View metadata, citation and similar papers at core.ac.uk



RUHR ECONOMIC PAPERS

Ansgar Belke Matthias Goecke Martin Guenther

> When Does It Hurt? The Exchange Rate "Pain Threshold" for German Exports



#148

Imprint

Ruhr Economic Papers

Published by

Ruhr-Universität Bochum (RUB), Department of Economics Universitätsstr. 150, 44801 Bochum, Germany Technische Universität Dortmund, Department of Economic and Social Sciences Vogelpothsweg 87, 44227 Dortmund, Germany Universität Duisburg-Essen, Department of Economics Universitätsstr. 12, 45117 Essen, Germany Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI) Hohenzollernstr. 1-3, 45128 Essen, Germany

Editors

Prof. Dr. Thomas K. Bauer RUB, Department of Economics, Empirical Economics Phone: +49 (0) 234/3 22 83 41, e-mail: thomas.bauer@rub.de

Prof. Dr. Wolfgang Leininger Technische Universität Dortmund, Department of Economic and Social Sciences Economics – Microeconomics Phone: +49 (0) 231/7 55-3297, email: W.Leininger@wiso.uni-dortmund.de Prof. Dr. Volker Clausen University of Duisburg-Essen, Department of Economics International Economics Phone: +49 (0) 201/1 83-3655, e-mail: vclausen@vwl.uni-due.de Prof. Dr. Christoph M. Schmidt RWI, Phone: +49 (0) 201/81 49-227, e-mail: christoph.schmidt@rwi-essen.de

Editorial Office

Joachim Schmidt RWI, Phone: +49 (0) 201/81 49-292, e-mail: joachim.schmidt@rwi-essen.de

Ruhr Economic Papers #148

Responsible Editor: Volker Clausen

All rights reserved. Bochum, Dortmund, Duisburg, Essen, Germany, 2009

ISSN 1864-4872 (online) - ISBN 978-3-86788-167-8

The working papers published in the Series constitute work in progress circulated to stimulate discussion and critical comments. Views expressed represent exclusively the authors' own opinions and do not necessarily reflect those of the editors.

Ruhr Economic Papers #148

Ansgar Belke, Matthias Goecke, and Martin Guenther

When Does It Hurt? The Exchange Rate "Pain Threshold" for German Exports



Bibliografische Informationen der Deutschen Nationalbibliothek

Die Deutsche Bibliothek verzeichnet diese Publikation in der deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über: *http://dnb.ddb.de* abrufbar.

ISSN 1864-4872 (online) ISBN 978-3-86788-167-8 Ansgar Belke, Matthias Goecke, and Martin Guenther¹

When Does It Hurt? The Exchange Rate "Pain Threshold" for German Exports

Abstract

This paper deals with the impact of the $f \in exchange rate on German exports in the period from 1995Q1 to 2008Q4. Our main aim is to identify "pain thresholds" for German exporters. We rely on a non-linear model according to which suddenly strong spurts of exports occur when changes of the EXR go beyond a kind of "play" area (analogous to a mechanical play). We implement an algorithm describing play-hysteresis into a regression framework. A unique "pain threshold" of the <math>f \in exchange rate$ does not exist, since the borders of the play area and, thus, also the "pain threshold" (as the upper border) depend on the historical path of the whole process. We come up with an estimate of a play area width of 24 US dollar cent per euro. At the end of our estimation period, the previous exchange rate movements had shifted the upper bound of the play area to about 1.55 US dollar per euro. In our interpretation, this is the current "pain threshold", where a strong spurt reaction of exports to a further appreciation of the euro is expected to start.

JEL Classification: C51, C63, E24, F41

Keywords: Exchange rate movements, play hysteresis, modelling techniques, switching regression, export demand

November 2009

¹ Ansgar Belke, University of Duisburg-Essen and IZA Bonn; Matthias Goecke, Justus Liebig University of Giessen; Martin Guenther, Justus Liebig University of Giessen. – We are grateful for valuable comments to Christian Dreger and Peter Winker. The usual caveats apply. – All correspondence to Ansgar Belke, University of Duisburg-Essen, Universitätsstr. 12, 45112 Essen, Germany, e-mail: ansgar.belke@uni-due.de.

"I'm not worried about a strong euro. I love a strong euro" – Peer Steinbrueck, former German finance minister, July 2007.

1. Introduction

European politicians and business persons are frequently concerned with the European currency. A leading pan-European business lobby said recently that the euro has reached its "pain threshold" and that the currency should be re-evaluated at the G7 meeting upcoming in autumn 2007. More concretely, it was BusinessEurope President Ernest-Antoine Seilliere who said to Jean-Claude Juncker, the chairman of Eurogroup, that he also agreed that the euro exchange rate had reached a "pain threshold" for European companies" (*Dow Jones International News* 2007).¹ Such kind of statements implicitly address the \$/€ exchange rate still is one of the most closely watched exchange rates in the world, much as the dollar/DM rate was in the past. Its gyrations, which are at times difficult to understand on purely economic grounds, are often perceived to be politically costly and, hence, closely observed and commented by analysts, business lobbies and governments (as, e.g. within the G-20).

Nevertheless, falling in love with a strong currency does not appear to be unusual for Germans. However, many would expect Peer Steinbrueck's fling (cited above) to be nasty, brutish and short (Munchau 2007). Since his statement in July 2007, the euro's exchange rate has risen even further (see Figure 1). In autumn 2007, it was assumed that if the euro continued to appreciate, Germany in particular would suffer from a sustained exchange-rate overshooting, as its economy remains as dependent as ever on a successful export sector. Even though the Germans tend to have a slightly higher exchange rate "pain threshold" than the French (because their exports are not as price sensitive as the French ones), Germany was nevertheless said to be not too far away from that threshold at that time. The question was raised what happened if the euro would rise above \$1.40. Germany's ability to improve its competitive position through a devaluation of the real exchange rate had run its course in

[&]quot;The exchange rate is a worry because we have gotten to the highest level of the euro since its creation," BusinessEurope President Ernest-Antoine Seilliere told a news conference. Seilliere said companies welcomed the ECB's decision to keep interest rates on hold on September 11, 2007, and praised the bank's actions over the summer of 2007, when it injected liquidity into the money market to keep it functioning amid a global credit crunch.

But BusinessEurope noted that the ECB had suggested it was only postponing a further rate rise. "Diverging monetary policies across the Atlantic have never been seen since the launch of the euro and could cause exchange rate volatility and an even more significant appreciation of the euro vis-à-vis the dollar and other currencies," it said. Seillière called on EU member states to intervene. In his words, "the euro cannot be a variable adjusting to reductions in the US foreign deficit" (<u>Strupczewski</u> 2007).

autumn 2007. With unemployment down sharply, conditions in the labor market were gradually returning to normal. The latest wage settlements and reports of capacity shortages at that time were a clear indication that Germany's competitive adjustment process had been completed. Hence, once the euro hits \$1.45, the guess was that it would be too strong even for Mr Steinbrueck's taste because it would endanger Germany's improvements in competitiveness (Munchau 2007). But at what level at all does the external value of the euro hit its "pain threshold", for instance, vis-à-vis the dollar?

In order to clarify issues in this respect, the paper proceeds as follows. In section 2, we motivate why it makes sense to investigate the question of a "pain threshold" exchange rate and give a brief historical overview of such claims referring to the $\ensuremath{\$/€}$ exchange rate. In section 3, we derive a simple model which serves to capture the non-linear hysteresis-type dynamics inherent in the relation between the exchange rate and exports. Taking this model as a starting point, we develop an algorithm describing play-hysteresis and implement it into a regression framework in section 4. In section 5, we estimate the exchange rate impacts on German exports to the US, differentiating between intervals of weak and strong reaction. Based on this regression we come up with an estimate of Germany's current "pain threshold" of the $\ensuremath{\$/€}$ exchange rate. Section 6 concludes.

2. Three episodes of \$/€ exchange rate "pain thresholds" and their main lessons

But at what level at all does the euro hit its "pain threshold", for instance, vis-à-vis the dollar? A closer look into the more recent episodes in which the fe rate reached "all-time highs" and, thus, at least local maxima might be helpful in this regard. According to Figure 1, the relevant periods are 2004 and somewhere in between 2007/2008 and the more recent months in 2009. Moreover, another important stylized fact is that in general German exports tend to move much slower than the fe exchange rate. If exports react to movements in the exchange rate, this reaction initially tends to be much less than proportional. Only if exchange rate changes are one-directional and steady (downward in the first part and upward in the second part of the sample period) also exports react more significantly.

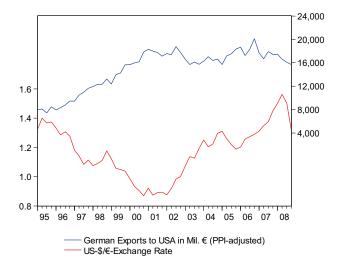


Figure 1 – \$/€ exchange rate and German exports to the US

Source: own calculation based on time series from Eurostat, WM/Reuters and OECD.

2.1 The history of local maxima of the \$/€ exchange rate

Figure 1 reveals that for three years before May 19, 2004, the exchange rate of the euro against the dollar has traveled a one-way street - upwards. From the perspective of business, the short correction in exchange rates which was taking place thereafter gave German industry some breathing room at best, but did nothing to solve the fundamental problems of a too high exchange rate. Since reaching its lowest level at the end of the year 2000, it has appreciated until May 19, 2004, in value against the dollar by approximately 40 percent.

From the start of 2004, the euro rushed from one all-time high to another within the same year. On January, 9th, the \$/€ rate had mounted to 1.28 in order to take values of nearly 1.36 at the end of the same year (December 31st: 1.35925). Since November 2003, the euro had re-valued in terms of the effective trade-weighted exchange rate in the midst of January 2004 by around five percent while the euro has appreciated by even ten percent in bilateral terms vis-à-vis the dollar since December 2003. This clear upward trend was interrupted only briefly when some members of the ECB council intervened verbally from January 12, 2004, on in favor of a lower euro ("brutal revaluation of the euro") (ECB Observer 2004). Thus, we feel legitimized to call this period episode one.

Indeed, on February 29, 2004, many central bankers still thought the present exchange rate was close to where it should be. In its monthly bulletin of January 2002, the ECB published a list of more than a dozen independent estimates for the equilibrium exchange rate of the euro. The average came to about \$1.17. Moreover, the exchange rate of 1.18 dollar per euro has often been called a "pain threshold" and a threat for German exporters closely corresponds to the starting exchange rate at the birth date of European Economic and Monetary Union in 1999 (ECB Observer 2004). The exchange rate prevailing at the end of February 2004 of around \$1.25 was thus said to be not that far off. Even though these estimates were at that time two years old, some ECB officials still looked to them as a rough guide. This means that their "pain threshold" was significantly higher than Mr. Schroeder's or Mr Raffarin's at that time. For instance, Munchau's (2004) guess was that the German and French governments would really start to squeal once the exchange rate hits \$1.30 whereas the ECB could have probably lived with \$1.40 or even \$1.50 quite comfortably. In this vein, the Federal Association of German Industry and some euro area politicians like the Belgian finance minister Didier Reynders already assessed a euro exchange rate vis-à-vis the dollar of 1.20 to 1.30 as a threat and a bottom line for interventions (ECB Observer 2004).

According to Figure 1, the $\ensuremath{\$}/\ensuremath{€}$ exchange rate exceeded its 1.40 threshold and thus reached its next local peak not earlier than on September 20th, 2007. What is more, it stayed above this threshold until October, 1st, 2008. From February, 27th, until August, 8th, 2008, the $\ensuremath{\$}/\ensuremath{€}$ exchange rate even reached values above a record high of 1.50. We would like to call this "episode two". Accordingly, in October 2007, leading EU business lobbies and the European Trade Union Council (ETUC) have urged the European Central Bank (ECB) to give markets a clear message that the continuing strengthening of the euro is no longer acceptable, and that there will not be a rise in interest rates in the foreseeable future (Europlatform 2007).

The background was that the euro rose to all-time highs above \$1.392 in the week before September 17, 2007, as investors priced in the likelihood of a rate cut by the U.S. Federal Reserve and a potential future rate rise by the ECB. For instance, the employers' federation BusinessEurope said that, by crossing 1.40 against the US dollar (and appreciating against the Chinese yuan and Japanese yen), the euro exchange rate had reached a "pain threshold" for European companies (Europlatform 2007). On 18 October 2007, the euro finally hit a new record level of 1.4310 against the dollar, thus breaking the previous record of 1.4283 per dollar set on 1 October 2007.

While France, concentrating more on the production of price-sensitive goods and services, at that time had repeatedly stated its concern over the effects of a continuously appreciating euro on the euro area's external competitiveness, Germany, the Netherlands and Austria seemed to be less troubled by the euro's high flight in autumn 2007.

Let us first take a step back to November 10th, 2007, because what ECB President Trichet said was intriguing. It was more than a signal that the ECB was opposed to further fast euro gains. He said that recent moves had been "undoubtedly sharp and abrupt" and he added that "brutal moves" were never welcome. The decisive watchword here was "brutal".² The use of this ECB code word clearly suggested that the ECB might have been going to launch a campaign of verbal intervention to dissuade currency speculators from pushing the euro any higher (MaBiCo 2007). However, as the euro approached the psychologically important \$1.50 level around 23 November 2007, German Finance Minister Peer Steinbrueck seemed to acknowledge that official acceptance of the moves in the foreign exchange market had limits. On November 21 he warned that there was a "pain threshold", even though he did not (could not?) define it, adding "I'm aware of the fact that there is a limit". He noted that the competitiveness of German companies ensured that the euro's strength was no cause of worry, but was unable to say whether that would be sufficient going forward.

The euro reached a next record high of \$1.5912 on April 10, 2008, which made German cars (and also French champagne) more expensive for American customers or forced exporters to squeeze their margins.³ But record sales figures from Europe's biggest automaker Volkswagen AG showed the trend. VW said on April 10th, 2008, it saw its best-ever quarterly car sales in the first three months of the year 2008, despite the dampening effect of the exchange rate. Sales to the U.S. fell slightly, though that was more than offset by surging demand in China and Brazil. Trade figures from 2007 show that euro nations, which include France and Germany, saw sales to the U.S. and Japan slip slightly but increased exports to most other major trading partners that year. Again, there was much evidence of weak reaction of exports to movements of the \$/€ exchange rate (see, among others, as a representative source from Associated Press, White 2008).

But the debate about exchange rate "pain thresholds" for EU exports continued with unfettered intensity also in 2008 because companies that rely on dollar-denominated sales -

² Also Guy Quaden, a governor of the ECB, has warned against "brutal" moves in the exchange rate (Evans-Pritchard 2007).

³ However, it also had an upside for Europe because it helps ease inflation by reducing the cost of dollarpriced oil imports.

such as Airbus - were feeling considerably more pain. For instance, Louis Gallois, chief executive of Airbus' parent European Aeronautic Defence & Space Co. warned that "we are at levels which are becoming unbearable". He argued that it could force the company to shift more of its costs into dollars by moving production outside of the euro area and making acquisitions in dollar-based countries. For Airbus, which sells its planes in dollars while many of its costs are in euros, each 10-cent rise in the euro against the dollar costs 1 billion euros (\$1.59 billion) (White 2008).

Furthermore, the euro currency zone's top official, Luxembourg Prime Minister Jean-Claude Juncker, said on April 10th, 2008, that the strong euro had not yet hit the "pain threshold" for European exporters. He did not see the strong euro hurting the European economy "for the time being". However, he clearly refused to say what level would hurt European exporters: "The export sector is developing quite well. The moment will come where the exchange rate level will start to cause serious harm to the European economy." (White 2008).

This debate was clearly overshadowed in the aftermath by the recent financial and economic crisis. Not earlier than on July 17, 2009, it became alive again. The recent appreciation of the euro against the dollar is not structural and therefore not worrying, the chairman of euro zone finance ministers, Jean-Claude Juncker said at that time. 'I don't think that we are facing a strengthening - in a structural sense - of the euro, so recent moves are not worrying,' he told Reuters. Before, the euro had firmed to \$1.4146 on July 9th, from \$1.3830 on July 8th, and has been rising steadily from \$1.2455 at the start of March 2009. European exporters said on the same occasion that the "pain threshold" for them is an exchange rate above \$1.40. However, the discussion even gained momentum in October 2009 when the \$/ \notin exchange rate climbed to values of around 1.50 and some ECB rhetoric against a "too high rate" started again (Strupczewski 2009).

2.2 Lessons from the episode of 2004: what is the significance of a strong euro (a weak dollar) for the German industry?

The exchange rate question has traditionally been of great importance for German industry. After all, more than a third of added value was exported in 2004.⁴ In 2008, this share increased to around 45 percent. It is true that this high export ratio is moderated by the fact

that Germany sent about 45 percent of its exports to the euro region in 2004, where exchange rates are stable.⁵ But there still remains a substantial proportion of German exports which is – at least potentially - sensitive to fluctuations of the \Re exchange rate. The share of Germany's exports destined for the United States, at 7.5 % in 2007, does not appear to be overly large. However, the exchange rate effect is amplified by competition with US-products on third markets and the fact that certain Asian emerging market economies which, in the past few years, have evolved into major export markets (accounting for 3.5 % of German exports at the last count), oriented their currencies very closely to the US dollar, at least in the past. Measured by a group of 19 major trading partners of Germany, the US dollar, including third-market effects, has a weight of around 15% from the point of view of the German economy (Deutsche Bundesbank 2008, pp. 34ff.).⁶

Business lobby groups argued that the massive drop in Germany's business with the US amounting to 10 % in 2003 proved how strongly exports react to the weakness of the dollar at that time. The price competitiveness of German industry had suffered badly in the years before 2004. German firm representatives and associations pushed the argument that the current euro-dollar exchange rate was at that time at about its long-term average and price competitiveness has to some extent remained unchanged fails to be convincing.

They also argued *that the frequently mentioned "pain threshold", which is supposed to generally apply to a specific and unique appropriate euro-dollar rate, simply does not exist.* This is because the "pain threshold", to adopt this expression, differs widely from company to company and is also highly product dependent (von Wartenberg, 2004). There is *heterogeneity of the exchange rate threshold across firms*, i.e. on the micro level. On the one hand, suppliers of niche products, such as in the field of specialized mechanical engineering or certain segments of the automobile business can perhaps shrug off the increase in value of the euro with comparative ease. On the other hand, firms with standard products which are exposed to the biting winds of international competition have a huge problem with a strong euro versus the dollar. Experience shows that when there is a sustained increase in the value of the euro against the dollar of around 10 percent, German exports drop – as a rule-of-thumb

⁴ Own calculations based on Bundesbank data published by Thomson Reuters Datastream. The share of exports in gross value added was 36.7 % in 2004 (annual average share in current prices, seasonally adjusted). In 2008, this share amounted to 44.67 %.

⁵ Own calculations based on Bundesbank data (annual average share in current prices, seasonally adjusted). In 2004, the share of German exports to the euro area in total German exports turned out to be 44.67 %. This value has slightly declined in 2008 to 43.03.

⁶ Own calculations based on Bundesbank data reveal that the respective values amounted to 8.87 in 2004 and 7.2 in 2008.

- by around 1 percent, von Wartenberg (2004) argued. In April 2004, many enterprises were only able to maintain their market shares by cutting prices. But there were limits, and the limits were visible. Above all, the main impact was felt by the sectors which are heavily export oriented, such as automobiles, mechanical engineering and pharmaceuticals.

The situation was no better with regard to medium-sized industry in Germany, quite the reverse, in fact. As a rule, medium-sized firms have few opportunities to adopt globalized strategies to cover themselves against undesirable exchange rate risks. So indeed there were quite a few companies which benefited from the strength of the euro which went along with the weakness of the dollar as, for instance, the tourism industry as are naturally all those sectors which derive a cost benefit from the cheap dollar but have their main market in the EU and/or Germany (Wartenberg 2004). Let us now discuss why - quite contradictory to the anecdotic evidence cited above - reactions of German exports to the US to \$/€ exchange rate movements tend to be rather weak unless a "pain" threshold is passed by the exchange rate.

2.3 Why is the reaction of German exports to small to medium-sized exchange rate movements so weak?

In contrast to its counterparts in most of the other euro area member countries, the German export industry has at least in the short run become much less vulnerable to exchange rate movements in the recent years. What are the reasons of *weak reactions* of German exports to exchange rate movements?

Hedging of exchange rate uncertainty

In the short run, i.e. in case of an only transitory appreciation of the euro, the choice of the invoice currency and the extent of cross-currency hedging play a role. Currently, around 80 percent of German exports are invoiced in euros and only 13 percent in US dollars. Moreover, three quarters of all foreign currency receivables from export business are hedged against exchange rate related losses for some time (Deutsche Bundesbank 2008). These exchange-traded or even tailor-made hedging deals are able to cushion the appreciation pressure only for a limited period. Even firms with a professional exchange rate management such as, for instance, Porsche, are hedged only for some years. Moreover, from the perspective of a single firm, hedging leads to additional fixed costs of exports (Deutsche Bundesbank, 2008, pp. 35ff.).

Many enterprises, especially most of the larger companies, have hedged their exchange rate risks, although as a rule only partially (around 30 to 50 %). Only a few companies take action to secure complete protection against foreign currency liabilities. In the case of most medium-sized firms, it is not usual practice to hedge foreign exchange risks. They simply lack experience in this field. Here the drop in the value of the dollar really hits home and heterogeneity of German exporting firms comes into the game again.

German export product line and price elasticity of exports

What is more, world demand of German products reacts only less than proportionally to a price increase since also the structure of German's product line is behind Germany's export success. The price elasticity of German exports has diminished markedly in recent years. Empirical studies show that, if domestic prices rise by 1% relative to foreign prices, real exports go down by 0.25%. This relatively small influence is due partly to the fact that the share of relatively price-inelastic goods in the range of German exports is quite high. Exports to non-euro area countries, in particular, respond relatively weakly to price competitiveness (Deutsche Bundesbank 2008).

Even more important: making up for a share of around 40% in 2007, machinery, equipment and vehicles dominate Germany's industrial production. Demand of these capital goods has been very strong globally until the current financial and economic crisis has fed itself into international trade. German firms are often highly specialized in these areas and in terms of technology maintained their position as the world market leader. As a consequence, importers are not able to or even do not want to switch to other suppliers even when the external value of the euro increases because switching costs would be too high for them. Foreign consumers are just stuck and "caught" in their relation to German suppliers. Even in the US, although it already headed into a record recession, the demand for capital goods kept quite stable until the late 2008 due to the sustained high corporate gains. Thereby, Germany displays a *more balanced export profile* than France with its focus on aeronautics and aerospace and Spain and Italy which export relatively many low tech-goods such as textiles and food.

Relief on the export side by cheaper imports

However, the current appreciation of the euro generally affects not only firms' export sales but also their costs by reducing the price of the imported intermediate inputs that go into the manufacture of exported goods. These imported inputs latterly made up 45% of exports, as against 31% in 1995. In addition, the prices of most commodities (including crude oil) in the world markets are quoted in US dollars. The appreciation-related cost relief, which has been particularly noticeable in Germany's energy bills, was a key factor in ensuring that, all in all, German exporters have coped relatively well with the strengthening euro in the past few years.

Adjusting intermediate inputs and outsourcing as a natural hedge

To some degree, exporters can offset exchange rate-related losses in price competitiveness by adjusting their intermediate inputs. Firms can, for instance, transfer their business at fairly short notice to suppliers from countries whose currencies have depreciated against the euro or otherwise provide cost advantages (examples are EADS, Boehringer Ingelheim and Böwe Syntec). Medium to long-term strategies are aimed more at restructuring production and revising the firm's internal policies for choosing production sites. In this way the share of imported intermediate inputs from low-cost countries can be increased at the expense of domestically generated value added – made more expensive by currency appreciation – or else manufacturing can be shifted partly to other, lower-cost countries in order to be able to sell the final products at competitive euro prices without any (major) losses in revenues (Axarloglou and Kouvelis 1999, Deutsche Bundesbank 2008).

An important element of this strategy is "natural hedging", which has been practised, for instance, by the German automotive industry (DaimlerChrysler) and its suppliers particularly in the past 15 years by establishing or at least envisaged (BMW) manufacturing capacity in the United States. This means not only that products are delivered to the local buyers without any exchange rate risk but that, if the euro appreciates, exchange rate-related losses from German exports to the USA are offset within the firm through exports to Europe. Such a hedge can also be achieved by buying equity stakes or existing manufacturing sites (Deutsche Bundesbank 2008).

Sunk costs

Recent research in international economics, employing theoretical analysis and assessment of firm level data clearly confirm that "sunk costs matter" (Godard, Goerg and Goerlich 2009). In a nutshell, this implies that setting up of global export networks coincides with substantial set up costs. These costs can to a large extent not be recouped once a firm leaves the export market or terminates its international customer-supplier relationships (the latter being especially significant in the German case of car producers which dominate German exports).

Examples of sunk costs of exporting are thought to be mainly those of information gathering on the new market (costs for market research), setting up new foreign distribution networks, marketing and possibly repackaging of the product to appeal to new consumers, paying for lawyers versed in the law of the foreign country, etc. While setting up a global export distribution network presupposes that the respective firm has covered these costs and got the knowledge, the value of this knowledge tends to depreciate rather quickly once the firm has left the export market (Roberts and Tybout 1997). The literature on German firm export decisions has found considerable persistence in export status over time. For example, Bernard and Wagner (2001) test for the role of plant characteristics and sunk costs in the entry decision, using a panel of German manufacturing plants. They find entry and exit among their German sample of 2.4 percent and 2.3 percent, indicating a huge impact of sunk costs.

All in all, the entry of German firms into export markets once included obtaining unknown information about consumption patterns and market potential, setting up distribution and service networks, bearing the costs of establishing a brand name through advertising, and bringing the foreign product into conformity with domestic health regulations. These costs are firm-specific and cannot be resold on exiting the market, at least in terms of their total value, being therefore regarded as irreversible or partially irreversible investments (Kannebley 2008). What is more, the great bulk of German firms which are willing and able to export to the US have entered the US market during the last peak of the world's business cycle in 2004 at the latest and, thus, are already in the US market. Hence, regarding the euro's appreciation, the market situation is currently such that German firms are currently prevented from exit rather than prevented from entry by sunk costs. Since significant sunk costs are usually associated with entry into each new export market, we would expect to see a high level of persistence in the firms' export activities on the US destination markets.

Price-setting of German exporting firms

Research in general has shown that German export prices display a weakening of cost passthrough and a *strengthening of pricing-to-market*. The results indicate that the price pick-up of German export goods compared with competitors' products in foreign markets resulting from an appreciation of the domestic currency has been smaller since German unification than it was in the 1980s. This implies that a permanently strong euro is being absorbed more through a reduction in the profit margin, thus at least temporarily lowering the return on capital. This will be especially valid for the German automobile and machinery sector. Enterprises are evidently now keener to avoid losses in sales volume following currency appreciation (and, thus, prefer an only weak reaction of export volumes) than they were in the 1980s. Tougher international competition in the wake of ongoing globalization, the establishment of the euro area or EU enlargement, but also advances in productivity and the deregulation of product and labor markets and the clearer prioritising of price stability by central banks worldwide, which pushed down inflation expectations, might be potential reasons for the *reduced price-setting range* of German firms (Deutsche Bundesbank 2008, pp. 44f.). Firms cannot roll over a permanent appreciation of the euro on to prices. On the whole, export prices calculated in euro, as a weighted average of all export sectors, are adjusted to include only around one-eighth of each respective change in exchange rates (Stahn 2007).

Overall assessment

Seen on the whole, we feel legitimized to argue that there are some sectors in which German exporters should display only a *weak reaction* to movements in the exchange rate. At the same time, however, it should be clear that firms are *heterogeneous* in their ability to withstand an adverse development of the exchange rate for a while. What is more, if there is a permanent upward trend in the external value of the euro, there will be a *threshold exchange rate* which will induce exporters in all sectors of the German economy *to react strongly* with reductions in exports. This is partly because whether or not firms will tolerate and will be able to cope with reduced export revenues owing to currency appreciation depends, among other things, on whether they see the shifts in exchange rates as being temporary or permanent.

What do these considerations imply for the future German export perspectives? Traditionally, experts make heavy weather of answering this type of question. It has become a stylized fact that different sectors of the economy are to a different degree sensitive to exchange rate movements, depending on their export shares and shares of imported raw materials and intermediates. It is also quite sure that the threshold has been steadily increasing in the recent years. While there was some talk of around 1.30 dollar per euro for some time, in the meantime many German exporters start to lament not until around 1.45 dollar per euro is reached. In the following, we try explain why this is so. For this purpose, let us now turn to our model of hysteresis in exports.

3. Hysteresis in exports

3.1 The emergence of a 'band of inaction' from a microeconomic perspective

Hysteresis in foreign trade generally occurs if sunk market-entry costs exist (Baldwin 1989, 1990). Potentially exporting firms must expend market-entry investments, e.g. in setting up a distribution and service network or for introductory sales promotion, in order to sell in the export market. These expenses are firm-specific and cannot be recovered if the firm later wants to leave the market; i.e. the entry costs are sunk. If the prices on the export market do not change in proportion to the exchange rate, the exporting firms have to bear revenue changes in their home currency when the exchange rate alters. If the foreign currency appreciates (i.e. the home currency depreciates), a market entry may become profitable, namely under consideration of the sunk entry costs.

After a firm has entered the export market, the foreign currency may again depreciate. However, as long as the variable costs are covered, once in the market, it is still profitable for the firm to sell. A previous entry is not fully reversed due to entry costs which have to be considered as sunk ex post. Analogous effects would result in the case of sunk exit costs. The resulting reaction pattern to exchange rate changes for a single exporting firm h is depicted in Figure 2. The exchange rate x is defined as the home currency price of foreign exchange. An exchange rate x_c exactly compensates for the variable unit costs of the firm. A devaluation (i.e. an increase of e) increases the unit revenues finally changed back into the exporters home currency. Since the sunk entry costs must be covered, a market entry requires an entry exchange rate xin which exceeds the variable costs (xc). A previously active firm will exit if the losses are larger than the sunk exit costs. Hence the exit trigger x_{out} must be located below x_c. Seen on the whole, thus, the entry and the exit triggers generally differ in a situation with sunk entry and exit costs. The microeconomic path-dependence occurs discontinuously if entry or exit trigger rates are passed.⁷ Combining both triggers results in a 'band of inaction'. Inside this band, the current exchange rate does not unambiguously determine the current state of the firm's activity.

⁷ According to Krasnosel'skii and Pokrovskii (1989), p. 263, this pattern corresponds to a so-called "non-ideal relay".

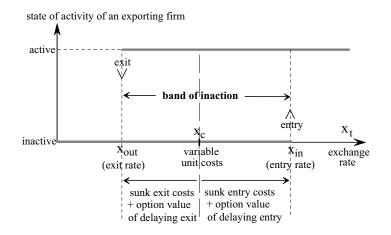


Figure 2 – Discontinuous micro hysteresis loop: export activity of a single firm h

Uncertainty, e.g. about the future exchange rate, reinforces the hysteresis characteristics via option value effects.⁸ Since an exit will destroy the market entry investments, an exporting firm may stay when the home currency devalues even if it is currently losing money. If the devaluation would prove to be only transitory, an immediate exit could turn out to be a mistake. Hence, under uncertainty the opportunity of a "wait-and-see"-strategy shifts the exit trigger to the left. Analogously waiting with an entry in a situation with uncertainty shifts the entry trigger to the right. Thus, the "band of inaction" is widened by uncertainty.

Exchange rate changes will result in substantial home currency revenue changes of the exporting firm if the price elasticity of demand in the export market is high. By implication, for a low price elasticity of demand exchange rate changes do not result in severe unit revenue changes. Thus, the band-of-inaction will be the wider, the lower is the demand elasticity, higher is the value of the sunk entry and exit costs and the higher is the uncertainty about the future situation of the exporter.

On a microeconomic level hysteresis occurs via a band of inaction, i.e. differences between both trigger/thresholds. Belke and Goecke (2005) focus on the shape and the location of a *macroeconomic* hysteresis loop, i.e. the *problem of aggregation*.⁹ Aggregation is not

⁸ For a comprehensive treatment of uncertainty effects see Dixit, Pindyck (1994). For an empirical application to trade see, based on macro time series, Parsley and Wei (1993) and Roberts and Tybout (1997) and Campa (2004) who work with micro panel data.

⁹ For an adequate aggregation procedure of micro to macro hysteresis see Amable et al. (1991), Cross (1994), and Belke and Goecke (2005).

trivial if heterogeneity regarding the value of sunk exit/entry costs and/or the level of uncertainty about future market situation and/or the elasticity of demand exist, i.e. if the entry and exit trigger rates are different for different exporting firms. In this (realistic) case of heterogeneity the transition from the micro to the macro level leads to a change of the hysteresis characteristics: the aggregate hysteresis loop shows no discontinuities (as known from magnetics). However, a pattern not very different to a "band of inaction" is remaining.

Belke and Goecke (2005) show that even the macro behavior can be characterized by areas of weak reactions which are – corresponding to mechanical play – called "play"-area.¹⁰ Persistent aggregate (export) effects do not result from small changes in the forcing (exchange rate) variables, as far as the changes occur inside a play area. However, if changes go beyond the play area, sudden strong reactions (and persistence effects) of the output variable (i.e. exports) occur.¹¹ The specific realization of the exchange rate reached instantly after the complete passing of the play area can be denoted as a "*pain threshold*", since, passing this rate, the reaction of exports to changes in the exchange rate becomes much stronger. However, play-hysteresis is in two aspects different to the micro-loop. First, as mentioned the play-loop shows no discontinuities. Second, analogous to the play in mechanics (e.g. when steering a car) the play area is shifted with the history of the forcing variable (exchange rate): Every change in the movement of the forcing variable starts with traversing a play area. Not until the play is passed, a spurt reaction will result, if the forcing variable continues move in the same direction.

In the following section, a straightforward empirical framework to test for a play-type impact of the exchange rate on exports is presented. We use an algorithm developed in Belke and Goecke (2001) describing play-hysteresis and implement it into a regression framework.

4. An empirical model of play-hysteresis

4.1 A linear approximation of exchange rate impacts on exports

In order to convey an impression of the simplified linearized play-dynamics – as theoretically developed by Belke and Goecke (2001, 2005) and described briefly in the introduction – we first illustrate the implications based on the interpretation of Figure 3. Here, we assume a constant width p of the play area to simplify issues. We start with an initial situation in point

¹⁰ For play hysteresis, see Krasnosel'skii and Pokrovskii (1989), pp. 6 ff. See Goecke (2002) for different phenotypes of hysteresis.

A (x_0) located on the upward leading (right) spurt line, a decrease in the forcing variable x results in entering the play area. A weak 'play' reaction results until the entire play area p is passed. The downward leading spurt line starts in point G with x_5 (with: $p=x_0-x_5$). In the play area only a weak reaction of the dependent variable y follows from changes in x. A further decrease of x would induce a strong response of y along the (left) downward leading spurt line.

Alternatively, one may think of an increase in x staring from x_0 (A) up to x_1 (point B) and a subsequent decrease to x_2 (C). The corresponding reaction of y initially evolves along the right spurt line. With an increase along the spurt line from $A \rightarrow B$ the relevant play area is vertically shifted upward from line GA to line DB ($p=x_1-x_3$). The decrease from x_2 (C) to x_3 (D) again takes place in a play area.¹² This play area is penetrated by an extent 'a' which is explicitly depicted. Consider next a decrease $x_2 \rightarrow x_3 \rightarrow x_4$ ($C \rightarrow D \rightarrow E$). After having passed the entire play p in point D (x_3), a strong reaction on the downward leading (left) spurt line up to point E results. In this situation, a decrease (i.e. a devaluation of the foreign currency) suddenly leads to a strong decrease of the exports. Thus, x_3 is a kind of "pain threshold". However, this "pain threshold" is not a constant trigger level as in the micro ("relais") loop, but path-dependent, since the play lines are vertically shifted by movements along the spurt lines. The play area is shifted in the opposite direction as before, so that for a subsequent increase again to x_3 (F) the reaction is described by line EF.

¹¹ For a empirical macro analysis of 'spurts' in investment implicitly based on *micro*-threshold models see Darby et al. (1999). See Pindyck (1988), pp. 980 f., Dixit and Pindyck (1994), pp. 15 f., for a non-technical description of 'spurts' based on a microeconomic sunk cost mechanism.

¹² In the case of 'mechanical play' there even would not be any reaction of y inside the play area. See Krasnosel'skii and Pokrovskii (1989), p. 8.

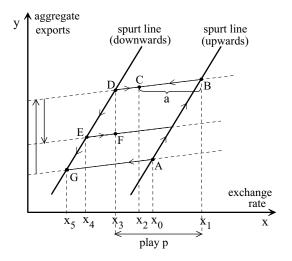


Figure 3 – Linear play-hysteresis and spurt areas

4.2 An algorithm capturing linear play

In the following, we present a *hybrid* version of a play algorithm which was originally developed by Belke and Goecke (2001, 2005) for the analysis of employment hysteresis and finally adapt it to our main research question, i.e. the identification of an exchange rate "pain threshold" for German exports. The change in the forcing variable x (Δx) may occur either inside the play area p inducing a weak reaction or on a spurt line resulting in a strong reaction of the dependent variable y (Δy). The movement of x inside the play area is Δa (and cumulated as a) and analogously the movement in the spurt area is Δs . We start with a special case, when Δx enters a play area. Let this change be denoted as Δx_j^s . according to Figure 3 this corresponds to the trajectory $B \rightarrow C \rightarrow E$. In the past the movement of x has led to j changes between the left and the right spurt line. The new change Δx_j^s may enter the play area to an extent of Δa_j or even pass the entire play p and enter the opposite spurt line by the fraction Δs_j . Due to starting from a spurt line the cumulated movement inside the play area a_j equals the change Δa_j . The trajectory $B \rightarrow C$ in Figure 3 might serve as an illustration of the distance "a". These considerations are usefully summarized by the formal expression:

(1)
$$\Delta x_j^s = a_j + \Delta s_j$$
 with: $\Delta s_j = \begin{cases} sign(\Delta x_j^s) \cdot (|\Delta x_j^s| - p) & \text{if } (|\Delta x_j^s| - p) > 0 \\ 0 & \text{else} \end{cases}$

The change in the independent variable $y(\Delta y)$ induced by Δx_j^s is composed of the weak play reaction $(B \rightarrow C)$ and – by occasion – additionally of a strong spurt reaction $(D \rightarrow E)$. Let the parameter α denote the weak play and $(\alpha + \beta)$ the strong spurt reaction:

(2)
$$\Delta y_j^s = \alpha \cdot a_j + (\alpha + \beta) \cdot \Delta s_j$$
 with: $|\alpha| < |\alpha + \beta|$

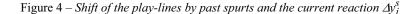
The play line is shifted vertically by spurt movements. The cumulated vertical displacement V_{j-1} of the relevant play line as a result of all previous movements on both spurt lines is:

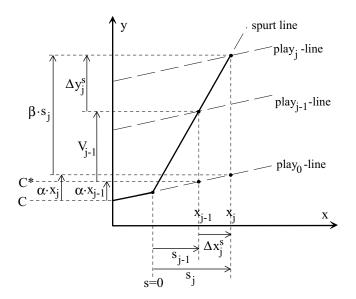
(3)
$$V_{j-1} = \beta \cdot \left[\sum_{i=0}^{j-1} \Delta s_i\right] = \beta \cdot s_{j-1}$$
 with: $s_{j-1} \equiv \sum_{i=0}^{j-1} \Delta s_i$

The dependent variable is determined by the shift V induced by past spurts and the current reaction Δy_i^s :

$$\begin{array}{ll} (4) & y_{j} = C^{*} + V_{j-1} + \Delta y_{j}^{S} = C^{*} + \beta \cdot \sum\limits_{i=0}^{j-1} \Delta s_{i} + \alpha \cdot a_{j} + (\alpha + \beta) \cdot \Delta s_{j} \\ \\ \Rightarrow & y_{j} = C^{*} + \beta \cdot \sum\limits_{i=0}^{j} \Delta s_{i} + \alpha \cdot \Delta x_{j}^{S} \\ \\ \Rightarrow & y_{j} = C^{*} - \alpha \cdot \sum\limits_{i=0}^{j-1} \Delta x_{i} + \beta \cdot \sum\limits_{i=0}^{j} \Delta s_{i} + \alpha \cdot (\sum\limits_{i=0}^{j-1} \Delta x_{i} + \Delta x_{j}^{S}) \\ \\ \Rightarrow & y_{j} = C + \alpha \cdot x_{j} + \beta \cdot s_{j} \end{array}$$
 with: $C \equiv C^{*} - \alpha \cdot \sum\limits_{i=0}^{j-1} \Delta x_{i}$

Figure 4 conveys an impression of the transformations of equation (4). As a result, the play hysteresis loop is captured by a simple linear equation based on an artificial variable s_j . The spurt variable s_j summarizes all preceding and present spurt movements leading to a shift of the current relation between x and y.





Of course, an accumulation by means of an index j describing the past changes between the spurt lines can be substituted by an accumulation over an explicit time index t. Additional non-hysteretic regressors (e.g. z_t) may be included to arrive at a suitably generalized presentation of the hysteretic process:¹³

(5)
$$y_t = C^* + \beta \cdot \sum_{k=0}^t \Delta s_t + \alpha \cdot \Delta x_t + \lambda \cdot z_t$$

 $\Rightarrow y_t = C + \alpha \cdot x_t + \beta \cdot s_t + \lambda \cdot z_t.$

5. Empirical analysis

The hypothesis of hysteresis in foreign trade was initially tested by Baldwin (1990) and Krugman and Baldwin (1987) based on macroeconomic time series for the U.S. economy by employing dummy variables associated with periods of exchange rate appreciation. Parsley and Wei (1993) came up with empirical models that try to capture the asymmetric effect of real exchange rate fluctuations and real exchange rate volatility on the imported quantities. However, they cast doubt on the validity of the hysteresis hypothesis as an explanation of the

¹³ For a detailed description of the algorithm calculating the artificial spurt variable s_t and for the implementation into an EVIEWS-batch program see Belke and Goecke (2001).

persistent U.S. trade deficits in the 1980s. Based on micro firm level data, and thus with a focus on the discontinous micro-hysteresis (however, emphasizing the heterogeneity of firms) Roberts and Tybout (1997) and Campa (2004) discovered sunk cost hysteresis to be an important factor in determining export market participation. Agur (2003) has found empirical evidence of structural breaks in the exchange rate import volume relation as a consequence of exchange rate extrema. Using a threshold cointegration model of Brazilian sectoral foreign trade data, Kannebley (2008) was able to identify an asymmetric (i.e. hysteretic) adjustment in 9 of 16 sectors.

Compared to existing studies of hysteresis in foreign trade, our approach is closer to the original concept of a *macroeconomic* "hysteresis loop", since (i) it is not based on the discontinuous non-ideal relay interpretation as in the microeconomic firm level case and since (ii) the path-dependent structural breaks in the macroeconomic relations are not added to the system as an exogenous information. On the contrary, in our approach the structural shifts are explicitly determined by the history of the exchange rate and the exports and are simultaneously estimated together with the (path dependent) relation of exports to the exchange rate.

5.1 Data and variables

In order to check for the empirical relevance of the hysteresis model for German exports, we now estimate equation (5) which generalizes hysteretic behavior of exports dependent on movements in the exchange rate. In our empirical application, we use German exports to the US as the dependent variable both at the aggregated level and disaggregated by product groups (SITC) and the \$/€ exchange rate as the hysteretic input variable. To be parsimonious as possible, we employ foreign real GDP, a linear trend and seasonal dummies as additional non-hysteretic explaining variables.

The exact definitions of the time series used are as follows. Nominal exports to the US are denoted in current € and taken from the Eurostat database. Our export series is deflated by means of the PPI taken from the OECD (Main Economic Indicators) database. Exchange rates are spot rates as documented by WM/Reuters and are deflated with the above mentioned PPI indices. The real GDP time series is also extracted from the Eurostat database. Our estimation period ranges from 1995Q1 to 2008Q4.

5.2 Characteristics of the regression model

The 'play' regression model displays the following characteristics: It is based on linear segments, where adjacent sections are linked (by so called 'knots', in Figure 3 these knots are e.g. points B, D, E in the case of the input path $x_1 \rightarrow x_3 \rightarrow x_4$.). The position of the linear partial function and the transition between the sections is determined by the past path of an input variable x. The model is a special case of a switching regression setting, since adjacent sections are joined.¹⁴ The positions of the knots are a-priori unknown and depend on the magnitude of the play area p, which has to be estimated by us. The knots divide the relation between x and y into sections with two different slopes (for $\beta \neq 0$). The number of parameters describing the complex dynamic is low: only the basic slope α , the slope difference β and the play width p are to be determined.

We suppose that the standard regression model assumptions hold: the error term is independently, identically and normally distributed with a constant finite variance over all sections, and the regressors are measured without any error and are not correlated with the error term.

Our model is non-linear in its parameters, since the knots are not known a-priori and since the play width p has to be estimated in order to determine the spurt variable s. The assumptions regarding the error term and the regressors guarantee that the OLS-estimators are best linear unbiased estimators (BLUE) in a standard regression model and allow the OLS-estimator to be regarded as a maximum likelihood estimator. If the knots are a-priori unknown, discontinuities and local maxima in the likelihood function result. However, if the adjacent sections are joined in a switching regression models the OLS-/ML-estimator leads to consistent and asymptotically normally distributed estimates.

However, the finite sample properties of the play regression model remain problematic: The parameter estimates are not even approximately normally distributed for small samples and local maxima in the likelihood function may occur.¹⁵ Moreover, the assumptions of the standard regression model may not be fulfilled. For example non-stationary variables might imply non-finite variances. Furthermore, the play dynamic represents a mixture of the short-term and the long-term dynamics, which obstructs the

¹⁴ For linear spline functions and linear switching regressions see Poirier (1976), p. 9 and p. 117.

¹⁵ See Hujer (1986), pp. 231 ff., Poirier (1976), pp. 108 ff., pp. 117 ff. and p. 129, Hudson (1966) and Hinkley (1969) for the small sample properties of the ML- (OLS-) estimates in a model with unknown but continuous switches.

application of standard cointegration analysis. Unfortunately, we are not aware of any technique which is directly applicable to our specific model and therefore delivers the distributions and the respective critical values of the relevant estimators. Thus, any solution to these problems is clearly beyond the scope of this paper.

We are now endowed with the necessary equipment to conduct an empirical application to German exports to the US of the play regression model. In accordance with section 2, we call the \Re exchange rate value at which a transition from a play to a spurt reaction of aggregate German US-exports occurs a "pain threshold". The empirical identification of these path-dependent switching thresholds must of course been seen with the necessary caution, due to the above mentioned imponderabilities with respect to inference.

We apply our algorithm to estimate the play-hysteretic model by minimizing the residual sum of squares i.e. by the OLS-method. For this purpose, we enact a grid search over the width of a time invariant play parameter $p_t = p = \gamma$. For every given point of the γ -grid the algorithm "recognize" the switches and for the given γ the spurt variable s is computed from the actual input (exchange rate) series. The size of γ is predetermined for each grid point. The respective OLS-estimation of α and β for each grid point is straightforward since s enters the regression equation (5) in a linear way. The final OLS-estimate of the play parameter is determined by choosing from the grid the γ -value with the minimum of the residual sum of squares (i.e. the maximum of the R-squared).

5.3 Hypotheses

In order to derive the hypotheses to be tested, we have to take into account that the width of the play p_t was not addressed up to now. In a simple case p_t is defined as a constant parameter, i.e. a time-invariant width of the play area $p_t = p = \gamma$, which has to be estimated. According to our description of the play dynamics in section 3, the relevant hypotheses to be tested must refer to the equations:¹⁶

(5')
$$y_t = C + \alpha \cdot x_t + \beta \cdot s_t(\gamma) + \lambda \cdot z_t$$
 with: $|\alpha| < |\alpha + \beta|$

(6)
$$p_t = \gamma$$
 with: $\gamma \ge 0$.

¹⁶ However, it appears to be quite straightforward how to generalize the model in a way where the play width p_t is not constant and determined by other variables. For instance, the higher an uncertainty variable u_t is, the more important are option value effects of waiting, and thus the play area is expected to widen. See eq. (12) in the appendix for this generalization.

If we want to check whether play is relevant at all, we have to test the hypothesis (H1) $\beta \neq 0$ against the alternative $\beta = 0$. According to Belke and Goecke (2001, 2005), the hypothesis to be tested might even be more restrictive, since a weaker play and a stronger spurt reaction (both with the same sign) are assumed as the "typical" hysteresis pattern.

If one (for the moment) neglects possible limitations on inference resulting from, for instance, non-finite variances of the variables, the OLS-estimates of the respective equations can – according to section 4 – be regarded as asymptotically unbiased (i.e. consistent) and asymptotically normally distributed. However, since the small sample properties remain problematic we refrain from further conclusions concerning exact inference and for the moment only convey a broad-brush view of the basic pattern of the results. Therefore, the following regression results serve more as a first illustration of the working of our algorithm and the main direction of results rather than a basis for exact inference. While there is evidence that our OLS approach is quite robust, we leave the latter task for future research.

5.5 Empirical results

Evidence for total exports

As a baseline we start with a standard regression of German total exports on the US on the PPP adjusted (PPI) \$/ \in exchange rate (RER) excluding play or spurt effects. In terms of the play model, thus, the restriction $\beta=0$ is applied. We display the corresponding results in Table 1.

Dependent Variable: total German Exports to the USA
Method: Least Squares
Sample: 1995Q1 2008Q4
Included observations: 56

	Coefficient	Std. Error	t-Statistic	Prob.
С	8529.546	2809.039	3.036464	0.0038
RER	-8460.892	971.8874	-8.705630	0.0000
US-GDP(-1)	17.76498	4.674016	3.800795	0.0004
D1	-326.4904	326.3298	-1.000492	0.3220
D2	-417.7190	324.2338	-1.288327	0.2037
D3	-399.7934	324.1422	-1.233389	0.2233
TREND	-186.2570	94.28472	-1.975474	0.0539
R-squared	0.946960	Mean depende	nt var	14878.20
Adjusted R-squared	0.940466	S.D. dependent var		3512.028
S.E. of regression	856.9225	Akaike info crite	erion	16.46104
Sum squared resid	35981496	Schwarz criterio	on	16.71421
Log likelihood	-453.9091	Hannan-Quinn	criter.	16.55919
F-statistic	145.8062	Durbin-Watson	stat	1.123707
Prob(F-statistic)	0.000000			

All variables are (according to the empirical realizations of the t-statistics) highly significant and display the theoretically expected sign. The US GDP variable enters with a lag of one quarter (best fit for lagged data and to avoid problems of reverse causation) whereas the real \$/€ exchange rate is considered contemporaneously (otherwise J-curve-effects might occur which might severely interfere with the hysteretic sub-system).

As a next step, we conduct a one-dimensional grid search in order to estimate γ for the simple case with constant play (see Figure 5). The sequence of R² dependent on different realizations of γ shows an absolute maximum at γ = 0.24 with an R² = 0.967891. The minimum of the R² plot at the realization γ =0 (R²=0.946960) exactly corresponds to the R-squared of the standard model as stated in Table 1. The respective OLS-estimates of the spurt/play regression with an artificial spurt-variable (SPURT) based on this constant width p= γ =0.24 of the play area is presented in Table 2.

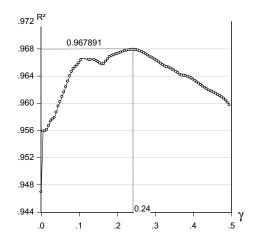


Table 2 – *Regression with constant play* $p = \gamma = 0.24$

Dependent Variable: total German Exports to the USA Method: Least Squares Sample: 1995Q1 2008Q4 Included observations: 56

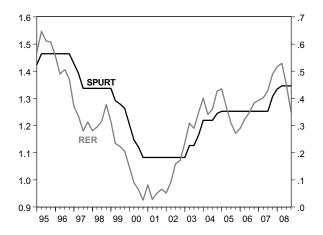
	Coefficient	Std. Error	t-Statistic	Prob.
С	9658.763	2217.468	4.355762	0.0001
RER	-2574.477	1300.446	-1.979688	0.0535
SPURT	-9385.067	1677.813	-5.593632	0.0000
US-GDP(-1)	18.51232	3.676795	5.034907	0.0000
D1	-417.7233	257.0544	-1.625038	0.1107
D2	-440.4651	254.9211	-1.727849	0.0904
D3	-424.5425	254.8551	-1.665819	0.1023
TREND	-227.2783	74.48162	-3.051469	0.0037
R-squared	0.967891	Mean depender	nt var	14878.20
Adjusted R-squared	0.963208	S.D. dependent var		3512.028
S.E. of regression	673.6494	Akaike info criterion		15.99486
Sum squared resid	21782566	Schwarz criterion		16.28420
Log likelihood	-439.8561	Hannan-Quinn	criter.	16.10704
F-statistic Prob(F-statistic)	206.6993 0.000000	Durbin-Watson	stat	1.637163

Again, all estimated coefficients display the theoretically expected sign. Note that the spurt-variable seems to substitute the effects of original the real \$/€ exchange rate. With respect to the hypothesis (H1) $\beta \neq 0$ the estimated coefficient of the spurt variable is $\beta = -9385.067$ with an empirical realization of the t-value of -5.59. However, since the

small sample properties of our regression model are still basically unknown, the t-values are most probably not student-t-distributed. Nevertheless, it seems fair to say that this high empirical t-realization (which is about three times as high as the 5% critical value in the case of the standard student-t-distribution) represents a strong hint at the relevance of play.

Figure 6 finally conveys a graphical impression of the time sequence of the artificial variables SPURT (right scale) which captures the strong impact of further exchange rate changes after passing through the play area (i.e. after passing a kind of "pain threshold") and of the original real $\ell \in \ell$ exchange rate (RER, left scale). Spurt was calculated based on the estimated play width $p=\gamma=24$. The path of the spurt variable across time shows of course similarities to the original RER path. However, periods of inaction exhibiting no variation of the spurt variable due to play effects also emerge. Expressed differently, short-term variability in the real exchange rate series is filtered via the play/spurt algorithm. Only large changes in the RER are mirrored by the spurt series.

Figure 6 – Real exchange rate and the resulting spurt variable ($\gamma = 24$)



If the spurt variable undergoes some changes, this simultaneously shifts the borders of the play area. The up to now most recent shift of this border corresponds to the exchange rate extremum reached in 2007/2008. Thus, this upper bound exchange rate of the play area should be still valid today, i.e. at the time this paper has been written. To be more specific, it is equivalent to the last RER maximum of our quarterly series and amounts to *about 1.55 US dollar per euro* in terms of the nominal exchange rate. We interpret this figure as the *current "pain threshold"*, where a strong spurt reaction of the export to a further appreciation of the

euro is expected to start again. What is more, *this point estimate is also identical with today's value*, i.e. in late October 2009, although it is derived from the data and estimations available at the end of our estimation period (December 2008). The reason is that no comparable exchange rate maximum which could have shifted the play area and thus the "pain threshold" has occurred in the meantime from January to October 2009.

Since we abstract from exchange rate volatility and, thus, calculate a constant play, we have been able to specify an upper border of the play based on an inspection of the previous exchange rate path. In our case, the latter corresponds with the absolute maximum of the exchange rate reached in 2008. Although in 2009 the euro (slightly) depreciated again for some time, this play has remained valid throughout 2009. The devaluation of the euro was too small to wipe out the effect of the previous maximum of the \$/€ exchange rate. In addition, we would like to point at the fact that inflation was – not at least due to the financial crisis and its transatlantic spillovers – moderate and nearly the same on both sides of the Atlantic since the start-of-year until today. Hence, inference with respect to the nominal \$/€ exchange rate.

Due to option value effects the path-dependent hysteresis effects will be the more important and, technically speaking the play area will be the wider, the more uncertain the economic environment is for the exporting firms. Vast uncertainty related to the financial crisis was highly important in 2008, but was substantially reduced in the meantime. Economic and financial uncertainty is now still slightly but steadily falling, at least according to all indicators of the financial fear factor as the DAX volatility index (VDAX) and the implied volatility on the S&P100. Obviously, the global policy response to the financial and economic crisis has calmed stock markets 'as the fears of an economic Armageddon have subsided'. Also political uncertainty has dropped after many world leaders had clarified the details of their stimulus packages (Bloem and Floetotto 2009). However, we did not explicitly incorporate uncertainty variables and correspondingly a time-variable play width in our spurt regression. Therefore, our estimation of the play width and thus the estimation of the "pain threshold" may be interpreted as a kind of a "maximum" estimate of the play width and of the upper border of the play area (interpreted as a "pain threshold"). Corrected for the reduction of uncertainty, *the current threshold may even take a value below* 1.55 \$/€.

Evidence on the sectoral level

Since sunk costs, uncertainty effects and the market structure are different for specific branches, we expect differences in the play-width for different sectors. Strikingly, the overall

empirical pattern continues to hold if we use German exports disaggregated by commodities (SITC-Groups) and conduct the same regression exercises once again. For instance, we find a slightly higher play γ =0.25 for the SITC-Group 7 consisting of machinery and transport equipment. 63% of the German Exports to the US (in the average of the period from 1995-2009) consist of goods from this SITC-Group. The quite detailed results stemming from this exercise can be found in Table 3, Table 4 and Figure 7.

Table 3 – *Standard regression without play / spurt (restriction* $\beta = 0$) *German Export to USA SITC-Group 7 (Machinery and transport equipment)*

Included observations: 5	6			
	Coefficient	Std. Error	t-Statistic	Prob.
С	9078.931	2342.025	3.876530	0.0003
RER	-7185.570	810.3073	-8.867709	0.0000
US-GDP(-1)	10.61871	3.896943	2.724883	0.0089
D1	-297.9861	272.0762	-1.095230	0.2788
D2	-502.5581	270.3286	-1.859064	0.0690
D3	-602.8220	270.2523	-2.230590	0.0303
TREND	-114.7852	78.60952	-1.460194	0.1506
R-squared	0.915305	Mean depende	nt var	9512.007
Adjusted R-squared	0.904934	S.D. dependen	t var	2317.200
S.E. of regression	714.4558	Akaike info criterion		16.09739
Sum squared resid	25011910	Schwarz criterie	on	16.35056
Log likelihood	-443.7269	Hannan-Quinn	criter.	16.19554
F-statistic	88.25789	Durbin-Watson	stat	0.929366
Prob(F-statistic)	0.000000			

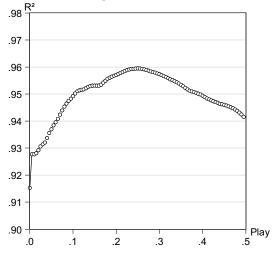
Dependent Variable: German Exports to USA SITC 7 Method: Least Squares Sample: 1995Q1 2008Q4 Included observations: 56

Table 4 – Regression with constant play $p = \gamma = 0.25$ German Export to USA SITC-Group 7

Sample: 1995Q1 2008Q4 Included observations: 56				
	Coefficient	Std. Error	t-Statistic	Prob.
С	10599.61	1649.952	6.424195	0.0000
RER	-1793.115	935.9492	-1.915825	0.0614
SPURT	-8908.247	1231.151	-7.235703	0.0000
US-GDP(-1)	11.35980	2.724951	4.168810	0.0001
D1	-374.4015	190.4089	-1.966302	0.0551
D2	-527.4124	188.9259	-2.791636	0.0075
D3	-629.4185	188.8771	-3.332424	0.0017
TREND	-155.7742	55.22047	-2.820951	0.0069
R-squared	0.959490	Mean depende	nt var	9512.007
Adjusted R-squared	0.953583	S.D. dependent	tvar	2317.200
S.E. of regression	499.2327	Akaike info crite	erion	15.39559
Sum squared resid	11963199	Schwarz criterio	on	15.68492
Log likelihood	-423.0764	Hannan-Quinn	criter.	15.50776
F-statistic	162.4150	Durbin-Watson	stat	1.619534
Prob(F-statistic)	0.000000			

Dependent Variable: German Exports to USA SITC 7 Method: Least Squares Sample: 1995Q1 2008Q4 Included observations: 56

Figure 7 – R^2 resulting from a one-dimensional grid search with constant play over γ German Export to USA SITC-Group 7 with an maximum $R^2=0.959490$ by play= $\gamma=0.25$



For the SITC-Subgroup 78 (Road vehicles including air-cushion vehicles) we able to identify a play of γ =0.23. These goods cover on average 26% of the German exports to the

USA over the observed period. We present the corresponding results in Table 5, Table 6 and Figure 8.

Table 5 – Standard regression without play / spurt (restriction $\beta = 0$) German Export to USA SITC-Group 78 (Road vehicles -including air-cushion vehicles)

Dependent Variable: German Exports to USA SITC 78 Method: Least Squares Sample: 1995Q1 2008Q4 Included observations: 56

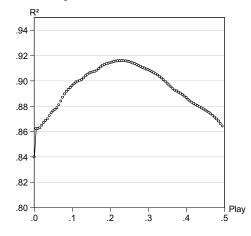
	Coefficient	Std. Error	t-Statistic	Prob.
С	1071.942	1786.412	0.600053	0.5512
RER	-3566.143	618.0731	-5.769775	0.0000
US-GDP(-1)	2.808519	2.972447	0.944851	0.3494
D1	-165.3654	207.5299	-0.796827	0.4294
D2	-411.4928	206.1969	-1.995630	0.0515
D3	-651.0405	206.1387	-3.158265	0.0027
TREND	0.156824	59.96050	0.002615	0.9979
R-squared	0.839841	Mean depender	nt var	4079.398
Adjusted R-squared	0.820230	S.D. dependent var		1285.308
S.E. of regression	544.9611	Akaike info criterion		15.55577
Sum squared resid	14552147	Schwarz criterio	on	15.80894
Log likelihood	-428.5617	Hannan-Quinn	criter.	15.65393
F-statistic	42.82447	Durbin-Watson	stat	0.620867
Prob(F-statistic)	0.000000			

Table 6 – Regression with constant play $p = \gamma = 0.23$ German Export to USA SITC-Group 78

Included observations: 56				
	Coefficient	Std. Error	t-Statistic	Prob.
С	1533.249	1310.301	1.170150	0.2477
RER	715.3072	792.3940	0.902716	0.3712
SPURT	-6596.096	1001.942	-6.583311	0.0000
US-GDP(-1)	3.310292	2.178451	1.519563	0.1352
D1	-237.0261	152.3909	-1.555382	0.1264
D2	-425.0628	151.0394	-2.814251	0.0071
D3	-666.1360	151.0001	-4.411494	0.0001
TREND	-27.15483	44.11253	-0.615581	0.5411
R-squared	0.915835	Mean depender	nt var	4079.398
Adjusted R-squared	0.903561			1285.308
S.E. of regression	399.1472	Akaike info criterion 14.9		14.94810
Sum squared resid	7647287.	Schwarz criterion 15.2		15.23744
Log likelihood	-410.5468	Hannan-Quinn	criter.	15.06028
F-statistic	74.61567	Durbin-Watson	stat	0.988597
Prob(F-statistic)	0.000000			

Dependent Variable: German Exports to USA SITC 78 Method: Least Squares Sample: 1995Q1 2008Q4 Included observations: 56

Figure 8 – R^2 resulting from a one-dimensional grid search with constant play over γ German Export to USA SITC-Group 78 with an maximum $R^2=0.915835$ by play= $\gamma=0.23$



Since the two sectors explicitly covered by us represent about three quarters of the German exports to the US, it turns out that the play of both groups of goods taken together nearly completely drives the play for the exports. Due to high market-entry cost emerging

especially in the machinery and automobile branches, it is not surprising that we could find the best fit of a play-spurt dynamics especially for these goods. As expected play-spurt dynamics are not important in sectors with homogenous goods where sunk market entry costs are negligible, we did not find any plausible play for the commodity groups 1 to 6 (SITC, the respective regression results are not explicitly stated in this paper).

6. Conclusions

The paper deals with the impact of the exchange rate on the relationship among German exports and its main determinants. Our main aim has been to identify an exchange rate "pain threshold" for German exporters. We rely on a nonlinear path-dependent model in which suddenly strong spurts of exports occur when changes of the exchange rate go beyond a so called "play area" (which is similar to the phenotype of play in mechanics). We capture this on-linear dynamics in a simplified linearized way and implement an algorithm describing linear play hysteresis into a suitable regression framework. Our non-linear model including play displays a much better performance than the standard linear model. Thus, the implications of some hysteretic macro models concerning the dynamics of aggregate labor demand are corroborated empirically for trade variables as well.

Moreover, we are able to show that the frequently mentioned notion of an exactly identified unique "pain threshold" of the \$/€ exchange rate simply does not exist. Instead, we come up with an estimate of a path-dependent play area width of 24 US dollar cent per euro for total German exports to the US. Taking into account that the borders of the play area and, thus, also the "pain threshold" (as the upper border) "today" depend on the historical path of the RER until "today", the location of the play area and the "pain threshold" is dependent on the observation point in time.

For instance, at the end of our estimation period, our estimation results imply a "pain threshold" of 1.55 \$/€ which continued to hold in October 2009. Compared to the more recent laments of business German business representatives and also to the more recent implicit assessment by the ECB this threshold is slightly but not substantially higher. On October 22nd, 2009, the dollar hit the 1.50 level, implying that we are currently not too far away from this threshold – especially if we take into account that macroeconomic uncertainty has shrunk slightly but steadily since the turn-of-the-year 2008/09 . However, laments appeared to be quite contained these days until very recently. The interesting question then is why this time "it was so different". We feel legitimized to preliminarily argue that maybe the awareness of

appreciation of the euro has been overlaid and dominated by the public reception of the economic and financial crisis.

But as in recent episodes of local maxima of the \$/€ exchange rate, also the ECB is also now apparently becoming increasingly aware that a stronger euro must absolutely be avoided. Further euro strengthening in the remainder of the year will have a significant impact on 2010 economic growth, for instance, in export dependent Germany, and make the ECB's own pessimistic forecasts for this year even more probable. Hence, this may become a new era of the ECB's rhetoric on exchange rates because (i) it will be the euro area which will have to bear the burden of the global adjustment and (ii) voting majorities in the ECB Council have changed in the meantime to the benefit of former weak currency countries which are inclined to enact central bank interventions in the FX markets in order to weaken their home currencies.

What does all this lead us? If, as a result of global imbalances, the external value of the euro increases even further, the demand for German exports will fall dramatically and (units of) German firms reduce or even stop trading internationally, then re-entrance into international trade will be severely hampered, even if the euro will devaluate again in the future. So, once there will be a zero entry in any export good category, the concern is that it is going to be hard for exporters in this goods category to re-establish their export nodes and get back in.¹⁷ A German firm may even be likely to decide not to re-establish global trading networks again, or, at a minimum, it might take some time before it well be capable of doing so. Hence, the "ever rising euro" may have consequences that go well beyond the prediction of any standard economic model incorporating a unique exports equilibrium, when the presence of global trading networks and sunk costs of for German exporters is acknowledged.

References

- Agur, I. 2003. Trade-volume Hysteresis An Investigation Using Aggregate Data. Research Memorandum WO no. 740, De Nederlandsche Bank, Amsterdam, August.
- Amable, B., Henry, J., Lordon, F. and Topol, R. 1991. Strong Hysteresis: An Application to Foreign Trade. OFCE Working Paper / Document de travail no. 9103, Observatoire Français des Conjonctures Economiques, Paris.

¹⁷ For a similar argument with respect to global production networks see Godart, Goerg and Goerlich (2009).

- Axarloglou, K., Kouvelis, P. 1999. Research Notes, Multinational Corporations and the Hysteresis in Foreign Direct Investment Flows. International Advances in Economic Research, Vol. 5 (2), pp.271.
- Baldwin, R. 1989. Sunk-Cost Hysteresis. NBER Working Paper, no. 2911, National Bureau of Economic Research, Cambridge/MA.
- Baldwin, R. 1990. Some Empirical Evidence on Hysteresis in Aggregate U.S. Import Prices. in: Gerlach, St., Petri, P.A. (eds.), The Economics of the Dollar Cycle, Cambridge/MA, pp. 235-268.
- Belke, A., Goecke, M. 2001. Exchange Rate Uncertainty and Employment: An Algorithm Describing 'Play'. Applied Stochastic Models in Business and Industry, Vol. 17(2), pp. 181-204.
- Belke, A., Goecke, M. 2005. Real Options Effects on Employment: Does Exchange Rate Uncertainty Matter for Aggregation?. German Economic Review, Vol. 62, pp. 185-203.
- Bernard, A.B., Wagner, J. 2001. Export Entry and Exit by German Firms. Weltwirtschaftliches Archiv, Vol. 137(1), pp. 105-123.
- Bloem, N., M. Floetotto, M. 2009. The Recession Will Be Over Sooner Than You Think. VoxEU, January, http://www.voxeu.org/index.php?q=node/2785.
- Campa, J. M. 2004. Exchange Rates and Trade: How Important is Hysteresis in Trade?. European Economic Review, Vol. 48(3), pp. 527-548.
- Cross, R. 1994. The Macroeconomic Consequences of Discontinous Adjustment: Selective Memory of Non-Dominated Extrema. Scottish Journal of Political Economy, Vol. 41, pp. 212-221.
- Darby, J., Hughes Hallet, A., Ireland, J. and Piscitelli, L. 1999. The Impact of Exchange Rate Uncertainty on the Level of Investment. Economic Journal, Vol. 109, C55-C67.
- Deutsche Bundesbank 2008. Macroeconomic Effects of Changes in Real Exchange Rates. Monthly Report, Frankfurt/Main, March, pp. 33-46.
- Dixit, A., Pindyck, R.S. 1994. Investment under Uncertainty. Princeton, NY.
- Dow Jones International News 2007. Business Lobby Says Euro Has Reached "Pain Threshold". October 3.
- ECB Observer 2004. Liquidity on the Rise Too Much Money Chasing too Few Goods, Part 1: A Case Against ECB FX Market Interventions. ECB Observer Report, No 6, Frankfurt/Main, 2 February.
- Europlatform 2007. Businesses and Trade Unions Fear Strong Euro. Eurotelegraph, October 9.
- Evans-Pritchard, A. 2007. Dollar Respite as Saudis Stick to Peg, for Now. Telegraph.co.uk, September 26.

- Godart, O., Goerg, H., Goerlich, D. 2009. Back to Normal The Future of Global Production Networks after the Crisis. Kiel Policy Brief, No. 9, Kiel Institute for the World Economy, September.
- Goecke, M. 2002. Various Concepts of Hysteresis Applied in Economics. Journal of Economic Surveys, Vol.16, n.2, pp.167-88.
- Hinkley, D.V. 1969. Inference about the Intersection in Two-Phase Regression. Biometrica, Vol. 56, pp. 495-504.
- Hudson, D.J. 1966. Fitting Segmented Curves Whose Join Points Have to Be Estimated. Journal of the American Statistical Association, Vol. 61, pp. 1097-1125.
- Hujer, R. 1986. Ökonometrische 'Switch'-Modelle: Methodische Ansätze und empirische Analysen. Jahrbücher für Nationalökonomie und Statistik, Vol. 201/3, pp. 229-256.
- Kannebley, Jr. S. 2008: Tests for the Hysteresis Hypothesis in Brazilian Industrialized Exports: A Threshold Cointegration Analysis. Economic Modelling, Vol. 25, pp. 171–190.
- Krasnosel'skii, M.A., Pokrovskii, A.V. 1989. Systems with Hysteresis. Berlin.
- Krugman, P. Baldwin, R. 1987. The Persistence of the US Trade Deficit. Brookings Papers on Economic Activity, Vol. 1, pp. 1-43.
- MaBiCo 2007. Have We Reached the Pain Threshold for EUR/USD?. MaBiCo Financial Company, News/Foreign Exchange, http://www.mabico.com/en/news/20071109/foreign_exchange/article94637/
- Munchau, W. 2004. A European Exchange Rate Policy. Financial Times, FT.com site, February 29.
- Munchau, W. 2007. The Case for an Intelligent Exchange Rate Policy for the Euro Area. EuroIntelligence, July 16.
- Parsley, D.C. and Wei, S.-J. 1993. Insignificant and Inconsequential Hysteresis: the Case of U.S. Bilateral Trade. Review of Economics and Statistics, Vol. 75, pp. 606-613.
- Pindyck, R.S. 1988. Irreversible Investment, Capacity Choice and Uncertainty. American Economic Review, Vol. 78, pp. 969-985.
- Poirier, D.J. 1976. The Econometrics of Structural Change With Special Emphasis on Spline Functions. Amsterdam.
- Roberts, M.J., Tybout, J.R. 1997. The Decision to Export in Colombia: an Empirical Model to Entry with Sunk Costs. American Economic Review, Vol. 87, pp. 545-564.
- Stahn, K. 2007. Has the Export Pricing Behavior of German Enterprises Changed? Empirical Evidence from German Sectoral Export Prices. Journal of Economics and Statistics, Vol 227(3), pp. 295 ff.
- Strupczewski, J. 2007. UPDATE 1-EU Business Group Says Euro Near Pain Threshold. Reuters, September 17.

- Strupczewski, J. 2009. Recent Euro Moves not Worrying Juncker. Thomoson Reuters, July 7
- von Wartenberg, L. 2004. Quo Vadis U.S. Dollar Consequences for Transatlantic Trade. Statement, Konrad-Adenauer-Stiftung e.V. (ed.), Washington, May 19.
- White, A. 2008. EU, Airbus Differ over Pain Threshold from Strong Euro. The Associated Press, April 15.

Annex: An algorithm to calculate the spurt variable

In the following we present a detailed algorithm in the spirit of Belke and Goecke (2001) to calculate the extent of the current penetration into the play area a_t and the cumulated spurts s_t . We define four dummy variables describing the current state of the system. For reasons of simplification, some special cases which become relevant if the change in x *exactly* meets the border between play and spurt (e.g. in point D) are not explicitly included below. However, these cases are taken into account in the Eviews version of the algorithm.

A dummy M_t^{\downarrow} indicates a movement starting in a left (downward leading) spurt line. Analogously, M_t^{\uparrow} indicates a start on a right (upward leading) spurt line. Corresponding to Figure 3 e.g. for point E, $M_t^{\downarrow}=1$ holds, and for point B $M_t^{\uparrow}=1$ is valid.

(7)
$$M_{t}^{\downarrow} = \begin{cases} 1 & \text{if } \Delta s_{t-1} < 0\\ 1 & \text{if } (\Delta s_{t-1} = 0) \land (\Delta x_{t-1} = 0) \land (\Delta a_{t-1} = 0)\\ 0 & \text{else} \end{cases}$$
$$M_{t}^{\uparrow} = \begin{cases} 1 & \text{if } \Delta s_{t-1} > 0\\ 1 & \text{if } (\Delta s_{t-1} = 0) \land (\Delta x_{t-1} = 0) \land (\Delta a_{t-1} = 0)\\ 0 & \text{else} \end{cases}$$

Due to the path dependence, information on the current reference spurt line has to transmitted to subsequent periods: The dummies B_t^{\downarrow} and B_t^{\uparrow} indicate the last (and maybe the current) spurt line. In Figure 3 e.g. for point F, $B_t^{\downarrow}=1$ is valid, and $B_t^{\uparrow}=1$ holds for point C.

(8)
$$B_{t}^{\downarrow} = \begin{cases} 1 & \text{if } \Delta s_{t-1} < 0 \\ 1 & \text{if } (\Delta s_{t-1} = 0) \land (B_{t-1}^{\downarrow} = 1) \\ 0 & \text{else} \end{cases}$$
$$B_{t}^{\uparrow} = \begin{cases} 1 & \text{if } \Delta s_{t-1} > 0 \\ 1 & \text{if } (\Delta s_{t-1} = 0) \land (B_{t-1}^{\uparrow} = 1) \\ 0 & \text{else} \end{cases} \text{ with: } B_{t}^{\uparrow} = 1 - B_{t}^{\downarrow}$$

Now, we calculate the extent a_t to which the play area p_t is penetrated. We first define an auxiliary variable b_t . Play penetration a_t is calculated based on a comparison of b_t and the play width p_t . (9) $b_t = B_t^{\downarrow} \cdot (1 - M_t^{\downarrow}) \cdot (a_{t-1} + \Delta x_t) + B_t^{\uparrow} \cdot (1 - M_t^{\uparrow}) \cdot (a_{t-1} - \Delta x_t)$

(10)
$$a_t = \begin{cases} b_t & \text{if } 0 < b_t \le p_t \\ \Delta x_t & \text{if } (M_t^{\downarrow} = 1) \land (\Delta x_t > 0) \land (\Delta x_t < p_t) \\ -\Delta x_t & \text{if } (M_t^{\uparrow} = 1) \land (\Delta x_t < 0) \land (-\Delta x_t < p_t) \end{cases}$$

Finally, we define changes in the spurt variable (Δs_t) induced by changes in the input variable (Δx_t) :

$$(11) \quad \Delta s_{t} = \begin{cases} b_{t} \cdot [B_{t}^{\downarrow} \cdot (1 - M_{t}^{\downarrow}) - B_{t}^{\uparrow} \cdot (1 - M_{t}^{\uparrow})] & \text{if } b_{t} < 0\\ (b_{t} - p_{t}) \cdot [B_{t}^{\downarrow} \cdot (1 - M_{t}^{\downarrow}) - B_{t}^{\uparrow} \cdot (1 - M_{t}^{\uparrow})] & \text{if } b_{t} > p_{t}\\ \Delta x_{t} & \text{if } [(M_{t}^{\downarrow} = 1) \land (\Delta x_{t} < 0)] \lor [(M_{t}^{\uparrow} = 1) \land (\Delta x_{t} > 0)]\\ \Delta x_{t} - p_{t} & \text{if } (M_{t}^{\downarrow} = 1) \land (\Delta x_{t} > p_{t})\\ \Delta x_{t} + p_{t} & \text{if } (M_{t}^{\uparrow} = 1) \land ((-\Delta x_{t}) > p_{t}) \end{cases}$$

The width of the play p_t was not addressed up to now. In a simple case p_t is defined as a constant parameter $p_t=p=\gamma$ witch has to be estimated. However, it is easy to generalize the model in a way where the play width p_t is determined by other variables. For instance, the higher an uncertainty variable u_t is, the more important are option value effects of waiting, and thus the play area is expected to widen. In technical term this can be expressed in a simple linear way as a function of, e.g., an uncertainty proxy variable u_t :

(12)
$$p_t = \gamma + \delta \cdot u_t$$
 with: $\gamma, \delta \ge 0$ and $u_t \ge 0 \implies p_t \ge 0$

Table A.1: Implementation	of the algorithm into an	FVIEWS-batch program
1 able 74.1. Implementation	or the argorithm muo an	L vill w S-baten program

'INPUT AREA	
GENR s_up=1	'set 1 for a maximum as an initial extremum (else 0)
!an = 73.3	'first estimation quarter (time of the first extremum in a
spurt area)	
!en = 96.1	'last estimation quarter
!n = 24*4+1	'number of sample point (calculated from !an to !en)
!g = 10	'precision of the grid search for the constant play
component	
!m = 0	'minimum of the grid search for the constant play component
!b = 20	'maximum of the grid search for the constant play component
!h = 10	'precision of the grid search for the variable play
component	
$\mathbf{y} = 0$	'minimum of the grid search for the variable play component
!v =30	'maximum of the grid search for the variable play component
GENR w = WRDM	
	determination of the uncertainty realisation
ST11= "BAI"	'dependent variable
	IP91(-2) PIOE(-4) TREND D1 D2 D3" 'independent variables of
the regression	EA

SMPL 69.1 98.4 GENR dw=na GENR d spurt=na GENR play=na GENR spurt=na GENR bs_do=na GENR s_do=na GENR bs_up=na GENR pb=na GENR pc=na GENR pa=na GENR punkt_do=na GENR punkt_up=na GENR dw = w - w(-1)C=0 matrix(!g,!h) R_2m =0 matrix(!g,!h) C_11m = 0 matrix(!g,!h) $C \ 12m = 0$ matrix(!g,1) P_CONSTA =0 matrix(1,!h) P_VARIA =0 SMPL !an !an GENR bs_up=s_up GENR s_do=1-s_up GENR bs_do=1-s_up SMPL !an-1 !an GENR pa=0 GENR pb=0 GENR pc=0 GENR d_spurt=0 GENR spurt=0 'END OF INITIALISATION 'START OF GRID SEARCH 'LOOP FOR P_CONSTA FOR !0=1 TO !g FOR !1=1 TO !h 'LOOP FOR P_VARIA SMPL !an !en GENR spurt=0 GENR play = !m+((!0-1)/(!g))*(!b-!m) + (!y+((!1-1)/(!h))*(!v-!y))*u $P_CONSTA(!0,1) = !m+((!0-1)/(!g))*(!b-!m)$ $P_VARIA(1,!1) = !y+((!1-1)/(!h))*(!v-!y)$ IF @MIN(play)>0 THEN 'LOOP FOR THE DETERMINATION OF THE SPURT VARIABLE FOR !2=1 TO !n SMPL !an+!2 !an+!2 GENR punkt_do=(pa(-1)=play(-1))*(pa(-1)<>0)*s_up(-1)+(pb(-1)=play(- $1))*(pb(-1) <> 0)*bs_up(-1)$ GENR punkt_up=(pa(-1)=play(-1))*(pa(-1)<>0)*s_do(-1)+(pb(-1)=play(- $1))*(pb(-1) <> 0)*bs_do(-1)$ GENR s_do=(pa(-1)<>play(-1))*(pb(-1)<>play(-1))*((d_spurt(-1)<0)+(s_do(-1)=1)*(d_spurt(-1)=0)*((dw(-1)=0)*(pa(-1)=0))) + punkt_do GENR s_up=(pa(-1)<>play(-1))*(pb(-1)<>play(-1))*((d_spurt(-1)>0)+(s_up(-1)=1)*(d_spurt(-1)=0)*((dw(-1)=0)*(pa(-1)=0))) + punkt_up GENR bs_do=(pa(-1)<>play(-1))*(pb(-1)<>play(-1))*((d_spurt(-1)<0)+(d_spurt(-1)=0)*(bs_do(-1))) + punkt_do GENR bs_up=(pa(-1)<>play(-1))*(pb(-1)<>play(-1))*((d_spurt(-1)>0)+(d_spurt(-1)=0)*(bs_up(-1))) + punkt_up GENR $pb=bs_do*(1-s_do)*(pa(-1)+dw) + bs_up*(1-s_up)*(pa(-1)-dw)$ GENR $pc=s_do^*(dw>0)^*dw + s_up^*(dw<0)^*(-dw)$ GENR pa=pc*(pc<=play) + bs_do*(1-s_do)*(pb>0)*(pb<=play)*pb + bs_up*(1s_up)*(pb>0)*(pb<=play)*pb GENR d_spurt=s_do*((dw<0)*dw+(dw>play)*(dw-play)) + s_up*((dw>0)*dw+((dw)>play)*(dw+play)) + bs_do*(1-s_do)*((pb<0)*pb+(pb>play)*(pb-play)) + $bs_up*(1-s_up)*((pb<0)*(-pb)+(pb>play)*(play-pb))$ GENR spurt=spurt(-1)+d_spurt NEXT ENDIF

c=0 SMPL !an !en IF @MEAN(spurt)=0 THEN EQUATION eq1.LS %ST11 %ST12 ELSE EQUATION eq1.LS %ST11 spurt %ST12 'OLS ESTIMATION ENDIF GENR EC = RESID $R_2m(!0,!1) = @R2$ $C_{11m(!0,!1)} = c(1)$ $C_{12m(!0,!1)} = c(2)$ c=0GENR RESID=na GENR EC=na NEXT 'END OF GRID SEARCH NEXT 'SEARCH FOR HIGHEST R² coef(2) c_und_d scalar r2_max=0 FOR !i=1 TO !g FOR !j=1 TO !h IF $(R_2m(!i,!j) > r2_max)$ THEN r2_max=R_2m(!i,!j) c_und_d(1)=p_consta(!i,1) c_und_d(2)=p_varia(1,!j) ENDIF NEXT NEXT

Transcriptions:

 $\begin{array}{l} a_t = pa \; ; \; B_t^{\downarrow} = bs_do \; ; \; B_t^{\uparrow} = bs_up \; ; \; b_t = pb \; ; \; M_t^{\downarrow} = s_do \; ; \; M_t^{\uparrow} = s_up \; ; \; p_t = play \; ; \; s_t = spurt \; ; \; \Delta s_t = d_spurt \; ; \end{array}$

 $u_t = u$; $x_t = w$; $\Delta x_t = dw$; $y_t = BAI$; $\gamma = c_und_d(1)$; $\delta = c_und_d(2)$.

Comments:

In order to apply the batch program, some information has to be delivered in the **'INPUT AREA**, since the starting point has to be characterized, due to the path dependence of the system. It is necessary to start in a spurt area (with either $M_t^{\uparrow}=s_up=1$ or $M_t^{\downarrow}=s_do=1$). Therefore, the sample has to be truncated on occasion and in the **'INPUT AREA** the variable s_up has to be set to 0 or 1.