Intrinsic Cycles of Land Price: A Simple Model*

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Abstract

The cyclicality and volatility of property prices have been extensively documented. Many explanations have been proposed. This paper builds a simple dynamic general equilibrium model in which these often cited channels are assumed away. Instead, the role of intertemporal elasticity of substitution is highlighted. In this model, the land price can exhibit price cycles. Moreover, the land price always fluctuates more than the aggregate output. The welfare of different cohorts depends crucially on the land price at the period they were born. The implications of these results are discussed.

Key words: land price cycle, oscillatory convergence, intertemporal substitution

JEL classification: E30, G12, R20

1 Introduction

Cyclicality and volatility of property and land prices have been extensively documented. For instance, Borio et al. (1994) and Renaud (1997) shows that there was a global real estate cycle among the OECD countries in between 1985 to 1994. Employing the Kalman Filter and data of more than a century, Ball et al. (1996) find very significant and long period cycles in non-residential property market, and Ball and Wood (1999) find similar results for the residential property market. Ortalo-Magne and Rady (1998) show that the real housing prices in the US and UK fluctuate more than the real GDP.[1]

Attempts have been made to explain the boom-bust cycles of property and land prices. There is a large literature on speculation and bubbles in the housing market as well as other asset prices such as stock prices (see for example, Case and Shiller (1989), Abraham and Hendershott (1995), Sato (1995), Ito and Iwaisako (1996), Levin and Wright (1997), and Muellbauer and Murphy (1997)). Many of these studies found evidence of speculative behavior to be a significant factor causing wide swings in property or land prices.

On the other hand, much research has focused on how house prices relate to market fundamentals. Stein (1995) rationalizes the appear-to-be excess house price volatility by changes in fundamentals. Given a down payment requirement and the initial distribution of household debt levels, the model demonstrates that there is a potential for multiple equilibria, and that a small change in fundamentals can generate within-equilibrium multiplier effects, leading to large, discontinuous jumps in prices. Kiyotaki and Moore (1997), Ortalo-Magne and Rady (1998), Chen (2001), among others, also generate the cyclical behavior of asset (land) prices. The crucial element of their results is that the interactions of credit constraint and collateral value (land/housing prices) generate large and persistent fluctuations in aggregate variables.[2] For empirical evidence, Chinloy (1996) finds that the real estate cycles

[1] See Leung (2004) for a review of the literature.

[2] Alternatively, property price fluctuations can also be generated by a search theoretic model, as demonstrated by Wheaton (1990).

depend on the lag in construction, planning, and entitlement, and also the lumpy cost of vacancy and releasing. Leung et al. (2002), Leung and Feng (2004) find that the short-term fluctuations of housing prices tend to be induced by the "down payment effect." Based on survey data and official data, Dokko et al. (1999), Edelstein and Paul (2000), Edelstein et al. (2001) find that the fluctuations of land price can be largely explained by the change in expectations (survey data) and changes of income generated by the corresponding land.

In this paper we study the dynamics of "land" prices and its welfare implications in an overlapping generations model.[3] It is well known that the land price fluctuations can have important implications on the well-being of the economic agents. Historical research such as King (1973), Macfarlane (1978) find that the land market has been developed and operated for many centuries and that the distribution of the land endowment (or inheritance) can have significant implications on the intra-generational inequality as well as inter-generational mobility. Recent studies on developing country generate similar results. For instance, Singh (1982) finds that land sale in India has important implications to both intra- and inter-generational distribution of income and welfare. This paper takes a preliminary step on this direction and focuses on the inter-generational dynamics. There are several reasons to employ an overlapping generations model. When the model is interpreted literally, it helps to explain the "long cycles" identified by the empirical literature.[4] Alternatively, it can be interpreted as "waves of myopic investors".[5] Moreover, overlapping generations models also generate transactions of land and real estate across cohorts naturally, which will prove to be an important mechanism of our results here.

Applying an overlapping generations model to the study of the land (real estate) market is not new,[6] however, this paper differs from the previous literature in the following

[3] There is a related literature on the relationship between housing and economic growth, such as Brito and Pereira (2002). Here the focus is very different. We study the relationship between housing price fluctuation and the aggregate economy (which is "constant" in the baseline model).

[4] For instance, see Ball et al. (1996), Ball and Wood (1999).

[5] Among others, see Bernanke and Gertler (1989) for more discussion on this interpretation.

[6] For instance, see Brueckner and Pereira (1997) and the reference therein.

aspects. First, land serves as an input for production (from the aggregate perspective) and also an investment vehicle (from an individual perspective). Second, the model is intentionally abstracted from many features which are considered in the previous studies that have been proposed to explain the cyclicality of land prices. They include the market frictions (informational asymmetry, collateral constraints, adjustment costs, and construction lag), uncertainty (persistent stochastic shocks), government intervention (taxation and policy changes), and bubbles. This paper complements the literature by showing that while these features are important, the equilibrium land price could still display "cycles" even without these features. It means that the land price oscillates above and below the steady state value, and even may not restore to the steady state values in some special cases. In particular, land prices exhibit cycles even when the aggregate output, wage and rental are constant over time. That says, the land price in this model fluctuates more than the aggregate output, as found in Ortalo-Magne and Rady (1998). Specifically, the equilibrium land price is either constant over time, or it exhibits oscillatory dynamics. In fact, the path of transitional dynamics can be indeterminate, even when the steady state is unique. The land price cycles are, in a sense, *intrinsic*.[7]

Moreover, we find that the nature of the dynamics of land prices depend crucially on certain combinations of preference parameters, such as the rate of time preference and the elasticity of intertemporal substitution. In addition, the model is constructed in a manner that, even when the aggregate output in the model is unaffected by the fluctuations of the land price, the welfare of different cohorts may vary and depend on the price of the land at the period they were born.[8] As an application, we also extend the analysis to a situation

[7]See Baumol and Benhabib (1989) for an exposition of different types of dynamics. See also Mountford (2002) where multiple steady states are possible without any cyclical dynamics, which seems to be complementary to our work.

Notice that we abstract from the fact that in practice, land lots are "discrete" and subject to the government regulations. We attempt to focus on the "intrinsic nature" of land price which does not depend on the particular details of the government regulations, which vary across countries and time periods.

[8]Empirically, the fluctuation of land price could affect the aggregate output, and that would strengthen the results here.

when the probability of second period survival depends on the first period consumption.[9] This can be interpreted as the case of a developing country, or a developed economy in earlier centuries, when land and labor are crucial factors of production.[10] Under this scenario, the movement of land price, which redistribute wealth across generations, will have an additional impact to the economy. A high land price could lead to a lower level of consumption for the young generation, which leads to a lower level of survival probability. It in turns affect the distribution of consumption in the following period. Under some parameter values, it can be shown that this extension can lead to multiple steady states and very rich dynamics in land price, as well as in population size. This may shed light on the widely documented historical experience of certain economies, especially when land played a very important in the aggregate production (Gottlieb (1976), Macfarlane (1979), and Sakolski (1932)).

It should be stressed that understanding the dynamics of land price can contribute to our understanding on the aggregate economy, as land is an important input even for modern economies. For instance, with time series data from 1961 to 1995, Kiyotaki and West (2004) report that the elasticity of substitution between land and capital in the aggregate Japan production function is close to unity. Borio et al. (1994), Kwon (1998), IMF (2000), and Gerlach and Peng (2003), to name a few, examine the relationship between the role of real estate prices behind the lending boom and bust during the late 1980s and 1990s, and find that bank lending is closely correlated with property prices in both developed and developing economies. In particular, recent empirical works confirm the prominent role of land serving as collateral in mitigating credit constraint of firms. For example, Kiyotaki and West (1996) and Ogawa and Kitasaka (1999) find that land value significantly affects aggregate investment of Japanese firms. Using firm-level data, Ogawa and Suzuki (1998, 2000), and Ogawa and Kitasaka (1999) find that the movement in land price significantly affects the investment behavior of credit-constrained firms in Japan.

[9] Among others, see Kalemli-Ozcan (2002) for evidence.

[10] For instance, MacFarlane (1979) documents that property market is indeed very active in England since 1400.

The rest of the paper is organized as follows. Section 2 presents the theoretical model and illustrate the possibility of oscillatory dynamics with numerical examples. We also investigate the welfare implications land price fluctuations for different cohorts. Section 3 extends the analysis to non-separable utility functions. Section 4 then extends our analysis to endogenous survival probability. The last section concludes.

2 A Simple Dynamic Model

Consider an overlapping generations (OLG) model in which there is a fixed quantity of durable asset, which is referred to as land thereafter.[11] The economy lasts forever but agents only live for two periods.[12] Each agent is endowed with one unit of labor when young, and nothing when old. The young generation of period t supplies labor inelastically,[13] receives wage w_t, consumes $c_{1,t}$, and purchases land l_t at unit price q_t, which is then rented out to the firms to serve as a productive input, with a rate of return r_{t+1} at period $t+1$. The land will also be resold at unit price q_{t+1}.[14] The old agent then consumes both the rental income

[11] OLG model has been widely used at least since the publication of Diamond (1965). For more discussions, for instance, see Azariadis (1993).

[12] Here, we implicitly assume that the population is constant over time to simplify the explosition. It will be relaxed in a later section.

[13] Empirically, the labor supply elasticity for men are small, and get even smaller as the lenght of period extends. Among others, see the survey by Blundell, R., MaCurdy, T., (1999), Pencavel, J., (1986)

[14] Even though land is often considered to be indivisible, in the paper we take the extreme position to assume that it is perfectly divisible. We defend our assumption as follows. In macroeconomic analysis, it is well known that fixed capital investment is lumpy, i.e., indivisible (for instance, see the survey paper by Caballero (1999)). Despite the "indivisible nature" of the capital stock, it is accustomed to assume that the capital stock to be "divisible", so that the researchers can focus on other issues. In other words, the "differentiability" can be interpreted as an approximation. It is true in other fields as well. For instance, measured "ability" is "discrete," but labor economist tend to model "ability" as a continuous variable in order to focus on more important issues (for instance, see Becker (1993) for a review). In microeconomics and general equilibrium theory, the total amount of consumer is also frequently modeled as a continuous variable while in reality we only have a finite number of people (for instance, see Hildenbrand (1974)). By the same token, for a country like U.S., which probably has millions of "lots" within the whole country, the "discreteness of land lot" seems to be too burdensome to carry around and it seems reasonable to use the differentiable assumption instead. In our view, the observed "indivisibility" of land is not intrinsic, but is rather created by the government regulation and "lot division". Our paper, as the title suggests, attempts to focus on the intrinsic nature of the land itself and thus assumes away regulations that may vary across

and resale value of land before exiting form the economy. Each generation maximizes the lifetime utility as follows:

$$\max \; u(c_{1,t}) + \left(\frac{1}{1+\theta}\right) u(c_{2,t+1})$$

$$\begin{aligned} \text{s.t. } c_{1,t} + q_t l_t &= w_t, &(1)\\ c_{2,t+1} &= (q_{t+1} + r_{t+1})\, l_t. &(2) \end{aligned}$$

where $c_{1,t}$ denotes the consumption of the young generation at period t, $c_{2,t+1}$ denotes the consumption of the old generation at period $t+1$, q_t is the price of land at period t, and θ is the rate of time preference. The utility function $u(.)$ is concave, $u'(.) > 0$, $u''(.) < 0$. Notice that this formulation emphasizes on the production role of land. Consumption in both periods are assumed to be non-negative,

$$c_{1,t}, c_{2,t+1} \geq 0.$$

This realistic restriction will help us to rule out certain equilibrium paths. Let $\lambda_{1,t}$, $\lambda_{2,t}$ be the Lagrangian multipliers of the constraints (1) and (2) respectively. The first order conditions are

$$\begin{aligned} u'(c_{1,t}) &= \lambda_{1,t}, &(3)\\ u'(c_{2,t+1})/\left(1+\theta\right) &= \lambda_{2,t}, &(4)\\ q_t \lambda_{1,t} &= \lambda_{2,t}\left(q_{t+1} + r_{t+1}\right). &(5) \end{aligned}$$

To rule out "bubbles," a transversality condition for the asset prices is imposed,

$\lim_{i\to\infty} \left(1+\theta\right)^{-i} q_{t+i} = 0$. Combining the equations from (3) to (5) delivers the following

nations.

expression:

$$\frac{u'(c_{1,t})}{u'(c_{2,t+1})/(1+\theta)} = \frac{q_{t+1}+r_{t+1}}{q_t}, \tag{6}$$

where $c_{1,t} = w_t - q_t l_t$, $c_{2,t+1} = (q_{t+1}+r_{t+1})\, l_t$. Implicitly, (6) is a non-linear difference equation of q_t, together with other variables (such as w_t, r_{t+1}, etc.), which describe the dynamics of the land price.

The production technology is constant returns to scale in land and labor. Recall that the labor is supplied inelastically by the young generation whose population is normalized to unity. Given that the factor market is assumed to be competitive, it gives rise to the familiar conditions:

$$r_t = f'(l_t),\, w_t = f(l_t) - l_t f'(l_t), \tag{7}$$

where $f(l_t)$ is the output per unit of labor given an input of l_t units of land, $f' > 0$, $f'' < 0$. At the equilibrium, the demand of land is equal to the supply, which is normalized to unity,

$$l_t = 1, \;\; \forall t. \tag{8}$$

Substituting (8) into (7) shows that the return to different factors of production are constant over time, $r_t = r$, $w_t = w$. Equipped with this, (6) can be simplified as

$$\frac{u'(c_{1,t})}{\left(\frac{1}{1+\theta}\right) u'(c_{2,t+1})} = \frac{q_{t+1}+r}{q_t}, \tag{9}$$

where $c_{1,t} = w - q_t$, $c_{2,t+1} = q_{t+1} + r$. Equation (9) can be interpreted as an asset-pricing equation. Notice that it can be re-arranged as $q_t = \left[(1+\theta)^{-1} u'(c_{2,t+1})/u'(c_{1,t})\right](q_{t+1}+r)$. Solving recursively, it can be expressed as a present value formula:

$$q_t = r \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \mathcal{M}_{t+j} \right), \tag{10}$$

where the intertemporal marginal rate of substitution $\mathcal{M}_{t+j} \equiv (1+\theta)^{-1} u'(c_{2,t+j})/u'(c_{1,t+j-1})$

is the pricing kernel in standard asset pricing models. Clearly, if $\mathcal{M}_{t+j}$ changes over time, then even if the rental income is constant over time, the land price can still fluctuate.[15] Hence, equation (10) demonstrates the following lemma:

Lemma 1 *Even if the rental income of land is constant over time, the land price could still fluctutate in a dynamic equilibrium model with perfect foresight agents.*

To generate more analytical results, we impose further restrictions on the preference henceforth. Following the literature, we assume that the utility function exhibits the typical constant intertemporal elasticity of substitution,

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},\ \sigma > 0,\ \sigma \neq 1, \tag{11}$$

where the elasticity of substitution is $1/\sigma$ in this case.[16] Now, (9) can be re-written as

$$q_{t+1} + r = q_t \left(1+\theta\right)^{1/(1-\sigma)} \left(\frac{w}{q_t} - 1\right)^{-\sigma/(1-\sigma)}, \tag{12}$$

which explicitly shows how the land price evolves over time. Differentiating both sides of (12) with respect to q_t will deliver a key equation for the land price dynamics,

$$\frac{dq_{t+1}}{dq_t} = \left(1+\theta\right)^{1/(1-\sigma)} \left(\frac{w}{q_t} - 1\right)^{-1/(1-\sigma)} \left[\left(1-\sigma\right)^{-1} \left(\frac{w}{q_t}\right) - 1\right]. \tag{13}$$

For the consumption in the first period must be positive, the unit land price must be lower than the wage, $q_t < w$. It means that $\left(\frac{w}{q_t} - 1\right) > 0$. The first order derivative thus depends

[15] It stands in contrast with Kiyotaki and Moore (1997), in which the fluctuations of land prices come from the changes in the expected future user cost as the agents' utility function are linear. Furthermore, since the rental rate of land is constant under our specification, the "asset-specific risk" correlation between the discount factor and asset-specific rate of return is zero.

[16] See Barro and Sala-i-Martin (1995) for more discussion of the intertemporal elasticity of substitution. When $\sigma = 1$, $u(c) = \log c$ and the dynamics becomes trivial. Since the results in this paper applies to any positive value of σ except when $\sigma = 1$, and the probability that the value of σ being exactly unity is measure-zero, this limiting case is not considered in the main text.

on whether σ is greater than or less than unity. The second order derivative can be easily calculated to be strictly positive for any $\sigma > 0$. By definition, the price does not change at the steady state, $q_t = q_{t+1} \equiv q$. Now, for the existence of a positive land price at the steady state, $q > 0$, it must be that

$$q\left(\frac{w}{q} - 1\right)^{-\sigma/(1-\sigma)} > r\,(1+\theta)^{-1/(1-\sigma)}. \tag{14}$$

However, the unique steady state may not exist, and if it does, it may not be stable (we will verify the existence and uniqueness in our numerical implementations). It means that the land price could eventually be too high (or too low) and violate some equilibrium conditions. It turns out that the inverse of the intertemporal elasticity of substitution σ plays a crucial role here. For exposition, we treat the two different cases separately:

1. If $0 < \sigma < 1$, then $-\sigma/(1-\sigma) < 0$, and thus (12) shows that as $q_t \to 0$, $q_{t+1} \to -r < 0$. Also, since $0 < \sigma < 1$, it is always true that $(1-\sigma)^{-1}(w/q_t) - 1 > 0$. By (13), we have $dq_{t+1}/dq_t > 0$, that is, land price is always increasing. Given that the second order derivative is strictly positive, the dynamics of the land price is unstable, as demonstrated in Figure 1a. Notice, however, that in this system, there is only a jump variable (q_t) and no sluggish variable. Therefore, as in the case of monetary economy in overlapping generations models, the equilibrium price of land will immediately jump to the steady state value and remain constant over time.[17] This is not a very interesting case and our attention will switch to the following case.

2. If $\sigma > 1$, then $-\sigma/(1-\sigma) > 1$, and thus (12) shows that as $q_t \to 0$, $q_{t+1} \to \infty$. By (13), $\sigma > 1$ leads to $dq_{t+1}/dq_t < 0$. As shown in figure 1, the dynamical system can be either "oscillatory and stable" (1b and 1c) or "explosive" (1d). In the former, when an unexpected temporary shock hits this economy (such as a temporary change in taste,

[17] Among others, see Azariadis (1993), Wallace (1980) for more discussion.

in tax rate, or even the productivity), the land price will oscillate above and below the steady state value. It will eventually converge to the steady state value but the convergence is not monotonic. Notice, however, that there are infinitely many paths to the unique steady state that are also consistent with the equation (12), as the initial land price is not dictated by any of the first order conditions. Thus, we encounter the situation of "*path indeterminacy*" which has been studied by the macroeconomics literature.[18]

On the other hand, if the system is "explosive", i.e. the land price will either be "too high" that it will eventually exceed the first period wage income, pushing the first period consumption to be negative, or it becomes "too low" (diverges to negative infinity) so that the second period consumption becomes negative.[19] Since neither of these are allowed, the price will jump to the steady state immediately. In other words, we have a constant price equilibrium path, as in the case with $0 < \sigma < 1$.

[Insert figure 1 here]

2.1 Discussion of the Results

The previous section shows that, even a simple model with land as a factor of production is capable of generating land price cycles under certain conditions, depending on whether the value of σ is larger than unity or not. This section attempts to provide some intuition behind these results. First, notice that when an agent is old, she is rather "passive": she simply collects the rent and sells all her land, and then consumes. The important economic decision is made when the agent is young. Furthermore, since labor is supplied inelastically when

[18] See Baumol and Benhabib (1989) for more discussion.

[19] The value of land cannot be negative because land is a productive input here.

young, the purchase of land should be the main focus. Thus, the task here is to characterize the young generation's demand function for land.

To facilitate the discussion, we define the rate of return of holding land,

$$R_t \equiv \frac{q_{t+1} + r_{t+1}}{q_t} = \frac{q_{t+1} + r}{q_t}. \tag{15}$$

Given the utility functional form (11), we can rewrite the equation (6) without imposing the equilibrium condition. After some manipulations, it can be shown that the function of demand for land by young agents is given by

$$l_t = \frac{w}{q_t}\left(1 + (1+\theta)^{1/\sigma}\,(R_t)^{(\sigma-1)/\sigma}\right)^{-1}. \tag{16}$$

Notice that the rate of return for holding land R_t is increasing in q_{t+1} (see (15)). Hence, other things being equal, it is feasible to compute the partial derivative of the demand of land with respect to the next period land price q_{t+1},

$$\frac{\partial l_t}{\partial q_{t+1}} = \left(\frac{w}{q_t}\right)(1+\theta)^{1/\sigma}(R_t)^{-1/\sigma}\left(1 + (1+\theta)^{1/\sigma}\,(R_t)^{(\sigma-1)/\sigma}\right)^{-2}\left(\frac{1-\sigma}{\sigma}\right).$$

Notice that all terms except $\left(\frac{1-\sigma}{\sigma}\right)$ in the above expression are always positive. It is thus clear that

$$\frac{\partial l_t}{\partial q_{t+1}} \begin{cases} > 0 & \text{if } 0 < \sigma < 1 \\ < 0 & \text{if } \sigma > 1 \end{cases},$$

which proves the following lemma:

Lemma 2 *Other things being equal, the demand of land increases (*decreases*) with the (expected) future land price if* $0 < \sigma < 1$ *(*$\sigma > 1$*).*

The intuition of the lemma is simple. Consider first the case when σ is low, which means that agents are more willing to substitute current consumption for future consumption. If they expect that the future land price will appreciate, then they will devote more resources to demand for land, which in turn raises investment, given the current price of land. In equilibrium, since the supply of land is fixed, the agents' willingness to intertemporal substitution also raises the current land price q_t. This gives an intuitive explanation why (13) exhibits a positive relationship between q_t and q_{t+1} given a low value of σ, which leads to an explosive dynamics. Since the agents are perfect foresight, the price will jump to the steady state and stays at that level from that period onwards.

On the other hand, when agents have a higher σ, which means that agents are reluctant to make intertemporal substitution, a higher future land price q_{t+1} would only suppress the current demand of land. To see why, note that a higher q_{t+1} raises period $t+1$ consumption for a given period t land holding. To maximize her lifetime utility, the agent will smooth out the consumption plan and increases period t consumption given the current price of land q_t. This results in a decline in the demand for land, and hence a decline in the current land price q_t. This also explains why in this case (13) exhibits a negative relationship between q_t and q_{t+1} given a high value of σ.

Notice that merely positive correlation between the present land price q_t and the future land price q_{t+1} is *not sufficient* to generate unstable dynamics. It must be coupled with the conditions that the future land price is negative when the present land price approaches zero, and that the magnitude of the "feedback" is large enough, i.e., $|dq_{t+1}/dq_t| > 1$. Otherwise, a steady state might not even exist. While $|dq_{t+1}/dq_t| \lesseqgtr 1$ depends on the combination of *all* of the parameter values and can only be verified numerically, the condition that the future land price is positive or negative as the present land price approaches zero, can be easily calculated. Figure 2 provides numerical examples and graphical illustrations. Figure 2a and 2b draw the movement of q_{t+1} against q_t with relatively large σ (5 and 6 respectively), and the former leads to constant-price equilibria and the latter shows cycles.

[Insert figure 2 here]

2.2 Welfare Implications

As it is shown in the previous sub-section, the land price could fluctuate even with a small temporary shock (the *"path indeterminacy"* case). Therefore, it is reasonable to suspect that the welfare of agents of different cohorts could also vary over time. This section studies the evolution of agents' welfare in this economy. Now define the life-time utility of a representative agent of the cohort born at time t to be $\mathcal{W}_t \equiv u(c_{1,t}) + (1+\theta)^{-1} u(c_{2,t+1})$. At the equilibrium, their consumption plan is $c_{1,t} = w - q_t$ and $c_{2,t+1} = q_{t+1} + r$. Combining the utility function of (11) with (12), the life-time utility of the period t cohort (or in short, the life-time utility) is given by

$$\mathcal{W}_t \equiv u(c_{1,t}) + (1+\theta)^{-1} u(c_{2,t+1}) = \left(\frac{(w-q_t)^{-\sigma}}{1-\sigma}\right) w.$$

Again, whether the life-time utility is increasing or decreasing in q_t depends on the value of σ,

$$\frac{d\mathcal{W}_t}{dq_t} = \left(\frac{\sigma}{1-\sigma}\right) \left[w\left(w-q_t\right)^{-\sigma-1}\right] \begin{cases} >0 & \text{if } 0<\sigma<1 \\ <0 & \text{if } \sigma>1 \end{cases}.$$

For the case of small value of σ ($0 < \sigma < 1$), the life-time utility of the period t cohort is increasing in the value of land. However, the earlier result has already established that this corresponds to the case where the land price immediately jumps to the steady state and remain constant over time. Thus, the welfare of different cohort will also remain constant at the equilibrium. On the other hand, for the case of large σ ($\sigma > 1$), the life-time utility of the period t cohort is decreasing with the price of land. We also know that the dynamics of land price can either be oscillatory and stable, or explosive, and that $dq_{t+1}/dq_t < 0$. This is the case when we have an "path indeterminacy" of land price. Consider the case when the equilibrium exhibits oscillatory convergence. Since the land price oscillates above and below the steady state value, the life-time utility of the period t cohort will be higher than the immediate next generation if the land price in period t is lower than that in the following period.

2.3 Numerical Examples

As shown in figure 2, the model is capable of generating oscillatory dynamics as the land price converges to the steady state value, or two-period cycles where the land price always oscillates between two values. In other words, land price cycle is a "possibility" in this model. However, one may wonder how large the "probability" of the cycle is. To put it in another way, with empirically plausible parameter values, how likely would these cycles occur? To answer these questions, it demands a precise estimation of the parameters from the data. This section presents numerical examples to investigate the dynamics of land price implied by the model.

The typical parameter value used in the macroeconomics literature on the time discount factor $\beta \equiv (1+\theta)^{-1}$ is about 0.96 for annual data.[20] It translates to a value of θ in between 1.2624 to 2.4014 in our two-period OLG model in which each period corresponds to a period of 20 to 30 years.[21] These calculations, however, are based on the infinite horizon model, and hence the discount factor include both life-cycle discounting and altruism for offsprings. The current model, on the other hand, focuses primarily on life-cycle discounting and thus the appropriate discount factor should be lower. Unfortunately, the empirical literature on the existence of altruism is controversial, and thus a widely accepted estimate of the altruism is unavailable. It should nevertheless be clear that even if the annual discount factor is adjusted to 0.85, the implied θ would be in the range of 24.8 (for a period of 20 years) to 130.05 (for a period of 30 years). The precise estimation of the pure life-cycle discount factor still awaits for future research.

The estimate for σ is even more problematic. In an infinite horizon asset pricing model

[20] For instance, see Cooley (1995).

[21] The calculation is simple. If the annual discount factor is 0.96. Thus, the discount factor for 20 and 30 years are 0.442 and 0.294 respectively. Then simply apply the formula that $\theta = \beta^{-1} - 1$ yields the result.

calibrated with quarterly data, Mehra and Prescott (1985) claims that σ is within a narrow range, $0 \leq \sigma \leq 10$. However, Kandel and Stanbaugh (1991), Abel (2002) forcefully argue that, even for quarterly data, the range of σ is in fact much larger. The issue is even more complicated here because each period of our two-period OLG model corresponds to 20 or even 30 years. It is not very clear how the previously mentioned estimates would apply to the current model. Due to these yet-to-be-solved problems, this paper simply experiments with a wide range of parameter values, θ and σ, between 0 and 100.

Figure 3a shows two regions of parameter combinations between σ and θ: the region delivering constant price equilibria (dark-shadowed area) and the region delivering oscillatory convergence (light-shadowed area). The 2-period cycle cases occur on the boundary of the two regions. It is clear that although the region which delivers the "land price cycles" is large, it is necessary (but not sufficient) to have $\theta > 10$ and $\sigma > 3$. It puts some severe restrictions on the parameter values and might be not satisfactory to some. In a sense, it might not be very surprising because this model has thus far assumes a time-separable utility function, and rational expectations asset pricing model with time-separable utility functions has been found to perform unsatisfactorily.[22] Recent research seems to suggest that allowing for time-non-separable utility functions can significantly improve the ability for the model to match the data (Constantinides (1990), Boldrin, Christiano and Fisher (1997)). Following this line of thought, the next section extends the analysis to different types of time-non-separable utility functions.

[Insert figure 3 here]

[22] For instance, see the survey by Kocherlakota (1996).

3 Non-Separable Utility Function

There are at least two types of time-non-separable utility function widely used in the literature. In the terminology of Carroll et al. (1997), one is "*inward-looking*" and the other is "*outward-looking*." In the former, the second period consumption is discounted by the amount of her *own* consumption in the first period ("habit formation"), whereas under the latter case, it is discounted by the *average* consumption in the first period ("catching up with the Joneses"). Under the outward-looking case, the agent fails to take into consideration that her consumption decision would have an impact to the aggregate/average consumption. Although at the equilibrium, the individual consumption coincides with the average consumption level, the dynamics displayed and the welfare implication can be different, as demonstrated by Carroll et al. (1997). Since it is empirically difficult to differentiate the "inward-looking" from the "outward-looking" preference, this paper considers both types for completeness.

3.1 Habit Formation

The basic structure retains in this section, except for a change in the utility function,

$$\max \; u(c_{1,t}) + \left(\frac{1}{1+\theta}\right) v(c_{2,t+1}, c_{1,t})$$

subject to the constraints (1) and (2), and that the consumption in both periods are non-negative, $c_{1,t}, c_{2,t+1} \geq 0$. The agent is aware of the effect of first period consumption $c_{1,t}$ on the periodic utility in both periods, $u(c_{1,t})$ and $v(c_{2,t+1}, c_{1,t})$. As before, $\lambda_{1,t}$ and $\lambda_{2,t}$ represent the Lagrangian multipliers of the constraints (1) and (2) respectively. The first order conditions (4) and (5) are still valid. However, (3) is modified as

$$u'(c_{1,t}) + \left(\frac{1}{1+\theta}\right) \frac{\partial v(c_{2,t+1}, c_{1,t})}{\partial c_{1,t}} = \lambda_{1,t}. \tag{17}$$

Combining the first order conditions and the market equilibrium conditions deliver a modified version of (6),

$$\frac{u'(c_{1,t})+\left(\frac{1}{1+\theta}\right)\frac{\partial v(c_{2,t+1},c_{1,t})}{\partial c_{1,t}}}{\left(\frac{1}{1+\theta}\right)\frac{\partial v(c_{2,t+1},c_{1,t})}{\partial c_{2,t+1}}}=\frac{q_{t+1}+r}{q_t}\equiv R_t, \tag{18}$$

where $c_{1,t}=w-q_t$, $c_{2,t+1}=q_{t+1}+r$. To examine the dynamics more explicitly, two special cases of time-non-separable utility function are considered.

1. Subtractive Habit Formation

This form of time-non-separable utility can be at least traced back to the seminal work of Constantinides (1990). In this case, the utility function takes the following form

$$u(c_{1,t})=\frac{(c_{1,t})^{1-\sigma}}{1-\sigma},\ \text{and}\ v(c_{2,t+1},c_{1,t})=\frac{(c_{2,t+1}-ac_{1,t})^{1-\sigma}}{1-\sigma},$$

where $0<a<1$, $\sigma>0$, $\sigma\neq 1$.[23] It is further imposed that $c_{2,t+1}-ac_{1,t}>0$. In the appendix, it is shown that equation (18) can be re-written as

$$\left(R_t\left(\frac{w}{q_t}-1\right)^{-1}-a\right)^{\sigma}=\frac{R_t+a}{1+\theta}. \tag{19}$$

2. Multiplicative Habit Formation

This form of habit formation has also been widely used in the asset pricing and macroeconomics literature.[24] In this case, the utility function takes the following form

$$u(c_{1,t})=\frac{(c_{1,t})^{1-\sigma}}{1-\sigma},\ \text{and}\ v(c_{2,t+1},c_{1,t})=\frac{\left(c_{2,t+1}\left(c_{1,t}\right)^{-\gamma}\right)^{1-\sigma}}{1-\sigma},$$

where $0<\gamma<1$, $0<\sigma$, $\sigma\neq 1$. It is clear that if $c_{1,t}$, $c_{2,t+1}>0$, then $c_{2,t+1}\left(c_{1,t}\right)^{-\gamma}>0$.

[23] When $\sigma\to 1$, $u(c_{1,t})\to\ln\left(c_{1,t}\right)$, and $v(c_{2,t+1},c_{1,t})=\ln\left(c_{2,t+1}-ac_{1,t}\right)$. The results and the derivations are similar and therefore skipped due to the space constraint.

[24] For instance, see Carroll, Overland, and Weil (1997, 2000), Carroll (2000), and the reference therein.

In the appendix, it is shown that equation (18) can be re-written as

$$(1+\theta)\,(q_t)^{\sigma}\,(w-q_t)^{\gamma(1-\sigma)-\sigma}\left[1+\gamma\left(\frac{w}{q_t}-1\right)^{-1}\right]^{-1}=(R_t)^{1-\sigma}\,. \tag{20}$$

In the appendix, it is further shown that under some conditions, there is a unique R_t that solves (19) ((20)) in the case of subtractive (multiplicative) habit formation. Figure 3b shows that, holding other parameter constant, there is a wider range of parameters which generates cyclical land price dynamics with multiplicative habit formation. Virtually any value of θ can generate oscillatory convergence, provided that a suitable value of σ is chosen. The restriction on σ, on the other hand, has not be relaxed. On the other hand, as Figure 3c shows, subtractive habit formation is even worse than the baseline case.

3.2 Catching up with the Joneses

The next class of time-non-separable utility function, "Catching up with the Joneses", can be at least traced back to Abel (1990). Formally, it means that the agent maximizes the life-time utility

$$\max\ u(c_{1,t})+\left(\frac{1}{1+\theta}\right)v(c_{2,t+1},\overline{c_{1,t}}),$$

subject to the constraints (1) and (2), where $\overline{c_{1,t}}$ is the average consumption level of the period t. The consumption in both periods are required to be non-negative, $c_{1,t}, c_{2,t+1}\geq 0$. As before, let $\lambda_{1,t}$ and $\lambda_{2,t}$ be the Lagrangian multipliers of the constraints (1) and (2) respectively. The first order conditions (4) and (5) are still valid. Condition (3) is also valid because the agent takes the average consumption $\overline{c_{1,t}}$ as given. Therefore, combining the first order conditions and the market equilibrium conditions, and the fact that $\overline{c_{1,t}}=c_{1,t}$, we have a modified version of (6),

$$\frac{u'(c_{1,t})}{\left(\frac{1}{1+\theta}\right)\frac{\partial v(c_{2,t+1},c_{1,t})}{\partial c_{2,t+1}}} = \frac{q_{t+1}+r}{q_t} \equiv R_t, \tag{21}$$

which is different from both the time-separable case (see (9)) and habit formation case (see (18)). To compare with the habit formation case, we also consider both subtractive and multiplicative cases.

1. Subtractive Catching Up with Joneses

   Similar to the subtractive habit formation case, the utility functions are assumed to be:

   $$u(c_{1,t}) = \frac{(c_{1,t})^{1-\sigma}}{1-\sigma}, v(c_{2,t+1}, \overline{c_{1,t}}) = \frac{(c_{2,t+1} - a\overline{c_{1,t}})^{1-\sigma}}{1-\sigma},$$

   where $0 < a < 1$, and $\sigma > 0$. It is further imposed that $c_{2,t+1} - ac_{1,t} > 0$. Equation (21) can be re-written as

   $$\left(R_t\left(\frac{w}{q_t} - 1\right)^{-1} - a\right)^{\sigma} = \frac{R_t}{1+\theta}, \tag{22}$$

   which is different from (19) by only a constant term.

2. Multiplicative Catching Up with the Joneses

   Similar to the multiplicative habit formation case, the utility functions take the following form:

   $$u(c_{1,t}) = \frac{(c_{1,t})^{1-\sigma}}{1-\sigma}, v(c_{2,t+1}, \overline{c_{1,t}}) = \frac{\left(c_{2,t+1}\left(\overline{c_{1,t}}\right)^{-\gamma}\right)^{1-\sigma}}{1-\sigma},$$

   where $0 < \gamma < 1$, $\sigma > 0$, and $\sigma \neq 1$. It is clear that if $c_{1,t}$, $c_{2,t+1} > 0$, then we have

$c_{2,t+1}\left(\overline{c_{1,t}}\right)^{-\gamma} > 0$. In the appendix, it is shown that equation (21) can be rewritten as

$$(1+\theta)(q_t)^{\sigma}(w-q_t)^{\gamma(1-\sigma)-\sigma} = (R_t)^{1-\sigma}, \tag{23}$$

which is only different from (20) by one term.

The qualitative as well as quantitative results of catching-up-with-the-Joneses preference, as shown in Figure 3d and 3e, are similar to the case of habit formation.[25] The multiplicative case produces a wider region of parameter combinations that deliver oscillatory land prices, while the subtractive is worse than the baseline case in terms of generating cyclical dynamics.

4 Extension: Endogenous Survival Probability

Thus far, we have assumed that the population is constant over time and all young agents will surely survive in the following period. However, this may not be a good assumption for some historical situations, or some developing countries in the modern time, as the possibility of infant death in those scenarios is not trivial. In light of this, we relax the assumption of constant population: the second period probability of survival is no longer unity but will depend on the first period consumption. This seems to be in agreement with the empirical evidence.[26] An agent may die due to diseases or insufficient "nutrition" she receives in the first period. The probability of survival is assumed to be positively correlated to the first period consumption. If the agent dies in the second period, she receives zero utility and the income from land re-sale and rental will be collected by the government and then

[25] The value of a and γ used in the numerical exercises is rather small and well within the estimate of the empirical literature. See Constantinides (1990), Campbell and Cochrane (1999), Li (2001) and the reference therein.

[26] For an review of the literature, among others, see Kalemli-Ozcan (2003).

re-distributed to the existing population. Formally, the agent maximizes

$$\max \frac{(c_{1,t})^{1-\sigma}}{1-\sigma} + \beta(c_{1,t})\frac{(c_{2,t+1})^{1-\sigma}}{1-\sigma}$$

$$\begin{aligned} s.t.\ c_{1,t} + q_t l_t &\leq w_t + \tau_{1,t}, \\ (q_{t+1} + r_{t+1})\, l_t + \tau_{2,t+1} &\geq c_{2,t+1}, \end{aligned} \tag{24}$$

where $\beta(c_{1t})$ is the probability for an agent born in time t to survive in time $t+1$ given an amount of consumption c_{1t} taken in period t, $0 \leq \beta(c_{1,t}) \leq 1$, $\beta'(.) > 0$, $\beta''(.) < 0$, $\tau_{1,t}$, $\tau_{2,t+1} \geq 0$ are the lump-sum transfer received from the government at the first and second period of time, which is taken as given by the individual. Essentially, the discount factor is endogenized. It is easy to show that

$$\frac{1}{\beta(c_{1,t})}\left(\frac{c_{2,t+1}}{c_{1,t}}\right)^{\sigma} + \frac{\beta'(c_{1,t})}{\beta(c_{1,t})}\frac{c_{2,t+1}}{1-\sigma} = R_t, \tag{25}$$

where $R_t = (q_{t+1} + r_{t+1})/q_t$. To determine the dynamics of the land price, it is necessary to have the exact functional form of the discount factor, and the details of the government budget. Following Kalemli-Ozcan (2002), we assume that

$$\beta(c_{1,t}) = a_0\left(1 - \exp\left(-a_1 c_{1,t}\right)\right), \tag{26}$$

which clearly satisfies the conditions stated previously. Using the World Bank data, Kalemli-Ozcan (2002) estimates that a_0 is in between 0.74 (1960) to 0.82 (1997) and a_1 is in between 0.36 (1960) to 0.44 (1997) for adult survival rate, and the estimates are statistically significant.

The behavior of the government is to tax 100 percent bequests of those who die early and redistributes to the rest of the population. All the income and asset of those who die at the end of the first period will be collected and redistributed in a lump sum manner.[27] We assume that the probability of dying is identically and independently distributed across agents. Thus, by the Law of Large Numbers, the proportion of dying in the aggregate is equal to the probability in the individual level. Since we have assumed that the population size of each cohort is unity, the government budget constraint takes the following form,

$$\tau_{1,t+1} + \beta(c_{1,t})\tau_{2,t+1} = (1 - \beta(c_{1,t}))\,(q_{t+1} + r_{t+1})\,,$$

as the market clearing condition implies $l_t = 1$. It remains to determine how these "accidental bequests" are to be distributed. For simplicity, we assume that all of these bequests go to the surviving old. In this case, $\tau_{1,t+1} = 0$, $\forall t$, and

$$\beta(c_{1,t})\tau_{2,t+1} = (1 - \beta(c_{1,t}))\,(q_{t+1} + r_{t+1})\,. \tag{27}$$

In the appendix, it is shown that the dynamics of the land price can be summarized by the following equation

$$q_{t+1} = \left\{ (\beta c_{1,t})^{\sigma} \left[\frac{\beta}{q_t} - \frac{\beta'}{\beta} \frac{1}{1-\sigma} \right] \right\}^{1/(\sigma-1)} - r. \tag{28}$$

Notice that the discount factor β depends on the first period consumption $c_{1,t}$, which in turn depends on period t land price q_t, as made clear in (24). It is indeed a highly non-linear difference equation. We can then proceed with some numerical examples.

Figure 4 shows that the dynamics and configuration of the steady state can be very rich. Figure 4a presents a case with a stable steady state. In Figure 4b, however, we actually have three steady states, in which one of them is stable (denoted P). Recall our earlier discussion that since there is no state variable and the price is a jump variable by definition,

[27] In Kalemli-Ozcan (2002, p.416-7), it is assumed that capital is not an input in the production and return to land is zero. Thus, the re-distribution of "accidental bequest" is not present in Kalemli-Ozcan (2002). In this paper, the manner of distribution will make a difference.

an "unstable" steady state corresponds to a constant land price in this economy. Therefore, the "mathematically unstable steady states" are in fact very "stable" in an economic sense, as the land price and other variables are constant over time.

On the other hand, a "stable" steady state (mathematically speaking) corresponds to a case of path indeterminacy, where the land price will oscillate around the steady state value. In other words, depends on the initial land price, this economy can stay at one of the two constant land price steady states, or experience cyclical fluctuations in land price. Notice that there is a "multiple steady state" issue here and depends on the initial condition and belief of economic agents, the economy may rest in different steady states.

Notice also that it is not straightforward to order the welfare of agent at different steady states in a Pareto sense. Consider, for example, the economy starts with a steady state at a higher land price. The young agents are then left with less income for food, resulting in a lower probability of survival. This further discourages the young generation to save. A positive death rate also means that, other things being equal, the lump sum transfer from the government to the "survivors" will be higher. For a young agent, she might prefer an economy with low land price and hence she can allocate more resource on food and increase the probability of survival. However, in ex post terms, the agent who survive may prefer to receive a higher payment from the government and that they can sell their land at a higher price. And clearly, with these inter-generational conflict of interest, it is not easy to compare the social welfare across different steady states.

In terms of dynamics, notice that the economy can also switch from one steady state to another when the economy is hit by a shock. In that case, there will be a very dramatic changes in land price and population size (due to the survival probability dependency on the consumption of young generation), on top of the "convergence dynamics" towards the steady state. While this model is highly stylized and should be interpreted with cautions, the results may shed light on the historical experience of certain economies documented in, among others, Gottlieb (1976), Macfarlane (1979), and Sakolski (1932).

[Insert figure 4 here]

5 Concluding Remarks

Land is a very important element in the economy. As the reference cited in the introduction suggests, land is an important factor of production even for modern economies. The land price fluctuation would significantly affect the credit constraints of firms and hence their investment. It could translate into changes in aggregate demand and have an impact to the macroeconomy. This paper illustrates that there is an intrinsic tendency to generate land price cycle in an overlapping generations model, even when factors such as capital market imperfection, informational incompleteness, non-convexity, uncertainty, etc. are abstracted away. It holds with typical time-separable utility function with a wide range of parameter values, and even wider range with multiplicative time-non-separable preference. It seems to suggest that land price cycle is inevitable in a simple general equilibrium OLG model, given a large class of utility functions and a wide spectrum of parameter combinations. Furthermore, we find that when the survival probability of old agents depends on the consumption of when the agents are young, we can have multiple steady states and very rich dynamics under certain parameter values. In particular, a change in some parameter or policy may move the economy from a constant-price steady state to another with land price cycles, or vice versa. Clearly, this model can potentially be extended for interesting policy analysis. In addition, this model also highlights that the interest of young agents may be very different from those who are old. Thus, it may be fruitful to introduce majority voting or other political process into the model. These efforts may contribute to the understanding of the cross-country difference in their historical development experience, especially when land played an important role in the production process.

This research can also be extended in several directions, such as an introduction of physical capital (for instance, see Mountford, 2002), endogenous fertility choice (for instance, see Kalemli-Ozcan, Ryder and Weil, 2000), endogenizing the supply of land, or including residential housing (for instance, see Wheaton 1999, Leung, 2003), the interaction between internet use and land use (for instance, see Quah (2000), Schlauch and Laposa (2001), Shibusawa (2000)). In particular, allowing the land both as an input of production and also as a collateral would enable us to have a deeper understanding of the interactions of cyclical behavior in different sectors of the economy.[28] In terms of welfare analysis, this model can also be extended to include both intra- and inter-generational heterogeneity among agents. It may be proved to be a convenient and plausible vehicle for different policy analysis.

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[28] For instance, see Chen and Leung (2003, 2004).

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A Appendix

A.1 Proof of (13)

Recall that (12) is

$$q_{t+1} + r = q_t \left(1+\theta\right)^{1/(1-\sigma)} \left(\frac{w}{q_t} - 1\right)^{-\sigma/(1-\sigma)}.$$

Now we differentiate both sides with respect to q_t. It gives the following expression:

$$\begin{aligned}\frac{dq_{t+1}}{dq_t} &= (1+\theta)^{1/(1-\sigma)} \left(\frac{w}{q_t} - 1\right)^{-\sigma/(1-\sigma)} \\ &+ q_t \left(1+\theta\right)^{1/(1-\sigma)} \left(\frac{-\sigma}{1-\sigma}\right) \left(\frac{w}{q_t} - 1\right)^{-\sigma/(1-\sigma)-1} \left[\frac{d}{dq_t}\left(\frac{w}{q_t}\right)\right].\end{aligned}$$

Notice that $-\sigma/(1-\sigma) - 1 = -1/(1-\sigma)$, and $\frac{d}{dq_t}\left(\frac{w}{q_t}\right) = -w/\left(q_t\right)^2$. Substituting all these into the expression, we have

$$\begin{aligned}\frac{dq_{t+1}}{dq_t} &= (1+\theta)^{1/(1-\sigma)} \left(\frac{w}{q_t} - 1\right)^{-\sigma/(1-\sigma)} \\ &+ q_t \left(1+\theta\right)^{1/(1-\sigma)} \left(\frac{-\sigma}{1-\sigma}\right) \left(\frac{w}{q_t} - 1\right)^{-1/(1-\sigma)} \left[\frac{-w}{\left(q_t\right)^2}\right].\end{aligned}$$

Clearly, we can group terms and further simplify that as

$$\begin{aligned}\frac{dq_{t+1}}{dq_t} &= (1+\theta)^{1/(1-\sigma)} \left[\left(\frac{w}{q_t} - 1\right)^{-\sigma/(1-\sigma)} \right. \\ &\left. + \left(\frac{\sigma}{1-\sigma}\right) \left(\frac{w}{q_t} - 1\right)^{-1/(1-\sigma)} \frac{w}{(q_t)}\right].\end{aligned}$$

Now notice that

$$\left(\frac{w}{q_t} - 1\right)^{-\sigma/(1-\sigma)} = \left(\frac{w}{q_t} - 1\right)^{-1/(1-\sigma)} \left(\frac{w}{q_t} - 1\right).$$

Thus, we can further simplify it as

$$\begin{aligned}\frac{dq_{t+1}}{dq_t} &= (1+\theta)^{1/(1-\sigma)} \left(\frac{w}{q_t} - 1\right)^{-1/(1-\sigma)} \\ &\cdot \left[\left(\frac{w}{q_t} - 1\right) + \left(\frac{\sigma}{1-\sigma}\right) \frac{w}{q_t}\right],\end{aligned}$$

which can be shown to be equal to the following expression,

$$\frac{dq_{t+1}}{dq_t} = (1+\theta)^{1/(1-\sigma)} \left(\frac{w}{q_t} - 1\right)^{-1/(1-\sigma)} \left[(1-\sigma)^{-1} \left(\frac{w}{q_t}\right) - 1\right],$$

which is indeed (13).

A.2 Proof of (14)

Again, we recall (12). Now, at the steady state, $q_{t+1} = q_t$. For simplicity, we call that value q. Thus, we have

$$q + r = q\,(1+\theta)^{1/(1-\sigma)} \left(\frac{w}{q} - 1\right)^{-\sigma/(1-\sigma)}.$$

Now,

$$\text{if } q > 0 \Longrightarrow q + r > r,$$

which means that

$$q\,(1+\theta)^{1/(1-\sigma)} \left(\frac{w}{q} - 1\right)^{-\sigma/(1-\sigma)} > r.$$

Re-arranging terms, we will have (14).

A.3 Proofs of the Cases of Non-separable Utility Functions

1. Subtractive Habit Formation
   This form of time-non-separable utility can be at least traced back to the seminal work of Constantinides (1990). In this case, the utility function takes the following form

   $$u(c_{1,t}) = \frac{(c_{1,t})^{1-\sigma}}{1-\sigma}, \text{ and } v(c_{2,t+1}, c_{1,t}) = \frac{(c_{2,t+1} - ac_{1,t})^{1-\sigma}}{1-\sigma},$$

   where $0 < a < 1$, $\sigma > 0$, $\sigma \neq 1$. It is further imposed that $c_{2,t+1} - ac_{1,t} > 0$. Equation (18) can be re-written as

   $$\frac{(c_{1,t})^{-\sigma} - a(1+\theta)^{-1} (c_{2,t+1} - ac_{1,t})^{-\sigma}}{(1+\theta)^{-1} (c_{2,t+1} - ac_{1,t})^{-\sigma}} = R_t,$$

   which means that

   $$(1+\theta) \left(\frac{c_{2,t+1}}{c_{1,t}} - a\right)^{\sigma} = R_t + a$$

   and

   $$\frac{c_{2,t+1}}{c_{1,t}} = \frac{q_{t+1}+r}{w-q_t} = \frac{q_{t+1}+r}{q_t} \bullet \frac{q_t}{w-q_t} = R_t \left(\frac{q_t}{w-q_t}\right).$$

   Thus, it delivers

   $$\left(R_t \left(\frac{w}{q_t} - 1\right)^{-1} - a\right)^{\sigma} = \frac{R_t + a}{1+\theta}.$$

Notice that with any fixed level of q_t, both sides of (19) are increasing in R_t. The right hand side is positive even when $R_t = 0$. In fact, it is linear in R_t, with a positive slope $(1+\theta)^{-1} < 1$. On the other hand, except when σ is an integer, the left hand side is not well defined for $R_t < a\,(w/q_t - 1)$. For $R_t > a\,(w/q_t - 1) > 0$, the left hand side is concave/linear/convex in R_t if $\sigma < 1/\ \sigma = 1/\ \sigma > 1$. Clearly, given a value of q_t and for $\sigma > 1$, there is a unique value of R_t which solves (19). For $\sigma < 1$, there might and might not be a solution, depending on the combination of the parameter values. Assuming that there is a solution for (19),[29] it is easy to show that

$$\frac{dq_{t+1}}{dq_t} = \frac{f_{1t}}{f_{2t}}, \tag{29}$$

where $f_{1t} = (1+\theta)^{-1}R_t\,(q_t)^{-1} \ \ +\sigma R_t q_t\,(w-q_t)^{-2}\left(R_t\left(\frac{w}{q_t}-1\right)^{-1} - a\right)^{\sigma-1} > 0$, and $f_{2t} = (1+\theta)^{-1}\,(q_t)^{-1} \ -\sigma\left(R_t\left(\frac{w}{q_t}-1\right)^{-1} - a\right)^{\sigma} * \left[R_t\,(q_t)^{-1} - a(w-q_t)\right]^{-1} = (1+\theta)^{-1}\,(q_t)^{-1} \ -\sigma(w-q_t)^{-1}\left(R_t\left(\frac{w}{q_t}-1\right)^{-1} - a\right)^{\sigma-1}$. Clearly, if $R_t > 1$, then $f_{1t} > f_{2t}$. Furthermore, when $f_{2t} > 0$, we have $dq_{t+1}/dq_t > 1$ (which means monotone and explosive), but when $f_{2t} < 0$, dq_{t+1}/dq_t is negative and $|dq_{t+1}/dq_t|$ can be either larger than unity (which means oscillatory and explosive) or smaller than unity (which means oscillatory and stable). Thus, whether the steady state exists, or whether it displays any oscillatory or monotonic dynamics can only be verified numerically for plausible parameter values.

2. Multiplicative Habit Formation
   This form of habit formation has also be widely used in the asset pricing and macroeconomics literature.[30] In this case, the utility function takes the following form

$$u(c_{1,t}) = \frac{(c_{1,t})^{1-\sigma}}{1-\sigma}, \text{ and } v(c_{2,t+1}, c_{1,t}) = \frac{\left(c_{2,t+1}\,(c_{1,t})^{-\gamma}\right)^{1-\sigma}}{1-\sigma},$$

where $0 < \gamma < 1$, $0 < \sigma$, $\sigma \neq 1$. It is clear that if $c_{1,t}$, $c_{2,t+1} > 0$, then $c_{2,t+1}\,(c_{1,t})^{-\gamma} > 0$. Equation (18) can be re-written as

$$\frac{(c_{1,t})^{-\sigma} - \gamma(1+\theta)^{-1}\left(c_{2,t+1}\,(c_{1,t})^{-\gamma}\right)^{-\sigma} c_{2,t+1}\,(c_{1,t})^{-\gamma-1}}{(1+\theta)^{-1}\left(c_{2,t+1}\,(c_{1,t})^{-\gamma}\right)^{-\sigma}(c_{1,t})^{-\gamma}} = R_t.$$

Note that the left hand side is equal to

$$(1+\theta)\left(\frac{c_{2,t+1}}{c_{1,t}}\right)^{\sigma}(c_{1,t})^{\gamma(1-\sigma)} - \gamma\left(\frac{c_{2,t+1}}{c_{1,t}}\right),$$

[29] For any given set of parameter values, the existence can be easily verified numerically.

[30] For instance, see Carroll, Overland and Weil (1997, 2000), Carroll (2000), and the reference therein.

and

$$\frac{c_{2,t+1}}{c_{1,t}} = \frac{q_{t+1}+r}{w-q_t} = \frac{q_{t+1}+r}{q_t} \bullet \frac{q_t}{w-q_t} = R_t \left(\frac{w}{q_t} - 1\right)^{-1},$$

thus we can rewrite the earlier equation as

$$(1+\theta)\left(q_t\right)^{\sigma}\left(w-q_t\right)^{\gamma(1-\sigma)-\sigma}\left[1+\gamma\left(\frac{w}{q_t}-1\right)^{-1}\right]^{-1} = \left(R_t\right)^{1-\sigma},$$

where $R_t = (q_{t+1}+r)/q_t$. Notice that the left hand side of (20) is invariant to R_t and the right hand side is monotonic increasing (decreasing) in R_t if $\sigma < 1$ ($\sigma > 1$). Notice also that the left hand side is always positive, while the right hand side is equal to zero when $R_t = 0$. Taking log on both sides of (20) and utilitize the fact that $dR_t/dq_t = \left(dq_{t+1}/dq_t - R_t\right)/q_t$, it is easy to show that

$$\left(\frac{1-\sigma}{R_t q_t}\right)\frac{dq_{t+1}}{dq_t} = \underbrace{\frac{1}{q_t}}_{+ve} - \underbrace{\left[\frac{\gamma w (w-q_t)^{-2}}{1+\gamma\left(\frac{w}{q_t}-1\right)^{-1}} + \frac{\gamma(1-\sigma)-\sigma}{w-q_t}\right]}_{+ve},$$

where $R_t q_t = q_{t+1} + r$ by definition. Hence, dq_{t+1}/dq_t can be either positive or negative. Thus, whether the steady state exists, or whether it displays any oscillatory or monotonic dynamics can only be verified numerically for plausible parameter values.

3. Subtractive Catching Up with the Joneses

Formally, it means that the agent maximizes the life-time utility

$$\max\ u(c_{1,t}) + \left(\frac{1}{1+\theta}\right) v(c_{2,t+1}, \overline{c_{1,t}}),$$

subject to the constraints (1) and (2), where $\overline{c_{1,t}}$ is the average consumption level of the period t. The consumption in both periods are required to be non-negative, $c_{1,t}, c_{2,t+1} \geq 0$. As before, let $\lambda_{1,t}$, $\lambda_{2,t}$ be the Langrange multipliers of the constraints (1) and (2) respectively. The first order conditions (4) and (5) are still valid. Condition (3) is also valid because the agent takes the average consumption $\overline{c_{1,t}}$ as given. Therefore, combining the first order conditions and the market equilibrium conditions, and the fact that

$$\overline{c_{1,t}} = c_{1,t},$$

delivers a modified version of (6),

$$\frac{u'(c_{1,t})}{\left(\frac{1}{1+\theta}\right)\frac{\partial v(c_{2,t+1}, c_{1,t})}{\partial c_{2,t+1}}} = \frac{q_{t+1}+r}{q_t} \equiv R_t, \tag{30}$$

which is different from both the time-separable case (see (9)) and habit formation case (see (18)). To fix the idea, we also consider both subtractive and multiplicative cases. To be comparable with the subtractive habit formation case, the utility functions are assumed to be similar: $u(c_{1,t}) = \frac{(c_{1,t})^{1-\sigma}}{1-\sigma}$, and $v(c_{2,t+1}, \overline{c_{1,t}}) = \frac{(c_{2,t+1} - a\overline{c_{1,t}})^{1-\sigma}}{1-\sigma}$, $0 < a < 1$, $0 < \sigma$. It is further imposed that $c_{2,t+1} - ac_{1,t} > 0$. Equation (30) can be re-written as

$$(1+\theta)\left(\frac{c_{2,t+1}}{c_{1,t}} - a\right)^{\sigma} = R_t.$$

And similar to the case of subtractive habit formation, it can be further simplified as

$$\left(R_t\left(\frac{w}{q_t} - 1\right)^{-1} - a\right)^{\sigma} = \frac{R_t}{1+\theta}, \tag{31}$$

which is different from (19) by only a constant term. Again, with any fixed level of q_t, both sides of (22) are increasing in R_t. The right hand side is zero when $R_t = 0$. In fact, it is linear in R_t, with a positive slope $(1+\theta)^{-1} < 1$. On the other hand, except when σ is an integer, the left hand side is not well defined for $R_t < a\left(\frac{w}{q_t} - 1\right)$. For $R_t > a\left(\frac{w}{q_t} - 1\right)$, the left hand side is concave/linear/convex in R_t if $\sigma < 1/\ \sigma = 1/\ \sigma > 1$. Clearly, with q_t being fixed and for $\sigma > 1$, there is a unique value of R_t which solves (22). For $\sigma \leq 1$, there might and might not be a solution, depending on the combination of the parameter values. We assume that there is a solution for (22).[31] As in the subtractive habit formation case, whether the steady state exists, or whether it displays any oscillatory or monotonic dynamics can only be verified numerically for plausible parameter values.

4. Multiplicative Catching Up with the Joneses

   To be comparable with the subtractive habit formation case, the utility functions assumed are again similar: $u(c_{1,t}) = \frac{(c_{1,t})^{1-\sigma}}{1-\sigma}$, and $v(c_{2,t+1}, \overline{c_{1,t}}) = \frac{\left(c_{2,t+1}(\overline{c_{1,t}})^{-\gamma}\right)^{1-\sigma}}{1-\sigma}$, $0 < \gamma < 1$, $0 < \sigma$, $\sigma \neq 1$. It is clear that if $c_{1,t}$, $c_{2,t+1} > 0$, $c_{2,t+1}\left(\overline{c_{1,t}}\right)^{-\gamma} > 0$. Equation (30) can be re-written as

$$\frac{(c_{1,t})^{-\sigma}}{(1+\theta)^{-1}\left(c_{2,t+1}\left(c_{1,t}\right)^{-\gamma}\right)^{-\sigma}\left(c_{1,t}\right)^{-\gamma}} = R_t.$$

   Since the left hand side is equal to

$$(1+\theta)\left(\frac{c_{2,t+1}}{c_{1,t}}\right)^{\sigma}(c_{1,t})^{\gamma(1-\sigma)},$$

   and

$$\frac{c_{2,t+1}}{c_{1,t}} = \frac{q_{t+1}+r}{w-q_t} = \frac{q_{t+1}+r}{q_t} \bullet \frac{q_t}{w-q_t} = R_t\left(\frac{w}{q_t} - 1\right)^{-1},$$

[31] For any given set of parameter values, the existence can be easily verified numerically.

the earlier equation can be re-written as

$$(1+\theta)\,(q_t)^{\sigma}\,(w-q_t)^{\gamma(1-\sigma)-\sigma}=(R_t)^{1-\sigma}\,,$$

which is only different from (20) by one term. The left hand side of (23) is invariant to R_t and the right hand side is monotonic increasing (decreasing) in R_t if $\sigma<1$ ($\sigma>1$). Notice also that the left hand side is always positive, while the right hand side is equal to zero when $R_t=0$. Taking log on both sides of (23) and utilitize the fact that $\frac{dR_t}{dq_t}=\frac{1}{q_t}\left(\frac{dq_{t+1}}{dq_t}-R_t\right)$, it is easy to show that

$$\left(\frac{1-\sigma}{R_t q_t}\right)\frac{dq_{t+1}}{dq_t}=\underbrace{\frac{1}{q_t}}_{+ve}-\underbrace{\frac{\gamma(1-\sigma)-\sigma}{w-q_t}}_{+ve},$$

where $R_t q_t=q_{t+1}+r$ by definition. Hence, $\frac{dq_{t+1}}{dq_t}$ can be either positive or negative. Thus, whether the steady state exists, or whether it displays any oscillatory or monotonic dynamics can only be verified numerically for plausible parameter values.

A.4 Some Derivations for the Case of Endogenous Survival Probability

Notice that by (26), we have

$$\begin{aligned}\beta'(c_{1t}) &= a_1 a_0 \exp\left(-a_1 c_{1t}\right)\\ &= a_1\left(a_0-\beta(c_{1t})\right).\end{aligned}$$

We can combine (24) with (27), we have

$$\beta(c_{1t})=\frac{q_{t+1}+r}{c_{2,t+1}}. \tag{32}$$

We substitute this into (25) and then we have

$$\frac{(c_{2,t+1})^{\sigma}}{\beta\,(c_{1,t})^{\sigma}}=(c_{2,t+1})\left[\frac{\beta}{q_t}-\frac{\beta'}{\beta}\frac{1}{1-\sigma}\right],$$

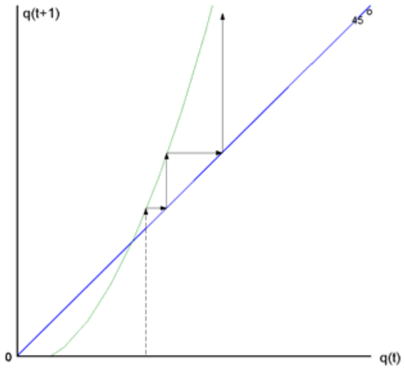
(with the argument of β suppressed), and combining this with (32) delivers (28)

$$q_{t+1}=\left\{(c_{1,t}\beta)^{\sigma}\left[\frac{\beta}{q_t}-\frac{\beta'}{\beta}\frac{1}{1-\sigma}\right]\right\}^{1/(\sigma-1)}-r.$$

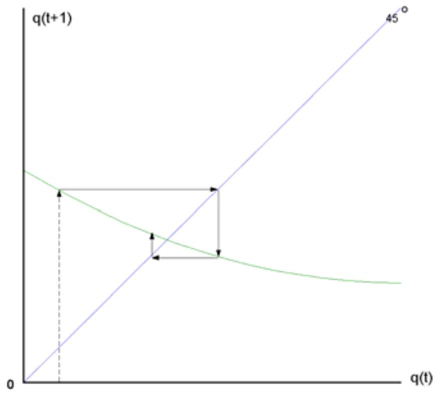
Notice that under this regime, $c_{1,t}=w-q_t$, and β, β' are functions of $c_{1,t}$ only. Thus, the right hand side of (28) is in terms of q_t only, and (28) describes a very non-linear relationship between q_t and q_{t+1}.

Figure 1 Dynamics of Land Prices

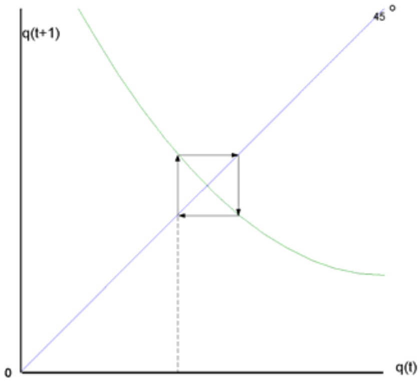
(1a) unstable ($0 < \sigma < 1$)



(1b) oscillatory convergence ($\sigma > 1$)



(1c) center ($\sigma > 1$)



(1d) oscillatory divergence ($\sigma > 1$)

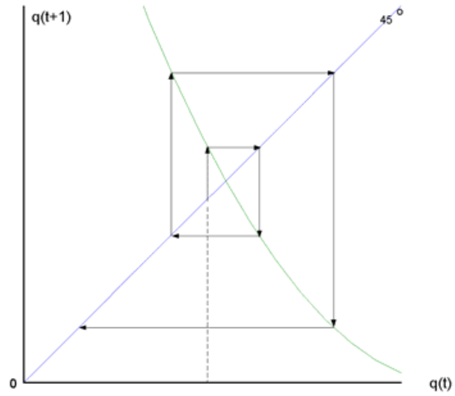
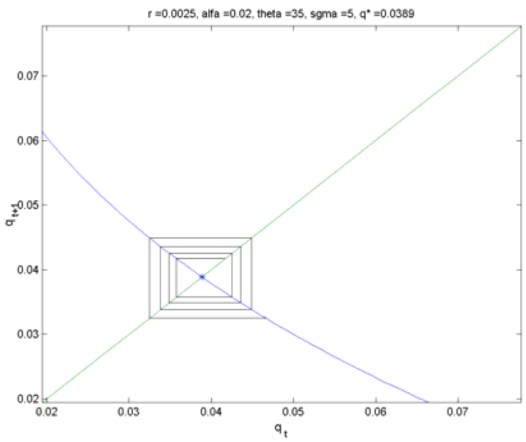


Figure 2 Numerical Examples for "oscillatory convergence" and "cycles"

(2a) oscillatory convergence



(2b) cycles

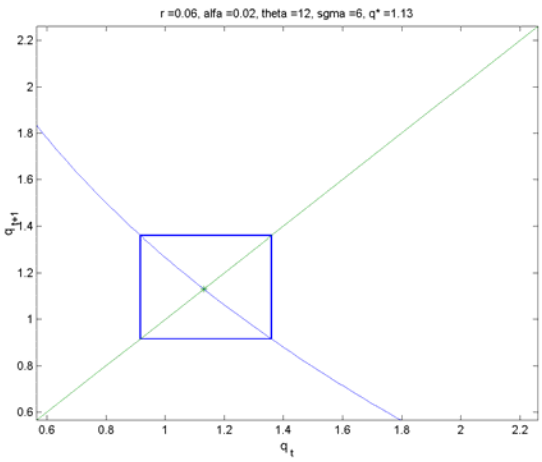


Figure 3a   Benchmark (r=0.04)

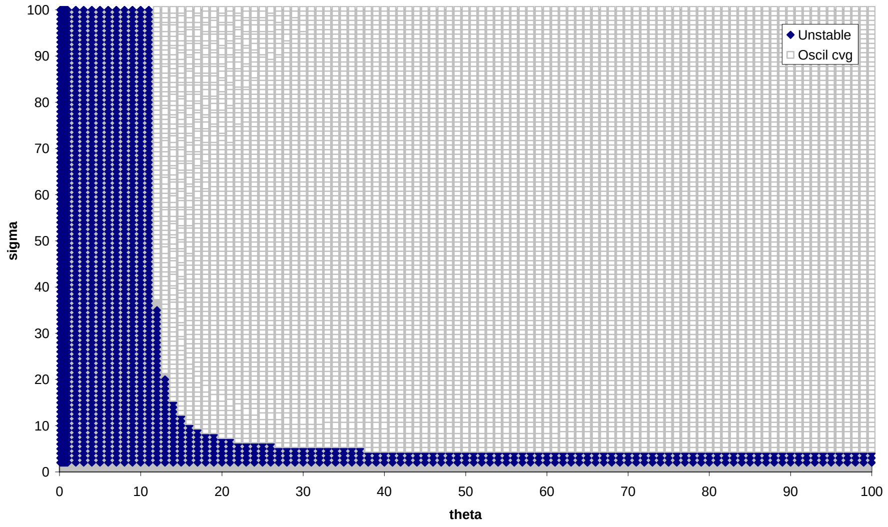


Figure 3b   Habit Multiplicative (r=0.04, gamma=0.3333)

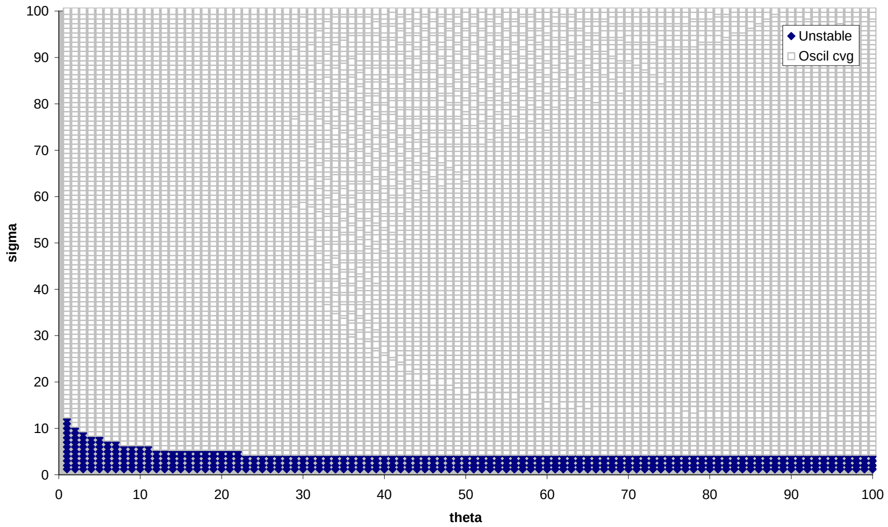


Figure 3c   Habit Subtractive (r=0.04, w=0.36)

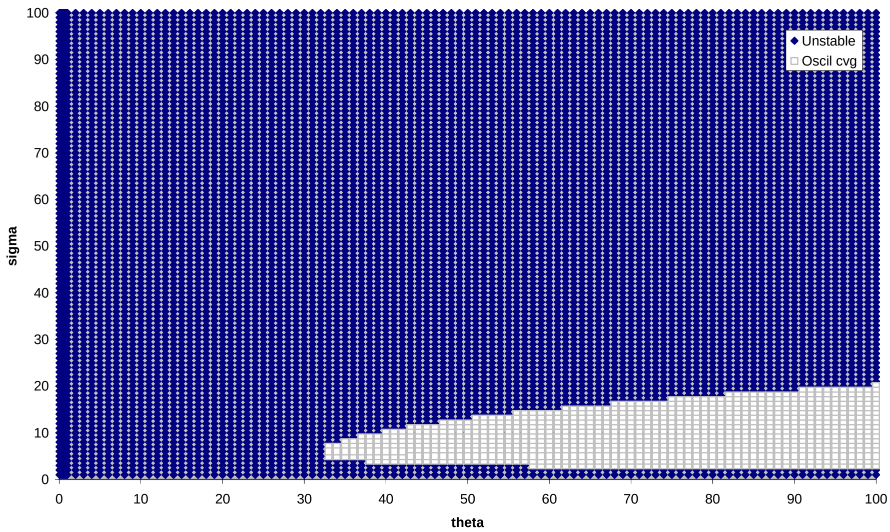


Figure 3d Jones Subtractive (r=0.04, w=0.36)

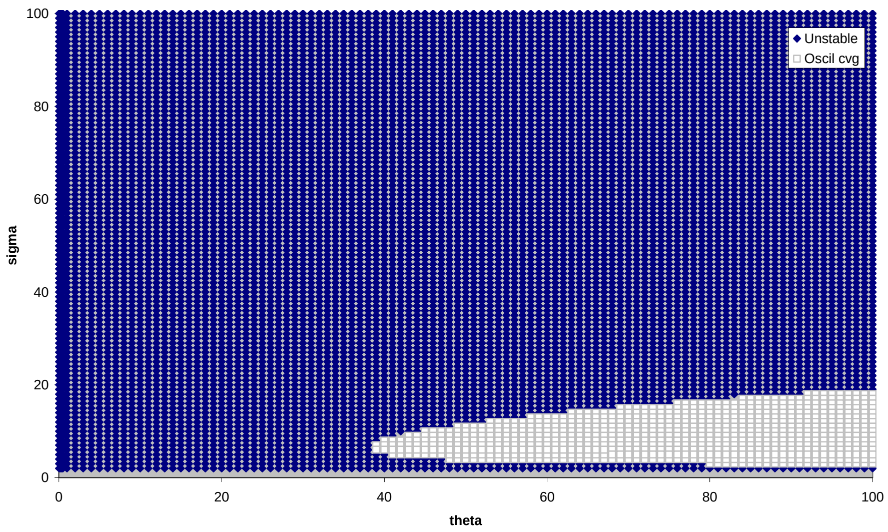


Figure 3e Jones Multiplicative (r=0.04, gamma=0.3333)

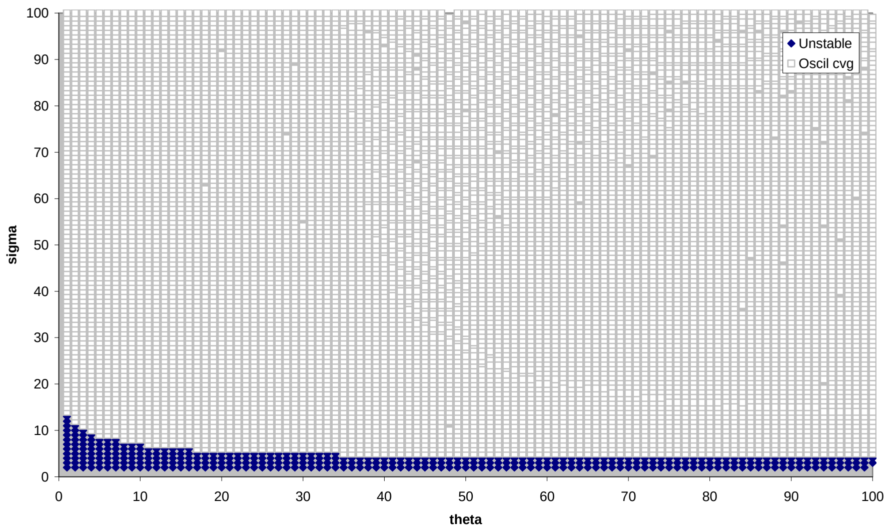
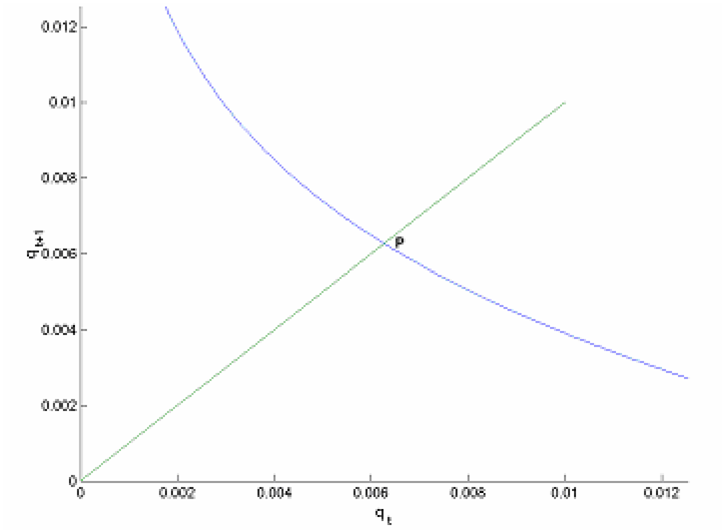


Figure 4 Dynamics of Land Price with Endogenous Survival Probability

(a) Stable cycle (r=0.02, sigma=8, w=0.3, tau1(t=1)=0.0001)



(b) Multiple steady states (r=0.04, sigma=2.5, w=0.5, tau1(t=1)=0.0001)