

Financial innovation and price volatility.[¶]

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Abstract

In a three-period finite competitive exchange economy with incomplete financial markets and retrading, we show the generic existence of financial innovation which decreases equilibrium price volatility (as well as innovation which increases it). The existence is obtained under conditions of sufficient market incompleteness. The financial innovation may consist of an asset which is only traded at time zero, or retraded, and with payoffs only at the terminal date. The existence is shown to be robust in the asset payoff space. *Journal of Economic Literature* Classification Numbers: C60, D52, G10. Keywords: incomplete markets, financial innovation, volatility.

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1. Introduction

This paper examines the robustness of the hypothesis that higher degrees of market incompleteness induce higher equilibrium asset price volatility on financial markets. Market volatility has been the focus of empirical tests of asset pricing models and business cycle theories (the literature is huge and cannot be summarized here; an example is Shiller (1981)). These studies mainly found excess volatility of stock returns, or deviations of asset price volatility from the one determined by "fundamentals". Traditionally the benchmark volatility is derived from a complete-market, infinite-horizon model of consumption and investment. Some authors (cfr. Allen and Gale (1994)) have attributed the excess volatility to the joint presence of liquidity effects and of restrictions to financial markets participation, hence suggesting that a possible misspecification of the general competitive model lies in the assumption that financial markets are perfect. Since market incompleteness is a form of restricted participation, one could ask whether the increase in volatility is a phenomenon that generally extends to incomplete versus complete (or less incomplete) financial markets. Related to these issues, questions have emerged about the effects of financial innovation on asset price stability. Few studies have already specifically examined financial innovation in relation to price volatility from a theoretical viewpoint (Detemple and Selden (1991), for option contracts, and Detemple (1996), Zapatero (1998), for general financial innovation in a general equilibrium setting, with asymmetric information or heterogeneous beliefs), although restrictions have been imposed on preferences or asset payoffs, that is, dividend processes, as asset payoffs are referred to in those models.

In this paper we take advantage of a very simple model of discrete-time dynamic trading of assets and multiple goods in a finite economy to allow for general preferences, endowments and asset structures. In our setup, financial innovation is not the result of optimizing behavior. We do not seek to explain why markets are incomplete or why new assets are introduced, but we take this as given and concentrate on the equilibrium effects of having different financial structures. We impose time and state separability of preferences, and use von Neumann-Morgenstern expected utilities. This is done for the sake of exposition, and because of the wide use of the separable case in the finance and macroeconomic literature. Any weaker version of separability (such as time nonseparability, or a habit formation assumption) also gives rise to the same results as the ones presented in this paper. The model we use covers any finite time horizon trading economy, even though here we focus on the three-period case. Although a new financial asset generally plays a role as a hedging device and as an information vehicle, the result in this paper does not address differential information economies. Rather, we concentrate on the spanning role of financial assets. Future work should introduce asymmetric information in the model to capture the

effects on price volatility of the information conveyed by financial innovation. Also, financial innovation in our model may not complete financial markets.¹

With these maintained general specifications, we show that, generically, there is some financial innovation leading to a volatility decrease, while some other leads to an increase in volatility. The intuition here is, along the lines of what is known in the constrained suboptimality literature (see for example Citanna, Kajii and Villanacci (1998)), that price effects induced by payoff changes may allow modifying asset prices in such a way to control volatilities at pleasure. To get the result, we have to impose some extra conditions on the economy. When uncertainty is represented by \bar{S} states of the world in each contingency, if H is the number of (types of) households in the economy and J is the number of preexisting assets, our theorem holds when $\bar{S}(\bar{S} + J) + \bar{S} + 1 \geq J \geq H + J$: If one allows for more periods, an analogous condition must hold. Generally, similar conclusions can be drawn provided the uncertainty in the economy is sufficiently large. This condition is generally sufficient, and also tight within our differential framework of analysis, in the sense that this condition is required to obtain robust (open sets of new assets have the same effect on open sets of economies, for almost all initial economies) and predictable (locally one-to-one) effects of financial innovation. The exact meaning of this will become apparent after Lemma 4.1.

Hence, other things equal, financial innovation allows the (robust) controllability of price volatility when uncertainty is high, and volatility-reducing financial innovation can be found more easily when there is more uncertainty in the economy. Since more states of nature typically induce more equilibrium volatility in financial markets to start with, the main theorem also shows that the more volatile prices are expected to be, the more controllable volatility is through financial innovation.

The last section of this paper shows that it is easier to reduce volatility through financial innovation when traders cannot rebalance their holdings of the new asset. More precisely, in the case of impossibility of retrading the condition linking the number of states, households and assets is weaker than in the other case. Now, suppose that over-the-counter financial contracts can be designed in a more customized way and retrading of these contracts is more difficult than on standardized contracts exchanged on organized markets. Then it can be argued that in the absence of greater levels of uncertainty, it may be convenient to introduce hedging instruments over the counter as opposed to widely retraded assets, when the intent is to control price volatility of existing financial assets (and in the absence of other effects, such as informational).

We give our results two interpretations. First, and regarding excess volatility and

¹So this paper addresses situations studied by Zapatero (1998), where traders differ in their beliefs, but there is no true informational difference in the sense that a rational expectations equilibrium would require, and not directly the results obtained in Detemple (1996) because we do not have an asymmetric information economy. In the following presentation, we choose to expose the case of heterogeneous preferences and homogeneous beliefs to simplify the exposition, although it is immediate to see that the main gist of our theorem holds even in the presence of heterogeneous beliefs.

market incompleteness, we give conditions under which nothing so general can be deduced as "the more financial markets are incomplete, the higher the volatility". Such statements may hold only in specific incomplete, or otherwise restricted, markets (as in Zapatero, for the first, and in Allen and Gale for the second case, e.g.). In fact, our results suggest that when there is considerable uncertainty in the economy, more incompleteness may be associated to lower volatility. Second, and regarding statements on the destabilizing effects of financial innovation, we remark that any strong conclusion against financial innovation strictly depends on the specific parametric assumptions. A small deviation from those assumptions may moderate the results (this is a direct consequence of genericity in endowments, security payoffs and utilities). We elaborate on a well-known example of preferences for which financial innovation always yields no change in volatility, motivating the perturbation of utilities as a necessary condition for robustness, and also showing that controlling volatility through financial innovation is not an obvious task even if we are allowed to introduce any asset we please.

From a normative viewpoint, our robust controllability result may suggest that financial innovation can be used as an instrument of volatility control, perhaps but not only by the monetary authority. This paper of course does not address the informational requirements needed for the implementation of this policy instrument, a topic linked to the recoverability literature in incomplete markets. Moreover, we can say at least that the new financial asset may not necessarily take the form of a futures contract or of a call or put option on the underlying asset. In this framework, an option is characterized by a fixed functional form and by only one extra parameter, the strike price. This seems not sufficient to guarantee robustness of volatility-reducing effects.²

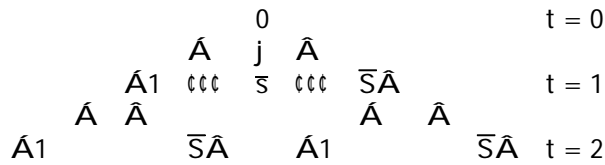
Technically, this paper extends to a multiperiod setting the differential framework developed in Cass and Citanna (1998) to study financial innovation in incomplete markets, itself a ramification of the long-debated issue of constrained suboptimality (see Citanna, Kajii and Villanacci (1998)). We believe that the study of the effects of financial innovation cannot be reduced to welfare comparisons, already addressed in the literature (see Cass and Citanna, (1998), or Elul, (1995), for example). We study the effects of innovation on price volatility, which cannot be defined in the standard two-period exchange economy. Moreover, the restrictions that naturally arise on the payoff matrix representing financial markets with dynamic trading are not encompassed by the previous theorems, and provide the structural motivation to this work. Finally, in the volatility context we can extend the controllability result to compare incomplete and (dynamically) complete markets, impossible in the case of welfare analysis. The analysis of equilibrium volatility is meant to be illustrative of more general issues whose study can be easily embedded in this framework, provided they can be represented by a smooth function defined over the equilibrium set.³

²In this sense, our results do not directly address the robustness of the work by Detemple and Selden (1991) on option contracts.

³An example of which is the study of the robustness of the differences in the price level of one asset depending on markets being complete or not, also known as the 'precautionary savings' effect.

2. The model

We consider a standard model of an intertemporal, competitive, pure-exchange economy with incomplete financial markets. Let t denote the time period, with $t = 0; 1; \dots; T$; where $t = 0$ is today, and $t = T$ is the terminal date. Although the formalization encompasses any finite-horizon economy, we will focus on the three-period case, i.e., $T = 2$. Uncertainty is represented by $1 < \bar{S} < 1$ states of the world in each period $t > 0$ and at each spot, or realization of previous uncertainty, indexed by \bar{s} : Although this is not strictly necessary, to simplify the notation we take the number of states to be constant over time and at each spot. The following tree structure represents uncertainty in this economy,



The total number of states in the economy is therefore given by $\prod_{t=0}^{T-1} \bar{S}^t$: In this paper, we assume that all the information in the economy is publicly available. We will assume that J financial instruments are tradable today and that $\bar{S} > J$; so financial markets are incomplete, even dynamically. These instruments are long-term securities, since they can be held until the terminal date T . Nevertheless, they can be retraded in any period $t < T$: It is notationally convenient also to represent the retraded instruments as independent assets i ; where $i = 0; \dots; I$; and $I = J \prod_{t=0}^{T-1} \bar{S}^t$. We will also index states in different periods all together as spots s ; and will write $S_{t+1} = \prod_{t=0}^t \bar{S}^t$:

There are H households (also referred to as traders) indexed by h : At each date and state, there are C commodities or goods indexed by c , with $C > 1$ ⁴. The commodity (and endowment) space is taken to be \mathbb{R}_{++}^G , where $G = C(S+1)$. A typical household's preferences are represented by the utility function $u_h : \mathbb{R}_{++}^G \rightarrow \mathbb{R}$, which is assumed to be smooth, differentially strictly increasing and differentially strictly concave, and to have the closure of indifference surfaces contained in \mathbb{R}_{++}^G . Moreover, the utility will be assumed of the form

$$u_h(x_h) = \sum_{s=0}^{\infty} \frac{1}{4^s} v_h(x_h^s), \quad (2.1)$$

with $\frac{1}{4^s} > 0$ and $\sum_{s=0}^{\infty} \frac{1}{4^s} = 1 + \bar{S}$. That is, we consider von Neumann-Morgenstern preferences, with objective probabilities and time separable utility. The case of non-separability (time and state) is easier to deal with, and follows from the proofs given

See Elul (1997), whose robustness conditions can be simplified using our framework.

⁴Contrary to Cass and Citanna (1998), robustness can be shown here also in the case when $C = 1$; but the equations considered are slightly different, and we do not give the computational details in this paper.

below. We choose to present the results with this specification because it is the most widely used, although maybe not the most economically plausible. As for using objective probabilities, again this choice derives from the need of comparison between our statements and those made in the related literature, and to simplify computations already burdensome. The conditions of Theorem 4.2 may be slightly altered by a subjective probability specification (due to the need of keeping track of volatility as perceived by each household, and therefore requiring a higher degree of market incompleteness for the theorem to hold), but the general framework of analysis does not change. Note that one can interpret $\frac{1}{2} \pi^s$ as derived from a (stationary) probability measure π on the states $s > 0$ in the following standard way:

$$\frac{1}{2} \pi^s = \begin{cases} \frac{1}{2} \pi(s) & \text{if } 0 < s \leq \bar{s} \\ \pi(s|\bar{s}^0) \pi(\bar{s}^0) & \text{if } s = 1 + \bar{s}^0 \bar{s} + \bar{s} \end{cases}$$

where $\pi(s|\bar{s}^0)$ is the conditional probability of state s in period $t = 2$ after state \bar{s}^0 occurs in period $t = 1$; and $\bar{s}^0 \in \bar{S}$. $\frac{1}{2} \pi^0$ is interpreted as a simple intertemporal preference parameter, not as a probability.

The space of households' endowments is $E = (\mathbb{R}_{++}^G)^H$. The space of households' utility functions is $U = U^H$, where U is a subset of the $C^3(\mathbb{R}_{++}^G; \mathbb{R})$ mappings⁵, endowed with the subspace topology induced by the compact-open topology assigned to the whole space. With $x_h \in \mathbb{R}_{++}^G$, $b_h \in \mathbb{R}^I$, $p \in \mathbb{R}^G$ and $q \in \mathbb{R}^I$ we denote the consumption bundle and the asset portfolio for household h , the commodity price vector and the asset price vector, respectively. It will be convenient to take quantity vectors as columns, and price vectors as rows.

The financial structure is represented by an $(S + 1) \times I$ -dimensional matrix of prices and payoffs R expressed in terms of a numeraire commodity, which we take to be the last at each spot s , i.e., $c = C$. It is apparent that we are dealing here with a special case of the usual standard incomplete market model, where the matrix R assumes the following form

$$R = \begin{matrix} & \begin{matrix} 2 \\ \vdots \\ \bar{s} \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} & & & & \begin{matrix} 3 \\ \vdots \\ \bar{s} \end{matrix} \\ \begin{matrix} i \\ \vdots \\ i \end{matrix} & \begin{matrix} q^0 \\ y^1(0) + q^1 \\ y^2(0) + q^2 \\ \vdots \\ y^{\bar{s}}(0) + q^{\bar{s}} \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 0 \\ i \\ 0 \\ 0 \\ 0 \\ Y(1) \\ Y(2) \\ \vdots \\ Y(\bar{S}) \end{matrix} & \begin{matrix} 0 \\ 0 \\ i \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ \ddots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ i \\ 0 \\ 0 \\ \vdots \\ 0 \end{matrix} \end{matrix}$$

where

⁵More precisely, we need the functions to be three times continuously differentiable locally around an equilibrium, although we consider them in the C^2 topology for our genericity statements.

$$a := \begin{pmatrix} p^{0;C} & 0 & \dots & 0 \\ 0 & \ddots & 0 & 0 \\ \vdots & 0 & p^{S;C} & 0 \end{pmatrix}$$

is an $S + 1$ -dimensional square matrix of prices of the numeraire commodity, and $Y(s)$; with $s = 0; 1; \dots; \bar{S}$ is an $\bar{S} \in J$ matrix of payoffs for the traded security. We will assume that each $Y(s)$ be in general position⁶. The argument of this paper adapts to the one-asset case ($J = 1$) with this condition becoming $Y(s) = Y \cdot \mathbf{1}$; which is consistent with intertemporal models of stock and bond trading. Denote by $\mathbb{R}^{\bar{S} \times J}$ the space of such matrices.

Note that an asset $j \in J$ in this economy promises to deliver $y^{s;j}(0)$ units of the numeraire good in state s in period $t = 1$, and $y^{s;j}(s)$ units of the numeraire in state s in period $t = 2$ after s occurred in period $t = 1$, with $s; s \in \{1; \dots; \bar{S}\}$. Hence $y^{s;j}(s) = (y^{s;j}(s))_{j \in J}$; is the s -th row of $Y(s)$; for $s = 0; 1; \dots; \bar{S}$.

Here q^s is a $J \in \mathbb{R}^+$ vector of prices, which are endogenously determined in equilibrium, and which correspond to 'different' assets as a function of the state. So, each asset $j \in J$ is exchanged at price $q^{0;j}$ at $t = 0$; and at price $q^{s;j}$ in state s and period $t = 1$; and $q^s = (q^{s;j})_{j \in J}$; for $s = 0; 1; \dots; \bar{S}$.

Finally, we denote by $b_h^{0;j}$ trader h 's holdings of the j -th asset at time zero; and by $b_h^{s;j}$ trader h 's holdings of the same asset at time $t = 1$ in state s . However, in what follows the natural identification of asset i with a pair $(s; j)$ for $s = 0; 1; \dots; \bar{S}$ will be used, and each asset j will generate $\bar{S} + 1$ 'different' assets, where b_h^i denotes the holdings of asset i for trader h .

We will parametrize each economy as an element of the pair $E \in U$, endowed with the product topology, with securities $Y(s)_{s=0}^{\bar{S}} \in \mathbb{R}^{\bar{S} \times J}$. We will later use the notation

$$y(s) = (y^{s;j}(0))_{j \in J}; \text{ for } s = 0; 1; \dots; \bar{S}; \text{ and}$$

$$y = [y(0); y(1); \dots; y(\bar{S})]$$

for the payoff vector of a newly introduced asset; an element of $\mathbb{R}^{\bar{S}}$.

This framework of analysis of dynamic trading in financial markets is common, apart from slight differences in the timing of trading or payoff payments, to several treatments of sequential trading in rational expectations models, and in particular to a model by Duane and Shafer (1986). It suffices to note at this stage that the economy is a dynamic one in the usual sense that trading occurs sequentially, but plans are made once at time $t = 0$. Indeed, in the first period ($t = 0$), households maximize utility given rational expectations about future prices, and the existing

⁶For a definition of general position, see Mas-Colell (1985, p.13).

financial structure, making plans for trading commodities and assets today and tomorrow. Then, today's trades are carried through, and households consume and hold portfolios to transfer wealth in the future. Tomorrow, given the state of the world, households fulfill their financial obligations, and then again trade commodities and financial instruments, and consume. At $T = 2$; again households fulfill their financial obligations, trade commodities and consume. Formally, a financial equilibrium is a vector $((x_h; b_h)_{h=1}^H; p; q)$ such that:

(H) given $p; q$; households optimize, that is, for every h , $(x_h; b_h)$ solves the problem

$$\begin{aligned} & \text{maximize}_{x_h; b_h} u_h(x_h) \\ & \text{subject to } \sum_{i=1}^S (x_{hi} - e_{hi}) = Rb_h, \end{aligned}$$

where

$$a = \begin{pmatrix} p^0 & 0 & \dots & 0 \\ 0 & \ddots & 0 & p^S \\ \vdots & 0 & p^S & \vdots \end{pmatrix}$$

is an $(S + 1) \times C$ price matrix, $e_h \in \mathbb{R}_{++}^C$ is the household's endowment, and (M) markets clear, that is,

$$\begin{aligned} \sum_h p_i (x_{hi} - e_{hi}) &= 0 \\ \text{and } \sum_h b_h &= 0. \end{aligned}$$

An equilibrium is represented in extended form by the system of equations⁷ consisting of both the households' Kuhn-Tucker conditions and the market-clearing conditions,

$$\begin{aligned} & \vdots \\ & Du_h(x_h) \lambda_{jh}^a = 0 \\ & \lambda_{jh}^a R = 0 \\ & \sum_i a_{ij} z_h + Rb_h = 0 \tag{2.2} \\ & \vdots \\ & \sum_h p_h z_h = 0 \\ & \sum_h b_h = 0, \end{aligned}$$

where $\lambda_{jh}^a \in \mathbb{R}_{++}^{S+1}$ is the household's vector of Lagrange multipliers (i.e., marginal utilities of wealth), $z_h = x_h - e_h$ is the household's vector of excess demands.

We define volatility of the j -th financial instrument as

$$v^j = \text{Var}_0(q^j) = \sum_{\xi} \frac{1}{\xi} (q^{\xi j} - E_0(q^j))^2$$

where $E_0(q^j) = \sum_{\xi} \frac{1}{\xi} q^{\xi j}$: In other words, we look at price volatility, as opposed to return volatility. Return volatility can be studied with a slight modification of the

⁷The analysis in terms of extended systems was first exploited by Smale (1974) for pure walrasian economies.

definition given above. Note that if $\frac{1}{4}$ depends on h ; more than one definition of volatility has to be used simultaneously.

Existence of equilibrium for this model has essentially been analyzed by Du±e and Shafer (1986). Retrading can potentially lead to a matrix R whose rank is less than I : To see this, let Q be the $S \times J$ matrix of security prices at time $t = 1$: The problem of loss of rank, i.e., redundancy, arises because $Y(0) + Q$ may not have full rank. This will hold generically in endowments and security payoffs. Note that for $J = 1$ the rank result follows immediately from the assumption that $Y \neq 0$, in which case the existence proof follows from Geanakoplos and Polemarchakis (1986):

To state the existence result for $J > 1$, we parametrize economies by endowments and securities only, fixing preferences once and for all.⁸ Let $\mathcal{E} \subset \mathcal{E}_j$ denote the parameter space.

Proposition 2.1. (Du±e and Shafer, 1986) For an open and full-measure subset \mathcal{E}^μ of \mathcal{E} ; a financial equilibrium exists. Moreover, if $\mu \in \mathcal{E}^\mu$; all the equilibria are such that $\text{rank } R = I$; and $Y(0) + Q$ is in general position.

The proof of this Proposition is in essence identical to the one in Du±e and Shafer, hence the reader is referred to that paper for the details. An argument already adapted to this paper's notation is available from this author upon request. We now turn to adjusting a framework formerly developed in Cass and Citanna for comparing equilibria before and after the introduction of new assets.

3. Introducing a new asset into the economy

For the model described in Section 2, the basic idea of the analysis of the impact of financial innovation on volatility can be easily reconduced to the framework for the study of the welfare impact of financial innovation as found in Cass and Citanna (1998). Although it will become apparent that the logic of the analysis follows very closely that paper, we stress that a straightforward application of the theorems provided there is not possible in this multiperiod setup with separable utilities. To reiterate, the steps of the analysis are identical, but the proofs differ because of the special structure of the payoff matrix R . Proofs of Lemmas 3.2, 4.1, of Theorem 4.2, case b), and of the lemmas and theorem in Section 4.2 are similar but not encompassed by statements contained in Cass and Citanna. For instance, in order to derive the condition on multipliers, Cass and Citanna assumed that either the last I rows of the matrix had full rank (general nonseparable case), or that the payoff matrix was in general position (additively separable utility, treated in their Appendix). Here we tied our hands as for the specification of the R matrix, given the dynamic structure of the model, and R is no longer in general position.

⁸So, for the time being, we keep $u \in \mathcal{U}$ as fixed; parameterization by utility functions will appear in the next section.

Obviously, our proofs in this paper show that the general position of R is not necessary to obtain results in this model⁹. Because of the similarities in the analyses of the two problems (welfare impact and volatility impact), we will show how to proceed with the general logic, we will leave the unchanged proofs to the reader, but will provide the differing proofs in the Appendix.

It is convenient to introduce some general notation for representing equations (2.2). We first normalize the numeraire commodity price at each spot, given that households' budget constraints are homogeneous of degree zero in $p^{s:C}$, all s . Hence, $p^{s:C} = 1$, all s (for simplicity, hereafter we will redefine $q = p^{0:C}$ as q). Moreover, we drop $S + 1$ commodity market clearing conditions, say, those corresponding to the numeraire commodity at each spot, by utilizing the analogue of Walras' law. Let z_h^n be z_h less the numeraire commodity at each spot. Let p^n be the commodity price vector without the normalized components.

Let $F_1 : \mathbb{Y}^{n_1} \times \mathbb{E}^n \rightarrow \mathbb{R}^{n_1}$ be the mapping representing the left-hand side of (2.2) after the above changes, where \mathbb{Y}^{n_1} is the n_1 -dimensional space of endogenous variables \mathfrak{y} , with

$$\mathfrak{y} = ((x_h; b_h; \dots; h)_{h=1}^H; p^n; q)$$

and the economy is again parametrized by endowments and securities only. An equilibrium in the original economy is then represented by the equation $F_1(\mathfrak{y}; \mu) = 0$.

We will consider a (smooth) function, defined over the equilibrium manifold of a (now multiperiod and) fictitious economy, i.e. the economy where the new asset is chosen so that at the original values of the endogenous variables it is redundant, but traded. For the original economy and given a fixed $\mu \in \mathbb{E}^n$, we can describe the equilibria by the zeros of a mapping

$$F : \mathbb{Y}^{n_1} \times \mathbb{T} \rightarrow \mathbb{R}^n$$

with $n > n_1$, such that

$$F(\mathfrak{y}; \zeta) = (F_1^0(\mathfrak{y}; \zeta; \mu); F_2(\mathfrak{y}; \zeta)),$$

where F_1^0 is going to be the mapping into \mathbb{R}^{n_1} describing the same equations as those of the original equilibrium, but modified according to the designer's intervention, while F_2 is going to be the mapping into $\mathbb{R}^{n - n_1}$ describing the arbitrage-pricing and market-clearing conditions for the new assets. \mathbb{T} is the space of instrumental variables, including both direct policy variables (new asset payoffs) and related market variables (new asset prices and holdings).

⁹Since we can prove the lemmas and theorems essentially without changing conditions on S ; I and H ; ex post this difference turns out not to substantially matter. Indeed, and to anticipate the presentation of our results, in the Cass and Citanna paper the controllability is obtained if $S + 1 \leq I \leq H + H$; while here if $S + 1 \leq I \leq H + J$ (Theorem 4.2). The first H conditions account for the new no arbitrage equations, and the last conditions (H there, J here) account for the number of objectives to control.

If we can show that there is a subvector of n_1 instrumental variables \bar{z}^0 for which a regularity-like result can be established, that is,

$$\text{rank } D_{\bar{z}^0} F(\bar{z}^0) = n_1 \quad (3.1)$$

where \bar{z}^0 is another subvector of \bar{z} fixed at the value \bar{z}^0 so to obtain the original equilibrium, then the set

$$M = \{(\bar{z}; \bar{z}^0) \in T_{\bar{z}^0} : F(\bar{z}; \bar{z}^0) = 0\}$$

where $T_{\bar{z}^0}$ is an open neighborhood of \bar{z}^0 in T , is a smooth, finite-dimensional manifold, by a straightforward application of the preimage theorem (see Guillemin and Pollack (1974, p. 21), for instance).

Our standard reference to \bar{z} as "endogenous variables" is justified by the following lemma, which is a well-known regularity result, and is therefore stated here without proof.

Lemma 3.1. $\text{rank } D_{\bar{z}} F_1(\bar{z}) = n_1$ on a generic subset of E^n .

Generic here means in an open, full-measure subset, although in the next section, when introducing volatilities in the analysis, we will use the term in a topological sense only. One property of the Lagrange multipliers and two of equilibrium commodity prices and asset price volatilities are summarized in the following lemma, and will be quite useful in the ensuing analysis. Their demonstration basically involves routine applications of a transversality theorem, hence the proof will be omitted. Let $\pi(s)$ be a permutation function of $\{0, 1, \dots, S\}$ into itself.

Lemma 3.2. Consider the solutions to $F_1(\bar{z}; \mu) = 0$: Then,

$$(i) \text{rank} \begin{bmatrix} \pi(s) & 1 & \dots & h & 0 & \dots & \pi(s) & \dots & H & 1 \end{bmatrix} = H \text{ if } H \cdot S + 1 \leq I \text{ and}$$

$$(ii) D_{q^{\pi(s)}} v^j = 2\pi^s(q^{\pi(s)}; E_0(q^j)) \neq 0$$

$$(iii) p^{s^0} \text{ is not colinear with } p^{s^0}; s^0 \neq s^0 > 0,$$

on a generic subset of E^n :

Notice that if $J = 1$; and it is assumed that $Y \neq 0$; we can always extract H spots out of the last S^2 ; since the first $S + 1 = I$ rows of R form the required full-ranked matrix.

It will be convenient to let

$$E_u = \{(\mu \in E^n) \text{ satisfies Lemmas 3.1, 3.2g},$$

which is therefore also a generic subset of E^n . From now on, $\mu \in E_u$:

Now let $G : M \rightarrow \mathbb{R}^k$ such that $m \in G(m)$ be a general function defined over the equilibrium manifold. Note that if $m \in M$ and $\zeta^0 = \zeta^1$, then $G(m)$ is precisely the value of the function at an equilibrium before innovation takes place. One such function could be the utility vector, with $k = H$ as in Cass and Citanna, or the price level of traded securities or, as hereafter, the price volatility, with $k = J$. So it is clear from their analysis that a (local) sufficient condition to obtain a decrease (or increase) of volatility due to financial innovation is that G be a submersion at every $m \in M$ with $\zeta^0 = \zeta^1$, that is, that

$$dG_m : T_m(M) \rightarrow \mathbb{R}^J$$

is onto for all such points. We can restate this condition in terms of the rank of a suitable matrix, which can be expressed using the complementary condition in terms of the system of equations

$$\begin{aligned} a^0 \begin{pmatrix} DF \\ DG \end{pmatrix} &= 0 \\ &F(\zeta^0, \zeta^0) = 0 \\ &\zeta^0 = \zeta^1 \\ \text{and } a^0 a_j &= 0, \end{aligned} \tag{3.2}$$

where a is an $(n + J)$ -dimensional vector.

Financial innovation generated by altering the yields from redundant assets can bring about an increase in volatility if the system of equations (3.2) has no solution.¹⁰ We will establish that this property obtains at every equilibrium for an open and dense set of economies. Having accomplished this, we will show robustness in the space of financial innovations, that is openness of the set of volatility-reducing innovations. It will simply follow from establishing that, for an open and dense set of economies, some altered equilibrium (after volatility-reducing innovation) is regular, just as was the original equilibrium.

4. Effects on market volatility

When an asset is redundant, it has no effect on the market allocation. If such a new asset is introduced into the economy, then (2.2) becomes

¹⁰Note that a necessary condition to have no solution to (3.2) is that $\dim T_m = n_j - n_1 + k$; and this will be verified in our case.

$$\begin{aligned}
& \vdots \\
& Du_h(x_h) \big|_{z_h^a} = 0 \\
& \vdots \\
& \sum_h R \bar{p} \cdot \mathbf{1} \\
& \sum_h z_h^a + [R r] \hat{b}_h = 0 \\
& \vdots \\
& \mathbf{P} \\
& \mathbf{P}_h z_h^n = 0 \\
& \mathbf{P}_h \hat{b}_h = 0 \\
& \vdots \\
& \sum_h r = 0 \\
& \vdots \\
& \mathbf{P} \\
& \mathbf{P}_h \hat{b}_h = 0,
\end{aligned} \tag{4.1}$$

where \hat{b}_h ; all h ; \hat{q} and y are the new asset holdings, price and yields, and $r = \frac{\mu \cdot \hat{q} \cdot \mathbf{1}}{y}$.

The left-hand side of the equations (4.1) corresponds to our function F , when $(\hat{b}; \hat{q}; y) = ((\hat{b}_h; \text{all } h); \hat{q}; y)$ is identified with ζ . A designer can introduce a new asset by choosing \hat{b} ; \hat{q} and y , with the constraint that \hat{b} and \hat{q} are equilibrium asset holdings (so they satisfy market clearing) and equilibrium price for given yields y (so they satisfy a no-arbitrage condition). That is, the constraints are

$$\begin{aligned}
& \vdots \\
& \sum_h r = 0 \\
& \vdots \\
& \text{and } \mathbf{P}_h \hat{b}_h = 0,
\end{aligned} \tag{4.2}$$

which would have to be appended to equations (2.2), while simultaneously modifying the households' budget constraints accordingly. The full set of constraints facing the designer is then described by equations (4.1). The dimension of the range of F just equals the number of equations defining an equilibrium with $H + 1$ assets. Note that the designer uses $H + 1 + S$ instruments (so that $T = \mathbf{R}^{H+1+S}$), of which (as many as) $H + 1$ cannot be chosen independently, given equations (4.2).¹¹

If the completely redundant asset $y = 0$ is introduced, arbitrage-pricing requires that $\hat{q} = 0$ as well, so that market-clearing is the only effective restriction on \hat{b} (and the planner is free to choose all but two of the remaining "policy" instruments, \hat{q} and \hat{b}_h ; some h). Thus, in the fictitious equilibrium, a natural choice for the subvector of

¹¹By this we mean that equations (4.2) restrict the choice of ζ , so that in order for these equations to be satisfied, $H + 1$ elements of ζ must be endogenously determined, once the others are fixed. The elements of ζ which are unrestricted are said to be 'independent'.

instrumental variables ζ^0 is $\zeta^0 = ((\hat{b}_h; h > 1); y)$, and for their particular values ζ^0 , say,¹²

$$\zeta^0 = ((\hat{b}_h; h > 1); y) = (1; 0).$$

With these choices, and given Lemma 3.2 (i), we will now prove condition (3.1) by selecting ζ^{00} as

$$\zeta^{00} = (\hat{b}_1; r^{3/4(s)}; 0 \cdot \frac{3}{4}(s) \cdot H_j - 1),$$

For this and later purposes it is very convenient to partition

$$r = (r^{00}; r^{000}) = ((r^{3/4(s)}; 0 \cdot \frac{3}{4}(s) \cdot H_j - 1); (r^{3/4(s)}; H \cdot \frac{3}{4}(s)),$$

to conform with ζ^{00} .

Lemma 4.1. For every $\mu \in \mathbb{R}_+$,

$$\text{rank} D_{\mu; \hat{b}; r} F_{\substack{F(\mu; \hat{b}; r) = 0 \\ \hat{b}_h = \hat{b}_h; h > 1 \\ y = y}} = \text{rank} D_{\mu; \hat{b}_1; r^{00}} F_{\substack{F(\mu; \hat{b}; r) = 0 \\ \hat{b}_h = \hat{b}_h; h > 1 \\ y = y}} = n,$$

provided $S + 1 \leq I = \overline{S}(\overline{S}_j - J) + \overline{S} + 1 \leq J \leq H$.

Proof. See the Appendix.

It is not difficult to see that, equipped with Lemma 4.1, we can apply the general methodology outlined in the previous section to this case, and therefore establish that G is a submersion at $(\mu; \zeta)$ with $\zeta^0 = \zeta^0$ for which $F(\mu; \zeta)_{\zeta^0 = \zeta^0} = 0$ if and only if the system of equations (3.2), now appearing as

$$\begin{aligned} a^0 \cdot \begin{pmatrix} DF \\ DG \end{pmatrix}_{\substack{F(\mu; \hat{b}; r) = 0 \\ \hat{b}_h = \hat{b}_h; h > 1 \\ y = y}} &= 0 \\ \text{and } a^0 a_j - 1 &= 0; \end{aligned} \tag{4.3}$$

has no solution. This last is the result that we now verify for an open and dense set of economies parameterized by both endowments and utility functions.

Intuitively, we are using $H + 1 + S$ instruments to achieve J objectives, the changes in volatility. The introduction of a new asset carries additional constraints in the form of H arbitrage pricing equations and one asset market clearing condition. Note that $\dim T = H + 1 + S \leq H + 1 + J$; where $H + 1 = n_j - n_1$; which is equivalent to $S \leq J$, obviously true in our context: However, the previous lemma indicates that this is not enough. H instruments (the new asset holdings) are indeed useless, because they only control market clearing. The remaining $S + 1$; the new asset price and yields, are also constrained to spread their effect through elements of the orthogonal space of R (through the ζ 's), losing I dimensions. Hence $S + 1 \leq I$ of these instruments must

¹²The argument, and only in Theorem 4.2, requires that $\hat{b}_h \notin 0$, all h .

be used to accomplish the control task, but they need to satisfy the no arbitrage equations as well. Therefore, $S + 1$ and H are really the independent instruments. The following theorem shows that condition $S + 1 \geq H + J$ itself is necessary (in our framework) and sufficient in order to show that the equilibrium volatility function is locally onto.

Theorem 4.2. On an open and dense set $\hat{E} \subset E \subset U$, at any original equilibrium, G is a submersion, so that there are new assets y^0 and y^{00} whose introduction decreases and increases volatility, respectively, provided $S + 1 \geq H + J$ (that is, again, $\bar{S}(\bar{S} + J) + \bar{S} + 1 \geq \bar{S} + H + J$).

The proof of Theorem 4.2 is rather technical and is deferred to the Appendix. From an economic viewpoint, only two aspects of the proof are worth mentioning here. First, the proof shows that it would be possible to choose the new asset payoffs out of the last spots, if these were in sufficient number. That is, if a stronger condition holds, and $\bar{S}^2 \geq J(\bar{S} + 1) \geq H + J$; then the new asset payoffs can be chosen only at the terminal date. When $J = 1$; no extra condition has to be explicitly imposed in order to establish the dependence of the payoff specification on the terminal date, because the condition occurs automatically. Second, in proving Theorem 4.2 we show that it is always possible to take as "independent" the subvector of instrumental variables

$$z^{000} = (r^{\frac{3}{4}(s)}; H \cdot \frac{3}{4}(s) \cdot H + J + 1).$$

and to $r^{\frac{3}{4}(s)}$; for $\frac{3}{4}(s) \geq H + J$; and b_h ; for $h > 1$.¹³ Taking advantage of this last observation, we can state and prove a corollary which shows the robustness of the existence of volatility-reducing innovation. It will be convenient hereafter to simply use z in place of $(b; r)$.

Corollary 4.3. On an open and dense set $\hat{E} \subset E$, at any original equilibrium, there is some altered equilibrium which is (i) volatility-reducing and (ii) regular, as well as some altered equilibrium which is (i) volatility-increasing and (ii) regular, so that there are open sets of new assets Y^0 and Y^{00} such that the introduction of $y^0 \geq Y^0$ or $y^{00} \geq Y^{00}$ can decrease or increase market volatility, respectively, provided $\bar{S}(\bar{S} + J) + \bar{S} + 1 \geq \bar{S} + H + J$.

Proof. See the Appendix. ■

4.1. Two examples of negligible economies

To convince the reader that utility perturbations are needed (that is, for density), we have elaborated on a well-known example for one-period trading models (see Magill and Quinzii, (1996)) of an economy with incomplete markets and linear-quadratic

¹³This is what "dropping the equations corresponding to z " means, as used in the course of the proof (see the Appendix).

utility functions. The example shows that fixing specific preferences leads to no changes in asset prices due to financial innovation, therefore to no changes in volatility.

A second example is provided which shows: a) how our result deals with incomplete vs complete asset markets; b) why in the absence of aggregate risk completing markets may reduce the volatility of the preexisting asset no matter what new asset is introduced, but again that the situation is not robust to parameter specifications.

First example

For this purpose, assume that u_h takes the form

$$u_h(x_h) = \frac{1}{4} v(x_h^0) + \sum_{s>0} \frac{1}{4^s} f_h(v(x_h^s)), \quad (4.4)$$

where $v : \mathbb{R}_{++}^C \rightarrow \mathbb{R}$ is a smooth, differentially strictly increasing and concave, homogeneous of degree one function, and

$$f_h(y) = \sum_{i=1}^I (1-\alpha_i) (\alpha_i y)^2$$

Then preferences are homothetic and spot-separable, and spot commodity equilibrium prices are independent of the income distribution across agents and across states. Then the portfolio choice only affects the level of consumption in each spot, but not the commodity prices. Let $w^s = \sum_h e_h^s$ be the level of aggregate endowments in spot s ; and consider the spot price normalization $p^s w^s = 1$; for all s : From $Dv(w^s)w^s = v(w^s)$; all s ; we get

$$p^s = Dv(w^s) = v(w^s)$$

for all s ; which shows that p^s only depends on aggregate resources, not on income distribution (hence, and a fortiori, not on financial innovation): Therefore one can reduce the trader's multi-commodity maximization problem to a one-commodity maximization: after defining $m_h^s = p^s e_h^s + r^s b_h$; and noting that the optimal consumption vector in spot s is given by $x_h^s = m_h^s w^s$; we transform (H) into

$$\max_{b_h} \frac{1}{4} v(x_h^0) + \sum_{s>0} \frac{1}{4^s} f_h(x_h^s) \\ \text{s.t. } x_h = l_h + R b_h$$

with $x_h^s = v(w^s) m_h^s$; $l_h^s = v(w^s) p^s e_h^s$ and the s_i th row in R is $r^s = v(w^s) r^s$: Define $l_h^1 = (l_h^1, \dots, l_h^{\bar{S}})$ and $l_h^2(s) = (l_h^{(s_1-1)\bar{S}+1}, \dots, l_h^{(s_1-1)\bar{S}+\bar{S}})$; for $1 \leq s \leq \bar{S}$: Also define $\frac{1}{4}^0 = \sum_h \frac{1}{4}^0$; $\frac{1}{4}^s = \sum_h \frac{1}{4}^s$; $l_h^1 = \sum_h l_h^1$ and $l_h^2(s) = \sum_h l_h^2(s)$: At this point, the linear-quadratic assumption on u_h leads to the following equilibrium asset prices

$$q^0 = \frac{1}{4}^0 \sum_{i=1}^I \alpha_i^{-1} \sum_{i=1}^I \alpha_i^{-1} (0) [Y(0) + Q]$$

and

$$q^s = \frac{1}{\prod_{i=1}^{\bar{S}} p_i(s)} \mathbf{1}^T Y(s)$$

where $p_i(s)$ is the \bar{S}_i dimensional square, diagonal matrix of conditional probabilities given state s has occurred, $s = 0; 1; \dots; \bar{S}$: Neither of these expressions changes as we add a new asset.

Second example

Consider an economy with $H = 3$; $\bar{S} = 2$ and $C = 2$. Traders have identical preferences described by the (state-dependent, but it does not matter for what we want to show) utility function $\sum_{c=1}^C (1-G^s) \log x_h^{s;c}$; and endowments such that $e_h^{s;c} = e_h$ for all pairs $(s; c)$ with $s > 0$; a fixed amount, while

$$e_h^{0;2} = [(G_i - 1)(G_i - SC) \quad i \quad SC] \quad e_h = G;$$

and no restriction is imposed on endowments at time $t = 0$ for $c = 1$. Assume that to start with there are $J = 2$ assets in the economy, with payoff matrix Y independent of s and with rank equal to 2, and satisfying the condition

$$\text{rank } Y = \text{rank} \left[Y + \begin{matrix} \mathbf{1}^T Y & \mathbf{1} \\ \mathbf{1}^T Y & \end{matrix} \right]$$

where $\mathbf{1}^T$ is a 2-dimensional row vector of ones.

Consider the walrasian model associated with this economy. It can easily be shown that the following is the unique walrasian equilibrium price \hat{p} : $\hat{p}^{s;c} = 1$ for all $(s; c)$ with $s > 0$; and for $(0; 2)$, while

$$\hat{p}^{0;1} = [(G_i - SC) \quad e_h \quad i \quad e_h^{0;2}] = e_h^{0;1}$$

Let $q^s = \mathbf{1}^T Y$; all s ; and $q^0 = \mathbf{1}^T [Y + Q]$, so that markets are complete at these prices (i.e., rank $R = 1$). Then there exist $b_h \in \mathbb{R}^2$; all h ; such that $\hat{p}^T (\hat{x}_h - e_h) = R b_h$ and $\mathbf{1}^T R = 0$; where \hat{p} is the usual commodity price matrix at \hat{p} ; and \hat{x}_h is the walrasian consumption: Letting $\hat{x}_h = \mathbf{1}_h \mathbf{1}$ for all h ; we see that \hat{p} is also the unique complete markets equilibrium for this economy.

If $J = 1$; the incomplete market economy will typically result in equilibria where $\hat{x}_h^s \in \hat{x}_h^{s^0}$ for some $s \in s^0$: That is, we keep preferences as given, and change endowments but always constrain them to be identical in the aggregate as explained above, and select only payoff matrices with strictly positive yields for all s . This is obtained through a standard round of transversality (here we need to have one commodity at time $t = 0$ not constrained to have fixed aggregate endowments).

Notice that in the complete market economy, $q^s = q^{s^0}$ for all $s; s^0$, so that asset volatility is zero when markets are complete, and will be typically positive for the

remaining asset when markets are incomplete. Finally, note that the absence of aggregate risk can be accommodated in our setup, but it is not generic, and this is why the example works, even though the condition for the application of the theorem is satisfied: $1 + S_j \leq H + J$; that is, $1 + 6_j \leq 3_j + 1$:

4.2. Innovation with retrading

In this section we consider the effects on volatility of the introduction of a new security that can be retraded at time $t = 1$: In the previous section we dealt with innovative instruments that could not be retraded between today, time of the innovation, and the terminal date. Although some hedging instruments which are traded over the counter present this one-time trading feature, most newly traded securities are marketed in exchanges where retrading is possible. It turns out that the general framework of Section 3 is applicable to this case in a fruitful manner, and almost the same analysis developed in Section 4 carries through. In particular Lemma 4.1 and Theorem 4.2 can be recovered when the new asset can be retraded, provided we change slightly Lemma 3.2 (i). Since the proofs of the results are similar to those presented in the previous sections, the details are omitted. The bottom line of this section is that retrading makes controllability more difficult, in the sense that the condition used to obtain volatility-reducing (or volatility-increasing) innovation is stronger than with no retrading. Introducing an asset with retrading corresponds to introducing $\bar{S} + 1$ new markets. Although we have more assets to use, the number of payoffs we control is unchanged (totalling S), while the number of constraints increases, because retrading requires more no arbitrage equations for the new asset.

Let \hat{R} be the matrix representing the payoffs and prices of the new financial instrument. \hat{R} is just a copy of R ; that is if we were to assume that $J = 1$; with arbitrary payoffs Y ; taken in general to be different across time and states. Let $r^s = (q^s; y^s)$; for $s = 0; 1; \dots; \bar{S}$: We need to append $\bar{S} + 1$ market clearing equations and $\bar{S} + 1$ H pricing equations to the original equilibrium system. As before, we will call F the equilibrium function with these appended equations.

First, we need to strengthen Lemma 3.2. Let \mathcal{H}^0 be a permutation of the states $s; s\bar{S} + 1; s\bar{S} + 2; \dots; s\bar{S} + \bar{S}$; for each $s = 0; 1; \dots; S$; with generic element $\mathcal{H}^0(s)$; that is, a permutation of the set including the direct successors of a state s and this state.

Lemma 4.4. Consider the solutions to $F_1(\mu) = 0$. Generically in \mathbb{E}^n , given any set of the \bar{S} direct successors of the state $s = 0; 1; \dots; \bar{S}$; and a permutation \mathcal{H}^0 ;

$$(ii) \text{rank} \begin{bmatrix} \mathcal{H}^0(s); 1 \cdot h \cdot H; 0 \cdot \mathcal{H}^0(s) \cdot H_j - 1 \end{bmatrix} = H,$$

if $1 + \bar{S} \leq H + J$:

Then we can prove the analogue of Lemma 4.1. Let $r^s = (r^{s00}; r^{s000}) = ((r^{\mathcal{H}^0(s)}; 0 \cdot \mathcal{H}^0(s) \cdot H_j - 1); (r^{\mathcal{H}^0(s)}; H \cdot \mathcal{H}^0(s))$ be a partition of each vector r^s ; for $s = 0; 1; \dots; \bar{S}$: Let $\hat{b}_1 = (\hat{b}_1^0; \hat{b}_1^1; \dots; \hat{b}_1^{\bar{S}})$; the vector of the new security holdings for trader $h = 1$:

After appropriately redefining the set E_u in the obvious way; we have the following lemma.

Lemma 4.5. For every $\mu \in E_u$,

$$\text{rank} D_{\mu; \hat{b}; r} F_{\substack{(\mu; \hat{b}; r)=0 \\ \hat{b}_n = \hat{b}_n; h > 1 \\ y = \hat{y}}} = \text{rank} D_{\mu; \hat{b}_1; r=0} F_{\substack{(\mu; \hat{b}; r)=0 \\ \hat{b}_n = \hat{b}_n; h > 1 \\ y = \hat{y}}} = n,$$

provided $1 + \bar{S} \leq H + J$.

The proof is based on the use of Lemma 4.4 and the fact that we can extract H states following and including each state $s = 0; 1; \dots; \bar{S}$ for which the multiplier matrix has rank H .

Finally, Theorem 4.2 can be modified accordingly. We state here only one side of the result, the volatility-reducing part, although the theorem shows that both directions for volatility are possible.

Theorem 4.6. On an open and dense set $\mu \in E \subseteq U$, at any original equilibrium, G is a submersion, so that in particular there is a new asset with retrading y , whose introduction decreases volatility provided $1 + \bar{S} \leq J \leq H + J$.

The proof of this theorem would show that only the terminal payoffs need to be chosen appropriately. Moreover, it is immediate to see that the condition $1 + \bar{S} \leq J \leq H + J$ needs to hold only for those many states following one state in period $t = 1$: In other words, if the number of states varied from node to node, a weaker condition than the one used in the theorem could be adopted.

5. Appendix

Proof of Lemma 4.1

The Jacobian matrix of F at a point as defined above is

eq. / var.	x _h	b _h	z _h	p	q	b̂ _h	r
⋮							
D _h u _h	D ² u _h	0	i ^a 0	i ^a x _h ⁿ	0	0	0
R _h μ	0	0	R ⁰	0	Q _h ¹	0	0
i ^a z _h + [R r]	i ^a	R	0	i ^a Z _h ⁿ	Q _h ²	j	b̂ _h
⋮						j	
P _h z _h ⁿ	I ⁿ	0	0	0	0	j	0
P _h b _h	0	I	0	0	0	j	0
⋮							
z _h r	0	0	0	0	0	0	z _h
⋮							
P _h b̂ _h	0	0	0	0	0	1	0

where

$$I^n = \begin{pmatrix} 2 & & 3 \\ 6 & & 7 \\ 6 & [I \ 0] & 7 \\ 4 & & 5 \end{pmatrix},$$

$$x_h^n = \begin{pmatrix} 2 & & 3 \\ 6 & & 7 \\ 6 & z_h & 7 \\ 4 & 0 & 5 \end{pmatrix}$$

and

$$Z_h^n = \begin{pmatrix} 2 & & 3 \\ 6 & & 7 \\ 6 & (z_h^{s_n})^0 & 7 \\ 4 & & 5 \end{pmatrix};$$

$$Q_h^1 = \begin{pmatrix} 2 & & 3 \\ 6 & i_{z_h}^0 & 7 \\ 6 & 0 & 7 \\ 4 & i_{z_h}^1 & 5 \\ & \vdots & \\ & 0 & \\ & 0 & i_{z_h}^s \end{pmatrix}$$

and

$$Q_h^2 = \begin{matrix} & \begin{matrix} 2 & & & & 3 \end{matrix} \\ \begin{matrix} 6 \\ 6 \\ 6 \\ 6 \\ 4 \end{matrix} & \begin{matrix} i & b_h^0 & 0 & \dots & 0 \\ 0 & i & b_h^1 + b_h^0 & \dots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \dots & \dots & 0 & i & b_h^S + b_h^0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \end{matrix} \begin{matrix} \\ \\ \\ \\ \\ 5 \end{matrix}$$

Lemma 3.1 implies that the block in the upper left corner (corresponding to $D_{\gg}F_1^0$) has full rank. But Lemma 3.2 (i) implies that, when $S + 1 \leq H$, the block in the lower right corner (corresponding to $D_{\hat{b},r}F_2$) also has full rank; simply consider the columns corresponding to the particular variables \hat{b}_1 and r^0 (that is, corresponding to $D_{\hat{b}_1, r^0}F_2$). Hence, since the block in the lower left corner (corresponding to $D_{\gg}F_2$) is 0, the matrix $D_{\gg, \hat{b}_1, r^0}F$, and, a fortiori, the matrix $D_{\gg, \hat{b}, r}F$ must have full rank. ■

Proof of Theorem 4.2

The proof is carried out in two steps.

Step 1 - Openness.

All we need to show is that the projection $\pi : \mathbb{R}^n \rightarrow \mathbb{R}^m \times \mathbb{R}^k$ is proper when restricted to the subset of the domain where

$$\begin{aligned} F(\gg; \hat{b}; r; e; Y(s)_{s=0}; u) &= 0 \\ \hat{b}_h &= \bar{b}_h; h > 1 \\ \text{and } y &= \bar{y}. \end{aligned} \tag{5.1}$$

This follows from the fact that the set of solutions to equations (4.3) yields a closed subset of the solutions to (5.1). Thus, given properness, the complement of its projection into the parameter space is open. But properness can be established through a well-known argument (see Citanna, Kajii and Villanacci, Lemma 1, e.g.), whose details we therefore omit.

Step 2 - Density.

Without loss of generality, we will assume that a $(\mu; u)$ is chosen in an open and dense subset of $\mathbb{R}^m \times \mathbb{R}^k$ such that $\mu \in \mathbb{R}^m$; with all the stated properties. Moreover, we will assume that at $(\mu; u)$ there is only one equilibrium. To establish density in $\mathbb{R}^m \times \mathbb{R}^k$, we show that the system (4.3) almost never has a solution for an open and full-measure subset of $\mu \in A$, where $\mu \in \mathbb{R}^m$ and $A = \{ \mu \in \mathbb{R}^m \mid \exists \text{ } \frac{1}{2} \text{ } (R^{G^2})^H \text{ is the space representing a finite-dimensional parametrization of the households' utility functions around } u \text{ (again, see Cass and Citanna, e.g.).} \}$

The reader should keep in mind that we need to be able to perturb utility spot-by-spot, at the same time keeping the functional form constant across spots. This could be done if we had

$$x_h^s \in \mathbb{R}^n; s^0 \in \mathbb{R}^k > 0, \text{ all } h.$$

But as we proved in Lemma 3.2 (iii), for differentially strictly concave (in fact, quasi-concave) utility functions and on a generic subset of E ,

$$p^{s^0} \text{ is not colinear with } p^{s^{00}}; s^0 \notin s^{00} > 0,$$

from which the required diversity in households' consumption follows directly.

Consider the system given by (4.3) in extensive form, that is, (5.1) and

$$\begin{array}{rcl}
 & & \vdots \\
 \text{HG} & & {}_h D^2 u_h \cdot i \cdot {}_h a + \pm I^n = 0 \quad (1) \\
 \text{HI} & & {}_h R + {}^2 = 0 \quad (2) \\
 \text{H}(S+1) & & i \cdot {}_h a^T + {}_h R^0 = 0 \quad (3) \\
 & & \vdots \\
 \text{(C } i \text{ - } 1)(S+1) & & P_h ({}_h \alpha_h^n + {}_h Z_h^n) = 0 \quad (4) \\
 \text{I} & & P_h ({}_h Q_h^1 + {}_h Q_h^2) + {}^1 D_q v = 0 \quad (5) \\
 & & \vdots \\
 \text{H [1]} & & \alpha = 0 \quad (6) \\
 & & \vdots \\
 \text{S + } 1 \text{ [H + J]} & & P_h (b_h^{\circ} \cdot h + \Delta_{h, s, h}) [P_h ({}^{\circ \#} b_h + \Delta_{h, s, h}^{\#})] = 0 \quad (7) \\
 \text{1} & & \text{and } {}^{01} 1_j = 0, \quad (8)
 \end{array} \quad (5.2)$$

where $a^0 = ({}_{hh}; ({}_{hh}; {}_h; {}_h); {}_{hh}; \pm; {}_{hh}; \Delta_{hh}; \alpha; {}_{hh}; 1; {}_{hh})$, ${}_h = ({}_{hh}^{\#}; {}_{hh}^{\#\#}) = (({}_{hh}^s; 0 \cdot \mathcal{H}(s) \cdot H + J; i - 1); ({}_{hh}^s; \mathcal{H}(s) \cdot H + J))$ and ${}_{s, h} = ({}_{s, h}^{\#}; {}_{s, h}^{\#\#})$ split accordingly (notation which is only used for the remainder of this argument) and on the far left side we have displayed the number of equations. Equation (5.2.8) replaces $a^0 a_{i-1} = 0$ without loss of generality due to Lemma 4.1. Since they are all identical, hence redundant, we drop $H - i - 1$ equations corresponding, in particular, to all but the first of (5.2.6). Given that $S + 1 - i \leq H + J$; and consequently that $S + 1 - i - H - J \leq 0$, it follows that the equations (5.2) still outnumber the additional variables a by this difference. So we can drop all but $H + J$ of (5.2.7) (as indicated in square brackets) and still have one more equation than variables. Now observe that the restriction on the domain in (5.2), that is, (5.1), is equivalent to

$$\begin{aligned}
 F_1(\mu; u) &= 0; \\
 \hat{b} = \hat{b} &= (i - (H - i - 1); 1; {}_{hh}; 1) \\
 \text{and } r = \hat{r} &= 0.
 \end{aligned}$$

But since $\mu \in E_u$, $\text{rank } D_{\mu} F_1 = n_1$, while by construction, $D_{a; A} F_1 \Big|_{F_1(\mu; u) = 0} = 0$. It remains to show that the Jacobian matrix of the truncated subsystem in (5.2) (with respect to $(a; A)$) has full rank, in order to apply the transversality theorem, and to conclude that, generically in parameterized utility functions, the full system (4.3)

has no solution. We need to consider few cases, since the matrix of derivatives of equation (5.2.1) with respect to the elements of the symmetric matrix A_h has full rank if $\alpha_h^s \neq 0$; all s ; but, for example, will have less-than-full rank if $\alpha_h^s = 0$; for some s ; h . Here is where the general position of R ; which we do not have, was heavily used in Cass and Citanna.

Case a - $\alpha_h^s \neq 0$, all s ; all h .

In this case it is straightforward to verify that equations (5.2.1), all h , can be perturbed independently by using the utility parameters A . Since α^1 only appears in equations (5.2.5) and (5.2.8), while, in light of equation (5.2.8), α^1 can never be equal to zero, this last can then be perturbed independently using α^1 . Similarly, since α_h only appears in equations (5.2.1), (5.2.8), (5.2.3) and (5.2.4), the last two, all h , can be perturbed independently using α : Use $\alpha_h^{s;c}$; all s , in (5.2.3), and $\alpha_h^{s;c}$; all s ; all $c \in C$, some h ; in (5.2.4). Continuing in the same manner, equations (5.2.5) can be perturbed independently using α_h ; some h , while, obviously, equation (5.2.6) can be perturbed independently using α . Finally, equations (5.2.2), all h , and (5.2.7) can be perturbed independently using $\alpha_h^{s;s}$, all h , and α_h^s ; all h ; $\alpha_h^{s;s}$, for J spots s and some h ; here we appeal to the assumptions that: a) the first I rows of R (including the relabelled spots $S + 1, \dots, S + H + J$ and therefore part $^{s;s}$ of the vectors) form an I^2 dimensional, full rank matrix; b) by assumption, we have at least J elements $\alpha_h^{s;s}$; and the $\alpha_h^{s;s}$'s selected include the ones forming a matrix of rank H ; c) $\alpha_h^s \neq 0$, all h .

Case b - $\alpha_h^s = 0$, some s ; some h .

First, note that $\alpha_h = 0$, some h cannot occur. In this case it is straightforward to verify that $\alpha_h = 0$; $\alpha_h = 0$ and $\alpha_h = 0$. Then $\alpha = 0$ and $\alpha = 0$; which implies $(\alpha_h; \alpha_h; \alpha_h) = 0$ from demand regularity for all other h , and this implies from (5.2.5) and Lemma 3.2.ii) that $\alpha^1 = 0$; which contradicts equation (5.2.8).

Let $S_h^0 = fs \ 2 \ f0; 1; \dots; Sg : \alpha_h^s = 0g$, with $S_h^0 \neq ;$, some h , and

$$\begin{aligned} \bar{S} &= [h S_h^0 \\ \underline{S} &= \setminus h S_h^0. \end{aligned}$$

We need to look at system (5.2) more closely. We rewrite its equations below, state

by state:

$$\begin{array}{ll}
 s \in \bar{S} & \textcircled{h}^s D_{s;s}^2 u_h i \textcircled{h}^s p^s + \pm^s [10] = 0 \quad (1a) \\
 s \in S_h^0; s \in \bar{S} & \textcircled{h}^s D_{s;s}^2 u_h i \textcircled{h}^s p^s = 0 \quad (1b) \\
 & \textcircled{h}^s R + \textcircled{h}^s = 0 \quad (2) \\
 s \in S_h^0 & i \textcircled{h}^s (p^s)^0 + \textcircled{h}^s (r^s)^0 = 0 \quad (3a) \\
 s \in S_h^0 & \textcircled{h}^s (r^s)^0 = 0 \quad (3b) \\
 s \in \underline{S} & \mathbf{P}_{h:s \in S_h^0} (\textcircled{h}^s \textcircled{h}^s + \textcircled{h}^s Z_h^{sn}) = 0 \quad (4) \\
 & \mathbf{P}_h (\textcircled{h}^s Q_h^1 + \textcircled{h}^s Q_h^2) + \textcircled{h}^s D_q v = 0 \quad (5) \\
 & \textcircled{h}^s = 0 \quad (6) \\
 s \in \underline{S} & \mathbf{P}_h (\textcircled{h}^s b_h^1 + \textcircled{h}^s \textcircled{h}^s) = 0 \quad (7a) \\
 s \in \underline{S} & \textcircled{h}^s \textcircled{h}^s = 0 \quad (7b) \\
 & \text{and } \textcircled{h}^s i = 0. \quad (8)
 \end{array} \tag{5.3}$$

We have substituted for $\textcircled{h}^s = 0$; discovering that this implies $\textcircled{h}^s = 0$ and $\pm^s = 0$ for these $s \in S_h^0$. This means that the corresponding equations (5.2.1) will drop for these spots $s \in S_h^0$. Note that also some equations among (5.2.4) have (possibly, for $s \in \underline{S}$) dropped out of the system. We have at least $\mathbf{P}_{h:S_h^0}$ extra equations. If we decide not to perturb the zeroed variables, we can drop some other equations, since now the equations still outnumber the unknowns by more than one. Notice also that equations (5.3.1) can be perturbed using the utility parameters A_h ; equations (5.3.3a) and (5.3.4) can be perturbed using the \textcircled{h}^s 's as before, and equations (5.3.6) using \textcircled{h}^s . We are left with perturbing equations (5.3.2), (5.3.3b), (5.3.5), (5.3.7) and (5.3.8).

Let R_h^{n0} be the submatrix of R with rows corresponding to those spots $s \in S_h^0$; and R_h^0 be the matrix with rows corresponding to spots $s \in S_h^0$; equations (5.3.3b) then read $\textcircled{h}^s R_h^{n0} = 0$. At this junction, the choice of the equations to be dropped depends on the rank of these two matrices. Let $\text{rank } R_h^{n0} = I_h^n = \min\{I; 1 + S_i | S_h^0\}$ and $\text{rank } R_h^0 = I_h^{n0} = \min\{I; S_h^0\}$; using the general position of Y : Of equations (5.3.3b), $S_h^0 - I_h^{n0}$ are redundant and must be thrown away. Therefore we have a surplus of at least $\textcircled{h}^s I_h^{n0}$ (i.e., fewer) equations.

If $S_h^0 = \emptyset$; some h ; then his equations (5.3.3b) disappear, and equations (5.3.3), (5.3.2) for this h and (5.3.7) are perturbed using this trader's \textcircled{h}^s ; \textcircled{h}^s and \textcircled{h}^s ; as in Case a). Then we can use this trader's \textcircled{h}^s to perturb equations (5.3.5). The remaining equations (5.3.2) and (5.3.3b) are perturbed as follows. Since there are $\textcircled{h}^s I_h^{n0} + \sum_{s \in \bar{S}} (C_i - 1)$ extra equations, if $I_h^{n0} = I$; we throw away equations (5.3.2) and use \textcircled{h}^s to perturb $\textcircled{h}^s = 0$; implied by (5.3.3b). If $I_h^{n0} < I$; then $I_h^{n0} = S_h^0$, and we can use \textcircled{h}^s to perturb equations (5.3.3b), and perturb (5.3.2) using \textcircled{h}^s possibly (if $I_h^n < I$) after throwing away S_h^0 of these equations. Equation (5.3.8) is perturbed using \textcircled{h}^s .

So consider the case when $S_h^0 \neq \emptyset$; all h :

1) If $I_h^{n0} = I$; all h , equations (5.3.3b) imply $\textcircled{h}^s = 0$: Then the system of equations for each household is

$$\begin{aligned}
s \geq \bar{S} \quad \mathbb{R}_h^s D_{s;s}^2 u_h i \quad \mathbb{R}_h^s p^s + \pm^s [10] &= 0 & (1a) \\
s \geq \bar{S}; s \geq S_h^0 \quad \mathbb{R}_h^s D_{s;s}^2 u_h i \quad \mathbb{R}_h^s p^s &= 0 & (1b) \\
\mathbb{R}_h^s R + \pm^2 &= 0 & (2) \\
s \geq \bar{S} \quad i \quad \mathbb{R}_h^s (p^s)^0 &= 0 & (3a)
\end{aligned}$$

First, we throw away J equations (5.3.7), perturbing the remaining H using $\hat{\alpha}$: Equation (5.3.8) is perturbed using $\hat{1}$. Note that equations (5.3.5) are now $\mathbb{R}_h^s Q_h^2 + {}^1 D_q v = 0$: They can be rewritten as

$$\mathbb{F} \begin{matrix} \mathbb{R}_h^0 \\ \vdots \\ \mathbb{R}_h^0 \end{matrix} \begin{matrix} \mathbb{R}_h^0 \\ \vdots \\ \mathbb{R}_h^0 \end{matrix} \begin{matrix} i \quad b_1^0 \\ \vdots \\ i \quad b_H^0 \end{matrix} \begin{matrix} \mathbb{R}_h^0 \\ \vdots \\ \mathbb{R}_h^0 \end{matrix} = 0$$

and

$$\mathbb{F} \begin{matrix} \mathbb{R}_h^s \\ \vdots \\ \mathbb{R}_h^s \end{matrix} \begin{matrix} \mathbb{R}_h^s \\ \vdots \\ \mathbb{R}_h^s \end{matrix} \begin{matrix} i \quad b_1^s + b_1^0 \\ \vdots \\ i \quad b_H^s + b_H^0 \end{matrix} \begin{matrix} \mathbb{R}_h^s \\ \vdots \\ \mathbb{R}_h^s \end{matrix} + {}^1 (D_{q^s} v) = 0$$

for $s = 1; \dots; \bar{S}$: For each s ; let $J^\#(s)$ be the rank of these matrices after deleting the rows corresponding to $\mathbb{R}_h^s = 0$. In order to perturb equations (5.3.5) using \mathbb{R}_h^s , we need to delete $\sum_{s=0}^{\bar{S}} [J \ i \ J^\#(s)]$ of them. We observe that it can never be that $\mathbb{R}_h^s = 0$ for all h ; for $s > 0$; since then $\hat{1} = 0$; contradicting Lemma 4.1. Hence $J^\#(s) \geq 1$; for $s > 0$ (this uses the fact that $b_h^s \neq 0$ or $i \quad b_h^s + b_h^0 \neq 0$; all $h; s$, generically in $\mathbb{F}^\#$; through an argument similar to the ones in the proof of Lemma 3.2). This implies $I > J + \sum_{s=0}^{\bar{S}} [J \ i \ J^\#(s)]$: Take $h = 1$ and leave $J + (J \ i \ J^\#)(\bar{S} + 1)$ of equations (5.3.2), and perturb them using $\hat{2}^i$: Then we can throw away the I equations (5.3.2) for $h > 1$; completing this subcase.

2) If $I_{h^0}^\# < I$; for some, but not all h : When $I_{h^0}^\# = I$ for some h^0 , for one of them we keep equations (5.3.2), which we perturb using $\hat{2}$. Equations (5.3.3b) are eliminated, thereby freeing the vector \bar{h}^0 for use in equations (5.3.5). For all other h^0 ; we can throw away the $I_{h^0}^\#$ equations (5.3.2), and use the vector \bar{h}^0 in equations (5.3.3b). For h such that $I_h^\# < I$; we throw away $I_h^\# = S_h^0$ equations (5.3.2), obtaining a submatrix of $R_h^{n^0}$ with $1 + S \ i \ S_h^0$ rows and $I \ i \ S_h^0$ columns, and since rows are more than columns, the rank of this submatrix is $I_h^\# = 1 + S \ i \ S_h^0$ using the general position of Y ; we use \mathbb{R}_h^0 to perturb the remaining equations (5.3.2). Equation (5.3.3b) is perturbed using \bar{h} ; and equation (5.3.8) using $\hat{1}$. For equations (5.3.7), we can perturb the J (5.3.7a) using \mathbb{R}_h^s , for some h such that $s \geq S_h^0$; and perturb the rest with $\hat{\alpha}$. Note that such a \mathbb{R}_h^s is free because $s \geq \bar{S}$ if s is part of these equations, and because either \mathbb{R}_h^s is not used to perturb (5.3.2), for h such that $I_{h^0}^\# = I$; or we can always avoid using one

such element if h is such that $I_h^{ss} < I$, since $S + 1 \leq S_h^0 > I \leq S_h^0$; completing the subcase.

3) If $I_h^{ss} < I$; all h : Then, as in the previous subcase, for $h > 1$ we throw away $I_h^{ss} = S_h^0$ equations (5.3.2), obtaining a submatrix of R_h^{n0} with more rows than columns, and we use ϕ_h to perturb the remaining equations (5.3.2). For equations (5.3.3b), we use $\bar{\phi}_h$. For $h = 1$; we keep equations (5.3.2) and perturb them with ϕ^2 : Then we throw away equations (5.3.3b), we use $\bar{\phi}_1$ to perturb equations (5.3.5). As for equations (5.3.7a), again we use ϕ_h^s for h such that $s \geq \underline{S}$; which we can always do as explained in the previous subcase, and for equation (5.3.8) we use ϕ^1 .

This ends the proof of case b).

In this way, we have established density. It now follows that, in the (weakly) generic set of economies for which G is a submersion, we can construct a new asset $y^0 = \phi y^0 \neq 0$ so that volatility increases in equilibrium, as well as a new asset $y^{00} = \phi y^{00} \neq 0$ so that volatility decreases. ■

Proof of the Corollary to Theorem 4.2.

We will focus on decrease of volatility, as the symmetric argument requires merely reversing an inequality. Fix $\hat{b}_h = \bar{b}_h = 1$; $h > 1$; and $y^{s(s)} = \bar{y}^{s(s)} = 0$; $s \in H + J$. Then, for $\hat{\mu} \in \hat{E}$, consider the system of equations and inequalities describing pairs of continuous and altered equilibria where the latter are both volatility-reducing and regular, that is,

$$\begin{aligned}
 & F(\mu; \hat{\mu}) = 0 \text{ or } F_1(\mu; \hat{\mu}) = 0 \\
 & F(\mu; \hat{\mu}) \begin{matrix} \hat{b}_h = \bar{b}_h; h > 1 \\ y^{s(s)} = \bar{y}^{s(s)}; s \in H + J \end{matrix} = 0 \\
 & G(\mu) \leq G(\hat{\mu}) \\
 & \text{and } \det D_{\mu; \hat{\mu}} F(\mu; \hat{\mu}) \begin{matrix} \hat{b}_h = \bar{b}_h; h > 1 \\ y^{s(s)} = \bar{y}^{s(s)}; s \in H \end{matrix} \leq 0.
 \end{aligned} \tag{5.4}$$

We will show that the projection of the solutions to (5.4) onto \hat{E} is open and dense. The proof is carried out in two steps.

First, it is straightforward to verify that, by virtue of the particular choice for \hat{b}_h ; $h > 1$, the projection $\pi : \mathbb{R}^{n_1} \times \mathbb{R}^{n_1} \times T \rightarrow \hat{E}$! \hat{E} restricted to the set defined by the first pair of equations in (5.4) is a proper mapping. Openness then follows directly from the fact that the denial of either of the second pair of inequalities in (5.4) is a closed property in the same set.

Now let $N_{\phi; \phi^0}$ be an open neighborhood of 0 in $\mathbb{R}^{n_1 + (H+1)}$, $N_{\phi; \phi^{00}}$ be an open neighborhood of 0 in \mathbb{R}^J and $N_{\phi; \phi^v}$ be an open neighborhood of 0 in \mathbb{R}^J ; intersected with \mathbb{R}_{++}^J . Then, for $\hat{\mu} \in \hat{E}$ with $\mu \in \mathbb{R}_u$, consider the system of $k^0 + 1 = (n_1 + J + 2n) + 1$ equations (representing differential volatility decrease with respect to a

critical equilibrium)

$$\begin{aligned}
 & F_1(\bar{y}; \bar{p}) = 0, \\
 & \sum_i DG(\bar{y}; \bar{z}) D_{y_i} F(\bar{y}; \bar{z}; \bar{p})^{-1} D_{z_i} F(\bar{y}; \bar{z}; \bar{p}) \Phi_i = \Phi_v, \\
 & F(\bar{y} + \Phi_y; \bar{z} + \Phi_z; \bar{p}) = 0, \\
 & a^0 D_{y; \delta; q} F(\bar{y} + \Phi_y; \bar{z} + \Phi_z; \bar{p}) = 0 \\
 & \text{and } a^0 a_{j-1} = 0
 \end{aligned} \tag{5.5}$$

in the k^0 "variables" $(\bar{y}; \Phi_y; \Phi_z; a) \in \mathbb{R}^{n_1} \times N_{\Phi_y; \Phi_z} \times N_{\Phi_z} \times \mathbb{R}^n$ and $\bar{p} = J + G^2 H J$ "parameters" $(\Phi_v; A) \in N_{\Phi_v} \times A$. It is only tedious to show that, by virtue of the particular choice for $y^{3/4}(s); z^{3/4}(s) \in H + J$, for $N_{\Phi_y; \Phi_z}; N_{\Phi_z}$ and N_{Φ_v} (as well as A and O) sufficiently small, the Jacobian matrix of this system has full rank (by perturbing each equation using the variables listed alongside). Hence we can once again apply the transversality theorem, and conclude that, generically in the "parameters", (5.5) has no solution. Density then follows from the fact that \bar{p} is open. The remainder of the argument is based on the implicit function theorem. Since it is always possible to restrict the analysis to the subset of utilities which have a compact domain containing the total resources of the given economy, and this subset is a Banach space, then (locally) the altered equilibrium depends smoothly on the yields from the volatility-reducing new asset. ■

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