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Theoretical and Practical Arguments for Modeling Labor Supply as a Choice among Latent Jobs

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Abstract

Models of labor supply derived from stochastic utility representations and discretized sets of feasible hours of work have gained popularity because they are more practical than the standard approaches based on marginal calculus. In this paper we argue that practicality is not the only feature that can be addressed by means of stochastic choice theory. This theory also offers a powerful framework for developing a more realistic model for labor supply choices, founded on individuals having preferences over jobs and facing restrictions on the choice of jobs and hours of work. We discuss and clarify how this modeling framework deviates from both the conventional discrete approach (Van Soest, 1995), as well as the standard textbook approach based on marginal calculus (Hausman, 1985). It is argued that a model based on job choice opens up for a more realistic representation of the choice environment, and consequently offers the possibility of conducting a richer set of simulations of alternative policies.

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1. Introduction

Discrete choice models of labor supply based on stochastic utility theory have gained widespread popularity, mainly because they are much more practical than the conventional continuous approach based on marginal calculus, see the survey by Creedy and Kalb (2005). The discrete approach differs from the corresponding continuous one in that the set of feasible hours of work is approximated by a suitable and finite discrete set. With the discrete choice approach, it is easy to deal with nonlinear and non-convex economic budget constraints, and to apply rather general functional forms of the utility representations. With particular distributional assumptions about the stochastic disturbances in the utility function one can derive tractable expressions for the distribution of hours of work, such as the multinomial - or the nested multinomial logit model. From a theoretical perspective, however, the conventional discrete choice model is similar to the standard textbook approach to labor supply in that it is essentially a version of the theory of consumer behavior. The only new assumption postulated is that the set of feasible hours of work is finite.[1] Some researchers, such as Keane (2010) for example, essentially ignore the discrete choice approach to labor supply altogether, and by others it is played down and referred to only as a somewhat crude approximate approach that makes estimation problems manageable (Blundell, MaCurdy and Meghir, 2007, and Heim, 2009).[2]

The main message we wish to convey in this paper is that the conventional discrete labor supply model can be extended and re-interpreted as a model that accounts for important but neglected aspects of the labor market, namely that individuals have preferences over jobs, and face restrictions in their choice of jobs and hours of work. For example, a typical feature of the labor market is that most jobs offer only full-time or a fixed fraction of full-time work (part-time). This feature is reflected in the data on the distribution of hours of work in many countries with peaks at full-time and part-time hours of work. See for example, Ilmakunnas and Pudney (1990), Kahn and Lang (1991), Dickens and Lundberg (1993), and Stewart and Swaffield (1997) who report evidence from surveys about differences between actual and desired hours of work. Traditional models of labor supply, including the conventional discrete choice model of labor supply, are silent about these restrictions.

[1] In addition, the random components of the utility function have c.d.f. which implies a tractable empirical model.

[2] Discrete choice approaches are also seen as useful in order to include programme participation in models, along the lines of Keane and Moffitt (1998) and Brewer et al. (2006).

This paper discusses how the notion of "job choice" and restrictions on hours of work can be accounted for within the framework of discrete choice and random utility representations. In this paper we discuss and clarify how this modeling framework deviates from both the conventional discrete approach (Van Soest, 1995), as well as the standard textbook approach based on marginal calculus (Hausman, 1985).

It should be noted that the choice of job and the notion of job attributes have been emphasized in other parts of the labor market literature, see for instance Sattinger (1993), van Ophem, Hartog and Vijverberg (1993), but the approach we follow in this paper differs from theirs. Here, the modeling approach departs from a formulation of the choice environment where individuals face latent choice sets of jobs. The jobs are characterized by fixed, job-specific hours of work and non-pecuniary attributes, such as the nature of tasks to be performed, location of the work place, working environment, etc. In this model, observed hours of work and disposable income, are thus interpreted as hours of work and consumption that follow from the *chosen* job. Although the sets of jobs that are available to the respective workers are latent (to the researcher), the discrete choice methodology enables us to represent the distribution of available job opportunities specified as a function of offered hours of work (offered by the firms). Thus, the type of model presented here is capable of representing the distribution of preferences on one hand, and economic and other type of choice restrictions, on the other. Restrictions on for example part-time hours of work will thus in this setting be interpreted as restrictions on the set of part-time jobs that are available to the individual. Similarly to the conventional discrete labor supply model, the alternative job choice model we propose is rather practical and user-friendly, in the sense that it is very easy to simulate behavioral effects from policy reforms. Thus, we believe that our alternative framework offers additional advantages to the conventional discrete choice approach that should be of particular relevance for practitioners. A version of labor supply model presented here is part of the model system that Statistics Norway have made available for Norwegian policy-makers to use, see Thoresen, Jia and Aasness (2010).

The paper is organized as follows. The next section discusses briefly the main features of other approaches that have appeared in the literature. In Section 3 we describe in detail our alternative job choice model. Section 4 discusses identification, empirical specification and model assessment, as well as some aspects related to policy simulations.

2. Main features of previous structural approaches

In this paper we focus solely on structural analysis of labor supply in a static setting, that is, analysis based on theories with explicit representations of preferences and budget restrictions, as well as other choice restrictions. Such models enable the researcher to quantify behavioral effects from counterfactual policy interventions, which are at the core of discussions on welfare effects of policy changes.[3]

An overwhelming majority of approaches in the static structural labor supply literature is based on variations of the standard textbook theory of consumer demand (standard approach). In this approach the individual agent is assumed to have preferences over total consumption and leisure, and is assumed to maximize utility under the economic budget constraint determined by the wage, nonlabor income and the tax system, see early contributions on econometric modeling and empirical analyses by Heckman (1974, 1979). Recent surveys of structural labor supply modeling, see Blundell and MaCurdy (1999), Blundell, MaCurdy and Meghir (2007), Keane (2010) and Meghir and Phillips (2010), focus almost entirely on analyses based on the standard approach. As mentioned in the introduction, the obvious fact that agents have preferences over qualitative job attributes, and face important quantitative restrictions when making labor market decisions, is typically neglected. A number of researchers have extended the standard labor supply model to the case with nonlinear (piece-wise linear) budget constraints that possibly imply non-convex budget sets. This type of budget sets follow from tax systems found in many countries, where deduction rules for different type of taxes imply that marginal taxes may not be monotonously increasing with income, but may in fact decrease in specific income intervals. Burtless and Hausman (1978) and Hausman (1979, 1980, 1985) and others, such as Blomquist (1983), have made important contributions to the modeling of labor supply in this type of situations.[4] Blundell, Duncan and Meghir (1998) is also based on a continuous approach, but relies on an

[3] With respect to analyses employing reduced form specifications and exploiting some type of exogeneously induced variation, the literature has recently witnessed a number of analyses using tax reform as "natural experiments" and obtaining measures of elasticities of taxable income, following Feldstein (1995); see the survey by Saez, Slemrod and Giertz (2009). Chetty (2009) argues that such studies very efficiently provide information about welfare effects of taxation and cannot therefore simply be discarded for not addressing key questions.

[4] MaCurdy, Green and Paarsch (1990) show that the econometric model may impose parametric restrictions that constrain estimates of substitution and income effects in applied work. Mroz (1987) also reviews the specifications employed in the (early) literature and finds that results are sensitive to the methodological choices, such as the measurement of wages. Similarly, Bloemen and Kapteyn (2008) report rather mixed experiences with the Hausman approach: even in the single agent case it is almost impossible to write down the true likelihood function of the empirical model given standard assumptions about unobservables, and considerable expertise and computer time is required to estimate this type of model.

identification strategy based on comparing labor supply over time (covering several tax reforms) for different groups defined by cohort and education level.[5]

Unfortunately, standard labor supply models based on marginal criteria, as well as the conventional discrete choice models applied in the studies referred to above, are silent about the potential importance of job attributes for labor supply behavior. Furthermore, these models cannot accommodate observed peaks at part-time and full-time hours, which is a typical feature of the hours of work distribution in most countries. There are however important modification of the standard labor supply models in the literature that address the problem of restrictions on labor market choices, such as Altonji and Paxson (1988), Ilmakunnas and Pudney (1990), Van Soest, Woittiez and Kapteyn (1990), Tummers and Woittiez (1991), Dickens and Lundberg (1993), and Bloemen (2000; 2008). Restrictions on the set of available hours of work have also received some attention in the *macro* literature on labor supply, see Hansen (1985), Keane and Rogerson (2011), Rogerson (1988) and Chang et al. (2011). Early attempts to model constraints on hours of work have taken different forms. For instance, in the model of Ilmakunnas and Pudney (1990) individuals are restricted in various degrees in their opportunities of choosing part-time and full-time work. They utilize actual information on person specific constraints in the labor market, and conduct policy simulations with and without constraints. Dickens and Lundberg (1993) assume a standard labor supply model with a particular rationing device that is somewhat similar in spirit to the one of Ilmakunnas and Pudney (1990), although the final empirical specification of the model is entirely different. Similarly, Bloemen (2000), Van Soest, Woittiez and Kapteyn (1990) and Tummers and Woittiez (1991) apply various specifications to account for hours of work restrictions. However, despite these earlier attempts to account for important labor market characteristics, there are few recent studies that accommodate choice restrictions in the analysis of labor supply.

2.1. The standard textbook approach

Typically, the standard approach to labor supply modeling, see Blundell and MaCurdy (1999), is to choose a specification of an individual labor supply function (hours of work function) consistent with the maximization of a quasi-concave utility function in disposable income and leisure, subject to the economic budget constraint. For simplicity, consider the

[5] To overcome the integrability problem at kinks Blundell et al. condition on the subsample where observations close to kinks are removed.

case with convex budget sets with constraint approximated by a suitable smooth representation. Suppose, for example, that the chosen labor supply function has the structure

$$h = \alpha + \beta \tilde{w}(h) + X\gamma + \delta \tilde{I}(h) + \varepsilon, \quad (2.1)$$

when

$$\alpha + \beta \tilde{w}(0) + X\gamma + \delta \tilde{I}(0) + \varepsilon > 0, \quad (2.2)$$

and $h = 0$ otherwise. Here $\tilde{w}(h)$ is the marginal wage, $\tilde{I}(h)$ is so-called virtual non-labor income, X is a vector of individual characteristics that affect preferences, ε is a random error term, $\alpha, \beta, \gamma,$ and δ are unknown parameters. The inequality in (2.2) represents the condition for working. In general, when the tax system is non-linear, the marginal wage rate and virtual income depend on hours of work and they are therefore endogenous. As a result, one cannot estimate the model by using the ordinary least squares method based on (2.1), and have to rely on appropriate instruments, which are not always easy to find. Additional complications follow from the fact that the wage is not observed for those who do not work (the selection problem).

After the parameters of this labor supply function have been estimated, one needs to solve for h (conditional on wage and non-labor income) in the non-linear equations given by (2.1) and (2.2), which may - or may not - be cumbersome. A more serious concern is that the linear relation in (2.1) may at best be a rather crude approximation of the "true" labor supply function. In fact, the linear functional form of the supply function is clearly *ad hoc* from a theoretical perspective. It is, however, possible to formulate and estimate more complicated labor supply functions in the case where the budget set is convex and smooth. However, in the presence of non-convex and kinked budget sets, which are quite typical in many countries, the analysis becomes very complicated in the case when more general and flexible model specifications are used, see Bloemen and Kapteyn (2008).

Thus, to summarize, the traditional textbook approach based on marginal calculus, including the extension proposed by Hausman (1985), has the following weaknesses. First, it becomes intractable to estimate except for the case with rather simple specifications. Second, empirical results obtained by this approach seem to be far from robust with respect to different specifications of functional form and distributional assumptions. The wide variation in labor supply estimates found in the literature testifies to this (Blundell and MaCurdy, 1999). Third, this approach is silent about other important aspects of the labor market, namely that workers have preferences over job types and may face important constraints in their labor market choices. Fourth, the approach is unable to accommodate observed peaks at part-time and full-time hours, which is typically found in data.

2.2. The conventional discrete choice model

During the last 15 years the adaptation of discrete choice models has become increasingly popular, mainly because this approach simplifies drastically the implementation of complicated nonlinear budget constraints. The work of Bingley and Walker (1997), Blundell et al. (2000), Van Soest, Das and Gong (2002), Creedy, Kalb and Scutella (2004), Haan and Steiner (2005), Labeaga, Olivier and Spadaro (2007) are examples of how empirical discrete choice labor supply models can be estimated and simulated for the purpose of assessing the effect of counterfactual tax reforms. The ability of discrete choice models to handle rather complex decision processes is also substantiated by the use of such models to analyze labor supply jointly with welfare programme participation, or use of non-parental care services (for preschool children). Examples of this type of work are provided by Hoynes (1996), Keane and Moffitt (1998), Brewer et al. (2006), and Kornstad and Thoresen (2007).

The discrete choice labor supply model departs from the theory of random utility models (see McFadden, 1984). We shall next give a summary description. Let $U(C,h)$ denote the agent's utility function of real disposable income and hours of work, (C,h), and assume that

(2.3) $$U(C,h)=v(C,h)+\eta(C,h),$$

where $v(C,h)$ is a positive deterministic term that represents the mean utility across observationally identical agents and $\eta(C,h)$ is a random term that is not correlated with the structural term $v(C,h)$ and with c.d.f. $\exp(-\exp(-x))$, defined for real x.[6] Moreover, $\eta(C,h)$ and $\eta(C',h')$ are independent for $(C,h)\neq(C',h')$. The budget constraint is given by

(2.4) $$C=f(hw,I),$$

[6] In the terminology of Resnick (1987) this c.d.f. is called the type III (standard) extreme value distribution, or Gumbel distribution. Other authors call this c.d.f. the type I extreme value distribution.

where I is non-labor income and $f(\cdot)$ is the function that transforms gross income into after-tax household income. The function $f(\cdot)$ can in principle capture all details of the tax and benefit system. Furthermore, the set D of feasible hours of work is a finite set. If (2.4) is inserted into (2.3) we obtain that

$$(2.5) \qquad \tilde{U}(h) \equiv U(f(hw,I),h) = v(f(hw,I),h) + \eta(f(hw,I),h) = \psi(h) + \tilde{\eta}(h),$$

where $\psi(h) = v(f(hw,I),h),$ and $\tilde{\eta}(h) = \eta(f(hw,I),h).$ For simplicity we have suppressed non-labor income in the notation. By well-known results from the theory of discrete choice (McFadden, 1984) it now follows that the probability $p(h),$ that the agent shall supply h hours of work, given D, the budget constraint in (2.4) and the wage rate and non-labor income (w, I), is equal to

$$(2.6) \qquad p(h) = P(\tilde{U}(h) = \max_{x \in D} \tilde{U}(x)) = \frac{\exp(\psi(h))}{\exp(\psi(0)) + \sum_{x \in D} \exp(\psi(x))}.$$

In empirical applications, the structural part of the utility function, $v(C,h),$ is assumed to have a convenient functional form and is allowed to depend on individual covariates, see for example Van Soest (1995) and Van Soest, Das and Gong (2002). Unfortunately, and similarly to the standard model above, the model given in (2.6) is unable to fit the data well in most cases due to observed peaks at full-time and part-time hours of work. Researchers have therefore replaced the systematic utility term $v(C,h),$ by a "modified" utility term $v(C,h) + \gamma(h),$ where $\gamma(h)$ is equal to one for hours of work different from part-time and full-time hours of work. For example, $\gamma(h) = a$ for h equal to full-time hours and $\gamma(h) = b,$ for h equal to part-time hours, and zero otherwise, where a and b are constants that are estimated from data. Under the modified specification the choice model takes the form

$$(2.7) \qquad p(h) = \frac{\exp(\psi(h) + \gamma(h))}{\exp(\psi(0)) + \sum_{x \in D} \exp(\psi(x) + \gamma(x))}.$$

After the introduction of the modified systematic term of the utility function, the model can be made to fit the data quite well. However, the problem of how this practice should be justified remains. One possibility is to interpret $v(C,h)+\gamma(h)$ as "true" representation of the systematic part of the utility function, but this implies that one believes that agents may have non-monotone utility in hours of work. In particular, this means that agents will have higher utility for part-time and full-time hours of work than for other hours. Although it cannot be ruled out that individuals may have particular preferences for full-time and part-time hours of work, for example, due to social conventions and habits, it seems more plausible to interpret the peaks found in the data as resulting from restrictions on hours of work. Indeed, some researchers have focused on this latter interpretation. Unfortunately, this interpretation lacks foundation because the modeling framework above has no theoretical rationing device that can rationalize quantity restrictions. This is because the conventional discrete labor supply model differs from the continuous one only on the assumption that the set of possible hours is discrete and finite. The discrete approximation of the set of feasible hours (D) of what is believed to be the ideal continuous choice set is not essential here because it does not change the basic theoretical setting, namely that the worker is assumed to be able to choose combinations of hours of work and consumption freely, as long as the budget constraint is met.

Among some researchers there seems to be a belief that the use of discrete choice models in the context of labor supply represents a somewhat crude approximation (Heim, 2009, Blundell, MaCurdy and Meghir, 2007). The reason is that the "true" choice setting is viewed as a continuous one, and consequently the discretization of the choice set of hours of work that follows by implementing a discrete choice model will induce approximation (measurement) errors. Further, it is often argued that commonly maintained assumptions about the error terms in the utility function yielding labor supply choice probabilities that satisfy the Independence from Irrelevant Assumption (IIA) property (Luce, 1959), are unrealistic. In our opinion, such attitudes are unjustified. The discrete approximation to what is believed to be the true continuous setting is hardly an important point. First, it may be argued (as we do) that the true choice setting is in fact a discrete one. Second, with today's computer capacity one may use a very fine-meshed partition: there is hardly any limit to the number of discrete alternatives that can be applied.[7]

[7] From a theoretical point of view, it is interesting to note that in the case when the set of discrete alternatives is infinite, the corresponding choice probability distribution will be a continuous one, see Dagsvik and Strøm (2006).

3. The job choice model

We shall now review essential features of our maintained job choice model. A more rigorous exposition and further details are found in Dagsvik and Strøm (2006), and Dagsvik and Jia (2011a).[8] As mentioned in the introduction, this model departs in an essential way from previous approaches in that we focus on a more comprehensive description of the choice environment in which job choice is the fundamental decision variable.

A job is characterized with fixed (job-specific) working hours, wages and other non-pecuniary attributes. Let $U(C,h,z)$ be the (ordinal) utility function of the household, where the positive indices, z = 1, 2,…, refer to labor market opportunities (jobs) and $z=0$ refers to the nonmarket alternative. For a market opportunity (job) z, associated hours of work is assumed fixed and equal to $H(z)$. In this paper, we will assume that the hours of work and wage rate take only discrete values in a given set. For simplicity, we shall only consider the special case where the wage only depends on individual qualifications and do not depend on jobs. See Dagsvik and Jia (2011a) for a treatment of the more general case with job-specific wages. The utility function is assumed to have the additive separable structure

$$(3.1) \qquad U(C,h,z) = v(C,h) + \varepsilon(z),$$

where the random taste shifters $\varepsilon(z)$ are assumed to account for unobservable individual characteristics and non-pecuniary job-type attributes that affect utility, and hence will vary both across households and job opportunities.

Similarly to the previous section, the particular distribution of the taste shifters $\exp(-\exp(-x))$ is consistent with the property that the choice of jobs satisfies (IIA), (Luce,1959). For given hours and wage rates, h and w, the economic budget constraint is given by (2.4). With the same notation as in the previous section, we realize that the term $\psi(h) = v(f(hw,I),h)$ is now to be interpreted as the representative utility of jobs with hours of work h, a given wage rate w and non-labor income I.

Agents in the labor market are likely to face restrictions on the set of available market opportunities. This is because there are job types for which the worker is not qualified and there may be variations in the set of job opportunities for which he or she is qualified. In addition, due to competition in the labor market, the most preferred type of job for which a

[8] Early versions of this approach are Dagsvik and Strøm (1994), and Aaberge, Dagsvik and Strøm (1995).

worker is qualified may not necessarily be available to her or him. Let $B(h)$ denote the agent's set of available jobs with hours of work h; that is, this set contains those jobs z for which $H(z)=h.$ Let $m(h)$ be the number of jobs in $B(h)$. There is only one nonmarket alternative, so that $m(0)=1$. The choice sets $\{B(h)\}$ are unobserved to the researcher. Here we treat the terms $\{m(h)\}$ as deterministic, which means that we neglect possible unobserved heterogeneity in choice sets. As already seen, the set D represents the see of feasible hours of work.

Further, let $\varphi(h)$ denote the probability that the agent chooses a particular job with offered hours h, wage rate w, given non-labor income and individual characteristics. Analogously to the previous section it follows from standard results in discrete choice theory that the agent will choose job z in $B(h)$ if the utility of this job, $v(f(hw,I),h)+\varepsilon(z),$ is higher than, or equal to the utility of all other jobs that are available, or, what is equivalent, equal to the highest utility that can be attained, given the choice restrictions. The corresponding probability that the agent shall choose this job can then be expressed as

$$(3.2) \qquad P\left(v(f(hw,I),h)+\varepsilon(z)=\max_{x\in D\cup\{0\},\, k\in B(x)} \max\left(v(f(xw,I),h)+\varepsilon(k)\right)\right)$$
$$=\frac{\exp(\psi(h))}{\sum_{x\in D,\, z\in B(x)}\sum \exp(\psi(x))+\exp(\psi(0))}.$$

We recognize the expression on the right hand side as the representative utility of job z divided by the sum of the representative utilities across all available alternatives. However, we are not particularly interested in this probability. Instead we want to derive an expression for the probability that the agent shall choose *any* job with hours of work and wage $\varphi(h)$, that is, the probability that the agent shall choose any job within $B(h).$ This probability is therefore obtained by summing the choice probability above over all alternatives within $B(h),$ that is,

$$(3.3) \qquad \varphi(h)=\sum_{z\in B(h)}\frac{\exp(\psi(h))}{\sum_{x\in D,\, z\in B(x,y)}\sum \exp(\psi(x))+\exp(\psi(0))}=\frac{\exp(\psi(h))m(h)}{\exp(\psi(0))+\sum_{x\in D}\exp(\psi(x))m(x)},$$

for positive h. When h = 0 we get

(3.4) $$\varphi(0) = \frac{\exp(\psi(0))}{\exp(\psi(0)) + \sum_{x \in D} \exp(\psi(x)) m(x)}.$$

Eqs. (3.3) and (3.4) yield choice probabilities that are analogous to multinomial logit ones with representative utility terms $\{\psi(h)\}$, weighted by the frequencies of available jobs, $\{m(h)\}$. Note that it is a consequence of our distributional assumptions about the random error terms in the utility function given in (3.1) that the respective numbers of available latent jobs, $\{m(h)\}$, represents a set of *sufficient* statistics for the corresponding choice sets. Unfortunately, $\{m(h)\}$ is not directly observable, but under specific assumptions, one can identify $m(h)$ and $\psi(h)$ and estimate their parameters. For the sake of interpretation, and with no loss of generality, let

$$\theta = \sum_{x \in D} m(x), \quad \text{and} \qquad g(h) = m(h) / \theta.$$

One can interpret $g(h)$ as the fraction of jobs available to the agent with offered hours of work equal to (h), whereas the parameter θ is the total number of jobs available to the agent. We shall call $\theta g(h)$ the *opportunity measure* and $g(h)$ the *opportunity distribution*. The interpretation of θ can be extended to include fixed cost; see Cogan (1981). To realize this, assume that a positive parameter c, representing the utility (disutility) of fixed cost, enters additively in the utility function given in (3.1) for positive hours of work. Then, evidently, the structure of the choice probabilities above remains the same, apart from θ which now transforms to $\theta \exp(c)$.

When inserting the opportunity measure into the expressions for probabilities, we obtain

(3.5) $$\varphi(h) = \frac{\exp(\psi(h)) g(h) \theta}{\exp(\psi(0)) + \theta \sum_{x \in D} \exp(\psi(x)) g(x)},$$

and

(3.6) $$\varphi(0) = \frac{\exp(\psi(0))}{\exp(\psi(0)) + \theta \sum_{x \in D} \exp(\psi(x)) g(x)}.$$

The expressions for the labor supply choice probabilities given in (3.5) and (3.6) form the point of departure for the corresponding empirical specification. Note that (3.5) can, alternatively be written as

(3.7) $$\varphi(h)=\frac{\exp(\psi(h)+\log(\theta g(h)))}{\exp(\psi(0))+\sum_{x\in D}\exp(\psi(x)+\log(\theta g(x)))}.$$

Note that the expression in (3.7) has the same structure as the expression in (2.7). There is, however, an essential difference between (2.7) and (3.7) because $\log g(h)+\log\theta$ in (3.7) is no longer a term that is added in an ad hoc manner but is motivated from theory and represents restrictions that stems from the demand side of the labor market.

The approach above corresponds to a similar formulation by Dickens and Lundberg (1993), and extensions thereof, see Tummers and Woittiez (1990), and Bloemen (2000). However, the approach by Dickens and Lundberg (1993) does not allow for non-pecuniary attributes of jobs, and it is fairly complicated compared to the simplicity of the job choice model above. In one sense the approach of Dickens and Lundberg (1993) is more general than the job choice model discussed so far, in that it allows for stochastic choice sets and thereby accommodates unobserved heterogeneity in job opportunities. One can, however, interpret the job choice model as a model with stochastic choice sets of available job opportunities. See Dagsvik (1994), Dagsvik and Strøm (2006) and Dagsvik and Jia (2011a) for discussions on how the approach can accommodate stochastic choice sets.[9]

An issue we have avoided so far is how the opportunity measure is determined in equilibrium. In Dagsvik (2000), and Dagsvik and Jia (2011b) it is demonstrated that the framework above can be interpreted as a particular two-sided matching equilibrium model. In particular, it is shown how the opportunity measure depends on the utilities of the firms (cost- or production functions). It is however beyond the scope of this paper to discuss the equilibrium setting, and we shall in the following resort to a reduced form representation.[10]

It seems reasonable to assume that the opportunity density $g(h)$ is fixed in the short run, at least as a first approximation. Institutional restrictions, such as centralized negotiations

[9] There may several reasons for treating choice sets as random, for instance that choice sets are unobserved for the researcher and that agent has limited capacity to identify and take choice sets into account (a type of bounded rationality).

[10] Peichl and Siegloch (2010) establish an equilibrium model by linking a labor demand model to the conventional discrete choice labor supply model.

between labor unions and employers' associations, may corroborate to this assumption. Moreover, in many firms it may be desirable to require the workforce to have more or less the same working hours because the production process requires workers to be present at the workplace simultaneously. The parameter θ, however, will, in addition to depending on individual qualifications, depend on business cycle variations.

Introducing random effects in the wage equation loosens the somewhat restrictive form of the conditional logit model given by the assumption that error terms are independent and identically extreme value distributed, implying the IIA property. In fact, IIA will be relaxed in typical empirical applications without introduced additional random effects because the wage is replaced by a wage equation that includes a stochastic error term, and thus a mixed multinomial logit model follows (McFadden and Train, 2000). See also Haan (2006) on this issue. This type of random effect specification to account for unobserved inter-individual heterogeneity in wages has been used by Dagsvik and Strøm (2006), Dagsvik and Jia (2011a) and Kornstad and Thoresen (2007).

4. Model specification and assessment

4.1. Empirical specification

This section contains a brief discussion about empirical specification and assessment. Note first that without further assumptions about the utility function and the opportunity measure the model is non-parametrically unidentified. In general, the opportunity distribution $g(h)$ may be dependent of the (individual) wage. We assume in the following that this is not the case. Assume furthermore for simplicity that the structural part of the utility function is additive separable in consumption and hours of work, that is $v(C,h)=v_1(C)+v_2(h).$ Then it follows from Dagsvik and Jia (2011a) that the function $v_1(C)$ is non-parametrically identified. However, one cannot without further assumptions separate $v_2(h)$ from $g(h)$.[11] This is evident from the structure of the choice probabilities in (3.5) and (3.6), because

$$\psi(h)+\log g(h)=v_1(f(hw,I))+v_2(h)+\log g(h),$$

[11] The identification result above can be proved without the conditions of additive separable of the utility function (Dagsvik and Jia, 2011a), but for expository simplicity this assumption is maintained here.

for positive h. To obtain full identification one must therefore make functional form assumptions. It is, however, important to note that for most type of policy simulations there is no need to separate $v_2(h)$ from $\log g(h)$. Specifically, this is true for changes in the budget constraints, represented by the function *f*, because $v_2(h)$ and $g(h)$ do not depend on *f*. Recall that here we are only concerned with supply effects under given assumptions about job restrictions, represented by the job opportunity measure and *not* about the corresponding equilibrium opportunity measure.

One type of specification of the opportunity distribution that seems reasonable and has been used in several empirical applications is to assume that $g(h)$ is uniform apart from a peak at full-time and part-time hours. That is, $g(h)$ is assumed to be constant apart from peaks at full-time and part-time hours.

As regards empirical specification of the deterministic part of the utility function the researcher must make functional form assumption. In this respect, there seems to be no consensus in the literature about what is the "best" choice. Although the consumption component, $v_1(C)$, is, under the separability assumptions discussed above non-parametrically identified, the functional form issue is still important because the data typically available only carry limited information about variations in the choice environment. Some researchers have applied a polynomial specification in leisure and disposable income, whereas others have applied translog specifications or Box-Cox type of functional forms. As is well-known, the translog and the polynomial functional form do not imply that the deterministic part of the utility function is globally concave. Dagsvik and Strøm (2006) and Dagsvik and Jia (2011a) assume that $v(C,h)$ is additive separable in consumption and hours of work, but allow for interaction between leisure for husband and wife. Each utility component is assumed to have a Box-Cox functional form, see Dagsvik and Jia (2011a) for details. A theoretical motivation for this particular functional form is given by Dagsvik and Strøm (2006) and Dagsvik and Røine Hoff (2011) and is based on particular invariance properties. Dagsvik and Strøm (2006) show that the Box-Cox functional form yields more or less the same fit as polynomial specifications. An advantage with the Box-Cox specification is that it is globally concave and increasing in disposable income and leisure.

Since the opportunity distribution of hours and the opportunity measure of the total amount of available jobs appears in the model as $\theta g(h)$, one does not have to impose the restriction that the sum of $g(h)$ should be equal to one when estimating the model, because one can easily obtain the desired normalization after the model has been estimated. Thus, the empirical specification of the job choice model therefore is similar to the model of Van Soest (1995) extended to include suitable dummy variables to account for hours constraints. The parameter, θ, representing the total amount of job opportunities available, can be allowed to depend on education (and a constant), and is estimated simultaneously with the choice model. Thus, more precisely, under the assumptions just discussed, one may conveniently represent $\log(\theta g(h)) = \log\theta + \log g(h)$ as a linear function in length of schooling and dummies for part-time and full-time hours of work.

4.2. Model simulations and performance

A much debated issue is how structural models should be assessed, see for example Hausman (1992). The problem is that the researcher usually does not have much information about model performance apart from goodness-of-fit measures. One key test of model performance is to examine the extent to which the model is able to predict out-of-sample labor supply behavior, for which we will provide an example; more details can be found in Dagsvik and Jia (2011a).

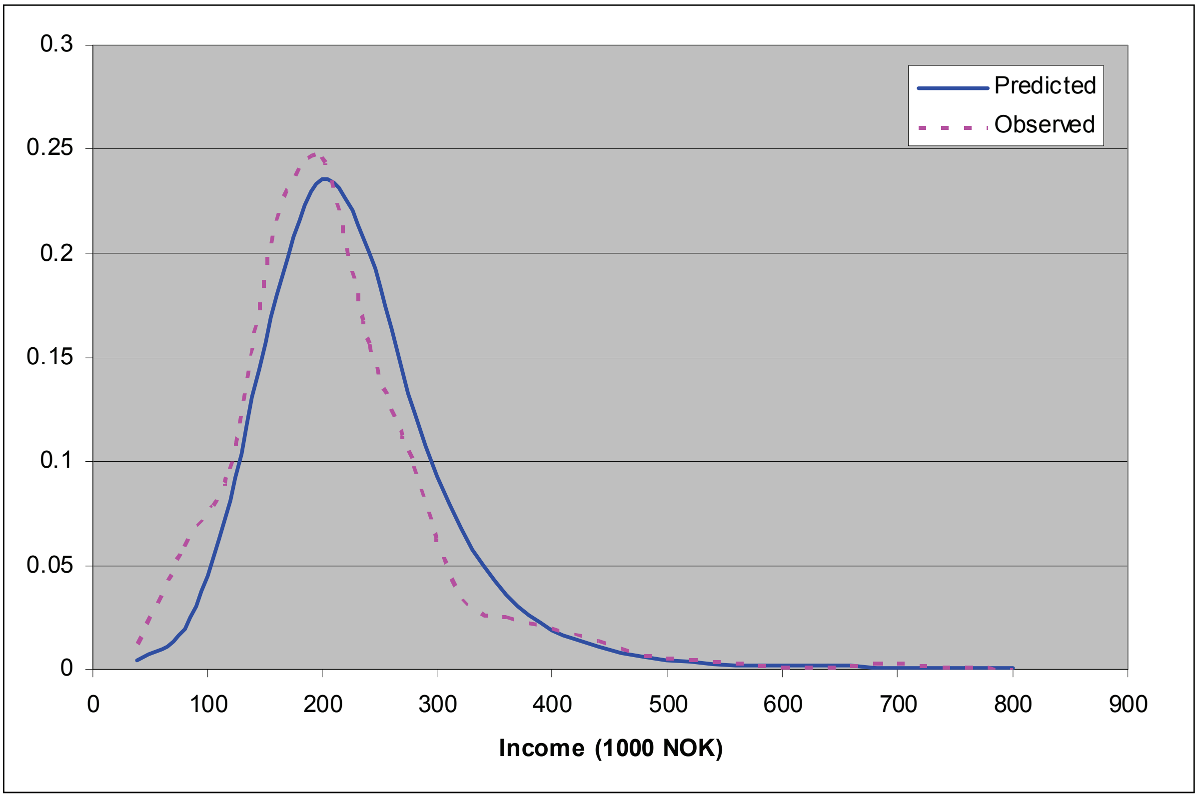
In the performance assessment reported here, we compare model prediction results for disposable income with income data for 2003. The model formulated and estimated in Dagsvik and Jia (2011a) is a version of the job choice model discussed above, extended to a joint (unitary) model for married couples. Thus, in this out-of-sample prediction exercise we have simulated how the model predicts the distribution of disposable income. In the simulation we keep the parameters of the opportunity measures fixed. Thus, the predictions from the model are, apart from trends in the mean wages, conditional on a stationary choice environment. The only sample selection criterion imposed for this simulation is the requirement that the individuals should be wage earners between 26 and 62 years of age.

Two parameters are important when using the estimated model based on data from one year (the base year) to predict labor market behavior and corresponding incomes of another year (the simulation year): these are the wage growth rate and the inflation rate, both measured from base year to simulation year. We use the observed wage growth rate from 1997 to 2003 and the wage regression for the base year to generate the wage rate in the simulation year. Similarly, incomes in the simulation year are adjusted by using the inflation rate.

Figure 1 shows the observed and predicted disposable income distribution, using information from the Income statistics (Statistics Norway, 2010) to establish the observed distribution. In this case, our model predicts well, which indicates good performance.

However, we need to be cautious when interpreting such results. In addition to depending on the labor supply model (conditional on the wages), the distribution of disposable income depends heavily on model of the wage distributions for males and females. Recall that the wage distributions are generated by lognormal distributions generated by estimated log wage equations. We have found that the shape of the distribution of disposable income appears to be quite robust with respect to moderate changes in the distribution of hours of work, conditional on wages. In contrast, it seems that model predictions are not very robust with respect to specification errors in the model of the wage distributions. Thus, a poor fit of the distribution of disposable income is not necessarily a sign of a poor fit of the underlying behavioral model, but could result from misspecified wage equations. We believe that the assumption of normally distributed error terms in the log wage equations may be restrictive. In simulations not reported, it is found that the wage equations are not capable of fully reproducing the right tails of the distribution of observed wages in the 2003 sample. This is not surprising because it is well-known that the right tail of the lognormal distribution is not heavy enough to capture the right tails of most income distributions. In fact, a closer look at Figure 1 reveals that the right tail of the empirical density seems fatter than the tail of the corresponding simulation.

Figure 1. Observed and predicted density of disposable income for married couples, 2003



Both the traditional discrete choice model and the job choice model presented in this paper are well suited for simulating the effects of changes in the economic budget constraint, determined by the tax system, wages and non-labor income. A particular feature of the job choice model is that it can also be used to study the effects of changes in the opportunity distribution of offered hours of work as well as the total number of job offers without introducing rather ad hoc assumptions about the "true" representation of preferences. Recall that in our framework, $g(h)$ represents the fraction of jobs available to the agent with offered hours of work equal to (h), whereas the parameter θ is the total number of jobs available to the agent. For example, there has recently been a heated discussion among politicians and labor unions to introduce a reform in which part-time positions are replaced by full-time positions. Within our framework one can readily simulate the effect of this reform on labor supply by changing the opportunity distribution of hours, see Dagsvik and Jia (2011a) for more details on the simulation of this type of reform. In contrast, within the conventional discrete choice framework one cannot simulate the effect of this type of reform because the quantitative choice restrictions are not explicitly represented in the model.

Note also that one cannot compute wage elasticities in the usual manner within the job choice framework, because the supply function is a random function, as is also the case for the conventional discrete choice model. In the job choice model it is convenient to calculate elasticities that accommodate the effect of both the systematic terms and the unobservables in the model. This means that one takes into account how the mean of the *distribution* of labor supply is affected by changes in, say, wage levels.

The elasticities that follow from the estimated model for married couples are of moderate magnitudes, with married females more responsive than males, and they are more or less of the same magnitudes as several other labor supply studies found in the literature. See Blundell and MaCurdy (1999) and Keane (2010) for a summary of elasticities from other studies. It is, however, a delicate issue how elasticities should be compared, in particular when the models are nonlinear. In nonlinear models the same model may produce very different elasticities on different samples. This stems from the inherent nonlinear properties of this kind of models. To illustrate this point, consider a simple logit model of working/not working, given by

$$P(w,X)=\frac{1}{1+\exp\left(-\alpha\log w-X\beta\right)}$$

where $P(w,X)$ is the probability of working, w is the wage and X is a vector of individual characteristics. This model implies that the wage elasticity of the probability of working is given by

$$\frac{\partial\log P(w,X)}{\partial\log w}=(1-P(w,X))\alpha.$$

Thus, in this model the wage elasticity depends crucially on the level of labor force participation, $P(w,X)$. For example, if the combination of wage and characteristics are such that $P(w,X)=0.6$, the corresponding wage elasticity becomes 0.4α, whereas if $P(w,X)=0.80$, the corresponding wage elasticity becomes 0.2α. Thus, when the fraction of individuals who works is 60 percent the wage elasticity is twice as high as the wage elasticity when the fraction who works is 80 percent. Note that the parameters α and β are kept constant across these two examples.

5. Summary

Specification of labor supply models continues to be a controversial issue, and there is no common consensus in the literature of what should be the preferred strategy. Theoretical labor supply models discussed in the literature are often highly stylized and the theory provides little guidance for specifying the empirical counterparts of the theoretical models. Moreover, there is a tendency of playing down the importance of specification issues related to functional form and distribution of unobservables. Since economic theory with few exceptions is silent about such issues, researchers have in practical empirical research resorted to various ad hoc specifications. As a result, there seems to be no consensus in the research community as to what should be the "right" specification, and consequently there is a large variety of specifications in the literature. This may be an important reason why labor supply wage elasticities in previous work are found to vary all over the map, see Blundell and MaCurdy (1999).

In this paper we have discussed a particular approach based on the notion of choice among job types. Concerns of practicality have often been the dominating motivation for applying the conventional discrete choice labor supply model derived from a particular random utility specification. In this paper we have discussed arguments for an alternative formulation based on the random utility that accommodates other important aspects of labor market behavior hitherto neglected, namely that individuals care about the nature and content of the jobs, and that the set of perceived jobs available to them may be limited. Although we as researchers do not observe the choice of jobs, nor the choice restrictions, we have demonstrated how one still can derive the corresponding model for the *observed* choice variables (hours of work of the chosen jobs and corresponding disposable income). Specifically, we have shown how our particular approach allows accounting for restrictions on the hours of work in a convenient way. Also, the job choice model provides a theoretical rationale for introducing dummy variables to accommodate peaks in the data at, for example, full-time and part-time hours of work.

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