BASEL II AND THE VALUE OF BANK DIFFERENTIATION

Eberhard Feess

Aachen University[*]

Ulrich Hege

HEC School of Management Paris[†]

October 2007

---

[*]Address: Aachen University (RWTH), Dept. of Economics, Templergraben 64, D-52056 Aachen, Germany. E-mail feess@rwth-aachen.de. Financial support from Center for Financial Studies is gratefully acknowledged.

[†]Corresponding author. Address: HEC School of Management, Department of Finance and Economics, F-78351 Jouy-en-Josas Cedex, France. E-mail hege@hec.fr.

BASEL II AND THE VALUE OF BANK DIFFERENTIATION

October 2007

Abstract

This paper analyzes optimal bank capital requirements when regulation can be differentiated according to banks' heterogeneous risk-assessment capabilities. The new Basel II Accord provides the opportunity to do by introducing distinct regulatory systems for banks authorized to apply internal ratings and externally rated banks. We show that optimal policies provide incentives to specialize: sophisticated banks should be directed towards low-risk loan portfolios and be allowed to grow, whereas banks with less developed rating systems should be regulated as niche players that absorb a maximum of the unavoidable systematic risk in the banking sector. The coexistence of two capital adequacy standards dominates a market structure in which all banks migrate to one regulatory regime. We analyze the moral hazard problem that sophisticated banks may misreport the true risk of their assets, and show that it will reduce the optimal level of differentiation between the two types of banks, but not eliminate the advantage of two coexisting standards. We address the problem of banks deploying internal rating systems without information sharing, and show that this may accelerate the adoption of differentiated regulation.

In a world of rapid financial innovation, the prudential regulation of banks faces numerous challenges. The scope and complexity of assets into which banks invest is now much larger than only a decade ago. In addition, sophisticated banks have developed advanced information systems to evaluate risks of loans and other assets, but they typically do not share with regulators the knowledge provided by these (increasingly electronic) systems. Banks are increasingly nimble at undertaking "regulatory arbitrage" in order to reduce the cost of capital requirements or other regulations that potentially put them at a disadvantage compared with less regulated financial entities. For example, they prefer to invest into assets that bear small capital requirements compared to the true risk levels (and risk premia), and they can structure funding commitments in ways that keep them off-balance sheet and out of the immediate reach of capital requirements.

The new Basel II Accord is set to modernize the prudential regulation of banks, in particular along two dimensions. First, by giving banks the right incentives to hold assets with appropriate risks in their portfolios. The new rules proposed by the Basel Committee[1] are intended to tailor the minimum capital requirements more accurately to the true credit risk afforded by each individual bank loan or other asset. Second, by giving incentives to share information about the quality of these assets with regulators in exchange for an attractive regulatory environment.

Central to the new Basel II Accord is the concept that regulators directly use the information contained in the risk-assessment systems of private banks in order to gauge the credit risk associated with theses loans and the necessary capital requirements. However, such a policy is currently appropriate only for the most advanced banks. The Basel Committee, therefore, proposes a two-layer approach by distinguishing between the Standard Approach (SA) and the Internal Ratings Based Approach (IRB). Under the SA regulation which applies to banks that are not yet ready for the full implementation of Basel II rules, certified credit rating agencies will assign risk coefficients to bank loans and other bank assets, and for commercial bank loans without an external rating, the risk weight will be assumed to be 100%. By contrast, under the IRB regulation, banks will be authorized to undertake the risk classification of assets themselves according to their own

---

[1] See Basel Committee (2004). The proposal provides for three distinct pillars of banking regulation, *(i)* standards for capital adequacy, *(ii)* banking supervision and *(iii)* market discipline. In this paper, we will exclusively focus on the most prominent of these pillars, the one on capital adequacy. While the pillars are probably regulatory substitutes to some extent (this has been pointed out e.g. by Acharya, 2003; Decamps, Rochet and Roger, 2004 or Morrison and White, 2004), our perspective is the optimal design of the capital adequacy pillar, once the optimal weight for each of the three pillars is determined.

credit scoring models. The Basel Committee plans to set a high regulatory standard for banks operating under the IRB approach, but wants to increase the attractiveness of investing into the IRB approval process. Therefore, the Committee will offer a reduction of 2-3% in the required capital, compared with the capital needed for the same risk-weighted asset portfolio under SA.

The two-layer approach is likely to profoundly affect the capacity to make informed lending decisions, the level of competition and of interest rates, the charter value and the failure risk of banks. The first objective of our paper is to analyze how such a two-layer approach will affect the market shares of banks operating under IRB relative to banks operating under SA rules, and to determine the optimal size of the IRB-bank segment. The proposed Basel II Accord has so far only been studied with respect to a single bank or in a partial equilibrium framework, i.e. the effect on the market structure of the banking sector has been neglected.

Many banks decided to invest into better risk-assessment systems independently of the anticipated regulatory environment, and the development of their systems is advanced at the time of the Basel II implementation. According to a widely held view, the result of these investments is a widening information gap between banks and regulators, and the main role of the Basel II approach is to narrow this gap by providing incentives for banks to share their internal ratings information with regulators. A third objective of our study is to determine under what circumstances banks would voluntarily disclose their private investments in improved credit scoring systems to the regulator in order to benefit from favorable capital requirements.

The Basel Accord will effectively delegate the power to determine the capital requirements for individual assets to the IRB-banks themselves. This raises genuine concerns about possible agency conflicts as banks can use this decentralization to reduce their overall required capital. Our final objective is thus to analyze the agency problem of misreporting portfolio risks in the context of two other incentive problems, bank risk-taking and banks' adoption of unregulated advanced scoring systems, and to study the possible interaction of these problems.[2]

We consider a model with two banks that choose between investing in safe projects character-

---

[2]Within the IRB-approach, the Basel Committee distinguishes between a foundation approach and an advanced approach. Under the foundation approach, the IRB-banks' own assessment is restricted to the probability of default (PD), whereas advanced IRB-banks use their own systems also to estimate the loss given default (LGD) and the exposure at default (EAD) (see Basel Committee (2004), Part 2, III). Though interesting, this distinction is neglected in this paper as in our model the expected loss, the product of PD and LGD, determines the contribution of each project to the social cost of bank failure.

ized by idiosyncratic risk and in risky projects with substantial systematic risk. Initially, neither banks nor regulators can discriminate between safe and risky projects. Banks, however, have the option to invest in an improved credit scoring system that enables them to screen between safe and risky projects. With this investment, banks qualify for the use of the IRB-approach. The interest rates for safe and risky projects are determined endogenously.

We assume initially that the regulator can perfectly verify the screening information obtained by banks, and we consider the case of one bank that invests to become an IRB-bank. The optimal regulation of risk-adjusted capital requirements provides incentives for the IRB-bank to adopt safe projects and to limit its risk exposure to the maximum that it can cover with its own equity. While the IRB-bank remains default-free, the SA-bank adopts a risky portfolio and is exposed to substantial failure risk. The reason is that there are economies of scale in the absorption of bank default risks: the more asymmetric the allocation of bankruptcy risk across banks, the lower the social cost of bank default. As a result, the regulator prefers to confine default risk to one bank and to keep that bank small. The two-layer approach of the Basel II regime allows the regulator to assign lower average capital requirements to the IRB-bank compared with the SA-bank.[3] This differentiation will increase the portfolio quality and market share of the IRB-bank, whereas quality and market share of the SA-bank decrease, providing a rationale that banks that can maintain a low default risk should be allowed to grow.

We then consider whether both banks should become IRB-banks. In this case as well, banks will adopt maximally differentiated portfolios, with one bank specializing in safe projects and the other bank in risky projects. The allocation with two IRB-banks, however, has two disadvantages: first, the same capital requirements apply to both banks, so that no size differentiation is possible. Second, the competition between banks is heightened as they are in an equal position on both loan markets, which will lead to lower lending rates and thus expose banks to a larger default risk. Both effects imply a strictly inferior overall allocation of portfolio and risks compared to the case with only one IRB-bank. Moreover, if the regulator has enough discretion to encourage or discourage investments in internal rating systems then the coexistence of SA-banks and IRB-banks can always be implemented.

This comparison highlights the diseconomies if all banks make the transition to the same regulatory status. In our model, the coexistence of SA-banks alongside IRB-banks is optimal as

---

[3]This is also the stated intention of the Basel Committee, but while the Basel Committee wants to apply favorable ratios to induce investment, we show that they should only be used to regulate portfolio size and composition.

it allows to confine a maximum of the unavoidable aggregate risk in the banking sector to a niche of tightly regulated SA-banks. While our analysis confirms that SA-banks will suffer from the transition to Basel II by moving to smaller and lower-quality loan portfolios, it emphasizes that this transition has positive effects overall. Our analysis suggests that there should be a yardstick that separates between more and less advanced banks, given their heterogeneous starting positions.

We then introduce the incentive problem that banks can misrepresent the risk of individual projects. We consider the extreme case where the regulator must fully rely on banks' reports. In this case, the regulator's capacity to tailor capital requirements to actual loan failure risks appears to break down since IRB-banks will be able to misreport their portfolios in favor of projects with the lowest risk weight. We show, however, that the consequences of this incentive problem for the overall allocation are far less dramatic than one might have expected since the beneficial differentiation effect still exists. The additional constraints caused by the incentive problem imply that the IRB-bank will often have a smaller portfolio than under perfect information, in particular if risk-taking is attractive or if the bank is poorly capitalized. Still, a banking sector with one IRB-bank is superior to two IRB-banks.

An important additional analysis concerns the possibility that banks invest in internal rating systems without applying for an IRB-status. We find that the regulator always prefers IRB-banks to unregulated banks with an internal rating system. This leads to a useful retrospective on the Basel I-regulation: Even in cases in which Basel II is exposed to problems of excessive investment in internal rating systems it is typically superior to the old Basel I rules as it allows to differentiate optimally between two segments of the banking sector.

There is a vast literature on the optimal regulation of bank capital and its relationship to risk-taking that is surveyed in Bhattacharya, Boot and Thakor (1998) and Allen (2004). Acharya (2001) endogenizes the choice of systematic risk in a bank loan portfolio and discusses the regulatory consequences. Tighter capital requirements have been argued to increase risk-taking incentives (Thakor, 1996; Besanko and Kanatas, 1996) or to reduce them (Repullo, 2004). Our study of the relationship between competition and risk-taking in banking is in accordance with a substantial literature. Matutes and Vives (1996) show that reduced banking competition mitigates risk-taking incentives. Hellmann, Murdock and Stiglitz (2000) also emphasize negative effects of strong competition, and Boot and Greenbaum (1993) argue that excessive risk-taking can be mitigated by reputation effects that in turn depend on imperfect competition.

The problem that banks may understate their portfolio risk when using internal ratings has

been addressed by Morrison and White (2005), Marshall and Venkataramaran (1999), Dangl and Lehar (2004) and Pelizzon and Schaefer (2005) who argue that the benefit from internal ratings depends on the regulator's capacity to monitor and to deter banks from cheating. Gersbach and Wehrspohn (2001) argue that banks will underinvest in their scoring models because they will identify bad loans more often, which leads to higher capital requirements with internal ratings.

There is little previous work on the impact of Basel II on the riskiness of IRB-banks and SA-banks. Rime (2005) discusses the adverse effect of Basel II on the loan quality of unsophisticated banks. Close in spirit is Repullo and Suarez (2004) who model a competitive banking sector where borrowers choose between IRB-banks and SA-banks. In their paper, IRB-banks will always specialize in riskier projects, and the average interest rate after Basel II decreases. In our model, both cherry-picking and risk-shifting is possible, and the interest rate increases if IRB-banks prefer cherry-picking, so that our conclusions are strikingly different from theirs. The reason is that, contrary to our analysis, they do not choose capital requirements to maximize social welfare and do not consider bank moral hazard, since their focus is on simulating the general equilibrium effects of Basel II. Hakenes and Schnabel (2007) emphasize competition for liabilities among banks with different screening capabilities, whereas we focus on competition for assets. Similar to our model, they find that the choice between the SA- and the IRB-approach may lead to more risk being adopted by less sophisticated banks.

In empirical work on potential impacts of the Basel II Accord, a substantial literature has focused on the likely procyclical effects, starting with Altman and Saunders (2001) who demonstrate that external ratings and hence the SA component of Basel II would likely to be procyclical. Peura and Jokivuolle (2004) find a comparable procyclical effect for internal rating systems. While our study is tangential to this discussion, it shows that SA-banks specializing in poor-quality loans are likely to absorb a disproportionate share of procyclical variations, because of optimal portfolio rebalancing between the two types of banks rather than because of rating changes. The literature investigating the consistency of internal and external ratings to predict default risks includes Carey (2002), Gropp and Richards (2001) and Claessens and Embrechts (2003).

The paper is organized as follows. Section I introduces the model. Section II lays the groundwork by analyzing bank profits and social welfare in the case of two SA-banks and by deriving general conditions for the portfolio choice of banks. Section III analyzes the scenario in which the banks' loan portfolio choice is observable to the regulator. Section IV analyzes the scenario of unobservable loan portfolios. In Section V we discuss possible extensions and Section VI concludes.

I. The Model

In our model, the banking sector consists of two banks, Bank A and Bank B. Banks are risk neutral and have identical equity of E. There is a continuum of projects with a total measure of one that they can finance. Costs of all projects are normalized to a unit of investment, i.e. funds worth n are needed to finance a portfolio of Lebesgue measure $n \leq 1$. $\frac{1}{2}$ of the projects are "safe", and $\frac{1}{2}$ of them are "risky". Safe projects yield a gross cash flow (rate of return plus one) $X_S > 2$ with probability $k \geq \frac{1}{2}$, and zero with probability $1 - k$. Safe projects are uncorrelated, meaning that the return of any measurable portfolio of n safe projects will be exactly knX_S. Risky projects yield a cash flow of X_R with probability $\frac{1}{2}$, and nothing with probability $\frac{1}{2}$. Risky projects are strongly correlated, so that they represent systematic risk. To capture this in a simple way, we assume that a portfolio of measure n of risky projects yields tnX_R, where $t \in [0, 1]$ is a uniformly distributed random variable. Thus, t indicates the realization of the systematic risk in the loan portfolio.[4]

A bank failure involves a real cost for society that consists of the disappearance of the organizational capital, know-how and proprietary knowledge of borrower relationships of the bank and the disruption of the financing and payment flows for the bank's borrowers and lenders. We assume that this type of social costs of a bankruptcy is proportional to the size of bank's assets, i.e. the number n of projects that it finances, and can hence be captured by zn. We denote the expected bankruptcy loss by $Z = zn \cdot \text{Prob(bankruptcy)}$.[5]

Initially, both banks A and B have low quality rating systems that cannot distinguish between safe and risky projects. Banks with low rating systems are called *SA-banks* as they will be regulated according to the Standard Approach. Each bank $i \in \{A, B\}$ can, however, make an investment $C_i \geq 0$ to build an internal rating system. We assume that $C_A < C_B$ to capture the idea that there is heterogeneity among banks concerning their readiness to adopt internal rating systems.[6] When investing C_i, Bank i acquires an internal rating system (IRS) that allows it to

---

[4] Since risky projects have a higher systematic risk and higher beta than safe projects, we would typically observe that $X_R \geq 2kX_S$ in a market equilibrium, but the relationship between X_R and X_S plays no role in our analysis.

[5] This assumption could be relaxed, for example to allow for increasing contagion effects that go beyond the assumed proportional bankruptcy costs. The essential assumption for our analysis is that it does not make a difference whether one bank with n projects goes bankrupt or whether two banks go bankrupt that jointly finance n projects.

[6] If $C_A = C_B$, mixed equilibria in pure strategies would sometimes obtain, but this would not change our results

perfectly screen between safe and risky projects and we will refer to it as an *IRS-bank*. Under Basel II, an IRS-bank can decide to be regulated according to the Internal Ratings Based (IRB) Approach. In this case, the bank will be called an *IRB-bank*.

We consider two different scenarios to analyze the impact of potential agency problems on the efficiency of Basel II: under the *observable scenario*, we assume that the regulator can observe the loan portfolio of IRB-banks. This is not possible under the *unobservable scenario* in which the regulator is entirely dependent on the banks' reports.

Under the Basel II approach, the regulator can set different capital requirements b_S and b_R for safe and for risky projects. We assume that $b_S \leq b_R$ to capture one of the main purposes of Basel II, namely to impose higher capital standards on riskier loans. The possibility to differentiate between b_S and b_R, however, applies only to IRB-banks. For SA-banks, neither banks nor external rating agencies can distinguish between projects so that the regulator needs to apply a uniform ratio b_U. For the most part, we also make the assumption that $b_S \leq b_U$, i.e. that the regulator cannot demand higher capital requirements for safe projects than for unidentified projects that represent a combination of risky and safe projects. This assumption, which is realistic in the context of the Basel II discussions, allows us to limit the complexity of the analysis.[7]

Besides the capital requirements b_S, b_R, and b_U, the regulator can encourage or discourage investments by a subsidy $s > 0$ or a tax $s < 0$ for IRB-banks.

Interest rates are determined endogenously in our model by competition between the two banks. Differences in interest rates will reflect different expected default rates.[8] We denote the interest factor (interest rate plus one) for risky projects by R_R and for safe projects by R_S, and by R_U the interest factor of SA-banks that cannot distinguish between safe and risky projects.

We assume that it is always possible for a safe project to masquerade as a risky project, even when screened by a bank that has an IRS-system (the converse does not hold, i.e. a risky project cannot masquerade as a safe project). This assumption ensures that safe projects must be offered weakly lower interest rates in equilibrium, $R_S \leq R_R$. This is a realistic implication, and we will also briefly discuss that it can be relaxed without affecting our main results.

---

qualitatively.

[7]Our main findings do not depend on this asumption, in particular since in most cases $b_S \leq b_U$ would emerge as the optimal regulation anyway. There is an exception that we briefly discuss below but that has no impact on our main results.

[8]Since banks are risk-neutral, the difference in systematic risk is not reflected in loan pricing.

To tie down the determination of interest rates, we assume that financial markets also provide non-bank funding offers (e.g. equity-based instruments, leasing, asset-backed loans, trade credit or factoring). The potential competition from such non-banks are effectively limits the interest rates that banks can demand, and this is the only role of non-banks in our model. Bank funding is more attractive for project owners as long as bank interest rates do not exceed a certain limit. We assume this limit to be uniform since non-banks cannot distinguish between the two project types, and for simplicity fix it at a level of $R = 2$, the minimum level that guarantees a break-even of non-banks on every project. This assumption is motivated by the two classical advantages of bank offers compared with non-bank lending, the larger expertise of banks in screening projects and their competence in monitoring projects. We assume that bank financing is the socially efficient funding choice for all projects in our model, say because it allows projects to generate (weakly) larger gross cash flows, so that it would be wasteful to tighten capital requirements so much that some projects take recourse to non-bank sources.

To formalize competition between banks, we assume that first SA-banks announce their interest rates simultaneously and then IRS-banks set their rates. This timing gives a second mover advantage to IRS-banks regardless of whether they are licensed as IRB-banks or not.[9] We assume that depositors of both banks are paid the risk-free interest rate of 0, independently of the actual failure risk. Thus, we implicitly assume that there is a deposit insurance scheme in place that effectively covers all credit risk for the depositors of the two banks.[10]

The timing of the game is as follows:

Stage 1. The regulator credibly commits to b and s.

Stage 2. Banks decide whether to invest, and if they do, whether to request an IRB-license.

Stage 3. SA-banks announce their interest rates simultaneously, followed by banks with an IRS that also announce their interest rates simultaneously. Then each project chooses a bank to which it makes a loan application. The bank makes a yes/no-decision on the loan. After a negative decision, the project can apply to the other bank, and finally (with no loan approval from either bank) to a non-bank financier.

---

[9]IRS-banks have an informational advantage over SA-banks that explains that they are more likely to win any bid competition for one of their preferred projects since they can undercut any competing offer. The second-mover advantage captures the idea of a bidding advantage in a simple way.

[10]We also assume that a bank's deposit insurance premia do not depend on its current loan portfolio decision. This allows us to ignore the impact of the endogenous default risk on the banks' payment for deposit insurance.

Stage 4. Nature decides upon the success of projects. This also determines if a bank is insolvent or not.

An essential consideration for the regulator is that an increase in bank lending also increases the bank failure risk. In order to tie down this relationship, we first assume that $E < \frac{1-k}{2}$. This implies, as we will discuss, that if all projects are bank-financed then at least one bank faces insolvency risk; we exclude the opposite case since it would be uninteresting. Moreover, for the essential part of our analysis, we assume that X_S and X_R are sufficiently large so that the regulator wants all projects to be funded. We will refer to this as *Assumption L.*[11] This allows us to avoid numerous case distinctions that would disrupt the flow of our exposition. It implies that we disregard any impact of banks' investments in screening technologies on the total lending volume in the economy. Rather, we focus on the efficient allocation of loans between banks. We relegate the interesting question how bank regulation and total lending volume interact to Section V. where we briefly discuss the consequences of relaxing *Assumption L.*

II.   Elements of the Analysis

A.   *Two SA-banks*

We start with the case where neither bank invests into an IRS. Both banks are SA-banks that cannot distinguish between project types, and they are regulated by b_U ("uninformed") and can fund $n_u \leq \frac{E}{b_U}$ projects. This case is in many ways similar to the situation under Basel I rules, but not quite: under Basel I rules, the regulator cannot distinguish between project types, but banks might nevertheless invest in superior screening skills. Basel I with investment is akin to our case of unregulated IRS-investments,[12] and we will discuss this case in Sections III..4 and IV..4.

Moving backwards, we first analyze profits and bankruptcy risks as a function of n_u (stage 3), and then turn to the regulator's choice of b_U (stage 1). Since banks cannot discriminate between safe and risky projects, they propose the same interest factor R_U to both types of projects, and each loan portfolio consists of $\frac{n_u}{2}$ safe and $\frac{n_u}{2}$ risky projects.

For given n_u and E, the bank will avoid insolvency if t is large enough so that the bank's

---

[11] A sufficient condition for *Assumption L* to hold in all situations is that $\min\{X_R, X_S\} \geq 2 + z$.

[12] The awareness of the growing screening skills of banks and their possibility to exploit the inconsistencies in the coarse Basel I grid of capital requirements is an important motivation behind the Basel II process.

assets are sufficient to cover its liabilities:

$$\frac{n_u}{2}kR_U + \frac{n_u}{2}tR_U + E - n_u \geq 0. \tag{1}$$

Let $\tilde{t}_u$ denote the minimum value of t satisfying (1). Because t is uniformly distributed over the unit interval, $\tilde{t}_u$ conveniently embodies also the bankruptcy probability. The expected profit per bank (we will always look at gross profits that include bank equity E) is

$$\Pi_u = (1 - \tilde{t}_u) \left(\frac{n_u}{2}kR_U + \frac{n_u}{2}E\left[t \,|\, t > \tilde{t}_u\right] R_U + E - n_u \right). \tag{2}$$

This equation says that if the bank avoids insolvency, which it does with probability $1 - \tilde{t}_u$, then the return from $\frac{n_u}{2}$ safe projects is kR_U, and the expected return from $\frac{n_u}{2}$ risky projects is $E\left[t \,|\, t > \tilde{t}_u\right] R_U = \frac{1+\tilde{t}_u}{2}R_U$, reflecting the conditional expectation of $t \geq \tilde{t}_u$. Furthermore, the bank owns E and invests n_u.

The insolvency threshold $\tilde{t}_u$ is reached when the return to the bank, including its capital reserves E, is just enough to satisfy (1),

$$\tilde{t}_u = \max\left\{ \frac{2n_u - 2E - n_u kR_U}{n_u R_U}, 0 \right\}. \tag{3}$$

Substituting into the bank's profit function (2) and rearranging yields

$$\Pi_u = \frac{1}{4n_u R_U} \left(2E - 2n_u + R_U n_u + R_U kn_u\right)^2. \tag{4}$$

Because banks do not have to pay for the losses if they become bankrupt, Π_u is strictly increasing in n_u which is straightforward to show. Hence, SA-banks fund as many projects as possible, meaning that in equilibrium $n_u = \frac{E}{b_U}$.

Following *Assumption L*, the regulator proposes $b_U = 2E$ or $n_u = \frac{1}{2}$, so that all projects will find bank financing. $b_U = 2E$ ensures that banks are capacity-constrained in equilibrium, and propose the maximum rate $R_U = 2$ that stems from the potential competition of non-bank financiers. Note that any $b_U < 2E$ would be inferior as banks could finance more than all available projects. Competition for projects would then lead to $R_U < 2$, and hence to higher expected bankruptcy costs. Thus, $b_U = 2E$ and $R_U = 2$ is the unique subgame perfect equilibrium (SPE). This leads to the following Proposition:

Proposition 1 *With two SA-banks, the regulator proposes $b_U = 2E$. Each bank funds $n_u = \frac{1}{2}$ projects and earns profits of $\Pi_u = \frac{1}{4}(2E + k)^2$. Total expected bankruptcy costs are $Z_u = z(1 - 2E - k)$.*

Proof. See Appendix.

Note that the expression for bankruptcy costs Z_u shows that the bankruptcy probability will always be strictly positive since we assumed that $E < \frac{1-k}{2}$. Our analysis shows that even though the two banks are symmetric and in Bertrand competition on the loan market, they earn a positive profit and thus have a charter value. The reason is that the regulator implements a quantity limit in lending via the capital requirement b_U, so that interest rates are not determined by Bertrand competition but by the ceiling imposed by non-bank financiers. Thus, banks make a profit as they benefit from the option value provided by a positive bankruptcy probability.

B. Choice of the Loan Portfolio

The portfolio choice of a bank that adopts an IRS is an important step of our analysis that we will use extensively below. We analyze whether the bank prefers safe projects, risky projects, or a mixed loan portfolio.

When choosing its loan portfolio, the bank faces a trade-off between safe projects that offer better quality expressed by $k > 0.5$ and risky projects that offer the option to benefit from the limited liability effect. Let us assume that an IRS-bank, say Bank A, finances a given number of projects n^A, of which n_S^A are safe and $n^A - n_S^A$ are risky. By analogy to the profit function (2) of a SA-bank, the IRS-bank's profit function is then[13]

$$\Pi^A = (1 - \tilde{t}^A) \left(kR_S n_S^A + E\left[t \mid t > \tilde{t}^A\right] R_R \left(n^A - n_S^A\right) + E - n^A \right) , \qquad (5)$$

where $\tilde{t}^A$ is the bankruptcy probability that increases in the fraction of the portfolio allocated to risky loans, $n^A - n_S^A$,

$$\tilde{t}^A = \max\left\{ \frac{n^A - E - kR_S n_S^A}{R_R \left(n^A - n_S^A\right)}, 0 \right\} . \qquad (6)$$

The bank maximizes its profit (5) subject to its equity constraint:[14]

$$n_R^A b_R + \left(n^A - n_S^A\right) b_S \leq E . \qquad (7)$$

Safe projects generate a deterministic return conditional on the bank remaining solvent, so that, independent of its overall portfolio and risk, the bank will only include safe projects in its portfolio if they earn a nonnegative return, $R_S \geq \frac{1}{k}$. We can safely limit attention to this case since $R_S < \frac{1}{k}$

---

[13] R_S and R_R denote interest factors for safe and risky projects.

[14] b_S and b_R denote capital ratios for safe and risky projects.

cannot occur in equilibrium. We can then identify the following fundamental portfolio allocation rule:

Lemma 1 *A Bank that adopts an IRS will strictly prefer either safe or risky projects. That is, it will finance the maximum feasible number* $\min\left\{\frac{1}{2}, \frac{E}{b_j}\right\}$ *of its preferred project type* $j \in \{R, S\}$, *and it will use its residual bank equity for projects of the other type if interest rates are sufficient to earn a profit.*

Proof. See Appendix.

Lemma 1 says that an IRS-bank first adopts only either safe or risky projects and then fills the remainder of its lending capacity with the other project type provided the latter earn a profit. We will refer to the first case as *cherry-picking*, and to the second case as *risk-shifting*. The intuition is that the bank's profit Π^A is a *call option* on the true value of the loan portfolio, and the expected value of this call option exhibits the usual convex shape as a function of the risk choice. Thus, there is no interior solution, and the bank will prefer either safe or risky projects.

The Lemma has two important implications: first, the bank will always prefer safe projects if there is no bankruptcy risk when choosing risky projects first. Second, choosing either safe or risky projects first yields strictly higher profits then the mixed allocation with two SA-banks, i.e. $\Pi^A > \Pi_u$ if R and n^A are identical. In the next sections, we will analyze how the decision between risk-shifting and cherry-picking depends on n^A, and how - in turn - this decision influences the regulator's choice of n^A in the SPE.

III.   Observable Loan Portfolios

We can now consider the portfolio choice of banks and the optimal regulation if at least one bank becomes an IRB-bank.[15]   We start with the case in which the regulator can observe the loan portfolio, and we proceed as follows: we analyze the optimal capital requirements separately for the scenario with one IRB-bank and with two IRB-banks, respectively, and then compare the two allocations and determine the optimal size of the IRB-bank segment.

---

[15]The case of uncertified investment is considered at the end of this Section.

A. One IRB-Bank

When portfolios are observable and there is only one IRB-bank, the regulator can implement any project split by adjusting capital requirements (b_R, b_S, b_U) appropriately. Total expected bankruptcy costs are minimized by differentiating between the IRB-bank and the SA-bank - the IRB-bank funds all safe projects and so many risky projects that it has no bankruptcy risk. The SA-bank funds all remaining projects. The regulator's objective function is (under Assumption L) to minimize the expected bankruptcy costs that we denote by $Z_o(1)$ for the case with one IRB-bank and observable loan portfolios. This leads to the following outcome:

Proposition 2 *With one IRB-bank and observable portfolios, $b_U = \frac{E}{1-E-k}$, $b_S = 0$ and $b_R = \frac{E}{E+k-\frac{1}{2}}$. The IRB-bank finances $n^A = E + k$ projects, and its portfolio comprises all safe projects and $E + k - \frac{1}{2}$ risky projects. The SA-bank funds $n^B = 1 - E - k$ projects that are all risky. Interest rates for safe and risky projects are $R_S = R_R = 2$. Only the SA-bank faces bankruptcy risk, and total expected bankruptcy costs are $Z_o(1) = z\frac{1-2E-k}{2}$.*

Proof. See Appendix.

The essential element of Proposition 2 is that in the optimal allocation, the IRB-bank will never be bankrupt; its payoff will be just zero in the extreme systematic risk event $t = 0$. The SA-bank, by contrast, has substantial failure risk even though it is the smaller bank, as a consequence of its risky-only loan portfolio. This allocation is optimal as it shelters as many projects as possible from bankruptcy risk. The social cost of default is minimized if a relatively small lending volume is allocated to the riskier bank and a relatively large portfolio to the less risky bank.[16]

Proposition 2 expresses an insight that plays an important role in our analysis: because the bankruptcy probability increases at a decreasing rate as a bank adopts more risky projects, there is an economies-of-scale effect in the allocation of bankruptcy risk, making an unequal allocation of overall failure risk optimal. A major advantage of Basel II is that it allows to differentiate between two segments of the banking sector: the IRB-bank should have regulatory rates that entice it to finance low-risk projects, and which avoid the risk of default. The fact that IRB-banks pick safer projects will inevitably deteriorate the average quality of remaining projects available for SA-banks so that poorly skilled banks are becoming more fragile. However, this

---

[16]This allocation is only optimal if the IRB-bank indeed prefers to fund safe projects. With the capital requirements of Proposition 2, this is the case as the IRB-bank cannot fund more than $E + k - \frac{1}{2}$ risky projects.

asymmetry caused by Basel II is just the consequence of the fact that all bank failure risk should be insulated in the SA-segment of the banking sector.

B. Two IRB-Banks

If both banks become IRB-banks and observe perfect signals on the quality of each project, there will be heightened competition for projects. We can quickly observe that there will be no symmetric equilibrium in which banks have the same project mix. This follows from Lemma 1 that shows that banks strictly prefer one project type. Hence, banks would undercut each other's interest rate to either attract all safe or all risky projects.

In an asymmetric equilibrium, the two banks' profits will be identical, $\Pi^A = \Pi^B$. Furthermore, to reduce the overall bankruptcy risk the regulator again sets capital requirements that lead to binding equity constraints so that banks cannot undercut each other. Since capital requirements are nonnegative, no bank can finance more than $\frac{3}{4}$ of all projects (otherwise the equity constraint would be slack for at least one bank).

It follows from the economies-of-scale effect in absorbing failure risk that total expected bankruptcy costs are minimized if one bank remains default-free and is larger than the other. This bank, say Bank A, should fund all safe projects and the maximum number of risky projects that allows it to remain default-free. Again, the regulator has an incentive to implement the maximum feasible interest rates $R_S = R_R = 2$ by making the equity capital ratios binding. Then, however, profits of the two banks would normally be unequal. This means that the regulator cannot in fact eschew competitive pressure on interest rates. There are two cases to be considered: in the first case, Bank A would earn higher profits when $R_S = R_R = 2$ so that Bank B would undercut Bank A. In equilibrium, the safe rate will adjust downwards to a level $R_S < 2$ that leads to profit equality, $\Pi^A = \Pi^B$, whereas the risky rate remains at the noncompetitive level $R_R = 2$. With the lower equilibrium interest rate $R_S < 2$, Bank A can finance at most $\min\left\{E + \frac{1}{2}kR_S, \frac{3}{4}\right\}$ while remaining default-free.

In the second case, Bank A would earn lower profits with $R_S = R_R = 2$ if Bank A is risk-free while Bank B finances only risky projects. This case will occur if k is below a certain threshold $\hat{k}(E)$ that we derive in the Appendix. Thus, the desired allocation with one risk-free and one risky bank would require $R_S < R_R$ to equilibrate profits. This, however, is impossible since firms having safe projects will not accept higher interest rates but will pretend that their projects are

risky. Furthermore, the regulator can not increase the profit of the risk-free Bank A by allocating more projects to the bank - as long as the bank is risk-free, its profits are independent of the number of risky projects. It follows that there is no asymmetric equilibrium where Bank A is risk-free: As long as projects are distinguishable, the risk free Bank A has always an incentive to acquire Bank B's portfolio by offering a lower interest rate for risky projects, $R_R < 2$. But then, safe project will imitate, project types are indistinguishable, and both banks fund identical mixed portfolios. We summarize:

Proposition 3 *With two IRB-banks and observable portfolios, profits are equal, $\Pi^A = \Pi^B$. There is a threshold $\hat{k}(E)$ that distinguishes two outcomes: (i) If $k > \hat{k}(E)$ then Bank A finances $\min\left\{E + \frac{1}{2}kR_S, \frac{3}{4}\right\}$ projects and is default-free, and interest rates are $R_R = 2$, $R_S \leq 2$. (ii) If $k \leq \hat{k}(E)$ then each bank funds half of all safe and half of all risky projects, and $R_S = R_R = 2$.*

Proof. See Appendix.

The equilibrium laid out in Proposition 3 is driven by the same logic that is behind the case of one IRB-bank in Proposition 2. The regulator wants to shelter as many projects as possible from bankruptcy risk by allocating them to one bank that remains default-free. However, when both banks are IRB-banks, then each bank can compete for the other bank's portfolio by cutting rates, and banks will undercut each other until in equilibrium their profits are equal. Case *(i)* of Proposition 3 depicts the situation in which the more attractive position is that of the default-free bank. Hence, rate competition between the two banks will lower the safe rate R_S until the profit of the default-free bank A is depressed down to that of Bank B. The regulator would clearly prefer a higher rate R_S but cannot prevent the rate competition between the two IRB-banks. This explains that the lending volume of the default-free bank is lower compared to the case of one IRB-bank.

From the regulator's point of view, the situation is worse in case *(ii)*, the situation in which a risky loan portfolio is more attractive than a safe portfolio. If banks were specializing in their loan portfolios as predicted by Lemma 1, then rate competition between the two banks would exert downwards pressure on the risky rate R_R. However, higher interest rates for safe projects are not possible in equilibrium, since safe projects would then masquerade as risky projects to benefit from a better rate. Therefore, specialized loan portfolios are no longer feasible, and we get the same allocation as with two SA-banks even though banks now have the heightened screening

capabilities of their IRS-systems. The role of the regulator is limited to reducing rate competition so that $R = 2$ in equilibrium for both project types.

C. Comparison and Optimal Regulation

We can now compare the expected minimal bankruptcy costs with one IRB-bank, two IRB-banks and two SA-banks, respectively. Denoting by $Z_o(2)$ the minimal expected bankruptcy costs in the case with two IRB-banks and observable loan portfolios, we find:

Proposition 4 *With observable portfolios, total expected bankruptcy costs are lower with one IRB-bank compared with two IRB-banks or two SA-banks, $Z_o(1) < \min\{Z_o(2), Z_u\}$.*

Proof. See Appendix.

The intuition for Proposition 4 is that, with two IRB-banks, the regulator cannot differentiate between the capital requirements of the two banks which leads to overall risk allocations that are strictly inferior to that with only one IRB-bank. In case *(i)* of Proposition 3, $R_S < 2$ implies a smaller portfolio for the default-free bank A and hence larger expected bankruptcy costs for Bank B.[17] And in case *(ii)* of Proposition 3, the allocation is the same as with two SA-banks. By contrast, when there is only one IRB-bank then the regulator can implement the optimal allocation of projects between banks, and $R_S = R_R = 2$.

While we have seen that bankruptcy costs are minimized with one IRB-bank, we need to take into account investment costs C_A (the cost of the bank that can become an IRB-bank at a lower cost) to determine the overall optimal regime. Depending on C_A, either one IRB-bank or two SA-banks are optimal.

As the criteria for the investment decision of Bank A do typically not coincide with the regulator's preferences, we now analyze whether the regulator can implement the optimal regime via subsidies ($s > 0$) and taxes ($s < 0$) for investments. To discuss this discrepancy of objectives, we will call an *overinvestment* situation any set of parameters that lead to $Z_u - Z_o(1) < C_A \leq \Pi_o(1) - \Pi_u$, i.e. a case in which the social benefit from investment is negative while Bank A will earn a positive net profit when investing. We call the opposite case, $Z_u - Z_o(1) \geq C_A > \Pi_o(1) - \Pi_u$,

---

[17]Both cases $Z_o(2) < Z_u$ and $Z_o(2) \geq Z_u$ can occur in this case depending on the parameters, but this case distinction has no impact on the equilibrium ranking.

an *underinvestment* problem. From the expressions derived earlier we find that:

$$(Z_u - Z_o(1)) - (\Pi_o(1) - \Pi_u) = z\frac{1 - 2E - k}{2} - \left(k + E - \frac{1}{2} - \frac{1}{4}(2E + k)^2\right).$$

This expression can be either positive or negative, as the following two boundary cases show: For k close to 1 and $z = 1$, the expression will converge to $-\frac{1}{4} - E(1 - E) < 0$; and for E close to 0, k close to $\frac{1}{2}$ and $z = 1$, it converges to $\frac{5}{16} > 0$. This ambiguity is intuitive since the objective functions of bank and regulator differ for two reasons: first, the regulator takes account of social costs of bank failure Z while the bank does not, and the bank is protected by limited liability while the regulator takes the downside risk into account; second, the bank's lending volume and profit will increase if it invests in an IRS (Propositions 2 and 3) whereas the regulator considers the constant aggregate lending volume in the economy. The first reason explains an underinvestment bias, but the second reason tends to lead to overinvestment.

Overinvestment can easily be deterred by a tax. An underinvestment situation is more difficult to remedy: when the regulator chooses a subsidy to induce Bank A to invest, it needs to avoid that at the same time this subsidy would lead Bank B to invest. We will show that this problem will not occur if:

$$\Pi_o^A(1) - \Pi_u > \hat{\Pi}_o^B(2) - \Pi_o^B(1). \tag{8}$$

Condition (8) says that Bank A's decision to invest (transition from two SA-banks, Π_u, to one IRB-bank, $\Pi_o^A(1)$) will increase A's profit by more than Bank B's decision to invest (transition from being the only SA-bank, $\Pi_o^B(1)$, to being a second IRB-bank, $\hat{\Pi}_o^B(2)$) would increase B's profit. The relevant capital requirements to be considered in this case are those for the case with one IRB-bank as we need to consider a deviation from the equilibrium, which is the regulator's preferred allocation, and this is reflected in our notation $\hat{\Pi}_o^B(2)$.

Since by assumption $C_A < C_B$, the regulator can always find subsidies or taxes such that Bank A will invest while Bank B will not if condition (8) holds. Therefore, inequality (8) is a sufficient condition for the existence of an equilibrium with one IRB-bank.

We briefly explain why condition (8) will always hold, leaving details to the Appendix. As $\Pi_o^A(1) > \Pi_u$, condition (8) holds if $\Pi_o^B(1) \geq \hat{\Pi}_o^B(2)$. The effective capital requirement $b_S = 0$ (see Proposition 2) implies that there is no capital cost on safe projects so that Bertrand competition between two IRB-banks would drive down interest rates for safe projects to the minimum $R_S = \frac{1}{k}$. Moreover, if $E + k \leq \frac{3}{4}$, each bank can only finance $E + k - \frac{1}{2}$ risky projects as $b_R = \frac{E}{E + k - \frac{1}{2}}$, so

that there is no competition for risky projects and $R_R = 2$. However, if it decided to remain a SA-bank, each bank could finance strictly more risky projects at the same interest rate, so that $\hat{\Pi}_o^B(2) < \Pi_o^B(1)$. And if $E + k > \frac{3}{4}$, then competition will reduce interest rates for both types of projects. There is a unique equilibrium outcome in mixed strategies, but in this equilibrium we have $\hat{\Pi}_o^B(2) = \Pi_o^B(1)$ (see the Appendix). The analysis can be summarized as:

Proposition 5 *With observable portfolios, the regulator can always uniquely implement the optimal outcome:*

(i) If $Z_u - Z_o(1) \geq C_A$, only Bank A invests and finances $n^A = E + k$. There is a subsidy if $C_A > \Pi_o^A(1) - \Pi_u$.

(ii) If $Z_u - Z_o(1) < C_A$, there will be two SA-banks. There is a tax on investment if $C_A < \Pi_o^A(1) - \Pi_u$.

Proof. See Appendix.

Proposition 5 confirms that the optimal allocation is always uniquely attainable if the portfolio allocations of the banks are observable.

D.   Unregulated Investment

So far, we have neglected that one or two banks might invest in an IRS *without* requesting an IRB-license. We will refer to this possibility as *unregulated investments.* An unregulated IRS corresponds to a situation in which the bank does not divulge its internal ratings information to the regulator. The situation in this case is similar to the Basel I regime where the regulator cannot differentiate capital requirements across banks or across stated project types, and where banks that invest and bank that do not will face the same uniform capital requirements. The same is true under Basel II when IRS-investments are unregulated.

The analysis of unregulated investments is relevant for the discussion of bank regulation because it addresses the view that the major effect of the IRB approach is to ensure that banks give regulators access to the information provided by their advanced internal rating systems, and that this effect dominates the impact on incentives to invest into such systems. According to this view, banks carry out investments in IRS-systems independently of regulation and are in fact well-advanced in developing such systems, and the Basel II regulation is largely a reaction to a largely accomplished change in bank industry practice. Thus, our analysis of unregulated

investments sheds light on the Basel committee's concern that the information gap between banks and regulators is widening, and to its objective to narrow it by providing incentives that banks share their internal rating information with regulators.

Hence, one insight of our model is that the objective of implementing full information sharing between banks and regulator is tantamount to analyzing the incentive constraints that banks submit their IRS-systems for IRB-approval.

Let us first consider the case in which only one bank (say Bank A) invests. We consider the case in which the regulator imposes a tax on a IRB-license because two SA-banks are optimal when investment costs are taken into account. If Bank A undertakes an unregulated investment, it can fund $n^A = \frac{1}{2}$, and it will do so if[18]

$$\max\left(\Pi_r^A, \Pi_s^A\right) - \Pi_u > C_A. \tag{9}$$

If condition (9) holds, the bank will invest even if $Z_u - Z_o(1) < C_A$, and the regulator will not be able to prevent *overinvestment* by Bank A. Then, the regulator's best response is to accommodate the investment and to give fiscal incentives to request an IRB-license. This dominates unregulated investment as total bankruptcy costs are lower if Bank A applies for the license and finances $E + k - \frac{1}{2}$ projects instead of only $\frac{1}{2}$. We thus find that investment without requesting an IRB-license can never be a subgame perfect strategy. The option to do so, however, remains important: it is the reason why the equilibrium outcome will be one IRB-bank whenever $\max\left(\Pi_r^A, \Pi_s^A\right) - \Pi_u > C_A$, even if the regulator prefers to have two SA-banks because $Z_u - Z_o(1) < C_A$.

We then consider the case in which *both* banks simultaneously consider to undertake unregulated investments. If they do, then each bank will be able to fund $\frac{1}{2}$ of the projects, and we get two possible cases: an asymmetric equilibrium as in case *(i)* of Proposition 3 in which bank A finances all safe and Bank B finances all risky projects, and where $R_S < R_R = 2$, and a symmetric equilibrium as in case *(ii)* of Proposition 3 in which each bank finances half of both project types. It is then straightforward to show that there is no equilibrium in which both banks undertake unregulated investments. If one bank deviates and requests an IRB-license instead, it would be subject to capital requirements that allow it to expand its lending to $n^A = E + k - \frac{1}{2} > \frac{1}{2}$ projects, and the equilibrium interest rates were $R_S = R_R = 2$. Thus, such a deviation would earn higher profits. A similar reasoning shows that if one bank becomes an IRB-bank, then the second bank

---

[18]Π_r^A and Π_s^A denote profits of Bank A when funding only risky or safe projects, respectively.

will either not invest or become an IRB-bank as well, but it will never undertake an unregulated investment: going from an unregulated investment to becoming an IRB-bank allows the second bank to increase its lending volume and has no adverse effect on interest rates. We summarize:

Proposition 6 *With observable portfolios and the possibility of unregulated IRS-investments, if two SA-banks are optimal but Bank A privately prefers to invest in an IRS (*$\max\left(\Pi_r^A, \Pi_s^A\right) - \Pi_u >$ C_A*) then the regulator implements one IRB-bank. In all other cases, the regulator can implement the optimal outcome. Unregulated investments do not occur in equilibrium.*

A comparison of Proposition 5 and Proposition 6 shows that there is exactly one case in which there is a change in the final outcome, namely if there is an overinvestment situation. The option of taking recourse to unregulated investment implies that investment cannot be prevented in this case, so that the regulator implements one IRB-bank even though two SA-banks were optimal.

IV.   Unobservable Loan Portfolios

Now we turn to the agency problems associated with improved rating capabilities. We consider the extreme case where the regulator cannot observe the projects chosen by banks. Banks have then full latitude to misreport the quality of their projects, and to declare risky projects to be safe and vice versa. It follows that banks will always report the project type that is more favorable for them, i.e. the type that is subject to the lowest capital adequacy ratio. That is, differentiating between the capital adequacy ratios of safe and of risky projects is now meaningless, and only $b_I = \min(b_S, b_R)$ matters. Hence, in this case there are effectively only two capital requirements that we need to consider, namely $b_I = \min(b_S, b_R)$ as the relevant ratio for IRB-banks, and b_U for SA-banks.

A.   One IRB-Bank

With observable portfolios, the regulator could prevent substitution between safe and risky projects by setting b_S and b_R appropriately. This is impossible when portfolios are unobservable, so that we need to answer two questions: When will the IRB-bank engage in risk-shifting? And what is the optimal response of the regulator in terms of fixing the unique capital requirement b_I that is relevant for IRB-banks? Concerning the first question, we find:

Lemma 2 *With one IRB-bank (say Bank A) and unobservable portfolios, there exists a unique threshold $\bar{n}^A$ such that Bank A chooses cherry-picking if $n^A \leq \bar{n}^A$ and risk-shifting if $n^A > \bar{n}^A$. The threshold $\bar{n}^A$ is strictly increasing in E and k over the interval $\bar{n}^A \in (0, E + k)$.*

Proof. See Appendix.

Lemma 2 shows that the IRB-bank's incentive to give priority to risky projects is increasing in the total number of projects funded: The smaller the portfolio, the lower is the bankruptcy probability if risky projects are chosen. And since the advantage of risky projects is precisely that limited liability allows to keep the upside and hedges against the downside of their riskier cash flows, it follows that the higher expected project return of safe projects dominates for small portfolios, whereas the higher variance of risky projects dominates for large portfolios. An increase in k means that safe projects are more attractive, and an increase in E means that the bank has more to lose in case of bankruptcy. Given Lemma 2, we find:

Proposition 7 *With one IRB-bank (say Bank A) and unobservable portfolios, the regulator's optimal choice of n^A depends on the parameters E and k:* (i) Region 1: *If $\bar{n}^A \geq E + k$, then the regulator sets $n^A = E + k$, Bank A opts for cherry-picking and will be risk-free. $\bar{n}^A \geq E + k$ holds if E and k are sufficiently large.* (ii) Region 2: *If $\frac{1}{2} \leq \bar{n}^A < E + k$, the regulator implements $n^A = \bar{n}^A$, Bank A opts for cherry-picking and will be risk-free. $\bar{n}^A \geq \frac{1}{2}$ holds for intermediate values of E and k.* (iii) Region 3: *If $\bar{n}^A < \frac{1}{2}$, the regulator cannot avoid risk-shifting. Bank A funds all risky projects, and Bank B funds all safe projects. $\bar{n}^A < \frac{1}{2}$ holds if E and k are small.*

Proof. See Appendix.

Proposition 7 divides the parameter space into three regions. In Region 1, risk-shifting is no problem and we get the same solution as with observable portfolios. But outside this region, the agency problems caused by unobservable portfolios lead to higher bankruptcy costs because the regulator cannot implement the optimal allocation: In Region 2, the IRB-bank would engage in risk-shifting for $n^A = E + k$, and the best the regulator can do is to implement $\bar{n}^A$, i.e. the maximum number of projects where risk-shifting is avoided. The IRB-bank is still the larger one, and all safe and some risky projects are sheltered from bankruptcy risk.

In Region 3, Bank A would opt for risk-shifting even if $n^A < \frac{1}{2}$. We can show that in principle it would then be optimal to make the IRB-bank A the *smaller* bank, and to allocate either

$n^A = \bar{n}^A$ safe or $n^A = \bar{n}^A + \varepsilon$ risky projects to Bank A. However, allocating less projects to an IRB-bank than to a SA-bank is excluded by the assumption that $b_S \leq b_U$. Since portfolios are unobservable, the bank can declare all projects as safe ones, so that $n^A \geq n^B$. And given *Assumption L*, $n^A \geq \frac{1}{2}$ follows.[19]

B. Two IRB-Banks

With two IRB-banks and unobservable portfolios, it is impossible to implement different capital requirements for different types of projects or for different banks. Hence, there will be a single effective capital requirement b_I and the lending volumes will be identical, $n^A = n^B = \frac{1}{2}$. The outcome is then the same as it would have been with two IRS-banks and unregulated investments (see Section 4.4)[20] - both banks fund half of the projects and portfolios and interest rates need to yield identical profits for both banks. We summarize the outcome as follows:

Proposition 8 *With two IRB-banks and unobservable portfolios, there are two possible outcomes depending on k and E. (i) If $k \geq \left(\frac{5}{8} - \frac{1}{2}E\right)$, then one bank (say bank A) funds all safe projects, and the other bank funds all risky projects. Both banks earn equal profits, and interest rates are $R_R = 2$ and $R_S \leq 2$. Total bankruptcy costs are those of Bank B, $Z = \frac{1}{2}\left(\frac{1}{2} - E\right)z$. (ii) If $k < \left(\frac{5}{8} - \frac{1}{2}E\right)$, then each bank finances one half of the safe and one half of the risky projects, and interest rates are $R_S = R_R = 2$. Total bankruptcy costs are the same as with two SA-banks, $Z_u = z\left(1 - 2E - k\right)$.*

Proof. See Appendix.

The logic behind Proposition 8 follows that of Proposition 3. The only difference is that both banks now finance the same number of projects because the regulator cannot differentiate the capital requirements for safe and risky projects when portfolios are unobservable. Since bank profits are higher when funding either safe *or* risky projects, banks will do so whenever this is compatible with the equilibrium requirement of identical profits. This is true in case *(i)* of

---

[19] The assumption $b_S \leq b_U$ is realistic but from a theory point of view not always optimal. For some parameter values the regulator would prefer $b_S > b_U$ to curtail the possibility that an IRB-bank engages in risk-taking. The IRB-bank would then be the smaller bank, and hence investment subsidies are typically needed. A full analysis of this case is available from the authors.

[20] As shown in section 4.4, however, this was no subgame perfect equilibrium as one bank will demand an IRB-licence.

Proposition 8, and competition will reduce the safe interest rate R_S until banks are indifferent between the two portfolios. In case *(ii)*, an adjustment of interest rates is not feasible, as the possibility that safe projects imitate risky ones imposes the condition $R_S \leq R_R$. Hence, as in Proposition 3, we get the same outcome as with two SA-banks.

C. Comparison and Optimal Regulation

The following Lemma summarizes the comparison of the bankruptcy losses in the different regimes (subscript n for "nonobservable"):

Lemma 3 (i) $Z_n(1) < Z_u$ if $\bar{n}^A$ is large (Region 1 and large values of E and k in Region 2) and $Z_n(1) > Z_u$ if $\bar{n}^A$ is small (Region 3 and small values of E and k in Region 2). (ii) Two IRB-banks are always dominated by either one IRB-Bank or two SA-banks.

Proof. See Appendix.

Part (i) of Lemma 3 shows that one IRB-bank is the dominant regime for large values of $\bar{n}^A$, and two SA-banks dominate if $\bar{n}^A$ is small, neglecting investment costs. By contrast, one IRB-bank was always superior with observable portfolios (Proposition 4). Thus, there are important differences between the cases of observable and unobservable portfolios. In fact, we find that in case *(i)* of Proposition 3 the allocation with two IRB-banks is dominated by one IRB-bank in all regions since two IRB-banks imply a lower safe interest rate R_S and a smaller portfolio of the default-free bank than one IRB-bank. In case *(ii)* of Proposition 3 on the other hand, two IRB-banks are dominated by two SA-banks, as they lead to an equivalent project allocation, and hence IRS-investments that would be wasted. It follows that two IRB-banks can never be optimal.

Next, we analyze whether subsidies or taxes can ensure that investment decisions are optimal. As in the case of observable portfolios, there may be over- or underinvestment incentives. As an example, consider Region 2: we know from Lemma 3 that in this case total bankruptcy costs may be lowest when there are two SA-banks. However, Bank A's profit is always higher when it invests ($\Pi_n^A(1) - \Pi_u > 0$) as $n^A > \frac{1}{2}$ by definition of Region 2. The best available outcomes, from the point of view of the regulator, are those summarized in Proposition 7 that takes account of the constraint that risk-shifting concerns impose on the lending volume of an IRB-bank. We obtain:

23

Proposition 9 *Suppose the portfolio choice is unobservable. Then, the regulator can always uniquely implement the best available outcome:*

(i) If $Z_u - Z_n(1) \geq C_A$, only Bank A invests and finances $n^A = \min \left\{ \max\{\frac{1}{2}, \bar{n}^A\}, E + k \right\}$ projects. There is a subsidy if $C_A > \Pi_n^A(1) - \Pi_u$.

(ii) If $Z_u - Z_n(1) < C_A$, there will be two SA-banks. There is a tax on investment if $C_A < \Pi_n^A(1) - \Pi_u$.

Proof. See Appendix.

Proposition 9 says that the regulator can again attain the optimal investment decisions. As in the case of observable portfolios, the main reason is that Bank B has no incentive to become a second IRB-bank as competition for projects would then reduce profits. The use of subsidies and taxes to implement the optimum is virtually the same as in Proposition 5. These instruments will optimally discriminate between Bank A's investment incentives and those of Bank B as $C_A < C_B$.

D.   Unregulated Investments

For the same reason as with observable loan portfolios, it will never occur that both banks invest in an IRS without applying for a license, or that one bank becomes an IRB-bank while the other makes an unregulated investment. Again, however, we need to analyze Bank A's incentive to engage in unregulated investment if Bank B is a SA-bank.

If so, Bank A can finance $n^A = \frac{1}{2}$, and it will fund either only risky or only safe projects. This closely resembles the case with unregulated investments and observable portfolios, so that Bank A prefers unregulated investment to no investment if condition (9), $\max\left(\Pi_r^A, \Pi_s^A\right) - \Pi_u > C_A$, holds. In this case, the regulator cannot avoid the investment anyway, and it is then better to solicit applications for an IRB-license, and to implement $\bar{n}^A \geq \frac{1}{2}$ rather than $n^A = \frac{1}{2}$. We can summarize as follows:

Corollary 1 *Suppose banks can undertake unregulated IRS-investments and the portfolio choice is unobservable. Whenever one IRB-bank is the optimal outcome, the regulator will implement this outcome. If two SA-banks are the optimal outcome, then the regulator will be able to implement that outcome if $\max\left(\Pi_r^A, \Pi_s^A\right) - \Pi_u \leq C_A$ and otherwise implement the outcome with one IRB-bank. In equilibrium, unregulated investments do not occur.*

The different outcomes in the case of observable and of unobservable portfolios are quite similar, as the comparison of Proposition 6 and Corollary 1 shows. In other words, if agency problems concerning the truthful reporting of the bank's information become more severe this will reduce the optimal lending volume of an IRB-bank but not fundamentally alter the regulatory trade-off.

From this vantage point, it is interesting to revisit the comparison to Basel I. Under Basel I rules, the regulator will not be able to prevent banks from investing, but such investments will always be tantamount to unregulated investments since there is no differentiation of regulatory regimes, and the allocation will always be $n^A = n^B = \frac{1}{2}$. Therefore, the transition to Basel II will strictly improve the allocation in two cases: first, if investment by one IRB-bank is optimal; second, if overinvestment in internal rating systems cannot be avoided so that Basel II allows for optimal accommodation. In summary, Basel II is superior whenever at least one bank invests, and Basel I and Basel II are equivalent otherwise.

V.   Possible Extensions and Discussion

In this Section, we briefly discuss how our results would be affected if we relaxed some of our key assumptions.

A.   Bankruptcy Costs

We begin with the assumption of linear bankruptcy costs made to ensure the formal tractability and closed form characterization of our analysis. We do not have a strong view whether a neoclassical assumption of convex bankruptcy losses, or rather concave costs reflecting some economies of scale in the social response to a bank's failure would be the natural extension, and so the linear case may be viewed as a compromise between these two conflicting views. As long as bankruptcy losses are assumed to be increasing (convex, linear, or concave), the qualitative message of our model should go through.

B.   Lending Volume

If we relax *Assumption L* that limits the number of necessary case distinctions in our analysis then the volume of financing becomes endogenous. In this case, it is easy to show that the regulator either implements all projects (the case analyzed) or the maximum number of projects

that avoids any bankruptcy risk for *both* banks. The intuition for this corner solution is that marginal expected bankruptcy costs are decreasing in the number of projects funded, so that there is no interior solution. The regulator will avoid the bankruptcy risk if z is high. Then, the total number of projects depends on the number of IRB-banks. However, with observable loan portfolios, it will normally still be the case that one IRB-bank is the optimal allocation. Allocating again $E + k$ projects to the IRB-bank, both banks together can fund $2E + k$ projects without facing any bankruptcy risk. All safe projects are funded. With two SA-banks, the two banks together can also finance $2E + k$ projects, but some of the projects will be risky, which reduces overall welfare. The exact welfare ranking between these two outcomes depends on the relative size of X_R and X_S on which we have not made assumptions. Thus, one IRB-bank will be preferable to two SA-banks under the (reasonable) assumption that safe projects are better than risky projects from the point of view of social welfare. With unobservable loan portfolios, the analysis is more complex as it also depends on the risk-shifting incentives, but it can still be shown that either one IRB-bank or two SA-banks are optimal.

Since this paper focuses on the consequences of the two-layer approach of Basel II and the possible differentiation that it affords, our analysis clearly neglects important aspects of Basel II: Improved screening capabilities may enhance the overall quality of loan portfolios and thus reduce bank failure risks if more safe projects and less risky projects are funded in the economy. Our model focuses on the heightened risk-taking incentives that will be the result of more competition among IRB-banks and lower interest rates, but positive effects of lower interest rates could be added to complete the picture. Along the lines of our model, such effects could be taken into account by assuming that the universe of available projects is larger than the unit interval. Assuming that the cash flow $X_S(n_S)$ is a declining function of the number n_S of safe projects funded, more safe projects will seek financing as R_S decreases. The competition effect of two IRB-banks would then lead to more lending to safe projects and corresponding positive welfare effects. This would shift the comparison between the scenario of one IRB-bank and two IRB-banks in favor of the latter, so that the former ceases to be the dominant outcome in all cases.

C.   *Heterogeneity and Endogeneity of Bank Equity*

Furthermore, we assume that equity E is identical for both banks and exogenously given. Relaxing this assumption, one could first assume that total equity is still $2E$, but unequally divided

between the two banks. Then, we have shown that again one IRB-bank or two SA-banks are optimal, and whether the regulator can implement the desired outcome with subsidies or taxes is a straightforward adaptation of the symmetric case. More interestingly, one could assume that the equity of a bank emerges endogenously as a function of the bank's profitability. Then, we would get two effects: First, market forces would reallocate equity from the SA-bank to the IRB-bank as the latter is more profitable. In equilibrium, the expected return on equity is the same in both banks, and the case can then be discussed along the lines with exogenously given, but unevenly distributed equity. Second, since the profitability of the banking sector is higher with one IRB-bank, one should observe an inflow of capital into the banking sector which further decreases expected bankruptcy costs.

D. Masquerading of Projects

We assumed that safe projects can always masquerade as risky projects. This assumption guarantees realistic loan pricing, viz. to loan rate for safe projects weakly below those for projects with higher default risk. We can show that when we remove this assumption, the essence of our analysis is unaffected. But interestingly, with two IRB-banks it may occur that $R_S > R_R$. While being counterfactual, this outcome is in fact an intuitive and robust feature of a general equilibrium model in loan markets if the incentives for risk-shifting are strong, which in our model corresponds to case in which the difference in expected returns, $k - \frac{1}{2}$, is small. Competition among multiple risk-neutral lenders will then typically imply an equilibrium in which risky projects that are preferred because of risk-shifting motives earn a lower expected return. However, this is unlikely to occur in reality for several reasons, including notably risk aversion, and our masquerading assumption is a simple way of capturing those.

VI. Conclusion

This paper analyzes consequences of the Basel II Accord by taking account of general equilibrium effects on the loan market. We consider a simple model in which two banks can invest to improve their internal credit screening capacities. The regulator wants to allocate a fixed amount of profitable projects among banks in order to minimize the overall expected costs of bank failure. These costs depend on two variables: the differentiation in size and composition of the loan portfolios, and the interest rates.

In this setting, our analysis reveals an original positive effect of the Basel II two-layer approach that differentiates the capital adequacy ratios between IRB-banks and SA-banks: it allows to better exploit economies of scale in the allocation of systematic risks by optimally adjusting the size and portfolio structure of the two segments within the banking sector. IRB-banks should receive incentives to keep their loan portfolios safe and bank failure risks should be confined to SA-banks. While the two-layer approach initially has been conceived as a transitional regime, our analysis reveals it as a very attractive feature of the Basel II architecture. There is no need to strive for homogeneity among banks. Ultimately, bank differentiation could give rise to more rather than less investment in internal rating systems if the yardstick that separates between the two layers of the banking sector adjusts dynamically to the state-of-the-art credit information technologies. In our model, the regulator never wants both banks to invest in internal rating systems as two IRB-banks reduce the differentiation advantage. While the portfolio composition remains fully differentiated, the size allocation is suboptimal.

The regulator can almost always implement the preferred outcome which is one IRB-bank for low and two SA-banks for high investment costs, with one important exception: if the bank can make an unregulated investment and if its incentive to invest is substantially larger than the regulator's and the bank, then the regulator optimally accommodates the bank's investment.

If the regulator can observe the banks' portfolios, it is possible to fully reap the differentiation advantage of Basel II. In this case, the regulator gives incentives to the IRB-bank to finance all safe projects and the maximum number of risky projects that keep it default-free. If the regulator cannot fully observe the IRB-bank's portfolio, however, the bank might opportunistically abuse its screening capabilities to fund risky projects and to misreport its portfolio to the regulator. The regulator may then be unable to fully exploit the size differentiation effect, since the possibility to underreport portfolio risks introduces new constraints. The regulator then needs to restrict the lending volume of the IRB-bank as risk-taking incentives grow in the size of bank assets and bank leverage. This constraint reduces social welfare compared to the observability case, but will leave the main findings of our analysis unchanged.

References

[1] Acharya, V.V. (2001) A Theory of Systemic Risk and Design of Prudential Bank Regulation, Working Paper, New York University, http://faculty.london.edu/vacharya/wp.htm.

[2] Acharya, V.V. (2003) Is the International Convergence of Capital Adequacy Ratios Desirable? *Journal of Finance*, 58, 27-45.

[3] Allen, L. (2004) The Basel Capital Accords and International Mortgage Markets: A Survey of the Literature, *Financial Markets, Institutions and Instruments*, 13, 41-108.

[4] Altman, E.I. and Saunders, A. (2001) An analysis and critique of the BIS proposal on capital adequacy and ratings, *Journal of Banking and Finance*, 25, 25-46.

[5] Basel Committee on Banking Supervision (2004) International Convergence of Capital Measurement and Capital Standards. A Revised Framework, Bank for International Settlements, June 2004.

[6] Besanko, D. and G. Kanatas (1996) The Regulation of Bank Capital: Do Capital Standards Promote Bank Safety? *Journal of Financial Intermediation*, 5, 160-183.

[7] Bhattacharya, S., Boot, A.W.A. and Thakor, A. V. (1998) The Economics of Bank Regulation, *Journal of Money, Credit, and Banking*, 30, 745-770.

[8] Boot, A.W.A. and Greenbaum, S.I. (1993) Bank regulation, reputation and rents: theory and policy Implications, in: Mayer, C.; Vives, X. (ed.) Capital Markets and Financial Intermediation, Cambridge: Cambridge University Press, 262-285.

[9] Carey, M. (2002) Some Evidence on the Consistency of Banks' Internal Credit Ratings, in Michael Ong (ed.), *Credit Ratings: Methodologies, Rationale and Default Risk.* London: Risk Books.

[10] Claessens, S. and G. Embrechts (2003) Basel II, Sovereign Ratings and Transfer Risk. External versus Internal Ratings. mimeo, University of Amsterdam and World Bank, http://ssrn.com/abstract=386480.

[11] Dangl, T. and Lehar, A (2004) Value-at-Risk vs. Building Block Regulation in Banking, *Journal of Financial Intermediation*, 13, 96-131.

[12] Decamps, J.P., Rochet, J.C., and Roger, B. (2004) The Three Pillars of Basel II: Optimizing the Mix, *Journal of Financial Intermediation*, 13, 132-155.

[13] Gersbach, H. and Wehrspohn, U. (2001) Lean IRB Approaches and Transition Design: The Basel II Proposal, Working Paper, University of Heidelberg, http://ssrn.com/abstract=290700.

[14] Gropp, R. and A.J. Richards (2001) Rating Agency Actions and the Pricing of Debt and Equity of European Banks: What Can We Infer About Private Sector Monitoring of Bank Soundness? European Central Bank Working Paper Series No. 76.

[15] Hakenes, H. and I. Schnabel (2007) Bank Size and Risk-Taking under Basel II, Working Paper, Max Planck Institute for Research on Collective Goods, Bonn.

[16] Hellmann, T.F., Murdock, K.C. and Stiglitz, J.E. (2000) Liberalization, Moral Hazard in Banking and Prudential Regulation: Are Capital Requirements Enough ? *American Economic Review*, 90, 147-165.

[17] Jensen, M. and Meckling, W. (1976) Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, *Journal of Financial Economics*, 3, 305-360.

[18] Matutes, C. and Vives, X. (1996) Competition for Deposits, Fragility, and Insurance, *Journal of Financial Intermediation*, 5, 184-216.

[19] Morrison, A.D. and White, L. (2005), Crises and Capital Requirements in Banking, *American Economic Review*, forthcoming.

[20] Pelizzon, L. and S. Schaefer (2006) Pillar 1 vs. Pillar 2 Under Risk Management, NBER WP 11666.

[21] Peura, S. and Jokivuolle, E. (2001) Simulation-based stress testing of banks' regulatory capital adequacy, *Journal of Banking and Finance,* 28, 1801-1824.

[22] Repullo, R. (2004) Capital Requirements, Market Power, and Risk Taking in Banking, *Journal of Financial Intermediation*, 13, 156-182.

[23] Repullo, R. and J. Suarez (2004) Loan Pricing Under Basel Capital Requirements, *Journal of Financial Intermediation*, 13, 496-521.

[24] Rime, B. (2005) Will Basel II Lead to a Specialization of Unsophisticated Banks on High-Risk Borrowers? *International Finance*, 8, 29-55.

[25] Thakor, A.V. (1996) Capital Requirements, Monetary Policy, and Aggregate Bank Lending: Theory and Empirical Evidence, *Journal of Finance*, 51, 279-324.

Appendix

Proof of Lemma 1. We use the notation $n_R^A = n^A - n_S^A$ for the quantity of risky projects that the bank finances in addition to n_S^A safe projects. The profit Π^A of the bank can then be written as:

$$\Pi^A = (1 - \tilde{t}^A) \left(E\left[t \mid t \geq \tilde{t}^A\right] R_R n_R^A + E - n_R^A + (kR_S - 1) n_S^A \right), \tag{10}$$

which will be maximized by choosing a pair $\left(n_R^A, n_S^A\right)$, subject to the bank's equity constraint (7). For the bankruptcy threshold and the conditional expectation we get from (10):

$$\tilde{t}^A = \max \left\{ \frac{n_R^A - E - (kR_S - 1) n_S^A}{R_R n_R^A}, 0 \right\}, \tag{11}$$

and

$$E\left[t \mid t \geq \tilde{t}^A\right] = \max \left\{ \frac{1}{2}, \frac{R_R n_R^A + n_R^A - E - (kR_S - 1) n_S^A}{2 R_R n_R^A} \right\}. \tag{12}$$

Eq. (11) implicitly defines a threshold value of $n_R^A = \bar{n} = E + (kR_S - 1) n_S^A$ such that $\tilde{t}^A = 0$ for all $n_R^A \leq \bar{n}$ and $\tilde{t}^A > 0$ for all $n_R^A > \bar{n}$. Substituting for $\tilde{t}^A$ and $E\left[t \mid t \geq \tilde{t}^A\right]$ in Eq. (10) and rearranging, profits can be rewritten as:

$$\Pi^A = \begin{cases} \frac{1}{2} R_R n_R^A + E - n_R^A + (kR_S - 1) n_S^A & \text{if } n_R^A \leq \bar{n} \\ \left(\frac{1}{2 R_R n_R^A}\right) \left(R_R n_R^A + E - n_R^A + (kR_S - 1) n_S^A\right)^2 & \text{if } n_R^A > \bar{n} \end{cases}. \tag{13}$$

Inspection of (10) shows that Π^A must be continuous at the point $\bar{n}$. If $R_R < 1$, Lemma 1 follows trivially: the bank will never adopt risky projects if $R_R < 1$, since in all states $t \in (0, 1)$, risky projects will lead to a reduced payoff. Therefore, we need only to consider $R_R \geq 1$.

Concerning the choice of safe projects, since $R_S \geq \frac{1}{k}$ and $R_R \geq 1$, it follows that profits are increasing in n_S^A, in both regions $n_R^A \leq \bar{n}$ and also if $n_R^A > \bar{n}$.

Concerning the choice of risky projects, consider first the region $n_R^A \leq \bar{n}$. If $n_R^A \leq \bar{n}$, Π^A is a linear function of n_R^A. Moreover, $n_R^A = \bar{n}$ cannot be a local maximum and if $R_R \leq 2$, then Π^A cannot be increasing in n_R^A. Thus, the bank will optimally either finance no risky projects, $n_R^A = 0$, or if $R_R = 2$ be indifferent among all $n_R^A \leq \bar{n}$, or the optimum lies outside the considered range $n_R^A \leq \bar{n}$.

Consider then the region $n_R^A > \bar{n}$. Over this region, with the equity constraint (7) and the two supply constraints $n_S^A \leq \frac{1}{2}$ and $n_R^A \leq \frac{1}{2}$, the Kuhn-Tucker problem is well defined and becomes:

$$L = \left(\frac{1}{2 R_R n_R^A}\right) \left(R_R n_R^A + E - n_R^A + (kR_S - 1) n_S^A\right)^2 - \lambda \left(n_R^A b_R + n_S^A b_S - E\right) - \mu_1 \left(n_S^A - \frac{1}{2}\right) - \mu_2 \left(n_R^A - \frac{1}{2}\right). \tag{14}$$

We obtain the necessary conditions:

$$\frac{\partial L}{\partial n_S^A} = \left(\frac{1}{R_R n_R^A}\right) \left(E - n_R^A + n_R^A R_R + n_S^A (kR_S - 1)\right) (kR_S - 1) - \lambda b_S - \mu_1 = 0 \tag{15}$$

$$\frac{\partial L}{\partial n_R^A} = \left(\frac{1}{2 R_R \left(n_R^A\right)^2}\right) \left(\left(n_R^A - n_R^A R_R\right)^2 - \left(E + (kR_S - 1) n_S^A\right)^2\right) - \lambda b_R - \mu_2 = 0. \tag{16}$$

31

Since $R_S \geq \frac{1}{k}$ and $R_R \geq 1$, from condition (15) it follows that either $\lambda > 0$ or $\mu_1 > 0$, or both.

First, consider the case $\lambda = 0$ (the equity constraint is slack). Then $\mu_1 > 0$, i.e. the bank will finance $n_S^A = \frac{1}{2}$ safe projects. To determine the bank's optimal quantity n_R^A, consider the unconstrained problem (13). The second derivative of Π^A with respect to n_R^A gives:

$$\frac{\partial^2 \Pi^A}{\partial \left(n_R^A\right)^2} = \frac{1}{R_R \left(n_R^A\right)^3} \left(E + (kR_S - 1) n_S^A\right)^2 > 0.$$

Thus, Π^A is strictly convex in n_R^A. We have already seen that Π^A is linear in n_R^A for $n_R^A \leq \bar{n}$. Hence only boundary solutions can be optimal. Thus, either $n_R^A \leq \bar{n}$ or $n_R^A = \frac{1}{2}$.

Second, consider the case $\lambda > 0$ (the equity constraint is binding). Constraint (7) can then be written as

$$n_S^A = \max \left\{ \frac{E - n_R^A b_R}{b_S}, 0 \right\}. \tag{17}$$

Substituting for n_S^A in (13) yields for values $n_R^A > \bar{n}$:

$$\Pi^A = \left(\frac{1}{2R_R n_R^A}\right) \left(R_R n_R^A + E - n_R^A + (kR_S - 1) \cdot \max \left\{ \frac{E - n_R^A b_R}{b_S}, 0 \right\}\right)^2.$$

Moreover, Π^A is continuous and piecewise differentiable in n_R^A. Then, taking the second derivative of Π^A, if $n_S^A = \frac{E - n_R^A b_R}{b_S}$:

$$\left.\frac{\partial^2 \Pi^A}{\partial \left(n_R^A\right)^2}\right|_{n_R^A > \bar{n}, n_R^A < \frac{E}{b_R}} = \frac{\left(E \left(1 + \frac{(kR_S - 1)}{b_S}\right)\right)^2}{\left(n_R^A\right)^3 R_R} > 0. \tag{18}$$

Moreover, if $n_R^A \geq \frac{E}{b_R}$ so that $n_S^A = 0$, from (17):

$$\left.\frac{\partial^2 \Pi^A}{\partial \left(n_R^A\right)^2}\right|_{n_R^A > \bar{n}, n_R^A \geq \frac{E}{b_R}} = \frac{E^2}{\left(n_R^A\right)^3 R_R} > 0. \tag{19}$$

Expressions (18) and (19) show that Π^A is piecewise strictly convex over the entire range $n_R^A > \bar{n}$. Only boundary points for n_R^A can be optimal. Hence, either $n_R^A \leq \bar{n}$ or $n_R^A = \frac{1}{2}$, or n_R^A is bounded by the budget constraint. Together with the result that the bank will always increase the number of safe projects until the budget constraint binds, we have shown Lemma 1. ∎

Proof of Proposition 2. We show that (i) the allocation given in Proposition 2 minimizes total bankruptcy costs and is hence optimal, and then (ii) that the allocation is implementable.

Part (i). First note that $R_S = R_R = 2$ because the marginal project is risky, and because the IRB-bank only needs to undercut the interest rate marginally to attract all safe projects. Also, $R_S = R_R = 2$ implies that safe projects do not have an incentive to masquerade as risky one.

We prove more generally for later use that allocating $n^A = \frac{1}{2} k R_S + E$ projects to the IRB-bank is optimal for $R_S \in \left(\frac{1}{k}, 2\right)$. Let Z^A (Z^B) be the bankruptcy loss from the IRB-bank A (the SA-bank B). From

$$\Pi^A = \left(1 - \tilde{t}^A\right) \left[\frac{1}{2}kR_S + E\left[t \mid t > \tilde{t}^A\right] 2\left(n^A - \frac{1}{2}\right) + E - n^A\right] , \qquad (20)$$

we get

$$\tilde{t}^A = \max\left\{\frac{\left(2n^A - 2E - kR_S\right)}{4n^A - 2}, 0\right\} ,$$

and for the SA-bank B we get

$$\tilde{t}^B = \max\left\{\frac{1 - n^A - E}{2\left(1 - n^A\right)}, 0\right\} . \qquad (21)$$

Total expected bankruptcy costs are

$$Z\left(n^A\right) = n^A \tilde{t}^A z + (1 - n^A)\tilde{t}^B z \qquad (22)$$
$$= \left(\frac{\left(2n^A - 2E - kR_S\right)}{4n^A - 2}n^A + \frac{1 - n^A - E}{2}\right) z .$$

Taking the derivative w.r.t. n^A gives

$$\frac{\partial Z}{\partial n^A} = z\frac{\left(2E - 1 + kR_S\right)}{2 + 8\left(n^A\right)^2 - 8n^A} > 0 ,$$

which proves that Z is increasing in n^A if $n^A > \frac{1}{2}kR_S + E$, i.e. if Bank A faces a positive bankruptcy risk. Otherwise, if only Bank B may go bankrupt, total bankruptcy costs are simply

$$Z = Z^B = \frac{1 - n^A - E}{2}z , \qquad (23)$$

as $Z^A = 0$. It follows that

$$\frac{\partial Z^B}{\partial n^A} = -\frac{1}{2}z < 0.$$

Since $\frac{\partial Z}{\partial n^A} > 0$ if $n^A > \frac{1}{2}kR_S + E$ and $\frac{\partial Z}{\partial n^A} < 0$ if $n^A < \frac{1}{2}kR_S + E$, it is optimal to implement $n^A = \frac{1}{2}kR_S + E$. For the special case in Proposition 2, we have $R_S = 2$ and thus $n^A = E + k$.

Part (ii). Suppose the regulator chooses $b_S = 0$, $b_R = \frac{E}{E+k-\frac{1}{2}}$ and $b_U = \frac{E}{1-E-k}$ as stated in the Proposition. Then, the IRB-bank needs no equity to fund all safe projects, and can fund exactly $E + k - \frac{1}{2}$ risky projects. Funding more risky projects is impossible as b_R is binding. For the SA-bank, there are no safe projects left, and it can fund $n^B = 1 - E - k$ risky projects. ∎

Proof of Proposition 3. *Part (i).* From Lemma 1, we know that profits are convex in the number of safe projects, so that there are no incentives for marginal deviations. Furthermore, we know from the proof of Proposition 2 that total bankruptcy costs are minimized for $\tilde{n}^A \equiv E + \frac{1}{2}kR_S$. By definition of case *(i)*, we have $\Pi^A(\tilde{n}^A) \geq \Pi^B$ for $R_S = R_R = 2$. For the minimum $R_S = \frac{1}{k}$ we have $\Pi^B \geq \Pi^A(\tilde{n}^A)$. We show that $\frac{d\left(\Pi^B - \Pi^A\right)}{dR_S} < 0$ to prove that there always exists a unique $R_S \in \left(\frac{1}{k}, 2\right)$ such that $\Pi^A = \Pi^B$. We have, if $R_R = 2$

$$\Pi^A = \frac{1}{2}kR_S + \frac{1}{2}\left(n^A - \frac{1}{2}\right)R_R + E - n^A = \left(\frac{1}{2}kR_S - \frac{1}{2} + E\right) ,$$

with $\frac{d\Pi^A}{dR_S} = \frac{1}{2}k > 0$. For Bank B, we have

$$\Pi^B = (1 - \tilde{t}^B) \left[E\left[t \,|\, t > \tilde{t}^B \right] 2 \left(1 - n^A \right) + E - \left(1 - n^A \right) \right] ,$$

and thus, with $\tilde{t}^B = \left(\frac{\left(1 - n^A - E \right)}{2 - 2n^A} \right)$,

$$\Pi^B = \frac{\left(1 + E - n^A \right)^2}{4 \left(1 - n^A \right)}.$$

Note that

$$\frac{d\Pi^B}{dR_S} = \frac{\partial \Pi^B}{\partial n^A} \frac{\partial n^A}{\partial R_S} = -\frac{\left(1 - n^A - E \right) \left(1 + E - n^A \right) k}{8 \left(1 - n^A \right)^2} < 0 ,$$

as $n^A < 1 - E$ is implied by the absence of bankruptcy risk of Bank A. Since $\frac{d\Pi^A}{dR_S} > 0$ while $\frac{d\Pi^B}{dR_S} < 0$, we have $\frac{d\left(\Pi^B - \Pi^A \right)}{dR_S} < 0$. Next, we show that there exist capital requirements that implement every $\tilde{n}^A \leq \frac{3}{4}$: $b_R = \frac{E}{1 - \tilde{n}^A}$ allows Bank B to exactly fund $1 - \tilde{n}^A$ risky projects, and $b_S \geq 0$ will be chosen so that Bank A can fund all safe and $\tilde{n}^A - \frac{1}{2}$ risky projects. Thus, $n^A = \min \left(\tilde{n}^A, \frac{3}{4} \right)$ can be implemented.

The threshold $\hat{k}(E)$ is derived as follows. If $R_S = R_R = 2$, and if Bank A finances the maximum default-free portfolio $\tilde{n}^A = E + k$, then $\Pi^A = k - \frac{1}{2} + E$ and $\Pi^B = \frac{(1 - k)^2}{4(1 - k - E)}$. This implies that $\Pi^A(E + k) \geq \Pi^B$ iff $k \geq \frac{4}{5} - \frac{4}{5}E - \frac{1}{5}\sqrt{-2E - 4E^2 + 1}$. Moreover, it is straightforward to show that $\hat{k}(E) < \frac{3}{4}$.

Part (ii). In this case, $\Pi^A(\tilde{n}^A) < \Pi^B$ for $R_S = R_R = 2$. We first construct the equilibrium and then show that it is unique. $R_S > 2$ is excluded by outside competition. $R_R < 2$ is also excluded as safe projects would masquerade as risky projects in order to get the lower interest rate. Hence, only equilibria with identical interest rates are possible, and from all these equilibria, the regulator prefers the one with the highest rates, hence $R_R = R_S = 2$. An equilibrium then requires that profits of both banks are identical. Since $\Pi^A(\tilde{n}^A) < \Pi^B$ for $R_S = R_R = 2$, each bank wants to fund all safe projects. However, in this equilibrium all safe projects masquerade as risky projects, thus becoming indistinguishable for the two IRB-banks. Thus, in equilibrium projects are randomly allocated and each bank funds half of the safe and half of the risky projects, leading to equal profits. This will only be the case if. Next, the equilibrium is unique: $R_R > R_S$ cannot be an equilibrium by definition of the case considered. $R_R < R_S$ is not feasible because of the masquerading assumption. $R_R = R_S < 2$ is not possible in equilibrium because the regulator prefers higher rates and can implement them by binding capital requirements. And in any asymmetric equilibria, either Bank A gets lower profits (when financing $n \leq \tilde{n}^A$ projects) and thus would undercut the rate for risky projects to take Bank B's position, or it faces positive bankruptcy risk (when financing $n > \tilde{n}^A$ projects). But then, total bankruptcy costs are higher than in the symmetric equilibrium. ∎

Proof of Proposition 4. We already know that $Z_o(1) = z\frac{1 - 2E - k}{2}$ and $Z_u = 1 - 2E - k$, thus $Z_u - Z_o(1) = z\frac{1 - 2E - k}{2} > 0$. With two IRB-banks, call total bankruptcy costs in case *(i)* of Proposition 3 $Z_o(2^i)$ and in case *(ii)* of Proposition 3, $Z_o(2^{ii})$. In case *(ii)* we have $Z_o(2^{ii}) = Z_u > Z_o(1)$. In case *(i)*, we have $Z_o(2^i) = z\frac{1 - n^A - E}{2} > Z_o(1) = z\frac{1 - 2E - k}{2}$ if $n^A < E + k$. And since $n^A = E + k$ is only feasible in the special case where $\Pi^A(n^A = E + k, R_S = R_R = 2) = \Pi^B(R_S = R_R = 2)$, the result follows. ∎

Proof of Proposition 5. Since all other elements of the proof are provided in the text, this proof only covers the remaining claim that $\hat{\Pi}_o^B(2) = \Pi_o^B(1)$ in the case in which with $n^A = E + k > \frac{3}{4}$. We need to consider the equilibrium profits of two IRB-banks with the capital requirements that are optimal in the case of one IRB-bank, $b_S = 0$ and $b_R = \frac{E}{E+k-\frac{1}{2}}$. Since $b_S = 0$, the two banks will compete for safe projects until the interest factor is so low that all profits from safe projects are eliminated, $R_S = \frac{1}{k}$. Moreover, with $b_R = \frac{E}{E+k-\frac{1}{2}}$ the capital constraints cannot be binding for both banks under any allocation in which they finance all risky projects. To see that there is no equilibrium in pure strategies, consider a bank i with a nonbinding capital constraint. There can be no pure strategy equilibrium in which R_R is so high that Bank i earns a positive profit. Suppose that R_R is so low that its payoff is equal to E, so that i is indifferent between funding the projects or not. This cannot be an equilibrium as Bank i would cut R_R further, attract more risky projects and gain a higher profit. But if its payoff Π^i is smaller than E, than the bank prefers to finance zero projects.

Thus, we need to consider equilibria where both banks choose mixed strategies concerning their stated interest factors R_R. In such an equilibrium, the bank that announces the lower rate finances the maximum feasible number of risky projects until its capital constraint is reached, $n^A = \frac{E}{b_R} = E + k - \frac{1}{2} > \frac{1}{2}$. The other bank funds $1 - n^A$ risky projects. The minimum interest factor $R_R^{\min} < 2$ is given by the condition that the bank just gets a payoff of E when financing $E + k - \frac{1}{2}$ risky projects. All interest factors over the feasible range $R_R = \left(R_R^{\min}, 2\right)$ must have positive support as otherwise there would be strategies R_R where banks could deviate to a marginally higher rate without altering the probability of becoming the large bank that finances $E + k - \frac{1}{2}$ risky projects. Hence, $R_R = 2$ has positive support. For $R_R = 2$, however, the bank becomes the small one with probability one as there are infinitely many interest rates. And since in any mixed strategy equilibrium the bank must be indifferent between all interest rates in the support, expected profits with all interest rates are $\hat{\Pi}^B(2) = \Pi^B(1) = \frac{(1-k)^2}{4(1-k-E)}$. $\blacksquare$

Proof of Lemma 2. To prove Lemma 2, we begin with the following Lemma that allows us to restrict attention to the properties of $\bar{n}^A$:

Lemma 4. There can be at most a single threshold $\bar{n}^A$ where Bank A switches from cherry-picking to risk-shifting.

Proof. We restrict attention to the case where $n^A \in (\frac{1}{2}, E + k)$ as $n^A < \frac{1}{2}$ cannot be implemented due to $b_S \leq b_U$. (The proof for $n^A < \frac{1}{2}$ is contained in a working paper version). Profits with cherry picking are

$$\Pi_S^A = \frac{1}{2}kR + \left(n^A - \frac{1}{2}\right)\frac{1}{2}R + E - n^A , \qquad (24)$$

as the bank faces no bankruptcy risk. For profits with risk-shifting, by using the profit expression from the proof of Lemma 1 and taking into account that with risk-shifting we have $\alpha n^A = \frac{1}{2}$, it follows:

$$\Pi_R^A = \frac{\left(\frac{1}{2}R + kR\left(n^A - \frac{1}{2}\right) + E - n^A\right)^2}{R}. \qquad (25)$$

Let $\Delta\Pi^A = \Pi_S^A - \Pi_R^A$ denote the profit difference. From Lemma 1, we know that in equilibrium there will be either cherry-picking or risk-shifting, so that $R = 2$ or $R = \frac{1}{k}$ are the only possible equilibrium

interest rates. Thus, it is sufficient to show that, for either of these two values of R we have $\frac{\partial \Delta \Pi^A}{\partial n^A} < 0$. Now

$$\frac{\partial \Delta \Pi^A}{\partial n^A} = \frac{1}{2}R + \frac{1}{R}\left(-2n^A + 2E\right) - k + 4kn^A - 2kE + R\left(k^2 - k - 2k^2 n^A\right). \tag{26}$$

If $R = 2$, we get:

$$\frac{\partial \Delta \Pi^A}{\partial n^A} = -\left(2k - 1\right)\left(E - n^A - k + 2kn^A + 1\right) < 0, \tag{27}$$

and if $R = \frac{1}{k}$, we get:

$$\frac{\partial \Delta \Pi^A}{\partial n^A} = -2kn^A + 2kE + 2n^A - 2E - \frac{1}{2k} < 0, \tag{28}$$

showing the claim.

Second, consider $n^A > E + k$. In this case, Bank A will no longer be defaultfree when cherry-picking, and hence we get as profit expression in the cherry-picking case:

$$\Pi_S^A = \left(1 - \tilde{t}^A\right)\left(\frac{1}{2}kR + \left(n^A - \frac{1}{2}\right)E\left[t \mid t \geq \tilde{t}^A\right]R + E - n^A\right) = \frac{\left(2E - R + Rk - 2n^A + 2Rn^A\right)^2}{4\left(2n^A - 1\right)R}.$$

As the residual project is risky, we have $R = 2$ and thus

$$\Pi_S^A = \frac{\left(E - 1 + k + n^A\right)^2}{\left(2n^A - 1\right)}.$$

Taking the derivative yields:

$$\frac{\partial \Pi_S^A}{\partial n^A} = 2\left(\frac{1}{2n^A - 1}\left(k + n^A + E - 1\right)\left(1 - \frac{1}{2n^A - 1}\left(k + n^A + E - 1\right)\right)\right) < 0.$$

On the other hand, for the profit in the risk-shifting case, the derivative is:

$$\frac{\partial \Pi_r^A}{\partial n} = \frac{\partial}{\partial n^A}\left(\left(1 + 2k\left(n^A - \frac{1}{2}\right) + E - n^A\right)^2\right)$$

$$= 2\left(2k - 1\right)\left(E - n^A - k + 2kn^A + 1\right) > 0.$$

Taking together, this shows again that $\frac{\partial \Delta \Pi^A}{\partial n^A} < 0$. ∎

Given Lemma 4, we can now derive a closed expression for $\bar{n}^A \geq \frac{1}{2}$ as stated in the text. Consider the case where $n^A = \bar{n}^A$ and where $R = 2$. Substituting into the profit expression yields for any $n^A = \bar{n}^A \leq E + k$:

$$\Pi_S^A = E + k - \frac{1}{2},$$

which is independent of n^A as risky projects just break even. If Bank A deviated to risk-shifting, then still $R = 2$, and from (25) we have:

$$\Pi_R^A = \frac{1}{2}\left(E - n^A + k\left(2n^A - 1\right) + 1\right)^2.$$

Since $\bar{n}^A$ is defined as the point where $\Pi_R^A = \Pi_S^A$, it follows

$$\frac{1}{2}\left(E - n^A + k\left(2n^A - 1\right) + 1\right)^2 = E + k - \frac{1}{2},$$

which we can solve as:

$$\bar{n}^A = \frac{(2E + 2k - 1)^{\frac{1}{2}} - E + k - 1}{(2k - 1)} \,. \tag{29}$$

Furthermore, note that the bank strictly prefers risky projects for $k = \frac{1}{2}$ as

$$\Pi_S^A - \Pi_R^A = \left(E + k - \frac{1}{2}\right) - \frac{\left(1 + 2kn^A - k + E - n^A\right)^2}{2} \,, \tag{30}$$

and hence

$$\Pi_S^A - \Pi_R^A \left(k = \frac{1}{2}\right) = -\frac{1}{2}E^2 + \frac{1}{2}E - \frac{1}{8} < 0. \tag{31}$$

It remains to show that $\bar{n}^A$ is monotonic in k and E. We take the expression for $\bar{n}^A$ in (29) and analyze the derivative with respect to k:

$$\frac{\partial \bar{n}^A}{\partial k} = \frac{1}{2k-1}\left(\frac{1}{\sqrt{2k+2E-1}} + 1\right) + \frac{2}{(2k-1)^2}\left(E - k - \sqrt{2k+2E-1} + 1\right).$$

A sufficient condition for this expression to be positive is:

$$\frac{\left(\sqrt{2k+2E-1} - 4E - 2k + 2E\sqrt{2k+2E-1} + 1\right)}{\sqrt{2k+2E-1}} > 0.$$

This is positive if the numerator is positive, or if

$$(1 + 2E)\sqrt{2k+2E-1} > 4E + 2k - 1.$$

After taking squares and rearranging, we find that this must be true since

$$\left(4E^2 - 2E - 2k + 1\right) = 2E\left(2E - 1\right) + 1 - 2k < 0,$$

which shows that $\bar{n}^A$ is monotonic in k.

Finally, taking derivatives of $\bar{n}^A$ with respect to E :

$$\frac{\partial \bar{n}^A}{\partial E} = \frac{1}{2k-1}\left(\frac{1}{\sqrt{2k+2E-1}} - 1\right) > 0. \ \blacksquare$$

Proof of Proposition 7. *Part (i).* We already know that $n^A = E + k$ is optimal in the class of cherry-picking allocations, and this allocation is feasible in Region 1 by definition. Finally, the proof of part *(ii)* below implies that $n^A = E + k$ is superior to any risk-shifting allocation.

Part (ii). Here, we consider situations where the optimal $n^A = E + k$ without risk-shifting is not feasible as $\bar{n}^A < E + k$. We prove that $n^A = \bar{n}^A > \frac{1}{2}$ is optimal. Consider first the case $n^A > \bar{n}^A$, meaning that Bank A is risk-shifting. Then, Bank B faces no bankruptcy risk, and Bank A earns profits of

$$\Pi_R^A(1) = (1 - \tilde{t}^A)\left[\left(n^A - \frac{1}{2}\right)kR + \frac{1}{2}E\left[t \mid t > \tilde{t}^A\right]R + E - n^A\right]. \tag{32}$$

37

Note that $R = \frac{1}{k}$, since we analyze bankruptcy costs in an equilibrium with risk-shifting, so that the projects *expected* by Bank B are all safe ones. Thus, the expected bankruptcy probability, again obtained as the marginal t where $[.] = 0$ in Eq. (32), is $\tilde{t}^A = k\,(1 - 2E)$. Hence,

$$Z_R(n^A) = n^A z k\,(1 - 2E), \tag{33}$$

and $\frac{\partial Z_R}{\partial n^A} = zk(1 - 2E) > 0$. Thus, *given* risk-shifting, $n^A = \bar{n}^A + \varepsilon$ would be optimal.

Bankruptcy costs with cherry-picking are $Z_S(\bar{n}^A) = \frac{1}{2}z\left(1 - \bar{n}^A - E\right)$, since only Bank B faces positive bankruptcy risk. Hence, we have

$$Z_S(\bar{n}^A) - Z_R(n^A) = z\left(\frac{1}{2}\left(1 - \bar{n}^A - E\right) - n^A k\,(1 - 2E)\right) \tag{34}$$

$$\leq z\left(\frac{1}{4}\left(1 - 2E\right) - \frac{k\,(1 - 2E)}{2}\right) < 0,$$

where we made use of $n^A \geq \frac{1}{2}, \bar{n}^A \geq \frac{1}{2}$ and $k \geq \frac{1}{2}$. Note that $n^A < \overline{n}^A$ cannot be optimal as the safe bank were unnecessarily small.

Part (iii). Since the IRB-bank A opts for risk-shifting for all feasible $n^A \geq \frac{1}{2}$, total expected bankruptcy costs are minimized for $n^A = \frac{1}{2}$ and are independent of whether Bank A funds all safe or all risky projects. Bankruptcy costs $Z(n^A)$ are

$$Z\,(1/2) = z\tilde{t}^A n^A = z\frac{n^A - E}{2n^A}n^A = z\left(\frac{1}{4} - \frac{1}{2}E\right). \ \blacksquare$$

Proof of Proposition 8. We know that $n^A = n^B = \frac{1}{2}$ and that profits need to be identical. *Case (i).* Suppose Bank A funds all safe projects and Bank B funds all risky projects. Then, for $R_S = R_R = 2$, we have $\Pi_A \geq \Pi_B \Leftrightarrow k \geq \left(\frac{5}{8} - \frac{1}{2}E\right)$. And since $\frac{\partial \Pi_A}{\partial R_S} > 0$, profits of the two banks are equalized in case *(i)* by $R_S < 2$. Total bankruptcy costs are those of Bank B and given by $Z = zn^B \Pr(t < \tilde{t}) = \frac{1}{2}\left(\frac{1}{2} - E\right)z$. *Case (ii).* For $\Pi_A < \Pi_B \Leftrightarrow k < \left(\frac{5}{8} - \frac{1}{2}E\right)$ profits of the two banks could only be identical if Bank A funded all safe projects and Bank B funded all risky projects, and if at the same time $R_S > R_R$. This outcome, however, is impossible beas safe projects would masquerade as risky projects and get the lower interest rate R_R. Therefore, in the unique SPE in case *(ii)*, both banks must fund identical mixed portfolios (the details are exactly as shown in the proof of Proposition 3, case *(ii)*). The outcome is the same as with two SA-banks, and total bankruptcy costs are $Z_u = z\,(1 - 2E - k)$. $\blacksquare$

Proof of Lemma 3. *Part (i).* We know from Proposition 4 that $Z_n(1) - Z_u = -z\,(1 - k - 2E) < 0$ in Region 1. In Regions 2 and 3, however, it can easily be shown that $Z_n < Z_u$ for the minimum $k = \frac{1}{2}$ whereas $Z_n > Z_u$ for the maximum $k = 1 - E$, hence $Z_n \lessgtr Z_u$.

Part (ii). As argued, two different allocations may arise with two IRB-banks, depending on the parameters: In case *(i)* of Proposition 8, Bank A is default-free and $R_S < 2$ whereas in case *(ii)* both banks fund identical mixed portfolios mix and have positive bankruptcy risk. Let us again denote by $Z_n(2^i)$ and

$Z_n(2^{ii})$ the respective bankruptcy costs in case *(i)* and *(ii)*. Then, in Region 1, $Z_n(1) - Z_n(2^i) = \frac{1-k-2E}{2}z - \frac{1}{2}\left(\frac{1}{2} - E\right)z = \frac{1}{2}\left(\frac{1}{2} - k - E\right) < 0$ as $k \geq \frac{1}{2}$. Let $n^{SUP} = \sup\left\{\bar{n}^A, 1 - \bar{n}^A\right\}$. Note that in Regions 2 and 3, $Z_n(1) \leq \frac{1}{2}z\left(1 - n^{SUP} - E\right)$. Then $Z_n(1) \leq Z_n(2^i)$ follows from $\frac{1}{2}\left(1 - n^{SUP} - E\right) - \frac{1}{2}\left(\frac{1}{2} - E\right) \leq 0$. Next, since the allocation in case *(ii)* with two IRB-banks is the same as with two SA-Banks, we have $Z_U = Z_n(2^{ii})$. And since investment costs accrue only with two IRB-banks, this is inferior to two SA-banks. It follows that that in case *(i)*, two IRB-banks are dominated by one IRB-bank, and case *(ii)* they are dominated by two SA-banks.∎

Proof of Proposition 9. Following Proposition 5, we know that $\hat{\Pi}_n^B(2) - \Pi_n^B(1) \leq 0$ is a sufficient condition that the optimal regulation can be implemented, where $\hat{\Pi}_n^B(2)$ is the profit of Bank B with two IRB-banks given that capital requirements are those that are optimal if there is one IRB-bank.

Any possible equilibrium with a positive number of projects funded requires that $\Pi^A = \Pi^B \geq E$. In equilibrium, the effective capital requirement for both banks will be $b_I = \min\{b_S, b_R\}$ as profits are increasing in the number of projects. Since $b_S \leq b_U$, and as we have $b_S = 0$, each bank can finance all projects.

This implies that in any possible equilibrium, $R_S = \frac{1}{k}$, since there must be at least one bank that does not finance all safe projects and that has a nonbinding capital constraint. $R_S = \frac{1}{k}$ implies that the marginal profit of financing safe projects is zero regardless of whether a bank faces positive bankruptcy risk or not. Hence, any equilibrium allocation of safe projects between the two banks is possible. A bank that finances only safe projects will earn E.

Suppose that one of the banks (Bank B, say) finances some risky projects in equilibrium and faces positive bankruptcy risk. Suppose that Bank A also finances some risky projects. In any equilibrium, R_R must be large enough to ensure $\Pi^B \geq E$. But then Bank B's profit would strictly increase in the number of risky projects it finances, hence B would attract some risky projects that A finances by undercutting R_R. It follows that if Bank B faces bankruptcy risk, there cannot be an equilibrium where both B and A finance risky projects, and one of the banks will in equilibrium finance all risky projects.

If Bank B finances all risky projects, then Bank A finances only safe projects and earns E. Therefore, if Bank B earns more than E, we would have $\Pi^A < \Pi^B$ and violate the equilibrium conditions: Bank A would undercut B to attract all risky projects. It follows that R_R will be such that both banks earn just E in equilibrium. Hence, $\hat{\Pi}_n^B(2) = E$, which implies $\hat{\Pi}_n^B(2) - \Pi_n^B(1) \leq 0$. ∎