

# Impacts of Selective Reductions in Labor Taxation

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## Abstract

To study the impacts of reductions in employer's social security contributions, we construct an intertemporal general equilibrium model with different types of workers (and wages), search unemployment and endogenous job destruction rates. Our model reproduces the empirical evidence that the impacts on employment, of reductions in contributions at the minimum wage level, go through a decrease in job destructions rather than an increase in job creations. We moreover find that, although it is prejudicial to average productivity, reductions targeted at the minimum wage create much more net employment than reductions targeted at other wages.

*Keywords:* Labor taxation, Job destruction rate, Employment.

*JEL classification:* E24, E62, H25, J38.

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# 1 Introduction

Over the last decades, the increase in labor taxation has been particularly significant in continental Europe. As a result, at the end of the 90's, the tax wedge on labor<sup>1</sup> in European countries was roughly twice as high than in Anglo-Saxon countries (figure 1). In a recent paper, Prescott (2003) emphasizes the effects of employee's taxation on the labor supply. From a business cycle model, he writes the labor supply as a function of the employee's labor tax rate. For different G7 countries, he shows that the predicted labor supply is close to the actual one, and concludes that tax rates alone account for most of the differences in labor supply in the major advanced industrial countries. But the labor taxation, especially the employer's taxation, also affects the labor demand and is often put forward to explain the high EU unemployment rate. In this paper, we construct a general equilibrium model to investigate the effects on labor demand of a reduction in employer's taxation.

In many EU countries, this type of policy has become popular over the last few years. For instance, in France, the "Juppé reform" implemented in the second half of the 90's, strongly reduced employer's contributions at the "SMIC" level<sup>2</sup>. Recently, the Belgian government also decreased employer's contributions for several categories of workers. Since the costs involved by these reductions are often important, it is crucial to have efficient implementations of the policies. It is well-known that labor demand for the low-skilled is more elastic to wages than labor demand for the high-skilled (see Hamermesh (1993) for a synthesis). As a result, a reduction in the contributions targeted at low wages may well be more effective in stimulating employment than reductions targeted at high wages, partially offset by rises in these bargained wages. Effects could still be larger if the reductions are targeted at rigid minimum wage<sup>3</sup>. On the other hand, favoring low-skilled employment (usually low wages employment) relatively to the high-skilled may harm the productivity of the economy and *in fine* the total employment.

In the French literature, several papers estimate the effects of reductions at the SMIC level. Using microeconomic data, Laroque and Salanié (2000) and Crépon and Desplatz (2001) find quite important increases in employment. Crépon and Desplatz (2002) also find that rises in employment are almost fully due to a sudden fall in job destruction rather an increase in job creation, and that most of the job adjustment is already realized after two years. These last observations are similar to those of Kramarz and Philippon (2000). They estimate the elasticity of transition probabilities (from employment to unemployment and from

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<sup>1</sup>The tax wedge is defined as the difference (in percentage of the gross wage) between the wage cost for the firm and the household's net available income. This includes the social contributions paid by the employer, the social contributions paid by the employee, and the personal income tax.

<sup>2</sup>The SMIC means *Salaires minimum interprofessionnel de croissance* and is the French minimum wage.

<sup>3</sup>See appendix 1 for a simple numerical evidence in a competitive labor market.

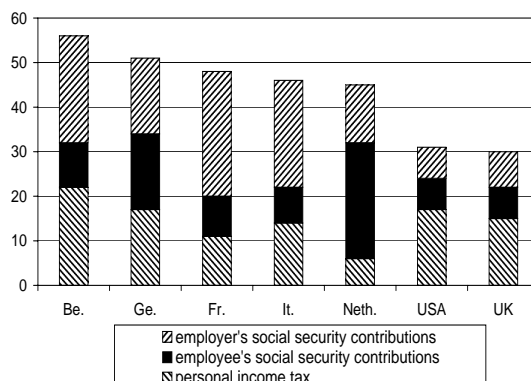


Figure 1: Tax wedge as a percentage of gross labor cost. Source: OECD (2000)

unemployment to employment) with respect to the minimum wage cost and obtain a highly positive elasticity for the employment to unemployment transition probability, and a not significant elasticity for the unemployment to employment transition probability. Several macroeconomic models, calibrated on the French economy, also try to produce estimations of the effects of contribution reductions at the minimum wage level<sup>4</sup>. More recently, Cahuc (2003) and Chéron, Hairault, and Langot (2003) develop models with matching, however they usually obtain smaller effects than those estimated by empirical micro studies. It is nevertheless worth noting that they assume an exogenous job destruction rate. In other words, in their models all employment effects go through job creation rather than job destruction, which seems, given Kramarz and Philippon (2000) and Crépon and Desplatz (2002), counter-factual.

In Belgium, Sneessens and Shadman (2000), Stockman (2002), Hendrickx, Joyeux, Masure, and Stockman (2003) or Burggraeve and Du Caju (2003), to only mention the more recent contributions, econometrically estimate the effects of reductions in social contributions on employment. They mainly confirm that employment increase is higher if the reduction is targeted at low wages. However, there is no Belgian specific estimation of reductions targeted at the minimum wage<sup>5</sup>.

The contribution of this paper is twofold. Firstly, we build on the existing theoretical matching literature by adding endogenous job destruction rate, to be able to compare our results with empirical evidences. Secondly, we calibrate our intertemporal general equilibrium model, on Belgian data<sup>6</sup>, and we evaluate

<sup>4</sup>See for instance Granier and Nyssen (1996) for an OLG model, Laffargue (2000) or Salanié (2000) for macro-econometric models,...

<sup>5</sup>Contrary to France where there is a unique SMIC, the minimum wage in Belgium may be different across sectors. However, it is generally estimated that 10% of the workers are paid at one of the sectoral minimum wages (see the calibration section and appendix 2).

<sup>6</sup>Economic situation in Belgium is close to those observed on average at the European level.

the effects of reductions in employer's contributions, targeted at different types of wages (very low wages or minimum wage, low wages, high wages, and all wages), not only on the employment but also on the welfare of individuals and on the economy's productivity. More precisely, we use the Pierrard and Sneessens (2003) general equilibrium framework with two types of workers (low- and high-skilled), two types of jobs (simple and complex) and possible crowding-out of the low-skilled by the high-skilled<sup>7</sup>. We add a minimum wage (all wages are bargained but workers are protected by a minimum wage if the bargained outcome is lower than the minimum wage) and endogenous job destruction rates, following Joseph, Pierrard, and Sneessens (2004).

Our main findings are: (i) Simulating a reduction in the minimum wage cost, we reproduce empirical facts showing that most of the effects on employment go through the lower job destruction rate rather than the job creation rate. Moreover, again as in empirical studies, adjustment is almost complete after ten periods (between two and three years). (ii) Our quantitative effects on employment when reductions are at the minimum wage level are higher than those of previous theoretical papers. This can be explained by the fact that they assume exogenous job destruction rates. (iii) Reductions targeted at the minimum wage create ten times more employment, mainly for the low-skilled, than reductions targeted at high wages. This policy seems therefore suitable in countries with high low-skilled unemployment rates. The productivity of the economy, however, highly decreases.

The model is developed in the next section and the calibration, as well as the simulation results, are reproduced in section 3. Section 4 provides some discussions and extensions, and the last section concludes.

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The analysis for the Belgian case may thus provide insights for other European countries as well.

<sup>7</sup>Pierrard and Sneessens (2003) show that job competition was important in Belgium to explain the relative unemployment rates. See also section 4.1 for a discussion.

## 2 Model

We construct a two-tier productive structure with three types of agents: intermediate firms, representative households, and a representative final firm. As in Mortensen and Pissarides (1994), intermediate firms are single-job<sup>8</sup> and are hit each period by an idiosyncratic productivity shock. They hold either a simple job, to produce a "low-tech" intermediate good, or a complex job, to produce a "high-tech" intermediate good. There are two representative households, one composed of the low-skilled and the other one of the high-skilled<sup>9</sup>. The low-skilled can only work on simple jobs while the high-skilled can work on simple jobs (crowding out the low-skilled) or on complex jobs. We allow for this flexibility (rather than perfectly segmented labour markets) because this "de-qualification" possibility has been highlighted by several studies as an important feature of the European labour markets (see for instance Hartog (2000) or Pierrard and Sneessens (2003)). Low-skilled household has labor and unemployment incomes that are entirely consumed. High-skilled household has labor income but also dividends (they own the intermediate firms) and capital income, which can be consumed or invested<sup>10</sup>. Finally, the representative final firm uses the low-tech goods, the high-tech goods, as well as capital to produce a *numéraire* final good. The model is developed in discrete time and detailed in the next subsections.

### 2.1 Labor Market Flows

Total labor force is normalized to 1 and consists of a fix proportion  $1-\alpha$  (resp.  $\alpha$ ) of low-skilled (resp. high-skilled) workers<sup>11</sup>. The low-skilled can be unemployed ( $U_t^l$ ) or work on a simple job ( $N_t^l$ ), and the high-skilled can be unemployed ( $U_t^h$ ), work on a simple job ( $N_t^h$ ) or work on a complex job ( $N_t^c$ ). This gives:

$$1 - \alpha = U_t^l + N_t^l \quad \text{and} \quad \alpha = U_t^h + N_t^h + N_t^c. \quad (1)$$

The high-skilled unemployed search for simple jobs (resp. complex jobs) with an endogenous search efficiency  $\mathcal{S}_t^s$  (resp.  $\mathcal{S}_t^c$ ). The high-skilled working on simple jobs also search for complex jobs with an endogenous search efficiency  $\mathcal{S}_t^o$ . The

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<sup>8</sup>The main motivation to the introduction of single job intermediate firms is to keep a tractable representation of the wage bargaining process.

<sup>9</sup>This representative households formulation amounts to assuming that, within each household, workers are perfectly insured against the unemployment risk. This simplification is common in the literature (see for instance Merz (1995) or Andolfatto (1996)) and reflects the current state of the art. Taking into account workers heterogeneity inside each household, due to imperfect insurance markets would make the model totally intractable.

<sup>10</sup>By simplicity, we assume that only high-skilled are capitalists.

<sup>11</sup>This paper does not focus on the education process. To illustrate the effects of labor market policies on vocational training, see for instance Chéron, Hairault, and Langot (2003).

search efficiency of the low-skilled worker is normalized to 1<sup>12</sup>.

If we denote the stock of simple (resp. complex) vacancies by  $V_t^s$  (resp.  $V_t^c$ ), the numbers of contacts at each period for simple and complex jobs are respectively defined by:

$$M_t^s = \mathcal{M}^s (V_t^s, U_t^l + \mathcal{S}_t^s U_t^h), \quad (2)$$

$$M_t^c = \mathcal{M}^c (V_t^c, \mathcal{S}_t^c U_t^h + \mathcal{S}_t^o N_t^h), \quad (3)$$

where  $\mathcal{M}^i$ , with  $i \in \{s, c\}$ , satisfies the usual Inada conditions. The probabilities for a firm to have a contact for a simple vacancy ( $q_t^s$ ) and a complex vacancy ( $q_t^c$ ) are:

$$q_t^s = \frac{M_t^s}{V_t^s} \quad \text{and} \quad q_t^c = \frac{M_t^c}{V_t^c}. \quad (4)$$

More precisely, for a simple vacancy, the probabilities to have a contact with a low-skilled ( $q_t^{sl}$ ) or a high-skilled ( $q_t^{sh}$ ) are:

$$q_t^{sl} = q_t^s \frac{U_t^l}{U_t^l + \mathcal{S}_t^s U_t^h}, \quad (5)$$

$$q_t^{sh} = q_t^s \frac{\mathcal{S}_t^s U_t^h}{U_t^l + \mathcal{S}_t^s U_t^h}. \quad (6)$$

In the same way,  $p_t^s$  (resp.  $p_t^c$ ) is the probability for an efficient job seeker to have a contact with a simple job (resp. complex job):

$$p_t^s = \frac{M_t^s}{U_t^l + \mathcal{S}_t^s U_t^h}, \quad (7)$$

$$p_t^c = \frac{M_t^c}{\mathcal{S}_t^c U_t^h + \mathcal{S}_t^o N_t^h}. \quad (8)$$

The timing on the labor market is the following: at time  $t$ , new matches are created; at the beginning of time  $t + 1$ , matches (new or already existing) are hit by idiosyncratic productivity shocks, wages are (re)negotiated and some matches are destroyed. By denoting  $\chi_t^l$ ,  $\chi_t^h$  and  $\psi_t$  the job destruction rates for respectively the simple jobs filled with the low-skilled, the simple jobs filled with the high skilled and the complex jobs, the dynamics of employment is described, in terms of the job seekers' search effort, by:

$$N_{t+1}^l = (1 - \chi_{t+1}^l)(N_t^l + p_t^s U_t^l), \quad (9)$$

$$N_{t+1}^h = (1 - \chi_{t+1}^h)((1 - p_t^c \mathcal{S}_t^o)N_t^h + p_t^s \mathcal{S}_t^s U_t^h), \quad (10)$$

$$N_{t+1}^c = (1 - \psi_{t+1})(N_t^c + p_t^c \mathcal{S}_t^c U_t^h + p_t^c \mathcal{S}_t^o N_t^h). \quad (11)$$

All these endogenous labor market flows are summarized in figure 2.

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<sup>12</sup>We do not introduce an endogenous participation rate (see for instance Engström, Holmlund, and Kolm (2001) or Garibaldi and Wasmer (2001) for such a modelization). This is consistent with empirical evidences (Piketty (1998) or Blundell and MaCurdy (1999)) showing that the elasticity of labor supply with respect to wage is weak.

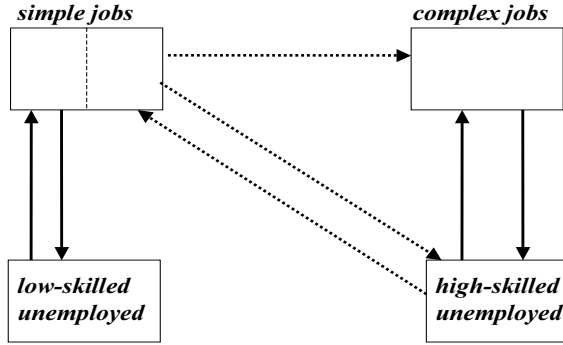


Figure 2: Labor market flows

## 2.2 Intermediate Firms

Intermediate firms are either "high-tech" firms (complex jobs with high-skilled workers) or "low-tech" firms (simple jobs with low- or high-skilled workers). At each period, each intermediate firm is hit by an idiosyncratic productivity shock  $x$ , drawn from a cdf  $F$ <sup>13</sup>.

### High-tech firms

A complex job is profitable if the idiosyncratic productivity shock  $x$  is higher than the reservation productivity for both the firm ( $R_t^{F,c}$ ) and the high-skilled household ( $R_t^{H,c}$ ), *i.e.* if  $x \geq R_t^c = \max\{R_t^{F,c}, R_t^{H,c}\}$ . The asset value of an intermediate firm with a complex vacancy is therefore<sup>14</sup>:

$$W_t^{V,c} = -b + \tilde{\beta}_t q_t^c E_t \left[ F(R_{t+1}^c) W_{t+1}^{V,c} + \int_{R_{t+1}^c}^{+\infty} W_{t+1}^{F,c}(z) dF(z) \right] + \tilde{\beta}_t (1 - q_t^c) E_t [W_{t+1}^{V,c}], \quad (12)$$

where  $b$  is the per-period complex vacancy opening cost and  $\tilde{\beta}_t$  is the rate at which the firm discounts future profits. The asset value of a firm producing a high-tech good is:

$$W_t^{F,c}(x) = x d_t^c - w_t^c(x) + \tilde{\beta}_t E_t \left[ F(R_{t+1}^c) W_{t+1}^{V,c} + \int_{R_{t+1}^c}^{+\infty} W_{t+1}^{F,c}(z) dF(z) \right], \quad (13)$$

<sup>13</sup>Without loss of generality, we assume the same  $F$  for both types of firms.

<sup>14</sup>Contrary to Mortensen and Pissarides (1994), we do not assume that jobs start at the highest available productivity, which would necessitate to make the distinction between "new jobs" and "old jobs". We also assume we have no employment protection (see section 4.2 for a discussion).

where  $d_t^c$  is the price of an intermediate high-tech good and  $w_t^c$  is the wage. The supply of high-tech goods is:

$$G_t^c = \frac{\int_{R_t^c}^{+\infty} z dF(z)}{1 - F(R_t^c)} N_t^c. \quad (14)$$

### Low-tech firms

If a simple job is held by a low-skilled, it is profitable if the idiosyncratic productivity shock  $x$  is higher than the reservation productivity for both the firm ( $R_t^{F,l}$ ) and the low-skilled household ( $R_t^{H,l}$ ), *i.e.* if  $x \geq R_t^l = \max\{R_t^{F,l}, R_t^{H,l}\}$ . If the simple job is held by a high-skilled, it is profitable if  $x$  is higher than the reservation productivity for both the firm ( $R_t^{F,h}$ ) and the high-skilled household ( $R_t^{H,h}$ ), *i.e.* if  $x \geq R_t^h = \max\{R_t^{F,h}, R_t^{H,h}\}$ . The asset value of an intermediate firm with a simple vacancy is therefore:

$$\begin{aligned} W_t^{V,s} &= -a + \tilde{\beta}_t q_t^{sl} E_t \left[ F(R_{t+1}^l) W_{t+1}^{V,s} + \int_{R_{t+1}^l}^{+\infty} W_{t+1}^{F,l}(z) dF(z) \right] \\ &\quad + \tilde{\beta}_t q_t^{sh} E_t \left[ F(R_{t+1}^h) W_{t+1}^{V,s} + \int_{R_{t+1}^h}^{+\infty} W_{t+1}^{F,h}(z) dF(z) \right] \\ &\quad + \tilde{\beta}_t (1 - q_t^{sl} - q_t^{sh}) E_t [W_{t+1}^{V,s}], \end{aligned} \quad (15)$$

where  $a$  is the per-period simple vacancy opening cost. The asset values of a firm producing a low-tech good with a low-skilled ( $W_t^{F,l}$ ) and a high-skilled ( $W_t^{F,h}$ ) are:

$$\begin{aligned} W_t^{F,l}(x) &= x d_t^s - w_t^l(x) \\ &\quad + \tilde{\beta}_t E_t \left[ F(R_{t+1}^l) W_{t+1}^{V,s} + \int_{R_{t+1}^l}^{+\infty} W_{t+1}^{F,l}(z) dF(z) \right], \end{aligned} \quad (16)$$

$$\begin{aligned} W_t^{F,h}(x) &= x d_t^s - w_t^h(x) \\ &\quad + \tilde{\beta}_t (1 - p_t^c \mathcal{S}_t^o) E_t \left[ F(R_{t+1}^h) W_{t+1}^{V,s} + \int_{R_{t+1}^h}^{+\infty} W_{t+1}^{F,h}(z) dF(z) \right] \\ &\quad + \tilde{\beta}_t p_t^c \mathcal{S}_t^o E_t [W_{t+1}^{V,s}], \end{aligned} \quad (17)$$

where  $d_t^s$  is the price of an intermediate low-tech good and  $w_t^l$  (resp.  $w_t^h$ ) is the wage for a low-skilled (resp. high-skilled) working on a simple job. The supply of low-tech goods is:

$$G_t^s = \frac{\int_{R_t^l}^{+\infty} z dF(z)}{1 - F(R_t^l)} N_t^l + \frac{\int_{R_t^h}^{+\infty} z dF(z)}{1 - F(R_t^h)} N_t^h. \quad (18)$$



### Reservation productivity and free entry conditions

The firms' reservation productivity is determined by:

$$W_t^{F,i} (R_t^{F,i}) = 0 \quad \text{with} \quad i \in \{l, h, c\}, \quad (19)$$

and the free entry condition gives:

$$W_t^{V,i} = 0 \quad \text{with} \quad i \in \{s, c\}. \quad (20)$$

## 2.3 Households

We have two representative households, one composed of the low-skilled and one composed of the high-skilled. Inside each household, members are perfectly insured against unemployment risk.

### Low-skilled household

The low-skilled representative household's welfare is:

$$W_t^{H,l} = \mathcal{U}(C_t^l) - \mathcal{D}^l(N_t^l) + \beta E_t [W_{t+1}^{H,l}], \quad (21)$$

where  $C_t^l$  is the low-skilled consumption,  $\mathcal{U}$  is an increasing and concave utility function,  $\mathcal{D}^l$  is an increasing and convex work disutility function, and  $\beta$  is the psychological discount factor. The low-skilled pay no tax and consume all their labor and unemployment incomes (equivalent to a liquidity constraint for low incomes):

$$C_t^l = w_t^u U_t^l + \bar{w}_t^l N_t^l, \quad (22)$$

where  $w_t^u$  is the unemployment benefits (determined in one exogenous policy process) and  $\bar{w}_t^l$  their average wage defined as:

$$\bar{w}_t^l = \frac{\int_{R_t^l}^{+\infty} w_t^l(z) dF(z)}{1 - F(R_t^l)}. \quad (23)$$

The marginal welfare  $W_{N_t^l}^{H,l}$  is developed in appendix 3.

### High-skilled household

The high-skilled representative household's welfare satisfies the Bellmann equation:

$$W_t^{H,h} = \max_{e_{c_t}, e_{o_t}, C_t^h} \left\{ \mathcal{U}(C_t^h) - \mathcal{D}^o(e_{o_t}) N_t^h - \mathcal{D}^c(N_t^c) + \beta E_t [W_{t+1}^{H,h}] \right\}. \quad (24)$$

The decision variables of the high-skilled household are the consumption level  $C_t^h$  and the search efforts of job seekers. High-skilled unemployed workers allocate

their total search time (normalized to one) between the simple ( $ec_t$ ) and the complex job markets ( $1 - ec_t$ ). Their search efficiency on the complex (resp. simple) job market, denoted  $\mathcal{S}_t^c$  (resp.  $\mathcal{S}_t^s$ ) is an increasing and concave function of the search time devoted to that market ( $ec_t$  and  $(1 - ec_t)$  respectively). High-skilled workers employed on a simple job may spend a fraction  $eo_t$  of their leisure time searching for a better paid complex job. Their on-the-job search efficiency  $\mathcal{S}_t^o$  is an increasing concave function of  $eo_t$ .  $\mathcal{D}^o$  is an increasing and convex on-the-job search disutility function, and  $\mathcal{D}^c$  is an increasing and convex complex work disutility function<sup>15</sup>. This maximization is subject to equations (1), (10), (11) and to the budget constraint:

$$C_t^h = \Pi_t^s + \Pi_t^c + w_t^u U_t^h + \bar{w}_t^h N_t^h + \bar{w}_t^c N_t^c + (r_t + \delta)K_t - K_{t+1} + (1 - \delta)K_t - T_t. \quad (25)$$

The high-skilled own the intermediate firms and  $\Pi_t^s$  (resp.  $\Pi_t^c$ ) represents the dividends paid by the simple (resp. complex) firms. They also lend their capital stock  $K_t$ , which provides a real interest rate  $r_t$  but depreciates at rate  $\delta$ . Finally, they are subject to a lump-sum tax  $T_t$  to finance the unemployment benefits. The average wages  $\bar{w}_t^h$  (resp.  $\bar{w}_t^c$ ) paid to a high-skilled working on a simple (resp. complex) jobs are respectively:

$$\bar{w}_t^h = \frac{\int_{R_t^h}^{+\infty} w_t^h(z) dF(z)}{1 - F(R_t^h)} \quad \text{and} \quad \bar{w}_t^c = \frac{\int_{R_t^c}^{+\infty} w_t^c(z) dF(z)}{1 - F(R_t^c)}. \quad (26)$$

The first order conditions (equality between marginal costs and expected marginal incomes) are, for respectively  $ec_t$ ,  $eo_t$  and  $C_t$ :

$$0 = p_t^c \mathcal{S}_{ec_t}^c \beta \left\{ \mathbb{E}_t \left[ \int_{R_{t+1}^c}^{+\infty} W_{N_{t+1}^c}^{H,h}(z) dF(z) \right] - p_t^s \mathcal{S}_{1-ec_t}^s \mathbb{E}_t \left[ \int_{R_{t+1}^h}^{+\infty} W_{N_{t+1}^h}^{H,h}(z) dF(z) \right] \right\}, \quad (27)$$

$$\mathcal{D}_{eo_t}^o = p_t^c \mathcal{S}_{eo_t}^o \beta \left\{ \mathbb{E}_t \left[ \int_{R_{t+1}^c}^{+\infty} W_{N_{t+1}^c}^{H,h}(z) dF(z) \right] - \mathbb{E}_t \left[ \int_{R_{t+1}^h}^{+\infty} W_{N_{t+1}^h}^{H,h}(z) dF(z) \right] \right\}, \quad (28)$$

$$\mathcal{U}_{C_t^h} = \beta \mathbb{E}_t \left[ \mathcal{U}_{C_{t+1}^h} (1 + r_{t+1}) \right]. \quad (29)$$

The marginal welfare  $W_{N_t^c}^{H,h}$  and  $W_{N_t^h}^{H,h}$  are developed in appendix 3.

<sup>15</sup>The simple work disutility is represented by the independent term of function  $\mathcal{D}^o$ .

## Reservation productivity, destruction rate and discount rate

The households' reservation productivity is determined by:

$$W_{N_t^l}^{H,l}(R_t^{H,l}) = 0, \quad W_{N_t^h}^{H,h}(R_t^{H,h}) = 0, \quad W_{N_t^c}^{H,h}(R_t^{H,c}) = 0. \quad (30)$$

We are now able to define the job destruction rates:

$$\chi_t^l = F(R_t^l), \quad \chi_t^h = F(R_t^h), \quad \psi_t = F(R_t^c), \quad (31)$$

and the rate at which future profits are discounted:

$$\tilde{\beta}_t = \beta E_t \left[ \frac{\mathcal{U}_{C_{t+1}^h}}{\mathcal{U}_{C_t^h}} \right]. \quad (32)$$

## 2.4 Final Firm

The representative final firm asset value satisfies the Bellmann equation:

$$W_t^F = \max_{K_{t+1}, G_t^c, G_t^s} \left\{ \mathcal{F}(K_t, G_t^c, G_t^s) - (r_t + \delta)K_t - d_t^c G_t^c - d_t^s G_t^s + \tilde{\beta}_t E_t [W_{t+1}^F] \right\}, \quad (33)$$

where the production function  $\mathcal{F}$  satisfies the usual Inada conditions. The demand for the three inputs is given by the first order conditions:

$$\mathcal{F}_{K_t} = r_t + \delta, \quad (34)$$

$$\mathcal{F}_{G_t^s} = d_t^s, \quad (35)$$

$$\mathcal{F}_{G_t^c} = d_t^c. \quad (36)$$

## 2.5 Wages Determination

At each period, wages are (re)negotiated between the firm and the corresponding household. These bargained wages (respectively for a low-skilled worker, for a high-skilled working on a simple job and for a high-skilled working on a complex job) are therefore determined by the following standard Nash product problems:

$$\max_{w_t^{b,l}(x)} \left( \frac{W_{N_t^l}^{H,l}(x)}{\mathcal{U}_{C_t^l}} \right)^{\eta^l} \left( W_t^{F,l}(x) - W^{V,s} \right)^{(1-\eta^l)}, \quad (37)$$

$$\max_{w_t^{b,h}(x)} \left( \frac{W_{N_t^h}^{H,h}(x)}{\mathcal{U}_{C_t^h}} \right)^{\eta^h} \left( W_t^{F,h}(x) - W^{V,s} \right)^{(1-\eta^h)}, \quad (38)$$

$$\max_{w_t^{b,c}(x)} \left( \frac{W_{N_t^c}^{H,h}(x)}{\mathcal{U}_{C_t^h}} \right)^{\eta^c} \left( W_t^{F,c}(x) - W^{V,c} \right)^{(1-\eta^c)}, \quad (39)$$

where  $\eta^i$ , with  $i \in \{l, h, c\}$ , is the corresponding households' bargaining power<sup>16</sup>. By using the asset value equations of section 2.2 (equations (12), (13), and (15) to (17)), the marginal welfare equations defined in appendix 3 (equations (61) to (63)), as well as the definitions of the reservation productivity (equations (19) and (30)) and the free entry conditions (equations (20)), we can solve equations (37) to (39) and rewrite the bargained wages:

$$w_t^{b,l}(x) = \eta^l(x - R_t^l)d_t^s + w_t^u, \quad (40)$$

$$w_t^{b,h}(x) = \eta^h(x - R_t^h)d_t^c + w_t^u, \quad (41)$$

$$w_t^{b,c}(x) = \eta^c(x - R_t^c)d_t^c + w_t^u. \quad (42)$$

Up to now, it was implicitly assumed that the minimum wage restriction is not binding. However, these negotiations are bounded downwards by a minimum wage  $w_t^m$  (exogenous policy process), *i.e.* there exists an endogenous productivity threshold  $Q_t^i$  such that:

$$w_t^{b,i}(Q_t^i) = w_t^m \quad \text{with} \quad i \in \{l, h, c\}. \quad (43)$$

Introducing this constraint into equations (40)-(42), we obtain the wage  $w_t^i$ , with  $i \in \{l, h, c\}$ :

$$w_t^i(x) = \begin{cases} w_t^m & \text{if } x \leq Q_t^i, \\ w_t^{b,i}(x) & \text{if } x > Q_t^i, \end{cases} \quad (44)$$

where

$$w_t^{b,l}(x) = \eta^l(x - Q_t^l)d_t^s + w_t^m, \quad (45)$$

$$w_t^{b,h}(x) = \eta^h(x - Q_t^h)d_t^c + w_t^m, \quad (46)$$

$$w_t^{b,c}(x) = \eta^c(x - Q_t^c)d_t^c + w_t^m. \quad (47)$$

### 3 Simulations

We first calibrate our model on Belgian data and discuss the exogenous policy process for  $w_t^u$  and  $w_t^m$ . We use it then to simulate reductions in employer's social security contributions.

#### 3.1 Calibration

We use the following specific functions:

$$\mathcal{M}^i(V, U) = \bar{m}^i V^{\lambda^i} U^{1-\lambda^i}, \quad \text{with} \quad i \in \{s, c\}, \quad (48)$$

---

<sup>16</sup>The firms asset values are expressed in the *numéraire* good while the households' welfare are in consumption utility. To normalize, we divide the marginal welfare by the marginal utility as, for instance, in Merz (1995).

$$\mathcal{F}(K, G^c, G^s) = K^\theta (G^c)^\mu (G^s)^{1-\theta-\mu}, \quad (49)$$

$$\mathcal{U}(C) = \ln(C), \quad (50)$$

$$\mathcal{S}^i(e) = \sigma_1^i \frac{e^{0.6}}{0.6}, \quad \text{with } i \in \{c, o\}, \quad (51)$$

$$\mathcal{D}^o(e) = \phi_0^o + \phi_1^o \frac{e^2}{2}, \quad (52)$$

$$\mathcal{D}^i(N) = \phi_1^i \frac{N^{1.2}}{1.2}, \quad \text{with } i \in \{l, c\}. \quad (53)$$

Matching functions are Cobb-Douglas with constant returns to scale. Following Manacorda and Petrongolo (1999), we also use a constant returns to scale production function with three inputs to represent the technological constraints faced by the final firm<sup>17</sup>. The utility function is logarithmic. Search functions are concave and disutility functions are convex. The independent term of  $\mathcal{D}^o$  represents the disutility of working on a simple job.

We calibrate the model on Belgian quarterly data and the reference years are 1995-1997<sup>18</sup>. As usual in this type of quarterly model, the depreciation rate  $\delta$  of capital is 2.5% and the psychological discount factor  $\beta$  is 0.99, which leads to an annual real interest rate of 4%. Using 1997 Belgian data, Van der Linden and Dor (2002) estimate the elasticity of matches with respect to vacancies at 0.4 (without skill distinction). We take this estimation for both the simple and the complex matches ( $\lambda^i = 0.4$ , with  $i \in \{s, c\}$ ). The household's bargaining power is usually calibrated as the coefficient of unemployment in the matching function ( $1 - \lambda^i$ )<sup>19</sup>. We therefore set the households' bargaining power, for both the low-skilled bargaining for a simple job and the high-skilled bargaining for a complex job  $\eta^s = \eta^c = 0.6$ . The high-skilled working on simple jobs have the same productivity as the low-skilled, but a stronger bargaining position (since they can search "on-the job" for complex jobs and leave). To keep identical wages for the low-skilled and the high-skilled (working on a simple job), we decrease the bargaining power of the latter to  $\eta^h = 0.55$ <sup>20</sup>. We also take the Van der Linden and Dor (2002) estimation of the replacement ratio (0.34), which gives  $w^u = 0.66$ <sup>21</sup>. As in Sneessens and Shadman (2000), the low-skilled have at most a lower secondary degree and the high-skilled have at least an upper secondary degree, and the estimated (for Belgium in 1996) skill fraction  $\alpha$  of the population

<sup>17</sup>This implies a unitary elasticity of substitution between low-tech and high-tech goods. For the French case, this elasticity is usually estimated between 0.7 and 2.5 (see for instance Cahuc (2003) for a synthesis). See section 4.3 for a discussion.

<sup>18</sup>Most data are available for this period. Moreover, Belgium was neither in a recession nor in a boom.

<sup>19</sup>See for instance Merz (1995) or Andolfatto (1996). Their motivation is that, in their simpler model, this sharing rule implies that the decentralized economy gives the same outcome that the social planner's problem (Hosios-Pissarides condition).

<sup>20</sup>We therefore assume that the wages on simple jobs depend on the job rather than the skill.

<sup>21</sup>This estimation may seem rather low with respect to others (see OECD (2002) for European estimations). The ratio we used here is however an average for all the unemployed and therefore takes into account the eligibility criteria.

	transition elasticity	duration elasticity	Kaitz index
empirical reference	1.5	0.2/1.0	0.58
$\bar{x} = 0$	1.7	0.2	0.46
$\bar{x} = 1$	1.4	0.9	0.59
$\bar{x} = 2$	1.3	1.3	0.61

Transition elasticity: elasticity of the transition probability (from the minimum wage employment to unemployment) with respect to the minimum wage cost, empirical reference given by Kramarz and Philippon (2000). Duration elasticity: elasticity of unemployment duration with respect to the replacement ratio, empirical reference given by Holmlund (1998). Kaitz index: ratio of the minimum wage to the average wage, empirical reference given by OECD (1998).

Table 1: Sensitivity analysis: model properties depending on the value of  $\bar{x}$

is 0.67. We also take their estimations to calibrate the production function, and set  $\theta = 0.35$  and  $\mu = 0.51$ . The legal nationwide minimum wage is very low in Belgium and not really binding. In practice, minimum wages constraints are fixed at the sectoral level. We estimate that 10% of the Belgian workers are paid these minimum wages (see appendix 2 for an extensive justification).  $w^m$  is calibrated at 1.14 to reproduce this result.

The matching efficiencies, the per-period vacancy costs and the parameters of the search and disutility functions ( $\bar{m}^s, \bar{m}^c, a, b, \sigma_1^c, \sigma_1^o, \phi_0^o, \phi_1^o, \phi_1^l, \phi_1^c$ ) are chosen to reproduce the observed values of the following 10 variables. In 1996, the low-skilled (resp. high-skilled) unemployment rate was 20% (resp. 7%) in Belgium (see Sneessens and Shadman (2000)). Van der Linden and Dor (2002) estimate a lower bound for the job destruction rate equal to 0.013 (monthly data). Using this estimate and the fact that simple jobs are more precarious than complex jobs, we choose the parameters such that the quarterly simple job destruction rate  $\chi^l = \chi^h = 0.05$  and the quarterly complex job destruction rate  $\psi = 0.025$ . We assume that the proportion of high-skilled on simple jobs is equal to 10%. If we keep in mind that distinguishing only two skill categories (rather than three or more) will mechanically and maybe drastically reduce the number of "overeducated workers", our assumption seems realistic compare to empirical estimations (see for instance Hartog (2000) European estimations and Delmotte, Van Hootegem, and Dejonckheere (2001) for Belgian ones). The probabilities to find a simple job and a complex job are based on Cockx and Dejemeppe (2002), and we choose the parameters to respectively have  $p^s = 0.2$  and  $p^c = 0.4$ . The probabilities to fill a complex job and a simple job are based on Delmotte, Van Hootegem, and Dejonckheere (2001), we choose the parameters to respectively have  $q^s = 0.5$  and  $q^c = 0.5$  (see Pierrard and Sneessens (2003) for an extensive explanation).

As in most of the related literature (see for instance Mortensen and Pissarides

Population and production					
$\alpha$	0.67	$\theta$	0.35	$\mu$	0.51
Matching and vacancy costs					
$\bar{m}^s$	0.30	$\lambda^s$	0.4	$a$	0.14
$\bar{m}^c$	0.45	$\lambda^c$	0.4	$b$	0.15
Search functions					
$\sigma_1^o$	0.47	$\sigma_1^l$	0.24		
Search and labor disutility functions					
$\phi_0^o$	0.26				
$\phi_1^o$	0.23	$\phi_1^l$	1.65	$\phi_1^c$	1.00
Psychological discount and depreciation rates					
$\beta$	0.99	$\delta$	0.025		
Wages determination and shocks distribution					
$\eta^l$	0.6	$\eta^h$	0.55	$\eta^c$	0.6
$w^u$	0.66	$w^m$	1.14	$\bar{x}$	1

Table 2: Numerical parameter values

(1994)), we assume the idiosyncratic shocks are drawn out of a uniform distribution on a unit interval  $[\bar{x}, \bar{x} + 1]$ , with  $\bar{x} \geq 0$ . The value of  $\bar{x}$  is crucial because it affects the wage distribution and transition probabilities<sup>22</sup>. In table 1, we compute the elasticity of the transition probability (from the minimum wage employment to unemployment) with respect to the minimum wage cost, the elasticity of unemployment duration with respect to the replacement ratio, and the Kaitz index (ratio of the minimum wage to the average wage), for different values of  $\bar{x}$  and keeping constant the above-mentioned calibration. Our model elasticity's are compared to reference empirical estimations by Kramarz and Philippon (2000), Holmlund (1998) and OECD (1998) for, respectively, the transition probability elasticity, the unemployment duration elasticity and the Kaitz index. Given this sensibility analysis, we eventually choose  $\bar{x} = 1$ , *i.e.* a uniform distribution  $F$  bounded between 1 and 2.

The model calibration yields reasonable values for the remaining endogenous variables. We obtain 0.05 for the on-the-job search intensity  $eo$  and 0.88 for the high-skilled complex job search intensity  $ec$ . Moreover, the minimum wage is not binding for the complex jobs ( $Q^c$  is strictly lower than  $R^c$ ), the average wage paid on simple jobs represents 55% of the average wage paid on complex jobs, which is similar to OECD (1996) estimation of wage dispersion in Belgium, and the steady state minimum wage represents 50% of the average high wage. As expected, the per-period cost of opening a complex job is slightly higher

<sup>22</sup>With a low  $\bar{x}$ , the relative difference between minimum and maximum productivity levels  $((\bar{x} + 1)/\bar{x})$  is large; while with a high  $\bar{x}$ , relative difference is narrow. As a result, the higher is  $\bar{x}$ , the flatter is the bargained wage curve. These differences in wage dispersion, depending on the value of  $\bar{x}$ , explain the variations in properties. Instead of increasing  $\bar{x}$ , we could obtain similar results by decreasing the dispersion around  $\bar{x}$ .

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Unemployment and crowding out							
$\frac{U^l}{1-\alpha}$	0.20	$\frac{U^h}{\alpha}$	0.07	$\frac{N^l}{N^l+N^h}$	0.10		
Probabilities to find and fill a job							
$p^s$	0.2	$p^c$	0.4	$q^s$	0.5	$q^c$	0.5
Job destruction rates							
$\chi^l$	0.05	$\chi^h$	0.05	$\psi$	0.025		
Search intensities							
$S^o$	0.05	$S^c$	0.88				
Wages							
RR	0.34	ind	0.50	WD	0.55	popmw	0.10
Consumptions and vacancy costs							
C ratio	0.47	AVC	0.04				

---

RR: replacement ratio (unemployment benefit over average wage). ind: minimum wage over average high wage. WD: wage dispersion (average wage paid on simple job over average wage paid on a complex job). popmw: share of the working population paid at the minimum wage. C ratio: the low-skilled consumption over the high-skilled consumption. AVC: average vacancy cost over average annual wage.

Table 3: Implied values for some variables

than the cost of opening a simple job and the average cost represents 3.7% of the average annual wage, similar to Mortensen and Pissarides (1999). Finally, the low-skilled per capita consumption represents 47% of the high-skilled per capita consumption. All the calibration, as well as the implied variable values, are summarized in tables 2 and 3.

### Exogenous deterministic policy process

Although the minimum wage is more rigid than other bargained wages, it is also regularly adapted (see appendix 2). To take into account both the short-run rigidity and the regular adaptations, we assume for the minimum wage an indexation policy with inertia:

$$w_t^m = ind (\alpha_0 \bar{w}_t^c + \alpha_1 \bar{w}_{t-1}^c + \alpha_2 \bar{w}_{t-2}^c + \alpha_3 \bar{w}_{t-3}^c + \alpha_4 \bar{w}_{t-4}^c), \quad (54)$$

where *ind* is the long run indexation parameter of the minimum wage on the high-skilled wages (see table 3 for the numerical value), and  $\sum_{i=0}^4 \alpha_i = 1$ . We indeed see in OECD (1996) that the D9/D1 ratio (ratio of the upper earning limit of the ninth decile of workers to the upper limit of the first decile, *i.e.* the ratio of the highest wages to the lowest wages) has remained fairly stable over the last years. We point out that the choice of the  $\alpha_i$  has only minor effects on the transitional dynamics and, of course, none on the steady state. We choose  $[\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4] = [0, 0.1, 0.1, 0.3, 0.5]$ .

In the same way, we assume that the unemployment benefits are regularly in-



dexed on the average wage, using the same policy process:

$$w_t^u = RR (\alpha_0 \bar{w}_t + \alpha_1 \bar{w}_{t-1} + \alpha_2 \bar{w}_{t-2} + \alpha_3 \bar{w}_{t-3} + \alpha_4 \bar{w}_{t-4}), \quad (55)$$

where  $RR$  is the long run replacement ratio (see table 3) and  $\bar{w}_t$  is the economy average wage. It is worth noting that, to simplify, we assume that  $w_t^m$  and  $w_t^u$  are taken as exogenous by the firms and the households.

### 3.2 Reductions in Employer's Social Security Contributions

The model presented in section 2 do not include labor taxation, *i.e.* the gross wage (paid by the firm) is equal to the net wage (received by the worker). Decreasing labor taxation for the employer is therefore modeled as a wage subsidy to the employer.

#### Transitional dynamics

In this subsection, we study a reduction targeted at the very low wages (minimum wage) and a reduction targeted at the high wages. For both policies, we assume a proportional reduction whose *ex ante* cost amounts to 0.2% of GDP.

To introduce a proportional reduction targeted at the minimum wage, we only have to modify wages paid by the intermediate firms. More precisely, if  $x < Q_t^l$ ,  $w_t^l(x)$  in equation (16) becomes  $(1 - t^m)w_t^m$  (no change if  $x > Q_t^l$ ), and if  $x < Q_t^h$ ,  $w_t^h(x)$  in equation (17) also becomes  $(1 - t^m)w_t^m$  (no change if  $x > Q_t^h$ ).  $(1 - t^m)w_t^m$  is the wage paid after the policy, by the concerned intermediate firms. The concerned workers still receive the wage  $w_t^m$  and the difference  $t^m w_t^m$  is paid by the government<sup>23</sup>. To close the government budget, the lump-sum tax  $T_t$  paid by the high-skilled household, in equation (25), is increased by the amount of the policy cost<sup>24</sup>. The effects of this policy are displayed in the graphs of figure 3.

Due to a decrease in the employer's contributions, targeted at the minimum wage, some low-productivity simple jobs, that were previously destroyed, are now retained, and the job destruction rate immediately decreases. After a small rise during the first period following the policy introduction, the job creation also decreases but less significantly than the destruction rate. As a result, net simple employment increases and we recover empirical results that most of the employment gains are due to a fall in job destruction rather than an increase in job creation, and that most of the adjustment occurs after two years<sup>25</sup>. Because we have more simple jobs paid at the minimum wage, the average wage

<sup>23</sup>The value of  $t^m$  is chosen to obtain an *ex ante* policy cost of 0.2% of GDP. Formally:  $0.2\% \mathcal{F} = t^m w^m \left( N^l \frac{F(Q^l) - F(R^l)}{1 - F(R^l)} + N^h \frac{F(Q^h) - F(R^h)}{1 - F(R^h)} \right)$ .

<sup>24</sup>In section 4.4, we replace the lump-sum tax by a proportional taxation on complex wages.

<sup>25</sup>Net employment change is defined as the job creation rate minus the job destruction rate and is represented in the graph as the area between these two curves.

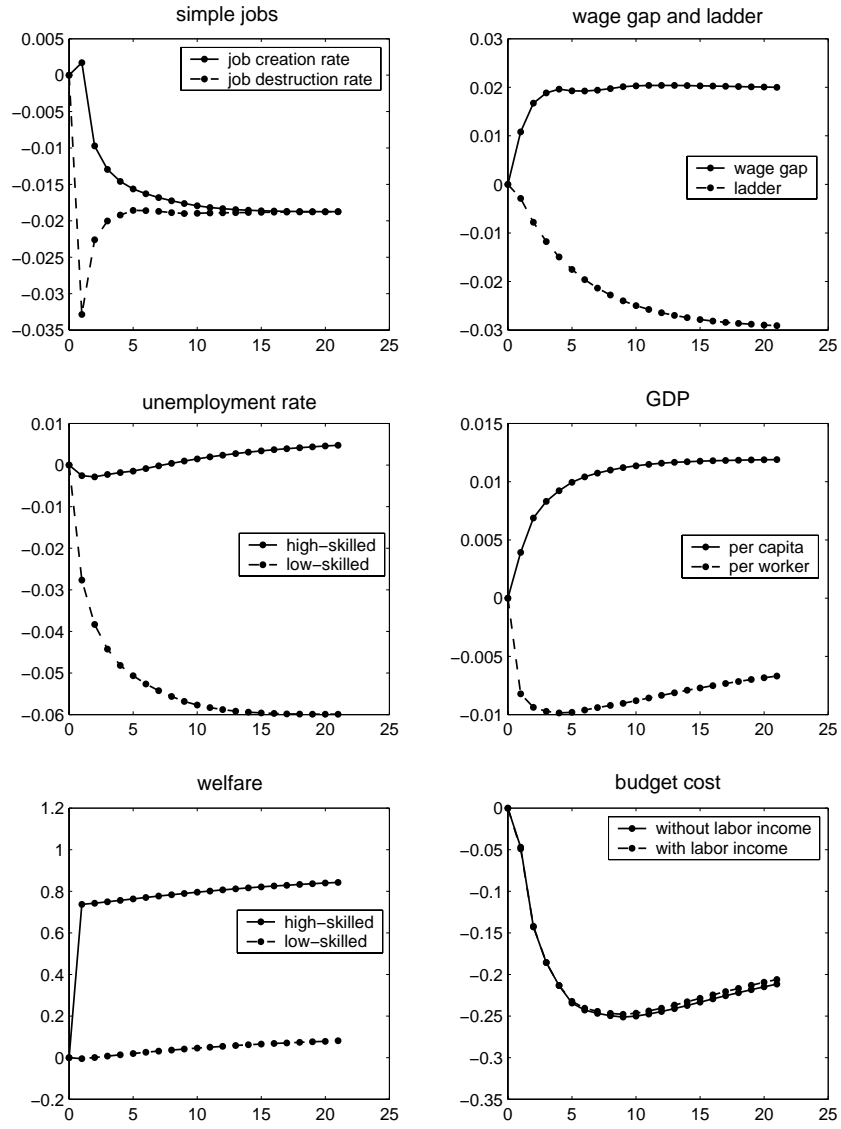


Figure 3: Short-run effects of reductions targeted at the minimum wage (deviations from the initial steady state)

paid on simple jobs falls (composition effect). On the other hand, the rise in simple employment (and low-tech goods) automatically improves the marginal productivity of the high-tech goods, which stimulates the demand for workers on complex jobs, and hence the upward pressure on complex wages. This widening of the wage gap reduces the job competition, *i.e.* decreases the ladder effect. With more jobs available and less competition, the low-skilled unemployment strongly decreases, while the increase in complex jobs is not sufficient to absorb the previously de-qualified high-skilled workers and the high-skilled unemployment slightly rises. This policy seems therefore particularly relevant in countries with high low-skilled unemployment rates. Overall, the higher employment stimulates the output per capita, but the share of low-skilled workers increases and this deteriorates the output per worker. The welfare of the low-skilled remains unchanged (more income but also more work), while the welfare of the high-skilled strongly increases, since they have much less unemployment benefits to finance<sup>26</sup>. Moreover, we see that the net relative cost (policy cost plus unemployment benefit cost, over GDP) strongly decreases. In other words, the policy cost is lower than the sum saved due to less unemployment benefits to finance. If we take into account the fact that higher employment increases labor taxation income for the government, we do not obtain a higher reduction in the budget cost because GDP increases more rapidly than the labor taxation income<sup>27</sup>.

To introduce a proportional reduction targeted at the high wages (all wages paid on complex jobs), we only have to modify equation (13), where  $w_i^c(x)$  becomes  $(1-t^c)w_i^c(x)$ , the wage paid after the policy by the concerned intermediate firms. However, the concerned workers still receive the same wage  $w_i^c$  and the difference  $t^c w_i^c$  is paid by the government<sup>28</sup>. Again, to close the government budget, the lump-sum tax  $T_t$  paid by the high-skilled household, in equation (25), is increased by the amount of the policy cost. The effects of this policy are displayed in the graphs of figure 4.

We observe that both the complex job destruction and job creation rates decrease, but the gains in employment are here much more subdued. The reason is that with Nash bargained wages, a decrease in employer's taxation leads to a higher bargained wage. On the simple job market, wages remain stable during the initial periods (inertia in the minimum wage indexation) and, as a result, the wage gap widens. However, this widening is short-lived because, after some periods, the minimum wage is being indexed, and the ladder remains mostly unchanged. The decrease in unemployment is therefore mainly concentrated among the high-skilled. Again, the GDP per capita is stimulated by higher

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<sup>26</sup>It means it is more efficient (increase in the welfare) to redistribute income by using fiscal transfers (from the high-skilled to the low-skilled) than by distorting the price mechanism (high minimum wage). See Saint-Paul (2000) for an extensive discussion about these concepts.

<sup>27</sup>We assume a tax of 40% on wages paid on complex jobs. This labor taxation income amounts to 20% of GDP, which seems a realistic assumption in a European economy.

<sup>28</sup>The value of  $t^c$  is chosen to obtain an *ex ante* policy cost of 0.2% of GDP. Formally:  $0.2\% \mathcal{F} = t^c \bar{w}^c N^c \left( \frac{F(Q^c) - F(R^c)}{1 - F(R^c)} \right)$ .

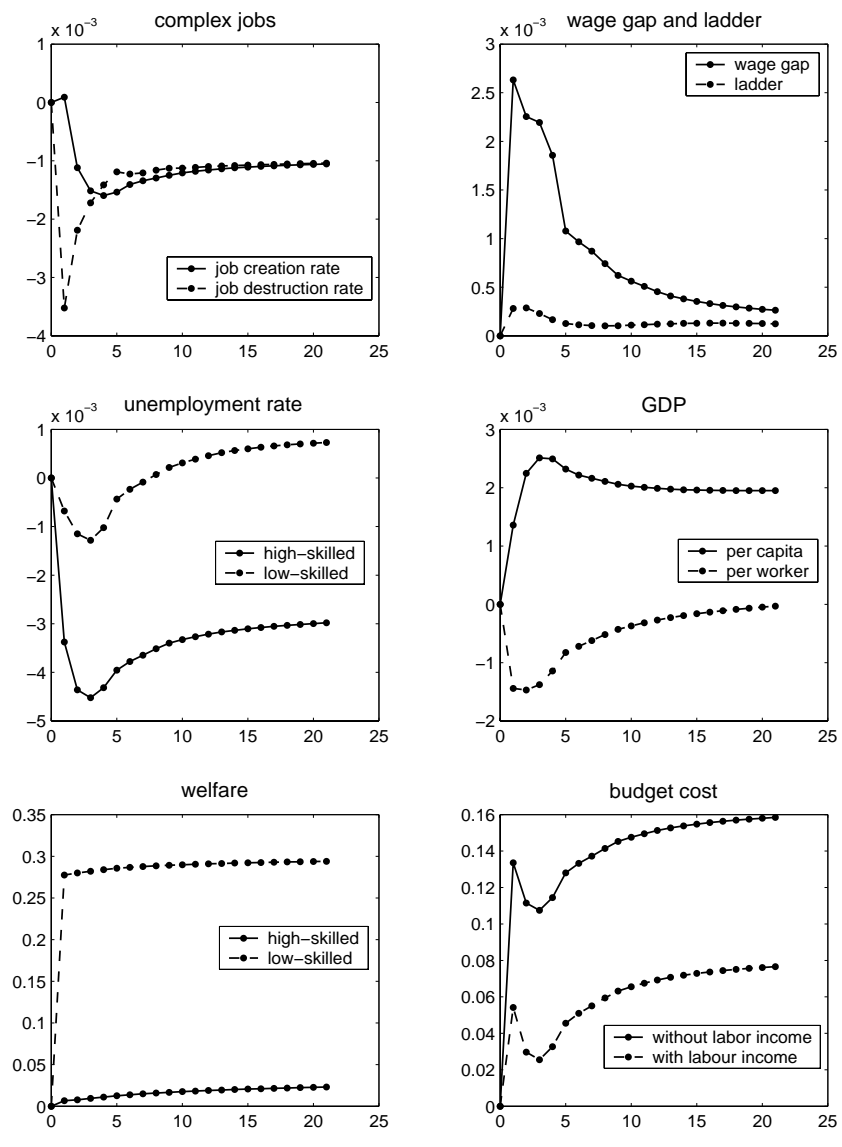


Figure 4: Short-run effects of reductions targeted at high wages (deviations from the initial steady state)

	"SMIC"	low wages	high wages	all
Sneessens & Shadman (2000)		+23.2		
Stockman (2002)		+9.5/+16.2	+4.4/+6.2	+2.2/+7.4
Burggraeve & Du Caju (2003)				+6.2/+9.4
Model	+59.6	+20.4	+6.3	+9.4

Table 4: Long-run effects on employment (in thousands) of reductions in contributions (of an *ex ante* cost of 0.2% of GDP), depending on the wages they are targeted at

employment, while the productivity (GDP per worker) is almost not affected. Welfare increases for the low-skilled (higher wages), but we see that the gains in employment (and the lower amount of unemployment benefits associated) are not sufficient to compensate the policy cost, even with the increase in the labor tax income for the government. The tax burden on the high-skilled rises and this affects their welfare.

### Long-run effects: comparison to previous literature

In this subsection, we compare our long-run results with those of other recent papers, also focused on the Belgian economy. Table 4 reproduces the long-run effects on total employment of reductions in employer's social security contributions. Depending on the papers, these reductions are targeted at the low wages (paid to all workers on "low-tech" jobs), the high wages (paid to all workers on "high-tech" jobs) or can be spread across all wages. For each policy of each paper, we assume that the *ex ante* cost is equivalent to 0.2% of GDP<sup>29</sup>. To evaluate the effects of the reductions, Sneessens and Shadman (2000), Stockman (2002) and Burggraeve and Du Caju (2003) use macro-econometric models and their effects may depend on the assumptions they introduce (see appendix 4 for a short explanation of their models). The last line of table 4 displays our results. With respect to the above-mentioned papers, we also simulate a reduction targeted at the very low wages (minimum wage or "SMIC"). We have already explained how we introduce policies focused on the minimum wage and high wages. To model a policy focused on the low wages, we simply add a proportional reduction tax  $t^s$  in equations (16) and (17). To model a policy focused on all wages, we add a proportional reduction tax  $t^a$  in equations (13), (16) and (17).

<sup>29</sup>The policy cost varies depending on the paper but we normalize it by assuming that the policy effects on employment are linear.

We see that our estimations lie at the top of the range of the previous estimations. We, moreover, confirm that policies focused on the low wages are more powerful, in term of employment stimulation, than policies focused on high wages. The main conclusion is that a policy focused on the very low wages (minimum wage) has ten times more effects on employment than a policy focused on high wages<sup>30</sup>.

Although the effects of reductions at the SMIC level have never been studied in Belgium, they have already been explored in France, both empirically and theoretically. If we transpose our results to the French economy, a reduction similar to those of the "Juppé reform" would create around 500 000 jobs. Our results are therefore in line with French empirical micro estimations. They are, nevertheless, more important than those predicted by the recent theoretical macro models with matching<sup>31</sup>. However, contrary to ours, these macro models assume exogenous job destruction rate (in other words, all employment gains are due to job creation), which seems contrary to the Crépon and Desplatz (2002) observations. By ignoring the effects through the decrease in the job destruction rate, they may well underestimate the total effect on employment.

## 4 Discussions and Extensions

### 4.1 Job Competition

Pierrard and Sneessens (2003) argue that job competition (also called ladder effect or crowding out) is important to understand the high low-skilled unemployment rate. This endogenous job competition is also important to understand the effects of a reduction in contributions at the minimum wage level, especially on the respective unemployment rates. Following the introduction of the policy and the widening of the wage gap (see figure 3), de-qualified workers exhibit a more intensive on-the-job search (their search intensity rises from 5% to 8%) and the skilled unemployed reduce their search for simple jobs (from 12% to 8%). As a result, the simple jobs held by high-skilled decrease by 32 000 units and this positively affects the low-skilled employment, which rises by 78 000 units. By ignoring the job competition effect, we would have a less significant reduction in the low-skilled unemployment rate, however, we would not have an increase in the high-skilled unemployment rate, as in figure 3.

### 4.2 Employment Protection

In our model, reductions targeted at the minimum wage strongly affect the job destruction and, as a result, employment. However, we assume no employment

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<sup>30</sup>See appendix 1 for a similar results with a simplified labor market.

<sup>31</sup>Empirical micro papers (Laroque and Salanié (2000) or Crépon and Desplatz (2001)) estimate an increase in employment of 475 000 units; while macro estimations are much more subdued (around 200 000 new jobs), even with a high elasticity of substitution between skills.

protection and this could explain the job destruction - over? - reaction. In this section, for simplicity, we start from a simplified model (only one type of workers and one type of jobs) but we add an employment protection (in a sense of a firing tax). We next look at the effects of a decrease of 10% in the minimum wage, on the job destruction rate and the employment, depending on the value of the firing tax (see figure 5).

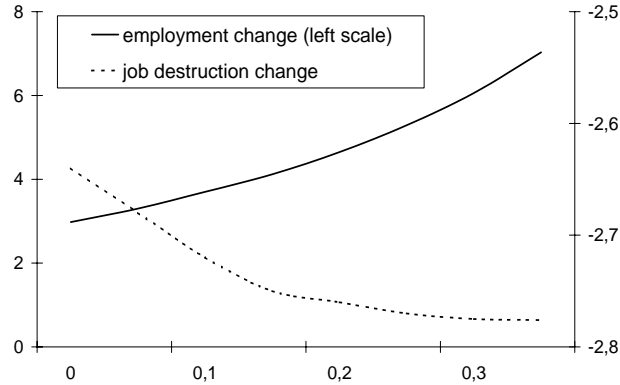


Figure 5: Effects of a decrease of 10% in the minimum wage, for different values of the firing tax

We see that adding employment protection *reinforce* the effects on the job turnover and the employment, *i.e.*, if  $f$  represents the firing tax,  $\frac{\partial^2 JT}{\partial w^m \partial f} > 0$  and  $\frac{\partial^2 N}{\partial w^m \partial f} > 0$ . The intuition is that a higher job protection increases the amount of low-productivity jobs and therefore the share of the population paid at the minimum wage. In this case, a shock on the minimum wage will produce strong employment effects.

Adding employment protection to soften the effects of a reduction in employer's contributions targeted at the minimum wage seems therefore counter-productive. An alternative solution could be to introduce firing restrictions *à la* Garibaldi (1998), although this is not trivial within our modelization.

### 4.3 Elasticity of Substitution Between Skills

In our model, with assume a unitary elasticity of substitution between the three production inputs (simple goods, complex goods and capital; see equation (49) and the calibration section for a justification). However, in the French literature<sup>32</sup>, it is generally assumed that the complex goods and capital are perfect

<sup>32</sup>See for instance Laffargue (2000), Salanié (2000), or Cahuc (2003).

complement and that the elasticity of substitution between this "complex aggregate" and the simple goods lies between 0.7 and 2.5. In this section, we follow the French literature and use the same production function (see appendix 5 for a detailed development).

	e.o.s.=0.7	e.o.s.=2.5	CD function
Simple employment	+38.0	+92.4	+46.5
Complex employment	+14.8	-14.8	+13.1
Total employment	+52.7	+77.6	+59.6

Table 5: Long-run effects on employment (in thousands) of reductions in contributions (for an *ex ante* cost of 0.2% of GDP) targeted at the minimum wage, depending on the elasticity of substitution between the skilled and the unskilled

We assume that the complex goods and capital are perfect complement. Table 5 reproduces the effects on employment of reductions in employer's contributions, targeted at the minimum wage (for an *ex ante* cost of 0.2% of GDP), for different values of the elasticity of substitution between the simple goods and the complex aggregate. In the second column, we assume a low elasticity of substitution (e.o.s.=0.7), while this elasticity is much higher in the third column (e.o.s.=2.5). These results are compared with those obtain with our initial production function (CD function).

We see that increasing the elasticity of substitution strengthens the positive effect of a reduction in contributions on total employment, as also emphasizes in the French literature. Moreover, with a low elasticity, complex jobs also benefit from a reduction targeted at the minimum wage, although this result no more holds with a high elasticity.

#### 4.4 Policy Financed by Labor Taxation

We assume in this paper a lump-sum taxation to finance the social policies (unemployment benefits and reductions in contributions). A lump-sum tax does not introduce distortions in the economy (it only affects the consumption level by the same amount) and therefore allows to isolate the effects of a reduction in contributions. This lump-sum tax is, however, not realistic and we instead assume in this section a proportional tax on the employees' complex wages to finance the policy. Figure 6 reproduces the effects on the unemployment rates, of reductions targeted at the minimum wage. The difference, comparing to figure 4, is that we use here a taxation on employees' complex wages rather



than a lump-sum tax on the high-skilled household.

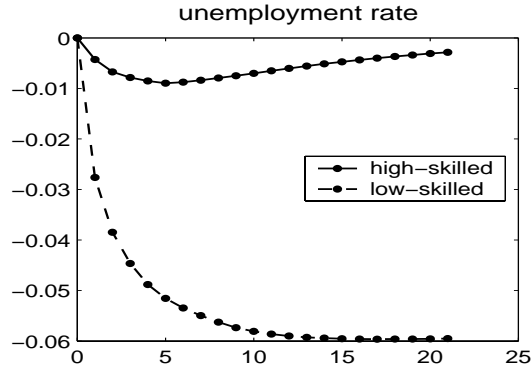


Figure 6: Short-run effects of reductions targeted at the very low wages, with high wage taxation (deviations from the initial steady state)

Since the reductions create a budgetary surplus, taxation on the employee's complex wages decreases and this also induces a decrease in the bargained wages. This is in turn favorable to the high-skilled employment. We therefore have a slight reduction in the high-skilled unemployment rate, which was not the case with a lump-sum tax.

#### 4.5 Budgetary Surplus

This paper studies the effects of selective reductions in labor taxation, for a given *ex ante* cost. We show that a reduction in the minimum wage cost by 5.5% (reduction allowed within an *ex ante* cost of 0.2% of GDP) creates a budgetary surplus, mainly because there is much less unemployment benefits to finance. Increasing the reductions will not necessarily lead to a higher budgetary surplus, since employment is progressively more difficult to create (tightness of the labor market) and a decrease in unemployment benefit expenses is no longer sufficient to compensate the policy costs. In figure 7, we compute the budgetary surplus for different values of reductions in the minimum wage cost. We see that the maximum budget surplus is obtained with a reduction of 16% in the minimum wage cost.

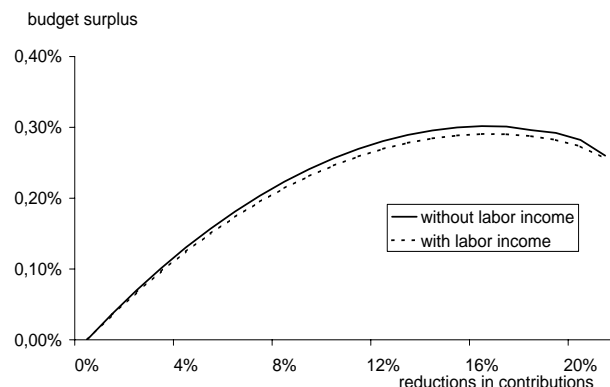


Figure 7: Budgetary surplus with respect to the level of reductions in contributions

## 5 Conclusion

We study in this paper the effects of selective reductions in employer’s social security contributions. To do so, we construct an intertemporal general equilibrium model with different types of workers (and wages) and search unemployment. With respect to previous literature, our main contribution is to add to this model endogenous job destruction rates. This seems important since it is empirically shown that most of the effects on employment, of reductions in employer’s contributions at the minimum wage level, go through lower job destruction rather than higher job creation.

Our model, calibrated on Belgian quarterly data, confirm this observation. It also shows that employment, and especially low-skilled employment, stimulation can be quite important and self-financed if the reductions are targeted at the minimum wage. Decreasing the minimum wage cost seems therefore a good answer to specific low-skilled unemployment problems, as it exists in many European countries. This should however be seen as a short-term policy, since it *decreases the destruction of the less productive jobs* and then strongly deteriorates the productivity of the economy. We then argue, along with this short-term policy, in favor of a long-term policy to *increase the creation of more productive jobs*.

Another interesting result is that reductions targeted at the minimum wage are supported (increase in the welfare) by both the low-skilled *and* the high-skilled households. It means that, within our modelization, the representative households prefer to achieve income equality through fiscal redistribution (from the high- to the low-skilled) rather than by rigid labor market institutions (high minimum wage).

This model could be easily extended to study other types of reductions in em-

ployer's contributions (we here only focus on proportional reductions), as well as more elaborated labor tax than the lump-sum tax. The modelization could also be further extended by adding some delay to firing, by combining endogenous *and* exogenous job destruction rates (these two extensions could attenuate the employment effects through less important decreases in the job destruction rate), or by introducing an endogenous labor market participation that could be combined with a more elaborated unemployment benefit representation. By simplicity, these extensions have been left aside in this paper but could be promising research venues.

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## Appendix 1: Effects of reductions in labor taxation in a simple model

We assume a competitive labor market with labor demand  $L^d$  a decreasing function of wage and labor supply  $L^s$  an increasing function of wage. The competitive equilibrium wage is given by  $L^d(w) = L^s(w)$ . If we introduce a tax reduction  $t$  in employer's contributions, the new equilibrium becomes:

$$L^d((1-t)w) = L^s(w). \quad (56)$$

By denoting  $\eta^d$  the elasticity of labor demand with respect to wage and  $\eta^s$  the elasticity of labor supply with respect to wage:

$$\eta^d = \frac{\partial L^d}{\partial((1-t)w)} \frac{(1-t)w}{L^d} \quad \text{and} \quad \eta^s = \frac{\partial L^s}{\partial w} \frac{w}{L^s},$$

and by log-differentiating equation (56), we obtain:

$$\eta^d \left( \frac{dw}{w} - \frac{dt}{1-t} \right) = \eta^s \left( \frac{dw}{w} \right). \quad (57)$$

The elasticity of wage with respect to tax reduction is therefore given by:

$$\frac{dw}{w} = -\frac{\eta^d}{\eta^s - \eta^d} \frac{dt}{1-t}, \quad (58)$$

and the elasticity of labor with respect to tax reduction by:

$$\frac{dL}{L} = \frac{dL^d}{L^d} = \frac{dL^s}{L^s} = -\frac{\eta^d \eta^s}{\eta^s - \eta^d} \frac{dt}{1-t}. \quad (59)$$

Labor supply elasticity to wage is usually estimated to be small and we assume  $\eta^s = 0.2$ . Labor demand for the low-skilled is more elastic to wage than labor demand for the high-skilled. We take  $\eta^d = -1$  (resp.  $\eta^d = -0.2$ ) for the low-skilled (resp. high-skilled). In the first case (low-skilled), a 1% decrease in employer's contributions increases wage by 0.8% and labor by 0.2%. In the second case (high-skilled), a 1% decrease in employer's contributions increases wage by 0.5% and labor by 0.1%. Reductions in contributions are therefore more effective, in employment term, if targeted at the low wages rather than at the high wages.

If we introduce a minimum wage in this economy,  $dw/w = 0$  and labor is completely determined by labor demand:

$$\frac{dL}{L} = \frac{dL^d}{L^d} = -\eta^d \frac{dt}{1-t}. \quad (60)$$

As a result, increase in labor with  $\eta^d = -1$  is now 1%.

## Appendix 2: Wage formation in Belgium

Wage formation in Belgium is quite complex and realized at three levels: the intra-sectoral (national) level, the sectoral level and the individual level. Every two years, negotiations at the intra-sectoral level determine the legal minimum wage, as well as the "wage norm". More regularly, wage negotiations are also held within the different economic sectors where the legal minimum wage and the wage norm are taken respectively as lower and upper bounds: sectoral minimum wages cannot be lower than the legal minimum wage and sectoral wage increases cannot exceed the limits fixed by the wage norm. Eventually, wages may, at each period, be (re)negotiated at the individual level, depending on the demand and supply of labor, or on individual characteristics. This justifies in our model the coexistence of a rigid minimum wage encompassing the legal and the sectoral ones (although this minimum wage may be regularly adapted), and of much more flexible bargained wages.

To try to estimate the percentage of the Belgian workers paid at one of the minimum wages, we base our analysis on the MET (1999) report. The monthly gross legal minimum wage (called RMMMG: *revenu mensuel minimum moyen garanti*) was of 41 660 BEF in 1995. Sectoral minimum wages are on average estimated to be 22% higher (although they may be much higher, *e.g.* up to 40% in the energy and construction sectors), which leads to an average gross minimum wage around 51 000 BEF. Still in 1995, 2% of the workers had a gross wage between 0 and 44 000 BEF; and 13% of the workers had a gross wage between 44 000 and 56 000. From these figures, we can conclude that if only a very small percentage of the workers is paid at the legal minimum wage, a much higher percentage is paid at one of the minimum wages.

In Belgium, the gross legal minimum wage represents 50.4% of the median gross wage, and the estimated gross minimum wage (taking into account the sectoral minimum wages) is therefore higher (around 60%). This ratio is one of the highest in the OECD countries. In France, the ratio is of 57.4% and it is usually estimated that 16% of the workers are paid at the "SMIC" minimum wage (see Cahuc and Zylberberg (2001)). Again this allows us to conclude that the percentage of Belgian workers paid at one of the minimum wages is important. We fix it at 10% in our calibration.



### Appendix 3: Marginal welfare

The first derivative with respect to  $N_t^l$  of equation (21), using equation (9), gives:

$$W_{N_t^l}^{H,l}(x) = \mathcal{U}_{C_t^l}(w_t^l(x) - w_t^u) - \mathcal{D}_{N_t^l}^l + (1 - p_t^s)\beta \mathbb{E}_t \left[ \int_{R_{t+1}^l}^{+\infty} W_{N_{t+1}^l}^{H,l}(z) dF(z) \right]. \quad (61)$$

The first derivatives with respect to  $N_t^h$  and  $N_t^c$  of equation (24), using equations (10) and (11), gives:

$$\begin{aligned} W_{N_t^h}^{H,h}(x) = & \mathcal{U}_{C_t^h}(w_t^h(x) - w_t^u) - \mathcal{D}^o(S_t^o) + \beta \left\{ (1 - p_t^c \mathcal{S}^o(S_t^o)) \right. \\ & \left. - p_t^s \mathcal{S}^c(1 - S_t^c) \right\} \mathbb{E}_t \left[ \int_{R_{t+1}^h}^{+\infty} W_{N_{t+1}^h}^{H,h}(z) dF(z) \right] \\ & + p_t^c (\mathcal{S}^o(S_t^o) - \mathcal{S}^c(S_t^c)) \mathbb{E}_t \left[ \int_{R_{t+1}^c}^{+\infty} W_{N_{t+1}^c}^{H,h}(z) dF(z) \right] \Big\}, \quad (62) \end{aligned}$$

$$\begin{aligned} W_{N_t^c}^{H,h}(x) = & \mathcal{U}_{C_t^h}(w_t^c(x) - w_t^u) - \mathcal{D}^c(N_t^c) \\ & + \beta \left\{ (1 - p_t^c \mathcal{S}^c(S_t^c)) \mathbb{E}_t \left[ \int_{R_{t+1}^c}^{+\infty} W_{N_{t+1}^c}^{H,c}(z) dF(z) \right] \right. \\ & \left. - p_t^s (\mathcal{S}^c(1 - S_t^c)) \mathbb{E}_t \left[ \int_{R_{t+1}^h}^{+\infty} W_{N_{t+1}^h}^{H,h}(z) dF(z) \right] \right\}. \quad (63) \end{aligned}$$

## Appendix 4: Some previous models calibrated on Belgian data

Sneessens and Shadman (2000) extended the original NAIRU model to take into account two types of workers: the high-skilled and the low-skilled. They econometrically estimate their model and use it to simulate the effects of a reduction in employer's contributions, targeted at the low-skilled. Our results are directly comparable to theirs because we use the same definitions for low-skilled (at most lower secondary education) and high-skilled (at least upper secondary education).

Stockman (2002) uses a new version of the sectoral macro-econometric model HERMES, developed at the Belgian Federal Planning Bureau. Workers are either paid at a low wage or at a high wage, and the elasticity of substitution between low and high wage workers is equal to 1 (as in our production function). They fix the cut-off monthly gross wage, between low and high wages, at 1562 €. As a result, they obtain 28% of the workers paid at a low wage and 72% paid at a high wage. This distinction is therefore not far from ours (respectively 33% and 67%). For each policy, two simulations are conducted: one assuming that gross wages are freely bargained and one assuming that gross wages are not affected by a reduction in employer's contributions. Reductions are implemented at time  $t$  and they estimate the effects from time  $t$  to time  $t + 6$  (yearly data). We take their time  $t + 6$  estimations to compare with our long-run results. More recently, still for the Belgian Federal Planning Bureau, Hendrickx, Joyeux, Masure, and Stockman (2003) develop an alternative macro-econometric model (the main difference is that the wage equation determines the wage cost, while in HERMES the wage equation determines the gross wage) but they only simulate a reduction distributed to all workers. Their estimations are slightly lower than those of Stockman (2002) (we do not reproduce their results in table 3).

To conduct their simulations, Burggraeve and Du Caju (2003) use as macro-econometric model the Belgian block of the eurosystem's multicountry model, developed, as a larger project, within the European System of Central Banks. They do not introduce skill and wage differentiation and they only simulate a reduction for all workers. They conduct several simulations. In table 3, we reproduce the case where all reductions are effectively used for labor cost reductions (we do not reproduce their two other cases, where reductions are entirely offset by increases in gross wage, and where they introduce a fiscal compensation (through VAT or production tax) for the reductions), considering two limiting scenario: a "real rule" scenario *vs.* a "nominal rule" scenario. Reductions are implemented at time  $t$  and they estimate the effects from time  $t$  to time  $t + 9$  (yearly data). We take their time  $t + 9$  estimations to compare with our long-run results.

## Appendix 5: A general CES production function

A general CES production function can be written as:

$$\mathcal{F} = \varepsilon [a_1(G^s)^v + a_2A^v]^{\frac{1}{v}}, \quad (64)$$

where

$$A = [a_3(G^c)^\tau + a_4(\gamma K)^\tau]^{\frac{1}{\tau}}. \quad (65)$$

$1/(1 - \tau)$  is the elasticity of substitution between  $G^c$  and  $K$ ,  $1/(1 - v)$  is the elasticity of substitution between  $G^s$  and the aggregate quantity  $A$ , and  $\varepsilon$ ,  $a_i$  and  $\gamma$  are technical parameters.

If we assume a unitary elasticity of substitution between inputs ( $\tau = v = 0$ ), it is easy to show that the production function becomes:

$$\mathcal{F} = \varepsilon (G^s)^{a_1} (G^c)^{a_2 a_3} (\gamma K)^{a_2 a_4}. \quad (66)$$

By setting  $\varepsilon = 1$ ,  $a_1 = 1 - \theta - \mu$ ,  $a_2 a_3 = \mu$ ,  $a_2 a_4 = \theta$  and  $\gamma = 1$ , we see that equation (66) is equivalent to equation (49). The Cobb-Douglas production function we use in our simulations is therefore a special case of the general CES production function (64).

In the French literature, it is usually assumed that complex goods and capital are perfect complement and that the elasticity of substitution between the simple goods and the complex aggregate lies between 0.7 and 2.5, *i.e.*  $1/(1 - \tau) = 0$  and  $0.7 \leq 1/(1 - v) \leq 2.5$ . In this case, the production function can be written as:

$$\mathcal{F} = \varepsilon [a_1(G^s)^v + a_2(G^c)^v]^{\frac{1}{v}}, \quad (67)$$

and the first order conditions (34) to (36) become:

$$\gamma K = G^c, \quad (68)$$

$$\mathcal{F}_{G^s} = a_1 \left( \frac{\mathcal{F}}{G^s} \right)^{1-v} = d^s, \quad (69)$$

$$\mathcal{F}_{G^c} = a_2 \left( \frac{\mathcal{F}}{G^c} \right)^{1-v} = d^c. \quad (70)$$

Given the values of  $v$ ,  $G^s$ , and  $G^c$ , the technical parameters  $\varepsilon$ ,  $a_1$ ,  $a_2$  and  $\gamma$  can be calibrated to obtain the wanted values for the variables  $\mathcal{F}$ ,  $K$ ,  $d^s$  and  $d^c$  (equations (67) to (70)).