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Firms' location under taste and demand heterogeneity

Toshihiro OKUBO [1] and Pierre M. PICARD[2]

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Abstract

In this paper we build a quality-augmented version of an economic geography model where consumers have heterogenous tastes for a set of manufacturing varieties. We discuss a footloose capital model and a footloose entrepreneur model. We show that firms selling the goods with higher values select the region hosting the largest number of consumers. Larger countries thus get better access to the higher quality products. We also show that the effect of spatial selection on firms' spatial distribution crucially depends on the properties of the taste distribution across varieties. Finally, we show that taste heterogeneity smooths the agglomeration patterns but that it should be considered neither as a dispersion force nor as an agglomeration force. Indeed, the introduction of taste heterogeneity makes an initially dispersed economy less dispersed and an initially agglomerated economy less agglomerated.

Keywords: heterogeneous taste and quality, spatial selection, economic geography, agglomeration, home market effect.

JEL Classification: F12, F15, R11, R12

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1 Introduction

Many economists like to think that the world is homogenous. But it is not. In particular, firms tend to specialize in product niches with unequal success. In some niches, firms are able to sell high value products whereas in other niches they can only offer lower value products to their consumers. Firms therefore turn out to be heterogenous with respect to demand for their specific products; that is, with respect to the consumers' taste for the specific characteristic of their products. Heterogeneity in demand and taste is known to impact on the structure of trade. Indeed, exporting firms are known to quote different export prices and to offer goods with higher values (e.g. Baldwin and Harrigan, 2008; Foster et al. 2008). However this type of heterogeneity is also likely to impact on firms' location and therefore on the regional composition of industries. It is likely that large regions and cities host a larger share of production and firms but also that they produce goods with higher value and higher demand. Conversely, peripheral regions are prone to host a smaller share of production and to produce goods with lower value.

The present paper studies the effect of taste and demand heterogeneity on firms' trade and location. In particular, it is concerned with the role of the size of countries (cities, or regions) on the spatial distribution of firms and product varieties. It is well-know that firms' mobility fosters discrepancies in economic activity (see Krugman 1991). It is however less clear whether and how firms' mobility fosters discrepancies in the value of goods supplied by regions. This is precisely the focus of the present paper: Do high value added firms self-select in some countries, giving a quality premium or a value added premium for those countries? Does such heterogeneity exacerbate or reduce the home market effect, according to which the share of firms is larger than the share of consumers in the larger region? Does the distribution of taste and demand across firms' product varieties have a significant impact on the spatial distribution of firms?

In this paper we build a quality-augmented version of Ottaviano *et al.* 's (2002) model where consumers have heterogenous tastes for a set of manufacturing varieties. As in Syverson (2008), the linear properties of the demand system of this model are particularly well suited for such an analysis. As is usual in the literature, each firm produces a distinct variety, competes under monopolistic competition and chooses its location in one of two countries. We envisage a footloose capital model and a footloose entrepreneurs model. In the footloose capital model, all consumers are immobile and capitalists allocate their capital to countries offering the highest return to capital. In the footloose entrepreneur model, firms are run by mobile entrepreneurs (or skilled workers) who choose the location that offers the best outcome in terms of earnings and consumer surplus. Because entrepreneurs move with their firms, the demand for manufacturing varieties follows the firms' movement and creates a demand linkage. In each set up, we derive the price equilibrium conditions and the firms' location equilibrium conditions. We then discuss the impact of the taste and demand distributions on the location equilibrium.

We obtain the following results. In both models, we firstly show that firms selling the goods with higher demand and higher value select the region hosting the largest number of consumers. As a result, *larger countries do not only get a better access to more varieties* as usually emphasized in the economic geography literature but they also get *a better access to the products for which their consumers put a higher value.* We secondly show that the skewness of the consumer's taste distribution has an important effect on the home market effect. A home market effect exists if a country's share of industry is more than proportional than its share of population. We show that, in the footloose capital model, *the introduction of taste heterogeneity reduces the home market effect only if the taste distribution is skewed towards high taste varieties.* That is, if the economy does not include too many highly demanded varieties. We further investigate the impact of changes in the taste distribution and show that such changes have ambiguous effects. Those results therefore suggest that the taste distribution is an important determinant of the location pattern and that it is should not be neglected in the theoretical and empirical work where taste and cost distributions are often assumed for their convenient analytical properties.[1]

We thirdly show that, in the footloose entrepreneur model, the taste distribution has important consequences on the number of an equilibrium, on the impact of trade costs and on the possibilities of catastrophic changes. We indeed establish the condition for a unique location equilibrium, a condition that crucially depends on the taste distribution. We also show that *heterogeneity smoothens catastrophic agglomeration processes* because the introduction of taste heterogeneity eliminates the possibilities of catastrophic changes.[2]

[1] Cabral and Mata (2001) study the distribution of firm size in more depth.

[2] Such a result about the impact of heterogeneity is not uncommon. For instance, it is well-know in the Industrial Organization literature that the dramatic outcome of Bertand price competition is smoothened by the introduction of consumer heterogeneity as done in the Hotelling model. Indeed,, homogeneous agent simultaneously change their decision whereas heterogeneous agents change their strategies for different pa-

Finally, we show that taste heterogeneity should be considered *neither as a dispersion force nor as an agglomeration force* (e.g. see Baldwin and Okubo 2006). Indeed, compared to the homogenous taste model, the introduction of heterogeneity has ambiguous effects on the dispersion and agglomeration of firms. In particular, *the introduction of taste heterogeneity makes an initially dispered economy less dispersed and an initially agglomerated economy less agglomerated*. It is a force that entices entrepreneurs to agglomerate *partially*. These results open an interesting empirical question about the impact on firms' location of the taste distributions across manufacturing varieties.

The paper is structured as it follows. The remaining part of this introduction presents a deeper review of the literature. Section 2 presents the model. Section 3 and 4 derive the spatial selection and the location equilibria in footloose capital and footloose entrepreneur models under taste heterogeneity. Section 5 extends the model to the simple case where higher taste and higher demand products are more expensive to produce. Section 6 concludes.

Related literature This paper is closely related to two strands of the International Trade literature on product quality and trade composition and on firms' heterogeneity. The first strand traditionally discusses vertical differentiation models in which higher quality products are more likely to be consumed and produced in high wage countries (Linden 1961, Falvey 1981, Falvey and Kierzkowski 1987 and Flam and Helpman 1987, Stockey 1991).[3] Murphy and Shleifer (1997) develop a model where high quality products end up being produced in high human capital countries. More recently, Feenstra and Romalis (2006) extend the Heckscher Ohlin model to product qualities. In contrast to our paper, none of those models study the (re-)location of firms. The relationship between product quality and location choice has remained an open research question. On the other hand, empirical studies show that the quality or the value of goods plays a crucial and important role in international trade pattern. For instance, using US commodity trade data, Schott (2004) finds that the

rameter or variable values.

[3]Falvey (1981) models different endowments. As a consequence, he finds that capital abundant country exports capital-intensive high quality products whereas it imports labor-intensive low quality products. Then, Falvey and Kierzkowski (1987) added technology difference to dissimilar factor endowments. Flam and Helpman (1987) use a Ricardian technology difference to model quality trade. Stockey (1991) suggests that consumers in the rich country consume more of the same quality products than consumers in the poor country.

unit value of trade within one product line is higher for high-wage countries. Hummels and Klenow (2005) find that richer countries export higher value goods. Hallak (2006) finds that rich countries import relatively more from the countries producing high quality goods.[4] To sum up, while there exist quantitative evidence of heterogeneity of product quality in the trade patterns, the analysis of the relationship between product quality and firms' location has been unexplored until the present contribution

The second strand of literature related to this paper concerns firms' cost heterogeneity (Melitz, 2003; Helpman, Melitz and Yeaple, 2004; Falvey *et al.,* 2004; Melitz and Ottaviano, 2008; Baldwin and Robert-Nicoud, 2008; Okubo, 2008). This literature responds to extensive empirical research on trade behavior at firm level[5] and focuses on the impact of trade liberalization on the average productivity and on the trade behaviors of firms with heterogenous productivity. Yet, cost heterogeneity is not the only critical characteristic that explains trade patterns. For some authors, trade is better explained by demand or quality heterogeneity than by cost heterogeneity (see Foster *et al.* 2008). Indeed, trade data suggest a positive correlation between product prices and exporting status of firms, which confirms the premise according to which exporting firms produce better quality or more demanded goods and which invalidates the idea that exporting firms have lower costs (see Baldwin 2005, Greenaway 1995 and Greenaway *et al.* 1995; Fukao *et al.* 2003; Okubo, 2007). So, a deeper investigation of the impact of demand and quality heterogeneity is welcome.

We note that our investigation on the quality or taste heterogeneity is not entirely new. Currently, Baldwin and Harrigan (2008) and Khandelwal (2007) study heterogeneous quality in international trade models. However, none of those papers study the issue of the firms' location and the self-selection of firms according to the value and/or the quality of their products. Instead, our paper is in line of economic geography models that discuss the relocation of firms between regions. We discuss the possibility of firms' spatial selection, in which high quality firms agglomerate in the large market and low quality firms locate in the small market. We furthermore pay careful attention to the way the value, the quality or the

[4] Hummels and Klenow (2005) use import data from 76 countries at the six digit level of the Harmonized System and then find that the quality margin is a function of the exporter size. Hallak (2006) analyzes bilateral trade flows among 60 countries.

[5] Many empirical studies give evidence on the relationship between trade and firm productivity through trade liberalization. See Bernard, Eaton, Jensen and Kortum (2003) Bernard, Jensen, and Schott (2004), Pavcnik (2002) and Tybout and Westbrook (1995).

taste for varieties is distributed across firms. Our geography model on heterogeneous quality extends the currently emerging trade literature on firm heterogeneity (Foster *et al.* 2008).

2 The model

Our model extends the framework of Ottaviano *et al.* (2002) and Ottaviano and Thisse (2004) by allowing for heterogenous tastes. In this section, we present the basic model and characterize the product market outcome for any given organizational structure and spatial distribution of firms.

2.1 Preferences

Consider a world with two countries, labeled H and F. Variables associated with each country will be subscripted accordingly. We assume that there is a mass L of consumers, with a share $1/2 \leq \theta_H < 1$ located in country H. In what follows, we refer to H as the large and to F as the small country.

All consumers in country $i = H, F$ have identical quasi-linear preferences over a homogenous good and a continuum of horizontally differentiated varieties, indexed by $v \in \mathcal{V} \equiv [0, 1]$. As in Ottaviano *et al.* (2002), the utility of a representative agent in country i is given by the following quadratic function:

$$U_i = \int_{\mathcal{V}} \widehat{\alpha}(v) q_i(v) \mathrm{d}v - \frac{\beta - \gamma}{2} \int_{\mathcal{V}} [q_i(v)]^2 \mathrm{d}v - \frac{\gamma}{2} \left[\int_{\mathcal{V}} q_i(v) \mathrm{d}v \right]^2 + q_i^o, \tag{1}$$

where $q_i(v)$ denotes the consumption of variety v in country i and q_i^o stands for the consumption of the homogenous good in that same country. As in Ottaviano *et al.* (2002), γ is a measure of the degree of substitution between varieties whereas $\beta - \gamma$ (> 0) measures the consumer bias toward a more dispersed consumption of varieties.

The new element in this model is the function $\widehat{\alpha}(v) : \mathcal{V} \rightarrow [\underline{\alpha}, \overline{\alpha}]$, $0 < \underline{\alpha} \leq \overline{\alpha}$, that measures the *willingness to pay* for variety v.[6] Willingness to pay is heterogenous and reflects the intensity of consumer's preferences for each differentiated product v with respect to the homogenous good. Without loss of generality we assume that varieties are ranked according to the consumers' willingness to pay, i.e. such that $v > v' \iff \widehat{\alpha}(v) > \widehat{\alpha}(v')$. Because

[6] In the spirit of Baldwin and Harrigan (2008) $\widehat{\alpha}(v)$ can be called as the quality of variety v. We will have a closer look at this interpretation in Section 5.

there is a unit mass of varieties, the inversion of the function $\widehat{\alpha}$ yields the taste cumulative distribution : $F_\alpha : [\underline{\alpha}, \overline{\alpha}] \rightarrow [0,1]$, $F_\alpha(x)$ =Proba$[v : \widehat{\alpha}(v) \leq x] = \widehat{\alpha}^{-1}(x)$. The taste distribution density is then the function $f_\alpha : [\underline{\alpha}, \overline{\alpha}] \rightarrow \mathcal{R}$, $f_\alpha(x) = F'_\alpha(x) = 1/[\widehat{\alpha}'(\widehat{\alpha}^{-1}(x))]$. Because of this close relationship, we will refer to $\widehat{\alpha}$ as the *taste* (function) and to F_α and f_α as the *taste cumulative distribution* and the *taste distribution density* across varieties. Note finally that the consumers have identical preferences: there is no priori 'regional preferences' (as in Tabuchi and Thisse (2001)) or 'local preferences' (e.g. Mossay, 2006). Using this vocabulary, we therefore call the *average taste* the parameter $\alpha \equiv \int_{\mathcal{V}} \widehat{\alpha}(v)\mathrm{d}v$.

Each agent maximizes his/her utility (1) subject to his/her budget constraint:

$$\int_{\mathcal{V}} p_i(v) q_i(v)\mathrm{d}v + p_i^o q_i^o \leq w_i + \overline{q}^o, \tag{2}$$

where $p_i(v)$ denotes the consumer price of variety v; w_i stands for the wage in country $i = H, F$. Following Ottaviano *et al.* (2002), we assume that consumers own a sufficiently large endowment $\overline{q}^o > 0$ of the numéraire. Consequently, consumers are not constrained in their consumption of the differentiated varieties and spend the rest of their income on the homogenous numéraire good. As a by-product, this eliminates the income effects. As will become clear in the sequel, free trade in the homogenous good market leads to price equalization across countries, thus making this good a natural choice for the numéraire ($p_i^o = 1,\ i = H, F$).

We assume that all varieties are consumed. Maximizing the utility (1) subject to the budget constraint (2) yields the following first order condition:

$$\widehat{\alpha}(v) - (\beta - \gamma)\, q_i(v) - \gamma \int_{\mathcal{V}} q_i(\xi) d\xi - p_i(v) = 0 \tag{3}$$

Integrating the left hand side of this equality yields the average taste

$$\alpha = \beta \int_{\mathcal{V}} q_i(v)\mathrm{d}v + \int_{\mathcal{V}} p_i(v)\mathrm{d}v \tag{4}$$

The last two expressions allows us to derive the individual demand for variety v as the following linear formula:

$$q_i(v) = \widehat{a}(v) - (b + c)\, p_i(v) + c\mathbb{P}_i \tag{5}$$

where

$$\mathbb{P}_i = \int_{\mathcal{V}} p_i(v)\mathrm{d}v$$

is the manufacturing price index in country i and where b and c are the following positive coefficients

$$b = \frac{1}{\beta} \quad \text{and} \quad c = \frac{\gamma}{\beta\left(\beta - \gamma\right)} \tag{6}$$

The parameter b measures the price sensitivity of demand and the parameter c as the degree of product differentiation. In particular, when $c \to 0$ varieties are perfectly differentiated, whereas they become perfect substitutes when $c \to \infty$.

In expression (5), the function $\widehat{a}(v)$ is equal to

$$\widehat{a}(v) = \widehat{\alpha}(v)\left(b + c\right) - c\alpha \tag{7}$$

and measures the *demand size* for variety v. This function is indeed equal to the consumer's demand when all prices are nil. At given prices and average taste, a change in $\widehat{a}(v)$ fully reflects a change in the taste $\widehat{\alpha}(v)$ of the variety v. Varieties for which consumers express a high taste have higher demands. For the sake of convenience, the minimum and the maximum demand size are defined as $\underline{a} \equiv \min_v \widehat{a}(v) = \underline{\alpha}(b+c) + \alpha c$ and $\overline{a} \equiv \max_v \widehat{a}(v) = \overline{\alpha}(b+c) + \alpha c$ while the average demand size is defined as

$$a \equiv \int_{\mathcal{V}} \widehat{a}(v)dv = \alpha b$$

The cumulative distribution of demand size across varieties is equal to the function F_a : $[\underline{a}, \overline{a}] \to [0,1]$, $F_a(y) =$Proba$[v : \widehat{a}(v) \leq y]$. This distribution is related the taste cumulative distribution by the relationship

$$F_\alpha\left(x\right) = F_a\left(x\left(b+c\right) - \alpha c\right) \iff F_a\left(y\right) = F_\alpha\left(\frac{by + ac}{b\left(b+c\right)}\right)$$

It is worth noting that the cumulative distribution of demand size F_a has an empirical content as this is the distribution that the econometrician is likely to measure. The taste cumulative distribution F_α cannot be measured.

To guarantee positive demand size $\widehat{a}(v) > 0$, we posit that $\widehat{\alpha}(v)/\alpha > c/\left(b+c\right)$ for all v. So, the preference for the lowest quality should not be too low.

The indirect utility is computed in Appendix 1 as it follows: $V_i = S_i + w_i + \overline{q}_i^o$ where

$$S_i = \begin{cases} \frac{a^2}{2b} - a\int_{\mathcal{V}} p_i(v)\mathrm{d}v + \frac{b+c}{2}\int_{\mathcal{V}}\left[p_i(v)\right]^2\mathrm{d}v - \frac{c}{2}\left[\int_{\mathcal{V}} p_i(v)\mathrm{d}v\right]^2 \\ +\frac{\mathrm{var}[\widehat{a}]}{2(b+c)} - \int_{\mathcal{V}}[\widehat{a}(v) - a]p_i(v)\mathrm{d}v \end{cases}$$

If varieties have the same demand size ($\widehat{a}(v) = a$), we get back to the Ottaviano *et al.'s* (2002) consumer surplus.

2.2 Price equilibrium and trade costs

Production takes place in two sectors. In the first sector, the homogenous good is produced under perfect competition using one unit of labor per unit of output. We assume that this good can be costlessly traded between countries, which implies that its price is internationally equalized and equal to wages. Normalizing wages to one we get $p_i^o = w_i = 1$ for $i = H, F$, which justifies our previous choice of this good as the numéraire.

In the second sector, called the manufacturing sector, each firm produces and sells a single differentiated manufacturing variety. Let $\mathcal{V}_i$ and n_i be the set and the mass of manufacturing firms located in country i. Naturally, we have that $n_i = \mu(\mathcal{V}_i) \equiv \int_0^1 d\mu_i(v)$ where $\mu(\mathcal{V}_i)$ is the measure of $\mathcal{V}_i$ and $\mu_i(v)$ is the measure of variety v if it is produced in country i $(\mu_H(v) + \mu_F(v) = 1)$. In this section we derive the price equilibrium for a *given* location structure $(\mathcal{V}_H, \mathcal{V}_F)$ and for a given distribution of taste and demand size across firms $(\widehat{\alpha}(\cdot))$.

The demand for each variety in each market depends on the set of varieties produced domestically and on the set produced abroad. In accord with empirical evidences (e.g., Head and Mayer, 2000; Haskel and Wolf, 2001), we assume that product markets are segmented. Firms are hence free to set prices specific to each national market they sell their product in. The profit of a manufacturing firm established in country i is given by

$$\Pi_i(v) = L\theta_i p_{ii}(v) q_{ii}(v) + L\theta_j (p_{ij}(v) - \tau) q_{ij}(v) - r_i(v), \quad v \in \mathcal{V}_i \tag{8}$$

where L is the total population, θ_i is the share of population in country i, $r_i(v)$ is the remuneration of firm v's fixed factors and $q_{ij}(v)$ and $p_{ij}(v)$ is the price and demand of variety v when it is produced in country i and consumed in country j. By (5), the individual demand writes as

$$q_{ij}(v) = \widehat{a}(v) - (b + c) p_{ij}(v) + c\mathbb{P}_j$$

for all $i, j \in \{H, F\}$. Under monopolistic competition, firms are too small to affect the aggregate variables. So they set their prices $p_{ii}(v)$ and $p_{ij}(v)$ taking $\mathbb{P}_i, \mathbb{P}_j$ and $\widehat{a}(\cdot)$ as given. The optimal prices are computed as it follows:

$$p_{ii}(v) = \frac{\widehat{a}(v) + c\mathbb{P}_i}{2(b + c)} \text{ and } p_{ij}(v) = p_{jj}(v) + \frac{\tau}{2} \tag{9}$$

which depend on the demand size of the variety offered.

At the equilibrium in the product market, the firm's prices $(p_{ii}(v), p_{ij}(v))$ are consistent with aggregate prices or price indices $(\mathbb{P}_i, \mathbb{P}_j)$. The latter are successively equal to

$$\begin{aligned}
\mathbb{P}_i &= \int_0^1 p_{ii}(v)d\mu_i(v) + \int_0^1 p_{ji}(v)d\mu_j(v) \\
&= \int_0^1 p_{ii}(v)d\mu_i(v) + \int_0^1 \left(p_{ii}(v) + \frac{\tau}{2}\right) d\mu_j(v) \\
&= \int_0^1 p_{ii}(v)dv + \int_0^1 \frac{\tau}{2} d\mu_j(v) \\
&= \frac{a + c\mathbb{P}_i}{2(b+c)} + \frac{\tau}{2} n_j
\end{aligned}$$

Solving for the fixed point yields

$$\mathbb{P}_i = \frac{a + (b+c)\tau n_j}{2b + c} \tag{10}$$

so that equilibrium prices are equal to

$$p_{ii}^*(v) = \frac{1}{2}\frac{2a + \tau n_j c}{2b + c} + \frac{\widehat{a}(v) - a}{2\,(b+c)} \text{ and } p_{ij}^*(v) = p_{jj}^*(v) + \frac{\tau}{2} \tag{11}$$

If varieties have the same specific demand sizes ($\widehat{a}(v) = a$), we get back to the Ottaviano *et al.'s* (2002) prices. Otherwise, the price is larger for any variety that offer a higher value to the consumer. The reader will observe that although the price of a variety depends specifically on the ideosyncratic taste for its own variety, it does not depend on where each other specific variety is produced. The price of a variety v depends only on the mass of varieties produced in each country. More formally, we mean that $p_{ii}^*(v)$ depends on (n_i, n_j) but not on the sets $(\mathcal{V}_i, \mathcal{V}_j)$. Indeed, in this model, the import price of a variety is adjusted to local price up to a constant equal to $\tau/2$, which does not depend on its demand size. The contribution of imported varieties in the local price index is therefore equal to $\tau/2$ times the mass of the importers, which is independent of any specific demand size. So, prices and price indices do not depend on the characteristics of each individual firm, but only on the mass of firms in each country. This independence of prices to the precise composition of local production turns out to be a useful property in the subsequent analysis of spatial selection.

Given the above prices, it is easy to show that production is equal to $q_{ii}^*(v) = (b+c)p_{ii}^*(v)$ and $q_{ij}^*(v) = (b+c)\left(p_j^*(v) - t/2\right)$ so that the profit of firm v located in country i can be written as

$$\Pi_i(v) = L(b+c)\left[\theta_i(p_{ii}^*(v))^2 + \theta_j\left(p_{jj}^*(v) - \frac{\tau}{2}\right)^2\right] - r_i(v) \tag{12}$$

Finally to assure that trade remains feasible, we must impose that the consumption for any variety never falls to zero: $\min_{\{i,j,v\}} q^*_{ij}(v) > 0$. This is equivalent to the condition:

$$\tau < \tau^{\text{trade}} \equiv \frac{2a}{2b+c} - \frac{a-\underline{a}}{b+c} = \frac{2\alpha b}{2b+c} - (\alpha - \underline{\alpha})$$

This is satisfied for a low enough trade cost τ and a high enough lower bounds for taste parameter $\underline{\alpha}$ and/or demand size $\underline{a}$.

In the two following sections, we analyze the footloose capital and entrepreneur models.

3 Footloose capital model

In this section, we consider a footloose capital model where consumers' location is exogenously given by the population shares (θ_i, θ_j) and where the home country has the largest population size: $\theta_H \geq \theta_F = 1 - \theta_H$. In this footloose capital model, a unit mass of capitalists invest their capital in the firms and collect their profits. More specifically, the timing is as follows. In a first stage, each immobile capitalist is endowed with a unit of capital that he/she invests in a variety before knowing the consumer's taste for this particular variety. Varieties are thus alike and capital is randomly spread across varieties. In a second stage, nature chooses the taste parameter of each variety according to the taste distribution F_α. Third, the capitalist sets up a firm and locate it to the country where his/her capital rent $r_i(v)$, $i \in \{H, F\}$, is the highest. Finally, labor and product markets clear. In accordance to the literature we define the location equilibrium in the manufacturing market as the distribution of firms such that product markets clear and no capital unit can earn a higher return in another location. Note that, in this model, the absence of income effect implies that capitalists consume the same set of manufacturing goods as other agents, irrespective of their actual ideosyncratic rents. The consumption of capitalists is thus simply reported in the consumers' shares (θ_H, θ_F).

3.1 Spatial selection and location equilibrium

In a footloose capital model, the equilibrium imposes that the capital rent r_i exhausts profits so that $\Pi_i(v) = 0$. The rent differential between the two countries writes as

$$\Delta r(v) \equiv r_H(v) - r_F(v) = L(b+c)\tau \left[\theta_H(p^*_{HH}(v) - \tau/4) - \theta_F(p^*_{FF}(v) - \tau/4)\right]$$

A *location equilibrium* is therefore a partition of the firms $(\mathcal{V}_H,\mathcal{V}_F)$ $(\mathcal{V}_H \cup \mathcal{V}_F = [0,1]$ and $\mathcal{V}_H \cap \mathcal{V}_F = \varnothing)$ such that capital owners do not wish to reallocate their capital. That is, it requires that $\Delta r(v) \geq 0$ if $v \in \mathcal{V}_H$ and $\Delta r(v) \leq 0$ if $v \in \mathcal{V}_F$.

Plugging equilibrium prices in the rent differential we get

$$\Delta r(v, n_H) = \frac{L\tau\,(b+c)}{2\,(2b+c)} \begin{bmatrix} (2b+c)\,(2\theta_H - 1)\,(\widehat{a}(v) - a) \\ +\,(2a - \tau b)\,(b+c)\,(2\theta_H - 1) \\ -c\tau\,(b+c)\,(n_H - 1/2) \end{bmatrix} \tag{13}$$

This rent differential increases with the trade cost τ. An increase in the latter rises the incentives to locate in the country with the largest population. As noted before, the rent differential of a variety v depends on its demand size $\widehat{a}(v)$ and on the mass of firms in the domestic country n_H. It does not depend on the spatial distribution of firms and their production sites $(\mathcal{V}_H, \mathcal{V}_F)$. As a result, it readily follows that high demand firms are enticed to self-select into the country with the large population. More formally, $\forall v, v'$, s.t. $\widehat{a}(v) > \widehat{a}(v')$, we have

$$\theta_H = 1/2 \Longrightarrow \Delta r(v, n_H) = \Delta r(v', n_H)$$
$$\theta_H > 1/2 \Longrightarrow \Delta r(v, n_H) > \Delta r(v', n_H)$$

This implies that at a given (θ_H, n_H), if $\theta_H > 1/2$, we encounter three situations: (i) $\Delta r(v, n_H) > 0$ for all v, (ii) $\Delta r(v, n_H) < 0$ for all v, or (iii) there exists a variety $\widetilde{v} \in \mathcal{V}$ such that $\Delta r(v, n_H) \geq 0$ iff $v \geq \widetilde{v}$. The variety $\widetilde{v}$ divides the set of firms between those that are willing to locate either in country H or F. As a result the spatial selection takes place according to the demand size of the variety produced by each firm.

Lemma 1 (Spatial selection) *If* $\theta_H > 1/2$, $\mathcal{V}_H = \{v \mid \widehat{a}(v) > \widehat{a}(1 - n_H)\}$ *and* $\mathcal{V}_F = \mathcal{V} \backslash \mathcal{V}_H$.

Let us define the function

$$G(n_H) = \widehat{a}\,(1 - n_H) - a = (b+c)\,[\widehat{\alpha}\,(1 - n_H) - \alpha]$$

which is decreasing in n_H under our monotonicity assumptions on $\widehat{\alpha}(v)$ and which crosses the zero axis for some $n_H \in [0, 1]$. Plugging this in the rent differential we get

$$\Delta r^{*}(n_H) \equiv \frac{L\tau\,(b+c)}{2\,(2b+c)} \begin{bmatrix} (2b+c)\,(2\theta_H - 1)\,G\,(n_H) \\ +\,(2a - \tau b)\,(b+c)\,(2\theta_H - 1) \\ -c\tau\,(b+c)\,(n_H - 1/2) \end{bmatrix} \tag{14}$$

which is a decreasing function of n_H.

Given the above lemma, we can deduce that a *location equilibrium* is represented by the mass of firms $n_H^* \in [0,1]$ such that (i) $n_H^* = 1$ and $\Delta r^*(1) > 0$, (ii) $n_H^* = 0$ and $\Delta r^*(0) < 0$ or, (iii) $n_H^* \in (0,1)$ and $\Delta r^*(n_H^*) = 0$. Because Δr^* decreases with larger n_H, the location equilibrium exists and is unique.

Proposition 2 *In the footloose capital model ($\theta_H > 1/2$), a unique equilibrium exists where high value varieties are produced in the larger country.*

We now analyze the impact of the taste distribution F_α.

3.2 Taste distribution and firms' location

The first question we ask is whether the home market effect is larger when varieties are homogenous. That is, we compare the case where $\widehat{a}(v)$ is equal to the its constant average a to the case where $\widehat{a}(v)$ is not equal to it. In the case of homogenous taste, we have that $G(n_H) = 0$ and the location equilibrium solves $\Delta r^*(n_H^o) = 0$ such that

$$n_H^o - 1/2 = \begin{cases} 2\frac{2a-\tau b}{c\tau}\left(\theta_H - 1/2\right) & \text{if} \quad \theta_H < \widetilde{\theta}_H \equiv \frac{1}{2}\frac{2a-\tau b+c\tau}{2a-\tau b} \\ 1 & \text{otherwise} \end{cases}$$

This is the Ottaviano and Thisse's (2004) result where, by the feasibility trade condition, the coefficient $2\frac{2a-\tau b}{c\tau}$ is larger than one. Hence a home market effect exists because the larger country attracts a share of firms that is larger than its share of population.

In the case of taste heterogeneity, $G(n_H)$ monotonically decreases in n_H and takes value within the interval $[\underline{a} - a, \overline{a} - a]$. Using expression (14), we observe that when $n_H^* \in (0,1)$, spatial selection reduces the home market effect ($n_H^* < n_H^o$) if and only if $G(n_H^o) < 0$, which is equivalent to the condition $\widehat{a}(1 - n_H^o) < a$. Since $n_H^o > 1/2$, this condition is more likely to be satisfied if the demand size of variety v, $\widehat{a}(v)$, increases less rapidly for lower demand varieties and more rapidly for higher demand varieties. So, *the role of spatial selection on the home market effect relies on the properties of the taste distribution.* More precisely, because the function $\widehat{a}$ is the inverse of F_a, spatial selection reduces the home market effect if the cumulative distribution of demand sizes F_a increases less rapidly for highly demanded varieties, which happens if the distribution density of demand size f_a is not too skewed towards low demand varieties. Since $\widehat{a}\,(1 - n_H) - a$ is proportional to $\widehat{\alpha}\,(1 - n_H) - \alpha$ we can infer from this observation that spatial selection reduces the home market effect if the taste distribution density f_α is not too skewed towards low taste varieties. This is shown in the panel (a) of Figure 1 that presents the graph of the taste cumulative distribution $F_\alpha(x)$ (that should be read from the horizontal axis to the vertical axis) and at the same time as the taste function $\widehat{\alpha}(v)$ (an inverse relationship that should be read from the vertical axis to the horizontal axis). It is readily seen that $\widehat{\alpha}(1-n_H^o) < \alpha$ when the distribution is not too skewed towards low taste varieties. Conversely, the panel (b) of Figure 1 displays the case of a distribution skewed towards low taste varieties so that $\widehat{\alpha}(1-n_H^o) > \alpha$.

Insert Figure 1 here

This argument is summarized in the following proposition.

Proposition 3 *Spatial selection reduces the home market effect ($n_H^* < n_H^o$) if and only if $\widehat{\alpha}(1-n_H^o) < \alpha$; that is, if the taste density distribution f_α is not too skewed toward low taste varieties.*

It is interesting to discuss the case of symmetric taste distributions, which are unskewed. Such distributions satisfy $F_\alpha(\alpha) = 1/2$ and $F_\alpha(\alpha + x) = 1 - F_\alpha(\alpha - x)$. Also, $\widehat{\alpha}(1/2) = \alpha = (\underline{\alpha} + \overline{\alpha})/2$ and $\widehat{\alpha}(1/2 - z) < \alpha < \widehat{\alpha}(1/2 + z)$ for $z \in [0, 1/2]$. Hence, because $n_H^o > 1/2$, we get $G(n_H^o) = (b+c)\left[\widehat{\alpha}(1-n_H^o) - \alpha\right] < (b+c)\left[\widehat{\alpha}(1/2) - \alpha\right] = 0$. Spatial selection therefore reduces the home market effect under any symmetric taste distributions. This includes uniform taste distributions. As a result, *any taste distribution that is skewed towards high taste varieties implies that spatial selection reduces the home market effect.* This is the case for log normal and Pareto distributions.

We are now equipped to discuss the impact of the taste distribution on firms' agglomeration. To our knowledge, the paper presents for the first time such results about the relationship between taste distributions and firms' location. We concentrate on three types of changes in the taste distribution. First, let us consider the impact of a parallel shift in the taste distribution. Suppose that two taste cumulative distributions F_α^1 and F_α^2 so that $F_\alpha^2(x) \equiv F_\alpha^1(x - \delta)$ $\forall x \in [\underline{\alpha}, \overline{\alpha}]$ and $\delta > 0$. This is shown in the panel (a) of Figure 2 that presents the graph of the taste cumulative distribution $F_\alpha(x)$ at the same time as the inverse relationship $\widehat{a}(v)$. It naturally comes that $\widehat{\alpha}_2(v) = \widehat{\alpha}_1(v) - \delta$ and $\alpha_2 = \alpha_1 - \delta$ so that $G_1(n_H^*) = (b+c)\left[\widehat{\alpha}_1(1-n_H^*) - \alpha_1\right]$ is equal to $G_2(n_H^*) = (b+c)\left[\widehat{\alpha}_2(1-n_H^*) - \alpha_2\right]$. So, a parallel shift in the taste distribution does not alter the location equilibrium. This result stems from the linear properties of the quadratic utility model we have explored.

Insert Figure 2 here

Proposition 4 *A parallel shift in the taste distribution does not alter the location equilibrium.*

Second let us consider a change in the taste distribution in the sense of first-order-stochastic dominance. We here show that such a change has ambiguous effects on the location of firms. Suppose again that two distributions F_α^1 and F_α^2 so that the second one has more low taste varieties and so that it first-order-stochastically dominates the first one: $F_\alpha^2(x) > F_\alpha^1(x)$, $x \in [\underline{\alpha}, \overline{\alpha}]$. Such a change in the taste distribution is depicted in the panel (b) of Figure 2. It naturally comes that $\widehat{\alpha}_1(v) > \widehat{\alpha}_2(v)$ and $\alpha_1 > \alpha_2$. The average taste and the idiosyncratic taste for a specific variety do not need to move in a parallel way. So, because firms are likely to change their price and location decisions according to how the idiosyncratic taste for their products compares to the average taste, this change in the taste distribution is ambiguous. The overall impact is presented in the following proposition.

Proposition 5 *There exists two numbers (n_1', n_2'), $1 > n_1' > n_2' > 0$, such that a first-order-stochastic change in the taste distribution F_α^1 to F_α^2 (with $F_\alpha^2 > F_\alpha^1$ and $\alpha_1 > \alpha_2$) reduces the number of firms in the large country ($n_{H1}^* < n_{H2}^*$) if and only if $n_{H1}^* \in [n_1', n_2']$.*

Proof. See Appendix. ■

To make things clear, let us consider the case where the taste distribution is not too skewed toward low taste varieties (see Proposition 3). In this case we know that the spatial selection reduces the home market effect ($G_1 < 0$). In addition it can be shown that the first root n_1' does not belong to the interval $[0, 1/2]$. We thus get the following simpler result. *A first-order-stochastic change in the taste distribution then reduces the home market effect if firms are initially not too strongly agglomerated ($n_{H1}^* < n_2'$, $G_2 < G_1 < 0$) and amplifies this effect if they are initially strongly agglomerated ($1 > n_{H1}^* > n_2'$, $G_1 < G_2 < 0$).*

Ultimately we study changes in the taste distribution in the sense of second-order-stochastic dominance. In other words we ask how a mean-preserving spread of the taste distribution affects the location equilibrium. There exist many ways to reallocate the consumer's taste across varieties while preserving the average taste. To make things simple, we here focus on what we call an "increase in spread around its mean". In this change of distribution, the taste parameter is reduced when it lies below the average taste and is increased when it lies above it. So, varieties with taste parameter below the average a positive probability to become worse (i.e. $F_{\alpha}^{1}(x) \leq F_{\alpha}^{2}(x)$ if $x < a \equiv a_1 = a_2$) and those with a taste parameter above the average have a positive probability to become better (i.e. $1 - F_{\alpha}^{1}(x) < 1 - F_{\alpha}^{2}(x) \iff F_{\alpha}^{1}(x) > F_{\alpha}^{2}(x)$ if $x > a$). This change in the taste distribution is depicted in panel (c) of Figure 2.

Using this definition and inverting the functions F_{α}^{1} and F_{α}^{2}, we readily get that $\widehat{\alpha}_2(v) < \widehat{\alpha}_1(v) \iff \widehat{\alpha}_1(v) < \alpha$, which is equivalent to $G_2 < G_1 \iff G_1 < 0$. So, the above change in the taste distribution reduces the home market effect if the difference $G_2(n_H^o) - G_1(n_H^o)$ is negative, which happens again when the distribution density of quality f_α is not too skewed toward low taste varieties ($G_1(n_H^o) < 0$).

Proposition 6 *An increase in the spread of the taste distribution around its mean reduces the home market effect if and only if $\widehat{a}(1 - n_H^o) < a$; that is, if the taste distribution density f_α is not too skewed toward low taste varieties.*

This result applies to uniform and Pareto taste distributions because they are (weakly) skewed toward high taste varieties. So, *an increase in the spread of those taste distributions reduces further the home market effect.*

4 Footloose Entrepreneur Model

We now discuss the properties of footloose entrepreneur model under taste heterogeneity. Let us assume now that the unit mass of firms is owned by a unit mass of mobile entrepreneurs (or skilled workers). The timing is as follows. In a first stage, each entrepreneur is endowed with a variety whose taste parameter is drawn from the taste distribution F_α. In the second stage, the entrepreneur sets up his/her firm and locates it to the country where his/her utility V_i, $i \in \{H, F\}$ is the highest. Finally, labor and product markets clear. In this model, each entrepreneur does not only consider the rent $r_i(v)$ that he/she collects from his/her firm but also the consumer surplus he/she obtains in his/her country. In addition, we assume a mass $A \equiv (L-1)/2$ of immobile consumers in each country; we naturally impose $L > 1$. Entrepreneurs and immobile consumers consume in the country they locate so the mass of consumers in country H (resp. F) is equal to $\theta_H L = A + n_H$ (resp. $\theta_F L = A + n_F$). The key feature of the entrepreneur models lies in the fact the entrepreneurs relocate their purchasing power when they move with their firms to another country.

4.1 Spatial sorting and location equilibrium

When entrepreneurs consider their location, they contemplate the utility differential between the two countries. Let us denote the utility differential as $\Delta V(v) = V_H(v) - V_F(v)$. A *location equilibrium* is then a partition of the entrepreneurs ($\mathcal{V}_H$,$\mathcal{V}_F$) ($\mathcal{V}_H \cup \mathcal{V}_F = [0,1]$, $\mathcal{V}_H \cap \mathcal{V}_F = \varnothing$) such that entrepreneurs do not wish to relocate: $\Delta V(v) \geq 0$ if $v \in \mathcal{V}_H$ and $\Delta V(v) \leq 0$ if $v \in \mathcal{V}_F$.

An entrepreneur's indirect utility is equal to $V_i(v) = S_i(v) + r_i(v)$ where his/her earning $r_i(v)$ and the consumer surplus $S_i(v)$ are given above. Because all entrepreneurs faces the same purchasing prices, they get the same consumer surplus $S_i(v) \equiv S_i$, irrespective of the consumer's idiosyncratic taste for the variety they produce (see Appendix). The utility differential between entrepreneurs within country i stem only from their earnings $r_i(v, n_i)$, which will again depend on the consumer's taste for the variety they sell and on the masses of firms in each country.

As a result we can apply the same spatial selection argument as in the footloose capital model. The only difference is that θ_H is now endogenous and given by $\theta_H L = A + (n_H - 1/2)$. Therefore a country that hosts more firms also hosts a larger population, which in turn will trigger firms selling varieties with high demand and high value products to locate there. So, we encounter three cases: first, $\Delta V(v, n_H) > 0$ for all v; second, $\Delta V(v, n_H) < 0$ for all v; and finally, if $n_H > 1/2$, there exists a entrepreneur $\widetilde{v} \in [0,1]$ such that $\Delta V(v, n_H) \geq 0$ if $v \geq \widetilde{v}$. The entrepreneur $\widetilde{v}$ divides the set of entrepreneurs between those that are willing to locate either in country H or in F. As a result the spatial sorting takes place according to the quality of the variety produced by each entrepreneur.

Lemma 7 (Spatial sorting) *If* $\theta_H > 1/2$*,* $\mathcal{V}_H = \{v \mid \widehat{a}(v) > \widehat{a}(1 - n_H)\}$ *and* $\mathcal{V}_H = \mathcal{V} \backslash \mathcal{V}_H$.

On the one hand, the entrepreneur's rent can now be written as $r_i(n_H)$ and his/her rent differential $\Delta r^*(n_H)$ as defined in expression (13). On the other hand, the consumer surplus can also be written as a function of the mass of entrepreneurs located in each country $(n_H, 1-n_H)$. In Appendix 1 we compute the differential in consumer surplus as

$$\Delta S^*(n_H) \equiv S_H - S_F = \frac{\tau}{2}(2n_H - 1)(2a - b\tau)\left(\frac{b+c}{2b+c}\right)^2 + \frac{\tau}{2}M(n_H) \tag{15}$$

where

$$M(n_H) \equiv \int_{1-n_H}^{1} (\widehat{a}(v) - a)\, dv = (b+c)\int_{1-n_H}^{1} (\widehat{\alpha}(v) - \alpha)\, dv \geq 0$$

Note that $M'(n_H) = G(n_H)$ so that the function $M(n_H)$ firstly increases from zero, then attains a maximum at n'_H where $G(n'_H) = 0$ and then decreases back to zero. Since $G(n_H)$ is a decreasing function, $M(n_H)$ is concave function.

The first term in expression (15) reflects the traditional demand linkage found in new economic geography. Domestic consumers indeed benefit from the agglomeration of firms in their country because they get lower prices on average. The second term in this expression reflects the role of taste heterogeneity and sorting of entrepreneurs. Consumers that belong to the larger country get access to the varieties they demand more because high demand entrepreneurs sort out in their country. As a result, taste heterogeneity rises the consumer surplus in that country and fosters further agglomeration.

Given the above argument, we can specify the entrepreneurs' utility differential by adding $\Delta S^*(n_H)$ to $\Delta r^*(n_H)$ and by using the relationship $(2\theta_H - 1)L = 2n_H - 1$. Let

$$\Phi(n_H) \equiv M(n_H) + G(n_H)(2n_H - 1)$$

If $n_H > 1/2$, the utility differential depends on the number of firms only, and can be written as

$$\Delta V^*(n_H) = \frac{\tau}{4}\frac{b+c}{(2b+c)^2}\left\{4a(3b+2c) - \tau\left[2b(3b+2c) + Lc(2b+c)\right]\right\}(2n_H - 1) + \frac{\tau}{2}\Phi(n_H) \tag{16}$$

If $n_H < 1/2$, the spatial sorting takes place in the other direction and we naturally get $\Delta V^*(n_H) \equiv -\Delta V^*(1-n_H)$. Because $\lim_{\varepsilon\to 0+}\Delta V^*(1/2+\varepsilon) \propto M(1/2) > 0 > \lim_{\varepsilon\to 0+}\Delta V^*(1/2-\varepsilon) \propto -M(1/2)$, the function ΔV^* has a discontinuity at $n_H = 1/2$. Lemma 7 does not give any information about the distribution of entrepreneurs in the case where $\theta_H = 1/2$ and therefore $n_H = 1/2$. Thus, without loss of generality, we simply assume that the distribution of entrepreneurs and varieties across countries is random when $n_H = 1/2$. As a consequence, consumer surpluses are equal everywhere and utility differential is nil: $\Delta V^*(1/2) = 0$.

The above specification of utility differential allows us to become more precise about the definition of location equilibria. A *location equilibrium* is represented by the mass of entrepreneurs $n_H^* \in [0,1]$ such that (i) $n_H^* = 1$ and $\Delta V(1) > 0$, (ii) $n_H^* = 0$ and $\Delta V(0) < 0$, or (iii) $n_H^* \in (0,1)$ and $\Delta V(n_H^*) = 0$. The latter will be *asymptotically stable* if any small deviation from the equilibrium distribution leads back to the equilibrium distribution according to the following dynamics of entrepreneurs:

$$\frac{dn_H}{dt} = \begin{cases} \Delta V(n_H) & \text{if} \quad n_H \in (0,1/2) \cup (1/2,1) \\ 0 & \text{if} \quad n_H \in \{0,1/2,1\} \end{cases}$$

This proves to be true iff $d(\Delta V)/dn_H < 0$ at any interior equilibrium location $n_H^* \in (0,1) \backslash \{1/2\}$. Any corner location equilibrium $n_H^* \in \{0,1\}$ is also stable. Finally, the symmetric equilibrium, $n_H = 1/2$, is stable if $\lim_{\varepsilon \to 0} \Delta V(1/2 + \varepsilon) - \Delta V(1/2 - \varepsilon) < 0$, which is equivalent to $\lim_{\varepsilon \to 0} \Delta V(1/2 + \varepsilon) < 0$.

Homogenous taste: It is instructive to begin by the description of the location equilibrium in the absence of heterogeneity. In that case, there is no sorting and we get $G(n_H) = M(n_H) = 0$ so that $\Delta V(n_H)$ is proportional to $2n_H - 1$. Therefore, firms will disperse in symmetric locations, $n_H^* = 1/2$, if $\tau > \tau^0$ where

$$\tau^0 \equiv \frac{4(3b + 2c)a}{2b(3b + 2c) + c(2b + c)L},$$

they fully agglomerate in one country, $n_H^* \in \{0,1\}$, if $\tau < \tau^0$, and they can allocate according to any spatial distribution if $\tau = \tau^0$. This is consistent with Ottaviano et al (2002).

In the presence of taste heterogeneity, firms may locate in quite a different way. Let us first check the case of full agglomeration where $n_H = 1$.

Full agglomeration: In this case, we have that $\theta_H L = A + 1$, $G(1) = \underline{a} - a < 0$ and $M(1) = 0$. So, $\Delta V(1) > 0$ if and only if

$$4(2c + 3b)a - \tau[c(2b + c)L + 2b(2c + 3b)] + 2\frac{(2b + c)^2}{b + c}(\underline{a} - a) > 0$$

which yields

$$\tau < \tau^{\text{sustain}} \equiv \tau^0 - \frac{2(a - \underline{a})(2b + c)^2}{(b + c)[c(2b + c)L + 2b(3b + 2c)]}$$

The threshold τ^{sustain} defines the sustain point as the trade cost below which full agglomeration is sustainable. It is equal to τ^0 when heterogeneity disappears ($\underline{a} \to a$) and decreases with any rise in heterogeneity as reflected in the value of $a - \underline{a}$. So, for $\tau \in (\tau^{\text{sustain}}, \tau^0)$, entrepreneurs do not fully agglomerate under taste heterogeneity whereas they fully agglomerate under homogenous taste. *Taste heterogeneity does not make full agglomeration more likely.*

Symmetric and interior equilibria: Let us now discuss the case of interior equilibria where $\Delta V(n_H^*) = 0$. In the presence of heterogeneity, the symmetric distribution of entrepreneurs is an equilibrium since $\Delta V(1/2) = 0$. It is however not stable because $\lim_{\varepsilon \to 0} \Delta V(1/2 + \varepsilon) \propto 2M(1/2) > 0$. If entrepreneurs locate symmetrically, entrepreneurs producing highly demanded goods have a *common* incentive to sort out in one country and thereby to attract more consumers there. A small perturbation in the distribution of entrepreneurs triggers this effect and some entrepreneurs start agglomerating.

The number of interior equilibria depends on the properties of the function $\Phi(n_H)$. Notice that the latter function is continuous on $(1/2, 1]$, has a positive value at $n_H \to 1/2$ and is equal to the negative value $\underline{a} - a$ at $n_H = 1$. Its shape depends on the taste distribution. As a consequence, the number of roots of $\Delta V(n_H)$ and the number of equilibria also depend on this properties of this distribution. Then, we derive the following Proposition. Let $\widetilde{n}$ be the solution of (16).

Proposition 8 *Suppose that $5\widehat{a}' > \widehat{a}''$, which means that taste function $\widehat{\alpha}$ is not a too convex function or equivalently that the taste cumulative distribution F_α is not too concave. Then, there exist four stable equilibria, two equilibria with full agglomeration (either $n_H^* = 1$ or $n_F^* = 1$) if $\tau < \tau^{sustain}$ and the two equilibria with asymmetric dispersion (either $n_H^* = \widetilde{n}$ or $n_F^* = \widetilde{n} \in (1/2, 1)$) otherwise. In the latter equilibrium, one country hosts a larger group of entrepreneurs who produce the more demanded varieties.*

Proof. See Appendix. ■

The condition of this Proposition applies for uniform and Pareto taste distributions (see Appendix). The traditional view in new economic geography models emphasizes the role of demand linkages which entice entrepreneurs to agglomerate and which make the larger country host the production of a wider range of varieties. When consumer's taste is heterogenous, the demand linkage entices the entrepreneurs producing the best varieties to locate in the larger market. In fact, taste heterogeneity is a factor working against dispersion when the distribution of firms is even but it is a factor working against agglomeration when the distribution of firm is uneven. To be more precise, under taste heterogeneity, the spatial sorting of entrepreneurs acts as a force that entices the low-demand firms *not* to co-agglomerate with high-demand firms in the core countries.

4.2 Trade costs

Given the above analysis, it is easy to discuss the impact of trade costs on the location of firms. We know that, when $\tau \in (\tau^{\text{sustain}}, \tau^0)$, entrepreneurs fully agglomerate under homogenous taste whereas they fully agglomerate under taste heterogeneity. Also, when $\tau \geq \tau^o$, firms evenly disperse under homogenous taste but never do so under taste heterogeneity. So, the equilibrium spatial distribution of firms implies less dispersion than under taste heterogeneity if τ is large enough and less agglomeration if τ is small enough.

In addition, it interesting to study the impact of trade costs on the equilibrium location pattern. In particular, it is known that the globalization process in the last century has triggered a fall in trade costs and trade barriers τ. Under homogenous taste for manufacturing varieties, a fall in trade cost around τ^o dramatically alters the location pattern of firms from even dispersion to full agglomeration. This is because expression (16) is multiplicative of $(2n_H - 1)$ when $\Phi = 0$. However, taste heteoregenity implies a term Φ in the expression (16) that is not multiplicative of $(2n_H - 1)$. As a result, dramatic changes in the location of firms vanish. We summarize those results in the following proposition:

Proposition 9 *Suppose that the location equilibrium is unique. Then,*
(i) as trade cost falls from $\tau = \tau^{trade}$ to 0, the equilibrium spatial distribution of entrepreneurs continuously moves from asymmetric dispersion to full agglomeration.
(ii) Compared to the case of homogenous taste ($\underline{a} = \overline{a} = a$), the equilibrium spatial distribution of entrepreneurs under taste heterogeneity involves less dispersion if $\tau > \tau^0$ and less agglomeration if $\tau \in (\tau^{sustain}, \tau^0)$.

We finally study the impact of changes in the distribution of taste across varieties.

4.3 Taste distribution and firms' location

We first discuss the impact of a parallel shift of the taste distribution. Suppose again that two taste distributions F_α^1 and F_α^2 so that $F_\alpha^2(x) \equiv F_\alpha^1(x-\delta)$ $\forall x \in [\underline{\alpha}, \overline{\alpha}]$ and $\delta > 0$. We know from our previous analysis that $\widehat{\alpha}_2(v) = \widehat{\alpha}_1(v) - \delta$ so that $\widehat{a}_1(v) - a_1 = \widehat{a}_2(v) - a_2$, $G_1(n_H^*) = G_2(n_H^*)$. From this it readily comes that $M_1(n_H) = \int_{1-n_H}^1 (\widehat{a}_1(v) - a_1)\, dv = \int_{1-n_H}^1 (\widehat{a}_2(v) - a_2)\, dv = M_2(n_H)$ so that $\Phi_1(n_H^*) = \Phi_2(n_H^*)$. As a consequence, a parallel shift in the taste distribution does not alter the location equilibrium.

Proposition 10 *In the footloose entrepreneur model, a parallel shift in the taste distribution does not alter the location equilibrium.*

We secondly study in more detail the impact of a mean-preserving increase in the spread of the taste distribution. First, it is readily observed that, holding a constant, the sustain point τ^{sustain} decreases with larger value for $a - \underline{a}$, which is an indicator of a larger spread of the demand size distribution. So, a larger spread in the demand size distribution reduces the set of parameters for which full agglomeration is sustainable.

Second, it is interesting to consider the case of a uniform taste distribution with density $f_\alpha = 1/\delta$ so that $\widehat{\alpha}(v) = \alpha + \delta\,(v - 1/2)$ where δ measures the 'spread' of the taste distribution and δ^{-1} its density. Then, one computes that $G(n_H) = \delta(b+c)\,(1/2 - n_H)$, $M(n_H) = \frac{1}{2}\delta(b+c)n_H\,(1-n_H)$ so that

$$\Phi(n_H) = \frac{1}{2}\delta\,(b+c)\left(5n_H - 5n_H^2 - 1\right) \tag{17}$$

Note that this polynomial is positive at $n_H = 1/2$ and negative at $n_H = 1$ and that it has a unique root $n_H = \frac{1}{2} + \frac{1}{10}\sqrt{5} \simeq 0.72$ on the interval $(1/2, 1]$. Therefore, we get $\Phi(n_H) \geq 0$ if $n_H \in (0.5, 0.72]$ and $\Phi(n_H) < 0$ if $n_H \in (0.72, 1]$. The value of this polynomial is multiplied by the spread of the taste distribution δ. So, an increase in δ raises the effect of taste heterogeneity in expression (16). The question is about the direction of this effect. The answer is given in the following proposition:

Proposition 11 *Suppose a uniform taste distribution. Then, an increase in the spread of this distribution increases the number of entrepreneurs and varieties in the larger country (up to $n_H^* = 0.72$) if $\tau > \tau^0$ but reduces it (down to $n_H^* = 0.72$) if $\tau^{sustain} < \tau < \tau^0$.*

Hence, the impact of a spread of the taste distribution depends on the value of trade costs. In any case, a larger spread in this distribution implies a relocation of entrepreneurs towards the *partial agglomeration* pattern where $n_H^* = 0.72$. This corresponds to the spatial distribution of firms when the trade cost is just equal to τ^0.

5 Heterogenous taste and heterogenous cost

In the above model, consumers have varying willingness to pay for the varieties whereas firms incur the same variable cost to produce each of them. In reality, more valuable varieties often offer more characteristics and have stronger technological and/labor content than less valuable ones. So, more valuable varieties cost more to firms and can become less attractive. One may wonder whether the combination of heterogenous taste and cost still yields the same qualitative results as in the previous section. Towards this aim, we focus on the particular case where the preference for and the cost of varieties are positively correlated.

For the sake of the argument, let the taste parameter $\widehat{\alpha}(v)$ represents the number of characteristics embedded in a specific products. Consumers demand more the products with higher $\widehat{\alpha}(v)$ because they offer a larger spectrum of characteristics. For simplicity, let $m \in [0, 1)$ be the production cost of a single characteritic. So, the marginal cost of producing variety v is equal to $\widehat{m}(v) = m\widehat{\alpha}(v)$.. Under this specification, one can compute that the profit maximizing price of a variety v (see expression (9)) is augmented by the constant $\widehat{m}(v)/2$ and each price indices $\mathbb{P}_i$ by $\alpha m(b+c)/(2b+c)$. In the footloose capital model, one computes the rent differential for the firm producing variety v as

$$\Delta r^*(v) = \frac{L\tau (b+c)}{2(2b+c)} \begin{bmatrix} (2b+c)(2\theta_H - 1)(\widehat{\alpha}(v) - a)(1-m) \\ +(2\theta_H - 1)(b+c)(2a(1-m) - bt) \\ -\tau c (b+c)(n_H - 1/2) \end{bmatrix}$$

This expression very similar to (14) and it is is equal to it when $m = 0$. It is obvious that firms' self-selection takes place in the same way as before even if higher value products are more costly. Results are thus qualitatively the same. The cost of characteristics m has nevertheless the following additional impact on the capitalists' location incentives. On the one hand, a higher cost m diminishes the firms' incentives to locate in the large country (see the second term). On the other hand, it also reduces the idiosyncratic advantage of best quality firms (see the first term). This is because the product value saved by locating in the larger market is smaller when quality is costly.

In the footloose entrepreneur model, location equilibrium is determined by the above rent differential plus the consumer surplus differential. As before, the entrepreneur's consumer surplus does not depend on the idiosyncratic taste of his/her variety. So, the spatial sorting of entrepreneurs is again given by the above rent differential. The consumer surplus differential can then be computed as

$$S_H - S_F = \frac{1}{2}\tau\left(2n_H - 1\right)\left[2a\left(1-m\right) + mac/b - b\tau\right]\frac{\left(b+c\right)^2}{\left(2b+c\right)^2} + \frac{\tau\left(1+m\right)}{2}M(n_H)$$

This expression similar to (15) and reduces to it when $m = 0$. The cost m introduces several additional effects. On the one hand, it reduces the aggregate economic value of goods and therefore the consumer surplus differential (see the term $a\left(1-m\right)$). However, this effect can be compensated by a pro-competitive effect. Indeed, the cost m increases the consumer surplus differential as the varieties become better substitutes (see the term mac/b for large enough c). This pro-competitive effect is explained by the fact that higher costs induce firms to raise their prices and hit a more elastic portion of their demand curves. So, the price increases are smaller than the respective increase in the cost m. On the other hand, an increase in the cost m exarcerbates the consumer's benefit of entrepreneurs' sorting process in the larger country (see the term $\frac{\tau(1+m)}{2}M(n_H)$). Although high value products are more costly to produce and sell at a higher price, the firms' mark-up and the consumer surplus of each high value product are larger. So, a closer supply of higher value goods benefit consumers and therefore entrepreneurs.

6 Conclusion

Product quality has been long studied as an important issue in the international trade literature. Existing theoretical and empirical studies have studied the relationship between trade patterns and the quality of imports. In contrast with this literature, this paper investigates the firms' location in an economic geography model where consumers value differently the product variety produced by each firm. Our model extends Ottaviano *et al.* (2002) to taste heterogeneity across varieties. Our line of research contrasts with the trade literature on quality and on cost heterogeneity as it stresses the role of spatial selection and firms' location rather than the role of export strategy and the resulting trade pattern. In our setting, large markets attract the firms that produce the most valuable varieties; the tougher competition in those markets entices the other firms to locate in small markets.

We obtain several interesting results. We show that firms selling the higher added value goods sort out into the region hosting the largest number of consumers. Larger countries thus get better access to the products that consumers value and demand more intensively. We also show that the effect of spatial selection on firms' spatial distribution crucially depends on the properties of the taste distribution across varieties. Finally, we show that taste heterogeneity smooths the agglomeration patterns but that it should be considered neither as a dispersion force nor as an agglomeration force. Indeed, the introduction of taste heterogeneity makes an initially dispered economy less dispersed and an initially agglomerated economy less agglomerated.

The present model can be extended in several ways. For instance, cost heterogeneity and beachhead type of export cost à la Melitz (2003) could be added to our taste heterogeneity model. The relationship between export behaviors and quality could be carefully explored. Also, empirical tests of our theoretical results could be performed with trade data or regional price data in the same way as Foster *et al.* (2008).

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Appendix 1: Consumer surplus

Consumer surplus: The consumer surplus in country i is equal to

$$S_i = \int_0^1 \widehat{\alpha}(v) q_i(v)\mathrm{d}v - \frac{\beta-\gamma}{2}\int_0^1 [q_i(v)]^2\mathrm{d}v - \frac{\gamma}{2}\left[\int_0^1 q_i(v)\mathrm{d}v\right]^2 - \int_0^1 p_i(v)q_i(v)\mathrm{d}v$$

This can be written as

$$S_i = \frac{1}{2}\int_0^1 q_i(v)\left\{2\widehat{\alpha}(v) - (\beta-\gamma)\,q_i(v) - \gamma\int_0^1 q_i(\xi)\mathrm{d}\xi - 2p_i(v)\right\}\mathrm{d}v$$

where, using the first order condition of the consumer decision (3) we successively get

$$S_i = \frac{1}{2}\int_0^1 q_i(v)\left\{\widehat{\alpha}(v) - p_i(v)\right\} dv$$
$$= \frac{1}{2(\beta-\gamma)}\int_0^1 \left\{\widehat{\alpha}(v) - \gamma\int_0^1 q_i(\xi)d\xi - p_i(v)\right\}\left\{\widehat{\alpha}(v) - p_i(v)\right\} \mathrm{d}v$$

Substituting the integral $\int_0^1 q_i(\xi)d\xi$ by (4) in the last expression and expanding it, we obtain the following expression on consumer surplus as a function of prices only:

$$2(\beta-\gamma) S_i = \int_0^1 \widehat{\alpha}(v)^2 \mathrm{d}v - 2\int_0^1 \widehat{\alpha}(v)p_i(v)\mathrm{d}v + \int_0^1 p_i(v)^2\mathrm{d}v - \frac{\gamma}{\beta}(\alpha - \mathbb{P}_i)^2$$

We now substitute β by $1/b$ and γ by $c/\left[b(b+c)\right]$ and we use the inverse of equality $\widehat{a}(v) = \widehat{\alpha}(v)(b+c) - c\alpha$ and we use the definitions of the means, $a = \alpha b$, and the variance, $\mathrm{var}[\widehat{a}(v)] \equiv \int_0^1 \left(\widehat{a}(v)^2 - a^2\right) dv = (b+c)^2 \int_0^1 (\widehat{\alpha}(v)^2 - \alpha^2)\mathrm{d}v$ to get

$$S_i = \frac{\mathrm{var}\left[\widehat{a}(v)\right]}{2(b+c)} - \int_0^1 [\widehat{a}(v)p_i(v) - a\mathbb{P}_i]\mathrm{d}v + \frac{a^2}{2b} - a\mathbb{P}_i + \frac{b+c}{2}\int_0^1 p_i(v)^2\mathrm{d}v - \frac{c}{2}\mathbb{P}_i^2$$

Because $\int_0^1 [\widehat{a}(v)p_i(v) - a\mathbb{P}_i]\mathrm{d}v = \int_0^1 [\widehat{a}(v) - a]p_i(v)\mathrm{d}v$, we can write

$$S_i = \frac{a^2}{2b} - a\mathbb{P}_i + \frac{b+c}{2}\int_0^1 p_i(v)^2\mathrm{d}v - \frac{c}{2}\mathbb{P}_i^2 - \int_0^1 [\widehat{a}(v) - a]p_i(v)\mathrm{d}v + \frac{\mathrm{var}\left[\widehat{a}(v)\right]}{2(b+c)} \tag{18}$$

Consumer surplus differential: We now derive the surplus differential $S_H(v) - S_F(v)$. One can write the consumer surplus (18) for the case of two countries as

$$S_i(v) = \begin{cases} \frac{a^2}{2b} + \frac{\mathrm{var}[\widehat{a}]}{2(b+c)} \\ -a\left[\int_{\mathcal{V}_i} p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} p_{ji}(\xi)\mathrm{d}\mu_j(\xi)\right] \\ +\frac{b+c}{2}\left\{\int_{\mathcal{V}_i} [p_{ii}(\xi)]^2 \mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [p_{ji}(\xi)]^2 \mathrm{d}\mu_j(\xi)\right\} \\ -\frac{c}{2}\left[\int_{\mathcal{V}_i} p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} p_{ji}(\xi)\mathrm{d}\mu_j(\xi)\right]^2 \\ -\left[\int_{\mathcal{V}_i} [\widehat{a}(\xi) - a]p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [\widehat{a}(\xi) - a]p_{ji}(\xi)\mathrm{d}\mu_j(\xi)\right] \end{cases}$$

or equivalently

$$S_i(v) = \begin{cases} \frac{a^2}{2b} + \frac{\mathrm{var}[\widehat{a}]}{2(b+c)} \\ -a\mathbb{P}_i \\ +\frac{b+c}{2}\left\{\int_{\mathcal{V}_i} [p_{ii}(\xi)]^2 \mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [p_{ji}(\xi)]^2 \mathrm{d}\mu_j(\xi)\right\} \\ -\frac{c}{2}\mathbb{P}_i^2 \\ -\left[\int_{\mathcal{V}_i} [\widehat{a}(\xi) - a]p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [\widehat{a}(\xi) - a]p_{ji}(\xi)\mathrm{d}\mu_j(\xi)\right] \end{cases} \tag{19}$$

where $\mu_i(\xi)$ is the measure of variety ξ on the set $\mathcal{V}_i$ and where $\mathbb{P}_i = \int_{\mathcal{V}_i} p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} p_{ji}(\xi)\mathrm{d}\mu_j(\xi)$. This expression is independent of the variety produced by the entrepreneur, ξ, so that his/her surplus and surplus differential are independent of ξ: $S_i(v) \equiv S_i$ and $S_H(v) - S_F(v) \equiv S_H - S_F$. We successively derive the surplus differential $S_H - S_F$ for each term of expression (19).

Since the first term in expression (19) is a constant it does not impact on the surplus differential. Using (10) and (11), the sum of the second term and fourth terms in expression (19) yields

$$\begin{aligned} -\left(a\mathbb{P}_H + \frac{c}{2}\mathbb{P}_H^2\right) + \left(a\mathbb{P}_F + \frac{c}{2}\mathbb{P}_F^2\right) &= -\left(\mathbb{P}_H - \mathbb{P}_F\right)\left(a + \frac{c}{2}\left(\mathbb{P}_H + \mathbb{P}_F\right)\right) \\ &= -\frac{(b+c)\tau\left(n_F - n_H\right)}{2b+c}\left(a + \frac{c}{2}\frac{2a + (b+c)\tau}{2b+c}\right) \\ &= \tau\left(n_H - n_F\right)\frac{\left(4a + c\tau\right)\left(b+c\right)^2}{2\left(2b+c\right)^2} \end{aligned}$$

Third, the last term in the bracket of expression (19) writes as

$$\int_{\mathcal{V}_i} \left[\widehat{a}(\xi) - a\right]\left[p_{ii}^o + \frac{\widehat{a}(v) - a}{2\left(b+c\right)}\right]\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} \left[\widehat{a}(\xi) - a\right]\left[p_{ii}^o + \frac{\widehat{a}(v) - a}{2\left(b+c\right)} + \frac{\tau}{2}\right]\mathrm{d}\mu_j(\xi)$$

where we have defined

$$p_{ii}^o \equiv \frac{1}{2}\frac{2a + \tau n_j c}{2b+c}$$

This can be re-written as

$$p_{ii}^o \int_{\mathcal{V}} \left(\widehat{a}(\xi) - a\right)\mathrm{d}\mu(\xi) + \frac{1}{2\left(b+c\right)}\int_{\mathcal{V}} \left[\widehat{a}(\xi) - a\right]^2 \mathrm{d}\mu(\xi) + \frac{\tau}{2}\int_{\mathcal{V}_j} \left(\widehat{a}(\xi) - a\right)\mathrm{d}\mu_j(\xi)$$

where $\mu(\xi)$ is the measure of variety ξ on the set $\mathcal{V}$ which is independent of ξ. The first term of this expression is nil and the second term is a constant. So, the contribution of this term in the surplus differential $S_H - S_F$ is simply equal to

$$-\tau M(n_H)$$

where

$$M(n_H) = \frac{1}{2}\int_{\mathcal{V}_H} \left(\widehat{a}(\xi) - a\right) d\mu_H(\xi) - \frac{1}{2}\int_{\mathcal{V}_F} \left(\widehat{a}(\xi) - a\right) d\mu_F(\xi)$$

Given that best highest demand varieties sort in country H, $\mathcal{V}_H = \{v : \widehat{a}(v) > \widehat{a}(1 - n_H)\} =$

$(1-n_H, 1]$, we can sequentially write

$$
\begin{aligned}
M(n_H) &= \frac{1}{2}\int_{1-n_H}^{1} (\widehat{a}(\xi)-a)\, d\xi - \frac{1}{2}\int_{0}^{1-n_H} (\widehat{a}(\xi)-a)\, d\xi \\
&= \int_{1-n_H}^{1} (\widehat{a}(\xi)-a)\, d\xi - \frac{1}{2}\underbrace{\int_{0}^{1} (\widehat{a}(\xi)-a)\, d\xi}_{0} \\
&= \int_{1-n_H}^{1} (\widehat{a}(\xi)-a)\, d\xi
\end{aligned}
$$

Finally, the curly bracket in the third term of expression (19) can be written as

$$
\begin{aligned}
\int_{\mathcal{V}_i} [p_{ii}(\xi)]^2\, \mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [p_{ji}(\xi)]^2\, \mathrm{d}\mu_j(\xi) &= \int_{\mathcal{V}_i} \left[p_{ii}^o + \frac{\widehat{a}(\xi)-a}{2\,(b+c)}\right]^2 \mathrm{d}\mu_i(\xi) \\
&+ \int_{\mathcal{V}_j} \left[p_{ii}^o + \frac{\widehat{a}(\xi)-a}{2\,(b+c)} + \frac{\tau}{2}\right]^2 \mathrm{d}\mu_j(\xi) \\
&= \int_{\mathcal{V}} \left[(p_{ii}^o)^2 + 2p_{ii}^o\frac{\widehat{a}(\xi)-a}{2\,(b+c)} + \left(\frac{\widehat{a}(\xi)-a}{2\,(b+c)}\right)^2\right] \mathrm{d}\mu(\xi) \\
&+ \int_{\mathcal{V}_j} \left[\tau p_{ii}^o + \tau\frac{\widehat{a}(\xi)-a}{2\,(b+c)} + \frac{\tau^2}{4}\right] \mathrm{d}\mu_j(\xi)
\end{aligned}
$$

This expression simplifies to

$$
(p_{ii}^o)^2 + 0 + \frac{\operatorname{var}[\widehat{a}]}{[2\,(b+c)]^2} + \tau p_{ii}^o n_j + \frac{\tau}{2\,(b+c)}\int_{\mathcal{V}_j} (\widehat{a}(\xi)-a)\, \mathrm{d}\mu_j(\xi) + \frac{\tau^2}{4} n_j
$$

So, the contribution of this term in the surplus differential $S_H - S_F$ is equal to

$$
(p_{HH}^o)^2 - (p_{FF}^o)^2 + \tau\,(p_{HH}^o n_F - p_{FF}^o n_H) - \frac{\tau}{b+c}M(n_H) + \frac{\tau^2}{4}(n_F - n_H)
$$

After plugging the values of (p_{HH}^o, p_{FF}^o) we get

$$
\tau\,(n_F - n_H)\,(b+c)\,\frac{2a + \tau\,(b+c)}{(2b+c)^2} - \frac{\tau}{b+c}M(n_H)
$$

Adding up those terms we get

$$
S_H - S_F = \frac{1}{2}\tau\,(n_H - n_F)\,(2a - b\tau)\left(\frac{b+c}{2b+c}\right)^2 + \frac{1}{2}\tau M(n_H) \tag{20}
$$

Appendix 2: Proof of Proposition 5

(i) For the sake of clarity, let us define n_1'' and n_2'' where $G_1(n_1'') = G_2(n_2'') = 0$. One can show that $n_1'' > n_2''$ and that $G_1(n_2'') > 0 > G_2(n_1'')$. So, the difference $G_2 - G_1$ is equal to $\alpha_1 - \alpha_2 > 0$ at $n_H = 0$ and $n_H = 1$ whereas it is equal to $-G_1(n_2') < 0$ at $n_H = n_2''$ and to $G_2(n_1'') < 0$ at $n_H = n_1''$. The difference $G_2 - G_1$ thus changes its sign (at least) two times (there should be a way to prove that it changes its signs *only* two times). Therefore let us define by n_1' and n_2', $1 > n_1' > n_2' > 0$, be the two zeros of $G_2 - G_1$. So, the difference $G_2 - G_1$ is negative if and only if $n_H \in [n_1', n_2']$. From this we can readily conclude that *the impact of a first-order-stochastic dominant change in the quality distribution reduces agglomeration* ($n_{H2}^* < n_{H1}^*$) *if and only if* $n_{H1}^* \in [n_1', n_2']$.

Appendix 3: Proof of Proposition 8

We need to show that the function $\Delta V(n_H)$ has at most one root on the interval $(1/2, 1]$. Because $\lim_{\varepsilon \to 0} \Delta V(1/2 + \varepsilon) > 0$ a sufficient condition is that the function $\Delta V(n_H)$ is concave on $(1/2, 1]$. This will be true if the function $\Phi(n_H)$ is also concave on this interval. Using $M' = G$, we successively get that

$$\Phi' = 3G + (2n_H - 1)\, G'$$

$$\Phi'' = 5G' + (2n_H - 1)\, G''$$

For $\Phi'' < 0$ on the interval $(1/2, 1]$, we must have that $-5G' > (2n_H - 1)\, G''$ for all $n_H \in (1/2, 1]$. Since $G' < 0$, this is true if $G'' < 0$. If $G'' > 0$, a sufficient condition is simply that $-5G' > G''$ for all $n_H \in (1/2, 1]$. Since $G'(n_H) = -(b + c)\widehat{\alpha}'(1 - n_H)$ and $G''(n_H) = (b + c)\widehat{\alpha}''(1 - n_H)$. As a result, a sufficient condition for a concave $\Phi(n_H)$ is that

$$5\widehat{\alpha}'(1 - n_H) > \widehat{\alpha}''(1 - n_H) \tag{21}$$

for all $n_H \in (1/2, 1]$. That is $\widehat{\alpha}$ is not a too convex function and, by the same token, F_α is not a too concave function. The sufficient condition (21) applies for uniform taste distribution since this implies that $F_\alpha'' = \widehat{\alpha}'' = 0$. It indeed applies for taste Pareto distribution $F = 1 - (x/\underline{\alpha})^{-k}$ provided that it has a finite average; that is, if $k > 1$. In this case, $\widehat{\alpha} = \underline{\alpha}\,(1 - v)^{-1/k}$ so that condition (21) becomes $5n_H > (1 + 1/k)$, which is true for all $n_H \in (1/2, 1]$ and $k > 1$.

7 Appendix 4: Proof of Proposition 11

To prove Proposition 11, suppose that $\tau = \tau^0$ so that the first term in expression (16) is equal to zero. This case is displayed in panel (b) of Figure 3. The location equilibrium is simply given by the zero of the above polynomial (17); that is, by $n_H^* = 0.72$. When the spread δ rises, the polynomial (17) increases for $n_H < 0.72$ and falls for $n_H > 0.72$. Suppose now that $\tau > \tau^0$ so that expression (16) smaller than (17) for any $n_H > 1/2$. As it can be seen in panel (a) of Figure 3, the number of entrepreneurs n_H^* is smaller than 0.72 and increases with larger spread δ. The opposite argument applies when $\tau < \tau^0$. So a rise in δ implies a convergence of the equilibrium distribution of entrepreneurs to $n_H = 0.72$.

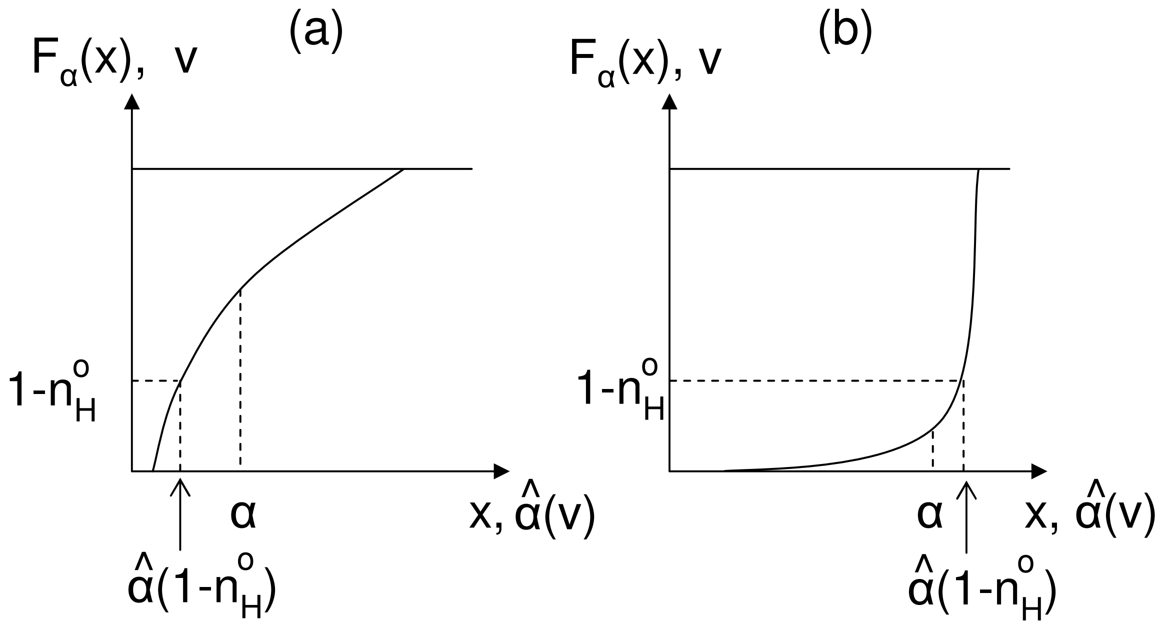

Figure 1: Taste distribution and firms' location: distribution skewed towards high taste (a) and towards low taste (b).

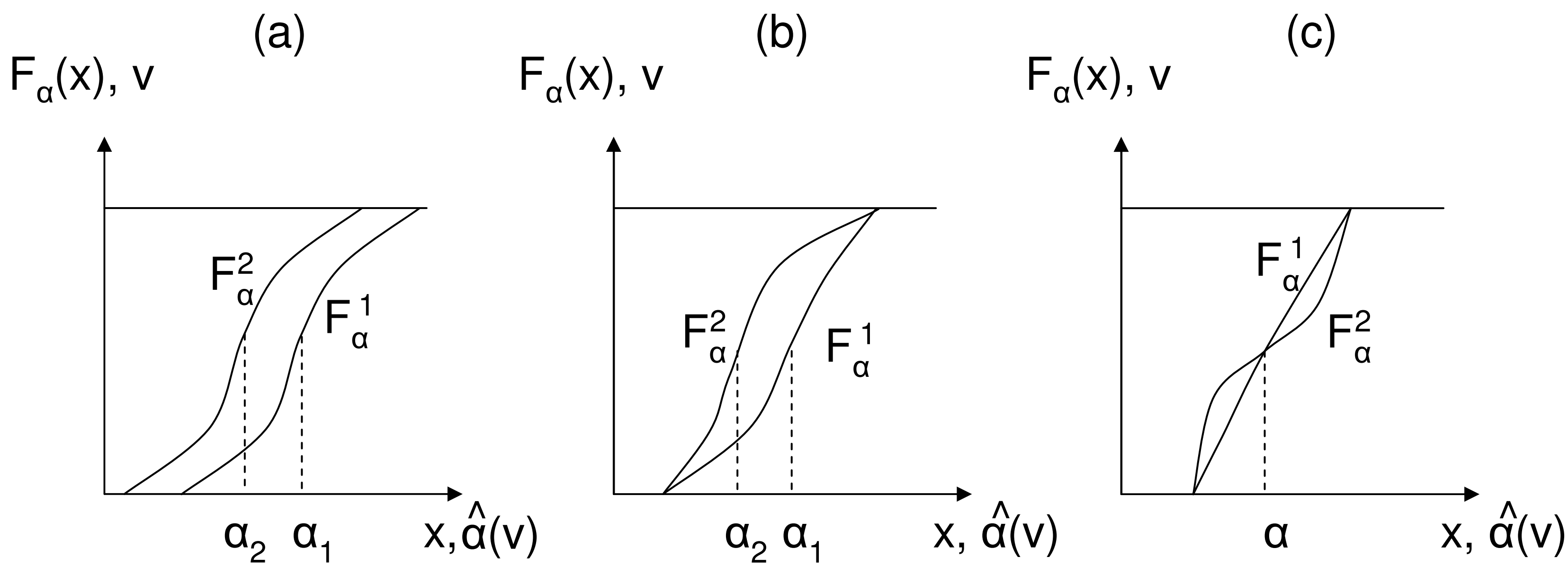

Figure 2: Changes in the taste distribution: parallel shift (a), first order stochastic dominant shift (b), and spread around the mean (c).

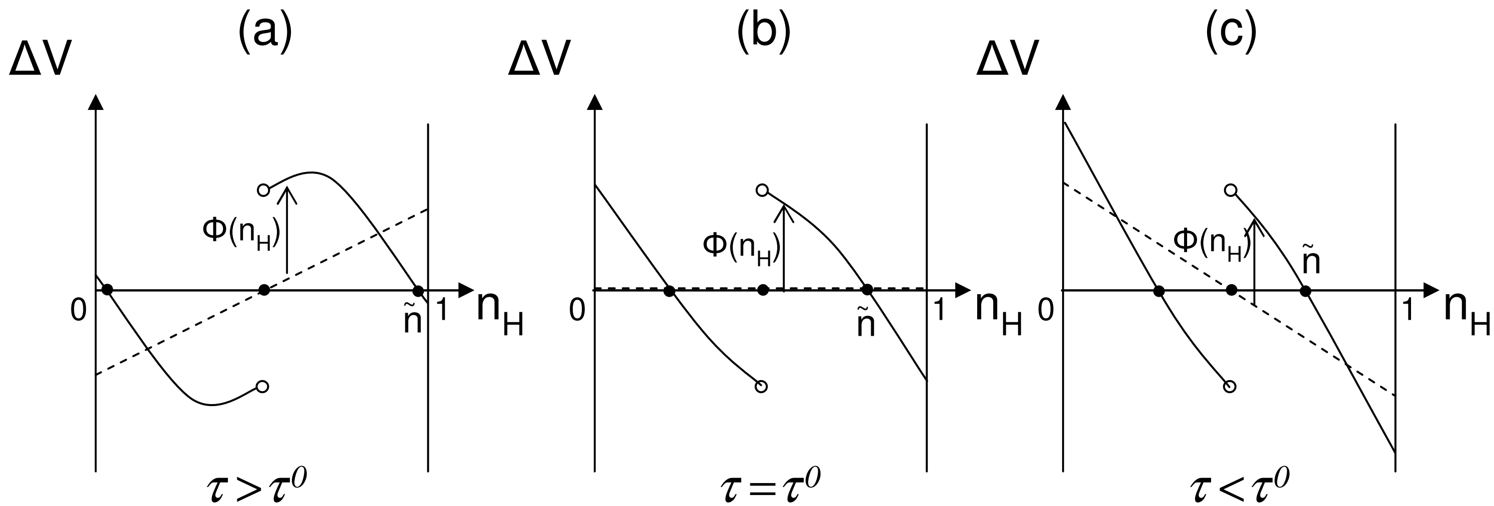

Figure 3: Location equilibrium in the entrepreneur model.

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