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of simple scoring rules

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Abstract

We study information aggregation in large elections. With two candidates, efficient information aggregation is possible in a large election (e.g., Feddersen and Pesendorfer [4, 5, 6], among others). We find that this result does not extend to large elections with more than two candidates. More precisely, we study a class of simple scoring rules in large voting games with Poisson population uncertainty and three candidates. We show that there is no simple scoring rule that aggregates information efficiently, even if preferences are dichotomous and a unique Condorcet winner always exists. We introduce a weaker criterion of informational efficiency that requires a voting rule to have at least one efficient equilibrium. Only approval voting satisfies this criterion.

Keywords: efficient information aggregation, scoring rules, Poisson games, approval voting.

JEL Classification: C72, D72, D81, D82

1 Introduction

We are interested in information aggregation in large elections when voters are strategic. Voters hold private information about the states of nature, but cannot communicate their private information to each other. In this context, voting rules can be judged according to their efficiency in aggregating private information.

Efficient information aggregation is clearly a relevant characteristic of a voting rule when the preferences of some of the voters depend on the state of nature. We refer to these voters as independent voters (as opposed to ideological, or partisan, voters, whose preferences do not depend on the state of nature). For the independent voters, one of the candidates to choose from is more suitable than the others (let us call her the 'best' candidate). Which one it is depends on the state of nature.

The general idea of efficient information aggregation (or informational efficiency) is that the result of the election is what it would be if all the relevant information were public information. Or, more precisely, it requires the probability that the elected candidate is the same as if information were public to tend to 1 as the size of the population tends to infinity. When there are two candidates, it means that the candidate who would be preferred by the majority of voters if information were public is elected in spite of the asymmetry of information. When there are three or more candidates, however, even if information is public, the elected candidate typically depends on the voting rule. That is the classical preference aggregation problem. We will therefore impose restrictions on preferences that guarantee that our results do not depend on the voting rule in this way.

Our assumption that voters are strategic follows the literature introduced by Feddersen and Pesendorfer [4], Myerson and Weber [11] and Myerson [9]. Voters vote in order to influence the outcome of the election. With uncertainty over the size of the voting population, each individual voter always has a strictly positive probability of being pivotal, independently of which voting strategy is chosen by the voters. Following Myerson [9], we assume that the size of the voting population follows a Poisson distribution.

To simplify matters, we make assumptions on preferences and the distribution of information in such a way that voters have ex-post dichotomous preferences and a unique Condorcet winner exists. Moreover, this Condorcet winner would be elected under plurality rule if the state of nature were known. Our definition of informational efficiency of a voting rule requires the outcome

of the election to coincide with the Condorcet winner in each equilibrium of the voting game.

The current wisdom is that efficient information aggregation is possible with strategic voters when there are two serious candidates and preferences of the independent voters are not too different. This wisdom is build upon three different sets of papers.

The first set assumes that independent voters have the same preferences, but differ in prior probability distributions or in the signals they receive about the states of nature. Myerson [8] and Feddersen and Pesendorfer [4] study the plurality rule and find that efficient information aggregation is possible. Bouton and Castanheira [2] use approval voting to obtain informational efficiency in a situation in which two serious candidates have to pass a vote-threshold of partisan votes of a third (not serious) candidate. To achieve this, voters have to give votes to more than one candidate.

Information aggregation in these models amounts to solving a coordination problem among independent voters with different posterior beliefs about the state of nature. However, even if these voters have different posterior beliefs, they all vote rationally conditional on the probability of being pivotal (i.e., of submitting a vote that changes the outcome of the election). This harmonizes their posterior beliefs, and they can coordinate their votes in such a fashion that the decisive fraction of voters consists of those with the correct information.

The second set, Feddersen and Pesendorfer [5, 6], generalizes the above result to cases in which the independent voters have different preferences. However, preferences still satisfy some monotonicity criterion, as the third set shows, which makes them similar enough to achieve efficient information aggregation.

The third set, Bhattacharya [1] (a singleton), seriously challenges the above results by showing that efficient information aggregation in a two-candidate election is a consequence of the assumption that preferences are monotonic in information. If belief updating leads to opposite changes in two voters' preferences over two candidates (non-monotonic preferences), efficient information aggregation is no longer possible in a two-candidate election.

In this paper, we challenge the current wisdom by showing that efficient information aggregation is impossible with three serious candidates, even if the independent voters have the same preferences.

Notice that this impossibility result is not a consequence of the typical difficulties with preference aggregation when there are more than three types

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(in our model, independent voters coexist with three types of partisan voters, one type for each candidate). By assuming that voters have dichotomous preferences, we remove the issue of preference aggregation and can focus exclusively on information aggregation. Any impossibility we discover is caused solely by a failure of the independent voters to coordinate their votes.

We study a class of simple scoring rules similar to those in Myerson [10]. Two of the most popular voting rules in our class are plurality rule and approval voting. Our first result is that none of the simple scoring rules in our class satisfies informational efficiency. There are two quite different reasons for this: First, for all scoring rules but approval voting, we can always find non-degenerate sets of parameters of the voter population for which no informationally efficient strategies exist.

Second, and more importantly, with three candidates there exist non-degenerate information sets (signals) such that the differences in posterior beliefs of voters with different signals are sufficiently large to create a coordination problem between them, even if they all vote conditional on being pivotal and have the same preferences. As a consequence, the decisive fraction of voters is not the one with the correct information, and the wrong candidate is elected. Even approval voting suffers from this problem.

Since all rules fail to satisfy informational efficiency (but not all for the same reason), we also define a weaker criterion of informational efficiency: A voting rule is weakly informationally efficient if it has at least one informationally efficient equilibrium for each set of parameters of the voter population. Our second result is that only approval voting satisfies this weak criterion.

This second result alone is not surprising. Brams and Fishburn [3] already find in a model of preference aggregation, rather than information aggregation, that approval voting always elects the Condorcet winner if preferences are dichotomous. Laslier [7], who studies preference aggregation in a large election with vote uncertainty (but not population uncertainty), also finds that approval voting always elects the Condorcet winner, if one exists. Nunez [12], however, mitigates this result in a model of preference aggregation with population uncertainty by constructing an economy with an equilibrium in which the Condorcet winner is not elected (this result does not hold if preferences are dichotomous).

The surprising result of this paper, therefore, is that approval voting is only *weakly* informationally efficient.

In Section 2, we describe the model. In Section 3, we examine all scoring

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rules, except approval voting, and show that none is even weakly informationally efficient. In Section 4, we develop the mathematical tools necessary to analyze approval voting in a large Poisson game with strategic voters. In Section 5, we show that there always exists an informationally efficient equilibrium with approval voting. In Section 6, we construct an economy for which not all equilibria with approval voting are informationally efficient. Combining the results in Sections 5 and 6, we infer that approval voting is the only weakly efficient rule, but that it is not efficient. In Section 7, we conclude.

2 The Setting

Consider an election in which voters elect one candidate into office from a set of candidates $\mathbf{K} = \{A, B, C\}$. The state of nature is uncertain and drawn from set $\mathbf{k} = \{a, b, c\}$ with prior probabilities π_a, π_b, and π_c.

Voters are either partisans or independent voters, and their type is private information. Partisan voters of candidate $X \in \mathbf{K}$, denote their type by t_X and their fraction in the population by p_X, care only about candidate X being in office, irrespective of the state of nature. And, as all other voters in the population, they derive utility from the outcome of the election alone.

$$
\begin{aligned}
u_X(X \mid z) &= 1 \quad \forall\, z \in \mathbf{k},\ X \in \mathbf{K}, \\
u_X(Y \mid z) &= 0 \quad \forall\, z \in \mathbf{k},\ Y \neq X \in \mathbf{K}.
\end{aligned}
\tag{1}
$$

Contrary to partisans, independent voters, denote their type by t_{IV} and their fraction by p_{IV}, have preferences that depend on the state of nature: In state x, candidate X is their best candidate, and they are indifferent between the other two candidates.

$$
\begin{aligned}
u_{IV}(X \mid x) &= 1 \quad \forall\, X \in \mathbf{K},\ x \in \mathbf{k}, \\
u_{IV}(Y \mid x) &= 0 \quad \forall\, Y \neq X \in \mathbf{K},\ x \in \mathbf{k}.
\end{aligned}
\tag{2}
$$

Before the election takes place, voters receive private signals. Let $\varnothing$ denote a non-informative signal. Let XY denote the signal that the best candidate is either X or Y, and let X denote the signal that X is the best candidate. In state $x \in \mathbf{k}$, the possible signals are $\varnothing, XY, XZ$, and X. Let $\varphi_z^{t,s}$ be the probability that a voter of type t receives private signal s in state z.

The partisan voters receive a non-informative signal:

$$\varphi_z^{t_X, \varnothing} = 1 \; \forall \; z \in \mathbf{k}, \; X \in \mathbf{K}. \tag{3}$$

Among the independent voters, there are uninformed, informed, and partially informed voters. The uninformed independents, denote their type by t_U and their fraction by p_U, receive a non-informative signal:

$$\varphi_z^{t_U, \varnothing} = 1 \; \forall \; z \in \mathbf{k}. \tag{4}$$

The informed independents, denote their type by t_I and their fraction by p_I, receive a signal such that they know the best candidate with certainty.

$$
\begin{aligned}
\varphi_x^{t_I, X} &= 1 \; \forall \; x \in \mathbf{k}, \; X \in \mathbf{K}, \\
\varphi_x^{t_I, Y} &= 0 \; \forall \; x \in \mathbf{k}, \; Y \neq X \in \mathbf{K}.
\end{aligned} \tag{5}
$$

The partially informed independents, denote their type by t_P and their fraction by p_P, receive signals that depend on the state of nature. In state x, a fraction of them receives signal XY, and the remaining fraction receives signal XZ.

$$\varphi_x^{t_P, XY} = 1 - \varphi_x^{t_P, XZ} \; \forall \; x \in \mathbf{k}, \; X \neq Y \neq Z \neq X \in \mathbf{K}. \tag{6}$$

As it will become clear below, our way of modeling partial information about the state of nature is crucial for the results. In addition, it is the only way to make a distinction between uninformed and partially informed voters in this model. If informative signals yield belief updating that does not eliminate one candidate for sure, then uninformed and partially informed choose the same strategy in the limit (and efficient information aggregation is possible).

Let us note that $\varphi_x^{t_P, XY}$ for $x \in \mathbf{k}$ and $X, Y \in \mathbf{K}$ are the only variable signal probabilities. This means that we can drop the type index and let $\varphi = (\varphi_a^{AB}, \varphi_a^{AC}, \varphi_b^{AB}, \varphi_b^{BC}, \varphi_c^{AC}, \varphi_c^{BC})$.

The type space is $\mathbf{T} = \{A, B, C, I, U, P\}$, and the signal space is $\mathbf{S} = \{\varnothing, AB, AC, BC, A, B, C\}$.

An economy $\mathcal{E}$ is a list (π, p, φ) that satisfies two restrictions. First, it must be true that $p_A + p_B + p_C + p_U + p_I + p_P = 1$.[1] And second, we assume

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[1] We could allow for strict inequality. This would imply that there exists a fraction of the population whose utility does not depend on the outcome of the election. We would assume that these voters do not vote.

that there is no aggregate uncertainty. If voters were to communicate their signals truthfully, the aggregation of signals would reveal the state of nature. This requires either the existence of some informed voters ($p_I > 0$), or, if $p_I = 0$, it requires the existence of some partially informed voters ($p_P > 0$) such that by combining the signals received by all of them it must be possible to deduce the state of nature: $\varphi_x^{XY} > 0$ for all $x \in \mathbf{k}$, all $Y \neq X \in \mathbf{K}$. All parameters of $\mathcal{E}$ are common knowledge.

We are interested in simple scoring rules. We define a V-scoring rule as follows: Voters choose between two ballot options, $(1, 0, 0)$ and $(1, V, 0)$. That is, they can either give one point to only one candidate, or they can give one point to one candidate and V points to another. A voter can choose one of these ballot options and distribute the points over the three candidates. Plurality voting corresponds to $V = 0$, and approval voting corresponds to $V = 1$.

This class of scoring rules is a subset of the (V, W)-scoring rules analyzed in Myerson [10]. These scoring rules are characterized by two parameters (V, W) with $0 \leq W \leq V \leq 1$ such that a voter chooses between ballot options $(1, V, 0)$ and $(1, W, 0)$.

We restrict our attention to rules with $W = 0$. In our setting, some voters (the partisans and the informed independents) can identify their preferred candidate and are indifferent between the two others. As we will show below, $W = 0$ implies that these voters have a dominant strategy. Consequently, information aggregation is a matter of coordination between the uninformed and the partially informed independent voters. Given that we will prove that coordination typically fails between these voters, allowing for a strictly positive W would only create additional problems. We do not prove it in this paper, but it is reasonably clear that the coordination problems increase with a strictly positive W, so that our main result holds for those voting rules, too.

Voters submit ballots $b \in \mathbf{B}$. A ballot is a vector $b = (b_A, b_B, b_C) \in \mathbf{B}$ which indicates that b_X points are given to candidate X. The voters' action set $\mathbf{B}$ includes all permutations of $(1, 0, 0)$ and $(1, V, 0)$, as well as $(0, 0, 0)$ to represent abstention. The candidate with the highest number of points is elected into office. We assume that a tie involving A is broken in favor of A, and a tie between B and C is broken in favor of B. This assumption has no impact on the results.

The population of voters is large. Following Myerson [9], among others, we assume that the number of voters who vote in the election is uncertain

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and follows a Poisson distribution with parameter n. The probability that exactly ν voters actually vote is

$$P(\nu|n) = \frac{e^{-n}n^{\nu}}{\nu!}. \tag{7}$$

Let V, the parameter of the voting rule, be given. Let $\mathcal{E}$ be an economy. For any expected size of the population n, a strategy is a function $\sigma_n : \mathbf{T} \times \mathbf{S} \to \Delta(\mathbf{B})$, associating a pair of type and signal with a probability distribution over $\mathbf{B}$. We let $\sigma_{n,b}(t,s) \geq 0$ denote the probability that a voter (t,s) chooses action b. It has to be true that $\sum_{b \in B} \sigma_{n,b}(t,s) = 1$ for all $(t,s) \in \mathbf{T} \times \mathbf{S}$. To save on notation, we occasionally suppress either the type or the signal, if it is not misleading.

Suppose that σ_n^* is a Bayesian Nash equilibrium of the voting game for some economy $\mathcal{E}$ with n expected voters.[2] We are interested in limit equilibria σ^* such that $\sigma_n^* \to \sigma^*$ as $n \to \infty$.

3 Informational Inefficiency of Scoring Rules

Before we can investigate the properties of information aggregation of V-scoring rules, we have to define efficient information aggregation. In an economy with partisans and independents, the notion of efficient information aggregation is not obvious. The partisans' utility does not depend on the state of nature, nor on the ability of the voting rule to reveal it, but the utility of the independent voters does.

Since we are mostly interested in the strategic interactions among informed, uninformed, and partially informed voters, we define informational efficiency only for cases in which the independent voters are part of a majority. In these cases only, the properties of information aggregation through a voting rule are worthwhile discussing. If the independent voters are part of a majority, a Condorcet winner exists and coincides with the candidate that would be elected by plurality rule, if all information were public. We require an informationally efficient voting rule to elect this Condorcet winner.

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[2]As it is typical in large Poisson games, strategies are defined type by type, and not agent by agent. This implies that all agents of the same type play the same strategy in an equilibrium. This corresponds to the restriction to symmetric equilibria in Feddersen and Pesendorfer [4].

Let V be a voting rule parameter, and let $\mathcal{E}$ be an economy. Let $P_{\sigma_n}(X|z)$ be the probability that candidate X is elected in state z given strategy profile σ_n.

Definition 1. *__Informational Efficiency__ A V-scoring rule is informationally efficient if and only if, for all $\mathcal{E} \in \mathbf{E}$ and all $X, Y, Z \in \mathbf{K}$ such that*

$$p_X + p_I + p_P + p_U > max\{p_Y, p_Z\}, \qquad (8)$$

and for all limit equilibria σ^*, we have $P_{\sigma^*}(X|x) = 1$.

Typically, there is more than one limit equilibrium for each V and each $\mathcal{E}$. Our definition of informational efficiency is very demanding because it requires all limit equilibria to be informationally efficient. As we will see later, this is never satisfied. So, we also define a weaker version of informational efficiency. Weak informational efficiency only requires a scoring rule to have at least one informationally efficient equilibrium for each economy.

Definition 2. *__Weak Informational Efficiency__ A V-scoring rule is weakly informationally efficient if and only if, for all $\mathcal{E} \in \mathbf{E}$ and all $X, Y, Z \in \mathbf{K}$ such that*

$$p_X + p_I + p_P + p_U > max\{p_Y, p_Z\},$$

there exists one limit equilibrium σ^* such that $P_{\sigma^*}(X|x) = 1$.

Our first result is that no voting rule with $V < 1$ is weakly informationally efficient. Theorem 1, actually, proves a much stronger result. There do not even exist strategies such that partisans only vote for their preferred candidate and the best candidate is sure to be elected when she should be. The reason why scoring rules with $V < 1$ are not informationally efficient is that we can construct economies in which voters do not have enough votes to give to the candidates to ensure they win when they ought to. If voters can only give V points rather than 1 point to a second candidate, this second candidate might not receive enough votes to win.

Theorem 1. *For any V-scoring rule with $V < 1$ there exist economies $\mathcal{E} \in \mathbf{E}$ for which there is no sequence of strategy functions σ_n with partisans voting for their candidate and*

$$\lim_{n \to \infty} P_{\sigma_n}(X|x) = 1.$$

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Consequently, no V-scoring rule with $V < 1$ (that is, different from approval voting) is weakly informationally efficient.

Proof. Let $V < 1$ be given. Let $\mathcal{E} = (\pi, p, \varphi)$ satisfy: $p_P = 0$ (so that $p_I > 0$), $p_A > p_B = p_C$, $p_I > 0$, $p_U > 0$, and

$$p_A < p_I + p_U + p_B, \tag{9}$$

$$p_A > p_I + \frac{1+V}{2} p_U + p_B. \tag{10}$$

Note that Eq. (9) implies that by weak informational efficiency B and C should be elected in state b and c, respectively, for at least one limit equilibrium. Let n be given. Let $\gamma_y^X(\sigma_n)$ denote the expected fraction of votes for X in state y when voters play σ_n. Let us assume that partisans only vote for their preferred candidate:

$$\sigma_{n,(b_X=1,b_{Y,Z}=0)}(t_X) = 1 \; \forall X, Y, Z \in \mathbf{K}.$$

Given that type t_U voters cannot condition their vote on the state of nature (whereas type t_I voters can), B receives the largest number of votes in b, and C in c, and A receives the lowest number of votes in these states, if

$$\sigma_{n,(0,1,0)}(t_I, B) = \sigma_{n,(0,0,1)}(t_I, C) = 1,$$
$$\sigma_{n,(0,1,V)}(t_U) + \sigma_{n,(0,V,1)}(t_U) = 1.$$

That is, informed voters give the maximum number of points to the best candidate, and uninformed voters distribute all their points between B and C. Consequently, we have that

$$\max_{\sigma_n}\{n(\gamma_b^B + \gamma_c^C)\} = n\left(2p_I + \frac{1+V}{2} p_U + p_B + p_C\right). \tag{11}$$

From Eqs. (10) and (11), we deduce that either $\gamma_b^B < \gamma_b^A$ or $\gamma_c^C < \gamma_c^A$, so that either

$$\lim_{n \to \infty} P_{\sigma_n}(B|b) = 0,$$

or

$$\lim_{n \to \infty} P_{\sigma_n}(C|c) = 0.$$

$\square$

The scoring rule with $V = 1$, approval voting, does not bear this particular insufficiency. This is why, in the following sections, we focus our attention on approval voting. It remains as the only rule possibly satisfying informational efficiency, weak or strong.

4 Magnitudes and Best Replies

In this section, we develop some tools necessary for the analysis of approval voting in a large Poisson voting game. Because we restrict our attention to this particular scoring rule, we make a few notational simplifications.

With approval voting, a voter's choice is between ballot options $(1, 1, 0)$ and $(1, 0, 0)$. A voter's action set is $\mathbf{B} = \{A, B, C, AB, AC, BC, \varnothing\}$, indicating which candidate(s) receive a point, or abstention ($\varnothing$).

Because voters derive utility solely from the outcome of the election, they pay attention only to pivotal events. In a pivotal event, a voter's ballot changes the outcome of the election from one candidate to another. Consider for example a tie between candidates A and B, and candidate C far behind. This is a pivotal event for a voter who considers ballot B. The additional vote for B changes the outcome of the election from A to B under our tie-breaking rule.

Pivotal event E_z^{XY} is the event in which one additional vote for X and no additional vote for Y changes the outcome of the election from Y to X in state z. The probability of this pivotal event is piv_z^{XY}.

Whether a voter prefers some ballot b over $\varnothing$ depends on the expected utility gain. For a partisan of candidate X, for example, the expected gain from choosing ballot X rather than $\varnothing$ can be written as

$$\pi_x piv_x^{XY} + \pi_x piv_x^{XZ} + \pi_y piv_y^{XY} + \pi_z piv_z^{XZ}. \tag{12}$$

The expected gain depends on the likelihood of those pivotal events in which ballot X changes the outcome of the election, while ballot $\varnothing$ does not. For the same partisan, the expected gain from choosing ballot XY rather than $\varnothing$ can be written as

$$\pi_x piv^{XZ} + \pi_z piv_z^{XZ}. \tag{13}$$

There is a difference between Eqs. (12) and (13) because ballot XY does not change the vote difference between candidates X and Y, but ballot X does. Both expressions are non-negative, and Eq. (12) is larger than Eq.

(13). It is easily verified as well that a partisan of X never chooses a ballot with zero points for X. Combining these facts, we conclude that ballot X is a dominant strategy for partisans of X.

Consider an informed voter with signal X. The expected gain from choosing ballots X or XY rather than $\varnothing$ can be written as

$$\pi_x piv_x^{XY} + \pi_x piv_x^{XZ}, \tag{14}$$

$$\pi_x piv_x^{XY}. \tag{15}$$

Both expressions are non-negative; Eq. (14) is larger than Eq. (15), and also larger than the expected gain from any other ballot over $\varnothing$. We conclude that informed voters have a dominant strategy to vote for X, and only for X, when they receive signal X.

Uninformed and partially informed independents have no precise information. Necessarily, they consider pivotal events in different states of nature. The utility gain from ballot b over $\varnothing$ depends on the state of nature: In state x, ballot X is a good choice, while in state y it is not.

For uninformed independents, the expected gain from ballots X or XY over $\varnothing$ can be written as

$$\pi_x piv_x^{XY} - \pi_y piv_y^{XY} + \pi_x piv_x^{XZ} - \pi_z piv_z^{XZ}, \tag{16}$$

$$\pi_x piv_x^{XZ} - \pi_z piv_z^{XZ} + \pi_y piv^{YZ} - \pi_z piv^{YZ}. \tag{17}$$

Eqs. (16) and (17) are not necessarily both non-negative. Moreover, it is not immediately obvious which of the two is largest. Uninformed independents do not have a dominant strategy, and they might even want to abstain.

Partially informed voters have updated beliefs about the state of nature. Let π_x^{XY} be the probability of state x conditional on receiving signal XY. For a partially informed voter with signal XY, the expected gain from ballots X or XY over $\varnothing$ can be written as

$$\pi_x^{XY} piv_x^{XY} - \pi_y^{XY} piv_y^{XY} + \pi_x^{XY} piv_x^{XZ}, \tag{18}$$

$$\pi_x^{XY} piv_x^{XZ} + \pi_y^{XY} piv_x^{YZ}. \tag{19}$$

Again, it is not immediately obvious which of Eqs. (18) or (19) is largest. It is clear, however, that Eq. (19) is non-negative. A partially informed voter

with signal XY always prefers ballot XY over ballot $\varnothing$. Partially informed independents never choose to abstain.

Nevertheless, we need more precise tools to evaluate expressions like Eqs. (16) through (19). In large elections, probabilities of pivotal events converge to zero. If best replies of uninformed and partially informed voters are to be meaningful, we have to evaluate utility differences between two different ballots, even though the expected utilities from both ballots eventually converge to zero as $n \to \infty$.

Probabilities of pivotal events do not converge to zero at the same speed. Events converging fast become negligible compared to events converging more slowly, and they can be ignored in expressions like Eqs. (16) through (19). Consequently, only those events that converge most slowly have to be considered.

Myerson [9] defines the magnitude of an event in a large Poisson game as a measure of its speed of convergence to zero. Let us denote the expected fraction of voters choosing ballot b in state z by

$$\lambda_z^b = \sum_{(t,s) \in \mathbf{T} \times \mathbf{S}} p_t \varphi_z^{t,s} \sigma_b(t,s), \tag{20}$$

and let $\lambda_z = (\lambda_z^b)_{b \in \mathbf{B}}$. The expected number of voters choosing ballot b in state z is $n\lambda_z^b$. The magnitude μ of an event E_z^{XY} is defined as

$$\mu = \lim_{n \to \infty} \frac{log(Prob(E_z^{XY} | n\lambda_z))}{n}. \tag{21}$$

The Magnitude Theorem (Myerson [10]) in the Appendix shows how to calculate the magnitude of a pivotal event. The magnitude of an event is non-positive. It is zero if the probability of an event does not converge to zero, or it is negative and indicates the speed of convergence. Events with larger magnitude converge to zero more slowly and are infinitively more likely in a large Poisson game than events with smaller magnitude.

Suppose events E_x^{XY} and E_y^{XY} have the largest magnitudes. If $\mu(E_x^{XY}) > \mu(E_y^{XY})$, a voter choosing ballot X is infinitively more likely to be pivotal in state x than in state y. Best replying certainly requires a point to candidate X, but not to candidate Y. Whether the voter also gives a point to candidate Z depends on the ranking of the remaining magnitudes. Generally, under approval voting, an uninformed or a partially informed voter votes for candidate X only if one of the most likely tie events involving X occurs in state x.

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In the following two sections, we establish the two main results on approval voting. In Section 5, we show that approval voting is weakly informationally efficient. In Section 6, we show that approval voting is not informationally efficient.

5 Approval voting is weakly informationally efficient

According to Definition 2, a V-scoring rule is weakly informationally efficient if it has at least one informationally efficient equilibrium. Theorem 2 states that this is true if $V = 1$.

Theorem 2. *Approval voting is weakly informationally efficient.*

Proof. Assume that $V = 1$. The proof is divided into two steps. In step 1, we show that for any $\mathcal{E} \in \mathbf{E}$ such that Eq. (8) is satisfied for all $X \in \mathbf{K}$, there exists a sequence of strategy profiles σ_n such that $P_{\sigma_n}(X|x) \to 1$ as $n \to \infty$ for all $X \in \mathbf{K}$ (we omit the similar and simpler proof that the claim is also true for economies such that Eq. (8) is true only for a subset of $\mathbf{K}$). In step 2, we deduce from step 1 that there exists a limit equilibrium σ^* that aggregates information efficiently.

Assume without loss of generality that $p_A \geq p_B \geq p_C$.

Step 1: Case 1: $p_I > 0$: Let σ_n be defined as follows (as it does not depend on n, we drop the index). Partisans and informed independents play their dominant strategies:

$$\sigma_X(t_X) = 1 \ \forall X \in \mathbf{K}, \tag{22}$$

$$\sigma_X(t_I, X) = 1 \ \forall X \in \mathbf{K}. \tag{23}$$

Both uninformed and partially informed voters mix between voting and abstaining such that their votes compensate the partisan difference between candidates A and C.

$$\sigma_\varnothing(t_U) = \sigma_\varnothing(t_P) = 1 - \min\left\{1, \frac{p_A - p_C}{p_U + p_P}\right\} \tag{24}$$

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Some uninformed voters who vote give votes to both B and C, but without increasing the votes for B above p_A. The remaining fraction of uninformed voters who vote gives their votes only to C.

$$\sigma_{BC}(t_U) = \min\left\{1 - \sigma_\varnothing(t_U), \frac{p_A - p_B}{p_U}\right\}, \tag{25}$$

$$\sigma_C(t_U) = 1 - \sigma_\varnothing(t_U) - \sigma_{BC}(t_U). \tag{26}$$

Partially informed voters with signals AC and BC vote for C, if they do not abstain. Those with signal AC who vote do not vote for A, but those with signal BC who vote might vote for B as well.

$$\sigma_C(t_P, AC) = 1 - \sigma_\varnothing(t_P), \tag{27}$$

$$\sigma_C(t_P, BC) + \sigma_{BC}(t_P, BC) = 1 - \sigma_\varnothing(t_P). \tag{28}$$

Partially informed voters with signal BC who vote give votes to B, but without increasing the vote for B above p_A in state c.

$$\sigma_{BC}(t_P, BC) = \begin{cases} \min\left\{1 - \sigma_\varnothing(t_P), \frac{p_A - p_B - p_U \sigma_{BC}(t_U)}{\varphi_c^{BC} p_P}\right\} & \text{if } \varphi_c^{BC} \neq 0, \\ 1 - \sigma_\varnothing(t_P) \text{ if } \varphi_c^{BC} = 0. \end{cases} \tag{29}$$

Partially informed voters who receive signal AB and vote give votes to B, but without increasing the vote for B above p_A in state a.

$$\sigma_B(t_P, AB) = \begin{cases} \min\left\{1 - \sigma_\varnothing(t_P), \frac{p_A - p_B - p_U \sigma_{BC}(t_U)}{\varphi_a^{AB} p_P}\right\} & \text{if } \varphi_a^{AB} \neq 0, \\ 1 - \sigma_\varnothing(t_P) \text{ if } \varphi_a^{AB} = 0. \end{cases} \tag{30}$$

Given σ, we can now verify that candidate X is elected in state x for all $X \in \mathbf{K}$. The expected fraction of votes for candidate X in state z is equal to

$$\gamma_z^X = \lambda_z^X + \lambda_z^{XY} + \lambda_z^{XZ} \quad \forall \ x \in \mathbf{k}, \ X, Y, Z \in \mathbf{K}. \tag{31}$$

Candidate A is elected in state a if and only if $\gamma_a^A \geq \gamma_a^B, \gamma_a^C$.
$\gamma_a^A = p_A + p_I$.
With Eqs. (25) and (30):
$\gamma_a^B = p_B + p_U \sigma_{BC}(t_U) + p_P \varphi_a^{AB} \sigma_B(t_P, AB) \leq p_A$.
If the constraint on $\sigma_{BC}(t_U)$ and/or $\sigma_B(t_P, AB)$ is not binding, so that $\sigma_{BC}(t_U) = 1 - \sigma_\varnothing(t_U)$ and/or $\sigma_B(t_P, AB) = 1 - \sigma_\varnothing(t_P)$, then $\gamma_a^B < p_A$. Otherwise, by construction, $\gamma_a^B = p_A$.
With Eqs. (25), (26), and (27) and the fact that $\varphi_a^{AC} \leq 1$:
$\gamma_a^C = p_C + p_U\{\sigma_{BC}(t_U) + \sigma_C(t_U)\} + p_P \varphi_a^{AC} \sigma_C(t_P, AC) \leq p_A$.

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Similar arguments apply to the cases below.

Candidate C is elected in state c if and only if $\gamma_c^C > \gamma_c^A, \gamma_c^B$.
$\gamma_c^A = p_A$.
With Eqs. (25), (26), and (29):
$\gamma_c^B = p_B + p_u\sigma_{BC}(t_U) + p_P\varphi_c^{BC}\sigma_{BC}(t_P, BC) \le p_A$.
With Eqs. (26), (28), and (27) and the fact that $\varphi_c^{AC} + \varphi_c^{BC} = 1$:
$\gamma_c^C = p_I + p_C + p_U\{\sigma_{BC}(t_U) + \sigma_C(t_U)\} + p_P\{\varphi_c^{BC}(\sigma_C(t_P, BC) + \sigma_{BC}(t_P, BC)) + \\ + \varphi_c^{AC}\sigma_C(t_P, AC)\} = p_I + p_C + p_U(1 - \sigma_\varnothing(t_U)) + p_P(1 - \sigma_\varnothing(t_P)) = p_I + p_A > p_A$,
the last equality stemming from Eq. (24).

Candidate B is elected in state b if and only if $\gamma_b^B > \gamma_b^A$ and $\gamma_b^B \ge \gamma_b^C$.
$\gamma_b^A = p_A$.
With Eqs. (26), (30), and(29):
$\gamma_b^C = p_C + p_U\{\sigma_{BC}(t_U) + \sigma_C(t_U)\} + p_P\varphi_c^{BC}\{\sigma_C(t_P, BC) + \sigma_{BC}(t_P, BC)\} \le p_A$.
With Eqs. (25), (30), and (30):
$\gamma_b^B = p_I + p_B + p_U\sigma_{BC}(t_U) + p_P\{\varphi_b^{AB}\sigma_B(t_P, AB) + \varphi_b^{BC}\sigma_{BC}(t_P, BC)\}$.
Clearly, if $\sigma_{BC}(t_P, BC) = \sigma_B(t_P, AB) = 1 - \sigma_\varnothing(t_P)$, then $\gamma_b^B > \gamma_b^A$.
Let us consider the case in which $\sigma_{BC}(t_P, BC), \sigma_B(t_P, AB) < 1 - \sigma_\varnothing(t_P)$.
Then $\gamma_b^B > \gamma_b^A$ if
$p_B + p_U\sigma_{BC}(t_U) + p_P\{\varphi_b^{AB}\frac{p_A - p_B - p_U\sigma_{BC}(t_U)}{\varphi_a^{AB}p_P} + \varphi_b^{BC}\frac{p_A - p_B - p_U\sigma_{BC}(t_U)}{\varphi_c^{BC}p_P}\} \ge p_A$,
or if $\frac{\varphi_b^{AB}}{\varphi_a^{AB}} + \frac{\varphi_b^{BC}}{\varphi_c^{BC}} \ge 1$. This is true since $\varphi_b^{AB} + \varphi_b^{BC} = 1$ and $\varphi_a^{AB}, \varphi_c^{BC} \le 1$.
It also follows that $\gamma_b^B > \gamma_b^A$ is true if either $\sigma_{BC}(t_P, BC) = 1 - \sigma_\varnothing(t_P)$ or $\sigma_B(t_P, AB) = 1 - \sigma_\varnothing(t_P)$, but not both.

Case 2: $p_I = 0$: note that in this case, with strategy σ defined as above, we have $\gamma_c^A = \gamma_c^C$, so that C is not elected in c, and we may have $\gamma_b^B = \gamma_b^A$, so that B may not be elected in b. To obtain an informationally efficient strategy function in these cases, only small changes are necessary. Observe that Eqs. (8) and (24) imply that $\sigma_\varnothing(t_P) > 0$. It is therefore possible to slightly decrease the probability of abstaining of partially informed voters, and slightly increase their probability of voting according to their signal. For each n, we replace σ with σ_n whose components are identical to σ except that

$$\sigma_{n,AC}(t_P, AC) = \epsilon^n,$$
$$\sigma_{n,BC}(t_P, BC) = \sigma_{BC}(t_P, BC) + \epsilon^n,$$

$$\sigma_{n,AB}(t_P, AB) = \epsilon^n,$$

where $\epsilon^n > 0$ is a sequence of strictly positive small fractions such that

$$\lim_{n\to\infty} \epsilon^n n = N_\epsilon > 0.$$

In state x, the expected number of voters for A, B and C are increased by $\epsilon^n n(\varphi_x^{AB} + \varphi_x^{AC}), \epsilon^n n(\varphi_x^{AB} + \varphi_x^{BC})$, and $\epsilon^n n(\varphi_x^{AC} + \varphi_x^{BC})$ respectively, which always breaks the possible ties in favour of the best candidate (recall that in state x all partially informed receive a signal including X).

Step 2: Let σ_n^* be a strategy such that Eqs. (22) and (23) hold for all n, and $\sigma_n^*(t_U)$ and $\sigma_n^*(t_P, \cdot)$ are defined as the strategies that maximize the ex-ante utility of the uninformed and partially informed voters (recall that they all have the same utility function). Such strategies exist, as they maximize a continuous function on a compact set. We claim they are equilibrium strategies. Indeed, the existence of a profitable deviation would contradict the fact that $\sigma_n^*(t_U)$ and $\sigma_n^*(t_P, \cdot)$ maximize expected utilities. Also, it is impossible that the expected utility from $\sigma_n^*(t_U)$ and $\sigma_n^*(t_P, \cdot)$ is lower than that from $\sigma(t_U)$ and $\sigma(t_P, \cdot)$ defined in Step 1. The expected utility of an independent agent tends to 1 if candidate X is elected in state x for all $X \in \mathbf{K}$. Given that $P_{\sigma_n}(X|x) \to 1$ as $n \to \infty$ for all $X \in \mathbf{K}$, so that the expected utility associated with σ tends to 1, and, given that, by construction, σ_n^* yields at least the same expected utility as σ, it has to be true that $P_{\sigma_n^*}(X|x) \to 1$ for all $X \in \mathbf{K}$. So, $\lim_{n\to\infty} \sigma_n^*$ is a limit equilibrium that aggregates information efficiently. $\square$

Combining Theorems 1 and 2, we conclude that approval voting is the only weakly efficient scoring rule. Indeed, it is possible to construct an informationally efficient strategy profile for every $\mathcal{E} \in \mathbf{E}$ only if $V = 1$. In the next section, we show that approval voting is not informationally efficient.

6 Approval voting is not informationally efficient

Unfortunately, even with approval voting there exist economies, and limit equilibria for these economies, for which the best candidate is not elected with probability converging to 1. This is what we prove in this section.

Theorem 3. *Approval voting is not informationally efficient.*

Proof. In the proof, we construct $\mathcal{E} \in \mathbf{E}$ satisfying Eq. (8) with a limit equilibrium σ^* in which A is not elected in state a. Consider $\mathcal{E} \in \mathbf{E}$ such that $p_A = p_B = 0.33$, $p_C = 0.14$, $p_U = 0.02$, $p_P = 0.15$, $p_I = 0.03$, $\varphi_a^{AB} = 0.9$, $\varphi_b^{AB} = 0.1$, and $\varphi_c^{AC} = 0.5$. Let σ be such that partisans and informed voters play the dominant strategy, uninformed voters vote only for C, partially informed voters with signals AC and BC vote only for C, and partially informed voters with signal AB vote only for B: $\sigma_X(t_X) = 1$ and $\sigma_X(t_I, X) = 1$ for all $X \in \mathbf{K}$, $\sigma_C(t_U) = \sigma_C(t_P, AC) = \sigma_C(t_P, BC) = \sigma_B(t_P, AB) = 1$.

This implies that $\gamma_a^A = 0.36$, $\gamma_a^B = 0.465$, $\gamma_a^C = 0.175$, $\gamma_b^A = 0.33$, $\gamma_b^B = 0.375$, $\gamma_b^C = 0.16$, $\gamma_c^A = 0.33$, $\gamma_c^B = 0.33$, $\gamma_c^C = 0.34$. Given the way σ is constructed, $\gamma_a^A < \gamma_a^B$, so that A is not elected in a for sufficiently large n. So, σ does not aggregate information efficiently.

Let μ_z^{XY} denote the magnitude of event E_z^{XY}. Computations based on Myerson [9] 's Magnitude Theorem yield (see Appendix)

$$\mu_c^{AC} = \mu_c^{BC} > \mu_c^{AB} > \mu_b^{AB} > \mu_a^{AB} > \mu_a^{BC} = \mu_a^{AC} > \mu_b^{BC} = \mu_b^{AC}. \tag{32}$$

To show that σ is a limit equilibrium, we only need to prove that uninformed and partially informed voters play a best response.

For the uninformed voters, the most likely pivotal events are the two relevant ties in state c. So, ballot C is a best response.

Partially informed consider different pivotal events. For those with signal AB, the most likely pivotal event occurs in state b. So, ballot B is a best response. For those with signal AC or BC, the most likely pivotal event occurs in state c, so ballot C is a best response. Indeed, σ^* is a limit equilibrium, but $\gamma_a^A < \gamma_a^B$, so that B is elected in a. $\square$

If information were public, the economy presented in the proof would have a unique equilibrium under approval voting, in which the Condorcet winner is elected (all independents vote for X in state x, even if they also vote for another candidate). The inefficiency we find, therefore, is not a consequence of difficulties associated with preference aggregation (as it is, for example, in the inefficient equilibria in Myerson [10]). It is rather caused by the failure of independent voters with differently precise information to coordinate their votes.

Inefficient equilibria of the type presented in the proof exist in our model because of two main features: The partially informed voters consider different states of nature than the uninformed voters, and the fractions of partially informed voters with different signals are state-dependent. These two factors

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create a coordination problem between the two voter types. Even though they have the same ex-post preferences, conditional on the state of nature, and are numerous enough to compensate the partisan difference between candidates, they are not able to coordinate their votes efficiently.

We conclude this section by insisting on the fact that this result follows from the presence of partially informed agents. If all independent voters are either informed or uninformed, then informational efficiency is possible, as stated in the following remark.

Remark 1. *Over the domain of economies without partially informed voters, approval voting is informationally efficient.*

Proof. See Appendix. □

Let us recall here that Theorem 1 also holds over the domain of economies without partially informed agents. The proof of Theorem 1 is indeed developed under the assumption that $p_P = 0$. Consequently, if we restrict our attention to economies without partially informed agents, then we can state that approval voting is the only scoring rule that is weakly informationally efficient, and, moreover, is informationally efficient.

7   Conclusion

This paper challenges the received wisdom that information aggregation is possible in large elections with simple voting rules when independent voters (that is, voters whose ranking of the candidates depend on the state of nature) have identical preferences. Here, independent voters differ in terms of the signals they receive. Partially informed agents, who are key for the result, receive the signal that one candidate out of three is, for sure, not the best one. This additional information turns out to create a coordination problem that no scoring rule can solve.

To conclude, let us note that our results are not restricted to the class of simple scoring rules we consider. We have assumed that agents choose between ballots $(1, 0, 0)$ and $(1, V, 0)$. Theorem 1 straightforwardly extends to scoring rules with ballot options $(1, 0, 0)$ and $(V_1, V_2, 0)$, which is obviously a more general class of rules. So, we reach the conclusion that weak information efficiency is only possible if $V_1 = V_2 = 1$, that is, with approval voting.

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Appendix

The Magnitude Theorem

Any pivotal event E_z^{XY} is characterized by a collection of linear inequalities. Consider, for example, event E_z^{BA}. According to our tie-breaking rule, this event is the collection of election outcomes such that candidates A and B have the same number of votes, and candidate C does not have more votes than they do, i.e. $n_z^A + n_z^{AC} = n_z^B + n_z^{BC}$, $n_z^A + n_z^{AB} \geq n_z^C + n_z^{BC}$, and $n_z^B + n_z^{AB} \geq n_z^C + n_z^{AC}$, where n_z^b is the number of voters choosing ballot b in state z. We use the Magnitude Theorem (Myerson [9]) to calculate the magnitude of a pivotal event E_z^{XY} by solving a maximization problem.

Theorem 4 (Myerson 2000, Theorem 1). *Let E_z^{XY} be an event, $(n_z^b)_{b \in \mathbf{B}}$ a specific election outcome, and $n\lambda_z$ the expected election outcome. Then*

$$\lim_{n \to \infty} \frac{log(Prob(E_z^{XY}|n\lambda_z))}{n} = \lim_{n \to \infty} \max_{n_z^b \in E_z^{XY}} log(Prob(n_z^b|n\lambda_z))/n$$

$$= \lim_{n \to \infty} \max_{n_z^b \in E_z^{XY}} \sum_{b \in \mathbf{B}} \lambda_z^b \psi\left(\frac{n_z^b}{n\lambda_z^b}\right) \qquad (33)$$

with $\psi(\theta) = \theta(1 - log(\theta)) - 1$, $\forall \theta > 0$.

The Magnitude Theorem gives the value of the magnitude of a pivotal event. The optimal n_z^b that solves the maximization problem is the most likely actual number of voters choosing each ballot b in state z, given the constraints of the pivotal event. In large Poisson games, almost all the probability mass of an event falls on this most likely subevent of E_z^{XY}.

Magnitudes in Theorem 3

Notice that in the equilibrium of Theorem 3, each voter gives a point to one candidate only. The only relevant fractions are, therefore, λ_z^A, λ_z^B, and λ_z^C $\forall z \in \mathbf{k}$ (that is, we have $\gamma_z^X = \lambda_z^X$ for all $z \in \mathbf{k}, X \in \mathbf{K}$).

Consider pivotal event E_z^{BA}, characterized by the constraints $n_z^A = n_z^B$ and $n_z^A \geq n_z^C$. Since the solution to the maximization problem in (33) depends on whether the constraint $n_z^A \geq n_z^C$ is binding or not, we consider the two cases separately.

Case 1: The constraint on the number of votes for C is not binding, i.e., for the optimal solution it is true that $n_z^A = n_z^B > n_z^C$.

Consider the maximization problem in (33), and let

$$M(n_z) = \sum_{b \in \mathbf{B}} \lambda_z^b \psi(\frac{n_z^b}{n\lambda_z^b}).$$

Maximizing $M(n_z)$ over $n_z \in E_z^{BA}$ and setting $n_z^A = n_z^B = k$ and $n_z^C = j$ yields

$$\frac{\partial M(n_z)}{\partial k} = \frac{1}{n}(ln(\frac{n\lambda_z^A}{k}) + ln(\frac{n\lambda_z^B}{k})),$$
$$\frac{\partial M(n_z)}{\partial j} = \frac{1}{n}ln(\frac{n\lambda_z^C}{j}).$$

The optimal solution to the maximization problem yields

$$k^* = n\sqrt{\lambda_z^A \lambda_z^B} > j^* = n\lambda_z^C,$$

$$\mu(E_z^{BA}) = -(\sqrt{\lambda_z^A} - \sqrt{\lambda_z^B})^2 = 2\sqrt{\lambda_z^A \lambda_z^B} - (\lambda_z^A + \lambda_z^B). \tag{34}$$

Case 2: The constraint on the number of votes for C is binding, i.e., for the optimal solution it is true that $n_z^A = n_z^B = n_z^C$.
Maximizing $M(n_z)$ over $n_z \in E_z^{BA}$ and setting $n_z^A = n_z^B = n_z^C = k$ yields

$$\frac{\partial M(n_z)}{\partial k} = \frac{1}{n}(ln(\frac{n\lambda_z^A}{k}) + ln(\frac{n\lambda_z^B}{k}) + ln(\frac{n\lambda_z^C}{k})).$$

The optimal solution to the maximization problem yields

$$k^* = \sqrt[3]{\lambda_z^A \lambda_z^B \lambda_z^C},$$

$$\mu(E_z^{BA}) = 3\sqrt[3]{\lambda_z^A \lambda_z^B \lambda_z^C} - (\lambda_z^A + \lambda_z^B + \lambda_z^C). \tag{35}$$

The values of λ_z^b in the equilibrium of Theorem 3 are given in the proof of Theorem 3. Using (34) and (35), we calculate the magnitudes of the relevant pivotal events.

For example, with (34), we get

$$\mu_c^{AC} = -(\sqrt{0.33} - \sqrt{0.34})^2 = \mu_c^{BC} \approx -7.46 * 10^{-5}.$$

And with (35), we get

$$\mu_c^{AB} = 3\sqrt[3]{0.33 * 0.33 * 0.34} - (0.33 + 0.33 + 0.34) \approx -9.93 * 10^{-5}$$

because in this case, the constraint on the votes for C in c is binding:

$$\sqrt{\lambda_c^A \lambda_c^B} = \sqrt{0.3 * 0.3} = 0.3 < 0.34 = \lambda_c^C.$$

Proof of Remark 1

Proof. Let $\mathcal{E} \in \mathbf{E}$ be such that $p_P = 0$ (so that $p_I > 0$). Let σ^* be a limit equilibrium for $\mathcal{E}$. We know that all partisans of X have a dominant strategy to vote for X only, for all $X \in \mathbf{K}$. We know that all informed voters vote for X in state x. We also know that uninformed voters cannot condition their vote on the state of nature. Consequently, *the total number of votes does not depend on the state of nature.* There are two immediate consequences of this fact:

1. The swing voter's curse (see Feddersen and Pesendorfer [4]: If some uninformed voters vote for X and Y, then none of them votes for Z, for all $X \neq Y \neq Z \neq X \in \mathbf{K}$.

2. A tie event between X and Y is more likely in state x than in state y if and only if the victory margin is smaller in state x than in state y, that is, if and only if $|\gamma_x^X - \gamma_x^Y| < |\gamma_y^Y - \gamma_y^X|$.

Let us assume that $p_X + p_I + p_U > max\{p_Y, p_Z\}$, whereas $P_{\sigma^*}(X|x) = 0$. Let us assume, w.l.o.g., that $P_{\sigma^*}(Y|x) > 0$. That implies $\gamma_x^Y > \gamma_x^X$. Given that

$$
\begin{aligned}
\gamma_x^X &= p_X + p_I + p_U(\sigma_X^*(t_U) + \sigma_{XY}^*(t_U) + \sigma_{XZ}^*(t_U)), \\
\gamma_x^Y &= p_Y + p_U(\sigma_Y^*(t_U) + \sigma_{XY}^*(t_U) + \sigma_{YZ}^*(t_U)), \\
\gamma_y^X &= p_X + p_U(\sigma_X^*(t_U) + \sigma_{XY}^*(t_U) + \sigma_{XZ}^*(t_U)), \\
\gamma_y^Y &= p_Y + p_I + p_U(\sigma_Y^*(t_U) + \sigma_{XY}^*(t_U) + \sigma_{YZ}^*(t_U)),
\end{aligned}
$$

we immediately get

$$\gamma_y^Y > \gamma_x^Y > \gamma_x^X > \gamma_y^X,$$

which, given Fact 2 above, yields

$$\mu_x^{XY} > \mu_x^{XY}. \tag{36}$$

We can also deduce that either not all uninformed vote for X, so that $\sigma_X^*(t_U) + \sigma_{XY}^*(t_U) + \sigma_{XZ}^*(t_U) < 1$ (Case 1), or that all uninformed voters vote for X, but some uninformed voters also vote for Y, so that $\sigma_Y^*(t_U) + \sigma_{XY}^*(t_U) + \sigma_{YZ}^*(t_U) > 0$ (Case 2).

In Case 1, Eq. (36) implies that one of the most likely ties involving X does not occur in x, which implies that

$$\mu_z^{XZ} \geq \mu_x^{XZ} \tag{37}$$

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and

$$\mu_z^{XZ} \geq \mu_x^{XY}. \tag{38}$$

Eq. (37) and Fact 2 above imply that

$$p_Z + p_U \sigma_{Z+XZ+YZ}^* \leq p_X + p_U \sigma_{X+XY+XZ}^*,$$

which makes Eq. (38) impossible: Indeed, in state z, we have $\gamma_z^Y > \gamma_z^Z, \gamma_z^X$, so that a tie between X and Z requires first that X and Z get the same number of votes, but also that X gets weakly more votes than Y, which itself is less likely than a tie between X and Y in state x. It is therefore impossible that not all uninformed vote for X.

In Case 2, Eq. (36) and the fact that some uninformed voters vote for Y imply that one of the most likely ties must occur in state y, and that it has to be a tie between Y and Z:

$$\mu_y^{YZ} \geq \mu_z^{YZ} \tag{39}$$

and

$$\mu_y^{YZ} \geq \mu_x^{XY}. \tag{40}$$

Eq. (39), however, implies that the difference in expected fractions of vote for Y and Z is smaller in state y than in state z, which, using the fact that only informed voters condition their vote on the state of nature, implies that $\gamma_y^Z \geq \gamma_z^Y$. With $\gamma_x^Z = \gamma_y^Z$ and $\gamma_z^Y = \gamma_x^Y$, it is also true that

$$\gamma_x^Z \geq \gamma_x^Y.$$

Given the fact that the economy satisfies Eq. (8), given the assumption that some uninformed voters vote for Y, and given the fact that $\sigma_X^*(t_U) + \sigma_{XY}^*(t_U) + \sigma_{XZ}^*(t_U) = 1$, there must be some some uninformed voters who vote for Z. This is a contradiction to Fact 1 above. $\qquad\square$

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