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## Abstract

We investigate and empirically estimate optimal hedge ratios, for the first time, in the EU ETS carbon market. Minimum variance hedge ratios are conditionally estimated with multivariate GARCH models, and unconditionally by OLS and the naïve strategy for the European Climate Exchange (ECX) market in the period 2005-2009. Also, utility gains are considered in order to take into account risk-return considerations.

Empirical results indicate that dynamic hedging provides superior gains (in reducing the variance portfolio) compared to those obtained from static hedging, when adjustment costs are not taken into account. Moreover, results improve when the leptokurtic characteristics of the data are into consideration through distributions. Results are always compared in and out of sample, suggesting also that utility gains increase with investor's increased preference over risk.

**Keywords**: $CO_2$ Emission Allowances; Dynamic Hedging; Futures Prices; Risk Management; Spot Prices

**JEL classification**: C32, G19, G32, Q54

# 1   Introduction

In the context of controlling greenhouse gas emissions, the EU-wide trading system for emission allowances may be considered one of the major steps towards reducing the environmental burden. For market participants, academics, policy makers and especially traders/hedgers understanding the price behavior and the links between spot and futures in the European Union Trading System (EU ETS) of this new asset class (carbon dioxide $CO_2$ emission allowances) is of particular interest.

Under the emission cap-and-trade system of the EU ETS, $CO_2$ has become a kind of tradable good. With the evolution of the carbon trading market, not only the carbon spot market but also some derivative markets such as the carbon futures market and option market have gradually emerged. Price risk arise when futures prices fluctuate, making agents to assume long or short positions in the forward and spot markets to hedge their exposure to price risk.

There exists a large number of studies in the hedging area, which consider the hedge ratio across financial (stocks and indices), agricultural, livestock, interest rates, foreign exchange, metal and energy markets (fuels and electricity), etc. However, research on hedging in the carbon market is very limited, if almost no existent. Given that this is a very recent market (trading started in the early 2005), market immaturity, efficiency issues, liquidity and lack of data availability have been commonly cited as restrictions (Daskalakis and Markellos, 2008; Paolella and Taschini, 2008; Uhrig-Homburg, 2008; Daskalakis, Psychoyios and Markellos, 2009; Chevallier, 2010). Chevallier (2008) researched Phase I of the EU-ETS extensively with the emphasis on banking[1], pricing and risk hedging strategies, but he does not discusses the possible use of the optimal hedge ratio, and we try to fill here the gap in the existing literature.

The appropriate way to calculate hedge ratios remains a controversial issue in the literature. The major methodologies for hedging with futures contracts have been OLS, VAR, VECM and multivariate GARCH (Moschini and Myers, 2002; Moulton, 2005; Pen and Sévy, 2007; Hua, 2007; Kumar, Singh and Pandey, 2008; Torró, 2008; among others). Modelling the asymmetric behavior of the covariance matrix in a multivariate setting and studying its consequences in the ECX $CO_2$ allowances spot-future systems is the main object of this paper. As such, this work is an attempt to calculate and evaluate the effectiveness of the minimum variance hedge ratio and expected utility in the EU-ETS carbon market, that as far as we know has never been tested before.

In order to capture the dynamic structure of second moments conditional on the underlying and price variations, recent studies have concentrated in the development of hedging ratios changing through time using modelling techniques based on conditional heteroskedasticity. Multivariate GARCH models capture the dynamic evolution of the variance covariance matrix and construct an estimate of the optimal hedge ratio using the conditional variances and covariances of spot and futures returns. Different authors use different specifications and use valid arguments to justify one or the other (Byström, 2003, Torró, 2008, among others).

Torró (2008) uses Minimum Variance Hedge Ratio estimated by OLS and Multivariate GARCH with a bivariate t-student distribution. Moulton (2005) and Byström (2003) also use this as the main objective function. Lien and Tse (2000) consider the optimal strategy for hedging the downside risk measured by the lower partial moments in the Nikkei stock exchange. Lien and Tse (2002) evaluate constant hedge ratios and time-varying hedge ratios, exploring different econometric implementations. They provide a survey that reviews some recent developments in futures hedging. However, there are superior gains including heteroskedasticity and time-varying variances in the calculation of hedge ratios. As such, multivariate GARCH models are useful in reducing the variance portfolio.

The conditional heteroskedastic autoregressive specification (ARCH) was first presented by Engle (1982). It has been extended by Bollerslev (1986) to the generalized conditional heteroskedastic specification (GARCH). In fact, the great part of financial series contradict the constant correlation hypothesis as explored by Tse and Tsui (2002). In order to capture the different conditional correlation characteristics between rates, Engle and Kroner (1995) develop the BEKK procedure for the multivariate GARCH estimation. The BEKK algorithm allows changes through time of the conditional covariance which assumes the positiveness of the conditional variance covariance matrix.

Some of the deviations of the optimal hedge ratio are based on the minimization of return variance or maximization of the expected utility. Other derivations of the optimal hedge ratio are based on the mean-Gini

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[1]Banking of allowances means the carrying forward of the unused emission allowances from the current year for use in the following year. The banking of allowances is now permitted within Phases (except for France and Poland), but it was prohibited from 2007 to 2008 (inter-phase). This had significant implications for the pricing of emission allowance and its underlying derivatives, where we have seen prices decreasing towards zero between both phases (Daskalakis and Markellos, 2008). Nevertheless, industries are allowed to bank the unused permit from Phase II to Phase III in France.

coefficient and generalized semivariance. A brief discussion is provided by Chen, Lee and Shrestha (2001).

Milliaris and Urrutia (1991) used weekly data to estimate the optimal hedge ratio and found hedging to be more effective when the hedging horizon was equal to the frequency of the data. Also by using weekly data, Benet (1992) found that shorter hedging horizons produced more effective hedging. Moreover, Chen et al. (2003) stress the potential problem of matching the length of the hedging horizon with data frequency, which leads to the loss of data observations. Our work evolves with respect to those of Byström (2003) and Torró (2008) in this respect, favouring the main point of Moulton (2005), although we consider both static (naïve and OLS) and dynamic hedging strategies.

Moschini and Myers (2002) reject the null of a constant hedge ratio and that time variation in optimal hedge ratios can solely be explained by deterministic seasonality and time to maturity effects, using weekly corn cash and futures prices. They develop modified BEKK parameterization for the Bivariate GARCH(q,r) model. Ripple and Moosa (2005) examine the effect of the maturity of the futures contract used as the hedging instrument on the effectiveness of futures hedging, using daily and monthly data on the WTI crude oil futures and spot prices (NYMEX).

Hua (2007) estimates the constant and dynamic hedge ratios from 3 alternative modeling frameworks: OLS, VEC and MGARCH for Chinese copper futures markets, to conclude that the Multivariate GARCH dynamic hedge ratios are superior to other hedge ratio estimates in terms of portfolio variance reduction. Pen and Sévi (2007) use as objective function the minimum variance hedge ratio and model the dynamic and distributional properties of daily spot and forward electricity prices across European wholesale markets. They doubt of the potential of forward markets for hedging purpose using multivariate Garch models, including the diagonal BEKK. They confirm the poor performance of these models since the variance reduction obtained was near zero or even negative. In opposition we obtained a good performance for the EU-ETS market using the same specification, thus contradicting their results. This makes us believe on the effectiveness of multivariate GARCH models, specially BEKK, for hedging purposes.

Data selection is a very important aspect for several reasons. Not only due to a required large number of observations, but also because non-overlapping futures contracts are preferable to avoid artificially introducing autocorrelation in the data series. Therefore, the present study focus on daily hedging with futures, taking one price per day, in the ECX allowances market. In this work, minimum variance hedge ratios are conditionally and unconditionally estimated with the multivariate GARCH model, the OLS and Näive models. Empirical results indicate that dynamic hedging provides superior gains compared to those obtained from static hedging.

The rest of the work evolves as follows. Section 2 presents the methodology, presenting optimal hedge ratios estimation based on minimum variance hedge and maximization of expected utility, while it also presents the six hedging strategies to be used. Section 3 presents the data to be used and its summary statistics, while section 4 presents and discusses the results attained. Finally, section 5 concludes.

## 2 Methodology

Hedging is a very common term in the financial world but a proper definition depends on the player of the industry. To some, hedge means eliminate the risk in a position or in a portfolio. To others it simply means limit the risk. A hedge is an action, which reduces risk, usually at the expense of potential reward.

The most simple way to hedge a position is to enter an identical, but opposite position to off-set all the risk (replicating hedge)[2]. For linear positions, whose price is linear in the underlying price, futures are generally the simplest hedging instrument. If the goal is to minimize the risk with a future that does not behave equivalent to the position that is to be hedged, it might not be optimal from a hedging point of view to enter a future with the same underlying amount as the position to be hedged. Under certain assumptions one can actually find the optimal future position that minimizes the risk.

### 2.1 The optimal hedge ratio and evaluation of hedging effectiveness

In the "Optimal hedge ratio" one assumes that a company holds a long spot position that it wants to hedge with a future. Let $\Delta S$ define the change in the spot price $S$, during the period of time equal to the life of the hedge. $\Delta F$ defines the change in futures price $F$, during the same period. The standard deviation of $\Delta S$ and $\Delta F$ are given by $\sigma_S$ and $\sigma_F$ respectively. The correlation between $\Delta S$ and $\Delta F$ is given by $\rho$ and the hedge ratio, defined as the position in the future divided by the position in the spot is given by $h$.

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[2]One tries to replicate the risky position that is to be hedged and takes a short position in that replication.

The change in value of the hedged position will be given by

$$\Delta S - h\Delta F \qquad (1)$$

The variance $\sigma^2$, of the change in value of the hedged position is

$$\sigma^2 = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F \qquad (2)$$

and the derivative with respect to the hedge ratio is

$$\frac{\partial\sigma^2}{\partial h} = 2h\sigma_F^2 - 2\rho\sigma_S\sigma_F \qquad (3)$$

and since $\frac{\partial^2\sigma^2}{\partial h^2} = 2\sigma_F^2$ is positive, the first order condition is sufficient to find the $h$ that minimizes the variance namely

$$\frac{\partial\sigma^2}{\partial h} = 0 \Rightarrow h_t = \rho\frac{\sigma_S}{\sigma_F} = \frac{Cov(\Delta S_t, \Delta F_t)}{Var(\Delta F_t)} = \frac{\sigma_{sf}}{\sigma_f^2} \qquad (4)$$

which shows that the amount of future $CO_2$ contracts that should be purchased to minimize the risk of holdings of spot $CO_2$ allowances is proportional to the covariance of changes in the spot and future price of $CO_2$ divided by the variance of change in future prices. As such, the hedge ratio is basically the slope coefficient in a regression of the spot price (the instrument) on the price of the future instrument. But, as expected, this also depends on the hedgers objective function, being the minimum variance the most widely used approach.

We have assumed that $\Delta S_t$ and $\Delta F_t$ define the change in the spot price $(S)$ and in futures price $(F)$ during the period of time equal to the life of the hedge, respectively. Defining this time between $t$ and $t+1$ we will end up with $\Delta S_t = s_{t+1}$ and $\Delta F_t = f_{t+1}$, thus providing

$$h_t = \frac{Cov_t(s_{t+1}, f_{t+1})}{Var_t(f_{t+1})} \qquad (5)$$

The minimum variance approach as been object of several criticisms, being the strongest the fact that it does not take into account the expected return. But, each participant in the carbon market has its own preferences. While investors desire to protect the investment portfolio from carbon price risk, they also need to ensure high returns at the same time. However, priority of risk management for emitters may be solely to hedge the carbon price risk. As such, their objective function can be, but not limited to, the achievement of a minimum variance of the hedged portfolio.

It is certain that the hedge ratio $h$, will minimize the variance, but it is debatable if it is optimal, since we implicitly state that variance is the risk measure of concern. If we assume that the spot price follows a geometric Brownian motion and that the good is storable, then the cash-and-carry strategy implies that also the future price will follow the same price process. The returns of both the spot and the future will therefore be normally distributed, while variance or standard deviation will be the natural risk measure, and a variance minimization is appropriate.

At the present work we will measure the effectiveness of each estimated hedge ratio based on the variance reduction and utility maximization, or else from a utility gains standpoint.

The degree of hedging effectiveness we will consider here, proposed by Ederington (1979), is measured by the percentage reduction in the variance of spot price changes. Therefore, the degree of hedging effectiveness, denoted as EH, can be expressed as

$$EH = \frac{Var(\Delta S_t) - Var(\Delta h_t)}{Var(\Delta S_t)} = \rho_{sf,t}^2 \qquad (6)$$

where $\rho_{sf,t}^2$ is the square of the correlation between the change in the spot and futures prices.

The variance metric (EH) measures the percentage reduction in the variance of a hedged portfolio as compared with the variance of an unhedged portfolio. The hedged portfolios are calculated by using the OHR's derived from the hedging models, with the best model being the one with the largest reduction in the variance. The performance metric can be re-written as:

$$EH = 1 - \left[\frac{Variance_{hedgedPortfolio}}{Variance_{unhedgedPortfolio}}\right] \qquad (7)$$

This gives us the percentage reduction in the variance of the hedged portfolio as compared with the unhedged portfolio. When the futures contract completely eliminates risk, we obtain EH = 1 which indicates a 100% reduction in the variance, whereas we obtain EH = 0 when hedging with the futures contract does not reduce risk. Therefore, a larger number indicates better hedging performance.

The variance is a standard measure of risk in finance and has become the dominant measure of hedging effectiveness used by hedgers. It has also been extensively applied in the literature on hedging and was used by Ederington (1979) to evaluate hedging effectiveness. The advantage of using the variance as a measure of performance is its ease of calculation and interpretation.

Hedging strategies considering the risk-return structure over the portfolio have appeared to fulfill the lack delivered by the inconsistency of the minimum variance strategies by not considering the expected return of the portfolio in the determination of the optimal hedge ratio, as shown by Howard and D'Antonio (1984), Cecchetti et al. (1988) and Hsin et al. (1994).

Even though the existence of proposals to define a heding strategy are mostly consistent with the mean-variance structure of the portfolio, others have look to strategies being consistent also with the agent utility function, trying to determine the optimal ratio maximizing this utility.

Looking to the utility function of a risk averse agent:

$$U\left[E(r_{p,t}), \sigma_{p,t}; \eta\left(r_{p,t}\right)\right] \tag{8}$$

where $\eta\left(r_{p,t}\right)$ is the absolute risk aversion coefficient, presented by Pratt (1964) and computed as

$$\eta\left(r_{p,t}\right) = -\frac{U''(r_{p,t})}{U'(r_{p,t})} \tag{9}$$

Hsin et al. (1994) assume that the agent that looks for an hedging strategy is risk averse. As such, his expected utility function is concave, conditioned on a constant absolute risk aversion measure. In using this method, the level of investor's utility will be computed differently from the hedged portfolio and after, compared and ranked by the degree of utility improvement from the unhedged portfolio.

Considering the return of the hedged portfolio, his variance and that transaction costs equal zero, the authors determine the optimal ratio in contracts on the futures market to hedge a position of an asset in the spot market, given by the maximization of the utility function relative to h, where the expected utility is:

$$E[U(r_{p,t})|\psi_{t-1}] = E[r_{p,t}|\psi_{t-1}] - \lambda Var[r_{p,t}|\psi_{t-1}] \tag{10}$$

and

$$\underset{h}{Max}U\left[E(r_{p,t}), \sigma_{p,t}; \eta\left(r_{p,t}\right)\right] = \underset{h}{Max}E\left(r_{p,t}\right) - 0,5\eta\left(r_{p,t}\right)\sigma_{p,t}^2 \tag{11}$$

where $r_{p,t}$ is the hedged portfolio (1), or else $\Delta S - h\Delta F$, $E(r_{p,t})$ is the expected return of the hedged portfolio, $Var(r_{p,t})$ its variance and $\eta = 2\lambda$ $(\lambda = \frac{1}{2}\eta)$ is the investor's level of risk aversion, which we will consider to be $\eta = 1$ (risk averse), $\eta = 2$ (risk neutral), and $\eta = 4$ (risk lover). Finally, $\psi_{t-1}$ stands for the information set at time $t-1$.

The extreme value of the expected utility function is given when the first derivative equals zero

$$\frac{\partial U\left[E(r_{p,t}), \sigma_{p,t}; \eta\left(r_{p,t}\right)\right]}{\partial h} = 0 \tag{12}$$

which yields

$$h = \frac{\sigma\left(r_s, r_f\right)}{\sigma^2\left(r_f\right)} - \frac{E\left(r_f\right)}{\sigma^2\left(r_f\right)\eta\left(r_h\right)} \tag{13}$$

This strategy incorporates the risk-return structure of the portfolio to determine the optimal hedge ratio, but for it to be consistent it is necessary the agent expected utility function to be quadratic or that the returns of the hedged portfolio would be normally distributed, once she assumes it explicitly.

Therefore, researchers tried to derive the optimal hedge ratio based on a structure that does not depends on such assumptions. An alternative was to use as a measure of portfolio risk the extended Gini coefficient, instead of the variance of the hedged portfolio, as it is consistent with the rules of the stochastic dominance.

Still, the MV hedge ratio is the most heavily used, analyzed, and discussed hedge ratio, and it can also be shown that, under some normality and martingale conditions, most of the hedge ratios based on other criteria (expected utility, extended mean-Gini coefficient, and generalized semi-variance) converge to the MV hedge ratio (Chen, Lee and Shrestha, 2001).

## 2.2 Hedge ratio estimation models

There are basically two hedging strategies categories: the static and the dynamic. By static hedging we mean that once the optimal hedging ratio is defined, the position in the futures market is kept constant until the end of the hedging period. Näsäkkälä and Keppo (2005) study partial hedging of electricity cash flows with static forward strategies. The dynamic strategy occurs when defined the optimal hedge ratio, this one is constantly monitored and the position in the futures market continuously rebalanced. However, the constant rebalancing becomes expensive to the hedger due to operational costs.

### 2.2.1 Static hedge ratio estimation models

We will assume that the market is incomplete, therefore not all the risks are hedgeable through trading the underlying stock. If the market were complete, given sufficient initial capital, all claims could be replicated by trading the stock dynamically. Static derivatives hedges do not add anything to dynamic hedges in complete markets, but of course they are very valuable tools in realistic incomplete market models, where there may be risk factors that cannot be eliminated just by dynamic trading of the underlying stock. By incorporating static hedges, we enlarge the set of feasible hedging strategies that the investor can choose from and allow for a better hedging performance.

When a hedge where the futures position have the same size but the opposite sign than the position held in the spot market is considered, we have what is called a naïve hedge ratio ($h_t = 1, \forall t$). The naïve model has a lower perceived value in practice, but will be used here for comparison purposes.

We have also estimated the hedge ratio through the OLS method. Empirically, the one period hedge ratio is estimated by the slope from the following ordinary least squared (OLS) regression equation:

$$s_{t+1} = \alpha + h^* f_{t+1} + \varepsilon_t \tag{14}$$

where $\varepsilon_t$ is the error term from OLS estimation, $s_{t+1}$ and $f_{t+1}$ are the changes in the spot and futures prices, respectively, between time $t$ and $t+1$, and $h^*$ is the minimum hedge ratio.

### 2.2.2 Time-varying (dynamic) hedge ratio estimation models

The static hedging strategy determines the equilibrium point or neutral point of the dynamic hedging strategy. If the position taken in derivatives changes over time, the hedging strategy is dynamic.

Multivariate models can be used for the computation of optimal hedge ratios. Selected multivariate models for this presentation are: the BEKK, the Diagonal BEKK, the CCC and DCC models. As such, we are also able to compare different parameterization.

Developed by Engle (1982) and then Bollerslev (1986), the autoregressive conditional heteroskedasticity model (ARCH) sparkled a substantial body of work which concerns with not only further examining the second moment of economic and financial time series, but also extending and generalizing the initial ARCH model to better fit the situation being investigated. Bollerslev, Engle and Wooldridge (1988) generalized the univariate GARCH to a multivariate dimension to simultaneously model the conditional variance and covariance of two interacted series. This multivariate GARCH model is thus applied to the calculation of dynamic hedge ratios that vary over time based on the conditional variance and covariance of the spot and futures prices. Engle and Kroner (1995) present various MGARCH models with variations to the conditional variance-covariance matrix of equations.

Generalized from GARCH(1,1), a standard M-GARCH(1,1) model is expressed as:

$$\begin{bmatrix} \sigma^2_{ss,t} \\ \sigma^2_{sf,t} \\ \sigma^2_{ff,t} \end{bmatrix} = \begin{bmatrix} c_{ss,t} \\ c_{sf,t} \\ c_{ff,t} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} \varepsilon^2_{s,t-1} \\ \varepsilon_{s,t-1}\varepsilon_{f,t-1} \\ \varepsilon^2_{f,t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \times \begin{bmatrix} \sigma^2_{ss,t-1} \\ \sigma^2_{sf,t-1} \\ \sigma^2_{ff,t-1} \end{bmatrix} \tag{15}$$

where $\sigma^2_{ss}, \sigma^2_{ff}$ are the conditional variance of the errors $(\varepsilon_{s,t}, \varepsilon_{f,t})$ from the mean equations, $c_{ij}, a_{ij}$ and $b_{ij}$ are coefficients. Where we have that:

$$\varepsilon_t | \phi_{t-1} \sim BN\left(0, H_t\right) \text{ with} \tag{16}$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{ft} \end{bmatrix} \text{ and } H_t = \begin{bmatrix} \sigma^2_{ss,t} & \sigma^2_{sf,t} \\ \sigma^2_{fs,t} & \sigma^2_{ff,t} \end{bmatrix} \tag{17}$$

To maintain a reasonable number of parameters and positive definiteness of the covariance matrix, different parametrization for the conditional covariances matrices are proposed.

**The BEKK and Diagonal BEKK models**  Here is presented the BEKK model of Engle and Kroner (1995) (named after an earlier working paper by Baba, Engle, Kraft and Kroner). In its full parametrization, the BEKK model can be written as

$$\Sigma_t = C'C + B'\Sigma_{t-1}B + A'\eta_{t-1}\eta'_{t-1}A \tag{18}$$

where C is a lower triangular matrix, and B and A are square matrices. Positive definiteness is guaranteed by the use of quadratic forms. Hence, strong restrictions that have to be made on the VEC model to ensure positive definiteness are bypassed. Restrictions of the BEKK model include the diagonal BEKK and the scalar BEKK. In the diagonal BEKK, matrices B and A are diagonal matrices. In the scalar BEKK, B and A are scalars. We will only look at the BEKK and diagonal versions.

Drawbacks from the BEKK parametrization are: (i) the remaining significant number of parameters to estimate which still grows with $O(n^2)$. For a BEKK model with one lag on ARCH and GARCH components, this give $(5n^2 + n) = 2$ coefficients. (ii) the impossibility to interpret estimated coefficients. Any covariability persistence is then difficult to characterize. (iii) the implicit hypothesis of a constant correlation structure. It is then useful to enrich the structure of the model by allowing for time-varying correlations.

Karolyi (1995) suggests that the BEKK (Baba, Engle, Kraft and Kroner) model allows the conditional variance and covariance of the spot and futures prices to influence each other, and, at the same time, do not require the estimation of a large number of parameters to be employed. The model also ensures the condition of a positive semi-definite conditional variance-covariance matrix in the optimization process which is a necessary condition for the estimated variance to be zero or positive. The BEKK parameterization for the MGARCH(1,1) model is written as:

$$
\begin{aligned}
\begin{bmatrix} \sigma^2_{ss,t} & \sigma^2_{sf,t} \\ \sigma^2_{fs,t} & \sigma^2_{ff,t} \end{bmatrix}
= \;&
\begin{bmatrix} c_{ss} & c_{sf} \\ 0 & c_{ff} \end{bmatrix}'
\begin{bmatrix} c_{ss} & c_{sf} \\ 0 & c_{ff} \end{bmatrix} + \\
&
\begin{bmatrix} a_{ss} & a_{sf} \\ a_{fs} & a_{ff} \end{bmatrix}'
\begin{bmatrix} \varepsilon^2_{s,t-1} & \varepsilon_{s,t-1}\varepsilon_{f,t-1} \\ \varepsilon_{f,t-1}\varepsilon_{s,t-1} & \varepsilon^2_{f,t-1} \end{bmatrix}
\begin{bmatrix} a_{ss} & a_{sf} \\ a_{fs} & a_{ff} \end{bmatrix} + \\
&
+\begin{bmatrix} b_{ss} & b_{sf} \\ b_{fs} & b_{ff} \end{bmatrix}'
\begin{bmatrix} \sigma^2_{ss,t-1} & \sigma^2_{sf,t-1} \\ \sigma^2_{fs,t-1} & \sigma^2_{ff,t-1} \end{bmatrix}
\begin{bmatrix} b_{ss} & b_{sf} \\ b_{fs} & b_{ff} \end{bmatrix}
\end{aligned} \tag{19}
$$

where $\sigma^2_{ss,t}, \sigma^2_{ff,t}$ and $\sigma^2_{sf,t}$ are the conditional variance and covariance of the errors $(\varepsilon_{st}, \varepsilon_{ft})$ from mean equations, so that we allow for the cointegration relationship in the series. Conditional variance and covariance only depend on their own lagged squared residuals and lagged values. The MGARCH model incorporates a time-varying conditional covariance and variance between the spot and futures prices and hence generates more realistic time-varying hedge ratios. The BHHH (Berndt, Hall, Hall and Hausman) algorithm is used to produce the maximum likelihood parameter estimates and their corresponding asymptotic standard errors.

Notice that the assumption of normality in allowances log-price variation is not a realistic one. Has we will see in the summary statistics of the data, one fact that characterizes allowances price distribution is its leptokurtosis. As such, as an alternative empirical distribution to the normal one we will also use the bivariate t-student distribution in the multivariate-GARCH BEKK  and Diagonal BEKK models used here:

$$\varepsilon_t|\phi_{t-1} \sim t\left(0, H_t, v\right) \tag{20}$$

where $v$ is the degrees of freedom parameter of a conditional bivariate t-student distribution.

Bivariate GARCH modelling allows to model the conditional second moments, but also the cross moments, with special relevance, in our case, to the contemporaneous covariance between electricity spot and futures. That's why the conditional, on time $t-1$ available information, error term vector follows a bivariate normal law, and for the comparison purpose also a bivariate $t$ distribution, being $H_t$ the positive definite variance covariance matrix dependent on time.

In view of the excessively large number of parameters needed to be estimated in the model, Bollerslev (1990) proposed an assumption that matrix $A_i$ and $B_i$ are diagonal and the correlation between the conditional variances are to be constant. Bollerslev, Engle and Wooldridge (1988) propose a parameterization of the conditional variance equation in the multivariate-GARCH model termed the Diagonal BEKK model which allows for a time-varying conditional variance. Like the constant correlation model, the off-diagonal in the matrices $A_i$ and $B_i$ are set to zero, i.e. the conditional variance depends only on its own lagged squared residuals and lagged values. Following Bollerslev, Engle and Wooldridge (1988), the diagonal representation of the conditional variances elements $\sigma^2_{ss}$ and $\sigma^2_{ff}$ and the covariance element $\sigma^2_{sf}$ can be expressed as:

$$\begin{aligned}
\sigma^2_{ss,t} &= c_{ss} + a_{ss}\varepsilon^2_{s,t-1} + b_{ss}\sigma^2_{ss,t-1} \\
\sigma^2_{sf,t} &= c_{sf} + a_{sf}\varepsilon_{s,t-1}\varepsilon_{f,t-1} + b_{sf}\sigma^2_{sf,t-1} \\
\sigma^2_{ff,t} &= c_{ff} + a_{ff}\varepsilon^2_{f,t-1} + b_{ff}\sigma^2_{ff,t-1}
\end{aligned} \tag{21}$$

This BEKK multivariate GARCH model employed in this paper explicitly incorporates a time varying conditional correlation coefficient between the spot and futures prices and hence generates more realistic time-varying hedge ratios.

### 2.2.3 The Bollerslev's (1990) CCC

In the Bollerslev's (1990) model, covariances between $i$ and $j$ are allowed to vary only through the product of standard deviations with a correlation coefficient which is constant through time (constant correlation model or CCC). The dynamic of standard deviations is governed by the GARCH(1,1) variances' dynamic or any univariate GARCH model. Keeping the covariance matrix $\Sigma_t = [\sigma_{ij,t}]$, we have

$$\sigma_{ii,t} = \omega_{ii} + \beta_{ii}\sigma_{ii,t-1} + \alpha_{ii}\eta^2_{i,t} \tag{22}$$

and

$$\sigma_{ij,t} = \rho_{ij}\sqrt{\sigma_{ii,t}\sigma_{jj,t}} \tag{23}$$

As pointed out by Bollerslev (1990), under the assumption of constant correlation, MLE of the correlation matrix and sample-based correlation matrix coincide. Because of the positive semi-definiteness of the sample-based estimate, the same is guaranteed for the conditional covariance matrix. The main advantage of this model is to greatly simplify computation by keeping out of the likelihood function the correlation matrix. The number of parameters to estimate when a GARCH(1,1) is retained is n(n+5)=2. The main drawback of this model is that the sign of the conditional correlation is constant over time once $\rho_{ij}$ is estimated. This may be a problem in the estimation of OHRs.

### 2.2.4 The DCC model of Engle (2002)

Correlations between returns may not be constant in time. They may be stronger when prices are falling. To model this feature of the series some dynamic correlation models can be employed in order to avoid an implicit loss of information when estimating conditional variances and covariances. Among dynamic correlation models is that of Engle (2002).

The general form of the dynamic conditional correlation (DCC) model introduced by Engle (2002) is defined by

$$\begin{aligned}
\Sigma_t &= D_t R_t D_t \\
\text{where } R_t &= Q_t^{*-1} Q_t Q_t^{*-1} \\
\text{and } Q_t &= \left(1 - \sum_{p=1}^{P}\alpha_p - \sum_{q=1}^{Q}\beta_q\right)\overline{Q} + \sum_{p=1}^{P}\alpha_p\left(\eta_{t-p}\eta'_{t-p'}\right) + \sum_{q=1}^{Q}\beta_q Q_{t-q}
\end{aligned} \tag{24}$$

where $D_t$ is a $n \times n$ diagonal matrix of time varying standard deviations defined by any univariate GARCH model, $D_t$ is a $n \times n$ time varying correlation matrix, $Q_t$ is defined above, $\overline{Q}$ is the unconditional covariance matrix using standardized residuals from the univariate estimates, and $Q_t^*$ is a diagonal matrix of the square root of the diagonal elements of $Q_t$. We then have the time varying correlation matrix defined as $R_t = [\rho_{ij,t}]$ with $[\rho_{ij,t}] = \frac{q_{ij,t}}{\sqrt{q_{ii}q_{jj}}}$. DCC differs from CCC mainly in that it allows the correlation matrix to be changed over time.

Interestingly, the DCC model can be estimated in two steps and the number of parameters to estimate is greatly reduced. The model is then manageable for a greater number of series. The model also keeps intuition in the interpretation of the parameters, which is lost by using a factor model where parameters describe an unobserved variable. Nevertheless, this simplification is made at a cost. Indeed, an implicit assumption of the DCC model is that $\alpha_p$ and $\beta_q$ being scalars, all correlations obey the same dynamic.

# 3  Data and Summary Statistics

The European Union (EU) clearly indicated its will against the fight of global warming when in 2005 they decided to trade European Union Allowances (EUAs), each representing the right to emit one ton of $CO_2$ in the atmosphere.

Established under Directive 2003/87/EC, the EU ETS (EU Emissions Trading Scheme) regulates the carbon dioxide emissions ($CO_2$) from installations across the EU, which includes power generation, mineral oil refineries, offshore installations, and other heavy industrial sectors in its first phase from 2005-2007 (Phase I or pre-Kyoto period) and in its second phase from 2008-2012 (Phase II or Kyoto period). Further 5-years phase will follow and $CO_2$ emission allowances are currently being traded on electricity power exchanges. We have decided to work with data from Powernext in France who trades $CO_2$ spots.

$CO_2$ has thus become a kind of tradable good where initially each member state decides, through the National Allocation Plan, how much EUAs to emit and how those will be distributed to each installation. If an installation emits bellow its level then at the end of the compliance year it can trade the excess EUAs; or it may need to buy EUAs due to excess emission in a given year, otherwise it will be forced to pay an excess emissions penalty. With the evolution of the carbon trading market, not only the carbon spot but also some derivatives markets such as the carbon futures and options markets have gradually emerged.

Due to the newness of this market, data of any useful size and quality has only recently become available. This article uses daily (Monday to Friday) $CO_2$ spot and futures prices for more than 4 years, June 24, 2005 to October 9, 2009, thus extending the data span considered by previous authors that mostly cover Phase I period contracts (Daskalakis and Markellos, 2008; Paolella and Taschini, 2008; Uhrig-Homburg, 2008; Chevallier, 2008, 2010).

From these daily prices quotes in Euro (€) per metric tonne, daily returns (log price first differences) are calculated. Data used comes from the French electricity market Powernext[3], whose trading of $CO_2$ allowances is performed on the Bluenext, the market place dedicated to $CO_2$ spot trading, based in Paris and created on June 24, 2005.

Trading of emission allowances futures contracts is primarily performed through the European Climate Exchange (ECX). Since the ECX does not allows spot EUA trading, it uses Bluenext spot prices as a reference for the futures contracts. ECX EUA Futures contracts were the first emissions products to be listed on the Intercontinental Exchange (ICE) Futures Europe platform in UK on April 22, 2005. ECX EUA Futures are based on underlying EU allowances (EUAs) and provide the market with standardized contract terms and a benchmark for price discovery. ICE/ECX continues to be the most liquid and transparent platform for EUA trading offering transparent screen trading with tight spreads as well as the clearing of over-the-counter positions. Contracts are listed on an quarterly expiry cycle such that March, June, September and December contract months are listed up to March 2013 and annual contracts with December expiries for 2013 and 2014. We choose to work with December contracts only, which are physically settled three days after expiry with the maturity date being the last business day of December in ECX.

As argued by Daskalakis, Psychoyios and Markellos (2009), the pricing mechanism and relationship between spot and futures allowances prices may vary considerably depending on if the futures contract is written and expires in the same phase or between different phases of the EU ETS, respectively. We have performed empirical tests using the methodologies presented before for all current December contracts traded on ECX (Futures December 2005 - FutDec05 - through Futures December 2012 - FutDec12). However, results turn out to be very similar in terms of general conclusions. As such, and in order to save space[4], we have decided to work only with the Future Contract maturing on December 2009 (FutDec09) given that for this specific contract we have data since June 24, 2005 until October 9, 2009, thus covering our entire data span[5]. As such, in the empirical application presented next, only one future contract (that maturing on December 2009) is considered to hedge the spot price variation at a daily scale.

Summary Statistics of the spot price and EUA futures contracts for all delivery dates (from 2005 to 2012) in the ECX market are provided in table 1.

Table 1: Descriptive Statistics of spot prices and futures price contracts, both in logarithmic returns for the ECX/Bluenext market

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[3] We would like to thank them for providing us with the necessary $CO_2$ spot data.

[4] Results for all these contracts, using the hedging strategies applied, will be provided upon request.

[5] Phase II contracts have started to be trading also during Phase I, and thus FutDec09 is the contract which allows us to have a complete picture of the whole scenario.

Table 1: Descriptive Statistics

| ECX Series | mean | variance | skewness | kurtosis |
|---|---|---|---|---|
| Spot CO2 | 0.044 | 4.045 | 0.671 | 45.072 |
| FutDec05 | 0.132 | 2.831 | -1.811 | 12.494 |
| FutDec06 | -0.223 | 4.864 | -0.292 | 44.226 |
| FutDec07 | -0.918 | 7.423 | -0.821 | 18.152 |
| FutDec08 | 0.110 | 2.944 | -1.558 | 10.310 |
| FutDec09 | -0.009 | 3.353 | -1.718 | 20.844 |
| FutDec10 | -0.002 | 3.322 | -1.660 | 20.104 |
| FutDec11 | 0.005 | 3.335 | -1.600 | 18.576 |
| FutDec12 | 0.011 | 3.404 | -1.564 | 16.965 |

Spot refers to ECX $CO_2$ Spot prices, FutDec05 to FutDec12 refer to ECX December 2005 to 2012 $CO_2$ Futures contracts; The variables are the standard ones.

Alberola and Chevallier (2009) show that banking restrictions[6] between 2007 and 2008 caused the disconnection of spot and futures prices between Phase I and Phase II.

Besides this also a structural break for carbon prices of all maturities occurred in April 2006 due to information revelation (Alberola, Chevallier and Chèze, 2008). The 2008 onwards decreasing EUAs prices are justified by the decreasing volume demand, a product of the worldwide financial crisis. EUAs were traded at €15 in March 2007, then stayed in the range of €19-25 until July 2008, and decreased steadily afterwards to achieve €8 in February 2009.

Futures of all maturities present negative skewness and excess kurtosis (for a normal distributed random variable skewness is zero and kurtosis is three). We may observe from table 1 the absence of normality in the returns, and data fat tail leptokurtic distributions. As such, we have heteroscedasticity presented on the series under analysis and MGARCH models are able to capture the data properties in a proper way.

Emission allowances are characterized by high historical volatility, as they were also previously in the literature (Paolella and Taschini, 2008; Daskalakis, Psychoyios and Markellos, 2009). Volatility is higher for FutDec06 and FutDec07, which should be expected given the immaturity of the ECX market during Phase I. However, future 2008 contracts through futures 2012 contracts evidence a much more similar volatile behavior between them, which may indicate a stabilized investors learning process, and when we compare Phase II futures contracts with the spot $CO_2$ allowances returns, we see that the latter is more volatile than the formers. As such, being $CO_2$ a commodity, its spot price is more volatile than futures[7].

In order to apply the methodologies presented previously, we also need to ensure the data stationarity.

Apart from the augmented Dicky-Fuller (ADF) tests, which attempt to account for temporally dependent and heterogeneously distributed errors by including lagged sequences of first differences of the variable in its set of regressors, the KPSS test can also be used. The null hypothesis for ADF test is that the variables contain a unit root or they are non-stationary at a certain significant level. In the KPSS tests, proposed by Kwiatkowski et al. (1992), the null hypothesis is that a series is stationary around a deterministic trend (TS) and the alternative hypothesis is that the series is difference stationary (DS).

We have performed ADF and KPSS tests for the market and strategies considered. We are working with spot and futures returns (log price first differences) and these tests confirm that series are stationary[8].

We also have to check cointegration in $CO_2$ allowances markets, and for this we use the Johansen's test. Although results are not presented here[9], correlation values revealed to be high, which will then ensure a good risk rduction for hedgers, as we will be able to confirm in the results of the empirical part.

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[6]According to the proposal of EU ETS, allowance banking and borrowing between Phasr I and II were prohibited. Hence, at the end of 2007, when the first phase of EU ETS came to its end, a palpable seem between the two phases appeared, which lead the carbon spot price to approach zero.

[7]As argued by Lien and Shrestha (2007): "*In the case of commodities, the futures markets are more liquid than the spot markets. Consequently, the variances of futures returns are much smaller than that of the spot returns for commodities*".

[8]Results will be provided upon request.

[9]Results will also be provided upon request.

# 4 Empirical Results

This paper presents empirical results about hedging allowances price risk with futures when an early daily cancellation of futures positions is made. As previously mentioned, to compare the hedging effectiveness and utility maximization obtained through the strategy, both risk reduction and utility gains are computed. Furthermore, ex post and ex ante results will be distinguished by splitting the data sample into two parts. In the first part, the hedging strategy is compared ex post, whereas in the second part, an ex ante approach is used. That is, in the ex ante study, strategies are compared using forecasted hedge ratios and models are estimated every time a new observation is considered by maximizing the log-likelihood function for multivariate GARCH BEKK models and quasi-likelihood maximization for the estimation of CCC and DCC models.

In the following we will present the results obtained using the empirical methodologies presented before.

Figures 1 to 4 show the estimated spot and futures volatility from each multivariate model (figures 1 and 2, respectively) and the estimated covariance and conditional correlations (figures 3 and 4, respectively).

Volatility estimated by the six different multivariate models adopted are presented in figures 1 and 2, being the spot $CO_2$ conditional volatility presented in figure 1, and Future December 2009 conditional volatility presented in figure 2.

Figure 1: Conditional volatility for the spot $CO_2$ allowances in the ECX market
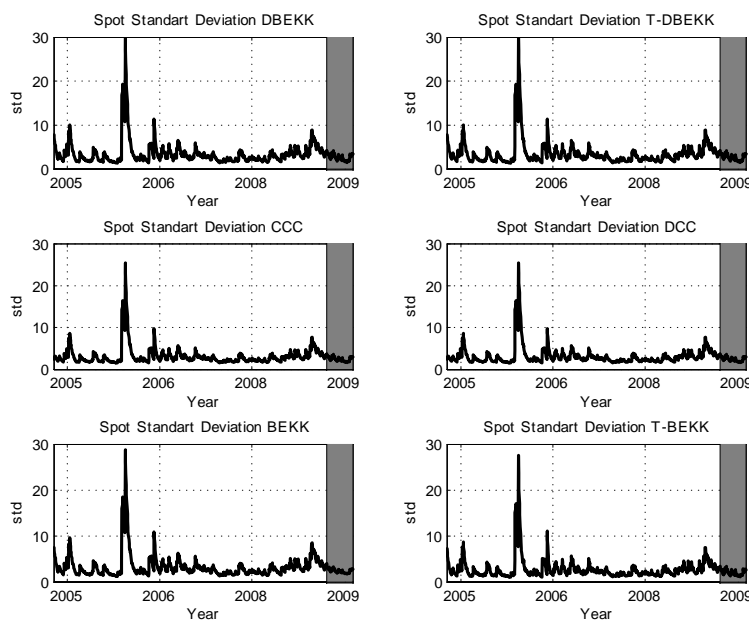


Comparing both figures we may see that conditional volatility estimated through the multivariate models is lower the December 2009 future contract with regard to its benchmark ($CO_2$ spot), which confirms the results obtained in the summary statistics.

Thus, it seems that portfolios which replicated the spot obtained lower volatility, i.e. risk levels, with this effect being particularly noticeable during periods of maximum volatility. These periods of maximum uncertainty started at the end of 2006 and the year 2007, while after we also have increased uncertainty in the second week of October 2008 which then ran to January 2009. It seems to have been caused by a growing lack of confidence of the agents operating in the stock market, caused by the worldwide crisis and the spread to all other financial and commodity markets around the globe.

Figure 2: Conditional volatility for the Futures December 2009 $CO_2$ allowances in the ECX market
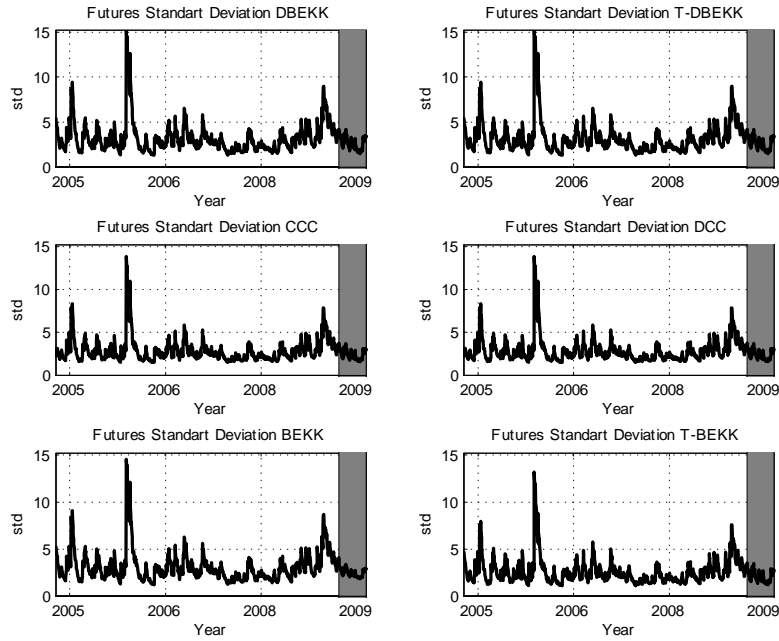
Results for the conditional covariance between EU ETS allowances and futures maturing in December 2009 are plotted in figure 3. This figure illustrates results of covariance estimated for in-sample prediction based on different econometric models that we have mentioned previously. Generally speaking, there are no significant differences in covariance forecasting performance, despite the MGARCH model used under the in-sample context. Both correlations (figure 4) and covariances are all positive and similar in absolute term (values) for all of these models. Moreover, by looking at the plots the only difference that seems to exist among models is the estimated correlation process. However, we can find that their covariance process have salient differences and accordingly it seems inappropriate to assume that the correlation parameter between $CO_2$ spot and futures is constant over time.

Figure 3: Conditional covariances between spot and futures $CO_2$ allowances for the ECX market during the period 2005-2009
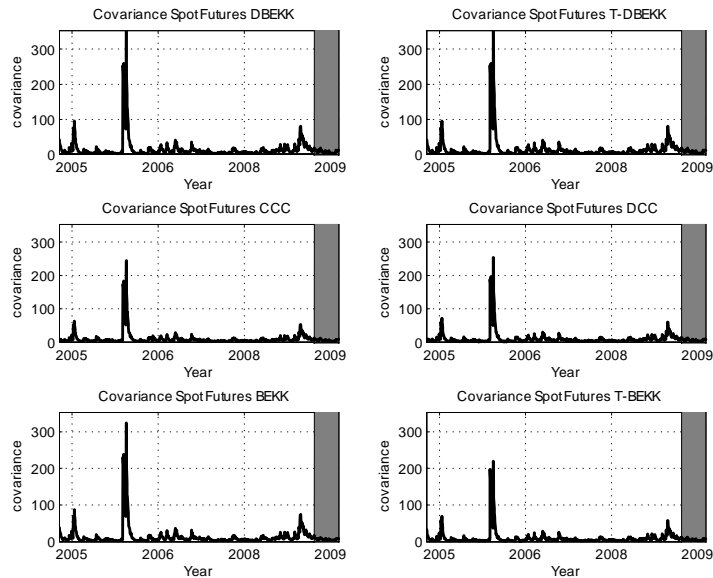


Figure 3 shows that covariance values are higher using BEKK models, with a peak value around 325, while being (most of the time) very close to zero. But, at the same time we see the conditional covariance approaching zero we also see conditional correlations very close to one (figure 4).

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Apart from such considerations it is remarkable that the evolution of returns estimated by the multivariate models for both $CO_2$ spot and FutDec09 are strongly correlated (figure 4) to an estimated value of near one most of the time. The exception is for the conditional correlations implied by the CCC model. In general during 2005 and 2006 we see conditional correlations deviate from the value 1 (perfect correlation), but still remained very high (between 0,5 and 1).

The price level and returns in 2008 hedging horizon has opened the way for a series of dynamic variances and covariances which are plotted has being fairly stable. Given that we can consider the Kyoto period a more mature phase when compared with the learning phase of the pre-kyoto commitment (Phase I), when increased and clustered volatility was evident, these calmer optimal hedge ratios for 2008 and 2009 are somehow expected.

Figure 4: Conditional correlations between spot and FutDec09 in the ECX $CO_2$ allowances market
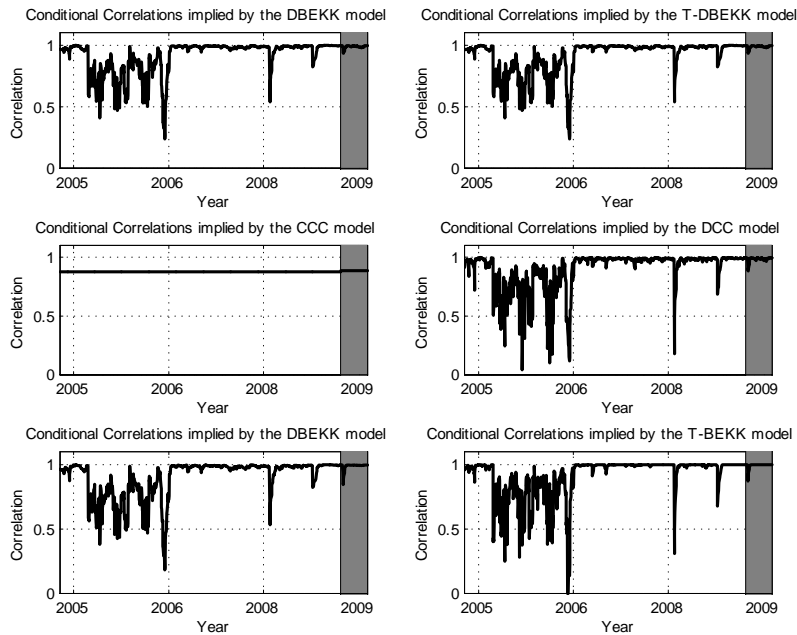


Obviously, the correlation results are important for EU ETS allowances price risk management, as they show that December Futures will provide a good risk reduction for hedgers. The time variation pattern documented in this study may carry some important implications for hedging. The instability in various aspects of market comovements may imply serious limitations to the investor's ability to exploit potential benefits from hedging with futures contracts in allowances markets. Much variation in the contemporaneous relationships among spot and futures prices may also highlight inadequacy in assuming (short-term) relationships in both markets, which might account for the difficulty in achieving profitable active trading.

The conditional hedge ratios derived by MGARCH models are graphed in figure 5. The computed values move around their unconditional values, and consequently, their performance is expected to be quite similar. In this figure, the dynamic optimal hedge ratio is plotted against the fixed optimal hedge ratio derived using OLS and Naive strategies.

Figure 5: Conditional Hedge ratios plot using FutDec09 to cover the spot $CO_2$ position in the ECX allowances market

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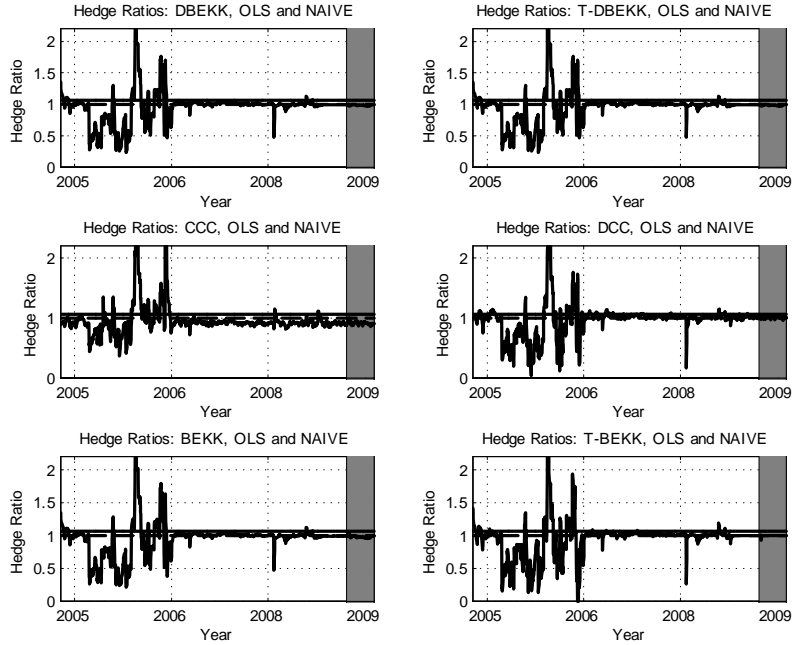
The horizontal axis indicates the hedging horizon while the vertical one represents the level of hedge ratios. The fluctuating line represents the conditional hedge ratio at each point in time obtained through the six considered dynamic MGARCH models (one plot for each), while the straight lines represent the constant hedge ratio (the solid straight line for OLS and the broken straight line being the naïve hedge ratio).

Results suggest that despite the volatile behavior evident during 2005 and 2006, for the rest of the time this hedge ratio clearly approaches its long run equilibrium value of one. The degree of hedging effectiveness approaches one because the shared permanent component ties both spot and futures in allowances markets. This also implies that the effect of the transitory components becomes weaker. As such, in the long run, the spot and futures prices are perfectly correlated in these newly markets (favouring In and Kim (2006, 2006a); and Fernandez (2008) results, for different commodities and financial assets).

Moreover, we see a very volatile behavior of estimated conditional hedge ratios for Phase I values, while an outlier at the beginning of 2008 is also observed. The former is explained by the investors uncertain expectations about the spot and futures $CO_2$ markets given the newness of the market. The latter (the sudden extreme jump in variance and covariance) deserves a more careful analysis and probably a structural break test would provide more insightful conclusions. In sum, optimal hedge ratios are very sensitive to changes in prices since the hedged portfolio is calculated on a daily basis.

Table 2 displays the variance reduction for the hedging combination spot $CO_2$ and Futures December 2009. The middle column reports in-the-sample results for the period June 24, 2005 to May 13, 2009. The last column reports out-of-sample results for the period May 14, 2009 to October 9, 2009 (around 100 observations). In this table the variance of a hedge strategy is calculated as the variance of the hedged portfolio. The risk reduction achieved for each strategy is computed by comparison with the variance of the spot position (the spot variance or else, assuming no hedging, $h = 0$).

Table 2: Hedging Effectiveness results in and out-of sample using ECX and Bluenext market data

In view of in sample and out of sample empirical results, we cannot clearly put all forecasting models in a proper order. However it is undoubted that the class of BEKK models possess the optimal forecasting power in covariance.

the dynamic hedging methods perform better than the static hedging strategies at a first look and not considering transaction costs[10]. One of the reasons for this result is the commodity we choose to work with,

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[10] Trasaction costs will not be considered when comparing hedging methods as the hedging theoretical framework is a one-period model for all hedging methods. As such, the individual must take futures positions at the beginning of the period and cancel them at the end of the period. As hedging ratio values revealed to be very similar in the considered methods, all will have similar transaction costs.

|  | In the Sample | Out of Sample |
|---|---|---|
| Spot variance (no hedging) $(h = 0)$ | 13.61 | 4.53 |
| | | |
| Hedging | Risk reduction | Risk reduction |
| Naive $(h = 1)$ | 74.32 | 98.62 |
| OLS $\left(h = \frac{\sigma_{FS}}{\sigma_F^2}\right)$ | 74.04 | 99.04 |
| Diag-BEKK $\left(h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$ | 85.53 | 99.04 |
| T-Diag-BEKK $\left(h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$ | 86.63[+] | 99.06 |
| CCC $\left(h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$ | 83.72 | 97.72 |
| DCC $\left(h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$ | 85.64 | 99.01 |
| BEKK $\left(h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$ | 85.34 | 99.00 |
| T-BEKK $\left(h_t = \frac{\sigma_{FS,t}}{\sigma_{F,t}^2}\right)$ | 85.03 | 99.07[+] |

The table displays the percentage of risk reduction achieved by each hedging strategy using Fut Dez 2009

The symbol + refers to the stategy with the largest risk reduction

Table 2: **H**edging Effectiveness

clearly indicating the dynamic relationship existing between spot and futures returns in $CO_2$ allowances.

Results can be summarized in the following way: 1) Hedge ratios vary from model to model but are extremely close to each other in most cases. Still, these minor differences may condition the hedge ratio performance evaluations being optimal hedge ratios one of the inputs for performance computations. 2) Naive and OLS strategies give worse statistical performance than dynamic hedging hedging strategies. However, adjustment costs of dynamic hedging strategies are higher given the daily adjustment. As such, the better statistical performance of MGARCH models should be expected. If those same costs were considered when OLS hedge ratio is used, probably results would point out a similar hedging effectiveness or variance reduction, although they are still very close to each other. This result implies that the better statistical performance of MGARCH models does not imply a better hedging strategy performance. 3) When MGARCH hedge ratio performances are compared, results are inconclusive in favour of any method as differences are quite small between strategies. However, the strategy with the largest risk reduction, for both in-sample and out-of-sample computations, is that obtained using the t distribution. This should also come at no surprise given that we have seen previously that one fact that characterizes allowances price distributions is its leptokurtosis. 4) For in-sample results the naïve hedging strategy provides better risk reduction than OLS although lower than that obtained using dynamic strategies.

As mentioned previously, the pure variance reduction approach of performance evaluation could be questioned by not taking into account the risk return trade-off, which is by opposition considered by utility maximization. As such, utility improvements (gains) of each considered model over the unhedged position are taken into account in the following. results are presented in table 3.

Table 3: Utility gains for alternative risk aversion levels and different models using spot and December 2009 futures $CO_2$ allowances for the ECX/Bluenext market

This table presents utility gains resulting from using different models with the risk aversion parameter $(\eta)$ ranging from 1 until 4. Utility gains values are presented in percentage terms for both in-sample and out-of-sample data span, as considered also in table 2.

|  | In Sample | | | | Out Sample | | | |
|---|---|---|---|---|---|---|---|---|
|  | Variance | Return | Exp Util[a] | Gain[b] | Variance | Return | Exp Util[a] | Gain[b] |
| $\eta = 1$ | | | | | | | | |
| Unhedge | 13.61 | −0.01 | −13.63 | − | 4.53 | −0.13 | −4.67 | − |
| Naive | 3.50 | 0.00 | −3.50 | 10.13 | 0.06 | 0.01 | 0.00 | 4.67 |
| OLS | 3.53 | 0.00 | −3.53 | 10.09 | 0.04 | 0.00 | −0.04 | 4.62 |
| Diag-BEKK | 1.97 | 0.01 | −1.96 | 11.66 | 0.04 | −0.00 | −0.04 | 4.62 |
| T-Diag-BEKK | 1.82 | 0.00 | −1.82 | 11.81 | 0.04 | −0.00 | −0.04 | 4.63 |
| CCC | 2.22 | −0.01 | −2.23 | 11.40 | 0.10 | −0.02 | −0.12 | 4.55 |
| DCC | 1.96 | −0.00 | −1.96 | 11.67 | 0.04 | −0.00 | −0.05 | 4.62 |
| BEKK | 2.00 | 0.01 | −1.99 | 11.64 | 0.05 | −0.00 | −0.05 | 4.62 |
| T-BEKK | 2.04 | 0.00 | −2.03 | 11.59 | 0.04 | −0.00 | −0.04 | 4.63 |
| $\eta = 2$ | | | | | | | | |
| Unhedge | 13.61 | −0.01 | −27.24 | − | 4.53 | −0.13 | −9.20 | − |
| Naive | 3.50 | 0.00 | −6.99 | 20.25 | 0.06 | 0.01 | −0.01 | 9.19 |
| OLS | 3.53 | 0.00 | −7.07 | 20.17 | 0.04 | 0.00 | −0.09 | 9.12 |
| Diag-BEKK | 1.97 | 0.01 | −3.93 | 23.31 | 0.04 | −0.00 | −0.09 | 9.12 |
| T-Diag-BEKK | 1.82 | 0.00 | −3.64 | 23.60 | 0.04 | −0.00 | −0.09 | 9.12 |
| CCC | 2.22 | −0.01 | −4.44 | 22.80 | 0.10 | −0.02 | −0.22 | 8.98 |
| DCC | 1.96 | −0.00 | −3.91 | 23.33 | 0.04 | −0.00 | −0.09 | 9.11 |
| BEKK | 2.00 | 0.01 | −3.98 | 23.26 | 0.05 | −0.00 | −0.09 | 9.11 |
| T-BEKK | 2.04 | 0.00 | −4.07 | 23.17 | 0.04 | −0.00 | −0.08 | 9.12 |
| $\eta = 4$ | | | | | | | | |
| Unhedge | 13.61 | −0.01 | −54.47 | − | 4.53 | −0.13 | −18.27 | − |
| Naive | 3.50 | 0.00 | −13.98 | 40.49 | 0.06 | 0.01 | −0.03 | 18.25 |
| OLS | 3.53 | 0.00 | −14.14 | 40.34 | 0.04 | 0.00 | −0.17 | 18.10 |
| Diag-BEKK | 1.97 | 0.01 | −7.87 | 46.60 | 0.04 | −0.00 | −0.17 | 18.10 |
| T-Diag-BEKK | 1.82 | 0.00 | −7.28 | 47.19 | 0.04 | −0.00 | −0.17 | 18.10 |
| CCC | 2.22 | −0.01 | −8.88 | 45.59 | 0.10 | −0.02 | −0.43 | 17.84 |
| DCC | 1.96 | −0.00 | −7.83 | 46.64 | 0.04 | −0.00 | −0.18 | 18.09 |
| BEKK | 2.00 | 0.01 | −7.98 | 46.49 | 0.05 | −0.00 | −0.18 | 18.09 |
| T-BEKK | 2.04 | 0.00 | −8.15 | 46.32 | 0.04 | −0.00 | −0.17 | 18.10 |

[a] Expected Utility: $E\left[U\left(r_{p,t}\right)|\psi_{t-1}\right] = E\left[r_{p,t}|\psi_{t-1}\right] - \lambda\, Var\left(r_{p,t}|\psi_{t-1}\right)$

[b] Utility Gain of Hedging Models over Unhedged Position

Table 3: **U**tility Gains for Alternative Risk Aversion Levels

As evidenced by the results, utility gains increase with the level of the risk aversion parameter. As such, for risk lovers ($\eta = 4$) utility gains are superior than those obtained for risk averse ($\eta = 1$) or risk neutral ($\eta = 2$) investors.

Moreover, the model which produces the highest utility gain over the unhedged position is the T-Diagonal-BEKK model for in-sample results. As for the out-of-sample results evidence is mixed with respect to the model providing the higher utility gain (value), but the highest utility gain, although similar, is obtained from the naïve hedging strategy. This could be used as an argument for full hedge, as the easiest and cheapest hedging strategy.

By all that was previously seen we may say that as more data for EU ETS allowances markets becomes available, a more careful analysis of hedging using $CO_2$ could provide insightful results for hedgers that participate in the allowances markets[11]. For now, we have provided evidence for the need to consider carbon instruments in the portfolio optimization. Moreover, given that allowances are traded in electricity markets, and are affected by fuel prices also, mixed portfolio strategies optimization could also be analyzed carefully, but we leave it for a future research.

## 5   Conclusions

As far as we know, this paper is a first attempt to empirically estimate optimal hedge ratios in the EU ETS $CO_2$ allowances markets (more specifically the EXC - European Climate Exchange - market). We analyze their hedging effectiveness applying both static (OLS and naïve) and dynamic (Multivariate GARCH - MGARCH) estimation models. Moreover, utility gains derived through the application of these models for different risk aversion parameters are also derived.

The contribution of this paper is fourfold: First, we calculate for the first time hedge ratios for the $CO_2$ allowances market. Second, we extend the data span considered by previous authors that mostly covered the Phase I period (2005-2007). Third, we use both static and dynamic hedging strategies which allows us to compare different specifications. Finally, we help to identify the internal dynamics of widely traded $CO_2$ emission allowances, essential in pricing of the contracts, while the implications of the study are expected to be functional for risk managers, individual investors and hedgers dealing with the carbon allowances trading markets.

Results indicate that taking into account transaction costs of rebalancing daily the hedged portfolio in dynamic MGARCH models will imply that their better statistical performance in the EU ETS market becomes seriously questioned. Taking into account the data leptokurtosis through the error distribution assumption indicates superior gains, measured by variance reduction, obtained from the multivariate model BEKK (Diagonal), for both in sample and out of sample results (BEKK). Moreover, utility gains increase with the investor's preference over risk.

Overall, there seems to be some gains from including heteroscedasticity and time-varying variances in hedge ratios calculations, although it is not completely guaranteed that improving statistical price modelling provides better performance. Correlation results are important for EU ETS allowances price risk management, as they show that December Futures will provide a good risk reduction for hedgers participating in EU ETS markets. As the market evolves and more data becomes available, it is expected more useful results obtained through dynamic models or even others given that empirical research is evolving constantly.

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[11]Despite the entire data span used we are still limited in terms of Phase II data.

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