

The Broken Chain of Comparative Advantage

E. Kwan Choi

Iowa State University

Henry Thompson

Auburn University

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The chain proposition of comparative advantage states that when factor prices differ between two countries producing many products with two factors, every export of the capital-abundant country is more capital intensive than any of its imports. The present paper shows that the full employment condition may break this chain. It is impossible for a capital-abundant country to produce only products with capital intensities greater than its capital-labor endowment ratio.

Choi, Dept of Economics, Iowa State University, Ames, IA 50011, 515-294-5999, fax 9913,
kchoi@iastate.edu

Thompson, Economics, Comer Hall, Auburn University, Auburn, AL 36849, 334-844-2910, fax
5639, thomph1@auburn.edu

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The chain proposition predicts the pattern of trade between two countries producing many products with two factors. In this $2 \times n$ world ($n > 2$) industry outputs cannot be uniquely determined from given factor endowments. In an attempt to resolve this indeterminacy, Jones (1956, p. 6) writes:

“Ordering the commodities with respect to the capital-labor ratios employed in production is to rank them in order of comparative advantage. Demand conditions merely determine the dividing line between exports and imports; it is not possible to break the chain of comparative advantage by exporting, say, the third and fifth commodities and importing the fourth when they are ranked by factor intensity.”

Bhagwati (1972) convincingly demonstrates that the chain proposition may not hold where factor price equalization occurs. However, he also suggests (p. 1052) that the “proposition, although correct for the case where factor prices are *not* equalized, is untenable as literally stated. When factor-price equalization is realized, a not unimportant case, a variety of crisscrossings are possible.”

Subsequently, Deardorff (1979) investigates Bhagwati’s conjecture and reasserted that the chain proposition holds where factor prices are not equalized. Specifically, Deardorff (p. 201) asserts that “every export of the capital abundant country, by either definition, must be more capital intensive than every one of its imports.” Deardorff’s result has been widely cited in the literature.¹ The chain proposition may be deemed to resolve production indeterminacy in the model with many products.

The present note shows that depending on factor endowments, the capital-abundant country may export products more labor intensive than some of its imports, and vice versa. The

capital-abundant country certainly cannot produce only the products with capital intensities greater than its capital-labor endowment ratio. Full employment requires that the cone of diversification generated by the expansion paths of produced goods spans the factor endowment. If the factor endowment point lies outside the diversification cone of the most capital-intensive goods, the chain of comparative advantage is broken to satisfy the full employment constraint.

1. A Break in the Chain

Consider the production decisions of two countries in the model with two factors and four products. Assume that

(i) country A is capital abundant, $k^A > k^B$, where $k^c = K^c / L^c$ is the capital-labor endowment ratio of country c ,

(ii) factor prices are different and $w^A / r^A > w^B / r^B$,

(iii) two industries have capital intensities greater than k^A and two other industries have capital intensities less than k^B , i.e.,²

$$k_1^A > k_2^A > k^A > k_3^A > k_4^A. \quad (1)$$

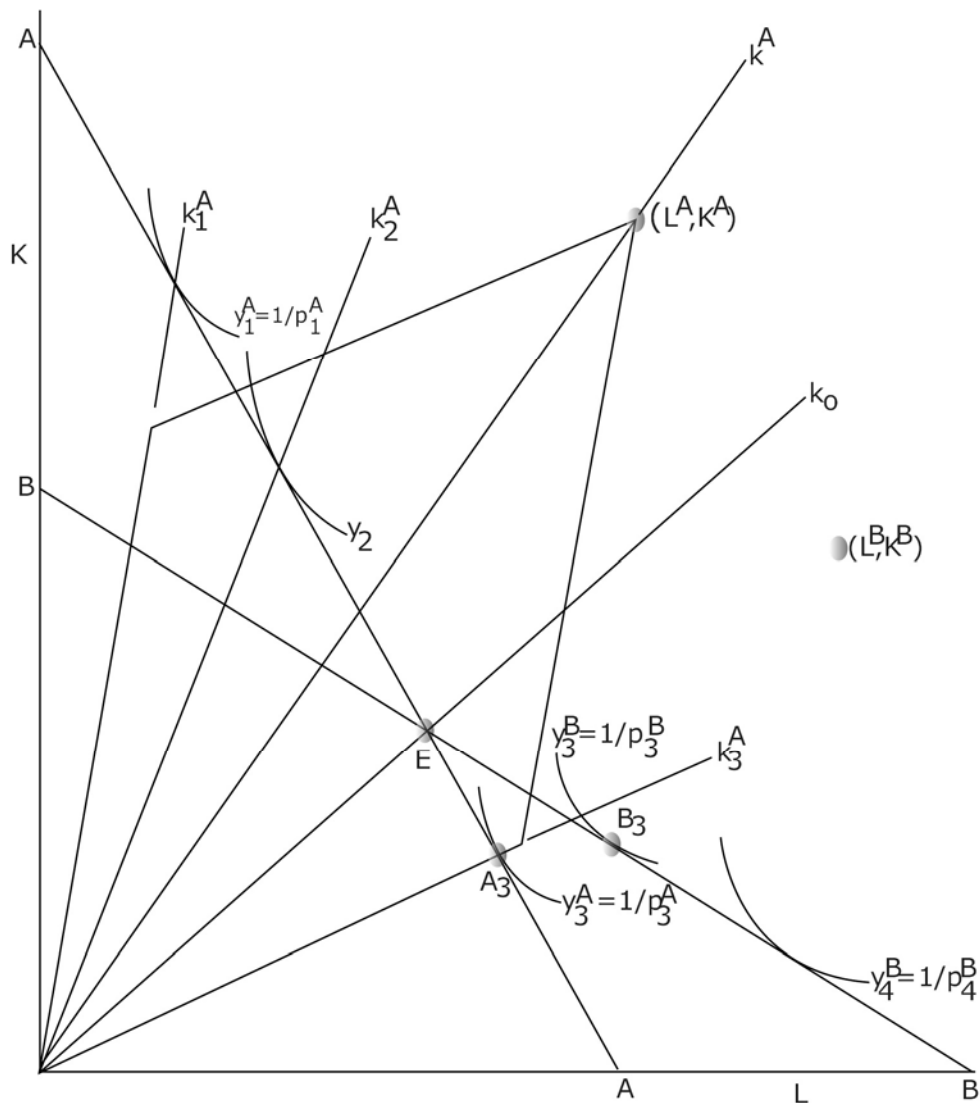
Assume there are no factor intensity reversals. Factor prices between the two countries differ and not all conditions for factor price equalization (FPE) are satisfied. Specifically, FPE is not guaranteed with more products than factors.

In Figure 1, unit value isoquants in country c are labeled $y_i^c = 1/p_i^c$, $i = 1, 2, 3, 4$. When output prices are independently chosen with two factors, each country would produce at most two products. The prices of these two products completely determine factor prices. The four products are ranked by capital intensity as in (i). Since the wage-rent ratio is higher in country A,

the capital intensity for each of its products is higher than in country B, $k_i^A > k_i^B$ for all i . The unit value isoquant $y_3^A = 1/p_3^A$ in country A lies below the unit value isoquant $y_3^B = 1/p_3^B$ in country B.

Given identical technologies, a higher isoquant implies higher input requirements and a higher output level. That is, $y_3^B = 1/p_3^B$ at B_3 is greater than $y_3^A = 1/p_3^A$ at A_3 , implying $p_3^B < p_3^A$, and country B has a comparative advantage in product 3.

Deardorff (1979) assumes that factor prices are different between two countries.³ Deardorff's (1979, p. 201) proof of the chain proposition is stated without regard to the capital-labor endowment ratio and the full employment constraint. It implicitly assumes the cone of diversification of the "most capital intensive" products spans the factor endowment point. The most capital-intensive products are those with capital intensities higher than $k^o = K^o / L^o$ in Figure 1, where the unit isocost lines intersect.



The capital abundant country cannot produce only the two most capital intensive products without violating the full employment constraint. If country A produces product 1, it also must produce another product with capital intensity less than k^A , breaking the chain and producing product 3 or 4.

The full employment condition breaks the chain of comparative advantage under the present assumptions. The capital-abundant country A must produce at least one product whose

capital intensity is less than its capital-labor endowment ratio, and may even produce the most labor intensive product 4. The capital-abundant country A cannot produce only the most capital intensive goods, those with capital intensities greater than k^o in Figure 1. To attain full employment, the capital-abundant country A must produce one of the labor intensive products.

In Figure 1, the capital abundant country A is able to produce combinations (1, 3), (2, 3), (1, 4) or (2, 4), but it cannot produce (1, 2) or (3, 4). Given each of these combinations, the labor-abundant country B would produce the other two products and the factor intensity chain is broken.⁴ The degree of indeterminacy increases with the number of industries.

2. Concluding Remarks

The chain of comparative advantage in a world of two factors and many products may have broken links when resources are fully employed. The capital-abundant country might produce some labor intensive products even though the labor-abundant country could produce them with fewer resources. The literature on the chain proposition implicitly assumes the capital abundant country's factor endowments lie within the diversification cone of the "most capital intensive goods," a condition that generally is not valid for arbitrary factor endowments and factor intensities.

A recent empirical study by Cavusoglu and Elmslie (2005) on nine U.S. manufacturing industries for three periods (1970-1985, 1970-1990, 1970-2000) finds no empirical support for the chain proposition that all export industries are more capital intensive than import competing industries.⁵ Specifically, they (p. 414) report that "the chain version of comparative advantage does not hold for any of the periods and any balance weight." This empirical result is consistent with breaks in the chain of comparative advantage.

Full employment and balanced trade are ingredients of a complete model with two factors, many goods, and two countries. As the number of products increases, international equilibrium is more likely to be characterized by breaks in the chain of comparative advantage.

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¹ For instance, see Bhagwati, Panagariya and Srinivasan (1998), Feenstra (2004), Romalis (2004), and Cavusoglu and Elmslie (2005).

² This is the minimum assumption for country A to break the chain. Similarly, $k_1^B > k_2^B > K^B / L^B > k_3^B > k_4^B$ is a sufficient condition for country B to break the chain.

³ Deardorff (p. 199) further assumes that (i) two countries trade with no trade impediments so that output prices are equal, (ii) production of each product requires that capital (K) and labor (L)

are available in fixed supply in each country, (iv) production functions are identical between countries, concave and (linearly) homogenous, (v) products are ranked in terms of capital intensity, $k_1 > \dots > k_n$, where $k_i \equiv K_i / L_i$, and hence there are no factor intensity reversals between any pair of products, and (vi) perfect competition prevails in both countries. These assumptions imply FPE in the 2×2 case, and can be extended to the even case where $n > 2$. These assumptions are not sufficient to imply FPE with more products than factors.

⁴ If output prices adjust to guarantee zero profits, country A in autarky can produce (1, 2, 3), (1, 2, 4), (1, 3, 4), or all four goods and still satisfy the full employment constraint. The output vector is remains indeterminate.

⁵ Cavusoglu and Elmslie (2005) noted a link between capital intensities and investment-labor rankings, a result not predicted by the chain proposition.

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