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Evaluating Alternative Methods of Forecasting House Prices: A Post-Crisis Reassessment

William D. Larson[*][†]

February 16, 2011

Abstract

This paper compares the performance of different forecasting models of California house prices. Multivariate, theory-driven models are able to outperform atheoretical time series models across a battery of forecast comparison measures. Error correction models were best able to predict the turning point in the housing market, whereas univariate models were not. Similarly, even after the turning point occurred, error correction models were still able to outperform univariate models based on MSFE, bias, and forecast encompassing statistics and tests. These results highlight the importance of incorporating theoretical economic relationships into empirical forecasting models.

1 Introduction

Despite the importance of the residential sector of the U.S. economy, relatively few studies have assessed the performance of alternative time series models that forecast the price of housing services. This is surprising, given the importance of house price forecasts in predicting mortgage defaults, property taxes, and a number of other consumption, investment, and policy decisions (Kochin and Parks, 1982; Demyanyk and Van Hemert, forthcoming) Furthermore, there is evidence that house prices are forecastable to a certain degree (Case and Shiller, 1989, 1990; Crawford and Fratantoni, 2003). Accordingly, it is useful to determine which forecasting models are best able to capture the future movements of house prices.

This paper reconsiders the existing evidence on which classes of simple time-series models best forecast house prices: theory-driven multivariate models or atheoretical univariate models. In particular, I ask three questions:

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- Which class of models was able to predict a turning point in the housing market before house price declines occurred?
- Which class of models forecast best from the peak of house prices from 2006, using multiple-step, dynamic forecasts, over the next three years?
- Which class of models had the lowest error and unique information in forecasts one step ahead during the period of house price declines?

In order to answer these questions, eight different empirical house price models are estimated using Federal Housing Finance Agency (FHFA, formerly OFHEO) house price data for California.[1] Pseudo *ex ante* forecasts are then computed and these forecasts are evaluated along a number of dimensions.[2]

Models are chosen to reflect different variable transformations (levels vs. first- or second-differences), information sets (incorporating non-house price series), and parameter restrictions (e.g. cointegration). In total, five univariate models are considered: two autoregressive (AR) models, an autoregressive fractionally integrated moving average (ARFIMA) model, an unobserved components (UC) model, and a random acceleration model. Three multivariate models are considered as well: a vector autoregression (VAR) in levels with house prices, personal incomes, and rental prices, a vector error correction (VEC) model with house prices and personal incomes, and a VEC model with house prices and rental prices. The literature suggests that there should be, *a priori*, a cointegrating relation between incomes and house prices and between rental and house prices (Malpezzi, 1999; Gallin, 2006). It is therefore plausible that VEC models would forecast quite well when the housing market is at a state of long-run disequilibrium such as at the peak of a house price bubble.

I find that multivariate models are best able to forecast the turning point in the California housing market. As far back as 2004, error correction models predicted that the growth in house prices in California was unsustainable, and retrenchment would occur. This suggests that expanding the model information set to include other variables and imposing cointegration restrictions helps to predict turning points.

I also find that from the peak in house prices in 2006, n-step forecasts constructed using multivariate models perform the best over the next three years. While all models exhibit bias, the multivariate models are substantially closer to the true decline in house prices than the univariate models.

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[1] California is the subject of study because three other works in the literature, Crawford and Fratantoni (2003), Miles (2008), and Gupta and Miller (2010), consider California as well. Additionally, data on personal incomes are available at the state level but not at the city level.

[2] Pseudo *ex ante* forecasts are constructed using only information that was available at the time of the forecast. For example, a model used to produce forecasts in 2005:Q4 would only include data up to 2005:Q4.

Finally, I find that multivariate models also have the lowest 1-step ahead forecast error during the period of house price declines, though the differences between these and the univariate models are small. Forecast encompassing tests show that in none of the 45 tests performed, does any multivariate model fail to encompass any univariate model. However, no model's 1-step forecasting error is as small as its in-sample estimation error, suggesting that either each of the forecasts considered leaves out some valuable information, or that structural shocks during the forecasting period were of higher variance than during the estimation period.

2 Stylized facts from the literature on housing markets

This section describes some of the broad characteristics of the housing market and several popular methods of modeling house prices. There are two stylized facts regarding the movement of house prices that motivate much of the house price literature. The first is that house price changes are highly persistent from one period to the next (Case and Shiller, 1989, 1990; Meese and Wallace, 1991; Glaeser and Gyourko, 2006). The second is that the housing market is prone to large boom periods followed by painful corrections (Muellbauer and Murphy, 1997; Glaeser and Gyourko, 2006).

Case and Shiller (1989, 1990) find that house price changes are highly persistent, and that house prices are forecastable over short time horizons. This has since become a well-known time series property of house prices, and has influenced much research on the subject, ranging from its implications for the efficiency of the housing market to the effect of this persistence on house price cycles. Glaeser and Gyourko (2006) remark that in the U.S. between 1980 and 2005, a $1 increase in real house prices in one year is associated with a $0.71 increase in house prices the following year. Various explanations exist as to why this is the case. Muellbauer and Murphy (1997), for example, argue that credit rationing, uncertainty, and transaction costs all contribute to the persistence of house price changes.

Muellbauer and Murphy (1997) go on to argue that it is this persistence in house price changes that contributes to the boom-bust nature of the housing market. While yearly house price changes are positively correlated, Glaeser and Gyourko (2006) show that five-year house price changes are negatively correlated, with a $1 increase in house prices in one year being correlated with a $0.32 decrease in house prices over the next five years.

Malpezzi (1999) shows that this reversion in house prices can be modeled as a reversion to a long-run relationship of house prices and incomes. Malpezzi estimates a panel equilibrium correction model of house prices in U.S. cities and finds that house prices correct to a long-run, area-specific, house price-income ratio. Similarly, it is a standard in user cost theory that house prices and rental prices share a long-run error correcting relationship. This link is related to the dividend pricing hypothesis in that the discounted stream of monthly user costs should approximate the price of the unit. Gallin (2006) estimates both a quarterly differenced model and a four-year differenced model and finds that house prices adjust to a constant long-run rental price-asset price ratio.[3]

Models where house prices correct to established long-run relationships may depend on the sample over which the relationship is measured. For example, while Gallin (2006) finds strong evidence of equilibrium correction in house prices and rents, Verbrugge (2008) describes how user costs have diverged from rents for quite some time, and that perhaps an equilibrium correcting relationship does not exist. However, Verbrugge's estimation sample ends at the peak of the house price bubble, making it clear in hindsight why he found no evidence of equilibrium correction.

3 Previous house price forecast comparisons

There are few papers comparing different house price forecasts in the literature. In general, forecast comparison exercises focus on univariate models and are compared on the basis of MSFEs. Many studies of house price dynamics have considered periods where prices were increasing and have not had to deal with problems created by major turning points. Under the circumstances considered, research has mostly found that univariate time series models perform better than multivariate VAR and error correction models motivated by economic theory. Other, more recent evidence suggests, however, that multivariate models may be able to predict turning points and produce good forecasts during periods of disequilibrium in the housing market.

One of the first house price forecast comparisons in the literature is Brown, Song, and McGillivray (1997). They consider UK house prices and consider a univariate time-varying coefficients model, an error correction model, an AR and a VAR model. Brown, Song, and McGillivray find that a time-varying coefficients model outperforms all others based on the MSFE of the different forecasts.

Crawford and Fratantoni (2003) consider house price forecasts of different U.S. states using the FHFA repeat-sales house price indices. They compute forecasts based exclusively on univariate house price models and evaluate their forecasts using MSFE and mean absolute deviation (MAD). Models considered include ARIMA, GARCH, and Markov-switching models. They find that ARIMA models are best able to forecast house price changes 1-step ahead based on MSFE comparisons. While Markov-switching models offer the best fit in-sample, this class of models performs quite poorly in out of sample forecasting tests compared to the other models considered. Miles (2008) closely follows Crawford and Fratantoni and finds that generalized autoregressive (GAR) models outperform ARIMA models.

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[3] Davis and Palumbo (2008) find that land prices drive house prices to a large degree, but this approach is difficult to implement because of the difficulty of measuring land prices.

Guirguis, Giannikos, and Anderson (2005) examine forecasts of the U.S. housing market using GARCH, AR, Kalman filters, and VEC models. They find the Kalman filters and the GARCH models forecast best on the basis of MSFE comparisons. However, the VEC forecasts they consider are based on a cointegrating relation covering the forecasting period as well as the estimation period. This approach is not a realistic forecasting exercise because forecasts are generated based on information unobtainable at the time of the forecast.

Gupta and Miller (2010) provide a recent study of MSA-level house prices in California. They consider a variety of specifications, including spatial VAR, spatial VEC, and spatial BVAR models. They find that different models forecast better in different locations, and that the best model in terms of overall RMSE in each location is able to predict turning points in the respective housing markets four quarters ahead with reasonable accuracy.

In general, the literature finds that univariate time series models are able to forecast better than theory-driven multivariate models. These evaluations are mostly performed over periods of increasing house prices. All of the papers' primary comparison measures are MSFEs, and they do not consider other forecast comparison metrics such as the ability to predict turning points (with the exception of Gupta and Miller) or the relative information content of rival forecasting models.

The approaches in the literature can be extended in two main ways. First, researchers who model house prices successfully do so using models that are able to capture both persistence and equilibrium correction. While all of the papers considered are able to model persistence, models able to represent equilibrium-correcting systems such as vector error correction models have not been fully explored. Many of those who have forecasted using VEC models have either done so over periods of increasing house prices (Brown, Song, and McGillivray, 1997) or they are conditional forecasts (Guirguis, Giannikos, and Anderson, 2005). Gupta and Miller's (2010) success in modeling house prices and predicting turning points shows that VEC models should be given a closer look, and that they may be able to predict turning points farther back in time and forecast well during periods of declining prices. Because error correction models are, by definition, able to model equilibrium-correcting systems, it is plausible that VEC models could perform well during periods of disequilibrium.

There are a number of different ways of comparing forecasts beyond MSFEs. For example, forecast encompassing tests can evaluate the relative information content of forecasts and parameter constancy tests can establish the adequacy of forecasting models during the forecasting period. These tools are readily available and should be utilized to compare and contrast the various characteristics of forecasts.

4 Statistical forecast comparison methods

Beyond the previously noted MSFE comparisons, there are several other ways of comparing rival forecasts. This section describes three of these means of comparison: bias tests, parameter constancy tests and forecast encompassing tests. When it is not obvious which forecasts are better than others, these comparison statistics, along with usual measures of MSFE and bias, can be used to establish rankings and evaluate the relative information content of a set of rival forecasting models.

Suppose a set of data over which a model is estimated on observations $t = 1, ..., T$ and forecasts $\tilde{Y}_t$ are generated from $t = T+1, ..., T+H$. Actual values are denoted as Y_t, and subtracting the actual from the forecast yields forecast errors ε_t. Mincer and Zarnowitz (1969) propose a bias test where, for a set of 1-step ahead forecasts, equation 1 is estimated and the joint hypothesis $\{\alpha = 0; \beta = 1\}$ is tested. A rejection of this null hypothesis is interpreted as evidence of bias in the forecasts.[4]

$$Y_{T+h} = \alpha + \beta \tilde{Y}_{T+h|T+h-1} + e_{T+h} \tag{1}$$

While the Mincer and Zarnowitz approach tests for *average* bias, Hendry's (1974) parameter constancy test establishes the *expected* bias of the individual forecast errors. This test can detect systematic biases that on average, are offsetting. Hendry's test statistic is the ratio of the mean squared forecast error and the in-sample residual variance.

$$\frac{MSFE}{\sigma^2} \sim F(H, T-k) \tag{2}$$

Under the null, the ratio of these variances follows an F distribution with H and $T-k$ degrees of freedom, where H is the number of forecasts used to compute the MSFE, T is the number of observations in the estimation sample, and k is the number of parameters estimated. This is equivalent to testing

$$E\left[(\varepsilon_{T+1}, \varepsilon_{T+2}, ..., \varepsilon_{T+H})'\right] = \mathbf{0} \tag{3}$$

A rejection of this null hypothesis indicates that 1), the parameters that define the relationship between exogenous variables and Y are non-constant between the estimation period and the forecasting period, and 2), that the expected bias of the individual forecast errors is nonzero.

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[4] Holden and Peel (1990) show that such a rejection is a sufficient but not necessary characteristic of a biased forecast.

Forecast encompassing tests are also used to evaluate and compare different forecasts. These tests were formalized by Chong and Hendry (1986) and extended by Ericsson (1992, 1993). They extend the Mincer and Zarnowitz's (1969) bias test to incorporate information from a rival forecast $\hat{Y}$, and test the relative information content of $\tilde{Y}$ versus $\hat{Y}$. When comparing two forecasts, if a first model contains information relevant to forecasting that a second model does not, the first model is said to "forecast encompass" the second.

Ericsson's (1993) test of forecast encompassing is selected over Chong and Hendry's version because it has higher power when Y, $\tilde{Y}$ and $\hat{Y}$ are I(1) and $Y - \tilde{Y}$ is I(0). Under these circumstances, the Chong and Hendry test involves estimating a regression of unbalanced order, whereas the Ericsson test addresses this issue. In the Ericsson test, the base equation is:

$$Y_{T+h} - \tilde{Y}_{T+h|T+h-1} = \alpha + \gamma(\hat{Y}_{T+h|T+h-1} - \tilde{Y}_{T+h|T+h-1}) + e_{T+h} \tag{4}$$

There are three tests that can be performed based on the above equation. Each of these tests represents slightly different assumptions, hypotheses, and has different resulting implications. The first two variants estimate equation 4, but the encompassing test is performed under different null hypotheses. The first is that $\hat{Y}$ contains no unique information that could be used to forecast Y that $\tilde{Y}$ does not provide. Under this null, $\{\gamma = 0\}$. The second test is the same as the first, but simultaneously tests that $\tilde{Y}$ is an unbiased forecast. Under this second null, $\{\alpha = 0; \gamma = 0\}$. Another way of interpreting the second test is as a forecast encompassing test versus *two* rival forecasts: a constant forecast and $\hat{Y}$.

The third test assumes *a priori* that $\alpha \equiv 0$ and tests that $\{\gamma = 0\}$. This forces the Ericsson regression through the origin and thus risks omitting a potentially critical deterministic component. However, this equation saves one degree of freedom relative to the regression in the first two tests, and thus has higher power, especially in small sample sizes and when $\tilde{Y}$ is an unbiased forecast.

A rejection of tests one or three indicates that the rival model contains information useful to forecasting that the encompassing model does not. This is classified as a *failure to encompass*. A rejection of test two may indicate a failure to encompass $\hat{Y}$, but may instead indicate that $\tilde{Y}$ is a biased forecast. A failure to reject, in any case, indicates that there is no unique information in the rival forecast and that $\tilde{Y}$ *encompasses* $\hat{Y}$.

5 Alternative forecasting models

This section describes the eight models used to compute the house price forecasts evaluated in this paper.[5] Of these models, five are univariate and three are multivariate, and each is shown in Table 1. The set of models considered is meant to reflect the wide number of choices based on the literature reviewed above, including different variable transformations (levels vs. first or second differences), information sets (incorporating non-house price series), and parameter restrictions (e.g. cointegration).

The first model considered is a random acceleration model. The forecast of a non-seasonal, second-differenced model is $\Delta^2\hat{Y}_{T+1|T} = 0$, or equivalently that $\Delta\hat{Y}_{T+1|T} = \Delta Y_T$, and $\hat{Y}_{T+1|T} = Y_T + \Delta Y_T$. Second-differenced models are robust to changes in deterministic constant terms, trends, and long-run equilibria. A random acceleration model's forecasts may be hard to beat in terms of RMSE because it immediately incorporates shifts in the first and second derivatives of house prices into forecasts for the next period, thus eliminating forecast bias due to deterministic shifts. However, this extreme adaptability comes at cost of a higher expected forecast error variance, as Hendry (2006) shows.

An AR(1) model of the level of house prices is the second model considered. Because house prices are highly persistent, this will be nearly equal to a random walk-with-drift model. The AR(1) model has some different characteristics than the random acceleration model. Assuming the data generating process is also AR(1), it has a lower expected forecast error variance provided there are no changes in the deterministic components of the model during the forecast period.[6] Additionally, the AR(1) model does not allow for persistence in growth rates. Because house prices exhibit persistent growth rates, as seen in Figure 2, the AR(1) model is likely to forecast poorly.

An AR(p) model shares many of the same characteristics as the AR(1) model, but is better specified because the optimal number of lags are included.[7] This eliminates autocorrelation that is likely to exist in an AR(1) model and therefore will produce more unbiased and consistent parameter estimates. Autocorrelation is likely to exist based on the well-established finding that house price changes are persistent.

An ARFIMA(2,d,2) model allows for fractional differencing and persistence in the error terms, in addition to autoregressive parameters. However, without second-differencing, its forecasts will not be robust to deterministic shifts in trend and equilibrium shifts. If $d > 1$, then the ARFIMA model's forecasts are robust to deterministic constant term shifts.

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[5] When models are of first- or second- differences, the log-level of house prices is constructed using the difference equation identities.

[6] However, if deterministic components change, then large biases could result, pushing the MSFE higher than the random acceleration model.

[7] The choice of $p = 6$ lags is made based on estimating the model with $p + m$ lags for some chosen p and m and then testing if each of the lags $\rho_{p+1} = \rho_{p+2} = ... = \rho_{p+m} = 0$. If this hypothesis is rejected, p is increased by one and the test is redone. This continues until the hypothesis is not rejected, which in this case, occurs when $p = 6$.

A standard unobserved components model includes level, trend, seasonal, and irregular components that are allowed to change over time. The trend is defined based on a linear dynamic system that encompasses a linear trend model, a random walk model, and a random walk with drift model. Without deterministic shifts, the UC model is likely to forecast well because of its incorporation of changes to the first derivative of house prices in the trend component. However, because the UC is explicitly based on deterministic (albeit evolving) components, it is particularly susceptible to deterministic shifts during the forecasting period.

VEC models of rental prices of housing services and house prices, and personal incomes and house prices impose a long-run relationship on the ratio of a particular variable with house prices. In VEC models, house price changes are modeled as a function of the current deviation from this relationship.[8] Because they are differenced with proper lag selection, these models are robust to constant shifts and can model persistence in growth rates. Also, due to the error correction term, these models incorporate long-run mean reversion. The fact that short-run growth persistence and long-run mean reversion are two of the defining characteristics of house prices indicates that VEC models should be used to forecast house prices.[9]

The final model is a VAR(5) consisting of the levels of house prices, rental prices of housing services, and personal incomes. This unrestricted VAR in levels generalizes the two VECs considered previously. Perhaps there are other interrelationships at work than error correction, or error correction is of an alternative form to that which is parameterized in the VEC models.

6 Data

The house price data are from the Federal Housing Finance Agency (FHFA, formerly OFHEO) and are quarterly from 1975:Q1 to 2009:Q4, yielding 140 observations. Personal income data is from the Bureau of Economic Analysis. The rental price data are from the Bureau of Labor Statistics' Rent of Shelter index for the West Urban geographic area.

Figures 1a and 1b illustrate the several stylized facts that were previously noted, including the tendency for persistence in growth rates and mean reversion over long periods. Figure

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[8] Justification for these restrictions is based on Johansen (1988) cointegration tests based on the literature described in Section 2, which confirm the presence of cointegration.

[9] Gupta and Miller (2010) consider a spatial VEC where city-level house prices converge to a regional equilibrium. This type of equilibrium correction is not considered in this paper.

1a indicates that there may be a long-run statistical relationship between house prices and incomes as Malpezzi (1999) finds. Figure 1b shows that there may be a similar long-run relationship between house prices and rents, as Gallin (2008) suggests. Case and Shiller (1989) show that house price growth rates show substantial autocorrelation across time, and Figure 2 shows this to be true in California, at least recently.

The FHFA price index is a repeat sales house price index. The construction method of this variable may affect certain test statistics, especially parameter constancy tests. A repeat sales house price index is based on transaction data for individual homes that have sold at least twice. The index is constructed by first estimating the change in appreciation for a pair of transactions for some unit i as a function of dummy variables set at -1 and 1 at the time of the first and second transactions.

$$\Delta V_i = \sum \beta_t D_{it} + \varepsilon \tag{5}$$

Errors are heteroskedastic as a function of the distance between transactions, so this regression is estimated using an FGLS procedure following Case and Shiller (1989)[10]. The index is then computed as

$$I_t \equiv e^{\hat{\beta}_t} \tag{6}$$

This correction does not address a different sort of heteroskedasticity: measurement error. Because the index relies on pairwise transactions, as new sales occur, past index values are revised. Therefore, a repeat sales price index calculated from time $t = 1...T$ has measurement error variance increasing with t and is highest at time T. [11]

$$var(I_t) = \sigma_t^2;\ d\sigma_t^2/dt > 0 \tag{7}$$

7 Forecast results and comparisons

7.1 Model fit

Table 2 presents a number of different model and forecast evaluation measures. The standard deviation of the errors of the RACC model is 0.012, which is only slightly larger than the standard deviation of the errors in models with many estimated parameters, which are .011 or .012 for six of the other seven models considered. In contrast to the other models, the

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[10] See Pennington-Cross (2005) for a concise explanation of repeat sales indices.

[11] Because the measurement error variance is predicted to be larger during the forecasting period than the estimation period, parameter constancy tests using repeat sales data may over-reject the null and a GLS procedure that addresses this known form of heteroskedasticity may perform better than unweighted tests.

AR(1) model fits the data relatively poorly, with an in-sample residual standard deviation of 0.025.[12]

7.2 Forecasting the turning point

Next, I consider which models were able to forecast the turning point in the housing market. The *turning point* is defined as the period in which house prices first decline, which occurs in 2006:Q3. Before this period, house prices in California had risen continuously since 1996. Figure 3 presents rolling-window, pseudo *ex ante* forecasts.[13] The figure is representative in that the error correction model was able to forecast a leveling-off of house prices, followed by a decline at some point in the future. These predictions of declines were made before declines were ever observed. These results are broadly consistent across the three multivariate models.

The AR(6) model forecasts house price increases in perpetuity until the quarter before the decline in house prices. While this model predicts a flattening of house prices in the 2006:Q2 and 2006:Q4 forecasts, it never predicts any significant house price declines until declines are actually observed. These results are similar to the other four univariate models considered, and suggest univariate models were unable to forecast the turning point in the housing market.

In general, error correction models were relatively successful in predicting that a turning point would occur, though concordance with the actual turning point is weak. The VEC-INC model is substantially better at forecasting the turning point than the AR(6) model, predicting a future turning point in every forecast going back as far back as 2003:Q4. Even in 2004, the high growth rate was pushing house prices on such a trajectory that future declines were inevitable. Predictions of the exact time of the turning point were fairly inaccurate, especially as the housing bubble inflated ever larger in 2005. However, some forecasts were on the mark, with the forecast made in 2004:Q2 predicting the turning point exactly.

What is important in these figures is that regardless of the date of the forecast, univariate models were unable to predict declines in house prices prior to actually observing a decline. While the forecast accuracy of the multivariate forecasts is mixed, some of them were able to forecast turning points in the housing market. In particular, each of the VEC-INC forecasts foreshadowed sustained future house price declines. A stylized interpretation of these results is that atheoretical time series models were unable to forecast turning points in the housing market, but theory-driven multivariate models were somewhat successful.

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[12] Full model results are available online at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1709647 in the appendix.

[13] A rolling window forecast is meant to simulate a sequence of forecasts generated by a researcher in real-time, using only data available at that point in time. In Figure 3, the first forecast is generated by estimating a model using data up until 2003:Q4, and forecasting each quarter from 2004:Q1 until 2009:Q4; the second forecast is generated by estimating a model with the same specification using data up until 2004:Q2 and forecasting each quarter from 2004:Q3 until 2009:Q4; and so on.

7.3 Quantitative forecasts from the turning point: Dynamic forecasts

After the turning point in 2006:Q3, I consider which forecasts were able to forecast well from a fixed point in 2006:Q4, and forecasting over the next three years. Figure 4 shows dynamic forecasts, multiple steps ahead, and Figure 5 shows some of these forecasts with confidence intervals.

Multivariate error correction models outperform the univariate models. Visually, it is immediately apparent that the VAR, VEC-INC, and VEC-RENT forecasts outperform the univariate forecasts, and both the VEC-INC and VEC-RENT forecasts do better than the VAR forecast. The RACC forecast outperforms all univariate models and is the only one to forecast even a slight decline in house prices over the forecasting window.

Table 3 shows estimates of the n-step ahead MSFEs, Theil's U statistics (relative to the RACC model) and average bias statistics. Multiple steps ahead, the VEC-RENT and VEC-INC models both have much lower MSFEs than any rival model. U-statistics for these forecasts are very low, both about 0.18. The VAR also does well relative to the naive forecast with a U-statistic of 0.46.

Multi-step forecasts show substantial systematic deviations from actual house prices. The bias of the RACC model is the lowest of the univariate models with an average bias of 19% of the value of an average home. Other univariate models perform even worse, with the AR(1) model over-forecasting by 32%. The multivariate models do better, with both VEC models showing an average bias of 8% and the VAR with an average bias of 13%. The bias in the multi-step forecasts serves to drive the MSFEs. Even the VEC-INC and VEC-RENT forecasts, which both have much lower MSFEs than other forecasts, have biases that are about six times that of the forecast error variance.

In general, these results are consistent with the previous exercise examining which models were best able to forecast the turning point. Theory-driven specifications outperform all atheoretical specifications across each of the comparison methods considered.

7.4 Quantitative forecasts from the turning point: one-step forecasts

The final forecasting exercise is to see which models are best able to forecast over the period of declining house prices, between 2007:Q1 and 2009:Q4. Figure 6 shows each model's 1-step ahead forecasts from just after the turning point in the housing market through 2009. Visually, all of these forecasts appear to be quite similar, with the exception of the AR(1) forecasts, which are biased. Because these forecasts are visually indistinguishable, it is necessary to proceed with analysis of forecast comparison statistics and tests.

MSFE comparison

Table 4 presents the different MSFEs of 1-step for each model, as well as MSFE comparisons versus the naive RACC model in the form of Theil's U-statistic. 1-step MSFEs indicate that the VEC-RENT model has the lowest MSFE among the models considered. The AR(6), ARIMA(2,d,2), VEC-RENT, VEC-INC and the vector autoregression with house prices, incomes, and rental prices (VAR) each have a lower MSFE than the naive RACC model. The AR(1) and the UC models both perform worse than the naive model, as indicated by U-statistics greater than one. Generally speaking however, each of the set of forecasts are indistinguishable from one another besides the AR(1) model, which performs much worse.

Bias test results

The 1-step forecasts appear to be mostly unbiased. Mincer and Zarnowitz's (1969) tests show that only the AR(1) model produces biased forecasts.

All F-tests reject Hendry's (1974) version of the Chow test for parameter constancy, indicating that there may be bias in the expectation of individual forecast errors in all models. These tests indicate that, while all of the models besides the AR(1) are similar in terms of MSFE and average bias, no model is able to forecast as well as the model fit in-sample. There are two possible reasons for this finding. Either some part of the data generating process changed over the forecasting period relative to the estimation sample, and modeling this change would produce better forecasts; or the structural error variance was higher in the forecasting period relative to the estimation period.[14]

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[14] As mentioned in Footnote 11, the nature of the repeat sales index may cause the structural error variance during the forecasting period to be higher then during the estimation sample.

Forecast encompassing test results

Table 5 presents a summary of the three different encompassing tests performed for each of the forecast pairs. Each of these individual tests is found in Tables 6-8. Because there are seven rival model for each forecast and three tests, there are $7 \times 3 = 21$ total encompassing tests for each forecast. The main concern, however, is to compare the relative performance of theory-driven multivariate and atheoretical univariate models. There are five univariate models and three multivariate models, so of the 21 possible tests, there are 15 relevant encompassing tests for the multivariate models and nine tests for the univariate models.

Of the univariate models, for only the AR(6) model is forecast encompassing of a rival multivariate model never rejected at the 10% level.[15] Each of the other univariate models fails encompassing in at least three of the nine tests. The UC and the AR(1) models perform the worst, with the RACC and the ARFIMA models in the middle.

No multivariate model ever fails to encompass a rival univariate model in any of the 15 encompassing test for the three multivariate models (45 tests, total). This indicates that incorporating additional non-house price information into the forecasts does not make forecasts any worse, and can add value relative to many of the univariate models when forecasting 1-step ahead during periods of declining house prices.

While MSFE, bias, and parameter constancy tests indicated that forecasts were indistinguishable (with the exception of the AR(1) model), forecast encompassing tests give a clearer picture of which models are better than others. Each of the multivariate models never fails to forecast encompass a rival model, and the AR(6) model fails to forecast encompass only one rival model and none of the multivariate models. The ARFIMA and the RACC models perform the next best, followed by the UC model. The AR(1) model, as with the other tests, performs the worst.

8 Conclusion

With the exception of the rolling window forecasts presented in Figure 3, all models listed in Table 1 are estimated using data on house prices from 1975:Q1-2006:Q4, and forecasts are generated over 2007:Q1-2009:Q4.[16] Each of these models is evaluated and compared using the battery of statistical measures described in Section 4. Five of these models were univariate models and three were multivariate models. The univariate models were selected to provide a range of different, commonly used time series specifications, and the multivariate models were selected based on theoretical economic relationships between house prices, incomes, and rental prices.

---

[15] Though there are three rejections out of nine tests at the 15% level, and the AR(6) model fails to encompass one of the univariate models in one of the tests.

[16] The rental price series begins at 1982:Q4, so if the model includes rental prices, the estimation sample starts in 1982:Q4. Results for other models estimated from 1982:Q4 to 2006:Q4 do not vary substantially from models estimated using the full sample, from 1975:Q1-2006:Q4.

There are three main results responding to the research questions presented in the introduction. First, error correction models of house prices and incomes and house prices and rental prices are able to forecast large house price declines multiple steps ahead before any house price declines were observed. In every forecast using the VEC-INC model, a future turning point in house prices was identified, though the concordance of the actual turning point with the predicted turning point was weak. In contrast, univariate models consistently predicted continued increases in house prices far out into the future. The best univariate forecasts predicted a flattening of house prices, but never significant declines.

Second, multivariate models all outperform univariate models when forecasting from the turning point multiple steps ahead over the next three years. While all forecasts considered under-predict the magnitude of house price declines, the multivariate models are not as bad as the univariate models.

Third and finally, several models are able to forecast well 1-step ahead during the period of falling housing prices. Each of the multivariate models, the RACC, and the AR(6) model are able to produce unbiased forecasts (on average) that are rarely encompassed by rival forecasts. The AR(1) and the unobserved components models each performs worse than the naive model and are often forecast encompassed. All models fail parameter constancy tests, indicating that the data generating process changed from the estimation sample to the forecasting period.

In general, the theoretically motivated multivariate models performed much better than the univariate time series models across a variety of comparison and evaluation metrics. The multivariate models were best able to forecast turning points in the housing market, were best able to forecast from the turning point over the succeeding three year window, and were best able to forecast 1-step ahead over the period of declining house prices.

This paper is a natural extension of the prior literature on house price forecast comparisons. The relative success of the VEC models with income and rental prices reinforces the finding of Gupta and Miller (2010), who show that spatial VEC models outperform non-spatial VAR models. It is clear from this work and theirs that it is crucial to model house prices using specifications that allow for long-run mean reversion in addition to short-run persistence of growth rates.

While past works compared MSFEs and other measures of fit across models, this research also compares forecasts multiple steps ahead, and evaluates forecasts based on average bias, tests of parameter constancy, and forecast encompassing. The past literature has also only compared forecasts over periods of growth in house prices, as opposed to periods of decline. Finally, this model considers an unobserved components model and a univariate random acceleration model, which past comparisons had not included.

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Table 1: Forecasting models

Model(short)	Model (long)	Specification
M1	RACC	$\Delta^2 Y_t = \alpha + \varepsilon_t$
M2	AR(1)	$Y_t = \alpha + \rho Y_{t-i} + \varepsilon_t$
M3	AR(6)	$Y_t = \alpha + \sum_{i=1}^{p} \rho_i Y_{t-i} + \varepsilon_t$
M4	ARFIMA(2,d,2)	$\Delta^d Y_t = \alpha + \sum_{i=1}^{p} \rho_i \Delta^d Y_{t-i} + \sum_{i=1}^{q} \theta_i \Delta^d \varepsilon_{t-i} + \varepsilon_t$
M5	UC	$Y_t = \mu_t + \psi_t + \varepsilon_t$
M6	VEC-RENT(4)	$\boldsymbol{\Delta Y}_t = \boldsymbol{\alpha\beta' Y}_{t-1} + \sum_{i=1}^{p} \boldsymbol{\Pi}_i \boldsymbol{\Delta Y}_{t-i} + \boldsymbol{\varepsilon}_t$
M7	VEC-INC(5)	$\boldsymbol{\Delta Y}_t = \boldsymbol{\alpha\beta' Y}_{t-1} + \sum_{i=1}^{p} \boldsymbol{\Pi}_i \boldsymbol{\Delta Y}_{t-i} + \boldsymbol{\varepsilon}_t$
M8	VAR(5)	$\boldsymbol{Y}_t = \boldsymbol{\alpha} + \sum_{i=1}^{p} \boldsymbol{\Pi}_i \boldsymbol{Y}_{t-i} + \boldsymbol{\varepsilon}_t$

Table 2: Model fit

Model	observations(T)	parameters(k)	σ
RACC	126	4	0.012
AR(1)	127	5	0.022
AR(6)	122	10	0.011
ARIMA(2,d,2)	128	10	0.011
UC	128	3	0.012
VEC-RENT	92	19	0.011
VEC-INC	122	15	0.011
VAR	92	13	0.011

Table 3: Multi-step forecast statistics

Model	RMSFE	Theil's U[1]	Average bias[2]
RACC	0.225	1	-0.190**
AR(1)	0.382	2.885	-0.329**
AR(6)	0.248	1.217	-0.210**
ARIMA(2,d,2)	0.298	1.752	-0.254**
UC	0.243	1.168	-0.206**
VEC-RENT	0.096	0.184	-0.080**
VEC-INC	0.097	0.188	-0.084**
VAR	0.154	0.467	-0.132**

One and two asterisks indicates significance at the 10% and 5% level, respectively.

[1] relative to RACC

[2] Rejections based on the null hypothesis that $\{\alpha = 0; \beta = 1\}$ in equation 1.

Table 4: 1-step forecast statistics

Model	RMSFE	Theil's U[1]	Average bias[2]	Parameter constancy[3]
RACC	0.024	1	0.007	4.133**
AR(1)	0.055	5.019	-0.049**	6.379**
AR(6)	0.022	0.834	0.004	3.986**
ARIMA(2,d,2)	0.023	0.905	0.002*	4.646**
UC	0.025	1.043	0.000	4.698**
VEC-RENT	0.020	0.675	0.002	3.487**
VEC-INC	0.023	0.872	0.002	4.451**
VAR	0.022	0.827	-0.011	4.368**

One and two asterisks indicates significance at the 10% and 5% level, respectively.

[1] relative to RACC

[2] Rejections based on the null hypothesis that $\{\alpha = 0; \beta = 1\}$ in Equation 1.

[3] Column presents the ratio $RMSFE^2/\sigma^2$ where σ is the in-sample residual standard deviation found in Table 2. Under the null that $RMSFE/\sigma = 1$, this ratio follows a χ^2 distribution with $(T - k, H)$ degrees of freedom, where $H = 12$ is the number of periods forecasted and T and K are found in Table 2.

Table 5: Forecast encompassing test summary

Encompassed by a multivariate model[1]	Test A	Test B	Test C	Total
RACC	2 of 3	1 of 3	1 of 3	4 of 9
AR(1)	2 of 3	3 of 3	3 of 3	8 of 9
AR(6)	0 of 3	0 of 3	0 of 3	0 of 9
ARIMA(2,d,2)	2 of 3	0 of 3	1 of 3	3 of 9
UC	2 of 3	2 of 3	2 of 3	6 of 9

Encompassed by a univariate model[1]	Test A	Test B	Test C	Total
VEC-RENT	0 of 5	0 of 5	0 of 5	0 of 15
VEC-INC	0 of 5	0 of 5	0 of 5	0 of 15
VAR	0 of 5	0 of 5	0 of 5	0 of 15

---

[1] at the 10% level

Table 6: Forecast-encompassing test statistics: test A

$$Y_{T+n|T+n-1} - \tilde{Y}_{T+n|T+n-1} = \alpha + \gamma(\hat{Y}_{T+n|T+n-1} - \tilde{Y}_{T+n|T+n-1}) + e_{T+n|T+n-1}$$

Forecast in row denoted $\tilde{y}$. Forecast in column denoted $\hat{y}$.
The value in each cell below is the p-value of the restriction $\{\gamma = 0\}$.

Encompassing	Model to be encompassed							
model	M1	M2	M3	M4	M5	M6	M7	M8
M1		0.240	0.199	0.547	0.399	0.072*	0.447	0.054*
M2	0.137		0.091*	0.083*	0.185	0.037**	0.132	0.029**
M3	0.560	0.394		0.892	0.038**	0.145	0.379	0.120
M4	0.750	0.167	0.303		0.691	0.091*	0.531	0.062*
M5	0.169	0.147	0.009**	0.206		0.017*	0.105	0.023**
M6	0.667	0.503	0.537	0.788	0.220		0.480	0.415
M7	0.975	0.352	0.220	0.782	0.349	0.085*		0.073*
M8	0.941	0.630	0.887	0.851	0.619	0.941	0.780	

Table 7: Forecast-encompassing test statistics: test B

$$Y_{T+n|T+n-1} - \tilde{Y}_{T+n|T+n-1} = \alpha + \gamma(\hat{Y}_{T+n|T+n-1} - \tilde{Y}_{T+n|T+n-1}) + e_{T+n|T+n-1}$$

Forecast in row denoted $\tilde{y}$. Forecast in column denoted $\hat{y}$.
The value in each cell below is the p-value of the joint restriction $\{\alpha = 0; \gamma = 0\}$.

Encompassing model	Model to be encompassed							
	M1	M2	M3	M4	M5	M6	M7	M8
M1		0.322	0.280	0.550	0.458	0.122	0.491	0.095*
M2	0.000**		0.000**	0.000**	0.000**	0.000**	0.000**	0.000**
M3	0.694	0.567		0.822	0.086*	0.273	0.553	0.233
M4	0.907	0.351	0.547		0.880	0.213	0.778	0.154
M5	0.370	0.331	0.028**	0.432		0.049**	0.251	0.068*
M6	0.869	0.757	0.784	0.922	0.435		0.737	0.675
M7	0.977	0.620	0.444	0.939	0.617	0.206		0.181
M8	0.244	0.217	0.243	0.241	0.215	0.244	0.235	

Table 8: Forecast-encompassing test statistics: test C

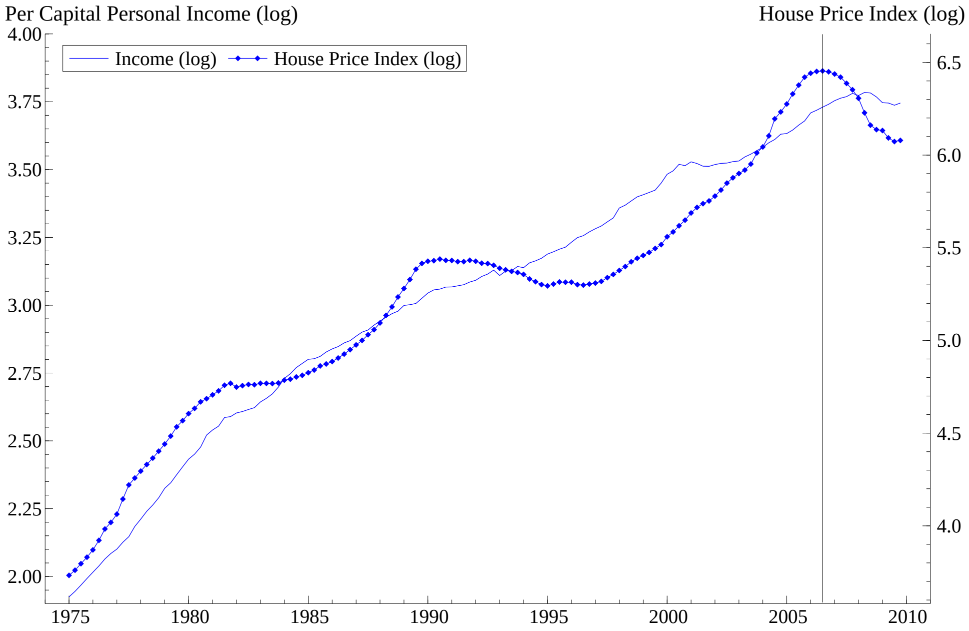
$$Y_{T+n|T+n-1} - \tilde{Y}_{T+n|T+n-1} = \gamma(\hat{Y}_{T+n|T+n-1} - \tilde{Y}_{T+n|T+n-1}) + e_{T+n|T+n-1}$$

Forecast in row denoted $\tilde{y}$. Forecast in column denoted $\hat{y}$.
The value in each cell below is the p-value of the restriction $\{\gamma = 0\}$.

Encompassing model	Model to be encompassed							
	M1	M2	M3	M4	M5	M6	M7	M8
M1		0.737	0.108	0.303	0.733	0.042*	0.225	0.140
M2	0.000**		0.000**	0.000**	0.000**	0.000**	0.000**	0.000**
M3	0.412	0.806		0.957	0.336	0.132	0.836	0.375
M4	0.916	0.746	0.354		0.773	0.078*	0.469	0.158
M5	0.453	0.547	0.070*	0.208		0.014**	0.100	0.086*
M6	0.892	0.597	0.824	0.883	0.229		0.469	0.564
M7	0.869	0.596	0.478	0.718	0.368	0.073*		0.184
M8	0.665	0.192	0.402	0.305	0.474	0.116	0.270	

Figure 1: House prices, personal incomes, and rental prices: California

(a) House prices and incomes



(b) House prices and rental prices

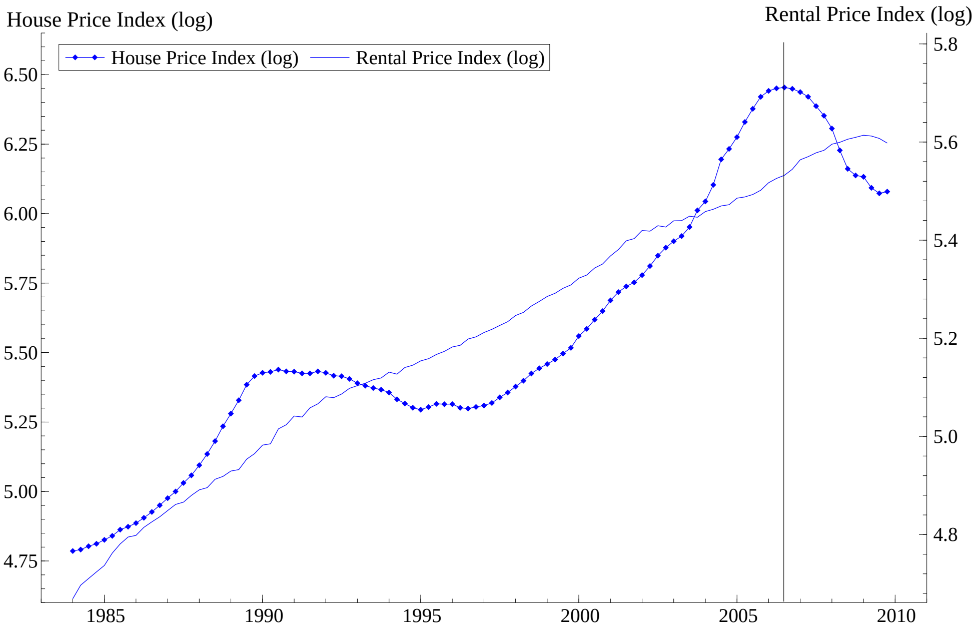


Figure 2: Change in house prices: California

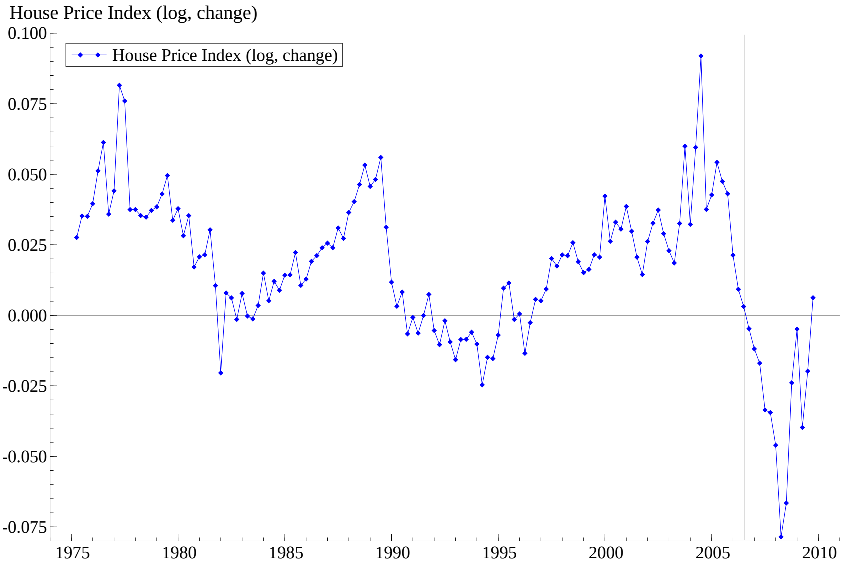
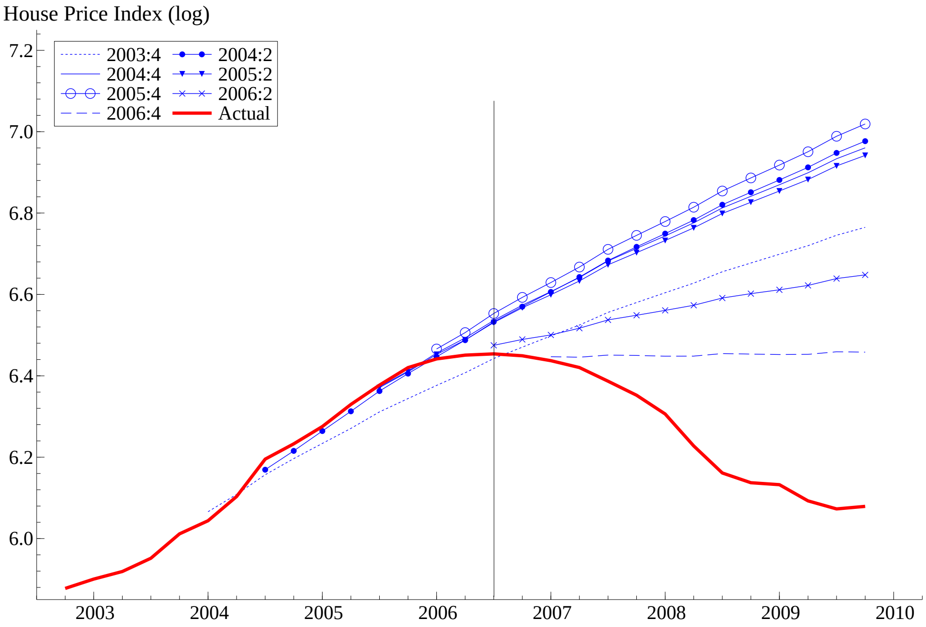


Figure 3: Pseudo ex-ante forecasts: univariate vs. multivariate models

(a) AR(6)



(b) VEC-INC

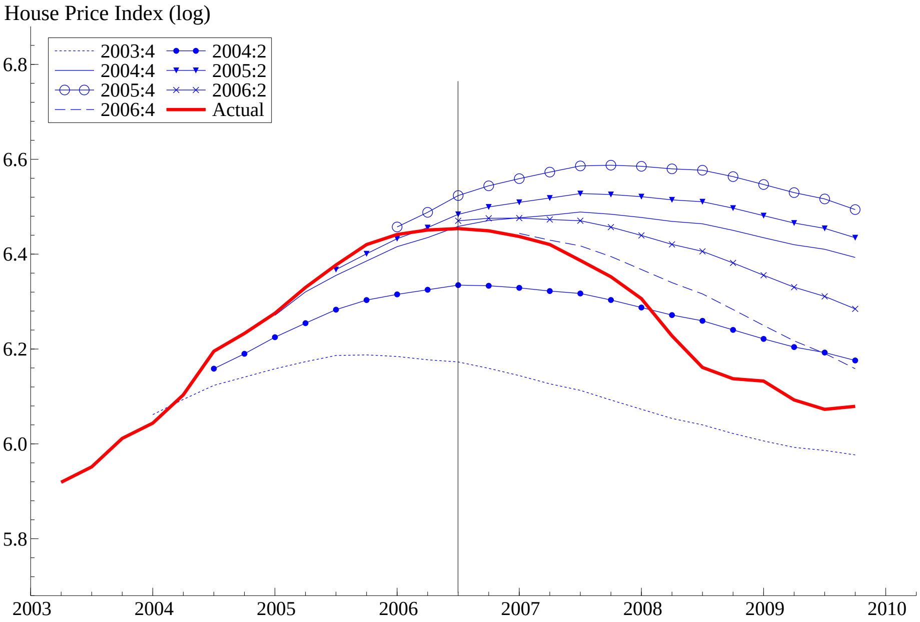


Figure 4: multi-step dynamic forecasts

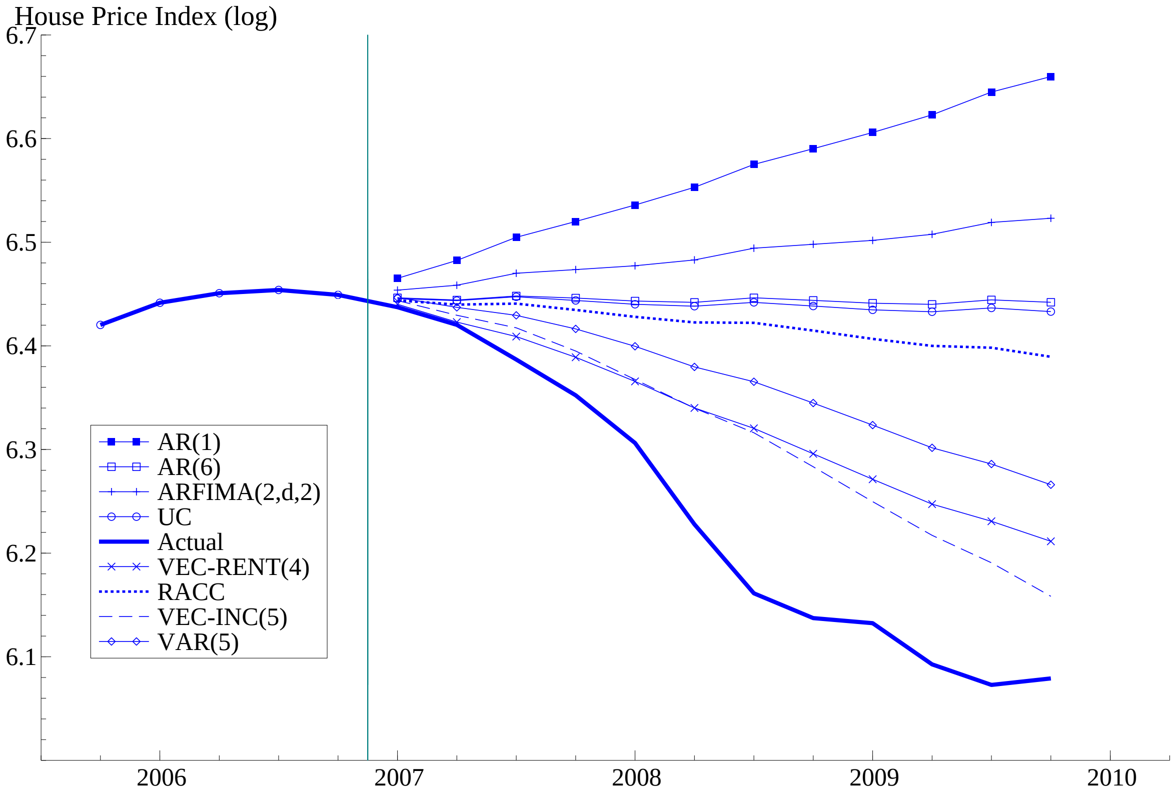


Figure 5: Confidence intervals of select multi-step dynamic forecasts

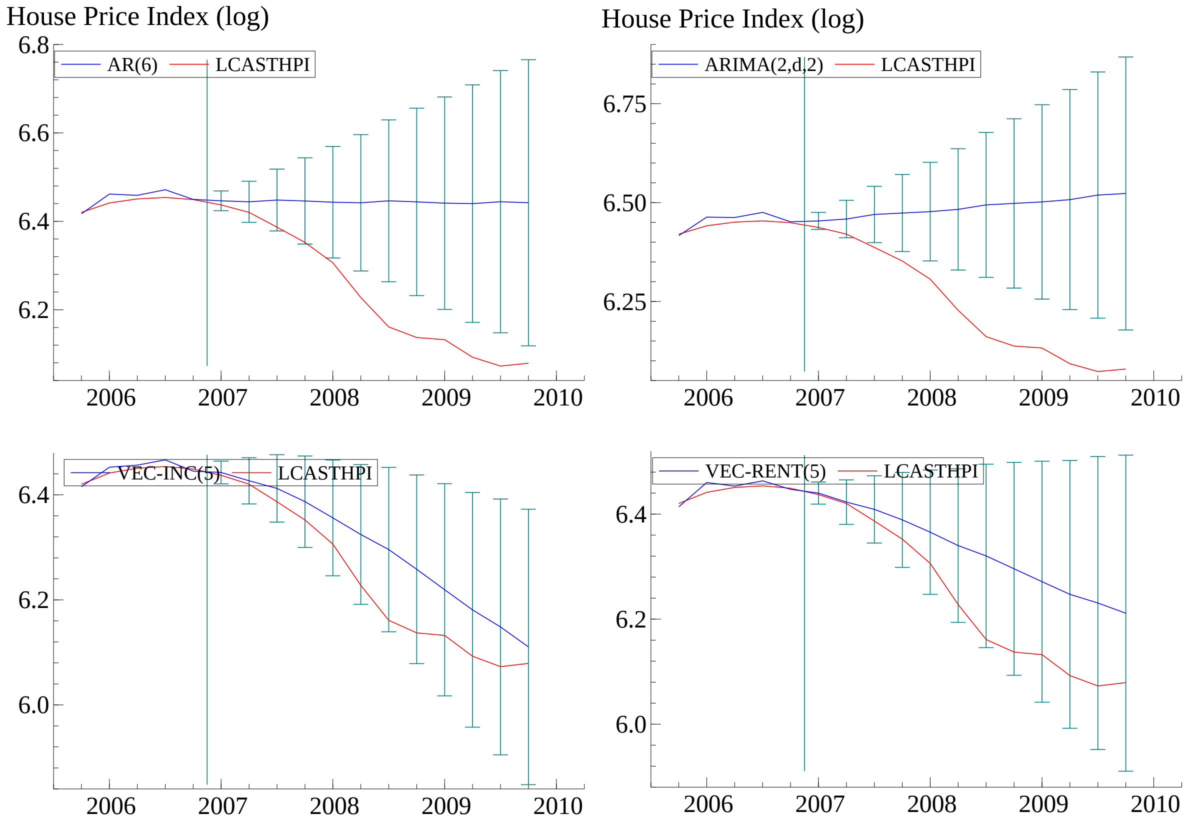
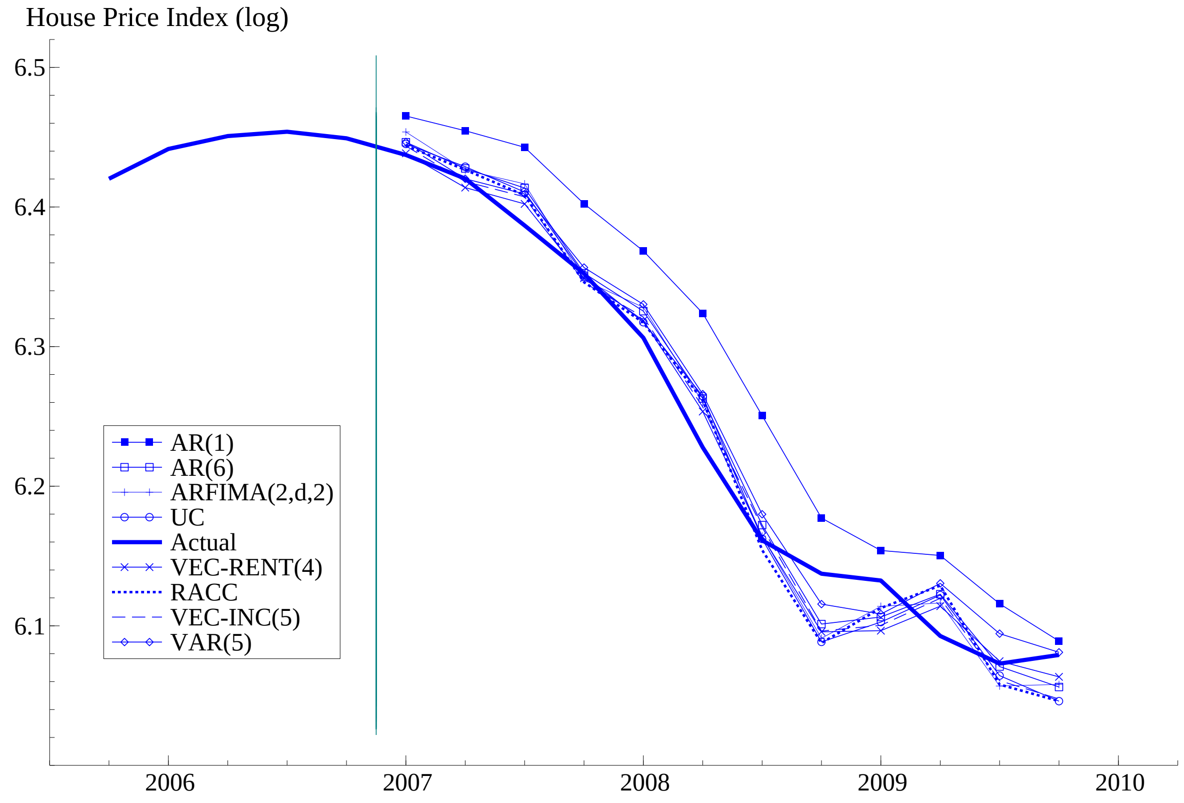


Figure 6: 1-step forecasts



A Appendix

Table A1: Random acceleration model estimation results

$$\text{DDCASTHPI} = \underset{(0.00107)}{-0.0002435} + \underset{(0.00303)}{0.006934}\ \text{CSeasonal}_t + \underset{(0.00303)}{0.008977}\ \text{CSeasonal}_{t-1} + \underset{(0.00301)}{0.01262}\ \text{CSeasonal}_{t-2}$$

$$N = 126,\ \sigma = [0.012024],\ LL = 380.273498,\ Smpl : 1975 : 3 - 2006 : 4$$

Table A2: AR(1) estimation results

$$\text{LCASTHPI} = \underset{(0.00306)}{0.9947}\ \text{LCASTHPI}_{t-1} + \underset{(0.0163)}{0.0463} + \underset{(0.00541)}{0.0002082}\ \text{Seasonal}_t + \underset{(0.00537)}{0.001881}\ \text{Seasonal}_{t-1} + \underset{(0.00537)}{0.007238}\ \text{Seasonal}_{t-2}$$

$$N = 127,\ \sigma = 0.02147,\ LL = 310.165,\ Smpl : 1975 : 2 - 2006 : 4$$

Table A3: AR(6) estimation results

$$\begin{aligned}\text{LCASTHPI} = {} & \underset{(0.0935)}{1.82}\ \text{LCASTHPI}_{t-1} - \underset{(0.192)}{0.9912}\ \text{LCASTHPI}_{t-2} + \underset{(0.203)}{0.5907}\ \text{LCASTHPI}_{t-3} \\ & - \underset{(0.202)}{0.7218}\ \text{LCASTHPI}_{t-4} + \underset{(0.194)}{0.4032}\ \text{LCASTHPI}_{t-5} - \underset{(0.0961)}{0.1027}\ \text{LCASTHPI}_{t-6} \\ & + \underset{(0.00979)}{0.003727} + \underset{(0.00326)}{0.006603}\ \text{Seasonal}_t + \underset{(0.00316)}{0.00536}\ \text{Seasonal}_{t-1} \\ & + \underset{(0.00317)}{0.01415}\ \text{Seasonal}_{t-2}\end{aligned}$$

$$N = 122,\ \sigma = 0.0111822,\ LL = 380.305,\ Smpl : 1976 : 3 - 2006 : 4$$

Table A4: ARFIMA(2,d,2) estimation results

	Coefficient		Std.Error	t-value	t-prob
d parameter	1.32477		0.2757	4.8	0
AR-1	0.483542		0.1911	2.53	0.013
AR-2	0.386105		0.1358	2.84	0.005
MA-1	0.147896		0.1335	1.11	0.27
MA-2	-0.60418		0.1798	-3.36	0.001
Constant	3.80971		0.1739	21.9	0
Seasonal	-0.00237		0.001324	-1.79	0.075
Seasonal_1	-0.0028		0.001857	-1.51	0.134
Seasonal_2	0.002409		0.001325	1.82	0.072
log-likelihood	400.1725				
no. of observations	128	no. of parameters	10		
AIC.T -	780.3449	AIC	-6.09644		
mean(LCASTHPI)	5.17594	var(LCASTHPI)	0.397383		
sigma	0.010787	σ^2	0.000116		

Smpl: 1975:1-2006:4

Table A5: Unobserved components model estimation results

T	128	Variances of disturbances:		
p	3		Value	(q-ratio)
std.error	0.011519	Level	3.46E-05	0.4476
Normality	10.55	Slope	7.74E-05	1
H(41)	1.4741	Seasonal	0.00E-05	0.00E-05
DW	1.909	State vector analysis at period 2006(4)		
r(1)	0.037426		Value	Prob
q	13	Level	6.44845	[0.00000]
r(q)	-0.00066524	Slope	-0.00135	[0.89524]
Q(q,q-p)	11.852	Seasonal chi2 test	16.34984	[0.00096]
$Rs\hat{2}$	0.71842			

Log-Likelihood: 537.546 Smpl: 1975:1-2006:4

Table A6: VEC-RENT(4) estimation results

$$
\begin{aligned}
\text{DLCASTHPI} \; = \; & \underset{(0.109)}{0.762}\ \text{DLCASTHPI}_{t-1} - \underset{(0.226)}{0.2916}\ \text{DLRENT}_{t-1} - \underset{(0.13)}{0.03325}\ \text{DLCASTHPI}_{t-2} \\
& - \underset{(0.23)}{0.3469}\ \text{DLRENT}_{t-2} + \underset{(0.131)}{0.434}\ \text{DLCASTHPI}_{t-3} - \underset{(0.228)}{0.006322}\ \text{DLRENT}_{t-3} \\
& - \underset{(0.119)}{0.2115}\ \text{DLCASTHPI}_{t-4} + \underset{(0.225)}{0.0223}\ \text{DLRENT}_{t-4} - \underset{(0.006)}{0.01054}\ \text{CIa}_{t-1} \\
& + \underset{(0.0186)}{0.03135} + \underset{(0.00379)}{0.004008}\ \text{Seasonal}_{t} + \underset{(0.00332)}{0.000686}\ \text{Seasonal}_{t-1} \\
& + \underset{(0.00372)}{0.01021}\ \text{Seasonal}_{t-2}
\end{aligned}
$$

$$
\begin{aligned}
\text{DLRENT} \; = \; & - \underset{(0.05)}{0.04321}\ \text{DLCASTHPI}_{t-1} - \underset{(0.103)}{0.1547}\ \text{DLRENT}_{t-1} + \underset{(0.0596)}{0.0761}\ \text{DLCASTHPI}_{t-2} \\
& + \underset{(0.105)}{0.1764}\ \text{DLRENT}_{t-2} - \underset{(0.0597)}{0.02048}\ \text{DLCASTHPI}_{t-3} + \underset{(0.104)}{0.04282}\ \text{DLRENT}_{t-3} \\
& + \underset{(0.0546)}{0.05458}\ \text{DLCASTHPI}_{t-4} + \underset{(0.103)}{0.399}\ \text{DLRENT}_{t-4} - \underset{(0.00274)}{0.004228}\ \text{CIa}_{t-1} \\
& + \underset{(0.00852)}{0.01516} + \underset{(0.00173)}{0.0004115}\ \text{Seasonal}_{t} - \underset{(0.00152)}{0.001019}\ \text{Seasonal}_{t-1} \\
& + \underset{(0.0017)}{0.000657}\ \text{Seasonal}_{t-2}
\end{aligned}
$$

$N = 92,\ \sigma = [0.0106267, 0.0048618],\ LL = 922.614282$

$Smpl : 1984 : 1 - 2006 : 4$

Table A7: VEC-INC(5) estimation results

$$
\begin{aligned}
\text{DLCASTHPI} = {} & \underset{(0.0925)}{0.7936}\ \text{DLCASTHPI}_{t-1} - \underset{(0.122)}{0.01286}\ \text{DLCAINC}_{t-1} - \underset{(0.116)}{0.1745}\ \text{DLCASTHPI}_{t-2} \\
& - \underset{(0.123)}{0.1025}\ \text{DLCAINC}_{t-2} + \underset{(0.11)}{0.4398}\ \text{DLCASTHPI}_{t-3} + \underset{(0.132)}{0.07195}\ \text{DLCAINC}_{t-3} \\
& - \underset{(0.116)}{0.2656}\ \text{DLCASTHPI}_{t-4} + \underset{(0.126)}{0.08464}\ \text{DLCAINC}_{t-4} + \underset{(0.0984)}{0.2033}\ \text{DLCASTHPI}_{t-5} \\
& - \underset{(0.124)}{0.2319}\ \text{DLCAINC}_{t-5} - \underset{(0.00806)}{0.02537}\ \text{CIa}_{t-1} + \underset{(0.0136)}{0.03665} \\
& + \underset{(0.00329)}{0.007836}\ \text{Seasonal}_{t} + \underset{(0.00323)}{0.006639}\ \text{Seasonal}_{t-1} + \underset{(0.00313)}{0.0145}\ \text{Seasonal}_{t-2}
\end{aligned}
$$

$$
\begin{aligned}
\text{DLCAINC} = {} & \underset{(0.0731)}{0.0788}\ \text{DLCASTHPI}_{t-1} + \underset{(0.0966)}{0.1441}\ \text{DLCAINC}_{t-1} + \underset{(0.0916)}{0.01208}\ \text{DLCASTHPI}_{t-2} \\
& + \underset{(0.0972)}{0.377}\ \text{DLCAINC}_{t-2} - \underset{(0.0867)}{0.04833}\ \text{DLCASTHPI}_{t-3} + \underset{(0.104)}{0.09225}\ \text{DLCAINC}_{t-3} \\
& + \underset{(0.0921)}{0.09231}\ \text{DLCASTHPI}_{t-4} + \underset{(0.0993)}{0.0449}\ \text{DLCAINC}_{t-4} - \underset{(0.0779)}{0.1023}\ \text{DLCASTHPI}_{t-5} \\
& - \underset{(0.0981)}{0.01506}\ \text{DLCAINC}_{t-5} + \underset{(0.00637)}{0.002279}\ \text{CIa}_{t-1} + \underset{(0.0107)}{0.002446} \\
& - \underset{(0.0026)}{0.0001563}\ \text{Seasonal}_{t} - \underset{(0.00256)}{0.003823}\ \text{Seasonal}_{t-1} - \underset{(0.00247)}{0.004077}\ \text{Seasonal}_{t-2}
\end{aligned}
$$

$N = 122,\ \sigma = [0.0108183, 0.00855774],\ LL = 804.117306$

$Smpl : 1976 : 3 - 2006 : 4$

Table A8: VAR(5) estimation results

$$\begin{aligned}
\text{LCASTHPI} = {} & \underset{(0.116)}{1.712}\ \text{LCASTHPI}_{t-1} - \underset{(0.217)}{0.7948}\ \text{LCASTHPI}_{t-2} + \underset{(0.226)}{0.4961}\ \text{LCASTHPI}_{t-3} \\
& - \underset{(0.218)}{0.6702}\ \text{LCASTHPI}_{t-4} + \underset{(0.125)}{0.2419}\ \text{LCASTHPI}_{t-5} + \underset{(0.146)}{0.04758}\ \text{LCAINC}_{t-1} \\
& - \underset{(0.214)}{0.08139}\ \text{LCAINC}_{t-2} - \underset{(0.212)}{0.1235}\ \text{LCAINC}_{t-3} + \underset{(0.222)}{0.3698}\ \text{LCAINC}_{t-4} \\
& - \underset{(0.17)}{0.07638}\ \text{LCAINC}_{t-5} - \underset{(0.293)}{0.4798}\ \text{LRENT}_{t-1} - \underset{(0.308)}{0.1447}\ \text{LRENT}_{t-2} \\
& + \underset{(0.314)}{0.2974}\ \text{LRENT}_{t-3} + \underset{(0.302)}{0.09472}\ \text{LRENT}_{t-4} + \underset{(0.267)}{0.0876}\ \text{LRENT}_{t-5} \\
& + \underset{(0.299)}{0.4052} + \underset{(0.00385)}{0.003177}\ \text{Seasonal}_{t} + \underset{(0.00349)}{0.0002053}\ \text{Seasonal}_{t-1} \\
& + \underset{(0.00381)}{0.01038}\ \text{Seasonal}_{t-2}
\end{aligned}$$

$$\begin{aligned}
\text{LCAINC} = {} & \underset{(0.0891)}{0.03273}\ \text{LCASTHPI}_{t-1} - \underset{(0.167)}{0.07873}\ \text{LCASTHPI}_{t-2} - \underset{(0.175)}{0.03144}\ \text{LCASTHPI}_{t-3} \\
& + \underset{(0.168)}{0.1795}\ \text{LCASTHPI}_{t-4} - \underset{(0.0964)}{0.1056}\ \text{LCASTHPI}_{t-5} + \underset{(0.112)}{1.069}\ \text{LCAINC}_{t-1} \\
& + \underset{(0.165)}{0.2645}\ \text{LCAINC}_{t-2} - \underset{(0.163)}{0.3988}\ \text{LCAINC}_{t-3} + \underset{(0.171)}{0.05721}\ \text{LCAINC}_{t-4} \\
& + \underset{(0.131)}{0.07809}\ \text{LCAINC}_{t-5} - \underset{(0.226)}{0.1991}\ \text{LRENT}_{t-1} - \underset{(0.237)}{0.06757}\ \text{LRENT}_{t-2} \\
& - \underset{(0.242)}{0.09053}\ \text{LRENT}_{t-3} - \underset{(0.233)}{0.1996}\ \text{LRENT}_{t-4} + \underset{(0.206)}{0.469}\ \text{LRENT}_{t-5} \\
& + \underset{(0.231)}{0.2679} + \underset{(0.00297)}{0.001981}\ \text{Seasonal}_{t} - \underset{(0.00269)}{0.002409}\ \text{Seasonal}_{t-1} \\
& - \underset{(0.00294)}{0.002643}\ \text{Seasonal}_{t-2}
\end{aligned}$$

$$
\begin{aligned}
\mathrm{LRENT} \;=\; & -\underset{(0.0456)}{0.07484}\ \mathrm{LCASTHPI}_{t-1} + \underset{(0.0856)}{0.1108}\ \mathrm{LCASTHPI}_{t-2} - \underset{(0.0893)}{0.08389}\ \mathrm{LCASTHPI}_{t-3} \\
& + \underset{(0.086)}{0.04549}\ \mathrm{LCASTHPI}_{t-4} + \underset{(0.0494)}{0.004924}\ \mathrm{LCASTHPI}_{t-5} - \underset{(0.0575)}{0.01308}\ \mathrm{LCAINC}_{t-1} \\
& + \underset{(0.0846)}{0.07082}\ \mathrm{LCAINC}_{t-2} + \underset{(0.0835)}{0.103}\ \mathrm{LCAINC}_{t-3} - \underset{(0.0875)}{0.06876}\ \mathrm{LCAINC}_{t-4} \\
& + \underset{(0.0672)}{0.06061}\ \mathrm{LCAINC}_{t-5} + \underset{(0.116)}{0.4888}\ \mathrm{LRENT}_{t-1} + \underset{(0.122)}{0.2222}\ \mathrm{LRENT}_{t-2} \\
& - \underset{(0.124)}{0.08982}\ \mathrm{LRENT}_{t-3} + \underset{(0.119)}{0.3959}\ \mathrm{LRENT}_{t-4} - \underset{(0.106)}{0.208}\ \mathrm{LRENT}_{t-5} \\
& + \underset{(0.118)}{0.4953} + \underset{(0.00152)}{0.0008408}\ \mathrm{Seasonal}_{t} - \underset{(0.00138)}{0.0005055}\ \mathrm{Seasonal}_{t-1} \\
& + \underset{(0.0015)}{0.0007145}\ \mathrm{Seasonal}_{t-2}
\end{aligned}
$$

$N = 92,\ \sigma = [0.0106387, 0.00820626, 0.00420011],\ LL = 1005.57355$
$Smpl : 1984 : 1 - 2006 : 4$
