The rest of this section shall nevertheless be dedicated to explaining the reply, to deflecting some common misunderstandings, and to arguing for its plausibility.

First of all, as in the previous discussion of the Zeno argument, it may be helpful to think of this style of reply in terms of languages.<sup>5</sup> The everyday and relevant notion of baldness seems close to the predicate “(is) bald<sub>O</sub>” in a language  $L_O$  which does not contain “a man with  $n$  hairs” for all  $n$ , but just for 0 and 5000 (for example), with a theory  $T_O$  containing the appropriate propositions concerning the application of “bald<sub>O</sub>” to these cases. Recall the importance of keeping in view *everything* the language can say: we hardly employ the entire sequence of “a man with  $n$  hairs” for all  $n$  on a regular basis!<sup>6</sup> The sorites argument, on the other hand, uses exactly this sequence: it uses all of the differentiations offered by the sorites scale  $S_S$ . It is thus couched in a language such as  $L_S$ , with terms “a man with  $n$  hairs” for each  $n$ , and a predicate “(is) bald<sub>S</sub>”, with a theory  $T_S$  containing the appropriate principles, and notably the premises of the sorites argument. Now,

<sup>5</sup>Once again, this is but a special instance of the reply. In particular, different, non-linguistic, interpretations of scale can be applied similarly to non-linguistic versions of the paradox.

<sup>6</sup>The simple language  $L_O$  is taken as the “ordinary language” only to aid discussion: the main points go through for any reasonable extension which does not contain the whole of the sorites sequence.

one might like to identify “bald<sub>S</sub>” and “bald<sub>O</sub>” and consider the language  $L_S$  as an extension of  $L_O$ , with  $T_S$  an extension of  $T_O$ . The sorites paradox implies that this extension is *non-conservative*, to say the least. In fact, it is *incompatible*: a certain gentleman, previously taken not to be bald<sub>O</sub> (according to  $T_O$ ), can now be argued to be bald<sub>S</sub> with a sorites styled argument (that is, according to  $T_S$ ). This leaves one with a choice. Either reject some of the previously accepted banalities expressed in  $T_O$  (sufficiently similar heads are either both bald or both not bald, the rules of classical logic apply); as discussed in the next section, this is the option taken by most theories of vagueness. Or deny that  $L_S$  is an *extension* of  $L_O$ . The second option suggests that “bald<sub>S</sub>” is simply not the same term as the everyday term “bald<sub>O</sub>”. It boils down to a rejection of the sorites scale for reasoning about baldness.

To some, it may seem that “prohibiting” the use of the sorites sequence with the term bald runs contrary to intuition.<sup>7</sup> However, under analysis, this intuition is not as sturdy as it may first seem. Firstly, as emphasised in the Zeno case, it is crucial to the sorites argument that it has available *all* the terms “a man with 0 hairs”, “a man with 1 hair”, . . . , “a man with 5000 hairs” at once, so to speak. Thus the usefulness of the notion of scale, and the importance of keeping track of everything the language can express. However, not only are some of these terms used rarely in everyday language (one seldom hears talk of “that man with 567 hairs”),<sup>8</sup> but one virtually never finds them all used together. This is consistent with the reply sketched above: the point is not that, for any  $n$ , the terms “bald” and “a man with  $n$  hairs” are not to be used together, but rather that there is a problem with employing “bald” with *all* the terms “a man with  $n$  hairs” for all  $n$  *conjointly*.

Secondly, it is incorrect to characterise this reply in terms of a “prohibition” (against using such and such terms together): it is rather the remark that, in using such and such terms together one has moved to another language. The attention to the exact power of languages allows the fine distinctions necessary for such a position, and it should be seen as an advantage of this accuracy that it can take the sorites paradox in its stride. Indeed, this approach to the paradox can be reformulated in quite a natural way: it suggests that the paradox rests on an *ambiguity* between the ordinary term *bald* (“bald<sub>O</sub>”) and some strengthened term, also noted *bald*, but belonging to a richer language (“bald<sub>S</sub>”). In a certain sense, modern pre-occupations with vagueness begin when Russell distinguishes it from ambiguity; this reply casts doubt on this distinction, suggesting that it may rest on an insuffi-

<sup>7</sup>The following two paragraphs address potential worries pointed out to the author by X.

<sup>8</sup>By the way, there is some linguistic evidence to support this claim: sorites-like terms such as hair often act as count nouns in the plural (one says “lots of hair” not “lots of hairs”), which would suggest there is a usage of “hair” which does not admit the precision of the sorites scale. I thank Z for the remark which drew my attention to this phenomenon.

ciently precise concept of language. In any case, the claim that the sorites paradox rests on a shifty ambiguity should not be shocking to anyone.

Before turning to the implications of the possibility of such a reply for theories of vagueness, let us note three possible arguments which might support such a reply. This should show that it is not devoid of validity. Firstly, one might cite the possibility of running the sorites argument itself. One might for example claim that one condition of the appropriateness of a concept or a scale is that, just by adding it to the language, one is not led to contradiction (compatibility), or stronger still, by adding it to the language, one does not permit any truths, expressed without the use of the new concept, which were not permitted previously (conservativity). One might defend this sort of position by affirming that, if new truths are permitted, the terms no longer have the same ‘meaning’, and so one is no longer discussing the same notion of baldness.<sup>9</sup>

Secondly, one might emphasise the arbitrariness of the choice of the sorites scale, by giving examples of scales which, although they satisfy the sorites styled predicates, do not lead to contradiction. In the example of baldness, one could introduce the notion of *degree of hirsuteness* of a head – the sum of the lengths of all its hairs – and take the scale  $S_D$  on  $\mathcal{H}$  with sets  $E_i$  containing all heads with degree of hirsuteness in the range  $[10 - \frac{10}{2^i}, 10 - \frac{10}{2^{i+1}})$ . This scale looks exactly like the Zeno scale in Figure 1 (relabeling the axis with degree of hirsuteness rather than time). Assuming that a head with degree of hirsuteness 10 is not definitely not bald, this scale does not permit a contradictory conclusion, even though it does permit one to run the sorites argument. The existence of scales permitting the sorites argument without leading to contradiction only serves to cast more doubt on the appropriateness of the sorites choice  $S_S$ . They emphasise the point made in the previous section, namely that the *existence* of a scale does not guarantee its *appropriateness*; after all, if different scales yield different conclusions, they cannot both be appropriate.

Thirdly, one might argue, on a case by case basis, that particular scales are inappropriate because they are in a certain sense incomplete or irrelevant – they fail to capture *all* the aspects necessary to properly understand the vague term. In the case of baldness, for example, one might note that aspects other than the number of hairs (such as the total length of hair, average hair thickness, surface area covered by hair) are equally, if not more, relevant to discussing baldness, so that one cannot reason solely in terms of follicle number, as in the sorites argument.

The important point to be made about this sort of reply to the sorites argument

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<sup>9</sup>The analogy with the well-known concept of conservativity in logic and the philosophy of mathematics should be evident.



is how it re-situates the debate between the defender and the critic of the argument. The argument as it stands is insufficient: *a justification of the use of the particular (sorites) scale is required*. The debate now revolves around the question of *appropriateness* of the sorites scale  $S_S$  to the argument and to the term *bald*. But if the problem with the vague predicate *bald* boils down to a question of the appropriateness or not of different scales for reasoning about baldness, then what one expects from a purported theory of vagueness has changed.

### 3 The problem of vagueness

Different diagnoses of the fault in the sorites argument inspire or imply different theories of vagueness. By placing the emphasis on the *appropriateness* of the scale used, the reply sketched above differs from a certain number of traditional approaches to the paradox. Consequently, it offers a different perspective on the problem of vagueness, and on what is required of a theory of vagueness.

Theories of vagueness usually take as their starting point the question of the truth of sentences featuring vague terms.<sup>10</sup> Whether they treat vagueness as a *logical* question, requiring a new logic, as do the many-valued logic or supervaluation theories,<sup>11</sup> or as an *epistemic* question, so that sentences featuring vague terms have a determinate (classical) truth values of which we are often ignorant,<sup>12</sup> or as a *pragmatic* question, where vague terms are ‘resolved’ in different ways in different contexts thus resulting in different truth values for sentences featuring these terms,<sup>13</sup> theories of vagueness have generally involved analyses of the truth of sentences featuring vague terms. Accordingly, each of these theories reply to the sorites argument by either denying the *truth* of one or more of the premises or the *validity* of the inference.

By contrast, the reply sketched in the preceding section does not challenge explicitly any of premises or the validity of the inference. Rather, it concentrates on the *scale* mobilised in the argument, the *architecture* on which it rests, the *language* in which it is couched. Whereas previous theories have accepted to work within a given language, in which “*bald<sub>S</sub>*” and “*bald<sub>O</sub>*” are identified, the reply proposed here calls for a theory which is concerned exactly with the relevance of the language, and which examines precisely this identification. To borrow Carnapian terminology, whereas previous theories are “internal”, the proposed reply

<sup>10</sup>Although we mainly discuss predicates in this paper, most of the discussion should apply to vague terms in general.

<sup>11</sup>Tye (1994), for example, defends a multi-valued logic approach; Fine (1975) develops a supervaluationist approach.

<sup>12</sup>Williamson (1994).

<sup>13</sup>Kamp (1981).

requires a theory which may be called “external”. A theory of vagueness which supports or advances such a reply would *not* therefore concern itself *primarily* with the analysis of the truth of sentences featuring vague terms.<sup>14</sup> Instead, this theory would work at a higher level of abstraction: it would attempt to understand, describe and characterise the *scales* which can be more or less closely associated with vague terms<sup>15</sup>, the languages featuring these terms and mobilising these scales, and the *relationships* between different scales and different languages.

The theory called for here may focus on a different aspect of vagueness, but it does not follow that it cannot deal with some of the problems which have preoccupied philosophers. For example, the “logic” of vague predicates has been a hot topic in the debate, given the tendency to prefer minimal deviation from classical logic<sup>16</sup>: a theory of the sort suggested conserves classical logic inside each particular language, or for reasoning with any given scale. To take another example, theories of vagueness should explain our hesitation over sentences such as “a man with 168 hairs is bald”<sup>17</sup>: the proposed sort of theory would do so by noting that the terms in the sentence do not belong to the same language (where language is understood in the precise sense introduced previously).<sup>18</sup>

A brief comparison with Fine’s supervaluation theory (1975) will clarify the relationship between the concerns of the theory envisaged here and traditional theories.<sup>19</sup> Inspired by the idea that vague terms may be rendered precise in a variety of ways (the line between bald and not bald heads may be at 576 hairs, or 577, or . . .), Fine imagines a structure such that, at each point, vague predicates are precise to such and such an extent (*bald* definitely applies to heads with more than 1067 hairs, and definitely not to heads with less than 455 hairs) but not to such and such an extent (*bald* does not have a definite application to other heads).

<sup>14</sup>Evidently, such a theory may have *implications* for the semantics of vague terms.

<sup>15</sup>It is not difficult to see that one can associate a (perhaps trivial) scale to any term.

<sup>16</sup>Williamson (1994).

<sup>17</sup>Graff (2000).

<sup>18</sup>For some, an important motivation of the sort of semantic theories of vagueness eschewed here is the understanding they purportedly provide of the use of vague terms in communication. For example, Stalnaker (1999) makes important use of such theories to extend his theories of communication to vague languages. Just as current theories of vagueness can be criticised for assuming a given notion of what counts as precise, and then trying to take account of vagueness (see below), theories of communication like Stalnaker’s can be criticised for assuming a fixed (precise) language, dealing with the question of communication for this language first, and then trying to extend the resulting theory to deal with vagueness. The author has criticised such aspects of Stalnaker’s theory, proposing an alternative theory (see Y). Indeed, this theory makes use of the an idea similar to that employed in the previous section: namely, the importance of keeping a strict control of the language involved in communication at any moment. The proposed theory of vagueness fits in most naturally with this theory of communication.

<sup>19</sup>Fine’s theory is taken only as an example; similar points apply to most other theories of vagueness.

At the points above any given point, the extension of the predicate is more determined (*bald* definitely applies and definitely does not apply to more heads) – these points determine precisifications of the predicate. At the top, so to speak, there are points where the predicates are entirely precise (the extension of *bald* is entirely determined). Fine calls such a structure a *specification space*, and uses it to determine the truth of sentences featuring vague terms. Indeed, for Fine, the specification space captures, to a large extent, the meaning of vague terms.<sup>20</sup> Formulated in Fine’s framework, the question of the appropriateness of scales and terms is essentially the question: what is the correct specification space?

The relationship can be seen rather easily. The specification space represents the ways in which a term can be made precise (the precisifications). However, if a certain term and a certain scale are not appropriate for use together, one would certainly not want the former to be rendered precise relative to the latter, and the specification space adopted should reflect this fact. For example, an appropriate scale for the term *bald* may distinguish according to a certain number of parameters (number of hairs, surface area covered, average length, arrangement on the head, etc.): one would expect that any “complete precisification” (that is, totally precise rendering) of the term *bald* applies definitely to any complete specification of all the parameters. In other words, one could tell whether the precisification applies to any individual, on the basis solely of his profile relative to these parameters. On the other hand, hair colour is inappropriate for determining baldness: given only a specific hair colour, one would not expect to be able say definitely whether individuals with that colour satisfy a particular “complete precisification” of *bald*. In both cases, these are properties expected of the specification space.

This clarifies the sense in which traditional theories are “internal” and the envisaged theory would be “external”: Fine takes the specification space as given, and studies the logic it implies; the theory envisaged here deals with the different possible specification spaces, and tries to discern which are relevant. However, this may be indicative of a more general difference of attitudes to the question of vagueness. Namely, there are two ways of understanding the relationship between a vague term and its precisifications. On the one hand, one could assume a fixed notion of what counts as precise; vague terms are gradually rendered more precise relative to this notion of precision. This is the conception implicit in the methodology of Fine and others: in assuming a specification space, they assume a notion of precision, with respect to which the vague terms may be rendered precise (complete precision corresponds to maximal precisifications, or the highest points in the specification space). On the other hand, one might *start* with the vague terms, and then *search* for the relevant more precise notions which capture what the vague term was meant to capture. The notion of precision which char-

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<sup>20</sup>(Fine, 1975, pp275-277).

acterises the rigorous ideal of what the vague term captures only inaccurately is not given, it is to be found. This is the vision that fits most comfortably with the perspective on vagueness suggested here. Indeed, it is not without its validity, and any argument for this conception serves to highlight the importance of the sort of questions that are being proposed here. Vagueness was seen as a defect, ideally absent from scientific languages; however, this should be taken to imply that one of the rôles of science is to work out a collection of precise terms which capture in a more rigorous manner what is alluded to by a vague term. In fact, many activities of scientists can be understood in this way: have they not rendered precise the vague notion of movement, in terms of a group of precise notions, such as speed, acceleration, and momentum? Do they not attempt to capture the important aspects of the vague notion of poor, by developing precise notions such as income per GDP, average cost of living and so on? To anyone sympathetic with this sort of view on the problem of vagueness, the call made here for a different sort of theory of vagueness should be welcome.

Several of the elements used in the response to the sorites offered in the previous section now come into focus. Once one renounces the idea of a pre-given notion of precision, and accepts that the question of what precise terms are relevant for a certain vague term is on a par with that of how the vague term can be rendered more precise, it is important to determine whether the notion of number of hairs (solely) is the relevant precise notion for the term *bald*. The innocence of the notion of scale allows it to abstract from the assumption that the number of hairs (alone) is the proper way to render baldness precise baldness; this assumption is implicit in the sorites argument. Recall that the notion of scale is philosophically neutral, hence the points made above, though formulated concerning vagueness in language, apply in other domains (for example, the application of vague terms to actual objects). Depending on where they are applied, the notion of *appropriateness* of scales will be understood in different ways: management of elements of language (scales as linguistic elements); relations between parts of the worlds (scales as capturing ontological distinctions); possibilities and limits of knowledge (scales representing the finest distinctions we can make<sup>21</sup>).

Such a theory of vagueness shall not be fully developed in this paper. The paper will close with several indication of what such a theory might look like. We shall firstly give three examples of the types of relations between scales and terms which should be important for such a theory, and secondly we shall make some comments about a possible formalisation.

Here are three sorts of relations which could hold between scales and which

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<sup>21</sup>In passing, note that this may give a rigorous sense to the oft muted idea of things we don't know and can never know (Williamson, 1994): if, for a given vague term, there are no "epistemic" scales relevant for rendering it more precise, then we cannot know it any more precisely.

such a theory of vagueness should attempt to understand:

**Bounds** A scale  $S_1$  may *bound* a part (or all) of another scale  $S_2$ , in that the union of (some of the) sets of  $S_2$  is contained in a certain union of sets of  $S_1$ . This is the case in Zeno's paradox, where a finite number of sets of the 'constant' scale (a finite number of ticks of the clock) bound the infinite number of sets of the Zeno scale.

**Matching** A scale or a (vague) term may *match* another scale or (vague) term, whilst still being distinct from it. A trivial example is the Zeno time-points  $t_i$  and the Zeno space points  $p_i$ : for each time-point in the Zeno time sequence, there is a corresponding position of the runner, and vice versa. More interesting examples arising in philosophy include the relationship between *similarity* and *counterfactuals* on which Lewis builds his theory of counterfactuals<sup>22</sup> or the relationship between *reliability* and *knowledge* which underpins Williamson's theory of knowledge.<sup>23</sup> This property is especially pertinent when it comes to proposing notions capable of rendering precise vague concept: to be able to say that a scale of precise terms captures a vague concept, one would expect the former to match the latter.

**Appropriateness** Certain scales may be more or less *appropriate* for discussing and constructing arguments relying on other scales or featuring other terms. As discussed in the previous section, there are several aspects which may contribute to the appropriateness of one scale to another, such as the compatibility or perhaps even conservativity of the scales (adding a scale does not lead to contradiction or to new truths respectively), or the 'completeness' of a scale (certain scales are only appropriate if used in conjunction with other scales). Furthermore, the relationships of bounding or matching between scales may be pertinent to their appropriateness: if two scales match, bringing one into play should not change what one can truly say only in terms of the other.

Such a theory of vagueness may find inspiration, or perhaps even a formalisation, in the mathematical theory of *topology*, which deals with abstract notions of *space* and notably *closeness*.<sup>24</sup> The problem of vagueness concerns scales, whose most important aspects are their ability to *distinguish* among objects or points. But intuitively, what cannot be distinguished is *close* in a certain sense, and so

<sup>22</sup>Lewis (1973, 92): "It often happens that two vague concepts are vague in a coordinated way: firmly connected to each other, if to nothing else."

<sup>23</sup>Williamson (2000, 100): "The concept *reliable* need not be precise to be related to the concept *knows*; it need only be vague in ways that correspond to the vagueness in *knows*."

<sup>24</sup>See any textbook on the subject, such as Sutherland (1975).

there is a natural affinity between mathematical topology and the notion of scale. In fact, the notion of scale introduced above is a primitive version of what mathematicians call a *topology*: a topology is commonly defined as a set of so-called “open sets”, satisfying certain conditions.<sup>25</sup> Indeed, several recent advances in topology, and notably the introduction of the field of formal topology,<sup>26</sup> where no sets of points are *supposed*, may prove useful. In particular, they may allow a notion of scale which does not have to rely on the supposition of a given set of objects  $\mathcal{O}$  (see the definition of scale in Section 1). Since such a supposition may threaten the innocence of the notion of scale, which has proved so important in preceding discussion, this would constitute a welcome development. The relationship with topology is currently intuitive and prospective; if it could be fruitfully developed, one would indeed obtain a ‘*topological*’ theory of vagueness.

Vagueness has been recognised as an important topic in philosophy. A large number of everyday and philosophical terms are vague, and are none the less useful for it. They can be used to say important things in many interesting cases; however, there are situations where they are less useful, there are questions to which they do not permit answers. Instead of attempting to force a truth value on these cases (of whatever logical flavour), perhaps a theory of vagueness should be more concerned with understanding where vague terms “work” and where they don’t, with which terms they can be used and with which ones they can’t. At worst, such a theory would introduce a different perspective on vagueness, and its relationship to precision.

## References

- Dummett, M. (1975). Wang’s paradox. *Synthese*, 30:301–24.
- Fine, K. (1975). Vagueness, truth and logic. *Synthese*, 30:265–300.
- Graff, D. (2000). Shifting sands: An interest-relative theory of vagueness. *Philosophical Topics*, 28:45–81.

<sup>25</sup>To be more precise, the notion of scale used here is reminiscent of the set of sets of points of a metric space which are less than a distance  $d$  from each other. The relationship between vagueness and this sort of metric structure seems to be a *leitmotiv* in the literature: the non-transitivity underlined by Dummett (1975) and Wright (1975) can be related to the non-transitivity of the relation “at a distance less than  $d$  from”, and the notion of distance figures explicitly in several of the analyses proposed in Williamson (1994).

<sup>26</sup>For a review, see Sambin (2003).

- Kamp, H. (1981). The paradox of the heap. In Mönnich, U., editor, *Aspects of Philosophical Logic*, pages 225–277. Reidel, Dordrecht.
- Lewis, D. K. (1973). *Counterfactuals*. Blackwell, Oxford.
- Sambin, G. (2003). Some points in formal topology. *Theoretical Computer Science*, 305:347–408.
- Stalnaker, R. C. (1999). *Context and Content: Essays on Intentionality in Speech and Thought*. Oxford University Press, Oxford.
- Sutherland, W. A. (1975). *Introduction to Metric and Topological Spaces*. OUP, Oxford.
- Tye, M. (1994). Sorites paradoxes and the semantics of vagueness. In Tomberlin, J., editor, *Philosophical Perspectives: Logic and Language*. Ridgeview, Atascadero, California.
- Williamson, T. (1994). *Vagueness*. Routledge, London.
- Williamson, T. (2000). *Knowledge and its Limits*. Oxford University Press, Oxford.
- Wright, C. (1975). On the coherence of vague predicates. *Synthese*, 30:325–63.