

Portfolio Allocation in Transition Economies

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Portfolio Allocation in Transition Economies

Abstract

Designing an investment strategy in transition economies is a difficult task because stock-markets opened through time, time series are short, and there is little guidance how to obtain expected returns and covariance matrices necessary for mean-variance portfolio allocation. Also, structural breaks are likely to occur. We develop an ad-hoc investment strategy with a flavor of Bayesian learning. An observation is that often an extreme event will herald a new state of the economy. We use this observation to re-initialize learning when unlikely returns materialize. By using a Cornell benchmark, we are able to show the usefulness of our strategy for certain types of re-initializations.

1 Introduction

In this work, we consider the difficulties involved with optimal portfolio choice in transition economies and propose a strategy based on Bayesian learning. This strategy is found to be of value for certain parameters.

The mean-variance framework of Lintner, Sharpe, and Markowitz assumes that investors have a measure of the expected returns and the covariance matrix. From a statistical viewpoint, the difficulty is to estimate these parameters. Estimation techniques may range from a simple constant parameter model to a model with time-varying expected returns and variances. For countries with a long tradition of relatively stable markets, such parameters may be obtained from rather sophisticated models, such as GARCH models or switching regressions.¹ For transition economies, where structural changes occur frequently, estimation of the inputs for a mean-variance model is rather complicated, since they are likely to be very unstable over time. As a consequence, the decision to invest in such countries is rendered difficult because of the great uncertainty about future performance of the stock markets.

Within the context of international portfolio choice, involving only developed economies, Solnik (1993) forecasts future risk premia and shows how a simple investment rule may improve portfolio performance. Another study in this line is by Kandel and Stambaugh (1996) who imbed predictability within a Bayesian framework. Further studies that consider predictability of asset returns are by Pesaran and Timmermann (1995), Kim and Omberg (1996), Brennan, Schwartz, and Lagnado (1997), Campbell and Viceira (1999), or Barberis (2000). In the first part of this paper, we will show that stock markets in transition economies are not predictable. A possible explanation is that structural breaks occur in such markets.

Another strand of the literature assumes Bayesian learning of the parameters. Such extensions may be found in Jorion (1985, 1986), Dumas and Jacquillat (1990), or Harvey and Zhou (1990). Pástor and Stambaugh (2001) show how, within a Bayesian framework, one may learn about multiple structural changes. Their model assumes, however, the availability of long time series. Comon (2000) shows how extreme realizations may affect portfolio allocation under learning. These contributions emphasize the importance of learning about parameters for portfolio allocation. An inherent problem of structural models such as GARCH models, switching regressions, or Bayesian learning, is that long time series are required for the para-

¹Recent complicated models such as by Chernov, Gallant, Ghysels, and Tauchen (2000) show that modeling returns even of a well-known series such as the DJIA is not a trivial matter.

meter estimation. For transition economies, such long time series are not available. Even if they were available, given the frequent structural breaks that occur in transition economies, where each new regime has little to do with the previous one, it may be expected that even a complex model may not be estimable. These observations emphasize the need for a parsimonious technique, developed in this paper, that allows estimation of portfolio parameters even if only very restricted information is available.

Other recent contributions have drawn attention to the fact that parameter uncertainty may directly affect the utility of investors. For instance, Barberis (2000) considers the case where there is predictability of future returns, and, moreover, where parameters are uncertain. See also Klein and Bawa (1976) for an early emphasis on this problem. Chamberlain (2000) shows how parameter and model uncertainty affects the utility of agents. Maenhout (1999) considers the case of an investor who hedges against worse-case mis-specifications of the model. Rather than focusing on these aspects, we investigate, here, what can be achieved within the traditional mean-variance model.

In the following paper, we consider, first, a large set of transition economies in Central and Eastern Europe. These countries include even very small and recently opened markets, such as Croatia or Estonia, for which only very few observations are available. In particular, for such countries, little more data is available than the exchange rate and a stock index. It is, therefore, not even possible to describe the risk premium with macroeconomic variables (such as in Bekaert and Harvey, 1995, or in the more direct way of Solnik, 1993). For this type of economy a model that is operational even if the sample-size is small is of particular value.

In this paper, we first address the issue of what can be learned from 3 to 8 years of history of transition economies. We carefully investigate whether there is some predictability of stock returns for the more evolved transition economies such as Poland, the Czech Republic, or Hungary. We also study the role of the few variables that are available for those countries in terms of predictive power of future risk premia. Our estimations show that risk premia are hardly predictable for such markets. This implies that models such as of Solnik (1993) or of Bekaert and Harvey (1995) cannot be used in the early stages of emergence of a market. Next, we investigate the performance of a portfolio that follows Bayesian updating rules such as by Jorion (1985) and Dumas and Jacquillat (1990). We show that Bayesian learning may be useful to improve portfolio performance, when transition economies are involved.

2 Description of data and a preliminary analysis

2.1 Data and notation

Given our interest in portfolio allocation, the horizon over which the data is sampled is important. Since emerging markets are subject to many shocks, we believe that investors will stick to a weekly rather than to the monthly horizon used in most papers on portfolio allocation involving developed economies.

We use data for stock-market indices, exchange rates, as well as short-term and long-term interest rates. Besides data for the UK and Germany, we use series for ten Central and Eastern European countries. These countries are the Czech Republic, Hungary, Poland, Russia, Slovakia, for which also short-term and long-term interest rates are available. Then there is Croatia, Estonia, Lithuania, Romania and Slovenia for which we could not obtain interest-rates. Essentially, the data covers the period from January 1991 to December 2000. Table 1 reports, for each country the retained label, the name of the stock index, and the date when each series becomes available. This table also provides some information on the availability of the exchange rates as well as interest rates. In our data base, the Hungarian and Polish stock markets became first available. For some countries, we use the one-week interbank interest rate as risk-free rate and the 6-month interbank rate as long-term interest rate. In most emerging markets, interbank rates are the only available market rates.

We define $r_{i,t+1} = \ln(P_{i,t+1}/P_{i,t})$ the weekly stock return of country i , over the period from t to $t + 1$, expressed in local currency. We denote by $s_{ij,t+1} = \ln(S_{ij,t+1}/S_{ij,t})$ the return of the foreign currency, with $S_{ij,t}$ the amount of (foreign) currency of country j that may be obtained for a unit of (local) currency of country i . Let $r_{i,t+1}^S$ and $r_{i,t+1}^L$ denote the short-term and the long-term interest rates of country i , respectively. These rates are weekly, cover the period from t to $t + 1$ and they are known at time t . We also express stock returns in a common currency and since we focus on European stock markets, we consider two reference currencies, the Sterling and German Mark. Thus, the stock return of country i , denominated in the currency of country j , is defined as $r_{i,t}^j = r_{i,t} - s_{ij,t}$. Last, the corresponding excess return is defined as $er_{i,t} = r_{i,t} - r_{i,t}^S$ in local currency and $er_{i,t}^j = r_{i,t}^j - r_{j,t}^S$ in common currency.

2.2 Descriptive statistics

As a first look at the data, we compute univariate summary statistics for stock returns, expressed in Sterling. Table 2 reports univariate moments and the test statistics for normality, serial correlation and heteroskedasticity. We find that mean returns range between -1% a week in Romania and 0.37% in Estonia. This compares with the 0.203% for UK and 0.275% for Germany. Asset volatility of the transition economies is high when compared to the UK and Germany. For instance, the volatility of the Lituanian and Czech markets, which are the least volatile, is nearly twice as high as for the UK or Germany. Six out of the ten East European stock indices are found to be left skewed. This result indicates that crashes are more likely to occur than booms. But, when standard errors are computed with the GMM procedure proposed by Richardson and Smith (1993), most of these skewness coefficients are found to be non-significantly different from 0. Contrary to what is usually found for mature markets, we obtain a positive skewness in Slovakia, Lituania, the Czech Republic, and Slovenia. On these stock markets, the largest increase in return exceeds the largest decrease. These positive outliers may be explained by political events that led to huge inflows of foreign capital. For all stock markets, we also obtain a significant positive excess kurtosis. Thus, stock-return distributions have fatter tails than the normal distribution. Finally, we test for normality, using the Wald statistic (Richardson and Smith, 1993). Under the null, skewness and excess kurtosis are jointly equal to zero. As reported in Table 2, stock returns in transition economies are not normally distributed, whereas normality of returns in the two developed markets over the given sample and frequency cannot be rejected.

We obtain a significant serial correlation in squared returns, as indicated by the Engle test statistics. In most countries, we also find a strong serial correlation in returns, when measured by the usual Ljung-Box test statistic. When this statistic is corrected to account for heteroskedasticity, however, we do not obtain such a strong serial correlation, except for Hungary. Changing the currency referential does not alter the main conclusions drawn using Sterling.

We also computed the correlation matrix between stock returns, expressed in local currency as well as in Sterling. Table 3 reports these correlations. For each pair of stock markets, correlation was computed over the largest sample available. The largest correlations which are indicated in bold for local currency and Sterling denominated currency returns, show that correlation patterns are very similar. An explanation for this similarity is advanced

by Rockinger and Urga (2000) who argue that most Eastern European countries adopted a crawling peg. For this reason, we focus now on the discussion of correlations using Sterling-denominated returns. First, we find a very strong link between the UK and the German stock indices, with a correlation as high as 0.6. Second, more developed stock markets in transition economies (the Czech Republic, Hungary, Poland, Russia, with the exception of Slovakia) are rather strongly interrelated, and they are also more connected with developed markets. This result is amplified when returns are expressed in common currency. Last, less developed markets are generally characterized by lower correlations, with the exception of Croatia. Over the period 1997-2000, the Croatian return has been strongly linked to the Czech, Hungarian, and Polish returns (with a correlation larger than 0.4). Since correlations between old economies and transition economies are, broadly speaking, rather low when compared with correlations across old economies alone, portfolio diversification involving transition economies could be very helpful to reduce portfolio risk.

2.3 Predictability of returns

Next, we address the issue of forecasting stock returns. We use a regression approach similar to the one suggested by Solnik (1993), who uses, however, developed economies. If predictability of returns is found, this could be incorporated in a dynamic mean-variance portfolio allocation. First, we consider domestic regressions: excess returns are denominated in local currency and information variables are the short-term and the long-term rates. We cannot include the dividend yield in the regression for practical reasons.² Therefore, we estimate the following regression

$$er_{i,t+1} = a_0^i + a_1^i er_{i,t} + a_2^i r_{i,t+1}^S + a_3^i r_{i,t+1}^L + a_4^i s_{i,t}^{UK} + \varepsilon_{i,t+1} \quad (1)$$

where $\varepsilon_{i,t+1}$ is a forecast error. All explanatory variables are known at date t .

We also consider these regressions from an asset-allocation perspective by using returns denominated in Sterling. In this case, we introduce in the regression stock returns of the German and the UK market. When domestic interest rates are available, we estimate the regression

$$er_{i,t+1}^{UK} = b_0^i + b_1^i er_{i,t}^{UK} + b_2^i (r_{i,t+1}^S - r_{UK,t+1}^S) + b_3^i (r_{i,t+1}^L - r_{UK,t+1}^L) + b_4^i er_{UK,t} + b_5^i er_{GE,t}^{UK} + \tilde{\varepsilon}_{i,t+1} \quad (2)$$

²Dividend yields are not available for most transition economies. For instance, MSCI publishes dividend yields for the Czech Republic, Hungary, Poland, and Russia from 1995 only.

and when there is no domestic interest rate in the data set we estimate

$$er_{i,t+1}^{UK} = b_0^i + b_1^i er_{i,t}^{UK} + b_2^i r_{UK,t+1}^S + b_3^i r_{UK,t+1}^L + b_4^i er_{UK,t} + b_5^i er_{GE,t}^{UK} + \tilde{\varepsilon}_{i,t+1}. \quad (3)$$

Results of regression (1) are reported in Table 4a, whereas results of regressions (2) and (3) are reported in Table 4b. The main conclusions drawn from these regressions are the following. Local variables (short-term and long-term rates) are useless to forecast stock returns of transition economies. In all cases, the adjusted R^2 from regression (1) is less than 7.8% and most parameters a_1 and a_2 are non significantly different from 0. Second, stock returns of developed countries appear to be helpful to forecast stock returns for some transition economies. The parameter b_3 is significantly positive in some cases. Yet, the R^2 associated to regression (2) remain very low, smaller than 4%. Adjusted R^2 are all less than 2.5%.

It is noteworthy that such a low predictability of stock returns is also found for developed markets. For instance, Campbell (1991) regresses the real return of the NYSE index on the lagged return, the dividend-price ratio, and the 1-month T-bill rate minus its past twelve-month average. Over the period 1927-88, he obtains an R^2 equal to 0.024.

The inability to obtain valuable forecasts of stock returns in transition economies is likely to be related to the bad statistical properties of the series, such as structural breaks. This does not mean that no attempt should be made to deal with this difficulty. Indeed, Kandel and Stambaugh (1996) show that stock returns can seem to be only weakly predictable according to usual statistical measures, such as the R^2 of the regression. Yet, if one considers nonetheless this weak predictability, it can substantially influence the investor's portfolio decision. Since predictability seems to be difficult to capture for transition economies using regression techniques, alternative methods may be relevant. Now, we use an ad-hoc learning procedure with some Bayesian flavor, in order to forecast the first two moments of returns.

3 Bayesian learning

3.1 The model

As shown in the previous section, techniques to forecast returns based on conditioning variables do not seem to work in transition economies. In this section, we outline a technique which aims at capturing the learning of asset-returns intrinsic parameters. This technique should take into account the specificities of transition economics. Among these specificities, we have

the fact that only a very small history of data exists, that the economies were subject to structural changes, that structural changes were likely to occur after a stock market reacted wildly, and that new economies emerged.

The first specificity, the short time series, implies that an investor must have some idea about the fundamental parameters of the economy. As time goes by and new observations become available, the investor will update these priors. This type of observation may be captured within a Bayesian framework. We will now illustrate how Bayesian updating works. To simplify, we assume that the vector of returns $y_t = (er_{1,t}, er_{2,t}, \dots, er_{N,t})'$ is distributed normally:

$$y_t \sim \mathcal{N}(\mu_t, \Sigma_t), \quad t = 1, \dots, T. \quad (4)$$

If μ_t and Σ_t were known, then they could be used in a mean-variance portfolio allocation. In practice, the investor has to learn the actual values of these parameters.³ Bayesian updating assures that μ and Σ follow a certain distribution. Learning about a new observation yields an update of the distribution. Care must be taken that the new distribution remains compatible with the prior. For the normal model (4), we can achieve this compatibility by choosing as conjugate prior distribution an inverted-Wishart distribution for the marginal prior pdf of Σ and a normal distribution for the conditional prior of $\mu|\Sigma$ (see Zellner, 1971, Zellner and Chetty, 1965, or Box and Tiao, 1992):

$$\begin{aligned} \Sigma &\sim \text{inverted-Wishart}(\Lambda_0, \nu_0) \\ \mu|\Sigma &\sim \mathcal{N}(\mu_0, \Sigma/\kappa_0) \end{aligned}$$

where prior parameters are Λ_0 , ν_0 , μ_0 and κ_0 . Hence, Λ_0/ν_0 and μ_0 are prior values for Σ and μ . The parameters ν_0 and κ_0 determine the strength of belief in Λ_0 and μ_0 . The parameter ν_0 is the degree of freedom of the inverted-Wishart distribution. Well-known computations give the following joint posterior density of μ and Σ

$$\begin{aligned} f(\mu, \Sigma | y) &= \frac{1}{(2\pi)^{k/2} \Gamma\left(\frac{\nu_0+T}{2}\right)} \left(\left| \frac{\Sigma}{\kappa_0 + T} \right|^{-\frac{1}{2}} |\Sigma|^{-(\frac{\nu_0+T}{2}-1)} \left| \frac{\tilde{\Lambda}}{2} \right|^{\frac{\nu_0+T}{2}} \right) \\ &\quad \times \exp \left[-\frac{1}{2} \left(\text{tr}(\tilde{\Lambda}\Sigma^{-1}) - (\mu - \tilde{\mu})' \left(\frac{\Sigma}{\kappa_0 + T} \right)^{-1} (\mu - \tilde{\mu}) \right) \right] \end{aligned}$$

³We assume in this study that the mean-variance analysis still holds. In other words, we assume that the investor does not change his objective function to explicitly take into account the randomness of the parameters μ and Σ .

where posterior estimates $\tilde{\mu}$ and $\tilde{\Lambda}$ are given by (see Gelman, Carlin, Stern, and Rubin, 2000, chap. 3):

$$\begin{aligned}\lambda &= \frac{\kappa_0}{\kappa_0 + T}, \\ \tilde{\mu}_T &= \lambda\mu_0 + (1 - \lambda)\bar{y}_T,\end{aligned}\tag{5}$$

$$\tilde{\Lambda}_T = \Lambda_0 + S_T + \lambda T(\bar{y}_T - \mu_0)(\bar{y}_T - \mu_0)',\tag{6}$$

where $\bar{y}_T = \frac{1}{T} \sum_{t=1}^T y_t$ and $S_T = \sum_{t=1}^T (y_t - \bar{y}_T)(y_t - \bar{y}_T)'$. The Bayesian estimate of the covariance matrix is given by $\tilde{\Sigma}_T = \tilde{\Lambda}_T / (\nu_0 + T)$. These estimates are equal, for large T , to those given, for instance, in Brown (1979) or Frost and Savarino (1986). Some authors provided estimates for prior parameters derived from the data (see also Morris, 1983). Using this empirical Bayesian approach, Jorion (1985) describes how to obtain an endogenous value for κ_0 and μ_0 . Frost and Savarino (1986) provide ML estimation techniques for estimating κ_0 and ν_0 .

In the model presented so far, it is assumed that, at time t , one uses all past data to compute the Bayesian estimates of the parameters. It is also possible to consider a version of the Bayesian updating approach, where the learning is sequential, and where one updates the prior with each new observation. In such a case, posterior estimates $\tilde{\mu}_t$ and $\tilde{\Lambda}_t$ are given by $\kappa_t = \kappa_{t-1} + 1$, $\nu_t = \nu_{t-1} + 1$, $\lambda_t = (\kappa_t - 1) / \kappa_t$, $\tilde{\mu}_t = \lambda_t \tilde{\mu}_{t-1} + (1 - \lambda_t) y_t$, and $\tilde{\Lambda}_t = \tilde{\Lambda}_{t-1} + \lambda_t (y_t - \tilde{\mu}_{t-1})(y_t - \tilde{\mu}_{t-1})'$, where $\tilde{\mu}_0$, $\tilde{\Lambda}_0$, κ_0 and ν_0 are prior parameters. The posterior estimate of the covariance matrix is now defined as $\tilde{\Sigma}_t = \tilde{\Lambda}_t / \nu_t$. It is known, e.g. Zellner (1971), that the estimates $\tilde{\mu}_t$ and $\tilde{\Lambda}_t$ obtained by this new approach are equal to those obtained with formulae (5) and (6) for date t .

We now turn to the other specificities of emerging markets that we treat simultaneously. Often a change in the structure of the economy occurs after a large movement of an index. Anecdotal evidence of this observation can be easily provided for transition economies. For instance, when Eltsine replaced Gorbachov, worldwide turbulences could be felt in financial markets. Clearly, Eltsine pursued a different policy than Gorbachov. Inspired by models where a change in structure occurs as a threshold is exceeded, such as in Tong (1993), we will reinitialize our learning model whenever a return is of a magnitude incompatible with a normal distribution. We specify a high quantile, and when the return at time t exceeds this threshold, we start a new learning process. The re-initialization has the advantage that it takes care of the possibility that completely new situations arise.⁴ Given that our final DGP

⁴In many models where learning occurs, it is assumed beforehand that only a given number of states may

will consist of a complex mixture of normal densities, we see that the resulting returns will be naturally non-normal.

Even though we re-initialize the learning procedure when an abnormal return occurs, we propose to re-initialize the learning only for a country where the abnormal event took place, rather than for all countries. This means that we are able to keep useful information, see Stambaugh (1999).

Analogously, when a new economy becomes available, we start learning about the parameters. Given the shortness of the time series, the statistician faces a dilemma in that tools such as switching regressions or GARCH models cannot be estimated. As an alternative, we suggest a rule-of-thumb learning procedure.

We now formalize these ideas. Consider the return of country i at time t . This return should be distributed marginally as a normal distribution with mean $\mu_{i,t}$ and variance $\Sigma_{ii,t}$, the i th element on the diagonal of the covariance matrix Σ_t . Assume that an extreme event occurs at time t on market i . For instance, that $|er_{it}|$ exceeds the 99% threshold of the normal density with mean $\mu_{i,t}$ and variance $\Sigma_{ii,t}$. In that case, we re-initialize the model as will be discussed below. Because, in a re-initialization we discard all the past observations concerning country i , it becomes necessary to perform a more subtle accounting of elements. In particular, weights for the mean vector, (λ_t) , and for the covariance matrix, (ν_t) , may differ from one market to the other. Concerning the weights, the updating of κ , ν , and λ is maintained: $\kappa_{i,t} = \kappa_{i,t-1} + 1$, $\nu_{i,t} = \nu_{i,t-1} + 1$, and $\lambda_{i,t} = \kappa_{i,t-1}/\kappa_{i,t}$. Furthermore, we need to take account of the fact that the weight of the observations of series i is not the same as for country j . This leads us to consider

$$\begin{aligned}\delta_{ij,t} &= \frac{\sqrt{\kappa_{i,t-1} \cdot \kappa_{j,t-1}}}{\sqrt{\kappa_{i,t} \cdot \kappa_{j,t}}}, \\ \tilde{\nu}_{ij,t} &= \sqrt{\nu_{i,t-1} \cdot \nu_{j,t-1}},\end{aligned}$$

so that the updating rules become

$$\begin{aligned}\tilde{\mu}_t &= \lambda_t \odot \tilde{\mu}_{t-1} + (I_{n,1} - \lambda_t) \odot y_t, \\ \tilde{\Lambda}_t &= \tilde{\Lambda}_{t-1} + \delta_t \odot (y_t - \tilde{\mu}_{t-1}) (y_t - \tilde{\mu}_{t-1})', \\ \tilde{\Sigma}_{ij,t} &= \frac{\tilde{\Lambda}_{ij,t}}{\tilde{\nu}_{ij,t}},\end{aligned}$$

occur. This is the case with Hamilton's (1994) switching regression. Models where the space of states may increase is given by Chib (1998). See also Kim and Nelson (1999) for a review of a large selection of models allowing several states. There, a large number of data points is, however, required in the estimation.

where \odot denotes the element by element multiplication of matrices and $I_{n,m}$ is the $(n \times m)$ matrix (possibly degenerated to a row or column vector) of ones.⁵ If an extreme value occurs on market i at time $t - 1$, moments associated with this market are (re-)initialized at time t : $\kappa_{i,t} = \kappa_{i,0}$, $\nu_{i,t} = \nu_{i,0}$, $\tilde{\mu}_{i,t} = \tilde{\mu}_{i,0}$, and $\tilde{\Lambda}_{ij,t} = \tilde{\Lambda}_{ij,0}$. We tried various initializing and re-initializing rules, which are described in the next section.

3.2 Initializing priors

Several methods to re-initialize priors are possible. It is possible to use one hyperprior from which some starting values could be drawn. Since the type of distribution which should be chosen for transition economies is not clear, we decided to use an ad-hoc rule as may be used on a trading floor. More precisely, we suggest that the investor under consideration waits for some time to see how the market evolves. His horizon is supposed to last 3 weeks.⁶ Therefore, $\tilde{\mu}_{i,0}$ and $\tilde{\Sigma}_{ij,0}$ are estimated, in the usual way, as the sample mean and the covariance over three observations. Given the way we construct our prior, we set $\kappa_{i,0} = \nu_{i,0} = 3$. During this learning period, it is assumed that the investor does not put his money in country i . We are aware that this assumption is strong. In particular, it implies that investors, who are rational in our economy, may by his action amplify negative movements. In other words, we place ourselves in a partial equilibrium framework. Indeed, if nobody should invest money for some time, this would naturally bring up the Grossman-Stiglitz no-trading paradox.

When country i experiences an extreme event at time $t - 1$, the mean, the variance, and all covariances have to be re-initialized. Given that the last observation is an extreme event, it is likely not to be valuable, see also Dumas and Jacquillat (1990). To construct estimates of future parameters, therefore, we initialize the mean return as $\tilde{\mu}_{i,0} = \alpha_M \bar{y}_i$, where $\alpha_M \in [0, 1]$ and \bar{y}_i is the sample mean over the last three observations before the crash. We discuss below the role played the α_M .

Concerning the covariance matrix, we use $\tilde{\Sigma}_{ii,0} = \alpha_V s_i^2$, where $\alpha_V \in [0, 1]$ and s_i^2 denotes the sample variance over the last three observations. Last, covariances are set up such that $\tilde{\Sigma}_{ij,0} = \alpha_C \rho_{ij,t-1} \sqrt{\tilde{\Sigma}_{ii,0} \tilde{\Sigma}_{jj,t}}$, where $\rho_{ij,t-1}$ denotes the correlation estimate just before the extreme event. The choice of α_C is quite challenging. On one hand, since an extreme event

⁵For instance, if $A = \{a_{i,j}\}$ and $B = \{b_{i,j}\}$ then $A \odot B = \{a_{i,j} b_{i,j}\}$ with A and B two conformable matrices.

⁶This is the lower bound to obtain a sensible covariance matrix. Although this assumption may appear drastic, Borensztein and Gelas (2000) report massive flows of institutional investors around crises. Notice that our reported results remain quantitatively the same if the time period is extended to several more weeks.

occurred on market i , we are reluctant to set a large parameter α_C , to avoid “contaminating” other stock markets. On the other hand, some empirical evidence obtained with various techniques indicates that correlation tends to increase in period of turbulence, so that stock markets are more related during crashes and booms (Ramchand and Susmel, 1998, Longin and Solnik, 2001). Possible values for α_C are $[-1/\rho_{ij,t-1}, 1/\rho_{ij,t-1}]$, but we typically tried values in the range $[0, 1]$. Finally, $\tilde{\Lambda}_{ij,0}$ is set equal to $\tilde{\nu}_{ij,0}\tilde{\Sigma}_{ij,0}$, with $\tilde{\nu}_{ij,0} = \sqrt{\tilde{\nu}_{i,0}\tilde{\nu}_{j,t}}$.

3.3 Assessment of Bayesian learning

We checked that, without any re-initialization, the last conditional expected excess return ($\tilde{\mu}_T$) and the last covariance matrix ($\tilde{\Sigma}_T$) equal the unconditional excess return (\bar{y}_T) and unconditional covariance matrix (S_T/T), respectively.

In order to assess our Bayesian-learning procedure, we performed several experiments. Table 5 reports some statistics on Bayesian learning. First, we indicate first and second unconditional moments of excess returns.⁷ We then report averages of first and second conditional moments of excess returns associated with various sets of re-initializations $\alpha = \{\alpha_M, \alpha_V, \alpha_C\}$. We also present the number of re-initializations for each stock market. A first result is that the number of re-initializations increases when we decrease the parameter α_V . A low value of α_V is associated with a low value of the variance in case of a re-initialization. This implies that, everything else being equal, a further re-initialization is more likely to occur since the standardized return is more likely to exceed the re-initialization threshold. For instance, when we chose $\alpha = (1, 1, 1)$, the number of re-initializations is 21 in Hungary, 8 in Russia, and 10 in Romania. When we choose $\alpha = (1, 0.5, 1)$, this number is as high as 32, 21 and 17, respectively.

In parallel, the conditional standard deviation also decreases with the parameter α_V . In most emerging markets, the conditional standard deviation is lower than the unconditional standard deviation whatever the re-initialization parameter. Such a result does not hold for the parameter α_M associated to the return re-initialization. The position of the unconditional mean with respect to the conditional mean is strongly related to the skewness of the distribution. Positive skewness indicates that booms are more likely to occur than crashes, so that re-initializing learning is likely to decrease the conditional mean. We observe such a phenom-

⁷The difference of the statistics displayed here and table 2 is that, now, we use excess returns rather than returns.

enon in Slovakia, Lithuania, and Slovenia. In emerging markets, reducing the parameter α_M from 1 to 0 generally leads to a conditional mean that is much closer to the unconditional mean. This translates the fact that extreme returns are not persistent. It is also noteworthy that, for some markets, reducing the parameter α_M implies a decrease in the number of re-initializations, as in the Czech Republic or Hungary.

4 Portfolio allocation under Bayesian learning

4.1 The asset allocation

Now, we use our Bayesian-learning procedure to construct a dynamic portfolio allocation. First, investors forecast the expected return and the covariance matrix, for the period between t and $t + 1$, with their current estimates $\tilde{\mu}_t$ and $\tilde{\Sigma}_t$, respectively. Second, they solve the mean-variance asset-allocation problem:

$$\max_{\{w_t\}} \theta w_t \tilde{\mu}_t - w_t' \tilde{\Sigma}_t w_t, \quad (7)$$

$$w_{j,t} \geq 0, \quad \forall j = 1, \dots, N_t, \quad (8)$$

$$\sum_{j=1}^{N_t} w_{j,t} \leq 1, \quad (9)$$

where w_t denotes the column vector of portfolio weights, chosen at date t for the period $[t; t + 1]$. The parameter θ denotes the coefficient of risk tolerance. Whenever we take a sum involving a varying number of elements, we assume that the ordering of the series is such that j runs over the existing series. We assume that there are no transaction costs. Given that short-selling is not allowed in many countries, we also do not allow it here. For this reason, all weights are constrained to be positive, as in (8). The weights are not assumed to sum to one, since a part of the wealth could be invested in the risk-free asset. In (9), we also impose the constraint that margin purchases are not allowed. Running the mean-variance program using the time-varying mean and covariance matrix yields a time series of asset-allocation weights.⁸ The sum of weighted returns gives a series of cumulative excess returns.

⁸We solve this quadratic optimization problem using the GAUSS QP module.

4.2 The benchmark

As stressed by Solnik (1993), theoretical international asset pricing models do not provide a benchmark portfolio that could be used to gauge alternative investment strategies. The reason for this is that hedging against currency risk requires holding a combination of the domestic risk-free asset and the world market portfolio plus a position in foreign risk-free assets. Therefore, the test of our Bayesian learning cannot be based on a predetermined benchmark. Hence, we apply the approach proposed by Cornell (1979). This is also the approach followed by Dumas and Jacquillat (1990). To understand the intuition of this approach, we assume two types of investors. First, Bayesian investors that use the ad-hoc learning procedure described above to forecast the expected return and the covariance matrix and who invest in assets using the investment rule (7). Second, naive (or uninformed) investors who assume that the return vector and the covariance matrix are not forecastable. Under this assumption, they use unconditional returns and covariance matrix to determine their optimal portfolio. Comparison of the cumulative expected return realized by the two types of investors allows us to gauge the value of the Bayesian technique. If Bayesian learning were worthless, the conditional distribution reduces to the unconditional distribution, and both optimal portfolios should be identical.

We first have to address the measure of unconditional returns and covariance matrix. Cornell (1979) measured the unconditional moments over the sample period preceding period t . Copeland and Mayers (1982) suggested that the whole sample period (including the period posterior to date t) would be better, if the forecasting model is estimated over the whole period. Here, the Bayesian learning only uses past information regarding returns. Solnik (1993), using highly developed economies, argued that biases due to the use of the whole sample are likely to be small and estimated an unconditional mean with the largest data sample.

An assumption behind Cornell's benchmark is stationarity. Clearly, our model is rather at odds with this assumption. For this reason, we will compare both strategies. As a matter of fact, since our sample is rather short, differences between mean returns computed using past data and using the whole sample are likely to be large, at least for some stock markets, with very agitated movements.

Let us proceed with a formal description of our test. To do so, we consider an investor with a Sterling referential using our learning rule. Using the optimal portfolio weights w_t , he

obtains the excess return, R_{t+1}^P , over the period between t and $t + 1$ as

$$R_{t+1}^P = \sum_{j=1}^{N_t} w_{j,t} er_{j,t+1}^{UK}.$$

For further convenience, we introduce the cumulative excess return for the Bayesian learning rule

$$\text{CER}_{t+1}^B = \sum_{s=1}^{t+1} R_s^p. \quad (10)$$

For the naive investors, as in Cornell (1979), we assume that the optimal weights are computed by Bayesian investors, but that the expected returns and the covariance matrix used to determine those weights are ignored. The expected excess return of the portfolio is therefore

$$E_t^{\text{COR}} [R_{t+1}^p] = \sum_{j=1}^{N_t} w_{j,t} \bar{er}_{j,t+1}^{UK} \quad \text{where} \quad \bar{er}_{j,t+1}^{UK} = \frac{1}{t_j} \sum_{s=0}^{t_j-1} er_{j,s}^{UK},$$

and where $\bar{er}_{j,t}^{UK}$ denotes the mean excess return of country j (denominated in Sterling) computed over the sample period up to date $t + 1$. Clearly, t_j represents the number of observations that are available for country j at date $t + 1$.

For the naive investors, as in Copeland and Mayers (1982), expectations are computed with

$$E_t^{\text{CM}} [R_{t+1}^p] = \sum_{j=1}^{N_t} w_{j,t} \bar{er}_{j,T}^{UK} \quad \text{where} \quad \bar{er}_{j,T}^{UK} = \frac{1}{T_j} \sum_{s=0}^{T_j-1} er_{j,s}^{UK},$$

with T_j the number of observations available for country j in the entire sample. From there on, we may define unexpected excess returns for the two benchmarks:

$$u_{t+1}^{\text{COR}} = R_{t+1}^p - E_t^{\text{COR}} [R_{t+1}^p] \quad \text{and} \quad u_{t+1}^{\text{CM}} = R_{t+1}^p - E_t^{\text{CM}} [R_{t+1}^p].$$

Under the null hypothesis that Bayesian learning is worthless, the unexpected return u_{t+1} should be zero. To construct a test of this hypothesis, we also define the uninformed variance of the portfolio return. For Cornell, this is computed using optimal weights w_t and the covariance matrix V_t , computed over the sample period up to date t . For Copeland and Mayers, this is computed using the variance-covariance matrix V , computed over the entire sample. We obtain

$$(\sigma_{t+1}^{\text{COR}})^2 = w_t' V_t w_t, \quad (\sigma_{t+1}^{\text{CM}})^2 = w_t' V w_t.$$

Then, we compute the standardized unexpected excess return and build a t-statistic. These are

$$\tau^{\text{CM}} = \frac{1}{\sqrt{T-1}} \sum_{t=1}^{T-1} \frac{u_{t+1}^{\text{CM}}}{\sigma_{t+1}}, \quad (11)$$

$$\tau^{\text{COR}} = \frac{1}{\sqrt{T-1}} \sum_{t=1}^{T-1} \frac{u_{t+1}^{\text{COR}}}{\sigma_{t+1}}. \quad (12)$$

For the case that the learning strategy does not add knowledge over a naive strategy, the null hypothesis is $\tau^{\text{CM}} = 0$ and $\tau^{\text{COR}} = 0$ can not be rejected. Both statistics are distributed, under the null, as a normal, $\mathcal{N}(0, 1)$.

It is useful to consider how the portfolios worth would have evolved through time. For this reason, we also define cumulative excess returns, CER. These are

$$\text{CER}_t^{\text{COR}} = \sum_{s=1}^t \sum_{j=1}^{N_s} w_{j,s} \overline{er}_{j,s+1}^{UK}, \quad (13)$$

$$\text{CER}_t^{\text{CM}} = \sum_{s=1}^t \sum_{j=1}^{N_s} w_{j,s} \overline{er}_{j,T}^{UK}, \text{ and} \quad (14)$$

$$\text{CER}_t^{\text{B}} = \sum_{s=1}^t \sum_{j=1}^{N_s} w_{j,s} er_{j,s+1}^{UK}, \text{ for } t = 1, \dots, T-1. \quad (15)$$

4.3 Results

In this section, we discuss the results of the implementation of our Bayesian model. In order to implement this model, it is necessary to select a level of the risk tolerance parameter. The choice of this parameter is arbitrary. For monthly data, Chepra and Ziemba (1993) use as a preferred coefficient of risk tolerance $\theta = 50$. They claim that this parameter corresponds to large U.S. pension funds and other institutional investors. Risk tolerances of 25 and 75 would characterize strongly conservative and aggressive investors, respectively.

4.3.1 Unconditional portfolio allocation

As a first case, for various levels of risk tolerance, we consider the allocations, w , obtained by using the mean and the covariance matrix estimated from the entire sample. In Table 6, we present various results for the unconditional framework.

Panel A displays the weights assuming that the weights are restricted to be positive. This means that investors may also invest in the riskless asset. For very conservative investors, i.e. with low θ , we find, as expected, that they invest very small amounts in equity. As risk

tolerance increases, the fraction of wealth invested in the risky assets increases. Interestingly, even for rather high risk tolerances, investors put at most 30% of their wealth in the UK and German indices. This comes from the fact that, during the period considered, stock markets offered a rather low excess return. We observe that no money would have been put in the set of transition economies. This can be explained by the fact that, given the relatively low level of expected returns, the transition economies do not offer sufficient diversification opportunities, as to offset the rather high level of volatility.

Panel B of Table 6 displays portfolio weights under the assumption that the weights sum to one. This means that all the wealth has to be invested into the risky assets. For a coefficient of risk tolerance of $\theta = 50$, we find that an investor would have put 82.8% of his wealth in the UK market and the rest in the German market. Nothing would have been invested in the transition economies.

So far, we considered the consequence on portfolio weights. An alternative question is how much a given strategy would yield in terms of cumulative excess returns (CER_T). To answer this question, we present in Panel C of Table 6, for various levels of risk tolerance, the cumulative excess returns, once for the optimal mean-variance allocation, and once for an equally weighed investment strategy.

As the level of risk tolerance increases, the mean-variance strategy yields a higher level of returns. On the other hand, the risk of the strategy increases. As a consequence, observation of the full sample CER only is misleading. A risk-adjusted measure is given by the Sharpe ratio. When we contemplate this statistic, we obtain a significant increase when we shift from equal weights to optimal weights, but only a marginal increase for higher levels of risk tolerance. We find that the investor who had invested according to mean-variance analysis would have realized a significant benefit over the equal-weight investor.

The results described so far are static. We now turn to investigate the contribution of the Bayesian learning rule.

4.3.2 Conditional portfolio allocation

In Table 7, we follow Cornell, as well as Copeland and Mayers, and present the cumulative excess returns that are required in the performance measurement. We present the full sample cumulative excess return, CER_T^{CM} , CER_T^{COR} and the bayesian one CER_T^B . Then, we present, in the last two columns, the τ^{CM} - and τ^{COR} -statistics (12) respectively (11). The first statistics

compares the ability of the portfolio based on Bayesian learning to outperform a naive strategy based on unconditional moments computed over a full sample (static measure). The second statistic compares the performance of the Bayesian portfolio to the one of a naive strategy based on unconditional moments computed over the sample period preceding the current period (dynamic measure). The two statistics are presented for various levels of risk tolerance and various levels of initialization. Given that the results are quantitatively the same as risk tolerances change, we focus in the discussion on the one for $\theta = 50$. For this level, we find that, whatever the level of initialization, the static measure provides a very small excess return. We find that the level is very different for the dynamic measure. We explain this result by the fact that in transition economies many events occurred that changed significantly the level of the mean returns. Using the Bayesian learning, we obtain very high cumulative excess returns for some of the initializations. We find that our Bayesian learning is significantly better than the static measure for low levels of variance re-initializations. It is marginally better than the dynamic measure.

We turn now to discuss the changes in performance as the initializations change. As we shift α_M from 1 to 0, meaning that we use as prior, for expected returns, 0 rather than a three-week average, our t-statistics drop. This shows that investors should, when they rebalance their portfolios, use past information.

When we compare the initializations for the variance, moving from $\alpha = (\alpha_M, \alpha_V, \alpha_C) = (1, 1, 1)$ to $(1, 0.1, 1)$, or to $(1, 0.05, 1)$, we notice an improvement in the t-statistics. As variance becomes smaller, it means that our learning model will consider more aggressively even moderate returns as trigger values for a re-initialization. This result indicates that careful listening to the market is necessary after a turbulent event occurred, and that, in transition economies, over the sample considered, it may be necessary to restructure the portfolio frequently. It also suggests that realizations that occur right after an extreme event should not be used in the computation of variances.

Last, we turn to the initialization of covariances by comparing the situation $\alpha = (1, 1, 1)$ with $(1, 1, 0.5)$. This means that we downweigh correlation across the markets after a crash. We find a relatively small increase in the t-statistics. Therefore, the impact of correlation changes, for the countries considered, will not be of major importance. This may be explained by the fact that, in emerging markets, changes in correlation are dominated by changes in return and for variance from an asset allocation viewpoint.

In Figure 1, we display the evolution of cumulative excess returns using the static and the dynamic measures as benchmarks and a Bayesian-learning model. The lowest curve represents the cumulative excess returns for an uninformed investor who uses all the sample information to compute averages. The formula has been presented in (14). This corresponds to the benchmark chosen by Solnik (1993). The curve in the middle corresponds to the knowledge assumed by Cornell (1979). The formula for this curve is given by (13). Last, the highest curve corresponds to the actual excess returns realized by the Bayesian strategy (10). The difference between the highest and the two other curves, when conveniently standardized, yields the statistics presented in Table 7.

The initializations correspond to $\alpha = (1, 0.05, 1)$. During the first 100 observations, from 1991 to the beginning of 1993, our informed strategy is comparable with the uninformed ones. Transition economies, namely Hungary and Poland, represented an interesting investment opportunity. The Bayesian learning would have to recognized this performance.

In Figure 2, we display the weights of an investment in the UK and Germany versus the weight of the global investment in all available transition economies, during this period. We notice that one should have invested aggressively in the transition economies during certain periods. Returning to Figure 1, we notice that before mid-1993, only small gains were realized. Figure 2 shows that during this early period wild fluctuations in expected returns occurred, leading to large variations of the investments. In other words, returns were hardly predictable, meaning that no information could be obtained from past returns.

From 1994 on, the dynamic measure remains rather stable, suggesting that the underlying parameters became more stable. Our Bayesian strategy had two periods of higher returns, the first one was due to a higher investment in Hungary in 1994. The second period, 1996-7 involved Hungary, Poland, Russia, and Slovakia. The gain of our strategy is, therefore, not only due to a single country but to a portfolio. We notice that the Bayesian learning rule yielded returns, which are increasing steadily with respect to the naive strategies. This suggests that our results are not driven by outliers, but reflect changes in investment opportunities in transition economies.

5 Conclusion

In this paper, we investigate first, in the spirit of Solnik (1993), whether excess returns in transition economies are predictable. Unfortunately, we find that stock-market indices are not predictable for these countries.⁹ This may be due to the fact that there exists no stable relation because of structural changes in the economies. This finding precludes a portfolio allocation strategy based on some predetermined variables.

In the light of this result, we address the issue what an investor (with some econometric background) could rationally do to implement a dynamic portfolio allocation. The difficulties are numerous when investing in transition economies. First, stock-markets only opened through time. Second, very little is known about these new markets, from the point of view of expected returns and covariance matrices. Third, structural breaks are likely to occur.

To overcome these difficulties, we consider a Bayesian-updating model. Our model is novel insofar that we force a re-initialization of the learning process as returns exceed a certain threshold. In other words, we follow the intuition that, in transition economies, extreme stock-market variations are accompanied by a change in expected returns and the covariance matrix.

We find that our Bayesian-learning model outperforms an equal-weight strategy. When compared with a static naive strategy, our model performs significantly better. When compared with an updated naive strategy, for certain initializations, our model remains better even though only marginally. In this light, we believe that Bayesian techniques may be of value in a portfolio allocation strategy involving transition economies.

⁹At least, not with the variables at hand.

References

- [1] Barberis, N. (2000), Investing for the Long Run When Returns Are Predictable, *Journal of Finance*, 55(1), 225–264.
- [2] Bekaert, G., and Harvey, C. L. (1995), Time-Varying World Market Integration, *Journal of Finance*, 50(2), 403–444.
- [3] Borensztein, E. R. and R. G. Gelos (2000), A Panick-Prone Pack? The Behavior of Emerging Market Mutual Funds, IMF working paper, WP/00/198.
- [4] Box, G. E. P, and G. C. Tiao (1992), *Bayesian Inference in Statistical Analysis*, John Wiley & Sons, New York.
- [5] Brennan, M., E. Schwartz, and R. Lagnado, (1997), Strategic Asset Allocation, *Journal of Economic Dynamics and Control*, 21(8-9), 1377–1403.
- [6] Brown, S. J. (1979), Optimal Portfolio Choice Under Uncertainty: A Bayesian Approach, in Bawa, V. S., S. J. Brown, and R. W. Klein, (eds.) *Estimation Risk and Optimal Portfolio Choice*, North-Holland, Amsterdam.
- [7] Campbell, J. Y. (1991), A Variance Decomposition for Stock Returns, *Economic Journal*, 101(405), 157–179.
- [8] Campbell, J. Y., and L. Viceira (1999), Consumption and Portfolio Decisions When Expected Returns Are Time-Varying, *Quarterly Journal of Economics*, 114(2), 433–495.
- [9] Chamberlain, G. (2000), Econometrics and Decision Theory, *Journal of Econometrics*, 95(2), 255–283.
- [10] Chopra, V. K., and W. T. Ziemba (1993), The Effect of Errors in Means, Variances, and Covariances on Optimal Portfolio Choice, *Journal of Portfolio Management*, 19(2), 6–11.
- [11] Chernov, M., A. R. Gallant, E. Ghysels, and G. Tauchen (2000), Alternative Models for Stock Price Dynamics, mimeo, Columbia University.
- [12] Chib, S. (1998), Estimation and Comparison of Multiple Change-Point Models, *Journal of Econometrics*, 86(2), 327–335.

- [13] Comon, E. (2000), *Extreme Events and the Role of Learning in Financial Markets*, mimeo, Harvard University.
- [14] Copeland, Th., and D. Mayers (1982), The Value Line Enigma (1965-1978): A Case Study of Performance Evaluation Issues, *Journal of Financial Economics*, 10(3), 289–322.
- [15] Cornell, B. (1979), Asymmetric Information and Portfolio Performance Evaluation, *Journal of Financial Economics*, 7(4), 381-390.
- [16] Dumas, B., and B. Jacquillat (1990), Performance of Currency Portfolios Chosen by a Bayesian Technique: 1967-1985, *Journal of Banking and Finance*, 14(2-3), 539–558.
- [17] Frost, P.A., and J. E. Savarino (1986), An Empirical Bayes Approach to Efficient Portfolio Selection, *Journal of Financial and Quantitative Analysis*, 21(3), 293–305.
- [18] Gelman, A., J. B. Carlin, H. S. Stern, and D. B. Rubin (2000), *Bayesian Data Analysis*, Chapman & Hall, London.
- [19] Hamilton, J. D. (1994), *Time Series Analysis*, Princeton University Press, New Jersey.
- [20] Harvey, C. R., and G. Zhou (1990), Bayesian Inference in Asset Pricing Tests, *Journal of Financial Economics*, (26), 221–254.
- [21] Jorion, Ph. (1985), International Portfolio Diversification with Estimation Risk, *Journal of Business*, 58(3), 259-278.
- [22] Jorion, Ph. (1986), Bayes-Stein Estimation for Portfolio Analysis, *Journal of Financial and Quantitative Analysis*, 21(3), 279–292.
- [23] Kandel, S., and R. F. Stambaugh (1996), On the Predictability of Asset Returns: An Asset-Allocation Perspective, *Journal of Finance*, 51(2), 385–424.
- [24] Kim, C. J., and C. R. Nelson (1999), *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*, MIT Press, Boston.
- [25] Kim, T. S., and E. Omberg (1996), Dynamic Nonmyopic Portfolio Behavior, *Review of Financial Studies*, 9(1), 141–161.
- [26] Klein, R. W., and V. S. Bawa (1976), The Effect of Estimation Risk on Optimal Portfolio Choice, *Journal of Financial Economics*, 3(3), 215–231.

- [27] Longin, F., and B. Solnik (2001), Extreme Correlation of International Equity Markets, *Journal of Finance*, 56(2), 649–676.
- [28] Maenhout, P. J. (1999), Robust Portfolio Rules and Asset Pricing, mimeo, Harvard University.
- [29] Morris, C. N. (1983), Parametric Empirical Bayes Inference: Theory and Applications, *Journal of the American Statistical Association*, 78(381), 47–55.
- [30] Pástor, L., and R. F. Stambaugh (2001), The Equity Premium and Structural Breaks, *Journal of Finance*, 56(4), 1207–1245.
- [31] Pesaran, M. H., and A. Timmermann (1995), Predictability of Stock Returns: Robustness and Economic Significance, *Journal of Finance*, 50(4), 1201–1228.
- [32] Ramchand, L., and R. Susmel (1998), Volatility and Cross Correlation across Major Stock Markets, *Journal of Empirical Finance*, 5(4), 397–416.
- [33] Richardson, M., and T. Smith (1993), A Test for Multivariate Normality in Stock Returns, *Journal of Business*, 66(2), 295–321.
- [34] Rockinger M, and G. Urga (2001), A Time-Variability Parameter Model to Test for Predictability and Integration in the Stock Markets of Transition Economies, *Journal of Business and Economic Statistics*, 19(1), 73–84.
- [35] Solnik, B. (1993), The Performance of International Asset Allocation Strategies Using Conditioning Information, *Journal of Empirical Finance*, 1, 33–55.
- [36] Stambaugh, R. F. (1999), Predictive Regressions, *Journal of Financial Economics*, 54(3), 375–421.
- [37] Tong, H. (1993), *Non-Linear Time Series: A Dynamical System Approach*, Oxford University Press, Oxford.
- [38] Zellner, A. (1971), *An Introduction to Bayesian Inference in Econometrics*, John Wiley and Sons, New York.

- [39] Zellner, A., and K. Chetty (1965), Prediction and Decision Problems in Regression Models from the Bayesian Point of View, *Journal of the American Statistical Association*, 60, 608–616.

Captions

Table 1: This table summarizes the names and availability of various financial series for the investigated economies. The short-term interest rate, and the long-term interest rate are given by the one-week, respectively the 6-month interbank rate.

Table 2: The first row indicates the date when a series start. All series end with June 29 2001. *nobs* is the number of observations in each series. Standard errors (std. err.) are computed using the GMM procedure suggested by Richardson and Smith (1993). The Wald statistic tests the null hypothesis that skewness and excess kurtosis are jointly equal to 0. Under the null, the statistic is distributed as a χ^2 with 2 degrees of freedom. $\rho(j)$ represents the j -th order autocorrelation.

Engle(K) represents the Engle-test statistic for heteroskedasticity obtained by regressing squared returns on K lags. Under the null hypothesis of homoskedasticity, this statistic is distributed as a χ^2 with K degrees of freedom. $Q(K)$ represents the Box-Ljung statistics without correction for heteroskedasticity. The statistic with correction for heteroskedasticity is denoted $QW(K)$. Under the null hypothesis of no serial correlation, the statistic is distributed as a χ^2 with K degrees of freedom. At the 95% level, we have the following critical values: $\chi_4^2 : 9.94$, $\chi_8^2 : 15.5$.

Tables 3a, 3b: The correlations are computed using for each pair of stock markets the largest available sample. Correlations larger than 0.2 are presented with bold figures.

Tables 4a, 4b: Here, we investigate whether excess returns are predictable. To do so, we present

$$\begin{aligned} \text{Equation 1: } er_{i,t+1} &= a_0^i + a_1^i er_{i,t} + a_2^i r_{i,t+1}^S + a_3^i r_{i,t+1}^L + a_4^i s_{i,t}^{UK} + \varepsilon_{i,t+1}, \\ \text{Equation 2: } er_{i,t+1}^{UK} &= b_0^i + b_1^i er_{i,t}^{UK} + b_2^i (r_{i,t+1}^S - r_{UK,t+1}^S) \\ &\quad + b_3^i (r_{i,t+1}^L - r_{UK,t+1}^L) + b_4^i er_{UK,t} + b_5^i er_{GE,t}^{UK} + \tilde{\varepsilon}_{i,t+1}, \\ \text{Equation 3: } er_{i,t+1}^{UK} &= b_0^i + b_1^i er_{i,t}^{UK} + b_2^i r_{UK,t+1}^S + b_3^i r_{UK,t+1}^L + b_4^i er_{UK,t} \\ &\quad + b_5^i er_{GE,t}^{UK} + \tilde{\varepsilon}_{i,t+1}, \end{aligned}$$

where the meaning of the variables is described in section 2.1. The numbers under the estimates are t-ratios. Significant t-ratios, at the 10% level, are presented with bold figures.

Table 5: We first display unconditional first and second moments of **excess** returns for various countries. Using our Bayesian-learning procedure we obtain series of conditional

returns $\tilde{\mu}_t$ and covariance matrices $\tilde{\Sigma}_t$. We present averages of these conditional means and associated standard deviations for various sets of $\alpha = (\alpha_M, \alpha_V, \alpha_C)$. The parameters α_M , α_V , and α_C weight, after a re-initialization, the 3-week mean, standard deviation and covariance used in the learning process. We also display how often in a given country learning is re-initialized.

Table 6: Here, we display the weights (fractions of wealth) to be invested in the various indices for various levels of risk tolerance θ , while using alternative portfolio allocation rules. In Panel A, investors may invest in stocks and the UK risk-free asset (we always impose a no-shortsale constraint). In Panel B, investors may only invest in risky assets. In Panel C we compare cumulative excess returns (CER) and Sharpe ratios for several strategies. ‘Equal weights’ corresponds to equal fractions of wealth in each index. ‘Optimal weights (positive)’ corresponds to the weights found in Panel A and ‘Optimal weights (sum to one)’ corresponds to Panel B.

Table 7: Here, we present the cumulative excess return at time T that may have been achieved using Copeland and Mayers (1982), Cornell (1979), or the Bayesian model: CER_T^{CM} , $\text{CER}_T^{\text{COR}}$, CER_T^{B} . We also present the statistics for a test of significance of the Bayesian learning over the Copelands and Mayers or the Cornell benchmarks

$$\begin{aligned}\tau^{\text{CM}} &= \frac{1}{\sqrt{T-1}} \sum_{t=1}^{T-1} \frac{u_{t+1}^{\text{CM}}}{\sigma_{t+1}}, \\ \tau^{\text{COR}} &= \frac{1}{\sqrt{T-1}} \sum_{t=1}^{T-1} \frac{u_{t+1}^{\text{COR}}}{\sigma_{t+1}}.\end{aligned}$$

Figure 1: Here, we display various cumulative excess returns over time. We have

$$\begin{aligned}\text{CER}_t^{\text{COR}} &= \sum_{s=1}^t \sum_{j=1}^{N_s} w_{j,s} \overline{er}_{j,s+1}^{\text{UK}}, \quad \text{CER}_t^{\text{CM}} = \sum_{s=1}^t \sum_{j=1}^{N_s} w_{j,s} \overline{er}_{j,T}^{\text{UK}}, \\ \text{CER}_t^{\text{B}} &= \sum_{s=1}^t \sum_{j=1}^{N_s} w_{j,s} er_{j,s+1}^{\text{UK}}, \quad \text{for } t = 1, \dots, T-1.\end{aligned}$$

Figure 2: Here, we present the aggregated weights invested either in the UK and Germany or in the set of transition economies. Weights are obtained by solving the mean-variance asset allocation problem (7) - (9) for each date t .

Table 1: Date of availability of asset prices

		Stock index		Currency		Short-term rate	Long-term rate
Czech Republic	CZ	PX 50	06/04/94	Koruna	01/01/91	22/04/92	22/04/92
Hungary	HU	BUX	02/01/91	Forint	01/01/91	18/01/93	18/01/93
Poland	PO	Warsaw General Index	16/04/91	Zloty	01/01/91	04/06/93	31/12/93
Russia	RU	RUR	01/09/94	Rouble	11/01/93	01/09/94	01/09/94
Slovakia	SL	SAX16	14/09/93	Koruna	11/01/93	27/04/93	29/12/95
Croatia	CR	Crobex	02/01/97	Kuna	03/06/94	-	-
Estonia	ES	Aripaev index	07/04/95	Kroon	12/10/92	-	-
Lituania	LI	Litin A	29/12/95	Lita	04/10/93	-	-
Romania	RO	BET	19/09/97	Leu	01/01/91	-	-
Slovenia	SV	SBI	03/01/94	Tolar	12/10/92	-	-

Table 2: Summary statistics for stock returns in Sterling

	UK	GE	CZ	HU	PO	RU	SL	CR	ES	LI	RO	SV
beginning date	91/01/01	91/01/01	93/09/14	91/01/08	91/04/16	94/09/06	93/09/14	97/01/07	95/04/11	96/01/02	97/09/23	93/09/14
nobs	522	522	381	521	507	330	381	208	299	261	171	381
mean	0.203	0.275	0.064	0.174	0.315	0.032	-0.113	-0.074	0.378	0.051	-1.009	-0.113
std. err.	0.082	0.102	0.318	0.233	0.344	0.621	0.385	0.393	0.443	0.334	0.540	0.385
standard deviation	2.091	2.803	4.462	4.525	6.693	8.759	4.796	5.427	5.985	4.197	6.721	4.796
std. err.	0.111	0.170	0.421	0.421	0.493	0.739	0.940	0.626	0.762	0.584	0.571	0.940
skewness	-0.210	-0.397	0.593	-0.152	-0.333	-0.386	2.716	-0.205	-1.670	1.653	-0.239	2.716
std. err.	0.256	0.218	0.391	0.575	0.227	0.274	1.255	0.365	0.741	0.828	0.332	1.255
excess kurtosis	1.753	1.645	3.016	6.279	2.925	2.436	22.929	3.228	10.302	11.933	1.797	22.929
std. err.	0.694	0.887	1.232	1.556	0.579	0.704	5.545	1.435	4.027	3.200	0.704	5.545
Wald stat.	6.468	3.565	6.520	16.464	25.684	12.148	19.121	12.124	6.548	14.216	6.523	19.121
p-value	0.039	0.168	0.038	0.000	0.000	0.002	0.000	0.002	0.038	0.001	0.038	0.000
minimum	-9.435	-14.133	-13.800	-26.533	-29.123	-38.242	-21.401	-27.300	-41.652	-18.408	-25.417	-21.401
median	0.291	0.312	-0.115	0.153	0.201	0.233	-0.132	-0.264	0.329	-0.214	-0.898	-0.132
maximum	8.722	8.708	23.987	26.900	25.052	31.586	41.835	19.802	18.548	27.387	20.767	41.835
$\rho(1)$	-0.107	-0.103	0.142	-0.003	0.081	0.115	0.424	0.005	0.109	0.329	0.028	0.424
$\rho(2)$	0.034	-0.029	0.176	0.140	0.040	0.158	0.265	0.055	0.168	0.119	-0.022	0.265
$\rho(3)$	-0.001	0.027	0.103	0.163	0.073	0.094	0.104	0.025	0.091	-0.009	-0.037	0.104
$\rho(4)$	-0.084	-0.076	0.048	-0.048	-0.051	0.007	0.059	-0.057	-0.044	-0.018	0.108	0.059
Engle(1)	0.108	12.567	33.201	13.280	25.947	1.516	51.045	28.483	0.802	19.851	2.181	51.045
p-value	0.742	0.000	0.000	0.000	0.000	0.218	0.000	0.000	0.371	0.000	0.140	0.000
Engle(4)	13.055	27.730	58.805	30.825	46.050	15.188	51.501	28.116	30.645	20.483	3.268	51.501
p-value	0.011	0.000	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.000	0.514	0.000
Q(4)	10.344	9.512	24.621	25.470	8.218	15.696	101.648	1.484	15.305	32.377	2.533	101.648
p-value	0.035	0.049	0.000	0.000	0.084	0.003	0.000	0.829	0.004	0.000	0.639	0.000
Q(8)	18.062	9.884	42.348	42.043	13.282	17.753	107.599	2.496	20.858	34.942	3.564	107.599
p-value	0.021	0.273	0.000	0.000	0.103	0.023	0.000	0.962	0.008	0.000	0.894	0.000
QW(4)	9.036	5.715	5.786	10.946	4.125	6.305	6.420	1.298	7.504	10.344	2.459	6.420
p-value	0.060	0.221	0.216	0.027	0.389	0.177	0.170	0.862	0.112	0.035	0.652	0.170
QW(8)	13.395	6.161	13.703	21.420	8.276	9.179	7.423	1.990	11.895	13.313	5.304	7.423
p-value	0.099	0.629	0.090	0.006	0.407	0.327	0.492	0.981	0.156	0.102	0.725	0.492

Table 3a: Cross-correlation between stock returns in local currency

	UK	GE	CZ	HU	PO	RU	SL	CR	ES	LI	RO
GE	0.652										
CZ	0.245	0.218									
HU	0.391	0.392	0.393								
PO	0.218	0.243	0.306	0.301							
RU	0.306	0.340	0.239	0.277	0.189						
SL	0.031	-0.022	0.149	0.181	0.116	0.073					
CR	0.376	0.344	0.424	0.485	0.511	0.306	0.067				
ES	0.233	0.262	0.238	0.278	0.234	0.322	0.079	0.223			
LI	0.000	0.002	0.125	0.168	0.172	0.083	0.071	0.158	0.187		
RO	0.043	0.149	0.189	0.241	0.214	0.128	-0.150	0.125	0.123	0.098	
SV	0.188	0.204	0.091	0.212	0.072	0.108	0.129	0.349	0.125	0.099	0.231

Table 3b: Cross-correlation between stock returns in Sterling

	UK	GE	CZ	HU	PO	RU	SL	CR	ES	LI	RO
GE	0.625										
CZ	0.246	0.271									
HU	0.407	0.406	0.408								
PO	0.248	0.281	0.353	0.327							
RU	0.394	0.375	0.262	0.393	0.280						
SL	0.068	0.061	0.201	0.231	0.163	0.113					
CR	0.351	0.398	0.478	0.513	0.544	0.317	0.218				
ES	0.221	0.262	0.256	0.276	0.276	0.343	0.146	0.247			
LI	0.061	0.072	0.177	0.205	0.229	0.159	0.136	0.239	0.216		
RO	0.073	0.146	0.203	0.255	0.265	0.203	-0.135	0.146	0.121	0.174	
SV	0.185	0.239	0.117	0.216	0.113	0.133	0.163	0.418	0.148	0.133	0.181

Table 4a: Estimation of excess returns in local currency

	<i>a0</i>	<i>a1</i>	<i>a2</i>	<i>a3</i>	<i>a4</i>		<i>see</i>	<i>R</i> ²	Adj. <i>R</i> ²	DW	<i>T</i>
Equation (1)											
Czech Republic	0.027	0.176	-6.780	6.378	0.090	-	4.137	0.040	0.030	2.026	380
(t-ratio)	0.046	1.866	-1.281	0.959	0.615						
Hungary	-1.701	-0.012	-7.174	12.981	0.042	-	4.708	0.013	0.004	2.001	414
(t-ratio)	-1.841	-0.143	-1.634	2.292	0.261						
Poland	0.082	-0.169	14.305	-14.631	-0.390	-	4.627	0.031	0.018	1.981	289
(t-ratio)	0.062	-2.149	0.812	-0.809	-1.227						
Russia	0.218	-0.071	-0.663	0.785	-0.408	-	9.539	0.089	0.078	1.997	329
(t-ratio)	0.210	-0.630	-0.200	0.271	-2.709						
Slovakia	1.281	0.059	-0.773	-6.275	0.160	-	2.823	0.060	0.049	2.011	334
(t-ratio)	2.873	0.849	-0.490	-2.638	1.474						

Table 4b: Estimation of excess returns in sterling

	<i>b0</i>	<i>b1</i>	<i>b2</i>	<i>b3</i>	<i>b4</i>	<i>b5</i>	<i>see</i>	<i>R</i> ²	Adj. <i>R</i> ²	DW	<i>T</i>
Equation (2)											
Czech Republic	-0.156	0.144	-9.156	11.992	-0.007	-0.093	4.422	0.035	0.022	2.015	380
(t-ratio)	-0.474	1.625	-1.321	1.401	-0.039	-0.737					
Hungary	-1.192	-0.019	-6.341	12.874	-0.168	0.106	4.844	0.018	0.006	1.982	415
(t-ratio)	-1.679	-0.230	-1.512	2.407	-0.925	0.760					
Poland	-0.798	-0.127	1.192	2.144	-0.185	0.210	5.238	0.018	0.000	1.970	289
(t-ratio)	-0.601	-1.806	0.049	0.083	-0.786	1.192					
Russia	-0.290	0.084	-6.743	4.893	0.098	-0.043	8.713	0.032	0.017	2.042	329
(t-ratio)	-0.323	1.094	-2.271	1.885	0.296	-0.190					
Slovakia	0.421	0.005	-0.002	-6.875	-0.102	0.061	3.139	0.039	0.025	2.009	334
(t-ratio)	1.540	0.070	-0.001	-2.686	-0.933	0.778					
Equation (3)											
Croatia	2.660	-0.020	-22.688	-2.561	-0.110	0.149	5.424	0.010	-0.015	2.010	207
(t-ratio)	0.816	-0.203	-0.375	-0.046	-0.377	0.834					
Estonia	5.781	0.115	-48.577	0.808	-0.014	-0.132	5.966	0.028	0.011	2.015	298
(t-ratio)	1.479	2.001	-0.972	0.018	-0.085	-0.772					
Lithuania	1.762	0.324	-43.217	26.151	0.046	-0.046	3.996	0.115	0.098	2.000	260
(t-ratio)	1.082	2.968	-1.300	0.697	0.418	-0.528					
Romania	7.033	0.007	-12.065	-56.400	-0.573	0.198	6.689	0.051	0.022	1.951	170
(t-ratio)	1.636	0.086	-0.174	-0.780	-2.511	1.195					
Slovenia	-0.247	0.022	11.331	-9.495	-0.082	0.022	4.036	0.002	-0.012	2.004	364
(t-ratio)	-0.142	0.324	0.465	-0.365	-0.608	0.217					

Table 5: Statistics on Bayesian learning

$\alpha=(\alpha_M, \alpha_V, \alpha_C)$	UK	GE	CZ	HU	PO	RU	SL	CR	ES	LI	RO	SV
Unconditional moments of excess returns												
mean	0.074	0.146	-0.048	0.045	0.189	-0.083	-0.226	-0.190	0.262	-0.063	-1.124	-0.105
standard deviation	2.092	2.806	4.468	4.531	6.701	8.775	4.804	5.441	5.997	4.206	6.744	4.007
$\alpha=(1,1,1)$												
Conditional moments												
mean	0.165	0.031	-0.234	0.237	-0.299	-0.505	0.553	-0.203	-0.304	0.282	-0.390	0.027
standard deviation	2.405	3.377	4.384	5.665	8.326	8.613	6.320	4.378	5.819	4.043	4.453	4.040
Reinitializations												
number	15	13	12	21	13	8	5	7	10	8	10	12
as a % of sample	0.029	0.025	0.032	0.041	0.026	0.024	0.013	0.034	0.034	0.031	0.060	0.033
$\alpha=(0.5,1,1)$												
Conditional moments												
mean	0.206	0.135	-0.254	0.332	-0.033	-0.475	0.198	-0.059	-0.067	0.029	-0.255	-0.095
standard deviation	2.346	3.238	4.305	5.502	8.010	8.528	6.013	4.376	5.616	3.860	4.408	3.860
Reinitializations												
number	15	11	12	19	12	7	4	6	10	7	10	12
as a % of sample	0.029	0.021	0.032	0.037	0.024	0.021	0.011	0.029	0.034	0.027	0.060	0.033
$\alpha=(0,1,1)$												
Conditional moments												
mean	0.212	0.227	-0.109	0.119	0.367	-0.418	-0.152	0.167	0.235	-0.209	-0.229	-0.145
standard deviation	2.457	3.370	5.196	6.110	8.476	8.861	6.787	4.565	6.322	4.343	4.971	4.167
Reinitializations												
number	12	8	3	15	12	6	5	5	6	6	5	10
as a % of sample	0.023	0.015	0.008	0.029	0.024	0.018	0.013	0.024	0.020	0.023	0.030	0.028
$\alpha=(1,0.5,1)$												
Conditional moments												
mean	0.776	-0.068	-0.886	0.832	-1.676	-2.147	2.191	-0.672	-0.098	0.807	-1.696	-0.633
standard deviation	8.719	11.472	14.915	19.093	29.005	30.244	24.386	14.343	17.880	14.604	14.668	13.342
Reinitializations												
number	21	28	19	32	21	21	7	9	23	11	17	21
as a % of sample	0.040	0.054	0.050	0.062	0.042	0.064	0.019	0.044	0.078	0.043	0.101	0.058
$\alpha=(1,1,0.5)$												
Conditional moments												
mean	0.595	0.267	-0.410	-0.039	0.595	-0.859	-0.534	-0.256	0.614	-0.197	-1.839	-0.469
standard deviation	3.674	5.294	5.941	8.183	10.883	10.201	5.066	9.343	5.796	3.172	6.280	5.376
Reinitializations												
number	246	238	156	200	224	166	180	81	159	152	83	148
as a % of sample	0.474	0.459	0.413	0.386	0.444	0.508	0.476	0.395	0.537	0.589	0.494	0.409
$\alpha=(0.5,0.5,1)$												
Conditional moments												
mean	0.613	0.077	-0.965	0.904	-1.239	-2.044	2.185	-0.829	-1.241	1.110	-1.572	0.081
standard deviation	9.619	13.506	17.536	22.661	33.305	34.454	25.279	17.512	23.275	16.173	17.812	16.160
Reinitializations												
number	15	13	12	21	13	8	5	7	10	8	10	12
as a % of sample	0.029	0.025	0.032	0.041	0.026	0.024	0.013	0.034	0.034	0.031	0.060	0.033

Table 6: Optimal weights computed using unconditional moments (sample: 1994:09-2000:12)

	UK	GE	CZ	HU	PO	RU	SL	SV
Panel A: Optimal weights when weights are only assumed to be positive								
$\theta=25$	0.031	0.067	0.000	0.000	0.000	0.000	0.000	0.000
$\theta=50$	0.062	0.134	0.000	0.000	0.000	0.000	0.000	0.000
$\theta=75$	0.093	0.201	0.000	0.000	0.000	0.000	0.000	0.000
Panel B: Optimal weights when weights sum to one								
$\theta=25$	0.831	0.088	0.000	0.000	0.000	0.000	0.017	0.063
$\theta=50$	0.828	0.172	0.000	0.000	0.000	0.000	0.000	0.000
$\theta=75$	0.766	0.234	0.000	0.000	0.000	0.000	0.000	0.000
Panel C: CER and Sharpe ratio								
	Equal weights		Optimal weights (positive)		Optimal weights (sum to one)			
	CER	Sharpe ratio	CER	Sharpe ratio	CER	Sharpe ratio		
$\theta=25$	-166.936	-0.814	65.092	0.424	13.758	0.742		
$\theta=50$	-166.936	-0.814	108.887	0.672	27.517	0.742		
$\theta=75$	-166.936	-0.814	112.743	0.690	41.275	0.742		

Table 7: Statistics on Bayesian learning - weights are less than or equal to one

$\alpha=(\alpha_M, \alpha_V, \alpha_C)$	Cumulative Excess returns			t-stat	
	Copeland/Mayers CER_T^{CM}	Cornell CER_T^{COR}	Bayesian CER_T^B	τ^{CM}	τ^{COR}
Tol = 25					
$\alpha=(1,1,1)$	0.219	3.875	6.328	1.290	-0.241
$\alpha=(0.5,1,1)$	0.310	3.524	4.487	0.670	-0.438
$\alpha=(0,1,1)$	0.333	2.837	2.892	0.531	-0.582
$\alpha=(1,0.1,1)$	0.069	6.362	17.157	3.326	1.503
$\alpha=(1,0.05,1)$	0.132	5.554	17.423	3.169	1.880
$\alpha=(1,1,0.5)$	0.215	4.036	6.609	1.330	-0.197
Tol = 50					
$\alpha=(1,1,1)$	0.270	5.099	8.147	1.441	-0.275
$\alpha=(0.5,1,1)$	0.485	4.394	6.376	0.852	-0.541
$\alpha=(0,1,1)$	0.628	3.757	4.254	0.618	-0.722
$\alpha=(1,0.1,1)$	0.061	7.510	18.859	3.348	1.341
$\alpha=(1,0.05,1)$	0.159	6.112	18.084	3.222	1.745
$\alpha=(1,1,0.5)$	0.269	5.316	8.356	1.529	-0.244
Tol = 75					
$\alpha=(1,1,1)$	0.257	5.986	9.329	1.553	-0.251
$\alpha=(0.5,1,1)$	0.537	4.980	6.956	0.938	-0.482
$\alpha=(0,1,1)$	0.804	4.190	4.671	0.696	-0.711
$\alpha=(1,0.1,1)$	0.070	7.785	18.872	3.309	1.267
$\alpha=(1,0.05,1)$	0.172	6.281	17.881	3.158	1.623
$\alpha=(1,1,0.5)$	0.256	6.168	9.375	1.616	-0.235

Figure 1: Cumulative Excess Returns using Various strategies

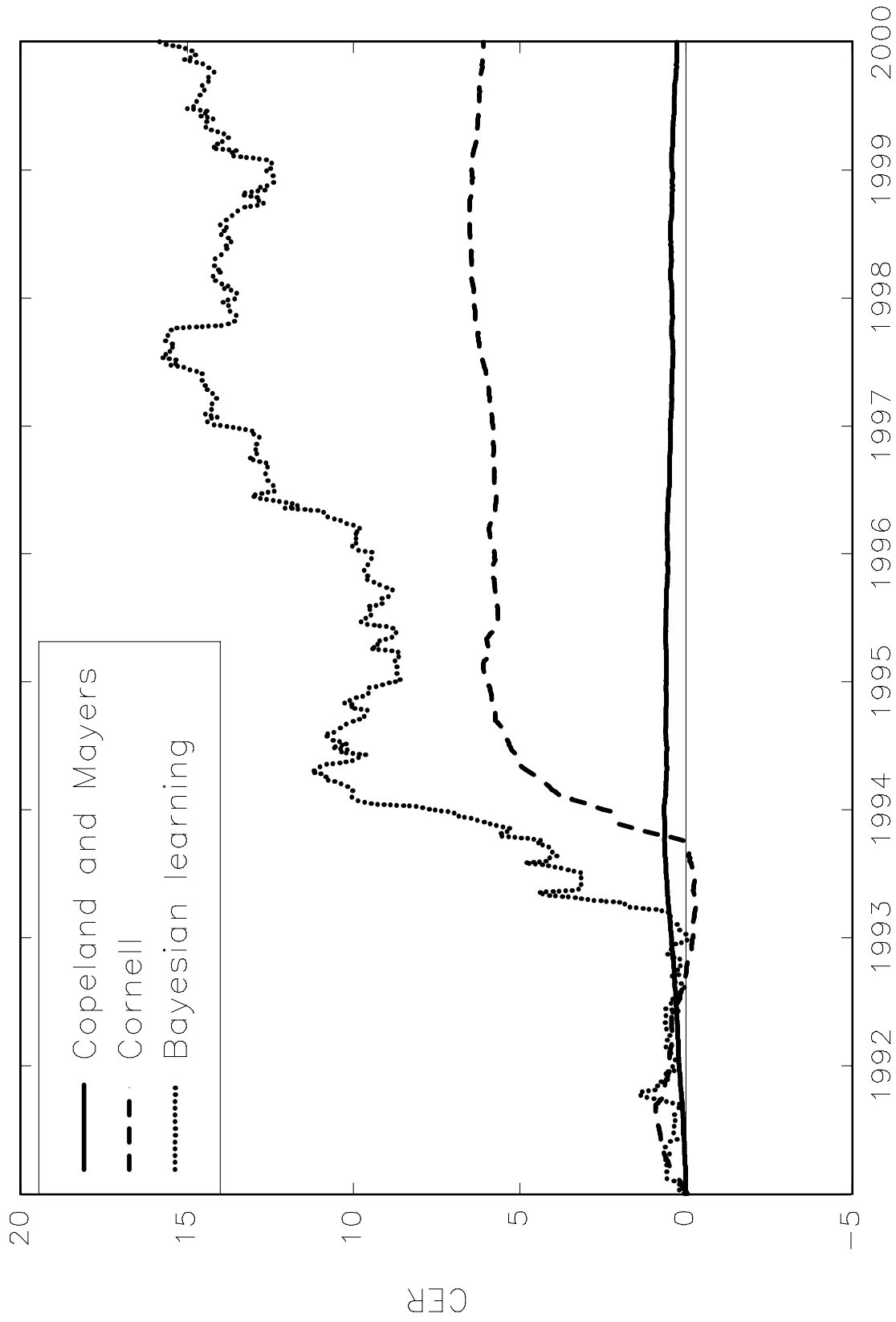


Figure 2: Weight of various areas

