Component-based Structural Equation Modelling

Michel Tenenhaus
HEC School of Management (GREGHEC),
1 rue de la Libération, Jouy-en-Josas, France [tenenhaus@hec.fr]

Abstract

Two complementary schools have come to the fore in the field of Structural Equation Modelling (SEM): covariance-based SEM and component-based SEM. The first approach has been developed around Karl Jöreskog and the second one around Herman Wold under the name "PLS" (*Partial Least Squares*). Hwang and Takane have proposed a new component-based SEM method named *Generalized Structured Component Analysis.* Covariance-based SEM is usually used with an objective of model validation and needs a large sample. Component-based SEM is mainly used for score computation and can be carried out on very small samples. In this research, we will explore the use of ULS-SEM, PLS, GSCA, path analysis on block principal components and path analysis on block scales on customer satisfaction data. Our conclusion is that score computation and bootstrap validation are very insensitive to the choice of the method when the blocks are homogenous.

Key words: Component-based SEM, Covariance-based SEM, GSCA, Path analysis, PLS path modelling, Structural Equation Modelling, Unweighted Least Squares

Introduction

Two complementary schools have come to the fore in the field of Structural Equation Modelling (SEM): covariance-based SEM and component-based SEM.

The first school developed around Karl Jöreskog. It can be considered as a generalisation of path models, principal component analysis and factor analysis to the case of several data tables connected by causal links. Covariance-based SEM is usually used with an objective of model validation and needs a large sample (what is large varies from an author to another: more than 100 subjects and preferably more than 200 subjects are often mentioned). The various methods of estimation used for covariance-based SEM, like Maximum Likelihood (ML) or Unweighted Least Squares (ULS), are full information methods.

The second school developed around Herman Wold under the name "PLS" (*Partial Least Squares*). It is a partial information method. It is a two-step method: (1) latent variables scores are computed using the PLS algorithm and (2) OLS regressions are carried out on the LV scores for estimating the structural equations. More recently Hwang and Takane (2004) have proposed a new full information method optimizing a global criterion and named *Generalized Structured Component Analysis* (GSCA). This second school can be considered as a generalisation of principal component analysis to the case of several data tables connected by causal links. Component-based SEM is mainly used for score computation and can be carried out on very small samples. A research based on 6 subjects has been published in Tenenhaus, Pagès, Ambroisine & Guinot (2005) and more recently another one on 21 subjects in Tenenhaus (2008).

Compare to covariance-based SEM, PLS suffers from several handicaps: (1) the diffusion of path modelling softwares is much more confidential than that of covariance-based SEM softwares, (2) the PLS algorithm is more an heuristic than an algorithm with well known properties and (3) the possibility of imposing value or equality constraints on path coefficients is easily managed in

1

covariance-based SEM and does not exist in PLS. Of course, PLS has also some advantages on covariance-based SEM (that's why PLS exists) and we can list some of them: systematic convergence of the algorithm due to its simplicity, possibility of managing data with a small number of individuals and a large number of variables, practical meaning of the latent variable estimates, general framework for multi-block analysis.

It is often mentioned that PLS is to covariance-based SEM as PCA is to factor analysis. But the situation has seriously changed when Roderick McDonald showed in his 1996 seminal paper that he could easily carry out a PCA with a covariance-based SEM software by using the ULS criterion and constraining the measurement error variances to be equal to zero. Furthermore, the estimation of the latent variables proposed by McDonald is similar to using the PLS mode A and the SEM scheme (i.e. using the "theoretical" latent variables as inner LV estimates). Thus, it became possible to use a covariance-based SEM software to mimic PLS. He concluded from this that he could in fact use the covariance-based SEM approach to obtain results similar to those of the PLS approach, but with a precise optimisation criterion in place of an algorithm with not well known properties.

When each block of variables is essentially uni-dimensional (the first eigenvalue of the correlation matrix is much larger than one and the second one much smaller) and homogenous (all the variables have the same scale, all the correlations are positive and the Cronbach alpha is large) it is a good procedure to summarize each block by the first principal component or more simply by the sum of the block items (scale). For this kind of data, path analysis of these summaries is a natural procedure and yields to the same results as the previous methods. First experiences have already shown that score computation and bootstrap validation are very insensitive to the choice of the method.

In the first section of this paper, it is reminded how to use the ULS criterion for covariance-based SEM and the PLS way of estimating latent variables for mimicking PLS path modelling. This methodology is applied to customer satisfaction data (the ECSI example) in the second section. In the third section PLS, GSCA, path analysis of block first principal components and path analysis of block scales are compared on this example. Listing the pluses and minuses of these various methods finally concludes the paper.

I. Using ULS estimation method for structural equation modelling and McDonald approach for LV estimates

We describe in this section the use of the ULS estimation method applied to the SEM parameter estimates and that of the McDonald estimation method for computing the LV values. In a first part we remind the structural equation model following Bollen (1989). A structural equation model consists of two models: the latent variable model and the measurement model.

The latent variable model

Let η be a column vector consisting of m endogenous (dependent) centred latent variables, and ξ a column vector consisting of k exogenous (independent) centred latent variables. The latent variable model connecting the vector η to the vectors η and ξ is written as

(1) $$\eta = B\eta + \Gamma\xi + \zeta$$

where $\mathbf{B}$ is a zero-diagonal $m \times m$ matrix of regression coefficients, Γ a $m \times k$ matrix of regression coefficients and ζ a centred random vector of dimension m.

The measurement model

Each latent (unobservable) variable is described by a set of manifest (observable) variables. The column vector $\mathbf{y}_j$ of the centred manifest variables linked to the dependent latent variable η_j can be written as a function of η_j through a simple regression with usual hypotheses:

(2) $$\mathbf{y}_j = \boldsymbol{\lambda}_j^y \eta_j + \boldsymbol{\varepsilon}_j$$

The column vector **y**, obtained by concatenation of the $\mathbf{y}_j$'s, is written as

(3) $$\mathbf{y} = \boldsymbol{\Lambda}_\mathbf{y} \boldsymbol{\eta} + \boldsymbol{\varepsilon}$$

where $\boldsymbol{\Lambda}_\mathbf{y} = \overset{m}{\underset{j=1}{\oplus}} \boldsymbol{\lambda}_j^y$ is the direct sum of $\boldsymbol{\lambda}_1^y,...,\boldsymbol{\lambda}_m^y$ and $\boldsymbol{\varepsilon}$ is a column vector obtained by concatenation of the $\boldsymbol{\varepsilon}_j$'s. It may be reminded that the direct sum of a set of matrices $\mathbf{A}_1$, $\mathbf{A}_2$,…, $\mathbf{A}_m$ is a block diagonal matrix in which the blocks of the diagonal are formed by matrices $\mathbf{A}_1$, $\mathbf{A}_2$,…, $\mathbf{A}_m$.

Similarly, the column vector **x** of the centred manifest variables linked to the latent independent variables is written as a function of $\boldsymbol{\xi}$:

(4) $$\mathbf{x} = \boldsymbol{\Lambda}_\mathbf{x} \boldsymbol{\xi} + \boldsymbol{\delta}$$

Adding the usual hypothesis that the matrix **I-B** is non-singular, equation (1) can also be written as:

(5) $$\boldsymbol{\eta} = (\mathbf{I} - \mathbf{B})^{-1}(\boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta})$$

and consequently (3) becomes

(6) $$\mathbf{y} = \boldsymbol{\Lambda}_\mathbf{y}[(\mathbf{I} - \mathbf{B})^{-1}(\boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta})] + \boldsymbol{\varepsilon}$$

Factorisation of the manifest variable covariance matrix

Let $\boldsymbol{\Phi} = Cov(\boldsymbol{\xi}) = E(\boldsymbol{\xi}\boldsymbol{\xi}')$, $\boldsymbol{\psi} = Cov(\boldsymbol{\zeta}) = E(\boldsymbol{\zeta}\boldsymbol{\zeta}')$, $\boldsymbol{\Theta}_\varepsilon = Cov(\boldsymbol{\varepsilon}) = E(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}')$ and $\boldsymbol{\Theta}_\delta = Cov(\boldsymbol{\delta}) = E(\boldsymbol{\delta}\boldsymbol{\delta}')$. Suppose that the random vectors $\boldsymbol{\xi}$, $\boldsymbol{\zeta}$, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\delta}$ are independent of each other and that the covariance matrices $\boldsymbol{\Psi}$, $\boldsymbol{\Theta}_\varepsilon$, $\boldsymbol{\Theta}_\delta$ of the error terms are diagonal. Then, we get:

$$\boldsymbol{\Sigma}_{xx} = \boldsymbol{\Lambda}_\mathbf{x}\boldsymbol{\Phi}\boldsymbol{\Lambda}_\mathbf{x}' + \boldsymbol{\Theta}_\delta,$$

$$\boldsymbol{\Sigma}_{yy} = \boldsymbol{\Lambda}_\mathbf{y}[(\mathbf{I} - \mathbf{B})^{-1}(\boldsymbol{\Gamma}\boldsymbol{\Phi}\boldsymbol{\Gamma}' + \boldsymbol{\Psi})][(\mathbf{I} - \mathbf{B})']^{-1}\boldsymbol{\Lambda}_\mathbf{y}' + \boldsymbol{\Theta}_\varepsilon,$$

$$\boldsymbol{\Sigma}_{xy} = \boldsymbol{\Lambda}_\mathbf{x}\boldsymbol{\Phi}\boldsymbol{\Gamma}'\left[(\mathbf{I} - \mathbf{B})'\right]^{-1}\boldsymbol{\Lambda}_\mathbf{y}'$$

From which we finally obtain:

(7)

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{yy} & \boldsymbol{\Sigma}_{yx} \\ \boldsymbol{\Sigma}_{xy} & \boldsymbol{\Sigma}_{xx} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Lambda}_\mathbf{y}[(\mathbf{I} - \mathbf{B})^{-1}(\boldsymbol{\Gamma}\boldsymbol{\Phi}\boldsymbol{\Gamma}' + \boldsymbol{\Psi})][(\mathbf{I} - \mathbf{B})']^{-1}\boldsymbol{\Lambda}_\mathbf{y}' + \boldsymbol{\Theta}_\varepsilon & \boldsymbol{\Lambda}_\mathbf{y}\left[(\mathbf{I} - \mathbf{B})\right]^{-1}\boldsymbol{\Gamma}\boldsymbol{\Phi}\boldsymbol{\Lambda}_\mathbf{x}' \\ \boldsymbol{\Lambda}_\mathbf{x}\boldsymbol{\Phi}\boldsymbol{\Gamma}'\left[(\mathbf{I} - \mathbf{B})^{-1}\right]'\boldsymbol{\Lambda}_\mathbf{y}' & \boldsymbol{\Lambda}_\mathbf{x}\boldsymbol{\Phi}\boldsymbol{\Lambda}_\mathbf{x}' + \boldsymbol{\Theta}_\delta \end{bmatrix}$$

Let $\theta = \left\{\boldsymbol{\Lambda}_\mathbf{x}, \boldsymbol{\Lambda}_\mathbf{y}, \mathbf{B}, \boldsymbol{\Gamma}, \boldsymbol{\Phi}, \boldsymbol{\Psi}, \boldsymbol{\Theta}_\varepsilon, \boldsymbol{\Theta}_\delta\right\}$ be the set of parameters of the model and $\boldsymbol{\Sigma}(\theta)$ the matrix (7).

Model estimation using the ULS method

Let **S** be the empirical covariance matrix of the MV's. The object of the ULS method is to seek the set of parameters $\hat{\theta} = \{\hat{\mathbf{\Lambda}}_x, \hat{\mathbf{\Lambda}}_y, \hat{\mathbf{B}}, \hat{\mathbf{\Gamma}}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \hat{\mathbf{\Theta}}_\varepsilon, \hat{\mathbf{\Theta}}_\delta\}$ minimizing the criterion

$$(8) \qquad \left\| \mathbf{S} - \mathbf{\Sigma}(\hat{\theta}) \right\|^2$$

The aim is therefore to seek a factorisation of the empirical covariance matrix **S** as a function of the parameters of the structural model. In SEM softwares, the default is to compute the covariance matrix estimates $\hat{\mathbf{\Theta}}_\varepsilon = \widehat{Cov}(\varepsilon)$ and $\hat{\mathbf{\Theta}}_\delta = \widehat{Cov}(\delta)$ of the residual terms in such a way that the diagonal of the reconstruction error matrix $\mathbf{E} = \mathbf{S} - \mathbf{\Sigma}(\hat{\theta})$ is null, even when it yields to negative variance (Heywood case).

Let's denote by $\hat{\sigma}_{ii}$ the i-th term of the diagonal of $\mathbf{\Sigma}(\hat{\mathbf{\Lambda}}_x, \hat{\mathbf{\Lambda}}_y, \hat{\mathbf{B}}, \hat{\mathbf{\Gamma}}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \mathbf{0}, \mathbf{0})$ and by $\hat{\theta}_{ii}$ the i-th term of the diagonal of $\hat{\mathbf{\Theta}}_\varepsilon \oplus \hat{\mathbf{\Theta}}_\delta$. From the formula:

$$(9) \qquad s_{ii} = \hat{\sigma}_{ii} + \hat{\theta}_{ii}$$

we may conclude that $\hat{\sigma}_{ii}$ is the part of the variance s_{ii} of the i-th MV explained by its LV (except in a Heywood case) and $\hat{\theta}_{ii}$ is the estimate of the variance of the measurement error relative to this MV. As all the error terms $e_{ii} = s_{ii} - (\hat{\sigma}_{ii} + \hat{\theta}_{ii})$ are null, this method is not oriented towards the research of parameters explaining the MV variances. It is in fact oriented towards the reconstruction of the covariances between the MV's, variances excluded.

The McDonald approach for parameter estimation

In his 1996 paper, McDonald proposes to estimate the model parameters subject to the constraints that all the $\hat{\theta}_{ii}$ are null. The object is to seek the parameters $\hat{\mathbf{\Lambda}}_x, \hat{\mathbf{\Lambda}}_y, \hat{\mathbf{B}}, \hat{\mathbf{\Gamma}}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}$ minimizing the criterion

$$(10) \qquad \left\| \mathbf{S} - \mathbf{\Sigma}(\hat{\mathbf{\Lambda}}_x, \hat{\mathbf{\Lambda}}_y, \hat{\mathbf{B}}, \hat{\mathbf{\Gamma}}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \mathbf{0}, \mathbf{0}) \right\|^2$$

The estimates of the variances of the residual terms ε and δ are integrated in the diagonal terms of the reconstruction error matrix $\mathbf{E} = \mathbf{S} - \mathbf{\Sigma}(\hat{\mathbf{\Lambda}}_x, \hat{\mathbf{\Lambda}}_y, \hat{\mathbf{B}}, \hat{\mathbf{\Gamma}}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \mathbf{0}, \mathbf{0})$. This method is therefore oriented towards the reconstruction of the full MV covariance matrix, variances included. On a second step, final estimates $\hat{\mathbf{\Theta}}_\varepsilon$ and $\hat{\mathbf{\Theta}}_\delta$ of the variances of the residual terms ε and δ are obtained by using again formula (9).

Goodness of Fit

The quality of the fit can be measured by the *GFI (Goodness of Fit Index)* criterion of Jöreskog & Sorbum, defined by the formula

$$(11) \qquad GFI = 1 - \frac{\left\| \mathbf{S} - \mathbf{\Sigma}(\hat{\mathbf{\Lambda}}_x, \hat{\mathbf{\Lambda}}_y, \hat{\mathbf{B}}, \hat{\mathbf{\Gamma}}, \hat{\mathbf{\Phi}}, \hat{\mathbf{\Psi}}, \hat{\mathbf{\Theta}}_\varepsilon, \hat{\mathbf{\Theta}}_\delta) \right\|^2}{\left\| \mathbf{S} \right\|^2}$$

i.e. the proportion of $\|\mathbf{S}\|^2$ explained by the model. By convention, the model under study is acceptable when the *GFI* is greater than 0.90.

The quantity $\left\|\mathbf{S}-\boldsymbol{\Sigma}(\hat{\boldsymbol{\Lambda}}_{\mathbf{x}},\hat{\boldsymbol{\Lambda}}_{\mathbf{y}},\hat{\mathbf{B}},\hat{\boldsymbol{\Gamma}},\hat{\boldsymbol{\Phi}},\hat{\boldsymbol{\Psi}},\hat{\boldsymbol{\Theta}}_{\varepsilon},\hat{\boldsymbol{\Theta}}_{\delta})\right\|^2$ can be deduced from the FMIN criterion given in the covariance-based SEM software AMOS (Arbuckle, 2005):

$$(12) \qquad FMIN = \frac{1}{2}\left\|\mathbf{S}-\boldsymbol{\Sigma}(\hat{\boldsymbol{\Lambda}}_{\mathbf{x}},\hat{\boldsymbol{\Lambda}}_{\mathbf{y}},\hat{\mathbf{B}},\hat{\boldsymbol{\Gamma}},\hat{\boldsymbol{\Phi}},\hat{\boldsymbol{\Psi}},\hat{\boldsymbol{\Theta}}_{\varepsilon},\hat{\boldsymbol{\Theta}}_{\delta})\right\|^2$$

Using the McDonald approach, the GFI given by AMOS is equal to

$$(13) \qquad GFI = 1 - \frac{\left\|\mathbf{S}-\boldsymbol{\Sigma}(\hat{\boldsymbol{\Lambda}}_{\mathbf{x}},\hat{\boldsymbol{\Lambda}}_{\mathbf{y}},\hat{\mathbf{B}},\hat{\boldsymbol{\Gamma}},\hat{\boldsymbol{\Phi}},\hat{\boldsymbol{\Psi}},\mathbf{0},\mathbf{0})\right\|^2}{\|\mathbf{S}\|^2}$$

The exact GFI computed with formula (11) is also equal to the following:

$$GFI = 1 - \frac{\left\|\mathbf{S}-\boldsymbol{\Sigma}(\hat{\boldsymbol{\Lambda}}_{\mathbf{x}},\hat{\boldsymbol{\Lambda}}_{\mathbf{y}},\hat{\mathbf{B}},\hat{\boldsymbol{\Gamma}},\hat{\boldsymbol{\Phi}},\hat{\boldsymbol{\Psi}},\hat{\boldsymbol{\Theta}}_{\varepsilon},\hat{\boldsymbol{\Theta}}_{\delta})\right\|^2}{\|\mathbf{S}\|^2}$$

$$(14)$$

$$= 1 - \frac{\left\|\mathbf{S}-\boldsymbol{\Sigma}(\hat{\boldsymbol{\Lambda}}_{\mathbf{x}},\hat{\boldsymbol{\Lambda}}_{\mathbf{y}},\hat{\mathbf{B}},\hat{\boldsymbol{\Gamma}},\hat{\boldsymbol{\Phi}},\hat{\boldsymbol{\Psi}},\mathbf{0},\mathbf{0})\right\|^2 - \sum_i \hat{\theta}_{ii}^2}{\|\mathbf{S}\|^2}$$

In practical applications of the McDonald approach, the difference between the GFI given by AMOS (formula (13)) and the exact GFI computed with formula (14) will be small as $\sum_i \hat{\theta}_{ii}^2 / \|\mathbf{S}\|^2$ is usually small. Furthermore, the exact *GFI* will always be larger than the *GFI* given by AMOS.

Evaluation of the latent variables

After having estimated the parameters of the model, we now present the problem of evaluating the latent variables. Three approaches can be distinguished: the traditional SEM approach, the "McDonald" approach, and the "Fornell" approach. As it is usual in the PLS approach, we now designate one manifest variable with the letter *x* and one latent variable with the letter ξ, regardless of whether they are of the dependent or independent type. The total number of latent variables is $n = k + m$ and the number of manifest variables related to the latent variable ξ_j is p_j.

The traditional SEM approach

To construct an estimate $\hat{\xi}_j$ of ξ_j, one proceeds by multiple regression of ξ_j on the whole set of the centred manifest variables $x_{11}-\overline{x}_{11},\dots,x_{np_n}-\overline{x}_{np_n}$. In other words, if one denotes as $\hat{\Sigma}_{xx}$ the implied (i.e. predicted by the structural model) covariance matrix between the manifest variables and as $\hat{\Sigma}_{x\xi_j}$ the vector of the implied covariances between the manifest variables *x* and the latent variable ξ_j, one obtains an expression of $\hat{\xi}_j$ as a function of the whole set of manifest variables:

$$(15) \qquad \hat{\xi}_j = \tilde{X}\hat{\Sigma}_{xx}^{-1}\hat{\Sigma}_{x\xi_j}$$

5

where $\tilde{X} = \left[x_{11} - \bar{x}_{11}, ..., x_{np_n} - \bar{x}_{np_n} \right]$. This method is not really usable, as it is more natural to estimate a latent variable solely as a function of its own manifest variables.

The "McDonald" approach for LV evaluation

Let $x_{j1}, ..., x_{jp_j}$ be the manifest variables relative to the latent variable ξ_j. McDonald (1996) proposes evaluating the latent variable ξ_j with the aid of the formula

$$(16) \qquad \hat{\xi}_j \propto \sum_k w_{jk}(x_{jk} - \bar{x}_{jk})$$

where $w_{jk} = \widehat{Cov}(x_{jk}, \xi_j)$ is the implied covariance between the MV x_{jk} and the LV ξ_j and where $\propto$ means that the left term is the standardized version of the right term.

The regression coefficient λ_{jk} of the latent variable ξ_j in the regression of the manifest variable x_{jk} on the latent variable ξ_j is estimated by

$$(17) \qquad \hat{\lambda}_{jk} = \widehat{Cov}(x_{jk}, \xi_j) / \widehat{Var}(\xi_j)$$

From this, we deduce that formula (16) can also be written as

$$(18) \qquad \hat{\xi}_j \propto \sum_k \hat{\lambda}_{jk}(x_{jk} - \bar{x}_{jk})$$

The McDonald approach thus amounts to estimating the latent variable ξ_j with the aid of the first PLS component computed in the PLS regression of the latent variable ξ_j on the manifest variables $x_{j1}, ..., x_{jp_j}$. This approach could enter into the PLS framework. In the usual PLS approach (Wold (1985), Tenenhaus, Esposito Vinzi, Chatelin & Lauro (2005)), under mode A, the outer weights are obtained by simple regression of each variable x_{jk} on the inner estimate z_j of the latent variable ξ_j. It is necessary to calculate expressly the inner estimate z_j of ξ_j to obtain these weights. Three procedures are proposed in PLS softwares: the centroid, factorial and structural schemes. The covariance-based SEM softwares, on the other hand, give directly the weights (loadings) that for each x_{jk} represent an estimate of the regression coefficient of the "theoretical" latent variable ξ_j in the regression of x_{jk} on ξ_j. Consequently, instead of the regression coefficient of the inner estimate z_j, the estimated regression coefficient of the "theoretical" latent variable ξ_j can be used. We have proposed this procedure for calculating the weights based simply on the outputs of a covariance-based SEM software in Tenenhaus, Esposito Vinzi, Chatelin & Lauro (2005). We called it the "LISREL" scheme and, without knowing it, found the choice of weights proposed by McDonald.

The "Fornell" approach

When all the coefficients $\hat{\lambda}_{jk}$ relative to a latent variable ξ_j have the same sign and the manifest variables are of a comparable order of magnitude, Fornell proposes building up a score taking into account the level of the manifest variables x_{jk}:

$$(19) \qquad \hat{\xi}_j = \sum_k \hat{\lambda}_{jk} x_{jk} / \sum_k \hat{\lambda}_{jk}$$

This approach is standard in customer satisfaction studies.

II.   Use of ULS-SEM on the ECSI data

We return to the ECSI (*European Consumer Satisfaction Index)* example presented in detail in Tenenhaus, Esposito Vinzi, Chatelin & Lauro (2005). The ECSI model is described in Figure 1. The data refer to a mobile phone provider. Table 1 lists the manifest variables associated with each latent variable. Variable h of block j is denoted by x_{jh}. The manifest variables x_{jh} are standardized to make the comparison easier with other methods. It it is a good practice to study in a first step the uni-dimensionality of each block (the first eigenvalue of the correlation matrix is much larger than 1 and the second one much smaller) and, when all the variables have the same scale and all the correlations are positive, its homogeneity (large Cronbach alpha). This information is given in Table 2. We detect two facts: block "customer expectation" is not very homogenous and item 2 of block "loyalty" is not a good representative of its block.
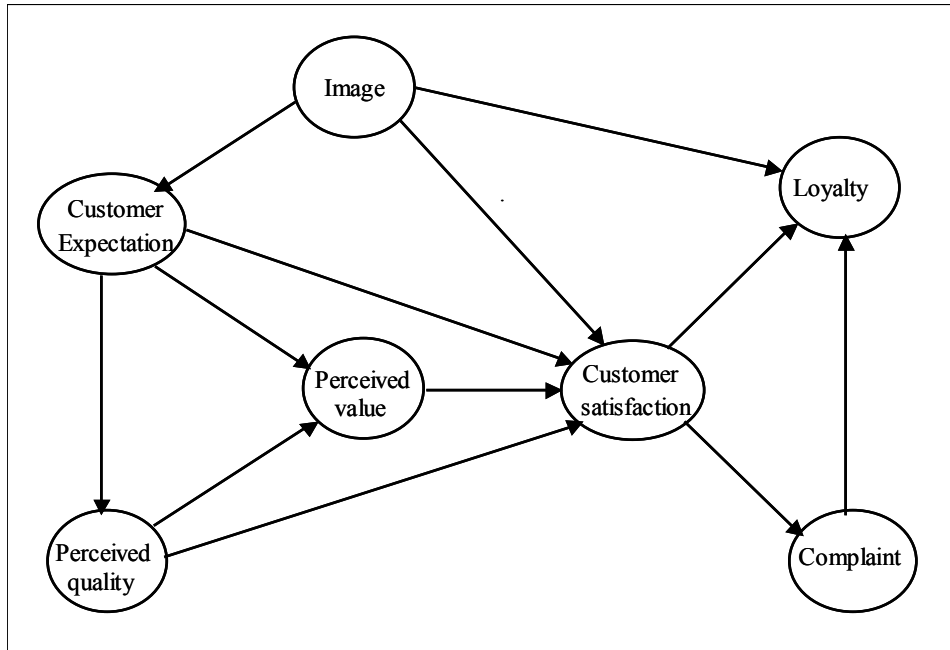

Figure 1: *ECSI model*

We have applied the usual constraint used in causality models to the weights of the manifest variables: for each latent variable, one of the manifest variables has a weight equal to 1. The estimation of the causality model of Figure 1 by minimisation of criterion (8) did not work because a (small) negative error variance was obtained. Therefore we have used the McDonald approach for parameter estimation. Figure 2 shows the results of the estimation of the causality model by minimisation of criterion (10). The outer model parameters normalized according to the Fornell approach are given in Table 3. The confidence intervals of the MV weights were calculated by bootstrapping (*Percentile confidence interval* option in AMOS 6.0). In Table 3, the non significant coefficients are shown in bold italic. LV scores are computed using these weights. For example the "Customer Satisfaction score" is computed as:

$$\text{Customer satisfaction score } = \frac{1 \times x_{51} + 1.02 \times x_{52} + 1.10 \times x_{53}}{1 + 1.02 + 1.10} = .320 \times x_{51} + .327 \times x_{52} + .353 \times x_{53}$$

Table 1: *Manifest variables associated with each latent variable*

All the items are on a scale of 0 to 100. The value 0 corresponds to a very negative point of view of the product, and a value of 100 to a very positive opinion.

Latent variables	Manifest variables
Image (ξ_1)	a) It can be trusted in what it says and does b) It is stable and firmly established c) It has a social contribution for the society d) It is concerned with customers e) It is innovative and forward looking
Customer Expectations of the overall quality (η_1)	a) Expectations for the overall quality of "your mobile phone provider" at the moment you became customer of this provider b) Expectations for "your mobile phone provider" to provide products and services to meet your personal need c) How often did you expect that things could go wrong at "your mobile phone provider"
Perceived Quality (η_2)	a) Overall perceived quality b) Technical quality of the network c) Customer service and personal advice offered d) Quality of the services you use e) Range of services and products offered f) Reliability and accuracy of the products and services provided g) Clarity and transparency of information provided
Perceived Value (η_3)	a) Given the quality of the products and services offered by "your mobile phone provider" how would you rate the fees and prices that you pay for them? b) Given the fees and prices that you pay for "your mobile phone provider" how would you rate the quality of the products and services offered by "your mobile phone provider"?
Customer Satisfaction (η_4)	a) Overall satisfaction b) Fulfilment of expectations c) How well do you think "your mobile phone provider" compares with your ideal mobile phone provider?
Customer Complaints (η_5)	a) You complained about "your mobile phone provider" last year. How well, or poorly, was your most recent complaint handled or b) You did not complain about "your mobile phone provider" last year. Imagine you have to complain to "your mobile phone provider" because of a bad quality of service or product. To what extent do you think that "your mobile phone provider" will care about your complaint?
Customer Loyalty (η_6)	a) If you would need to choose a new mobile phone provider how likely is it that you would choose "your provider" again? b) Let us now suppose that other mobile phone providers decide to lower their fees and prices, but "your mobile phone provider" stays at the same level as today. At which level of difference (in %) would you choose another mobile phone provider? c) If a friend or colleague asks you for advice, how likely is it that you would recommend "your mobile phone provider"?

Table 2: *Block uni-dimensionality and Cronbach alpha on the standardized items*

	Block correlation 1st eigenvalue	Block correlation 2nd eigenvalue	Cronbach alpha
Image	2.394	0.913	0.723
Customer expectation	1.444	0.903	0.452
Perceived quality	4.040	0.771	0.877
Perceived value	1.700	0.300	0.824
Customer satisfaction	2.082	0.518	0.779
Customer loyalty	1.561	0.983	0.472
Customer loyalty (without item 2)	1.542	0.458	0.703

For the inner model (structural equations) we don't use the ULS-SEM results because the obtained parameters are related to theoretical LV's and not to LV scores. Therefore, these parameters are not comparable to the regression coefficients obtained with the other methods. We prefer to use the path model build on the standardized LV scores shown in Figure 3. The parameters of this model have been estimated using the maximum likelihood method available in AMOS 6.0 and are given in Table 4. For each block, we compute the AVE (Average Variance Extracted) given by the formula

$$AVE_j = \frac{1}{p_j} \sum_{h=1}^{p_j} Cor^2(x_{jh}, \hat{\xi}_j)$$

The AVE's are given in Table 5. Then, for each endogenous LV ξ_j, we compute the R-square between $\hat{\xi}_j$ and the other LV's $\hat{\xi}_k$ explaining $\hat{\xi}_j$. These R-square are given in Table 6. Finally the absolute goodness-of-fit (GoF) defined by the formula

$$GoF = \sqrt{\frac{1}{\sum_{j:p_j>1} p_j} \sum_{j:p_j>1} p_j AVE_j} \times \sqrt{\frac{1}{\text{Nb of endogenous LV}} \sum_{\text{Endogenous VL}} R^2(\hat{\xi}_j; \hat{\xi}_k \text{ explaining } \hat{\xi}_j)}$$

is given in Table 7. Using the maximum likelihood method of estimation on the path model constructed on the ULS-SEM LV scores, we obtain the modification indices given in Tables 8 and 9. These results suggest a strong new link from Image to Perceived quality and maybe another one from Perceived value to loyalty.
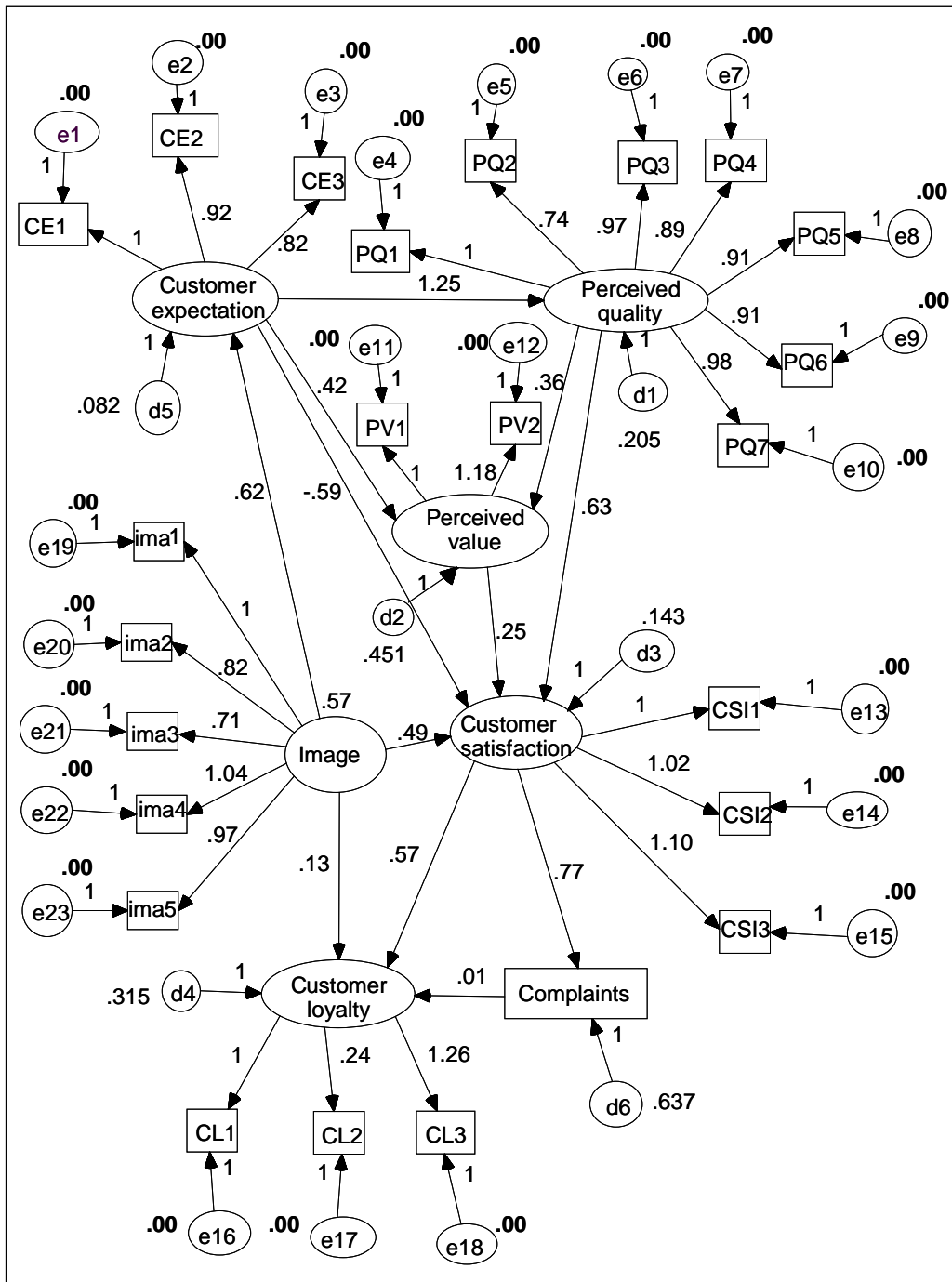
Figure 2 : *Estimation of the ECSI model using ULS-SEM*

Table 3: *Estimation of the outer model parameters* (*Fornell normalization*)
Non-significant (5% level) parameters in bold italic

Outer model

	Parameter	ULS-SEM	PLS	GSCA	PCA	SCALE
x11	←- Image	0.220	0.208	0.218	0.218	.200
x12	←- Image	0.180	0.183	0.169	0.170	.200
x13	←- Image	0.156	0.153	0.174	0.175	.200
x14	←- Image	0.229	0.229	0.225	0.223	.200
x15	←- Image	0.214	0.227	0.214	0.214	.200
x21	←- Customer expectation	0.364	0.363	0.389	0.384	.333
x22	←- Customer expectation	0.336	0.325	0.349	0.347	.333
x23	←- Customer expectation	0.300	0.312	0.262	0.269	.333
x31	←- Perceived quality	0.156	0.163	0.150	0.150	.143
x32	←- Perceived quality	0.116	0.110	0.123	0.122	.143
x33	←- Perceived quality	0.152	0.152	0.145	0.146	.143
x34	←- Perceived quality	0.139	0.136	0.148	0.147	.143
x35	←- Perceived quality	0.142	0.138	0.142	0.143	.143
x36	←- Perceived quality	0.142	0.138	0.147	0.148	.143
x37	←- Perceived quality	0.153	0.164	0.145	0.144	.143
x41	←- Perceived value	0.459	0.448	0.500	0.500	.500
x42	←- Perceived value	0.541	0.551	0.500	0.500	.500
x51	←- Customer satisfaction	0.320	0.314	0.321	0.322	.333
x52	←- Customer satisfaction	0.327	0.318	0.345	0.344	.333
x53	←- Customer satisfaction	0.353	0.368	0.334	0.334	.333
x71	←- Customer loyalty	0.401	0.363	0.446	0.439	.500
x72	←- Customer loyalty	*0.095*	0.106	*0.103*	*0.117*	0
x73	←- Customer loyalty	0.505	0.531	0.451	0.445	.500

Table 4: *Estimation of the inner model parameters* (*Standardized LV scores*)
Non-significant (5% level) parameters in bold italic

Inner model

| Parameter			OLS		ML		
			PLS	GSCA	Path ULS-SEM	Path PCA	Path SCALE
Cust. Expectation.	←-	Image	0.505	.487	.505	.504	.508
Perc. Quality	←-	Cust. Expectation	0.557	.533	.555	.551	.553
Perc. Value	←-	Perc. Quality	0.557	.545	.553	.545	.538
Perc. Value	←-	Cust. Expectation	*0.051*	*.041*	*.052*	*.053*	*.062*
Cust. Satisfaction	←-	Perc. Value	0.192	.173	.190	.186	.187
Cust. Satisfaction	←-	Cust. Expectation	*0.064*	*.076*	*.069*	*.072*	*.076*
Cust. Satisfaction	←-	Image	0.179	.186	.179	.178	.172
Cust. Satisfaction	←-	Perc. Quality	0.513	.506	.509	.511	.513
Complaints	←-	Cust. Satisfaction	0.526	.509	.523	.520	.519
Cust. Loyalty	←-	Cust. Satisfaction	0.483	.463	.476	.456	.464
Cust. Loyalty	←-	Image	0.195	.212	.198	.210	.206
Cust. Loyalty	←-	Complaints	*0.071*	*.044*	*.067*	*.063*	*.054*
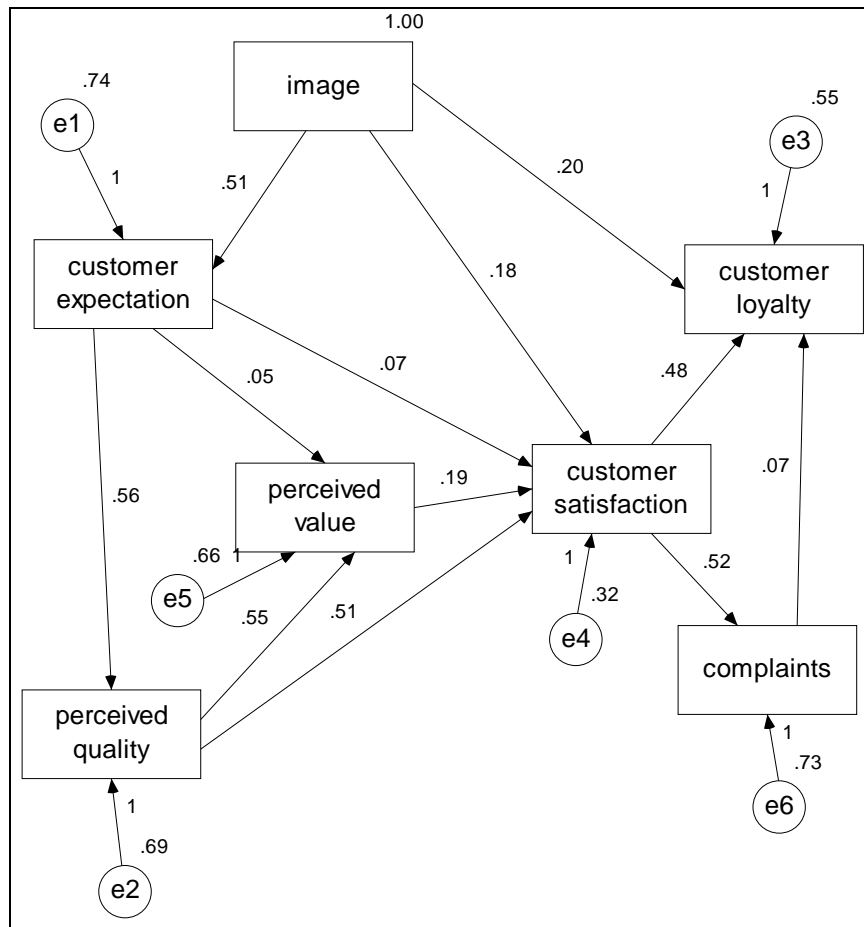
Figure 3 : *Estimation of the ECSI model using Path analysis on ULS-SEM LV scores*

Table 5: *Average Variance Extracted computed on McDonald LV estimates*

	AVE				
	ULS-SEM	**PLS**	**GSCA**	**PCA**	**SCALE**
Image	0.479	0.478	0.479	0.479	0.478
Customer expectation	0.481	0.480	0.481	0.481	0.479
Perceived quality	0.577	0.577	0.577	0.577	0.577
Perceived value	0.849	0.849	0.850	0.850	0.850
Customer satisfaction	0.694	0.693	0.694	0.694	0.694
Complaints	1.000	1.000	1.000	1.000	1.000
Customer loyalty	0.519	0.517	0.520	0.520	0.516
Weighted average AVE (Complaints not included)	0.575	0.574	0.575	0.575	0.574

Table 6: *R Square for structural equation computed on McDonald LV estimates*

	R-Square				
	ULS-SEM	**PLS**	**GSCA**	**PCA**	**SCALE**
Customer expectation	0.255	0.255	0.253	0.254	0.258
Perceived quality	0.308	0.310	0.303	0.304	0.306
Perceived value	0.341	0.345	0.331	0.331	0.330
Customer satisfaction	0.677	0.680	0.673	0.672	0.671
Complaints	0.274	0.277	0.271	0.270	0.270
Customer loyalty	0.447	0.457	0.431	0.431	0.427
Average R-Square	0.384	0.387	0.377	0.377	0.377

Table 7: *GoF for the various models*

	ULS-SEM	PLS	GSCA	PCA	SCALE
GoF	0.470	0.471	0.465	0.465	0.465

Table 8: *Path analysis on standardized ULS-SEM LV scores*
Model FIT and Modification Indices (> 4) for LV structural model

CMIN				172
df				9
CMIN/df				19.111
RMSEA				.270
GFI				.859
MI	Perceived quality	$\leftarrow$	Image	78.5
	Loyalty	$\leftarrow$	Perceived value	6.0

Table 9: *Path analysis on standardized ULS-SEM LV scores*
Revised model with an arrow added from Image to Perceived quality
Model FIT and Modification Indices (> 4) for LV structural model

CMIN				34.8
df				8
CMIN/df				4.354
RMSEA				.116
GFI				.963
MI	Loyalty	$\leftarrow$	Perceived value	5.96

III.   Use of PLS path modelling, GSCA, path analysis of block principal component and path analysis of block scales on the ECSI data

1.   PLS path modelling

The PLS estimation method for structural equation modelling proposed by Wold (1982) and Lohmöller (1989) and also fully described in Tenenhaus, Esposito Vinzi, Chatelin & Lauro (2005) is now used on the ECSI data. We have run XLSTAT-PLSPM (Addinsoft, 2008) on the standardized data, using mode A and centroid scheme. Weights are standardized according to the Fornell approach. Results are given in Tables 3 to 7. They are similar to ULS-SEM followed by path analysis of ULS-SEM LV scores. The only difference between both analyses is that the weight related to item 2 of the loyalty block is not significant in ULS-SEM and considered as significant in PLS. The 95% confidence interval for this weight in PLS is equal to [.012 - .245]. So, PLS fails to detect that this weight is not significant. This result suggests that item 2 of block "loyalty" should have been deleted from its block after inspection of the Cronbach alpha (see Table 2) and consequently not used in the analysis.

2.   Generalized Structured Component Analysis

In 2004, Hwang and Takane have proposed a new method named GSCA (*Generalized Structured Component Analysis*). This is a competing method with PLS path modelling. Its main advantage is that it is based on a global criterion to be minimized. Let's describe the criterion. All the latent variables are denoted as ξ_j. However, endogenous and exogenous latent variables as well as reflective and formative ones are distinguished. A standardized estimate of the latent variable ξ_j is denoted by $\hat{\xi}_j = \mathbf{X}_j \mathbf{w}_j$. The objective of GSCA is to search vectors of weights $\mathbf{w}_j$, $\mathbf{c}_j$ and regression coefficients b_{jk} minimizing the criterion

$$(20) \qquad \sum_{\xi_j \text{ réflective}} \left\| \mathbf{X}_j - \hat{\xi}_j \mathbf{c}_j' \right\|^2 + \sum_{\substack{\xi_j \text{ endogenous,} \\ \xi_k \text{ explaining } \xi_j}} \left\| \hat{\xi}_j - \sum_k b_{jk} \hat{\xi}_k \right\|^2$$

where $\mathbf{c}_j$ is a column vector of weights related to the manifest variables x_{jh} belonging to block j. The first term of criterion (20) corresponds exactly to PCA and the second term to OLS regressions on variables similar to "principal components". GSCA is here a compromise between PCA and OLS regressions. We have used the software program VisualGSCA1.0 (2007) of Heungsun Hwang. This software program is downloadable free of charge from the following site http://www.psych.mcgill.ca/perpg/fac/hwang/VisualGSCA.html. Results are given in Tables 3 to 7. MV weights have been standardized according to the Fornell approach. All the results are similar to ULS-SEM results.

3.   Path analysis of block principal components

If the blocks are good blocks − uni-dimensional and homogenous − we can use path analysis on the standardized first principal components of each block. We have used AMOS 6.0 with the ML method of estimation. MV weights have been standardized according to the Fornell approach. All the results given in Tables 3 to 7 are similar to ULS-SEM results.

_4._   _Path analysis of block scales_

On a good block, there is practically no difference between the standardized first principal component and the "scale" defined here by the standardized sum of the standardized items. Block "loyalty" has been made good by deleting item 2. MV weights have been standardized according to the Fornell approach. All the results given in Tables 3 to 7 are similar to ULS-SEM results.

IV.   Comparison between PLS, GSCA, ULS-SEM path analysis, PCA path analysis and Scale path analysis

We may compare the block components computed with the five methods. We give in Table 10 the correlation matrix for each block. The conclusion is clear: all methods yield to comparable components. The results seem a little less comparable for block "loyalty". But if we compute all the loyalty components without using item 2 from this block, we obtain the same kind of correlation matrix as the other ones. We may conclude two points: (1) when the blocks are good, the computation of the components does not depend upon the method used, and (2) path analysis of the components seems also a very simple and promising approach.

V.   Conclusion

Roderick McDonald has thrown a bridge between the SEM and PLS approaches by making use of three ideas: (1) using the ULS method, (2) setting the variances of the residual terms of the measurement model to 0, and (3) estimating the latent variables by using the loadings of the MV's on their LV's. The McDonald approach has some very promising implications. Using a SEM software such as AMOS 6.0 makes it possible to get back to the analysis of multi-block data and to a "data analysis" approach for SEM completely similar to the PLS approach. However, this approach is limited to reflective blocks. Heungsun Hwang and Yoshio Takane have proposed a full information method named GSCA and based on a global criterion to be minimized. This method can be used for reflective and formative blocks. We have also considered PCA path analysis and Scale path analysis. We have illustrated these five methods with one classical example on customer satisfaction: the ECSI data. On these "good" data, all the methods give practically the same results. On more general data, ULS-SEM, PLS, GSCA will probably still give close results. PCA path analysis will probably give similar results as the previous methods, except when a first principal component is not related to the other blocks. We end this paper with a wish: that all these methods are included in a component-based SEM software. The user would then have access to a very comprehensive toolbox for a "data analysis" approach to structural equation modelling.

Table 10: *Correlation matrices between the LV scores computed with five methods*

Image					
	ULS-SEM	PLS	GSCA	PCA	SCALE
ULS-SEM	1.000				
PLS	1.000	1.000			
GSCA	1.000	.999	1.000		
PCA	1.000	.999	1.000	1.000	
SCALE	.998	.997	.998	.998	1.000

Customer expectation					
	ULS-SEM	PLS	GSCA	PCA	SCALE
ULS-SEM	1.000				
PLS	1.000	1.000			
GSCA	.998	.997	1.000		
PCA	.999	.997	1.000	1.000	
SCALE	.998	.999	.993	.994	1.000

Perceived quality					
	ULS-SEM	PLS	GSCA	PCA	SCALE
ULS-SEM	1.000				
PLS	1.000	1.000			
GSCA	1.000	.999	1.000		
PCA	1.000	.999	1.000	1.000	
SCALE	.999	.999	1.000	1.000	1.000

Perceived value					
	ULS-SEM	PLS	GSCA	PCA	SCALE
ULS-SEM	1.000				
PLS	1.000	1.000			
GSCA	.999	.999	1.000		
PCA	.999	.999	1.000	1.000	
SCALE	.999	.999	1.000	1.000	1.000

Customer satisfaction					
	ULS-SEM	PLS	GSCA	PCA	SCALE
ULS-SEM	1.000				
PLS	1.000	1.000			
GSCA	1.000	.999	1.000		
PCA	1.000	.999	1.000	1.000	
SCALE	1.000	.999	1.000	1.000	1.000

Loyalty					
	ULS-SEM	PLS	GSCA	PCA	SCALE
ULS-SEM	1.000				
PLS	.999	1.000			
GSCA	.998	.995	1.000		
PCA	.998	.995	1.000	1.000	
SCALE	.991	.986	.992	.989	1.000

References

Arbuckle J.L. (2005): *AMOS 6.0.* AMOS Development Corporation, Spring House, PA.

Bollen K. A. (1989): *Structural Equations with Latent Variables,* John Wiley & Sons.

Hwang H. (2007): *VisualGSCA 1.0, A Graphical User Interface Software Program for Generalized Structured Component Analysis,* Dept of Psychology, McGill University.

Hwang H. & Takane Y. (2004) : Generalized structured component analysis, *Psychometrika*, 69, 1, 81-99.

Lohmöller J.-B. (1989): *Latent Variables Path Modeling with Partial Least Squares*, Physica-Verlag, Heildelberg.

McDonald R.P. (1996): Path analysis with composite variables, *Multivariate Behavioral Research*, 31 (2), 239-270.

Noonan R. & Wold H. (1982): PLS path modelling with indirectly observed variables: a comparison of alternative estimates for the latent variable. In: Jöreskog, K.G., Wold, H. (Eds.), *Systems under Indirect Observation*. North-Holland, Amsterdam, pp. 75–94.

Tenenhaus M. (2008): Structural Equation Modelling for small samples. *Working paper* n° 885. HEC Paris, Jouy-en-Josas.

Tenenhaus M., Esposito Vinzi V., Chatelin Y.-M., Lauro C. (2005): PLS path modeling. *Computational Statistics & Data Analysis,* 48, 159-205.

Tenenhaus M., Pagès J., Ambroisine L. & Guinot C. (2005): PLS methodology to study relationships between hedonic judgements and product characteristics, *Food Quality and Preference,* vol. 16, n° 4, pp. 315-325.

XLSTAT (2008): XLSTAT-PLSPM module, XLSTAT software, Addinsoft, Paris.

Wold H. (1982): "Soft Modeling: The Basic Design and Some Extensions", in *Systems under indirect observation, Part 2*, K.G. Jöreskog & H. Wold (Eds), North-Holland, Amsterdam, pp. 1-54.