

International trade, migration and investment with horizontal product differentiation and free entry and exit of firms

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Received: January 2006 – Approved: February 2006

Abstract. This paper builds a circular road model of the world with horizontal product differentiation and free entry and exit of firms, to show that freer international trade increases welfare –with ideal variety preferences– through the exploitation of economies of scale and better allocative efficiency, that all participating countries gain from trade, and that smaller countries have more to win from free trade than larger countries. Political resistance to trade liberalization, international migration and foreign direct investment are also analyzed with the model. Finally, the model provides a microfoundation for the use of demand curves with constant and negative slopes.

Key words: monopolistic competition, horizontal product differentiation, international trade, international migration, foreign direct investment.

JEL classification: F12, F13.

Resumen. Este documento construye un modelo de carretera circular del mundo con diferenciación horizontal de producto y libre entrada y salida de firmas, para mostrar que un comercio internacional más libre aumenta el bienestar –con preferencias de variedad ideal– por medio de la explotación de economías de escala y de una mejor asignación de recursos, que todos los países participantes ganan con el comercio y que los países más pequeños tienen más que ganar del libre comercio que los países grandes. La resistencia política a la liberación del comercio, la migración internacional y la inversión extranjera directa también son estudiados con el modelo. Finalmente, el modelo provee una microfundamentación para el uso de curvas de demanda con pendientes constantes y negativas.

Palabras clave: competencia monopolística, diferenciación horizontal de producto, comercio internacional, migración internacional, inversión extranjera directa.

Clasificación JEL: F12, F13.

*I am grateful to Rafael Santos, Jorge Tovar and Pietro Bonaldi for their suggestions, to Luis Ernesto Taborda for his research assistance, to Carolina Mejía for her feedback, and to an anonymous referee for his detailed comments. Support from Universidad de los Andes and its Department of Economics is also gratefully acknowledged. All remaining errors are mine.

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1. Introduction

This paper builds on the circular road model of horizontal product differentiation of Eaton et al. (1975) with free entry and exit of firms, to derive results that can be applied in industrial organization, international trade and political economy.

The model shows that freer international trade increases welfare –with ideal variety preferences– through the exploitation of economies of scale and improved allocative efficiency, that all participating countries gain from trade, and that smaller countries have more to win from free trade than larger countries. Furthermore, the model explains that there may be adjustment costs when liberalizing trade and thus, political resistance to trade liberalization. International migration can also be analyzed with the model, showing the possibility of suboptimal migration flows and political barriers to the exit of national citizens. The model suggests that foreign direct investment will be welfare improving for the source country in the short run and for the receiving country in the long run.

2. Previous literature

Many of the insights generated in this paper have been derived before in a monopolistic competition model within a general equilibrium setting, by Lancaster (1979) –with ideal variety preferences– and by Krugman (1979) –with love of variety preferences–. In fact, the model presented in this paper can be interpreted –at least in part– as a formal and partial equilibrium representation of the model presented in an intuitive manner by Lancaster (1979). A similar approach, but with some differences in the parameters used and emphasising the effects of tariffs, was followed by Schmitt (1990). Other authors that have written in this area are, among others, Salop (1979), Schmitt (1995), and Bocard and Wauthy (2000).

The main contributions of this paper are that the results here generated are derived within a partial equilibrium framework, highlighting the different effects of trade in goods between larger and smaller countries, providing insights for the political economy of trade and migration, and exploring the short and long run impacts of foreign direct investment. The paper also provides a microfoundation for the use of demand curves with constant and negative slopes.

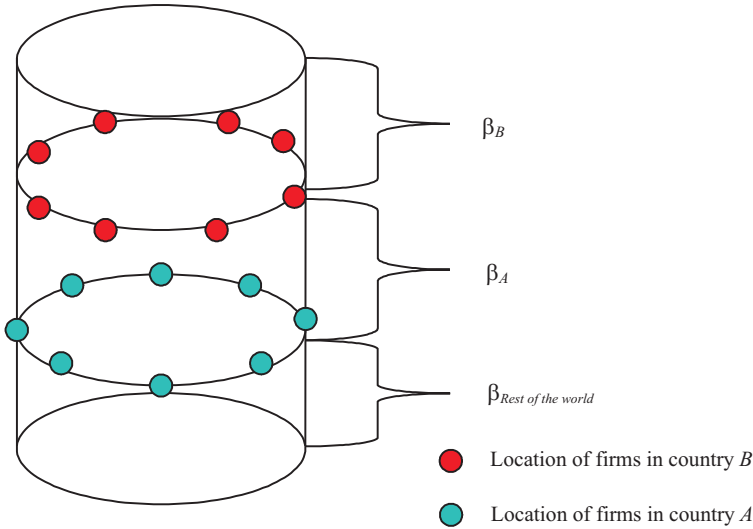
3. The model

3.1. Assumptions and notation

The basic assumptions of the model presented in this paper are:

- i. The world can be represented as a circular road of extension equal to a , where all countries are located one on top of the other (see Figure 1).

Figure 1. A circular road world



- ii. There is one industry (this is a partial equilibrium model).
- iii. The good produced is homogeneous in quality, but not in location.
- iv. There are N producers of the homogeneous good, that represent N varieties of that good in terms of location.
- v. Firms play a two stage game: on the first stage they determine locations, and on the second stage they determine their prices.
- vi. To solve the model by backward induction, in the second stage of the game firms are assumed to be located at a distance $\frac{\alpha}{N}$ of each other.
- vii. Each firm has the same cost structure, $TC = f + ex$, where f is the fixed cost and e is the constant marginal cost.
- viii. Consumers are homogeneous and uniformly distributed along the circular road (there are β consumers on every unit of distance of the road, with $\beta > 0$).
- ix. Consumers have identical ideal variety preferences and they all consume one unit of the good, as long as the utility they receive from that consumption is non-negative (assuming that $u^* = 0$ when there is no consumption). Thus, the representative consumer will have the following utility function:

$$U = u^* - p - td \tag{1}$$

where

- u^* = utility derived from consuming one unit of the good,
- p = unit price of the good,
- t = unit transport cost for the consumers, with $t > 0$, and
- d = distance to the nearest producer.

- x. There are no international transport costs.

3.2. *Autarky equilibrium*

In this paper, a symmetrical equilibrium is searched. Thus, the second stage of the game identifies what a symmetric Nash equilibrium in prices is. Then, by backward induction, the first stage of the game identifies a Nash equilibrium in locations. The paper also identifies what the number of producers is, what their sales are, and what the level of welfare per capita is at equilibrium.

To do so, the model works with the long run monopolistic competition conditions as follows: first, the sales of a typical firm in a circular road model are derived. Then, the optimal strategy price and the corresponding quantities are obtained from the profit maximization condition. These price and quantities –and the locations initially assumed– are shown to be not only optimal strategies for the firm, but also a Nash equilibrium for the market. The next step is to find the endogenous number of firms (varieties) from the zero profit conditions. Finally, the autarky equilibrium is completed by expressing the utility function of the average consumer in terms of the parameters of the model, and analysing the impacts that changes in the parameters of the model have on key variables.

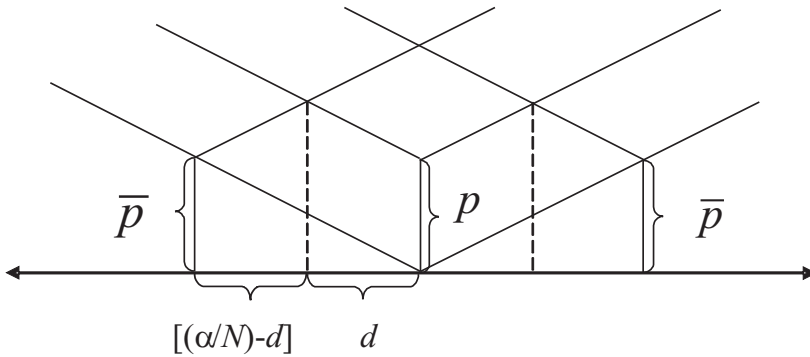
3.2.1. *Deriving the sales of a typical firm in a circular road model*

Given the assumptions, it is possible to construct Hotelling's umbrellas,¹ as shown in Figure 2.

The vertical bases of the umbrellas show the location of each producer, and the extension of the vertical bases represent the prices charged by the producers located at each base. The slope of the arms of the umbrella is the uniform unit transport cost for the consumers. Given that the total cost incurred by a consumer located at a given point over the circular road –when buying a unit of the good– is the price paid plus the unit transport cost multiplied by the distance travelled, the heights of the arms of Hotelling's umbrellas show the total cost for a given consumer buying the good from the producer located at the base corresponding to each umbrella.

¹This version of Hotelling's model is based on the class notes taken by the author from the lectures of Professor John Sutton. For an alternative explanation of Hotelling's model see Cabral (2000).

Figure 2. The circular world model and Hotelling's umbrellas



The equation that describes the marginal consumer, i.e., the consumer that is indifferent between buying from the producer located nearest to the right hand side and the producer located nearest to the left hand side, is

$$p + td = \bar{p} + t \left[\frac{\alpha}{N} - d \right]$$

Solving for d ,

$$2td = \bar{p} - p + \frac{t\alpha}{N}$$

$$d = \frac{\bar{p} - p}{2t} + \frac{\alpha}{2N}$$

Given that the model is symmetric and, in particular, given that the per unit transport costs are uniform, the sales of the representative producer x and, in particular, of the producer located in the middle of Figure 2, are

$$x = 2d\beta \tag{2}$$

$$2d\beta = \left[\frac{\bar{p} - p}{t} \right] \beta + \frac{\alpha\beta}{N}$$

$$x = \left[\frac{\bar{p} - p}{t} \right] \beta + \frac{\alpha\beta}{N} \tag{3}$$

3.2.2. Profit maximization condition

For simplicity and clearness of exposition, it is of interest to analyse if the case where all firms charge identical prices is a Nash equilibrium, regardless of whether it is unique or optimal for society as a whole.

To show this, note that the profit maximization condition for the firm under consideration, given that all other firms charge the same price \bar{p} , is

$$\begin{aligned}\Pi &= px - f - ex \\ &= p \left[\left[\frac{\bar{p} - p}{t} \right] \beta + \frac{\alpha\beta}{N} \right] - f - e \left[\left[\frac{\bar{p} - p}{t} \right] \beta + \frac{\alpha\beta}{N} \right] \\ &= \left[\frac{\bar{p}p - p^2}{t} \right] \beta + \frac{p\alpha}{N} \beta - f - \left[\frac{\bar{p}e - pe}{t} \right] \beta - \frac{\alpha\beta e}{N}\end{aligned}$$

$$\frac{\partial \Pi}{\partial p} = \frac{\beta}{t} \left[\bar{p} - 2p + \frac{\alpha t}{N} + e \right] = 0$$

$$\bar{p} - 2p + \frac{\alpha t}{N} + e = 0$$

Now assume that the firm under consideration maximizes profits and, also, charges the same price as all other firms $p = \bar{p}$. Thus,

$$p = \frac{\alpha t}{N} + e \quad (4)$$

Now, this price is charged by the firm maximizing profits, given that all the other firms are charging this same price. Thus, it is an optimal strategy for that firm. Repeating the same process for all other firms, the symmetrical equilibrium in prices is found to be a Nash equilibrium, regardless of whether it is unique or optimal for society as a whole.

In such equilibrium and from equation (3), the optimal quantities sold by each firm are

$$x = \frac{\alpha\beta}{N} \quad (5)$$

Note that in the circular road model, if all firms charge identical prices, locating at a distance $\frac{\alpha}{N}$ in the first stage of the game is a Nash equilibrium in location. Thus, a Nash equilibrium is found for location in the first stage and for prices in the second stage of the game.

3.2.3. *The zero profit condition*

Free entry and exit of firms ensures that, in the long run, firms will have zero profits. The zero profit condition of profit maximizing firms is met when

the average costs equal the per unit price, as follows:

$$\frac{f}{x} + e = \frac{\alpha t}{N} + e$$

$$N^2 = \frac{\alpha^2 \beta t}{f}$$

Thus, the total number of firms (varieties), expressed in terms of the parameters of the model, is

$$N = \alpha \sqrt{\frac{\beta t}{f}} \quad (6)$$

Replacing (6) in (4), the equilibrium price can also be expressed in terms of the parameters of the model by

$$p = \sqrt{\frac{ft}{\beta}} + e \quad (7)$$

Since the price includes a mark-up over marginal cost, resources are not allocated efficiently. The lower the fixed and transport costs, and the higher the market density, the better the allocative efficiency and the lower the price. Falls in the marginal cost will also lower the equilibrium price.

Replacing (6) in (5), the equilibrium quantities can be expressed as

$$x = \sqrt{\frac{\beta f}{t}} \quad (8)$$

This means that higher population density and fixed costs, and lower unit transport costs lead to higher equilibrium quantities. Note also that, in the symmetric equilibrium and also from (3), the demand function faced by a typical firm is

$$x = \left[\frac{2\sqrt{f\beta}}{\sqrt{t}} + \frac{\beta e}{t} \right] - \frac{\beta}{t} p$$

which is a demand curve with a constant and negative slope. In this sense, the model developed in this paper provides a microfoundation for the use of this simple type of demand curves.

From (2) and (8),

$$d = \frac{1}{2} \sqrt{\frac{f}{\beta t}} \quad (9)$$

The utility function of the marginal consumer, in terms of the parameters of the model, can now be obtained by substituting (6), (7) and (9) in (1) yielding

$$U = u^* - \sqrt{\frac{ft}{\beta}} - e - \frac{t}{2} \sqrt{\frac{f}{\beta t}}$$

$$= u^* - \frac{3}{2} \sqrt{\frac{ft}{\beta}} - e \quad (10)$$

Now, in the symmetric Nash equilibrium in prices, the utility of the consumer located exactly in the same place where a producer is located is $u^* - p$. This means that there are transactions in this economy only as long as

$$u^* \geq \sqrt{\frac{ft}{\beta}} + e$$

On the other hand, the utility of the marginal consumer is given by (10). This implies that all consumers buy the good only if

$$u^* \geq \frac{3}{2} \sqrt{\frac{ft}{\beta}} - e$$

Besides, the utility of the average consumer is given by

$$\begin{aligned} U &= \frac{u^* - p + u^* - p - td}{2} \\ &= u^* - \frac{5}{4} \sqrt{\frac{ft}{\beta}} - e \end{aligned} \quad (11)$$

Thus, a nation's welfare per capita in autarky will increase (decrease), the lower (greater) the fixed, marginal and transport costs, and the greater (lower) the population density.

Note that, since individuals have ideal variety preferences, and since the number of varieties is endogenous, the impact of the number of varieties available on welfare can be expressed in terms of the parameters of the model. Note also that the size of the circular road (α) has a direct effect on the number of firms, but it does not have an impact on the level of production per firm and the level of equilibrium prices and so, it does not affect the level of welfare.

3.3. *International trade*

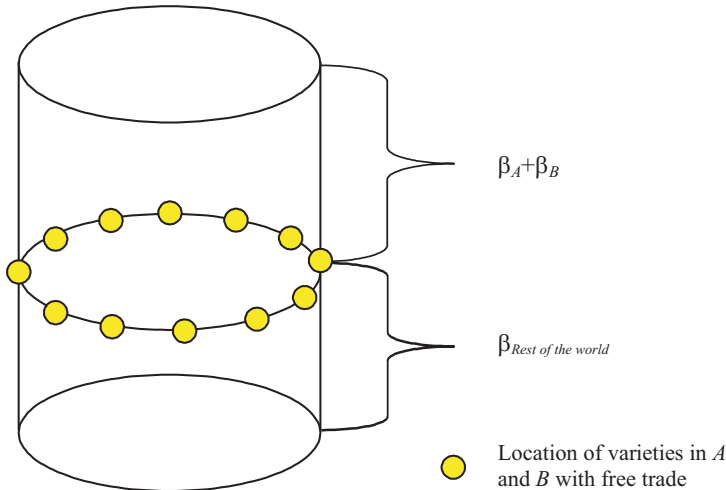
International trade can be introduced in this model as an increase in population density perceived by the producers of the good.² To do so, assume that there are two countries as follows.

Country	Population density
A	β_A
B	β_B

The circular road world with free trade between A and B will look as in Figure 3.

²The size of the circular road (α) is kept constant and equal in all countries since, as mentioned above, it affects the number of producers, but it does not affect output per firm, prices and welfare.

Figure 3. A circular road world with free trade between *A* and *B*



To start with, note that if both countries are identical, their autarky equilibrium conditions are also identical. However, if one country is smaller than the other, the larger country in autarky has more varieties, lower prices, higher output of each variety produced, and greater welfare per capita, than the smaller country.

Note also that, with free trade and no international transport costs between *A* and *B*, the change in population density perceived in *A* and *B* will lead to a new equilibrium that can be expressed as

$$N_{FT} = \alpha \sqrt{\frac{(\beta_A + \beta_B)t}{f}} \tag{12}$$

$$p_{FT} = \sqrt{\frac{ft}{(\beta_A + \beta_B)}} + e \tag{13}$$

$$x_{FT} = \sqrt{\frac{(\beta_A + \beta_B)f}{t}} \tag{14}$$

$$\bar{U} = u^* - \frac{5}{4} \sqrt{\frac{ft}{(\beta_A + \beta_B)}} - e \tag{15}$$

This means that in both *A* and *B*, and regardless of whether *A* and *B* are large and/or small, the number of varieties available increases, per unit prices fall, output of each variety increases, and welfare per capita rises. These gains from trade are generated by the better exploitation of economies of scale

(higher output per firm) and the improved allocative efficiency (lower mark-up of prices over marginal costs), thanks to the free entry and exit of firms (lower unit costs of production and zero profits at equilibrium).

Furthermore, the free trade varieties, prices, output per variety and welfare per citizen are all identical in both countries, regardless of their original size. As such, the gains from free trade are larger for smaller countries, than for larger countries.

It has been shown that $N_{FT} > N_A$ and that $N_{FT} > N_B$. Under this conditions, the pattern of trade will be intra-industry –local varieties are exported and foreign varieties of the same good are imported–.

3.4. Political economy of trade policy

The numbers of varieties in autarky for country A and for country B are

$$N_A = \alpha \sqrt{\frac{\beta_A t}{f}}$$

and

$$N_B = \alpha \sqrt{\frac{\beta_B t}{f}}$$

The sum of the varieties available in autarky in country A plus the number of varieties available in autarky in country B is

$$N_A + N_B = \alpha \left[\frac{\sqrt{\beta_A t} + \sqrt{\beta_B t}}{\sqrt{f}} \right]$$

The number of varieties with free trade in both A and B is

$$N_{FT} = \alpha \sqrt{\frac{(\beta_A + \beta_B)t}{f}}$$

$N_A + N_B$ is higher than N_{FT} if

$$\left(\sqrt{\beta_A} + \sqrt{\beta_B} \right) > \sqrt{(\beta_A + \beta_B)}$$

That is, if

$$2\sqrt{\beta_A \beta_B} > 0$$

which is true as long as $\beta_A > 0$ and $\beta_B > 0$, i.e., as long as countries A and B exist.

Thus,

$$N_A + N_B > N_{FT}$$

The number of varieties available with free trade is less than the sum of varieties available in A and in B in autarky. This means that since the remaining firms produce more, and the total density of the countries and the consumption per capita do not change, some firms have to exit the market.

This result is important for political economy reasons. In particular, there will be resistance to trade liberalization, because some firms will disappear –and some factors will have to move from some firms to other firms– introducing adjustment costs. However, such resistance may be milder with intra-industry trade than with inter-industry trade, since factors are moving within the same sector, as is the case in this paper. Besides, in the long run resources will remain fully employed with free trade, just as they were in autarky.

3.5. *International migration*

International migration can be introduced in this model as a shift in population density between countries.³ With no trade in goods and services, and *ceteris paribus*, if countries A and B free up population movements, the following effects will occur:

- i. As pointed out earlier, if one country has a higher population than the other, its citizens have a higher welfare. For example, if $\beta_A > \beta_B$, then $\bar{U}_A > \bar{U}_B$ because

$$\frac{5}{4} \sqrt{\frac{ft}{\beta_A}} < \frac{5}{4} \sqrt{\frac{ft}{\beta_B}}$$

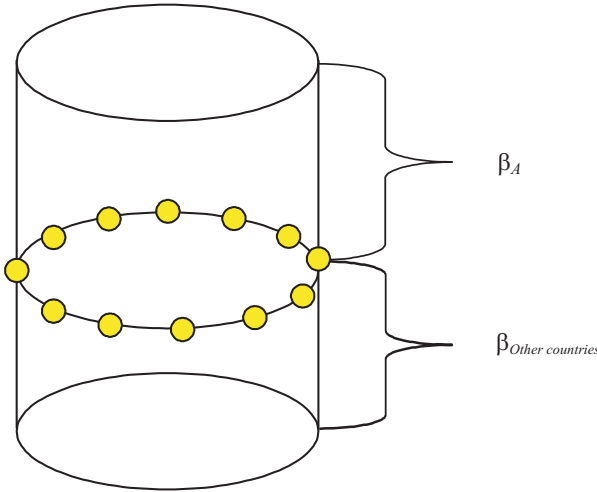
Thus, if population movements are allowed *ceteris paribus*, all the people move from the small country (B) to the large country (A). This situation is shown in Figure 4.

- ii. If A and B are identical, there are no movements, since per capita welfare is identical. However, this equilibrium is unstable, since all that is required is that one consumer move from one country to the other (to make welfare in the receiving country higher than in the country of origin) for all his fellow countrymen to do the same, just as in the previous case.
- iii. If one country has a technological advantage (lower f , e and/or t) –assuming that there are no technological transfers between countries– and both countries are identical in all other respects, the technologically advanced country has higher autarky welfare. For example, if $f_A > f_B$, then $\bar{U}_A < \bar{U}_B$ because

$$\frac{5}{4} \sqrt{\frac{f_A t}{\beta}} > \frac{5}{4} \sqrt{\frac{f_B t}{\beta}}$$

³If marginal consumers move from one country to the other, there is a reallocation of the population such that the density is reduced, but remains uniform.

Figure 4. A circular road world with free movements of population between *A* and *B*, where $\beta_A > \beta_B$



and all consumers will go from the less technologically advanced country (*A*) to the more technologically advanced country (*B*).

- iv. If one country has a technological advantage (lower f , e and/or t) –assuming that there are no technological transfers between countries– and, at the same time, that country is smaller than the other country, the size advantage may more than compensate the technological advantage in terms of welfare. For example, if $\beta_A > \beta_B$, $f_A > f_B$ and $\frac{f_A}{\beta_A} < \frac{f_B}{\beta_B}$, then $\bar{U}_A > \bar{U}_B$ because

$$\frac{5}{4} \sqrt{\frac{f_A}{\beta_A}} < \frac{5}{4} \sqrt{\frac{f_B}{\beta_B}}$$

In this case, all consumers go from the more technologically advanced country (*B*) to the less technologically advanced country (*A*) and the final equilibrium is pareto dominated, i.e., there are suboptimal migration flows. All the consumers in the world would be better off if they managed to live in the technologically advanced country.

3.6. Political economy of international migration

In this model, if international migration occurs at all, it leads to emptying one country. Although this is an unlikely outcome, in this case, such movements could lead political leaders –and maybe some economists– of the country whose consumers are leaving, to put barriers to the exit (rather than barriers to the

entry) in order to avoid the massive departure of citizens. To understand why, assume that A has a higher autarky welfare than B and that all consumers in B want to migrate to A . If all consumers in B go to A , all consumers from B have higher welfare than before (the gross national income is then higher in both countries A and B). But the citizens located in B (none), will have zero welfare (the gross domestic product in B is zero and lower than in autarky). Now, think of a dramatic, but not complete, migratory movement for ad-hoc reasons (such as language, climate and religion).⁴ Then, the citizens that for whatever reason remain in country B are worse off than in autarky (again, B then has a lower gross domestic product).

This could explain –at least in part–, although it may not justify, the exit restrictions that have prevailed in certain countries, or those that prevailed in former communist countries, given the welfare incentives for consumers and policy makers.

3.7. Foreign direct investment

In this model, with identical cost structures, without international trade and *ceteris paribus*, if firms have identical technologies, there are no incentives for foreign direct investment (FDI) flows, regardless of the size of the countries. However, if firms in country B have a technological advantage over firms in country A , there is an incentive for FDI flows to move from B to A since, at the prices in A , firms from B would be able to make a positive profit in the short run. However, if there is free entry and exit of foreign investors, eventually, the firms from B investing in A would have zero economic profits in the long run. On the other hand, the long run gains in welfare per capita in A from receiving the FDI flows

$$U_A^{FDI} - U_A$$

would be

$$\frac{5}{4} \sqrt{\frac{f_{At}}{\beta}} - \frac{5}{4} \sqrt{\frac{f_{Bt}}{\beta}} > 0$$

4. Conclusions

This paper has built on the circular road model of horizontal product differentiation. The model derived shows that, in autarky, larger countries have higher welfare than smaller countries. The model also shows that freer international trade increases welfare –with ideal variety preferences– through the exploitation of economies of scale and through better allocative efficiency, that benefit consumers thanks to the free entry and exit of firms.

In autarky, larger countries have higher welfare than smaller countries and, although all countries that take part in international trade gain from trade,

⁴Future work may focus on modelling the reasons why some consumers decide to stay in the smaller country.

with free trade all of the world's countries have the same welfare per capita, regardless of their population size. This implies that smaller countries have more to win from free trade than larger countries. As is usual in the literature of trade with imperfect competition, the pattern of trade in this model is intra-industry.

Furthermore, the model explains that there may be adjustment costs when liberalizing trade, because some producers will exit the market and some factors will have to move from some firms to other firms. These adjustment costs are likely to generate political resistance to trade liberalization, although such resistance may be milder with intra-industry trade than with inter-industry trade, since factors will be moving within the same sector. Besides, in the long run, all resources will be fully employed with free trade, as they were in the autarky equilibrium.

Allowing for international migration within the model has highlighted the possibility of both suboptimal migration flows and the existence of political barriers to the exit of national citizens. Besides, foreign direct investment would provide short run gains for the source country and long run gains for the receiving country. As a by product, this paper has provided microfoundations for the use of demand curves with constant and negative slopes.

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