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by

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Fundamental Uncertainty, Portfolio Choice, and Liquidity Preference Theory

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Abstract

One of Keynes' core issues in his liquidity preference theory is how fundamental uncertainty affects the propensity to hold money as a liquid asset. The paper critically assesses various formal representations of fundamental uncertainty and provides an argument for a more boundedly rational approach to portfolio choice between liquidity and risky assets. The choice is made on the basis of individual beliefs which are subject to mental representations of the underlying economic structure. Self-consciousness arises when the agent is aware of the fact that beliefs are dispersed among agents due to the absence of a "true" model. Responding to this fact by increasing liquidity preference is rationalized by the higher ex post performance of choice. Moreover, we analyze the case that the portfolio is partially financed by debt. It is explored how fundamental uncertainty affects the volume of the portfolio and hence money and credit demand as well as the probability of debt failures.

Keywords: liquidity preference, portfolio choice, self-confidence, self-consciousness, fundamental uncertainty, bounded rationality, Keynes, Knight.

JEL Classification: G11, D81, E41, B31

1 Introduction

According to Keynes (1936) uncertainty plays a crucial role for holding liquidity, especially money. Uncertainty is understood not as risk which could be presented as a singleton probability measure on the set of events, but as *fundamental* uncertainty about the underlying structures, economic relationships, the inferences that could be drawn from past experience, etc. Such fundamental uncertainty arises due to the uniqueness of an event, the novelty of an economic activity or technology as well as to the lack of knowledge about the underlying economic causal relationships or the fact that knowledge is inconclusive for probabilistic inferences. Some investment decisions are unique in the sense that they are not repetitive and have no long record of experience regarding the distribution of returns. Most investment decisions incorporate a specific "new" element so that accumulated knowledge is of limited use to form expectations about future outcomes. The agent has simply no objective basis to determine reasonable probability measures. In a similar fashion also Knight (1921) called for fundamental uncertainty in the sense that the agent doubts his own probability measures since they are based on vague and subjective knowledge. It is widely discussed that this fundamental uncertainty as opposed to risk and also to ambiguity requires a new route in the theory of decision making (for example Dequech (2000*a*), Dequech (2000*b*), de Carvalho (1988), Fontana and Gerrard (2004), Rosser (2001), Wray (2006)).

Some economists claim that fundamental uncertainty is an omnipresent and unavoidable phenomenon since the economic process evolves in historical time, the unknown underlying economic structures may change over time and produce therefore a non-ergodic trajectory of data (Davidson (1987), de Carvalho (1988)). This would exclude any Bayesian rationality. It should be noted, however, that non-ergodicity is not proven to be an overall empirically relevant phenomenon and that it should not be invoked as a "nihilistic" argument against any form of expectation formation (Rosser (2001)). Also Keynes' main point was to think about how a rational agent behaves in presence of uncertainty, not to disprove any rationality. But the point is not (only) the question of non-ergodicity. It is typical that among economic agents as well as among economists there exist different and partially conflicting views about reality, and the existing empirical data does not clearly rule out most of them and does not give a clear evidence for only one point of view. Thus, *beliefs are dispersed* which reflects fundamental uncertainty, and it is a matter of rationality that agents will respond to this fact in some way.

While Knight emphasizes the lack of self-confidence in own probability measures, which is more close to the problem of ambiguity, Keynes points out the uncertainty about inferences in case of inconcludent or missing knowledge (see Hoogduin (1987) for a detailed discussion). Or in other words: it is the uncertainty regarding the weight of empirical evidence for probability judgements versus the uncertainty regarding the weight of an argument or conclusion. The Keynesian view is a broader perspective which also includes self-consciousness about the own expectations. The latter could also be related to econometric estimation risk where the "true model" is known but its parameters are estimated from data in an unbiased way. As we will discuss later, this would open ways to deal with estimation risk in a proper econometric way. Keynesian fundamental uncertainty with absence of a ficticious "true" model is more difficult to deal with.

In Keynes' view holding liquidity is a kind of hedging instrument against fundamental uncertainty. The interest rate indicates the marginal willingness to waive for this kind of hedging. In the presence of fundamental uncertainty the existence of money (as well as the possibility to create money via credit) and the liquidity preference have an important impact e.g. on investment behavior which results in non-neutrality of money (Runde (1994), Davidson (1988)).

Understanding fundamental uncertainty and holding liquidity as a response to it is therefore an important and broadly discussed issue from a micro- as well as from a macro-perspective which deserves a closer look from a bounded rationality perspective. We will first briefly review different ways how fundamental uncertainty is incorporated into decision theory and put forward an argument why a boundedly rational approach is more appropriate to explain behavior (chapter 2). In chapter 3 we provide a simple model how an agent allocates his financial ressources to risky asets and riskless liquidity. It is shown how this portfolio choice is affected by fundamental uncertainty and the degree of self-confidence or self-consciousness. The rationale for adopting heuristic modifications of the portfolio approach is that it provides a higher ex post performance than naive rational decision making. In chapter 4 we analyse the case that the portfolio is partially financed by debt. We show how (the response to) fundamental uncertainty affects the debt size and the probability of debt failures. Chapter 5 concludes.

2 On Formal Representations of Knightian and Keynesian Uncertainty

At a first sight it is hard or even impossible to integrate fundamental uncertainty into the logic of rational economic decision making. It is especially incommensurable with expected utility theory. Expected utility theory (EUT), however, has been challenged by a large and even growing body of robust empirical evidence which contradicts the predictions of EUT (Camerer (1995)). As a response, different types of non-expected utility theories have been developped where some of them also capture ambiguity and uncertainty (Gilboa (1987), Gilboa and Schmeidler (1989), Camerer and Weber (1992), Chateauneuf (1991), or recently Hill (2009)). These approaches represent fundamental uncertainty e.g. by non-additive probability measures or multiple priors, and they represent the response of the decision maker to uncertainty in terms of preferences, e.g. ambiguity aversion. Almost all approaches are probabilistic in that uncertainty is represented as ambiguity: Subjective probability measures may be unreliable and the decision maker has not perfect confidence in his measures or he assigns different probability measures a different degree of (im)plausibility. In addition to these representations the theory has also consider the attitude to ambiguity (Ghirardato and Marinacci (2002), Ghirardato et al. (2004)). These approaches usually have an axiomatic foundation, and many economists accept them as a reasonable answer to the problem of Keynesian or Knightian uncertainty (see Basili and Zappia (2009), Fontana and Gerrard (2004), or Dequech (2000*b*) for a critical discussion).

One of the most prominent concepts is the multiple prior approach. While the subjective risk is expressed as a singleton probability measure, fundamental uncertainty is represented by a set of different priors which also may be taken into consideration as an appropriate description of the situation. The agent is therefore uncertain about the expected utility, so that we need further (ambiguity) preferences to describe how the agent responds to this kind of uncertainty, i.e. to the set of different priors. A reasonable assumption is that the decision should be in some sense *robust* against belief errors. One example is maxmin behavior where the alternative with highest expected utility under the most pessimistic prior is chosen. Another approach is to relax the axiomatic foundations to allow for non-additive measures. Fundamental uncertainty could then be interpreted as decision weights or distortions of probabilities. The non-additivity allows for a consistent description of choice behavior without referreing to second-order probabilities (Basili and Zappia (2009)). These concepts has been widely used to create new insights into investment behavior (Nishimura and Ozaki (2007)), the dynamic allocation of income to consumption and savings (Miao (2003)), or the behavior of agents on financial markets (Epstein and Wang (1994), overview in Basili (2001)), especially in portfolio decisions (Orszag and Yang (1995), Ma et al. (2008)).

A similar, but complete non-probabilistic approach is the Fuzzy Set theory (Zimmermann (1996)). Fundamental uncertainty is represented as vague or fuzzy knowledge about possible states of the world. The "degree of possibility" of a state or a parametrization of a model is captured by a membership function which assigns a certain "weight" to each state. The main advantage is that strong requirements of a probabilistic theory such as the existence of a σ-algebra of the set of states are not neccessary. Also this approach has been used to characterize agent's behavior under fundamental uncertainty (e.g. Arnold et al. (2000), Cherubini (1997)).

One key element of uncertainty is that agents do not know the "true" model which generates the economic data. Moreover, it is not possible by principle to know the "true" model. This is in opposition to rational expectations where we have common knowledge about the underlying structure where all parameters are known or could be learned in a consistent way. From an epistomological point of view things are even worse: A "true model" is an *oxymoron* since a model is per se an abstraction, an explanatory device which is constructed by an observer, communicated between observers, and – in the best case – does not contradict the observed data. Whether a model is "true" is an undecisable question as a matter of principle, hence rational decisions must be made without reference to "truth". This epistomological banality is a point for Keynesian fundamental uncertainty: knowledge is about data, but the causal relationships which produce the data are subject to different and eventually conflicting hypothesis. Or as Loasby (2003) notes: "Knowledge is an open system of selected relationships and the adequacy of our representations of phenomena are always subject to Knightian uncertainty" (p.285). The best possible case is *negative knowledge* about false hypothesis which contradict empirical observations. Thus, the data might give some evidence, or "weight" in Keynesian terms, to propositions about the underlying structure. But rational decisions have to be done in *absence of "truth"*.

We adopt the viewpoint that subjective beliefs, formed on the basis of a subjective mental representation of the world, should in the mean be conformed by observed data. To be more specific: Subjective ex ante beliefs are dispersed but should not systematically deviate in the mean from ex post realizations. Even in the case of rational expectations (RE) with its very strong and questionable common knowledge assumptions it is possible that a wide range of different models may be *observationally equivalent* (Beyer and Farmer (2003), Beyer and Farmer (2008)). Therefore, even in the RE paradigm we have an argument for the non-existence of the "true" model. Agents trust in their model not because it is regarded to be the "truth" but because it provides some consistency. But if there is a multiplicity of observationally equivalent models why should agents trust in their exclusive explanatory power? Why should an agent believe that other agents adopt the same view? Therefore, in the following we specify fundamental uncertainty with an irreducible dispersion of beliefs or diversity of opinions. Of course these dispersed beliefs result in dispersed expectations and decisions.

Our assumption that beliefs, formed on subjective representations of the world, should be compatible with the observed data is related to the theory of *rational beliefs* (Kurz (1994*b*)). An equilibrium in rational beliefs (see Kurz (1994*a*)) is then a state where all agents act according to their beliefs, and the beliefs are consistent with the data which are driven by the individual decisions. There is no need for the knowledge (and even the concept) of a "true" description of the world to derive choice behavior consistently. On the aggregated level the dispersion of different rational beliefs has a similar effect as the multiple priors on the individual level. It can be shown that in such rational belief equilibria money is non-neutral (Motolese (2001)).

All briefly discussed ways to represent fundamental uncertainty – multiple priors, non-additive measures, fuzzy set representations – require heavy additional assumptions about non-observable entities, e.g. multiple priors, preferences about how to deal with ambiguity, membership functions and defuzzification strategies in the fuzzy set approach, etc.. These entities constitute the core of the explanans for understanding observable decision making. The argument put forward in Pasche (2008) is that a theory loses explanatory power when imposing a rich and specific structure for non-observable antecedence conditions. For almost all kind of behavior under fundamental uncertainty it is possible to construct sets of priors, ambiguity preferences, decision weights, distortions of probability measures and so forth in that the deduced rational decision behavior fits the observed behavioral evidence. In what sense the behavior is "explained" if most of explanatory variables are non-observable and subject to arbitrary assumptions about them? If the resulting equilibria are ambigous so that the real evolution is driven by invoking "animal spirits" (like in Epstein and Wang (1994)), the question arises why the reader is confronted with complex mathematical representations when in the end such vague behavioral concepts are neccessary to drive the results. Much simpler hypothesis about boundedly rational decision making seem to be more appropriate. In the Keynes exegesis it is a discussed question whether any kind of Bayesian rationality is in line with Keynesian ideas or whether behavioral approaches are a more appropriate framework to represent them (Leijonhufvud (1993), Rosser (2001)).

We identify "behavioral approaches" with heuristic or rule-governed behavior (Vanberg (1994), Vanberg (2004)). This is opposed to the broadly accepted but nevertheless misleading use of the term "behavioral" in economic theories where the rationality concept – deriving decisions from axioms on preferences – is still intact but preferences are enriched by intrinsic motives, social attitudes or, like in our case, by measures of fundamental uncertainty and attitudes to them. Rule-governed behavior, instead, means that decision behavior is not derived from a closed calculus. Agents are assumed to follow their material preferences only indirectly within an adopted set of behavioral patterns. These may include simple rules of thumb as well as commitments to social norms, and also more or less sophisticated procedures. The lack of explanatory power of invoking rich structured sets of non-observable variables has been critizized before. However, the same criticism holds true in case of behavioral hypothesis. If we are decoupling (material) preferences and decision behavior by assuming rule-guided behavior we are also free do assume *any* behavioral heuristic that fits observed data. This alleged loss of internal consistency provoces scepticism about the explanatory power of behavioral economics (Pesendorfer (2006), Gul and Pesendorfer (2005)). But we argue that behavioral rules have to be justified in that their adoption leads to equilibrium outcomes which are superior in terms of the agent's material preferences. So we are *not* free to assume *any* behavioral rules but only those which represent an equilibrium in the sense that a single agent cannot benefit from adopting another rule (for details see Pasche (2008)).

As Aumann (2008) argues, rationality can be interpreted as *rule rationality* rather than *act rationality* as in classical economis. Such an economic rationale for rules provides explanatory power since the logic why some rules are adopted and others not follows traditional economic reasoning. In contrast, a similar rationale is not as easy possible to justify assumptions about ambiguity preferences, decision weights, or distortions of probability measures. If behavior is consistent with complex preferences involving non-material motives and attitudes, then the outcome of behavior should be evaluated with *these* preferences. It is then questionable whether the welfare of an equilibrium outcome should be assessed in material terms. An observable poor payoff or a loss of material welfare may possibly be beneficial in terms of complex preferences including biased probability measures, specific subjective beliefs on the plausibility of inferences, ambiguity preference parameters etc.. This problem does not arise in the proposed behavioral approach where behavioral performance is simply measured in terms of material preferences. In the following chapter we adopt this view of rule rationality as we justify slight heuristic deviations from rational portfolio decisions – as an expression of self-consciousness in case of fundamental uncertainty – by their superior performance ex post.

3 Portfolio choice and fundamental uncertainty

3.1 The dispersion of beliefs

A portfolio P consists of a risky part R and a risk-free asset which is considered to be money. The share of wealth which is hold as money indicates the liquidity preference of the agent. The risky portfolio R consists of different risky assets i with subjectively expected returns μ_i, variances σ_i^2 and covariances σ_{ij}^2. Standard portfolio theory assumes that these values are "given" or "known" by the individual. In an uncertain world these values have to be estimated in some way on the basis of observed data and considerations about the underlying data generating process. Obtaining the parameters is not only a demanding data retrieving and information processing task. But moreover, there is fundamental uncertainty regarding the underlying structural relations and processes and their stability over time. Therefore, the ex post parameter values of the distribution cannot be "known" ex ante by principle. We will assume that the returns of the risky assets are generated by a distribution which could be described by $\bar{\mu}_i, \bar{\sigma}_i^2, \bar{\sigma}_{ij}^2$ for all i, j. These values should not be considered as "true" values since they are not generated by nature but by an economic process which is driven by decisions made under fundamental uncertainty. They incorporate the individual responses to fundamental uncertainty like self-confidence or self-consciousness.

We consider individual beliefs as "rational" in the sense that they are in the mean consistent with the ex post realized data:

$$\mu_i = \bar{\mu}_i + \epsilon_{1,i}, \qquad \sigma_i^2 = \bar{\sigma}_i^2 + \epsilon_{2,i}, \qquad \sigma_{ij}^2 = \bar{\sigma}_{ij}^2 + \epsilon_{ij} \qquad \forall i, j$$

where all ϵ have a zero mean. The variances $Var[\epsilon]$ (we suppress specific indices for ϵ) and covariances are a measure for the dispersion of beliefs. The individual is assumed to be aware of the fact that beliefs are dispersed. This means that the agent knows that there exists other considerations about the undelying process than his own, and that his beliefs will differ from the ex post realizations with the same probability than the beliefs of any other agent. This reflects uncertainty, since he has no objective basis to assume that *his* beliefs are closer to the ex post realized values than the beliefs of others.

One could object that under these assumptions it is possible to generate a statistic on all beliefs, to average them and to replace the original beliefs with the averaged one. This is, however, a misleading idea for three reasons: (a) An "averaged belief" is inconsistent with the individual considerations about the underlying process, i.e. with the mental model of the economy. (b) The empirically determined "average belief" is only an estimator of $\bar{\mu}_i, \bar{\sigma}_i^2, \bar{\sigma}_{ij}^2$. This estimator itself is dispersed around the average values. (c) If decisions are made upon these "averaged beliefs" the nature of the generating process may change and the ex ante averaged values may not be confirmed by the ex post realizations. We will henceforth assume that decisions are made upon $\mu_i, \sigma_i^2, \sigma_{ij}^2$. Agents may respond to fundamental uncertainty but they can not "correct" their beliefs ex ante.

For low $Var[\epsilon]$, the ex ante beliefs are similar which indicates that the underlying mental models of the world may not be too different. In the extreme case of $Var[\epsilon] \to 0$ all agents have identical expectations, based on identical or equivalent beliefs regarding the underlying structure. Since the subjectively expected values are then equal to the ex post realized values, the beliefs about the economic model must be "true", and the model collapses to a rational expectations approach.

We have to distinguish between the matter of fact that individual beliefs are fundamentally uncertain and dispersed around a mean, and the *awareness* of the individual that his beliefs are formed on an uncertain basis. An individual who is not aware of this fact will not respond to uncertainty. He will make naive rational decisions based on $\mu_i, \sigma_i^2, \sigma_{ij}^2$. We call such an agent *self-confident*. An agent who feels uncertain and responds to this uncertainty in some way is called *self-conscious*. For the reasons discussed in chapter 2, we will describe this self-consciousness not by a Bayesian multiple-prior framework, but with boundedly rational heuristics.

The outlined description of uncertain beliefs is very similar to the well investigated case of *estimation risk*. If we interpret $\bar{\mu}_i, \bar{\sigma}_i^2, \bar{\sigma}_{ij}^2$ as the "true" parameters of the underlying process, and $\mu_i, \sigma_i^2, \sigma_{ij}^2$ as unbiased estimators, then these estimators are also dispersed around the mean, and their variance depend on the sample size (among other things). It is well understood that conventional estimation risk leads to biased portfolio decisions and loss of performance (Siegel and Woodgate (2005), Michaud (1998), Barberis (2000), Chopra and Ziemba (1993), Kan and Zhou (2005)). In general, this problem cannot be avoided completely, but it could be alleviated by different adjustment procedures. The core idea of these procedures is that both, the estimation of values from data and the portfolio decisions made upon these estimations, should be determined in *one* optimization approach instead of two sequential steps. This leads to different (adjusted) estimators the usual portfolio approach is applied to (e.g. Fomby and Samanta (1991), Jorion (1991)). The portfolio performance will be increased *because* the estimators are not unbiased anymore. Compared to Bayesian approaches which have a similar logic than the multiple prior approaches as discussed in chapter 2, Siegel and Woodgate (2005) prove that certain non-Bayesian procedures may have just a slightly higher performance than Bayesian ones. Furthermore, they show that such procedures are also supported by empirical evidence. While the authors prove the asymptotic characteristics of the adjustment procedures, it is clear that there exists an open class of adjustment procedures how to deal with estimation risk which lead to performance improvements.

It has to be noted that estimation risk is different from fundamental uncertainty even if it has similar implications. To go a step further one can assume model uncertainty in addition to estimation risk. Tu and Zhou (2004) show that this kind of uncertainty also has a biasing effect on portfolio choice. It has to be noted, however, that all these studies start from a given "true" process and an ideal portfolio which is based on the exact knowledge of the parameter of this process. Performance losses or gains from adjusting procedures are calculated by comparison with the ideal situation. This is impossible when the "true" model is unknown as a matter of principle. But it is always possible to consider a portfolio which is ex post optimal and serves as a benchmark.

3.2 Portfolio decisions and response to uncertainty

First, we analyze the portfolio decision based on the subjective beliefs without being aware of fundamental uncertainty (case of complete self-confidence). The agent will calculate the set of efficient portfolios and determines the optimal risky portfolio R which should be combined with riskless liquidity. Since we are only interested in the share of money, interpreted as the liquidity preference, we will neglect all biasing effects on the composition of the risky portfolio. To keep the analysis as simple as possible we assume that the subjectively expected performance of R, described by μ, σ^2 is in the mean identical with the performance of the ex post optimal portfolio which is based on $\bar{\mu}_i, \bar{\sigma}_i^2, \bar{\sigma}_{ij}^2$:

$$\mu = \bar{\mu} + \epsilon_1 \tag{1}$$

$$\sigma^2 = \bar{\sigma}^2 + \epsilon_2 \tag{2}$$

with $E[\epsilon_1] = E[\epsilon_2] = 0$ and $V_1 = Var[\epsilon_1]$, $V_2 = Var[\epsilon_2]$, $COV = Cov[\epsilon_1, \epsilon_2]$. These variances reflect the dispersion of the beliefs which are transformed into variances of the expected performance. Again, the beliefs about the portfolio performance are "rational" in the sense that they are ex post confirmed in the mean.

This risky portfolio is now combined with riskless liquidity. We assume that holding liquidity yields no return. Let λ be the share of financial wealth V which is invested into the risky portfolio R. The realized value of the portfolio after the investment period is therefore $\tilde{V} = V(1 + \lambda r)$ with r as the realized return with $E[r] = \mu, Var[r] = \sigma^2$. The agent subjectively expects a mean wealth $E[\tilde{V}] = V(1 + \lambda\mu)$ and a variance $Var[\tilde{V}] = V^2\lambda^2\sigma^2$.

Since we want to seperate the decisions about the portfolio structure (λ) and decisions about the invested wealth V we have to choose a utility function which allows for a separate maximization. It is well known that only utility functions with constant relative risk aversion (CRRA) have this property. The usual quadratic (μ, σ)-approach does not meet this requirement. We adopt the Power function which is widely used in micro- and macroeconomics: $u(\tilde{V}) = \tilde{V}^{(1-\theta)}/(1-\theta)$ where $\theta > 0$ is the constant Arrow-Pratt measure for relative risk aversion. Maximizing $E[u(\tilde{V})]$ with respect to λ leads to the well established result from portfolio theory (details see appendix a)):

$$\lambda^* = \min\left\{\frac{\mu}{\theta\sigma^2}, 1\right\} \tag{3}$$

Given the beliefs and the behavior of all agents and therefore the ex post values $\bar{\mu}, \bar{\sigma}^2$ the optimal composition would be $\bar{\lambda} = \bar{\mu}/(\theta\bar{\sigma}^2)$. Since the individual expectations are dispersed and σ^2 is in the denominator of λ^* the averaged decision is obvioulsy biased due to the Jensen inequality. Developping $E[\lambda^*]$ as a second order Taylor expansion around $\bar{\lambda}$ we have (details see appendix c)):

$$E[\lambda^*] = \bar{\lambda} + \underbrace{\frac{\bar{\mu}V_2 - \bar{\sigma}^2 COV}{\theta\sigma^6}}_{\text{bias}(\lambda)} \tag{4}$$

It is reasonbale to assume the covariance COV is negative, i.e. if the expected return is overestimated then it is more likely that the variance is underestimated (case of being "too optimistic") and vice versa (being "too pessimistic"). Alternatively, also $COV = 0$ may be regarded. Then the bias is always positive which implies that due to dispersion of beliefs the ex ante determined fraction of the risky portfolio λ is too large. The ex post experienced risk is larger than expected ex ante. This result is in line with the literature (e.g. Muller (1993)).

As we have seen, the choice of λ^* is not robust against uncertainty as reflected in dispersion of beliefs. The ex post performance of the portfolio cannot be optimal since the "ideal" portfolio is characterized by $\bar{\lambda}$. This creates benefits of deviating from the standard portfolio choice. According to Pasche (1997) agents benefit from choosing heuristic deviations from rational choice since these decisions are more robust against "errors". Since it is impossible to derive an optimal adaption to uncertainty in a closed form – this would require knowledge about ex post values $\bar{\mu}, \bar{\sigma}^2$ – these heuristic adaptions (rules) should be referred to as boundedly rational.

In the present case a simple adaptive rule is

$$\lambda^a(\beta) = (1 - \beta)\lambda^*, \qquad \beta \in [0, 1)$$

With $\beta = 0$ the agent is completely self-confident or not aware of the fact that beliefs are dispersed and decisions are biased. For $\beta > 0$ he tries to compensate the bias by partially deviating from his own beliefs. Obviously, $\beta^* = \text{bias}(\lambda)/(\text{bias}(\lambda) + \bar{\lambda})$ would be the "optimal" debiasing rule, but this cannot be determined ex ante due to the lack of knowledge of $\bar{\lambda}$. But it can be argued that a single agent may learn β^* at least in a stationary environment. Since the ex post expected utility (performance) is a monotone strictly concave function in λ with $\bar{\lambda}$ as a global maximum, it follows from the epigraphe theorem that the set B of all β for which $\lambda^a(\beta)$ leads to a higher portfolio performance than the naive maximization is convex (Rockefeller (1997)):

$$B = \{\beta | E[u(\lambda^a(\beta))] \geq E[u(\lambda^*)]\}$$

With $\beta = 0$ the agent starts from the border of this convex set and learns to adapt β e.g. by reinforcement learning. Obviously, it is $\beta^* \in B$. For the purpose of the paper we are not interested in the learning dynamics. We confine to the fact that this behavior reflects increasing liquidity preference as a response to fundamental uncertainty.

If an agent responds to fundamental uncertainty by choosing $\beta > 0$ and hence deviating from naive maximizing he shows a higher liquidity preference. It would be possible to "rationalize" any $\lambda \neq \lambda^*$ by introducing additional unobservable structures like multiple priors, decision weights, ambiguity aversion preferences. But the boundedly rational policy rule $\lambda^a(\beta)$ points out that the agent *compromises* his own beliefs and the resulting optimal decisions with the fact that other agents have other beliefs and draw other conclusions. Due to the fact that beliefs and opinions about the underlying structure are dispersed and that there are no a priori reasons that his own point of view is "correct" or "superior", the agent will be self-conscious. Moreover, he knows that there is no way to robustify his decision via a closed calculus that guarantees a priori a maximum performance. But the agent is able to learn that it may be beneficial to compromise subjective optimality considerations with heuristic adaptations.

The higher the self-consciousness the higher the chosen β. The reason of becoming more concerned with uncertainty may be psychological but could also be driven by an increasing dispersion of beliefs, denoted by $V_1, V_2, |COV|$. This would negatively affect the ex post experienced performance and makes it beneficial to adapt β. It should be noted, however, that an adaption of β may be performance-improving only ceteris paribus, i.e. the behavior of all other agents is unchanged. An unilateral adaption of behavior creates benefits for the individual. But if, in contrast, *all* agents adapt their β this would induce a change in ex post realizations $\bar{\mu}, \bar{\sigma}^2$. Without an economic model and assumptions regarding the learning dynamics it is not possible to draw conclusion about the performance in the equilibrium. But nevertheless it is clear that increasing (concern about) uncertainty will lead to a change in liquidity preference.

4 A debt-financed portfolio

4.1 Determining optimal debt size

Portfolio theory is about the optimal structure of the portfolio. Since we assumed a CRRA utility function we are now able to analyse the decision about the invested financial wealth V (henceforth called portfolio volume). We assume that the invested volume could be expanded by debt-financed financial funds. It is rational to expand the portfolio via debt as long as the marginal utility of a portfolio unit exceeds the marginal cost of refinancing this unit, i.e. the interest rate to be paid for the debt.

Assume that the individual could invest own capital C and borrowed financial ressources D. The total portfolio volume is hence $V = C + D$. Let i be the fixed interest rate to be paid for the debt D. We assume that the utility from holding the portfolio and the interest payments are separable. The utility function has to be extended by subtracting the interest payments iD, and it is maximized over D:

$$\max_{D \geq 0} \xi \cdot E\left[\frac{((C+D)(1+\lambda r))^{(1-\theta)}}{1-\theta}\right] - iD$$

The first order conditions equalize the marginal expected utility of the portfolio with the interest rate:

$$\frac{\partial E[u]}{\partial V} = \frac{\partial E[u]}{\partial D} = \xi \cdot (C+D)^{-\theta}(1+\lambda\mu - \frac{1}{2}\theta\lambda^2\sigma^2)^{1-\theta} = i \tag{5}$$

It has to be noted, that in this case the utility function is cardinal. Otherwise the marginal utility would be arbitrary since an ordinal function u is unique up to a positive affine transformation. Therefore we have added $\xi > 0$ in order to choose a proper scale for marginal utility for obtaining realisitic values. Due to risk aversion the marginal utility is decreasing in the portfolio volume, and the intersection point $\partial E[u]/\partial D = i$ determines the optimal volume and hence the optimal debt D (details see appendix b))

$$D^* = \left(\frac{\xi \cdot ((1+\lambda\mu - \frac{1}{2}\theta\lambda^2\sigma^2)^{1-\theta}}{i}\right)^{1/\theta} - C \tag{6}$$

To study the bias we have to take into account that also λ depends on μ, σ^2. Inserting $\lambda = (1-\beta)\lambda^*$ leads to

$$D^* = \left(\frac{\xi \cdot ((1 + \frac{(1-\beta^2)\mu^2}{2\theta\sigma^2})^{1-\theta}}{i} \right)^{1/\theta} - C \quad (7)$$

The effect of an increasing portfolio performance μ on D^* is negative (positive) for $\theta > 1$ ($\theta < 1$) as it can seen by differentiating (5) with respect to μ. Increasing portfolio performance leads to higher total expected utility, eventually accelerated by an increase of λ, but this implies a lower marginal utility due to the concavity of $u(\cdot)$. Hence the portfolio has to be sized down. Only in case of low risk aversion ($\theta < 1$) the marginal utility of the last portfolio unit and therefore the optimal debt size increases (see figure 1).

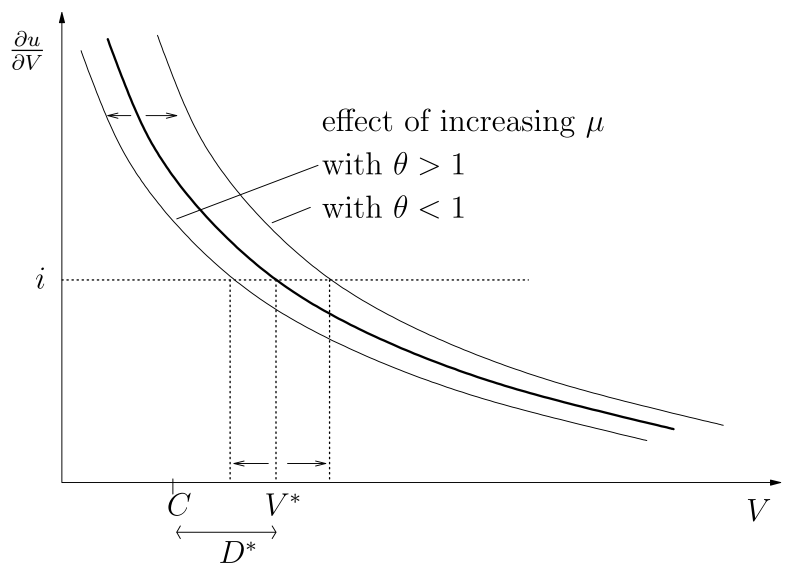

Figure 1: *Effect of increasing expected returns on debt size*

We study the effect of belief dispersion on the expected debt size, including the indirect effect on λ when $\theta > 1$. In case of $\theta < 1$ we have typically a strict boundary solution $\lambda^* = 1$ so that we will not observe additional variance from λ^*. We develop (7) as a second degree Taylor expansion around the ex post optimal $\bar{D}$ which depends on $\bar{\mu}, \bar{\sigma}^2$ and apply the expectation operator (see appendix d)).

$$E[D^*] = \bar{D} + \text{bias}(D)$$

where bias(D) < 0 for $\theta > 0.5$. For $\theta < 0.5$ and hence the boundary solution $\lambda = 1$ less risk-averse or almost risk-neutral agents tend to oversize the debt volume, more risk-averse agents underestimate the marginal utility of their portfolio and choose D below the optimal level.

As we have discussed above, agents who are uncertain regarding their beliefs will be self-conscious and adapt the portfolio structure in favor of liquidity. We assumed a simple boundedly rational procedure $\lambda^a = (1-\beta)\lambda^*$ for such an adaption. A lower λ decreases the expected utility of the portfolio but enhances its marginal expected utility. Therefore, an adaption of λ via β alleviates both, the bias(λ) as well as the bias(D). Only in case of very low risk-aversion with a boundary solution $\lambda^* = 1$ we assume that responding to the uncertainty will not keep the agent away from the boundary. But it might be reasonable to assume a separate heuristic to adapt the debt volume D in case of self-consciousness.

4.2 Probability of debt failures

Since the adapting procedure reduces the bias and thus leads to a higher ex post perfor-mace of the portfolio it has also an effect on the probability of debt failures. Different cases could be distinguished (see also figure 1 where $F(r)$ denotes the cumulative probability distribution of the returns):

- The current return of the portfolio may be not sufficient to cover the interest payments. In this case we have a negative return on the own capital C:

$$prob(\lambda^a(C+D^*)\cdot r < iD^*) = prob\left(r < \frac{iD^*}{\lambda^a(C+D^*)} = Z_1\right) \qquad \text{(Case 1)}$$

- The ex post value $\tilde{V}$ is not sufficient to pay back debt including interest payments:

$$prob((C+D^*)(1-\lambda^a\cdot r) < (1+i)D^*) = prob\left(r < \frac{iD^*-C}{\lambda^a(C+D^*)} = Z_2\right) \quad \text{(Case 2)}$$

Since λ^a and D^* have a certain bias, also the expected values $E[Z_1], E[Z_2]$ will be biased. We are not interested in calculating these expected values. Obviously, Z_1, Z_2 depend negatively on λ and positively on D. A self-confident agent has therefore c.p. a lower probability for debt troubles than a self-conscious risk-averse agent who adapts to uncertainty by increasing β. Inserting D^* and λ^a into the expression Z_1, it can easily seen

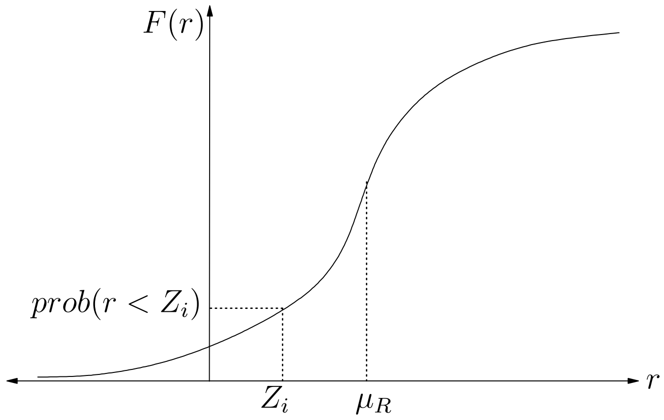

Figure 2: *Probability of debt troubles*

that

$$\frac{\partial Z_1}{\partial \beta} = \frac{2i(\theta - 1)\beta\mu\sigma^2 C}{V(1-\beta^2)\mu^2 + 2\theta\sigma^2}$$

which is positive for $\theta > 1$ (analogously for Z_2). Hence there is a trade-off between the performance-enhancing effect of increasing liquidity preference and the probability of facing debt failures. This may induce a problematic self-enforcing effect: Increasing uncertainty leads to higher liquidity preference (increasing β), therefore the risk of debt troubles increases. This enhances the uncertainty again, etc.

For a less risk-averse agent with $\theta < 1$, however, the opposite holds true. If he does not adapt λ because it is a boundary solution, also Z_1, Z_2 will not respond. If we assume also for this type of agent an increasing β the values Z_1, Z_2 will decrease. This is explained by the effect that for low θ-values the expected debt size is too large in contrast to risk-averse agents. A downwards adaption of λ would then help to reduce debt size.

If we assume that agents are typically characterized by $\theta > 1$ we should expect that with increasing self-consciousness they will adapt β and therefore reduce portfolio risk and favor to hold money. Depending on the debt interest rate they will, however, also slightly increase debt demand. Both effects are complementary: Increasing debt demand stimulates the endogenous creation of money which is accompanyied by the increased preference for money as a part of the portfolio. In a phase of increasing self-confidence portfolios become more risky and will be more equity financed. This result is in line with models of banking behavior and with empirical evidence (see Krainer (2009)).

5 Concluding Remarks

We have argued that typical methods to account for Knightian or Keynesian uncertainty in formal decision models have their merits, but lack explanatory power since large parts of the explanans are related to non-observable variables. We favor a more boundedly rational approach to deal with fundamental uncertainty. The paper provides a simple approach how agents respond to fundamental uncertainty by using a simple adaptive rule which modifies portfolio choice. It shows that the structure as well as the debt-financed volume of the portfolio is affected by the degree of self-confidence/self-consciousness. Especially the liquidity preference depends positively on the degree of fundamental uncertainty, as Keynes had argued. The rationale for these adaptions is that the ex post experienced performance of the portfolio will increase. This increasing desire to hold money is accompanyied by a larger debt demand and hence expansion of money supply. We have seen that these performance enhancing effects have negative effects on the probability to get in troubles of paying back the debt. We also find differences in behavior between more risk-averse and less risk-averse agents. It would be desirable to investigate the effects of becoming more self-confident in a population of more risk-averse and less risk-averse agents e.g. in a boom phase. The model would propose that there would be a shift of debt demand from the risk-averse to the less risk-averse agents, resulting in higher overall risk and higher probability of debt failures. The results would be affected by possible correlations between the degree of risk-aversion and the degree of self-confidence.

The aim of the approach is incorporate fundamental uncertainty into macro models. In most macroeconomic models the financial markets and especially the behavior of commercial banks are far from having a rich and realistic structure. As discussed in Georg and Pasche (2008) the behavior of a commercial bank as the hinge between central bank and the financial markets is driven by portfolio considerations. Incorporating fundamental uncertainty into such a model would provide an explanation why in the current financial crisis banks are more self-conscious and restructure their portfolio in favor of more safe and more liquid assets. Therefore a monetary policy impulse of lowering the refinancing costs will not neccessarily stimulate the debt supply.

Appendix

a) Optimal portfolio structure:

To separate decisions about portfolio structure and invested financial wealth, we use the Power function as a CRRA-utility function: $u(\tilde{V}) = \tilde{V}^{(1-\theta)}/(1-\theta)$ where θ is the Arrow-Pratt measure for constant relative risk aversion. The uncertain value of the portfolio is given by $\tilde{V} = V + \lambda V r$ where V is the invested financial wealth, λ is the share which is invested in the risky part of the portfolio (the secure part has zero interest rate for simplicity), and r is the realized return which is normally distributed with $E[r] = \mu, Var[r] = \sigma^2$. It is not trivial to compute directly the expected utility $E[u(\tilde{V})]$ so we calculate the risk premium ψ and maximize the utility of the security equivalent:

$$\max_{\lambda \in [0,1]} u(V - \psi) = E[u(\tilde{V})]$$

Since $E[\lambda V r] = \lambda V \mu$ and $Var[\lambda V r] = \lambda^2 V^2 \sigma^2$ it is well known from literature that the risk premium is approximatevly

$$\begin{aligned} \psi &\approx -\frac{1}{2}\frac{u''(V)}{u'(V)}\lambda^2 V^2 \sigma^2 - \lambda V \mu \\ &= \frac{1}{2}\theta \lambda^2 V \sigma^2 - \lambda V \mu \end{aligned}$$

Inserting ψ into the utility function yields

$$u = \frac{(V(1 + \lambda\mu - \frac{1}{2}\theta\lambda^2\sigma^2))^{(1-\theta)}}{1-\theta} \tag{8}$$

Maximizing this expression with respect to λ leads to the well known portfolio result:

$$\lambda^* = \min\left\{\frac{\mu}{\theta\sigma^2}, 1\right\}$$

which is independend from the invested portfolio volume V.

b) Optimal debt size:

To obtain the marginal utility of the last portfolio unit we differentiate (8) with respect to V. Since we can expand $V = C + D$ only by debt (own capital C is fixed), first order condition requires that the marginal utility must be equal to the interest rate i. Observe, that in this case we have to interpret u as a cardinal utility function so we are free to multiply marginal utility with $\xi > 0$ to obtain values on a reasonable scale. First order condition thus reads

$$
\begin{aligned}
i &= \xi \frac{((1+\lambda\mu - \frac{1}{2}\theta\lambda^2\sigma^2)^{1-\theta}}{V^\theta} \\
\Rightarrow \quad V &= \left(\frac{\xi((1+\lambda\mu - \frac{1}{2}\theta\lambda^2\sigma^2)^{1-\theta}}{i} \right)^{1/\theta} \\
D^* &= \left(\frac{\xi((1+\lambda\mu - \frac{1}{2}\theta\lambda^2\sigma^2)^{1-\theta}}{i} \right)^{1/\theta} - C \qquad (9)
\end{aligned}
$$

Depending on θ we have to distinguish two cases: For relative large values of θ we have interior solutions $\lambda^* \in (0,1)$. In this case we consider agents to choose $\lambda^a = (1-\beta)\lambda^*$. For relative small values of θ we have the boundary solution $\lambda^* = 1$. In this case we assume that agents do not adapt their decisions since in most cases $\lambda^a = \arg\max\{(1-\beta)\lambda^*, 1\}$ is $\lambda^a = 1$ again. According to these cases we have two expressions for D^*_i, D^*_b (i=interior, b=boundary).

$$
D^*_i = \left(\frac{\xi \left(1 + (1-\beta^2)\frac{\mu^2}{2\theta\sigma^2}\right)^{1-\theta}}{i} \right)^{1/\theta} - C \qquad (10)
$$

$$
D^*_b = \left(\frac{\xi \left(1 + \mu - \frac{1}{2}\theta\sigma^2\right)^{1-\theta}}{i} \right)^{1/\theta} - C \qquad (11)
$$

c) Bias of λ^*:

Since λ^* depends nonlinearily on dispersed beliefs, the expected value $E[\lambda^*]$ will differ from that $\bar{\lambda}$ which is optimal when the ex post values would have been known ex ante. We call the difference bias(λ) $= E[\lambda^*] - \bar{\lambda}$. For determining bias($\lambda$) we compute the expected value of a second degree Taylor expansion around $\bar{\lambda}$. With $E[\epsilon_i] = 0$ and $V[\epsilon_i] = V_i, COV[\epsilon_e, \epsilon_2] = COV$ we have

$$
\begin{aligned}
E[\lambda^*] &= \bar{\lambda} + \frac{1}{2} E\left[\frac{\partial^2\lambda}{\partial\mu^2}\epsilon_1^2 + \frac{\partial^2\lambda}{\partial(\sigma^2)^2}\epsilon_2^2 + 2\frac{\partial^2\lambda}{\partial\mu\partial\sigma^2}\epsilon_1\epsilon_2 \right] \\
&= \bar{\lambda} + \underbrace{\frac{\bar{\mu}V_2 - \bar{\sigma}^2 COV}{\theta\sigma^6}}_{\text{bias}(\lambda)}
\end{aligned}
$$

Since we have assumed $COV \leq 0$ (see text) the bias is always positive.

c) Bias of D^*:

Also the optimal D^* depends nonlinearily on dispersed beliefs so that we should expect a deviation from the ex post optimal value $\bar{D}$. Analogoulsy to appendix c) we call this difference bias(D). In case of an interior solution, D_i^* depends directly on μ, σ^2 as well as indirectly due to λ^a. In case of a boundary solution, D_b^* has no indirect effects. For both cases we calculate the bias as a second degree Taylor approximation of (7) around $\bar{D}$ (assuming $COV = 0$ for simplicity):

$$E[D^*] = \bar{D} + \frac{1}{2}\left(\frac{\partial^2 D}{\partial \mu^2} V_1 + \frac{\partial^2 D}{\partial (\sigma^2)^2} V_2\right) \tag{12}$$

The derivatives on the r.h.s. are a little bit too elaborate to print them out here. Because an interior solution requires a sufficiently large θ we have $\theta > 1$ as a condition that the second order derivatives on the r.h.s. of (12) are negative. Hence the bias(D_i^*) is negative. In case of a boundary solution the sign of the second order derivatives on the r.h.s. is negative (positive) for $\theta > 0.5$ ($\theta < 0.5$). This implies that agents with very low or almost no risk aversion will choose in the mean a debt level (far) above the level $\bar{D}$ which turned out to be optimal ex post.

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