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an Extended Matching Model

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Entrepreneurship and the Hidden Economy: an Extended Matching Model [*]

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Abstract

This paper develops a standard matching model to address the problem of the hidden sector (including non-registered firms but producing for legal markets), as it is characterised in Italy, i.e. framed in a rather advanced economic and institutional setting, but also linked to the socio-economic regional dualism. The main novel features of the model are that entrepreneurial ability is heterogeneous, and that regular firms receive negative externalities from the hidden sector, and positive externalities from the other regular firms. Not only does an interior equilibrium emerge, but two stable equilibria are possible, thus accounting for Italy's dualism. The "bad" equilibrium with respect to the "good" one is in fact characterised by a larger hidden sector, lower levels of overall productivity, output, entrepreneurial ability used, extra-profits, skilled employment, wages, and investment in education, as well as positive externalities; while the negative externalities, which may capture the pressure typically exerted by organised crime, are relatively greater.

JEL classification: E26, J23, J24, J63, J64, L26

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1 – Introduction and summary

"If you hit a mule to make it go, you may succeed, or you may not, since the mule may die". This anonymous maxim can be applied to the resources employed in those economic activities which escape fiscal and normative duties although producing for legal markets. These activities, which remain hidden to the public authorities, are blamed for the consequent losses in public revenues, and for the deterioration of tax morale. Unemployment benefits, heavy taxation, bureaucratic redundancy are seen as the underlying reasons for firms and workers "to go underground". Therefore, a more flexible labour market, lighter taxes, less bureaucratic procedures, but also some controls and fines are recommended to make these activities move "overground". Public authorities should thus subject all resources available to the pressure of the market on an equal footing (Schneider and Enste, 2000).

An alternative view argues that valuable entrepreneurial talent can more freely grow in hidden activities, thereby training workers and possibly incubating successful enterprises, or even infant industries (De Soto, 1989). The economy is thus described as dualistic, and even as producing two different kinds of goods on the final market (Agenor and Aizenman, 1999; Bandyopadhyay and Gupta, 1995; Rauch, 1991). A state that rigorously imposes taxes and regulations puts the economy at risk of losing resources which may be no longer employed for production, so that the expected revenues may not be even retrieved.

Put briefly, it can be said that the first view considers the quality of the resources available in the economy to be given and inter-sectorally homogeneous, and it argues for improving their efficient allocation. The second view recognises that the quality of the resources employed in the hidden sector is specifically worse, and it addresses the issue of how this quality can be improved.

The first view often refers to the developed economies, and especially to the phenomenon of moonlighting, while the second view usually refers to the less developed economies, and especially to informal activities like homework.

However, the empirical evidence is not clear-cut. Indeed, within the group of the developed countries, those with the highest tax burdens and strictest regulations and labour protection legislation, i.e. the European Nordic countries, are not those with the largest hidden sector. The US instead exhibits both the lowest intrusion of the state into the economy and the lowest proportion of the hidden sector (Bovi and Dell'Anno, 2007). On the whole, several cross-country studies find a negative correlation between the tax marginal rate and the size of the hidden economy (e.g. Johnson et al., 1998, 1999; Friedman et al., 2000). Notwithstanding, there is widespread consensus that overall taxation and social security contributions are crucial in explaining the rise of the shadow economy. However, many authors support the view that also moral corruption and bribery lead potential employers into the irregular economy (Loayaza, 1996; Fortin et al., 1997; Agenor and Aizenman, 1999; Johnson et al., 2000; Sarte, 2000; Fugazza and Jacques, 2004), while the provision of public services financed by relatively high taxation and income has been found to be significantly and positively correlated (Johnson et al., 1998). Finally, no agreement in the literature emerges about the effects of the shadow economy on GDP (Eilat and Zinnes, 2000).

This paper is theoretical, but it addresses the problem of the hidden sector as it is characterised in Italy, i.e. framed in a rather advanced economic and institutional setting, but also localised and linked to the economy

of specific regions (see *Appendix A*). In fact, the paper pays particular attention not only to the typical features of the irregular firm but also to the very different contexts in which firms operate.

Almost by definition, reliable data on the irregular firm, on the hidden sector, and on the links between the two sectors are not available. Most data regard estimations on the proportion of the hidden sector in the entire economy, with serious problems of disentangling the illegal sector (Schneider and Enste, 2000; Zizza, 2002). Some other data on Italy provide information only on irregular workers, and on tax evasion. However, pilot studies, case studies, and occasional observations are able to characterise the typical irregular firm and its links with the other firms (Donolo and Capparucci, 2002; Meldolesi and Aniello, 1998; Meldolesi, 1998).

The typical entrepreneur running an irregular firm adopts old techniques in a small-sized organisation, employs unskilled labour, pays low wages, and produces intermediate goods for bigger official firms, competing on a price basis. His entrepreneurial ability is typically based on family connections, with no cooperation with the other firms, largely eschewing information on legislation even when it may be advantageous. Networking with the other firms is local; orders are signed after a fierce price competition which is likely to be unfair. When irregular workers are employed through some sort of selection on the market, the relationships are usually based on distrust, which provokes frequent micro-conflicts. In some Southern Italian regions, irregular firms are often embedded in a social system where criminal organisations operate (Fazio, 2004). Economic transactions comply with the usual market laws even less. Public authorities are corrupted, and the entire economy is affected. Official firms must pay bribes and protection money. Therefore, entrepreneurs must deal with a context in which market distortions impose specific costs.

Some recent empirical studies have shown the adverse effects of organized crime on economic growth and on the local institutional system (e.g., Centorrino and Ofria, 2008; Peri, 2004). A survey conducted by Marini and Turato (2002) on a panel of entrepreneurs from the North-East of Italy reported that almost the entire sample of interviewees (92.6%) regarded criminality to be the main obstacle against the development of enterprises in the Mezzogiorno area. A survey conducted on behalf of the Ministry of Economy in 11 countries confirms that entrepreneurs perceive the Mezzogiorno as an area which lacks security (Gpf-Ispo, 2005) (see again *Appendix A*).

The paper addresses this complex issue on fairly usual assumptions, and on new key assumptions. The former are the following: irregular firms adopt techniques which are relatively less efficient than those of official firms in order to produce the same product, they evade taxes despite the risk of being detected and punished, they do not pay start-up costs. The new key assumptions are the following: firstly, regular firms enjoy positive externalities from the other regular firms, but they also suffer from negative externalities produced by the irregular firms. Secondly, entrepreneurial ability is heterogeneous across individuals, while also workforce skills is assumed as heterogeneous.

In particular, the paper adopts a matching model which shares several features that can be found in the literature on matching models and hidden economy (Boeri and Garibaldi, 2002, 2006; Bouev, 2002, 2005; Kolm and Larsen, 2003), but it departs because of the assumption of the heterogeneous entrepreneurial ability. Since this is not a tradeable input for firms, entry into the market is not completely free, so that the standard zero-profit conditions hold only for the marginal entrepreneurs, i.e. those endowed with the minimum ability. The other, abler, entrepreneurs earn extra-profits. This new assumption provides a new solution to the problem of determining the allocation of vacant jobs between the regular and the irregular sector.

The paper is thus able to give a theoretical account of a number of facts: at the macroeconomic level, the persistence of a substantial proportion of the hidden sector with detrimental effects on overall output and underemployment, and on investment in education; at the microeconomic level, some key characteristics of irregular firms, such as their relatively lower entrepreneurial ability used, lower-skilled workers employed, lower wages earned.

When the analysis concentrates on the role of externalities, the paper shows other results by recognising the particular non-linearity of externalities in diffusing themselves. Two final aggregate outcomes may emerge within the same institutional structure and with the same economic potential. The "bad" outcome consists of a relatively large hidden sector, important negative externalities, and reduced positive externalities; the "good" outcome consists of a relatively small hidden sector, important positive externalities, and reduced negative externalities.

This approach to the problem of the hidden economy makes it possible to extend the opportunity of policy actions from the fine tuning of the institutional duties (Kolm and Larsen, 2003), from larger individual benefits of participating in the regular sector (Fugazza and Jacques, 2004; Valentini, 2007), and from labour-market liberalisation (Boeri and Garibaldi, 2002, 2006; Bouev, 2002, 2005), to actions aiming at increasing positive externalities and reducing negative ones.

The paper is organised as follows: sections 2 and 3 present the simple benchmark model and, respectively, the extended model, where externalities are endogenised; section 4 performs some numerical simulations, while section 5 concludes with some policy implications. The appendices *B*-*F* set out the relevant proofs and math details.

2 – The benchmark model

The model proposes a general equilibrium model of the matching type (Mortensen and Pissarides, 1994; Pissarides, 2000).[1] This means that both firm's equilibrium and aggregate equilibrium are studied, and that there are frictions in the labour market because firms and workers do not perfectly match. The general equilibrium character is particularly stressed, because the model comprises two types of firms, thus forming two sectors, and because each firm is affected by the sectoral composition. The matching character of the model enables study of joint decision-making by entrepreneurs and workers, thus yielding non-market clearing wages in the two sectors, and unemployment.

The environment is characterised by a non-competitive labour market with wage bargaining.[2] Numerous firms competitively produce a homogeneous product,[3] but adopt different institutional and technological set-ups. They may be registered, and therefore pay a production tax and adopt a relatively advanced technology; or they may not be registered, and therefore evade taxes and adopt a less efficient

[1] Hence, as usually assumed, time is continuous, and individuals are risk neutral, live infinitely, and discount the future at an exogenous discount rate r.

[2] In this work we abstract from goods and capital markets (both of which are assumed to clear) in order to highlight the joint effect of search frictions and rent sharing on job composition, rather than on prices of both capital input and final output.

[3] Bouev (2002) also assumes that goods produced in both regular and irregular sectors are perfect substitutes. Indeed, if different goods are being produced, one could ask how come the workers and consumers are able to locate the irregular firms whereas the tax authorities are not fully able to.

technology. Hence non-registered firms form the hidden sector of the economy, which is not legal because of the process employed, not because of the good being produced.[4] Each firm is run by an entrepreneur who is an individual endowed with an extra-ability to run a new firm.

An unexpected result of the model is that equilibrium is not necessarily a corner solution, as a competitive market for a homogeneous product might suggest. The key assumption for obtaining at least one interior solution is that entrepreneurs are endowed with different abilities. This is a new assumption in the family of matching models, and it allows us to concentrate on the conditions inducing entrepreneurs to enter one sector or the other.[5]

2.1 Entrepreneurs' expected profitability and workers' expected wages

As is usual in matching-type models (Pissarides, 2000; Petrongolo and Pissarides, 2001), let us assume that the meeting of vacant jobs and unemployed workers is regulated by a matching function with constant returns to scale. Let us denote the number of vacancies in the official (or regular) sector and in the hidden (or irregular) sector with v_r and v_s respectively, and the number of unemployed with u. The unemployed workers are the only job seekers in the labour market.[6] The matching function of filled jobs (m) in the two sectors (r,s) is thus as follows:[7]

$m_i = m\,(v_i, u\,)$ with $i = r,s$

which is positive and concave in the two arguments. Since the function performs constant returns to scale, it implies that:

$f\,(\theta_i) = m_i\,(v_i, u\,) / v_i \qquad g\,(\theta_i) = \theta_i \cdot f(\theta_i) = m_i\,(v_i, u\,) / u$ with $i = r,s$

where $\theta_i = v_i / u$ represents the tightness of the labour market in each sector from the firm's standpoint; $f(\theta_i)$ indicates the instantaneous probability of a firm filling a vacancy in the official and the hidden sector respectively. Conversely, $g(\theta_i)$ indicates the instantaneous probability of an unemployed person finding a job in the official and the hidden sector respectively. The properties of the functions indicate that the greater the vacancy rate or the smaller the unemployment rate, the smaller the probability of filling a vacancy for firms, and the greater the probability of finding a job for the unemployed, i.e.:

$f'(\theta_i) < 0$ and $g'(\theta_i) > 0$ with $i = r,s$.

The modelling of the match between firms and workers depends on the form of matching technology used. Nevertheless, a Cobb-Douglas functional form is generally favoured by empirical studies.[8] In this case:

$f''(\theta_i) > 0$ and $g''(\theta_i) < 0$ with $i = r,s$.

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[4] This is also the definition usually used for the hidden, shadow, or underground economy (OECD, 2002).

[5] The literature that employs matching models instead concentrates on the study of the individual's choice between running a firm or working as an employee (Fonseca, Lopez-Garcia and Pissarides, 2001; Pissarides, 2001 and 2003; Uren, 2007).

[6] In terms of flows, the model ignores on-the-job-search and direct transitions from shadow to legal employment without intervening unemployment spells.

[7] Unlike Bouev (2002, 2005), Kolm and Larsen (2003), Albrecht and Vroman (2002) and Albrecht et al. (2006), we assume a directed search. In the presence of an undirected search both formal and informal vacancies have the same probability of meeting workers; then it is the total number of vacancies that enters the matching function.

[8] For a review see Petrongolo and Pissarides (2001), while Stevens (2004) provides microeconomic foundations for it.

and the *Inada*-type conditions hold (Pissarides, 2001; Bouev, 2002, 2005; Uren, 2007):

$\lim_{\theta\to\infty} f(\theta)=0$, $\lim_{\theta\to 0} f(\theta)=\infty$, $\lim_{\theta\to\infty} g(\theta)=\infty$, $\lim_{\theta\to 0} g(\theta)=0$.

Therefore, when $f(\theta_i)$ and $g(\theta_i)<\infty$, matching is not instantaneous and takes some time (Bouev, 2005).

The usual Bellman equations for the expected values of a vacancy, and for a filled job over the infinite horizon, specified for each sector, are as follows:[9]

Value of …	***Hidden sector***	***Official sector***
a vacancy	$r\cdot V_s = f(\theta_s)\cdot[J_s - V_s]$	$r\cdot V_r = -c + f(\theta_r)\cdot[J_r - V_r]$
a filled job	$(r+\rho)J_s = x_s y_s - w_s - \rho\phi\tau + \delta[V_s - J_s]$	$rJ_r = (px_r y_r + k) - w_r - \tau - s + \delta[V_r - J_r]$
searching a job	$r\cdot U_s = g(\theta_s)\cdot[W_s - U_s]$	$r\cdot U_r = g(\theta_r)\cdot[W_r - U_r]$
being employed	$r\cdot W_s = w_s + (\delta+\rho)\cdot[U_s - W_s]$	$r\cdot W_r = w_r + \delta\cdot[U_r - W_r]$

where V_i is the value of a vacancy, J_i is the value of a filled job, c is the start-up cost in the regular sector, $p>1$ is the exogenous productivity premium in the official sector, x_i is the entrepreneurial ability level (discussed in 2.2), y_i is the labour productivity (discussed in 2.3), w_i is the wage rate, τ is an exogenous production tax, ρ is the exogenous instantaneous probability of a firm being discovered as unregistered (or monitoring rate), ϕ is the exogenous multiplier of the tax due to be levied, δ is an exogenous bankruptcy rate of firm,[10] U_i is the value for seeking a job (the unemployed workers can't search for a job in both sectors at the same time),[11] W_i is the value for being employed.

The symbols k and s denote the specific advantages and disadvantages for regular firms, like the benefits of participating in a larger information network and of receiving specific public services, and conversely, of paying bureaucratic and administrative costs, including bribes and money protection if imposed by criminal organisations. Both s and k are temporarily assumed as parameters in this section, but they will be considered as variables in the next section on the extended model.

The vacancy equations state that the return to the vacancy is equal to the potential change in value in the case of a successful match minus the cost of posting the vacancy, i.e. the start-up cost, which is zero in the hidden sector. The filled job equations state that the return to the firm is equal to the difference between the job's productivity and costs, plus the potential change in value in the case of a breakdown in the match (bankruptcy of firm). A job's productivity depends on entrepreneurial ability and the worker's skill; and in the case of a regular firm, it also depends on a productivity premium, thereby capturing a technological advantage, and on receiving positive externalities. A job's costs are wage costs, taxes or fines (where reasonably $\rho\phi<1$), and in the case of a regular firm, the negative externalities deriving from the other firms. If the irregular firm is discovered, then the job is destroyed, and it does not turn into an unfilled vacancy. The equations for the unemployed state that the return on being unemployed equals the potential change into employment in the case of a match. The equations for the employed workers state that the return on employment is equal to the wage

[9] Note that Bellman functions are specified to find infinite horizon steady-state solutions.

[10] We ignore the possibility of voluntary quits by workers.

[11] No unemployment benefit is considered because it is not quantitatively important in Italy.

income plus a potential change into unemployment in the case of a breakdown in the match (bankruptcy of the firm).

The Bellman equations thus yield:

$$V_r = \frac{(p \cdot x_r \cdot y_r + k) - w(\theta_r) - \tau - s - c(\delta + r)/\, f(\theta_r)}{r(1 + (\delta + r)/\, f(\theta_r))} \qquad [1]$$

$$V_s = \frac{x_s y_s - w(\theta_s) - \rho\phi\tau}{r + \rho + r(r + \rho + \delta)/\, f(\theta_s)} \qquad [2]$$

as well as the value of a filled job in the two sectors, thus defining the different profitability of running a regular firm (J_r) with respect to an unregistered firm (J_s):

$$J_r = \frac{(p \cdot x_r \cdot y_r + k) - w(\theta_r) - \tau - s + \delta \cdot V_r(\theta_r)}{r + \delta} \qquad [3]$$

$$J_s = \frac{x_s \cdot y_s - w(\theta_s) - \rho \cdot \phi \cdot \tau + \delta \cdot V_s(\theta_s)}{r + \delta + \rho} \qquad [4]$$

Successful matches in each sector enjoy a pure economic rent, and wages are assumed to be the outcome of a Nash bargaining problem, with workers earning a share of the total surplus of a filled job, i.e. the sum of values for both the firm and the employed worker net of outside options. For the regular firms, we thus have:

$$w_r = \arg\max (J_r - V_r)^{1-\beta} \cdot (W_r - U_r)^{\beta}$$

where $\beta \in (0,1)$ is the surplus share for labour. Analogously, the wage rate in the irregular firm is obtained with a share γ, which is assumed to be smaller than β because workers' bargaining power in the hidden sector is lower than in the official sector, owing to the absence of protective legislation and unions.

Simple manipulations thus yield the formulae for wages (see *Appendix B* for details):

$$w_r = \frac{p \cdot x_r \cdot y_r + k + c - \tau - s}{[(1-\beta)/\beta][r + \delta + f(\theta_r)]/[r + \delta + g(\theta_r)] + 1} \qquad [5]$$

$$w_s = \frac{x_s \cdot y_s - \rho \cdot \phi \cdot \tau}{[(1-\gamma)/\gamma][r + \delta + \rho + f(\theta_s)]/[r + \delta + \rho + g(\theta_s)] + 1} \qquad [6]$$

Wages in the official sector are advantaged over wages in the hidden sector for the following reasons: $\gamma < \beta$; $p > 1$; and $v_r > v_s$ and hence $\theta_r > \theta_s$. The latter condition is usual (e.g. see Boeri and Garibaldi's (2006) calibrations), but it is not always accepted in the literature. One reason is that the official sector is larger, or even much larger, than the hidden sector, so that the unemployed have fewer chances of meeting irregular vacancies.

2.2 Entrepreneurial ability and the career choice

An entrepreneur runs a one-job firm after having posted a vacancy. A key feature of the model is that the comparison of expected profitability of running firms in the two sectors depends on the entrepreneurial ability of individuals (x). Each individual is in fact assumed to be endowed with a specific entrepreneurial ability which cannot be used on the job as a worker, and which may have a level different from that of the other individuals.

Formally, entrepreneurial ability x is distributed over a continuum of infinitely-living individuals who expect to enter the market, with population size normalised to one. It can be measured in continuous manner, $]0, x_{max}]$, following the known cumulative distribution function $F(x)$, so that:

$$F(0) = 0\,,\ F(x_{max}) > 0 \text{ and } \int_0^{x_{max}} x\, dF(x) = 1\,.$$

Let us assume that individuals with an entrepreneurial ability equal to 1 (and below) do not find it convenient to post vacancies in the hidden sector because $V_S = 0$. Hence, $y_S - \tau\phi\rho = w_S$, and also running an irregular firm is not profitable, since $J_S = 0$ from [4]. All the entrepreneurs thus find it convenient to post vacancies and run irregular firms because they earn a rent from their extra-ability, i.e. with $x > 1$. Also the positive sign of $w_S > 0$ is thus determined from [6].

An analogous, though not so immediate, reasoning can be applied to regular vacancies and firms. In fact, $V_r = 0$ requires that $p\, y_r + k - \tau - s = w_r + c\cdot(\delta + r) / f(\theta_r)$, but this depends on θ_r. Plugging [5] in this equation, and rearranging yields:

$$\frac{p \cdot y_r + k - \tau - s}{[\beta/(1-\beta)]\dfrac{r+\delta+g(\theta_r)}{r+\delta+f(\theta_r)}+1} - \frac{c}{[(1-\beta)/\beta]\dfrac{r+\delta+f(\theta_r)}{r+\delta+g(\theta_r)}+1} - \frac{c(r+\delta)}{f(\theta_r)} = 0$$

The limit of the left-hand-side for v_r tending to 0 gives $py_r + k - \tau - s = 0$. Therefore, higher than minimum ability, i.e. with $x > 1$, enables entrepreneurs to have the condition $V_r > 0$ and to cover their start-up costs. Similar conditions govern $J_r > 0$. Positive wages in the official sector are also warranted.

Accordingly, individuals endowed with an entrepreneurial ability equal to 1 or below become workers (l) and then do not post any vacancy. Formally:

$$l \equiv F(1) = \int_0^1 x\, dF(x)$$

Therefore, we get:

$$u = l - n_r - n_s \qquad [7]$$

where n_r and n_s are the steady-state employment in the official and hidden sectors respectively (n_r and n_s will be determined in subsection 2.4).

Further, let us assume that a threshold level of entrepreneurial ability $T \in [\, x > 1, x_{max}\,]$ exists such that two firms drawn from the two sectors yield equal expected profitability. More precisely, let us assume that:

$$J_r(T, \theta_r, y_r) = J_s(T, \theta_s, y_s)$$

Therefore, T can be derived from [1]-[6]:[12]

[12] This formalisation is similar to the one adopted by Fonseca, Lopez-Garcia and Pissarides (2001) and Pissarides (2001), who, however, study the individuals' choice between entering the market as an entrepreneur or becoming a worker.

$$T = \frac{\frac{\tau + s - k}{\varsigma^{-1}+1} + \frac{c}{\varsigma+1} - \frac{\psi\tau\phi\rho}{\vartheta^{-1}+1} - \frac{\delta\cdot\left[k-\tau-s-c\cdot(\delta+r)/\,f(\theta_r)\right]}{\varepsilon_r} + \frac{\delta\cdot(k+c-\tau-s)}{\varepsilon_r\cdot(\varsigma+1)} - (\delta\psi\rho\phi\tau)\cdot\left[\frac{\vartheta}{\varepsilon_s\cdot(\vartheta+1)}\right]}{p\cdot y_r\cdot\left(\frac{1}{\varsigma^{-1}+1}+\frac{\delta}{\varepsilon_r}-\frac{\delta}{\varepsilon_r\cdot(\varsigma+1)}\right)-\psi\cdot y_s\cdot\left(\frac{1}{\vartheta^{-1}+1}+\frac{\delta}{\varepsilon_s}-\frac{\delta}{\varepsilon_s\cdot(\vartheta+1)}\right)}$$

[8]

where: $\psi = \frac{r+\delta}{r+\delta+\rho} < 1$, $\varsigma = \frac{(1-\beta)\cdot(r+\delta+f(\theta_r))}{\beta\cdot(r+\delta+g(\theta_r))}$, $\vartheta = \frac{(1-\gamma)\cdot(r+\delta+\rho+f(\theta_s))}{\gamma\cdot(r+\delta+\rho+g(\theta_s))}$,

$\varepsilon_r = r\cdot\left(1+\frac{\delta+r}{f(\theta_r)}\right)$, $\varepsilon_s = r+\rho+r\cdot\left(\frac{r+\rho+\delta}{f(\theta_s)}\right)$.

Each entrepreneur assumes u as given. Let us here determine v_i. The entrepreneur's indifference condition between running firms in the two sectors implies that the share of entrepreneurs in the hidden sector is $F(T)-F(1)$, while the remaining share of entrepreneurs $1-F(T)$ opens a vacancy in the official sector.[13] Formally:

$$v_r = 1-F(T) = \int_T^{x_{\max}} x\,dF(x) \qquad [9]$$

$$v_s = F(T)-F(1) = \int_{x>1}^{T} x\,dF(x) \qquad [10]$$

Entrepreneurs, i.e. those individuals endowed with entrepreneurial ability $x>1$, may post a vacancy and fill the job, or fail to fill it, in one of the two sectors, so that it can be simply stated that $v_r = 1-(v_s+l)$.[14] Hence, equation [8] can be re-written in a more general form as follows:

$$T = T(v_s, y_r, y_s) \qquad \text{with} \qquad T'(y_r) < 0,\ T'(y_S) > 0 \qquad [11]$$

Equations [10] and [11] can thus be studied in the diagram with axes [v_S, T] like Fig.3. Equation [11] is monotonically decreasing in θ_S, and hence in v_S (depicted as a continuous line in Fig.3), because productivities in the two sectors are assumed to be sufficiently great, as guaranteed by the conditions for $V_r>0$, $V_s>0$ discussed above, and also because $\partial v_r/\partial v_S < 0$, $V_r'(\theta_r)<0$, $V_s'(\theta_s)<0$. Equation [10] is monotonically rising in T, from $x=1$ up to x_{max} and its form depends on the distribution of ability across entrepreneurs.

[13] Note that this is a approach different from the usual one, which uses $V(\theta_i)=0$ as the condition to determine θ_i together with the wage equation. In this model, in fact, entrepreneurs have heterogeneous ability.

[14] This pattern may imply a negative relationship between the shadow economy and economic growth. Although there is no agreement in the literature on the sign of this relationship, there is some empirical evidence to support the hypothesis of a negative relation between the Italian shadow economy and the official growth rate of GDP (Dell'Anno and Schneider, 2003).

Equation [11] has been built for $T \in [\, x > 1 \,, x_{max} \,]$, so that the vertical start-point of [11] is higher than the intercept of [10].

========== Fig. 3 about here (now at the end) ==========

Remarks. *(i) A unique couple of (v_S , T) exists; (ii) the solution of the system [10]-[11] in the dynamic form yields a stable solution.*

This is the key result of the benchmark model, which will be proved in *Appendix C* and completed in the next subsections. However, an interesting conclusion can be drawn here: that entrepreneurs will prefer the hidden sector if they are endowed with $x < T$, and they will prefer the official sector if endowed with $x > T$. Some entrepreneurial ability may thus remain hidden, but it will also be of the worst quality (see also Pugno, 2000a; Carillo and Pugno, 2004; Rauch, 1991, Levenson and Maloney, 1998). This conclusion runs counter to the argument that the shadow sector is an incubator of infant industries.

Secondly, wages in the official sector have a further advantage besides those mentioned above with respect to wages in the hidden sector. In fact, regular firms are more productive because they are run by more able entrepreneurs.

2.3 The workers' skill and the endogenous schooling investment

Although the workers have inadequate entrepreneurial ability, they are endowed with a positive level of productivity which can be used on the job. Empirical findings show that $y_r > y_s$ (e.g. Busetta and Giovannini, 1998; Boeri and Garibaldi, 2002, 2006), so that we would expect, on the firms' side, to find that official firms employ high-skill workers, while low-skill workers are sufficient for irregular firms.[15]

Given that the labour force supplies labour time inelastically,[16] let us assume that workers enter the labour market either unskilled or skilled, thus achieving two different levels of labour productivity if they choose to invest in education properly (Acemoglu, 1996): indeed, formal education enhances the worker's skill (Laing et al. 1995):

$$y_r \equiv y_{skilled} = (1+e) \cdot y_{unskilled}$$

where e denotes investment in education. This choice is linked to the job for which workers expect to be employed because they may work with two different technologies, as captured by the premium $p > 1$ for the official sector.

We now consider the optimal choice of investment in education (e^*) by workers in the economic environment described beforehand. Investment in education is beneficial to workers in two respects: it raises their initial stock of human capital and their ability to accumulate additional human capital once employed (Laing et. al., 1995).

We assume that schooling investment is costly, as a result of either a direct pecuniary cost or the disutility from scholastic effort. Let $c(e)$ denote the utility cost to workers from investment in education. The

[15] Firms' productivity is supposed to be known by the entrepreneurs, including workers' skill.
[16] In a matching framework this hypothesis has also been used by Bouev (2002, 2005).

function $c(e)$ is strictly increasing, twice continuously differentiable, and convex in *e*, and satisfies $\partial c(0)/\partial e = 0$.

Each individual is conceived of as solving the following programme (see e.g. Laing et. al., 1995; Decreuse and Granier, 2007):

$$\max_{e\geq 0}\{r \cdot U_i - c(e)\} \qquad \text{with } i = r,s$$

Accordingly, in the official sector, optimising *e* implies:

$$\max_{e\geq 0}\{g(\theta_r)\cdot \beta \cdot S_r - c(e)\}$$

being $[W_r - U_r] = \beta \cdot S_r$. Surpluses S_i arising from the match in each of the two sectors can be derived from the Bellman equations. Knowing that $S_i = (J_i - V_i + W_i - U_i)$, straightforward algebra gives:

$$S_r(e) = \frac{p \cdot x_r \cdot y_r(e) + k - s - \tau + c}{r + \delta + (1-\beta)\cdot f(\theta_r) + \beta \cdot g(\theta_r)}$$

First-order condition thus yields:

$$\frac{\partial\{g(\theta_r)\cdot \beta \cdot S_r(e^*)\}}{\partial e^*} = \frac{\partial c(e^*)}{\partial e^*}$$

This is the optimalization standard condition: the individual's marginal revenue consequent on investment in education must be equal to its marginal cost. It states that workers expecting to be employed in the official sector find it optimal to invest in education. From a macroeconomic point of view, it also follows that investment in education is proportional to the official sector, because the higher θ_r is, the greater is the marginal revenue of investment in education.

The same procedure applies to the hidden sector, so that:

$$S_s = \frac{x_s \cdot y_s - \rho \cdot \phi \cdot \tau}{r + \delta + \rho + (1-\gamma)\cdot f(\theta_s) + \gamma \cdot g(\theta_s)}$$

Eventually, we get:

$$0 = \partial c(e^*)/\partial e^*$$

from the properties of cost function, i.e. $\partial c(0)/\partial e = 0$, first-order condition for the hidden sector implies that $e^* = 0$, i.e. workers expecting to be employed in the hidden sector find it optimal to do not invest in education.

These results are confirmed by empirical findings in Italy (Cappariello and Zizza, 2009).

2.4 Closing the model in the labour market

Finally, the model is closed by determining the stock of workers into the official and hidden sectors. We have argued that jobs arrive to unemployed workers at rate $g(\theta_i)$ and regular and irregular filled jobs break up at rate δ and $(\delta + \rho)$, respectively.

Therefore, the equations for the evolution of employment in the two sectors in terms of the worker's transition rates are the following:

$$\dot{n}_r = u \cdot g(\theta_r) - \delta \cdot n_r$$

$$\dot{n}_s = u \cdot g(\theta_s) - (\delta + \rho) \cdot n_s$$

Steady-state implies that $\dot{n}_r = \dot{n}_s = 0$, so that:

$$n_r = \frac{u \cdot g(\theta_r)}{\delta} \qquad [12]$$

$$n_s = \frac{u \cdot g(\theta_s)}{\delta + \rho} \qquad [13]$$

Employment in the model has two components: entrepreneurs and workers. Entrepreneurs are never unemployed, they always manage either vacant or filled jobs. Therefore total employment in the model is given by $[1 - F(1)] + n_r + n_s$.

Steady-state unemployment is thus given by [7], [12] and [13]:

$$u = \frac{l}{\frac{g(\theta_r)}{\delta} + \frac{g(\theta_s)}{\delta + \rho} + 1} \Rightarrow u = \frac{l \cdot \delta \cdot (\delta + \rho)}{g(\theta_r) \cdot (\delta + \rho) + g(\theta_s) \cdot \delta + \delta \cdot (\delta + \rho)} \qquad [14]$$

The temporary assumption that *u* is given to the entrepreneurs can be dropped. In order to show clearly that equation [14] closes the model, it can be re-written in the general form $u = \Gamma(u)$, by also considering that changes of *u* shift the function [8] in fig. 1 and then changes of v_s.

Proposition. *An aggregate equilibrium with positive* u *in the model exists and it is unique.*

Proof. In order to proof the uniqueness of the equilibrium it suffices to study the slope of the *l.h.s* and the *r.h.s* of equation [14]. The *l.h.s* side of [14] is an increasing linear function of *u*, whereas the *r.h.s.* of [14] is a rising and concave function in *u* for given v_r and v_s, because $\partial g(\theta_i)/\partial u < 0$ and $\partial^2 g(\theta_i)/\partial u^2 > 0$. Note that being a rate, *u* ranges between 0 and 1, i.e. $(0 < u < 1)$. Since $\lim_{u \to 0} \Gamma(u) = 0$ and $\lim_{u \to 1} \Gamma(u) < 1$, a unique intersection exists between the *l.h.s.* and the *r.h.s.* Induced changes of $v_r(u)$ and $v_s(u)$, through changes in *T*, cannot cumulate themselves because they are complementary; therefore, we also get $|\Gamma'(u)| < 1$.##

With knowledge of v_s, T, v_r, and u, the number of regular and irregular entrepreneurs, workers, total employment and the overall unemployment rate, given in (14), are all uniquely determined.

Therefore, the equilibrium of the model can be defined thus:

Definition. *The solutions for the five key variables* v_s, v, T, e *and* u *are obtained by considering: 1) the Bellman equations; 2) the Nash bargaining rule; 3) the entrepreneur's indifference condition between running firms in the two sectors, given their ability distribution; 4) the optimalization of skill accumulation; 5) the equilibrium condition of the transition flows in the labour market.*

2.5 Discussion

The main result is that an interior solution exists where both the hidden sector and the official sector survive in equilibrium (see also Pugno, 2000a, and Carillo and Pugno, 2004). As claimed by Boeri and Garibaldi (2006), this may explain the "shadow puzzle", i.e. the persistence of the hidden sector despite advances in detection technologies and organisation by public authorities to reduce irregularities.

A number of other important results can be drawn from exercises of comparative statics. A general exercise concerns the effects of the shift of the *T*-curve [11] due to changes in some parameters. Its downward shift decreases the equilibrium v_S in Fig. 3, but it also decreases the equilibrium v_S in the entire model, since the feedback through u is of minor importance. Therefore, the downward shift of the *T*-curve [11] squeezes the proportion of the hidden sector and expands the proportion of the official sector, as clearly emerges from [9] and [10] and as implied by [7], [12] and [13] together.

The downward shift of the *T*-curve [11], and the squeeze of the hidden sector increases unemployment if v_r is sufficiently large (see the *Appendix D*); otherwise it reduces unemployment. Showing this requires use of the *Beveridge Curve*, i.e. equation [14]: from this we obtain $\partial u / \partial v_r < 0$, $\partial u / \partial v_s < 0$ and $\partial u / \partial v_r < \partial u / \partial v_s$ (see *Appendix E*). As a result, if v_r is sufficiently large, the *Beveridge Curve* of the hidden sector is steeper than the *Beveridge Curve* of the official sector, which means that when the hidden sector decreases the unemployment rate increases.[17] The effect is not large, because the effects of the two sectors partially offset each other. More generally, therefore, the result is that a change in the hidden sector can have an ambiguous but small effect on unemployment. This ambiguity reflects the lack of consensus in the literature, both theoretical and empirical, on this issue (see Tanzi, 1999; Giles and Tedds, 2002).

The downward shift of the *T*-curve [11] may very likely increase overall output, because the official sector achieves greater productivity than the hidden sector for three reasons: first, the official sector exhibits the premium p, which captures its greater technological level; second, it employs skilled workers; third, the most able entrepreneurs prefer this sector. The possible reduction of the hidden sector must be very substantial to compensate for these effects.

Another result claimed by Boeri and Garibaldi (2006) can also be drawn from our model: the shadow wage gap, i.e. the wage gap, is wider, the better the aggregate economic conditions. In fact, for a greater y_r, the *T*-curve is shifted downwards, so that w_r is greater both directly and indirectly through the rise in v_r. Therefore, gap $w_r - w_S$ widens.

3 – The extended model

The performances of the regular firm and the irregular firm differ not only because of their technological level and other specific economic features but also because of the contexts in which they operate. If regular firms are diffused and pervasive in the economy with respect to the irregular firms, they operate more efficiently than in the case where they are relatively few. In fact, information flow more easily, trust is more widespread, networking is more diffused, and a more efficient use of public services, including information and

[17] This conclusion run counter to Bouev's (2002, 2005) idea that scaling down the unofficial sector can lead to a decrease in the level of unemployment, whereas it agree with the idea of Boeri and Garibaldi (2002, 2006) that attempts to reduce, in the first place, shadow employment will result in higher open unemployment.

assistance from the public authorities and agencies, becomes possible. Large positive externalities are at work in this case.[18]

By contrast, if the hidden sector is widespread, large negative externalities on the regular firms may be at work. The unfortunate case of the Southern regions of Italy provides the clearest example of these externalities, because in those regions the hidden sector is linked to the illegal sector and to criminal organisations. Transaction costs become greater in this case, market networking becomes distorted, and tax morality worsens.[19]

Both positive and negative externalities can be characterised by a non-linearity which is typical of diffusion of the contagion-type. In the case of positive externalities, the diffusion of information and of trustful entrepreneurial behaviour typically follows the bandwagon effect, which characterises the acceleration of the central phase of the diffusion process (Minniti, 2005). A similar pattern seems to follow criminal behaviour (Glaeser et al., 1996) and criminal enterprises (Pugno, 2000b), which exert negative externalities onto regular firms. The underlying mechanism of the S-shape pattern of diffusion is Schelling's (1978: ch.3) argument of critical mass in imitative behaviour on spatial basis (see also Granovetter 1978). The non-linear diffusion also emerges if imitation simply follows costs reduction because of strategic complementarities on spatial basis, thus explaining the geographical concentration (Krugman, 1991; Puga and Venables, 1996).

Our model is able to capture these phenomena with interesting results. Let us cease considering s and k as fixed parameters and treat them as functions of v_r and v_s as follows:

$$s = \frac{\Phi_1}{1 + e^{\Phi_2 - \Phi_3 v_s}} \qquad [15]$$

$$k = \frac{\Omega_1}{1 + e^{\Omega_2 - \Omega_3 v_r}} \qquad [16]$$

The key property of [15], which is monotonically increasing with respect to v_s, is the convexity in the first trait and then the concavity. The function [16] has the same properties with respect to v_r, but opposite properties with respect to v_s, so that the algebraic sum s–k reinforces the non-linear effect in the same direction. Greek capital letters give the horizontal position of the inflection point, if numbered with 2, and the slope of the function, if numbered with 3. The adoption of these specifications fixes the ideas without losing in generality.

The parameter Φ_1 captures the administrative and bureaucratic burdens and the maximum burden imposed by the criminal context, while Φ_3 gives the acceleration effect when the critical density of the criminal activity has been approached. Similarly, Ω_1 captures the maximum possible effect of the positive externalities arising from the diffusion of regular firms, while Ω_3 gives the acceleration effect of these externalities.

[18] There is a large body of evidence for the spillover effects on productivity. See Cooper and Haltiwanger (1996) for a survey on this literature. For the importance of social networks for entrepreneurship see Aldrich and Zimmer (1986), and Granovetter (1985).

[19] Cross-section analysis of developed and developing countries shows that the size of the hidden sector is significantly negatively correlated with generalised trust (D'Hernoncourt and Méon, 2008), and that generalised trust is negatively correlated with corruption. Although the connection between trust and corruption is reciprocal, the effect of trust on corruption is greater than the reverse (Uslaner, 2002). Further, hidden activity is larger in countries where managers are more likely to pay bribes, where managers pay for mafia-type protection, where managers have less faith in the legal system (Johnson et al., 2000), and where corruption is generally more widespread (Buehn and Schneider, 2009).

If the functions $s(v_S)$ and $k(1-v_S)$ as in [15] and [16] are plugged into [8], then the relationship between T and v_S can change significantly, since a "hump" arises. For v_S close to zero, negative externalities tend to the minimum, and positive externalities tend to the maximum. For rising v_S, the threshold value of entrepreneurial ability, i.e. T, is declining when v_S remains low, but it rises when the density of the irregular firms accelerates the negative externalities and largely reduces the positive externalities, since greater entrepreneurial ability is required. After this acceleration of the externalities, the usual forces that reduce function [8] once again prevail, thus going towards the conditions where negative externalities are at the maximum, and positive externalities are at the minimum, since v_S becomes predominant.

If the accelerations and decelerations were irrelevant and externalities diffused themselves smoothly, then the slope of the extended variant of [8] would be less steep or steeper than the slope of the benchmark function, depending on the importance of the negative with respect to the positive externalities. This captures two distinct facts: that control over the territory by criminal organisations discourages the establishment of regular firms, thus reducing the proportion of the official sector, and that efficient networking requires numerous official partner firms. Crossing the extended variant of [8] with [10] determines the unique equilibrium in a similar way as in the benchmark case. The only difference is that the proportion of the hidden sector is greater if negative externalities are greater than the positive ones.

However, if accelerations and decelerations are significant and externalities diffuse themselves roughly, then three intersections between the extended variant of [8] and [10] become possible, as depicted in Fig.3 (dotted line).[20] The extreme intersections are the relevant ones, because they define two stable equilibria. The intersection in the middle is generally unstable, since it can act as a *saddlepoint* (see *Appendix F*). The two relevant equilibria may be labelled as "good" and "bad" because they define two different conditions where the proportion of the hidden sector is small and, respectively, large; production is high and, respectively, low; the entrepreneurial ability is used efficiently and, respectively, inefficiently; skilled workers are many, and, respectively, few; negative externalities are limited, and, respectively, widespread; positive externalities are exploited, and, respectively, scarce.

========== Fig. 3 about here (now at the end) ==========

According to the model, unemployment in the "good" equilibrium should be substitutable with employment in the hidden sector because the official sector would be large. But in the "bad" equilibrium, the official sector may be not sufficiently large, so that unemployment may be complementary to employment in the hidden sector. A more reliable result on unemployment, however, rests on the fact that several workers in the irregular firms appear as unemployed in the official statistics, so that they will be more numerous in the "bad" with respect to the "good" equilibrium.

This result is interesting because it can represent an economy characterised by a uniform structure, including the institutional structure, as captured by the same parameters of the model, but with two regions and two populations that differ in their histories alone, as captured by different initial levels of v_s. The region starting with a greater proportion of the hidden sector may converge towards the "bad" equilibrium, while the region starting with a smaller proportion of the hidden sector may converge towards the "good" equilibrium.

[20] Also Minniti's (2005) model of entrepreneurship and non-linear externalities, but without the hidden sector, exhibits multiple equilibria.

Distortions, both costly and beneficial, develop differently, and eventually establish a dualism in both economic and social aspects. The Italian North-South divide, which is special but not unique in the world, can thus find an explanation.

Therefore, these results strengthen the idea of the persistence of shadow activities, since they emerge as non-inevitable phenomena insofar as the hidden sector is really small in the "good" equilibrium. Economic development can be represented by growth of *y* in both sectors, and it thus may not change greatly in the relative proportion of the hidden sector. Symmetrically, these results weaken the idea of shadow activities as incubators of infant industries because they attract the less qualified entrepreneurs and workers even in the case of a substantial hidden sector.

4 – Simulations and comparative statics

In order to substantiate the main analytical predictions of the theoretical model, some simple numerical simulations are performed. The baseline specification of the model's parameters has been drawn from Boeri and Garibaldi (2006), and it is described in Table 1.

parameter	***calibration value***
β	*0.50*
r	*0.03*
ρ	*0.06*
δ	*0.15*
τ	*0.20*
u	*0.121*
c	*0.40*
$b = a$ [21]	*0.50*

Table 1

Further, ϕ=2 and $\gamma = 0.2$ because in our model $\beta > \gamma$.

The simulation confirms that the function $T(v_s)$ is monotonically decreasing in v_s. Moreover, although the function [8] is indeterminate for $v_s \to 0$, the simulation shows that for $v_s \approx 0$ the vertical starting-point of [8] is clearly higher than the intercept of [10].[22] Regarding function [10], we use a distribution for the entrepreneurial ability *x* that is negative exponential.[23] As a result, a unique couple of (v_s, T) exists in the benchmark model.

[21] Elasticity of matching function (Cobb-Douglas functional form) with respect to unemployment rate.

[22] For example, for $v_s = 0.001$, T is equal to 5.216.

[23] A negative exponential distribution is also used by Boeri and Garibaldi (2006) for the distribution of productivity.

========== Fig. 4 about here (now at the end) ==========

The effects of parameter changes, which are interesting for policy purposes, on the stable interior equilibrium are summarized in Table 2 below:

Policy \ *Effect on*	T	v_s	v_r	$\Theta = \frac{\theta_r}{\theta_s}$
$\Delta\rho > 0$	–	–	+	+
$\Delta\phi > 0$	–	–	+	+
$\Delta\tau > 0$	+	+	–	–
$\Delta c > 0$	+	+	–	–
$\Delta(p \cdot y_r - y_s) > 0$	–	–	+	+

Table 2

The effects of a change in the intensity of monitoring ($\Delta\rho > 0$), the severity of punishment for concealing business underground ($\Delta\phi > 0$)[24], as well as the effect of higher taxes ($\Delta\tau > 0$), confirm the results obtained in other studies of shadow economies (Friedman et al., 2000; Johnson et al., 2000; Sarte, 2000).

In our model, c reflects the burden on start-up. Hence, a decrease in c can possibly be achieved by restructuring bureaucracy. As a result, the outcome is a drop in the equilibrium value of v_s and an increase in the equilibrium value of v_r. A shadow employment that is increasing in labour market regulations is common to other models of the shadow economy and holds in many cross-sectional studies (Bouev, 2005).

Finally, the productivity differential, $\Delta(p \cdot y_r - y_s) > 0$, contributes to reduction of the hidden sector because the greater is $p \cdot y_r$ (or the lower is y_s), the wider are the surplus differentials and wage gap between regular and irregular sectors.

The calibration of the extended model considers the parameter values in equations [15] and [16] such as to ensure that $T > 0$. The result is depicted in Fig.5.

========== Fig. 5 about here (now at the end) ==========

In particular, the simulation shows the special role played by the parameters which regulate the acceleration/deceleration of the externalities: in fact, the greater is Φ_3 and the lower is Ω_3, the higher is the "hump" of the extended function [8], because the negative externalities rise faster and the positive ones end up quicker.

5 – Final remarks on policy implications

[24] This effect on *T* is very small.

Policies to reduce the hidden sector emerge as advisable from our analysis of both the benchmark and the extended model.

Any policy action that discourages the profitability of irregular firms will very likely improve the overall production level and productivity. Both entrepreneurs and workers take advantage of their qualities, while tax morality is strengthened. But the extended model yields a further result in this regard, since it suggests policy actions from the sectoral perspective, rather than from the firm perspective alone, with possible powerful effects when irregular practices are widespread.

Firm-perspective policies are a reduction of taxes (τ), of start-up costs (*c*), and increased controls (ρ) and penalties (ϕ). All these policy actions reduce the proportion of the hidden sector because the reduction of τ or *c*, and the rise of ρ or ϕ shift function [8] in Fig. 5 downwards, thus reducing the equilibrium level of v_s. The efficacy of each of these policy actions is difficult to evaluate, since much depends on the relative amount of changes in the parameters, for which there is no yardstick. Indeed, the model is not built for determination of the optimal policy.

The extended model provides further parameters which are eligible for policy actions, i.e. the parameters that specify the externalities from sectors on firms. The parameters in [15] suggest reducing administrative and bureaucratic burdens and also detecting and punishing criminal and illegal activities. But parameter Φ_3 plays a special role, and it should suggest something more specific. This parameter, in fact, regulates the acceleration of the negative externalities, and if it is great, it may be responsible for the equilibrium of the "bad" type. This parameter suggests policy actions intended to combat the contagion effect of the negative externalities, as appear from the diffusion of the bad practices like bribes and protection money. If Φ_3 is sufficiently reduced, then the "hump" of the extended function [8] in Fig. 6 is smoothed out, and the system spontaneously converges to the "good" equilibrium without any further policy action.

The parameters in [16], on the other hand, suggest the provision of infrastructure, services, and network facilities. Development policies of this kind also emerge as very high-return actions because they may work to smooth out the "hump" of the extended function [8].

An interesting parameter for policy action would be *x*, i.e. entrepreneurial ability. Education, in fact, may be important for increasing entrepreneurial ability (Draghi, 2006), although the evidence on this point is not robust (Cappariello and Zizza, 2009). Education to improve entrepreneurial ability is especially favourable because it raises efficiency more in regular firms than in irregular ones and raises efficiency more in the regular sector than in the irregular one. Moreover, the model tells us that the effect of greater education works through the increase of y_r and the downwards shift of function [8]. This reduces the equilibrium level of v_s and increases firm's productivity and production beyond and above the simple composition effect between the two sectors.

Appendices

Appendix A: The geography of the hidden economy, and of the organised crime in Italy

Figure 1 shows how the hidden economy, measured by regional employment rates of irregular labour, is localised in Italy. Figure 2 shows the geographical distribution of organized crime, measured by an index calculated as the sum of typical crimes committed by criminal organisations, i.e. extortion and the creation of criminal associations (including mafia-type ones), per 10,000 inhabitants. The similarity between the two figures is striking.

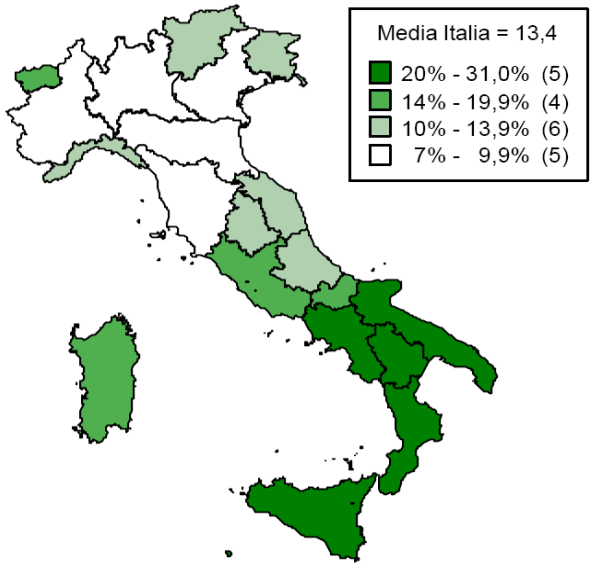

Figure 1: The geography of hidden labour. Source: ISTAT (2005)

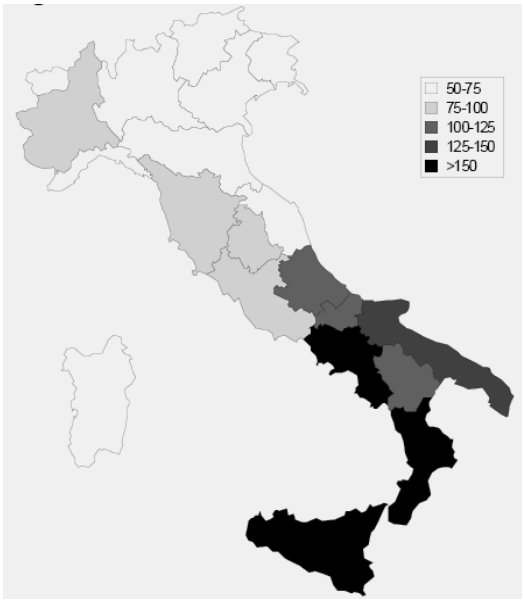

Figure 2: The geography of the organized crime Source: Daniele and Marani (2008)

Appendix B: Wage setting

The first order condition of the optimal sharing of surplus in the official sector yields:

$$w_r = \arg\max\left(J_r - V_r\right)^{1-\beta} \cdot \left(W_r - U_r\right)^{\beta}$$

$$\Rightarrow \left(W_r - U_r\right) = \frac{\beta}{1-\beta} \cdot \left(J_r - V_r\right)$$

Using the Bellman equations this becomes:

$$\Rightarrow \frac{w_r}{r+\delta+g(\theta_r)} = \frac{\beta}{1-\beta} \cdot \frac{p \cdot x_r \cdot y_r + k - w_r + c - \tau - s}{r+\delta+f(\theta_r)}$$

Simple manipulations give the final formula for the wage in the official sector [5]:

$$w_r = \frac{\beta \cdot \left[r+\delta+g(\theta_r)\right] \cdot \left(p \cdot x_r \cdot y_r + k + c - \tau - s\right)}{\left\{\beta \cdot \left[r+\delta+g(\theta_r)\right] + (1-\beta) \cdot \right\}\left[r+\delta+f(\theta_r)\right]} \Rightarrow w_r = \frac{p \cdot x_r \cdot y_r + k + c - \tau - s}{1 + \frac{(1-\beta) \cdot \left[r+\delta+f(\theta_r)\right]}{\beta \cdot \left[r+\delta+g(\theta_r)\right]}}$$

The same procedure applies for wages in the hidden sector:

$$\left(W_s - U_s\right) = \frac{\gamma}{1-\gamma} \cdot \left(J_s - V_s\right)$$

$$\Rightarrow \frac{w_s}{r+\delta+\rho+g(\theta_s)} = \frac{\gamma}{1-\gamma} \cdot \frac{x_s \cdot y_s - w_s - \rho \cdot \phi \cdot \tau}{r+\delta+\rho+f(\theta_s)}$$

$$\Rightarrow (1-\gamma) \cdot \left[r+\delta+\rho+f(\theta_s)\right] \cdot w_s = \gamma \cdot \left[r+\delta+\rho+g(\theta_s)\right] \cdot \left(x_s \cdot y_s - w_s - \rho \cdot \phi \cdot \tau\right)$$

So that the final formula for wages in the hidden sector [6] is:

$$w_s = \frac{\gamma \cdot \left[r + \delta + \rho + g(\theta_s)\right] \cdot \left(x_s \cdot y_s - \rho \cdot \phi \cdot \tau\right)}{\left\{\gamma \cdot \left[r + \delta + \rho + g(\theta_s)\right] + (1-\gamma) \cdot \left[r + \delta + \rho + f(\theta_s)\right]\right\}}$$

$$\Rightarrow w_s = \frac{x_s \cdot y_s - \rho \cdot \phi \cdot \tau}{1 + \dfrac{(1-\gamma) \cdot \left[r + \delta + \rho + f(\theta_s)\right]}{\gamma \cdot \left[r + \delta + \rho + g(\theta_s)\right]}}$$

Appendix C: Equilibrium in the benchmark model

In order to prove the existence of one stable equilibrium in the benchmark model, the associated dynamic equations are required, i.e.:

$$\dot{v}_s(t) = \int_{x_{\min}}^{T(t)} x \, dF(x) - v_s(t) \cdot f(\theta_s(t)) \qquad [C.1]$$

$$\dot{T}(t) = T(v_s(t)/u, y_r, y_s) - T(t) \qquad [C.2]$$

[C.1] shows the transition flows in and out of irregular vacancies. As regards [C.2], because T is not characterised by flows, but is a threshold value, we consider the simple difference in continuous time between the new and the previous value of T. From [C.1] and [C.2] we get:

$$\frac{\partial \dot{v}_s}{\partial v_s} = -f(\theta_s) < 0 \; ; \; \frac{\partial \dot{T}}{\partial T} = -1 .$$

While, we know that:

$$\frac{\partial \dot{v}_s}{\partial T} > 0 \; ; \; \frac{\partial \dot{T}}{\partial v_s} < 0 .$$

In this very simple case, the equilibrium emerges from study of the "phase diagram" as depicted in Fig. 3.

Appendix D: Threshold value of regular vacancies

From equation [14], we get:

$$\frac{\partial}{\partial v_r} \frac{l \cdot \delta \cdot (\delta + \rho)}{g(v_r/u) \cdot (\delta + \rho) + g\left((1 - v_r - l)/u\right) \cdot \delta + \delta \cdot (\delta + \rho)}$$

because we are interested in the value of v_r such that $\partial u / \partial v_r > 0$:

$$-\left[l \cdot \delta \cdot (\delta + \rho)^{-2}\right] \cdot g'(v_r/u) + l \cdot \delta^{-2} \cdot (\delta + \rho) \cdot g'\left((1 - v_r - l)/u\right) > 0$$

$$g'\left((1 - v_r - l)/u\right) > \left[(\delta + \rho)/\delta\right] \cdot g'(v_r/u)$$

in the *Cobb-Douglas* case, i.e. $m = v^{1-a} u^a$, we obtain:

$$(1-a) \cdot \left((1 - v_r - l)/u\right)^{-a} > (1-a) \cdot \left[(\delta + \rho)/\delta\right] \cdot (v_r/u)^{-a}$$

$$(1 - v_r - l)^{-a} \cdot u^a > \left[(\delta + \rho)/\delta\right] \cdot (v_r)^{-a} \cdot u^a$$

$$\left[\delta/(\delta + \rho)\right] > \left((1-l)/v_r\right)^a - 1$$

$$1 + \left[\delta/(\delta + \rho)\right]^{\frac{1}{a}} > (1-l)/v_r$$

Eventually, we get:

$$\Rightarrow v_r > (1-l) \cdot \left[1 + \left(\delta/(\delta + \rho)\right)^{\frac{1}{a}}\right]^{-1}$$

Appendix E: Beveridge Curves analysis

From equation [14], we get[25]:

$$\frac{\partial u}{\partial v_r} = -\left\{ \frac{l \cdot \delta \cdot (\delta+\rho)^2 \cdot \overset{>0}{g'(\theta_r)}}{\left[(\delta+\rho)\cdot g(\theta_r) + \delta \cdot g(\theta_s) + \delta \cdot (\delta+\rho)\right]^2} \right\} < 0$$

$$\frac{\partial u}{\partial v_s} = -\left\{ \frac{l \cdot \delta^2 \cdot (\delta+\rho) \cdot \overset{>0}{g'(\theta_s)}}{\left[(\delta+\rho)\cdot g(\theta_r) + \delta \cdot g(\theta_s) + \delta \cdot (\delta+\rho)\right]^2} \right\} < 0$$

Assuming that $\theta_r > \theta_s$, i.e. $v_r > v_s$, and knowing that $g'(\theta_i) > 0$, $g''(\theta_i) < 0$, we obtain $g'(\theta_s) > g'(\theta_r)$ and eventually we have $\partial u / \partial v_r < \partial u / \partial v_s$.

Appendix F: Equilibrium in the extended model

Linearizing the dynamic equations around the steady state (v_s^*, T^*) yields:

$$\dot{v}_s = \Theta \cdot [T - T^*] - f(\theta_s) \cdot [v_s - v_s^*]$$

$$\dot{T} = -\Psi \cdot [v_s - v_s^*] - [T - T^*]$$

where $\Theta \equiv \frac{\partial \dot{v}_s}{\partial T} > 0$ and $-\Psi \equiv \frac{\partial \dot{T}}{\partial v_s} < 0$.[26]

In matrix form, we get:

$$\begin{pmatrix} \dot{v}_s \\ \dot{T} \end{pmatrix} = \begin{pmatrix} -f(\theta_s) & \Theta \\ -\Psi & -1 \end{pmatrix} \cdot \begin{pmatrix} v_s - v_s^* \\ T - T^* \end{pmatrix}$$

where $\begin{pmatrix} -f(\theta_s) & \Theta \\ -\Psi & -1 \end{pmatrix} \equiv \Lambda$ is the *coefficients matrix*.

And thus, we obtain:

$$\begin{pmatrix} -f(\theta_s) - \lambda & \Theta \\ -\Psi & -1 - \lambda \end{pmatrix}$$

$$[-f(\theta_s) - \lambda] \cdot (-1 - \lambda) - (\Theta \cdot -\Psi)$$

$$[-f(\theta_s) - \lambda] \cdot (-1 - \lambda) + \Theta \cdot \Psi$$

$$\Rightarrow \lambda^2 - \lambda \cdot tr\{\Lambda\} + det\{\Lambda\}$$

where $tr\{\Lambda\} = -f(\theta_s) - 1$ and $det\{\Lambda\} = f(\theta_s) + \Theta \cdot \Psi$ are the trace and the determinant of the matrix respectively.

Stable equilibrium conditions require that:

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[25] Equation [14], like the standard *Beveridge Curve*, is a convex function with respect to both v_r and v_s:

$$\frac{\partial^2 u}{\partial v_r^2} = \frac{-l \cdot \delta \cdot (\delta+\rho)^2 \cdot \overset{<0}{g''(\theta_r)} \cdot \mathrm{H}^2 - \left\{ -l \cdot \delta \cdot (\delta+\rho)^2 \cdot g'(\theta_r) \cdot 2 \cdot \mathrm{H} \cdot (\delta+\rho) \cdot g'(\theta_r) \right\}}{\mathrm{H}^4} > 0$$

$$\frac{\partial^2 u}{\partial v_s^2} = \frac{-l \cdot \delta^2 \cdot (\delta+\rho) \cdot \overset{<0}{g''(\theta_s)} \cdot \mathrm{H}^2 - \left\{ -l \cdot \delta^2 \cdot (\delta+\rho) \cdot g'(\theta_s) \cdot 2 \cdot \mathrm{H} \cdot \delta \cdot g'(\theta_s) \right\}}{\mathrm{H}^4} > 0$$

where: $\mathrm{H} \equiv [(\delta+\rho) \cdot g(\theta_r) + \delta \cdot g(\theta_s) + \delta \cdot (\delta+\rho)]$.

[26] In the extreme intersections (or lateral equilibria), we know that: $\partial T / \partial v_s < 0$.

$$tr\{\Lambda\} = \{- f(\theta_s) - 1\} < 0$$

Indeed, in our case the trace is negative, and then the lateral equilibria are stable.

When we analyse the intersection in the middle, we must recall that:

$$\Psi \equiv \partial \dot{T} / \partial v_s > 0$$

and thus, in this case, we obtain:

$$det\{\Lambda\} = f(\theta_s) - \Theta \cdot \Psi$$

if $\Theta \cdot \Psi > f(\theta_s)$, then $det\{\Lambda\} < 0$.

If the determinant has a negative sign, then a *saddlepoint* exists in the equilibrium point between the lateral ones, as depicted in Fig. 4.

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Figures

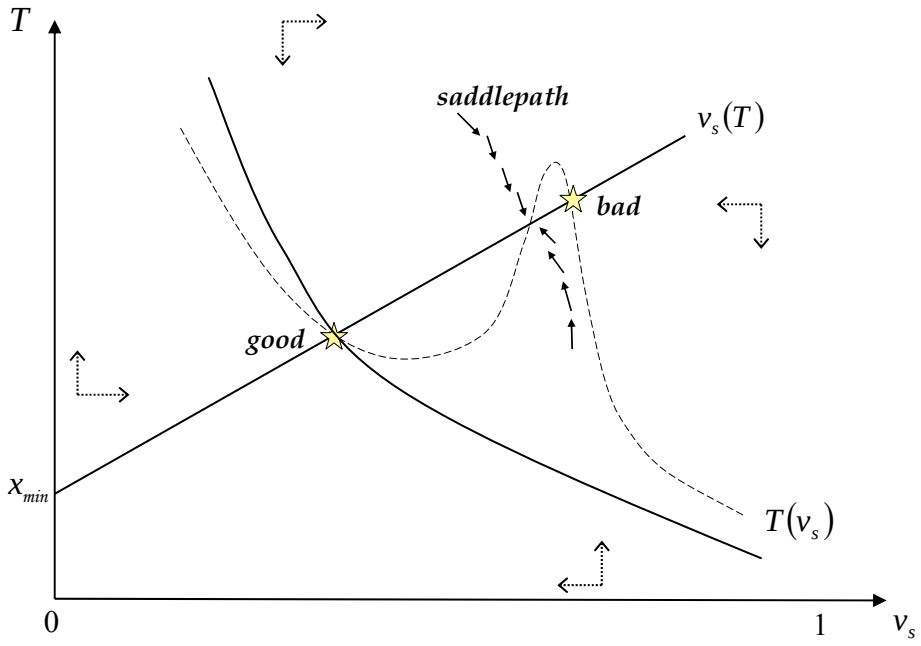

Figura 3

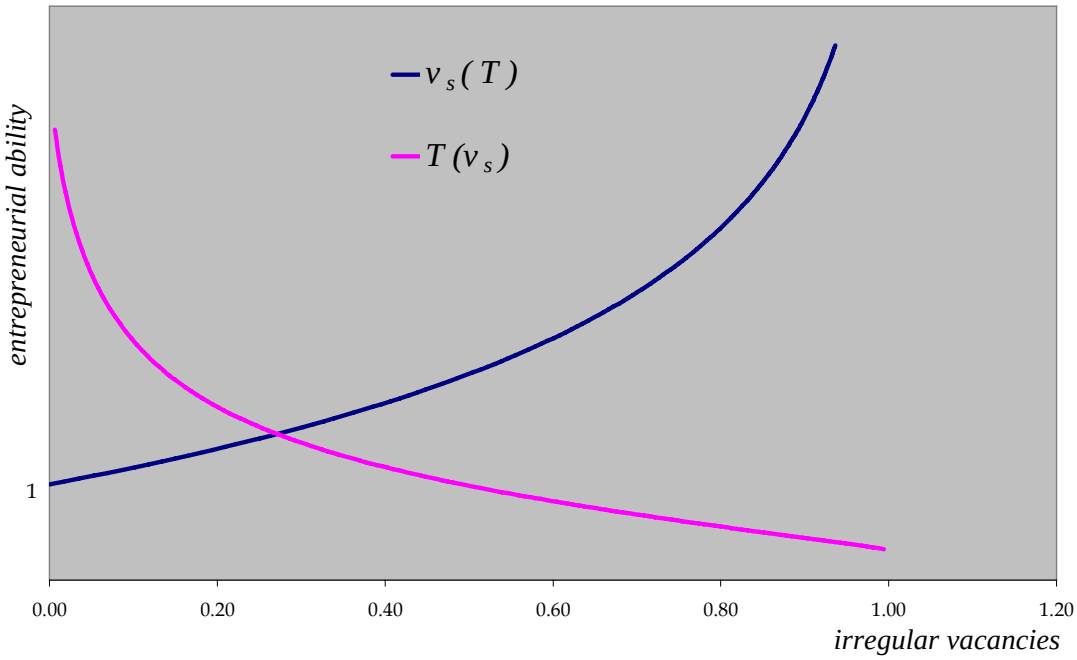

Figure 4

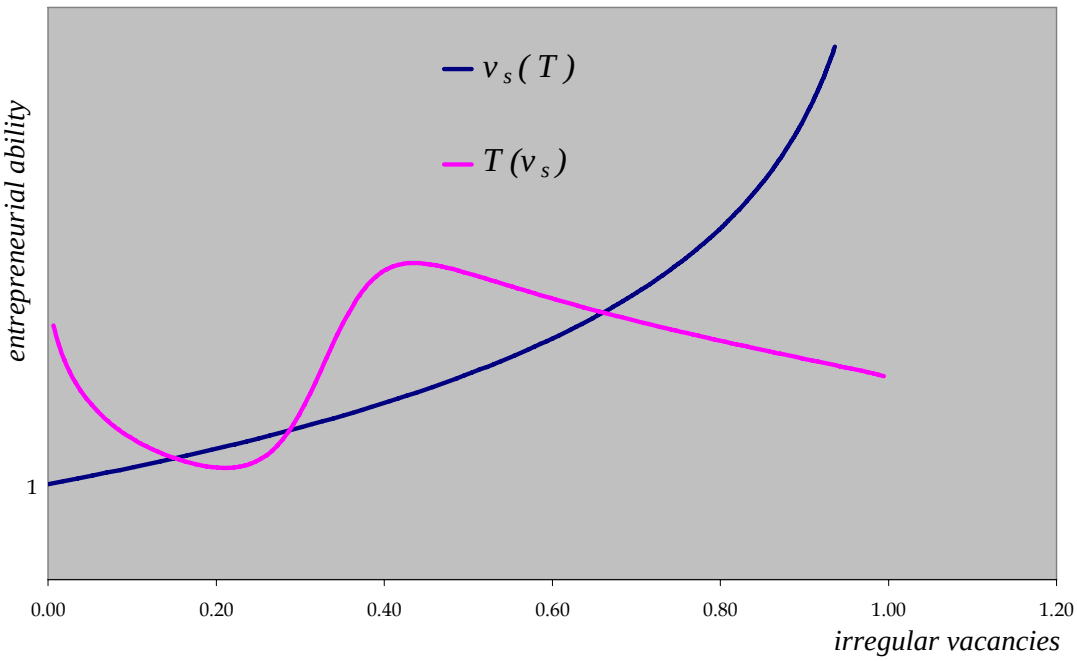
$v_s(T)$
$T(v_s)$
entrepreneurial ability
1
0.00
0.20
0.40
0.60
0.80
1.00
1.20
irregular vacancies

Figure 5