What Does It Take to Achieve Equality of Opportunity in Education ? An Empirical Investigation Based on Brazilian Data

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What Does It Take to Achieve Equality of Opportunity in Education?

An Empirical Investigation Based on Brazilian Data [ℑ]

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Preliminary version. Do not quote.

Abstract

Roemer's' 1998 seminal work on equality of opportunity has contributed to the emergence of a theory of justice that is modern, conceptually clear and easy to mobilize in policy design. In this paper, we apply Roemer's theory to *education policy*. We first analyze the reallocations of educational expenditure required to equalize opportunities (taken to be test scores close to the end of compulsory education). Using Brazilian data, we find that implementing an equal-opportunity policy across pupils of different socio-economic background, by using per-pupil spending as the instrument, and ensuring that nobody receives less that 1/3 of the current national average, requires multiplying by 8.6 the current level of spending on the lowest achieving pupils. This result is driven by the extremely low elasticity of scores to per-pupil spending. As such, it implies large reallocations that are probably politically unacceptable. By exploiting our knowledge of the education production function, we then identify ways of reducing financial reallocations needed to achieve equality of opportunity. We show that the simultaneous redistribution of monetary and *non-monetary inputs,* like peer group quality (i.e., desegregation) and school effectiveness (i.e., equalizing access to the best-run schools), considerably reduces – by almost 50% – the magnitude of financial redistribution needed. Implementing an EOp policy would not come at any particular cost (or benefit) in terms of efficiency.

JEL classification: I28 (Education: Government Policy), H520 (National Government Expenditures and Education).

Key works: Equality of Opportunity, Education, Formula Funding**.**

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Introduction

In a nutshell, John Roemer's' seminal book on equality of opportunity (1998) defends the view that while some fraction of inequalities of outcome/achievement is determined by morally acceptable factors, another fraction is caused by morally unacceptable factors. Roemer's conception of equity and justice does not rest on gross outcome variables. He prefers instead to choose as relevant attributes *conditional outcome variables*, which somehow take into account the reasons underlying the achievement of a certain outcome. Inequalities caused by morally unacceptable factors (typically circumstances beyond an individual's control like gender, race or socio-economic origin) should give rise to compensations in order to be eliminated. Inequality caused by legitimate factors (effort, autonomous choice etc.), in turn, should not call for compensation. The aim of an equal-opportunity policy is thus to equalize achievements across groups of individuals with similar circumstances. It is not to equalize achievements within these groups.

This simple idea has been discussed and developed in the philosophical literature over the last forty years, and the debate has turned, to a great extent, around the demarcation of the frontier between the acceptable and the unacceptable, between what outcome gaps are to be compensated for and what advantages constitute legitimate claims by individuals. Economists have also contributed to the shaping of conceptions of justice of this kind, but on a different way, usually working at a more abstract level, making use of the mathematic language.

In his celebrated book of 1998, Roemer has, not only translated that widespread conception of justice into a precise mathematical formulation, but he has also provided a simple algorithm ready for policy use. He has labeled his theory with the appealing name of "equality of opportunity" (EOp for short). He has not tried to spell out what is acceptable and what is not. Instead, he has worked with a general and pluralistic demarcation, according to which inequalities due to circumstances – what is out of control of the individual – are considered unacceptable, while inequalities due to choices made by the individual – what is under control of the individual – are acceptable, and that the precise boundary is to be set by each society in the political arena.

The aim of this paper is to apply such a theory to the particular domain of *education policy*. In section 1 we spell out in what sense our paper provides an original contribution to the EOp literature and we discuss how Roemer's ideas can be applied to education. Section 2 contains a brief presentation of the Brazilian data used. In section 3, we compute the reallocations of educational expenditure required to equalize opportunities. We find that to implement an equal-opportunity policy by using per-pupil spending as the instrument requires multiplying the current level of spending of the lowest achieving type of pupils by a factor 8.6, while the level of spending on the highest achiever should be reduced. In section 4 we then identify ways of reducing financial reallocations needed to achieve equality of opportunity. We show that the simultaneous redistribution of monetary and *non-monetary inputs,* like peer group quality and school effectiveness, considerably reduces the magnitude of financial redistribution required. In section 5, we assess efficiency under alternative allocations. Our results suggest that implementing EOp would not come at any particular cost (or benefit) in terms of efficiency. Section 6 concludes.

1. Equality of opportunity and education policy

Before turning to our subject matter, some words on the originality of this paper and on how it is related to the existing literature. On the one hand, there is a large literature discussing normative issues, both in economics (welfare economics) and in political philosophy (theories of distributive justice). Usually, this literature remains on theoretical and conceptual grounds. On the other hand, there is another strand of literature which is more policy-oriented, and its main concern is to propose formula funding schemes for the (re)distribution of inputs leading to different relevant outcomes (fiscal policy, health-care policy, education policy and so on). Generally, there is little, if any, explicit description of the underlying conception of justice in which they are based. The most important feature of this paper is that it bridges a link between a particular conception of justice that has been developed within the normative literature – namely, John Roemer's theory – and a formula funding scheme in the domain of education. Thus we try to fill the gap by contributing to the development of an application of Roemer's theory of equality of opportunity in the particular area of education.

There are also some more specific features of this paper which make of it an original contribution to the literature. Firstly, while there are good reasons to care about income – possibly the "ultimate educational achievement" – such as Betts & Roemer (2004) and Roemer et al. (2003) do, we believe it is also important to focus on intermediate educational achievement. Thus, an original feature of this paper is that we focus on *educational achievements in terms of test scores* (score-EOp) as the outcome, and – consequently – on *education policy* as the instrument. A second original contribution of our paper is our refusal to focus exclusively on financial reallocations of educational resources. By exploiting our knowledge of the education production function, we enlarge the set of policy instruments, investigating how the simultaneous redistribution of monetary and *non-monetary inputs* – like peer group quality and school effectiveness – can reduce the magnitude of financial redistribution needed. Finally, a third important feature of this paper is that we do not limit our analysis to a first-best situation, but we explore some second-best situations, namely by imposing some restrictions on the extent of the redistribution[1]. This amounts to say that we investigate situations in which inequality of opportunity is not intended to be fully eliminated, but only attenuated, following the proposal made by Moreno-Ternero (2004).

Concretely, how should education policies be specified to equalize opportunity? Although Roemer's framework is relatively easy to mobilize in policy design and can provide guidelines for many real life problems, implementation in the context of education still requires a gradual transposition.

1.1. Circumstances, types and effort

Betts & Roemer (2004) usefully remind us that five keywords constitute the vocabulary of the EOp theory: circumstances, type, effort, objective, and instrument. A *type* is the set of individuals with similar circumstances; the *objective* is the condition for which opportunities are to be equalized; and the *instrument* is the policy intervention used to effect that equalization. The equal-opportunity (EOp) policy is the value of the instrument which makes it the case that an agent's expected value of the objective is a function only of his effort and not of his circumstances. Thus, in order to equalize opportunities for young people to acquire basic (compulsory) education, the schooling system should be organized in such a way that a pupil's score in math, science or reading be a function only of his *effort* and not of his *circumstances*.

The reasoning starts with the observation that pupils will vary in their propensity of attaining some goal (e.g. get a certain score in math), due to *circumstances* – such as their race, or the socio-economic status of their parents. And the bedrock of the EOp is to consider that they should not be held accountable for these circumstances-related achievement differences.

Equal-opportunity ethics maintains that differences in the degree to which individuals achieve the goal in question that arise from their differential expenditure of *effort* are, morally speaking, perfectly all right. The partition of causes into circumstances and effort (or autonomous choice) is the central move that distinguishes Roemer's EOp ethic from a strictly outcome-egalitarian conception of educational justice. While the latter vision implicitly holds the individual responsible for nothing, EOp emphasizes that an individual has a claim against society for a low outcome only if he expended sufficiently high effort.

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[1] Betts & Roemer (2003) also simulate different scenarios, which include limited redistributions.

Before moving to the following point, we must add a comment on the nature of the agents we are dealing with in education. The center of our attention in this paper is scores of pupils whose age is typically 14. Following Roemer (1998), we shall divide these pupils into types assuming that all pupils within a type face the same set of circumstances. All the variation of scores of pupils within a given type shall be assumed to be caused by differential personal effort, and given that the amount of effort expended is a choice made by the individuals, there shall be no compensation for scores inequalities within type. An obvious question we might pose is whether it is reasonable to held pupils accountable for their effort, given that they are not adults, but kids or teenagers. Can we consider them to be fully able to take autonomous and informed decisions? Can they be held totally accountable for important choices they have to make in their schooling years (e.g., allocation of their time between leisure vs. studying)? During a large fraction of their school lives, individuals can not be said to be perfect judges for what is good for themselves, kids are possibly "economically myopic", since they are unable to evaluate all the future benefits that are made available if he or she acquires education in the present time, and they make choices according to other, non-monetary, motivations (Akerlof & Kranton, 2002). To sum up, if we push the argument far enough, we could conclude that circumstances account for virtually all the variability of educational outcomes, that is, that all inequality is unacceptable, which would amount to say that Roemer's theory is useless in this case – we could trivially conclude that the policy objective must be one which consists of equalizing pupils' scores.

The objection makes sense. Indeed a great part of inequalities in educational outcomes could be attributed to circumstances. Notwithstanding, we defend the applicability of Roemer's theory to our problem for both fundamental and pragmatic reasons. The fundamental reason is that a fraction of educational achievements can be credited to the pupil itself, at least to those of a certain minimal age. While it is clear that considering a 5 year-old pupil accountable for his efforts is not reasonable, the claim loses strength when we are talking about a 14-15 year-old youngster, who lives, and is being further prepared to live, in societies where people are, at least partially, held accountable for their acts. For his own benefit, he should be prepared to respond for his acts. Acquiring knowledge and skills depends upon natural and social circumstances (talent, quality of family support etc.), but it also requires personal commitment and effort, and these variables can be considered to be under control of the individual to a certain extent.

There are at least three pragmatic reasons for conditioning inequalities upon choices and circumstances. Firstly, if all are circumstances, score-EOp would require the equalization of scores across all individuals, possibly at a low level, given that the low-talented would never be able to reach high levels of achievements[2]. The second pragmatic argument involves efficiency issues. Even if a given society may be willing to reduce educational inequalities for ethical reasons, reducing them could be so costly that it refrains from pursuing this objective. The equalization objective may be costly, not only because of large talent gaps, but also because detailed information accounting for all relevant circumstances may be costly to gather (Trannoy, 2003). Alternatively, that society may refrain from pursuing full equalization, but to pursue instead, some less costly compromise (partial equalization). In any case, efficiency concerns may constitute obstacles to a full neutralization of the effect of circumstances. The third pragmatic reason is that some side-condition may be imposed, such as stating that all individuals have the right to a minimum level of education, regardless of their circumstances and choices. If every kid (of a given age, for example) really reaches a pre-defined minimal level of education, then society will have done a contribution, be it moderate, to neutralize the effects of circumstances. The objection on the relevance of the demarcation circumstances versus choices would only be applicable beyond that pair age-achievement.

1.2. Outcome and instrument(s)

In principle, any outcome variable is compatible with the EOp agenda when applied to education. A possible candidate is individuals' earnings or *income*, since they reflect, to a certain extent, his well-being.

In a paper connected to this one, Bettts & Roemer (2004) have been trying – regressing wages on per pupil spending – to assess what would have been the necessary redistribution and/or increase of spending-per-pupil (i.e., the *instrument*) in the US in order for the income-EOp objective to be achieved across *types* of individuals (i.e., race and/or socio-economic groups). They conclude that the extent of the necessary redistribution would be quite substantial, especially when circumstances are defined in terms of race, in which case the per-pupil spending ratio between white and black kids would oscillate between 8 (lower bound) and 80 (upper bound). These results are found because of the extremely low income-elasticities to per-pupil-spending obtained in their regressions. When circumstances are defined simultaneously in terms of parents' education and race, the per-pupil spending ratio between the better-off and the worse-off types would be about 14.

[2] What is known as the "leveling down" objection.

A logical variant to this approach consists of using another instrument than per-pupil spending, but at a later stage: income taxation and transfers[3]. Roemer et al. (2003) have tried to asses how well fiscal regimes of eleven industrialized countries perform as far as the income-EOp objective is concerned. They find that fiscal regimes of some countries in Northern Europe do very well in terms of income-EOp objective. They conclude by raising an efficiency issue, namely, on whether redistributive taxation is more or less effective than educational policies as an EOp instrument. They are quite skeptical about education as a means for implementing income-EOp.

In this paper, in turn, neither do we take income to be the relevant outcome variable, nor do we focus on post-education instruments (e.g. tax-and-transfers). Rather, we focus on *educational achievements in terms of test scores* (score-EOp) as the outcome, and – consequently – on *education policy* as the instrument. Such a choice requires some justification. While there are good reasons to care about earnings – what could be called the "ultimate educational achievement" – such as Betts & Roemer (2004) and Roemer et al. (2003) do, we believe it is also important to focus on intermediate educational achievement such as scores in tests of cognitive ability.

First of all, because there is evidence on the existence of positive links between the performance of students in tests and their future earning capacity (Currie & Ducan, 2001). If this is true, by aiming at equality of opportunity for achieving scores, we would be setting the seeds for achieving income-EOp years later.

Secondly, for efficiency or political feasibility reasons, it may be relevant to focus on the distribution of test scores instead of that of income. For a given society to achieve income-EOp, the two papers cited above show it would be necessary, either to massively change the allocation of school resources, or to redistribute income massively (with efficiency costs and well-known disincentive effects). If reshaping the distribution of test scores involves less dramatic reallocations of resources, that line of action may be a good policy instrument that would contribute for the achievement of income-EOp in 10- to 15-years time.

The third reason for focusing on pupils' skills is related to the widely recognized importance of educational achievements. At least since the seminal works of Schultz (1963) and Becker (1964) economists recognize that education has an important economic value. It is a means, or resource, for

[3] Possibly motivated by the idea that achieving the income-EOp objective through redistribution of educational resources would require radical – probably politically unfeasible – reallocations.

achieving a wide array of personal goals. Educational achievements can be good predictors of the access to college, of future earnings capacity and of the social position an individual holds. But education is also likely to be positively correlated to outcome variables or “advantages” valued by various theories of justice, and not only within the normative framework usually adopted by economists (i.e. welfarism). Being more educated might enhance the probabilities that an individual scores higher in the distribution of primary goods defined by John Rawls (1971), of functionings (achievements) and capabilities (freedom) defined by Sen (1985), but also of other “mesojustice” attributes, such as health status, for example (Grossman, 2005). Finally, beyond all the doors education opens, it can also be seen as an end in itself, as an attribute of a "good life" (Sen, 1985). That is, being educated may have an intrinsic value, regardless of the effect it may have (and will have) on other objectives.

Having agreed on taking test scores as the advantage/outcome, we now turn to the issues of which aspects of education policy are relevant for achieving score-EOp. Like Betts & Roemer (2004) we focus on *per-pupil spending*. Our aim, with this paper, is also to provide estimates of the required changes in the distribution of spending per pupil securing EOp. However, we argue that it is useful to enlarge the scope of instruments that can be used and not limit ourselves to reallocations of monetary resources. We argue that policymakers can, and even should, intervene by redistributing non-monetary resources.

Our aim is to better exploit the results highlighted by the abundant literature on education production function (Hanushek, 1986 and 1997; Belfield, 2000). Of course per-pupil spending and its components (teacher salary, class-size, capital expenditure etc.) will always be central to education policy design. Yet, we believe that the production function literature largely legitimizes integrating non-monetary inputs to the EOp

Several authors (Monk, 1992; Haveman & Wolfe, 1984; Vandenberghe, 2002) have shown that a pupil's achievement could indeed be influenced by variables with no immediate monetary expression: the pupils themselves and their human capital background. Education is one of those services wherein outputs depend partially on the customers as inputs. In addition, the presence of other customers (as inputs) often contributes to the output 'experienced' by each customer individually (Rothschild & White, 1995). Human capital endowment of pupils and their aggregation – the student body composition – apparently condition the productivity of more classical inputs (teacher-pupil ratios, teacher salary, capital, sport and scientific facility…). The point here is that peer quality – due to well known segregation phenomena – can be unequally distributed and contribute to (in)equality of opportunity.

Several case studies (Monk, 1992), but also nation-wide empirical research (Hanushek, 1986, 1997) and international studies (Vandenberghe & Robin, 2004) also highlight the critical role played by intra-organizational attributes. The technological relation between inputs and outputs is conditional to the presence of organizational assets. These cannot be directly related to the amount of monetary resources made available by the public authority. There is some evidence that, in many countries, pupils attending privately-run schools benefit from a higher level of organizational effectiveness than those enrolled in public schools. The point, again, is that school effectiveness can be unequally distributed among pupils and contribute to (in)equality of opportunity.

2. Data

The data we use come from the 2001 wave of SAEB (Basic Education Assessment System), a survey on pupils' achievement carried out by INEP, a research bureau subordinated to the Brazilian Ministry of Education. While the SAEB is not suitable for international comparisons, its objectives and statistical design, and the procedures employed in the application of the test, have been inspired by, and do not differ very much from, well-known cross-country assessments of pupils' performance, such as PISA, TIMMS/PIRLS, and LLECE[4].

SAEB consists of countrywide tests that evaluate pupils' cognitive abilities in Portuguese[5] but also in Mathematics. Test score information is coupled with data on relevant features of pupils and their family, as well as teachers', principals' and schools' characteristics. The global database consists of repeated cross-sections (not panels) of representative samples of schools and students. Firstly, schools are randomly chosen to take part in the SAEB. Secondly, one or two classes inside each school are randomly selected. All students of a given selected class have to pass the SAEB exam, but only in one of the subjects.

SAEB focuses on the evaluation of pupils at three key stages of their formal education: 4th and 8th year of primary school, and 3rd year of secondary school. Schooling is mandatory in Brazil for children up to 14 years, regardless of the grade they are attending. The 8th grade sample constitutes

[4] The INEP website (http://www.inep.gov.br) contains useful information concerning evaluation of students in Brazil, most of which in Portuguese, in the section "Avaliação e Exames". INEP (2002) provides specific information about the 2001 wave of SAEB. INEP (2004) provides information about the SAEB exam in English.

[5] Portuguese is the official language in Brazil and it is the native language of nearly all Brazilians.

a good approximation for the end of compulsory schooling, since most of its students are in fact around 14 years old[6]. Moreover, 8th grade pupils are less likely to have dropped out than 3rd grade of secondary school pupils. Finally, the 8th grade datasets have fewer missing data in key questions (e.g., mother's education) as compared to the 4th grade. For these reasons, we focus exclusively on the 8th grade sample.

Pupils' test scores correspond to subject-specific scales elaborated by INEP staff together with teachers, researchers, and national and international survey experts. Possible scores range from 0 to 500, and are supposed to evaluate skills and abilities of students. The SAEB scale is continuous and hierarchical, which means that a pupil who achieves a certain score – say, 400 in the Portuguese test – has all the literacy skills held by students who scored, say, 150, 300 or 380, plus some additional skills. For example, he might be able to understand and interpret more complex texts than his peers who scored lower. Because of the invariance of the scale, pupils' scores are comparable across years and across grades. Scores are not comparable across subjects, but the distributions of scores do not look very different in Portuguese and in Mathematics[7].

This data set contains information about teachers' gross monthly wages expressed in "salários mínimos" (*SM*), an index frequently used in Brazilian administrative data[8], as well a number of pupils in the classroom where the test was implemented (i.e., the teacher-to-pupils ratio). The product of the two gives a reasonable proxy of per-pupil spending (at the classroom level) expressed in units of *SM* per pupil.

Ideally, a *type* should be defined as a set of individuals facing the same circumstances. In practice, however, it is impossible to define types so perfectly, and so we have to turn to some proxy which allows us to define types as sets of individuals facing similar circumstances. The SAEB data set contains a series of socio-economic variables, one of them being the highest degree obtained by the pupil's mother. We assume here that such variable is a good proxy for pupils' circumstances, since it is known to be highly correlated with a number of past, current and future advantages an

[6] In the final samples used in this study, a majority of 8th grade pupils (71%) were 15 or less by the time they did the SAEB exams. However, the range is actually quite wide – maximum pupil age in the sample is 19 – especially because of grade repetition and irregular school attendance.

[7] SAEB scales have been built in such a way that the mean and the dispersion were identical across subjects, for the 8th grade, in the 1997 wave. Averages were set to 250, and standard deviations were set to 50.

[8] Literally it means minimum wage, but actually, more than defining the actual value of the Brazilian wages, it is used as an economic index. In October 2001, when SAEB tests took place, one unit of SM was worth 68 US dollars.

individual faces. So the highest degree obtained by the pupil's mother is the variable we choose to define pupil's type (*t*).

We also use the highest degree obtained by the pupil's mother to compute some proxy – in fact a simple average – of the quality of the peers (*PEER*) from which the pupil might benefit/suffer at the classroom level. Finally, SAEB tells us about the public vs. private nature of the school attended. A public school is a school managed directly by a public authority (the state or the municipality). A private school is a school managed directly by a non-government organization (e.g. a church, business or any other private institution). In brief, the underlying classification is not that of the origin of financial resources, but the legal status of the board. When that variable is used as a dummy (*PRIV*) in regression, it can help us quantify the importance of the school effectiveness as an input, and assess its potential role in achieving EOp.

Descriptive statistics are reported in table 1. We notice that the average score (*S*) varies considerably from type to type. The amounts of resources, both monetary (*X*) and non-monetary (PEER and PRIV), which are available for each type are also very different. The higher the education level of pupils' mothers, the higher are the level of pupils' educational inputs and output.

Table 1 – Descriptive statistics

Mother's highest degree (which defines *types*)	**Observations**	**Pct.**	**Average score in Portuguese test (*S*)**	**Per-pupils spending (*X*)**	**Index of peer quality (*PEER*)**	**Probability of attending a private school (*PRIV*)**
All	45,030	100.00	246.45	0.153	3.25	0.36
At most primary education 1	3,054	6.78	212.92	0.121	2.54	0.06
Lower secondary 2	11,652	25.88	226.57	0.131	2.67	0.08
Upper secondary 3	9,950	22.10	236.11	0.143	3.00	0.20
Tertiary short 4	11,741	26.07	258.65	0.158	3.55	0.52
Tertiary long 5	8,633	19.17	280.47	0.200	4.15	0.82

3. Per-pupil spending and score-EOp

3.1. The EOp algorithm and its specification

Following the strategy of Betts & Roemer (2004), but defining the outcome variable as Portuguese test scores (S) at the age of 14, we first compute the reallocation of *spending per pupil* (X) that would be necessary to equalize opportunities. We consider reallocations of spending per pupil across types of pupils, given a *fixed educational budget per pupil* (R).

However, since such reallocations are virtually guaranteed to reduce spending per pupil dramatically for certain types, we also calculate EOp solutions where the constraint is that each type receives at least a fraction, α, of the per-pupil educational budget, R.

Similar circumstances are used to partition student data into types (t). In this paper, we define only five types using information on *the highest degree of the mother*[9]. The idea of effort, in Roemer's framework, is captured by the rank of the student in the within-type conditional distribution of effort. In statistical terms, this rank (and thus the level of effort) can be adequately captured by the quantile, π, of the type-specific distribution of score.

Supposing that pupils of type t and quantile π are allocated an amount X^t_π of the resource, and assuming that the education production function connecting resources to (the natural log of) score (S) is of the form:

$$lnS(X^t,\pi) = a_\pi + b_\pi . X^t_\pi + Z^t c + \varepsilon \qquad [1]$$

where:

- t: pupil's type (i.e., mother's highest degree);
- S: Portuguese score,
- π: within type score quantile to which the pupil belongs;
- X : per-pupil spending;
- Z : a vector of control variables

The core of the EOp allocation problem consists of identifying, for each quantile π, the vector $X_\pi = (X^1_{\pi,} X^2_\pi \cdots X^T_\pi)$ that equalizes (expected) scores, subject to the following budget constraint:

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[9] Isced classification.

$R = \sum p^t X^t_\pi$ [2]

where:

R: the average per-pupil spending

p^t: the share of type t pupils in total population

As results will make it clear later on, equalizing within-quantile scores will virtually never be possible without the absurd implication of imposing negative values of the educational input to some types (that is, pupils belonging to high-performing types would have to be "taxed", transferring resources to their low-performing peers). In this case the EOp agenda will, *de facto*, have to become one which aims at minimizing score inequalities as much as possible – that is, a maximin approach replaces an egalitarian one[10]. In this paper it will be the automatic consequence of imposing the additional constraint that nobody receives resources which are less than a fraction α of the current national average (R).

$X^t_\pi \geq \alpha R$ [3]

Finally, it is worth emphasizing that the EOp algorithm as we exposed it so far amounts to maximizing several objectives simultaneously (one per quantile). To come closer to a solution that is practically feasible, one second-best approach must be taken. Like Roemer, our compromise will be to *take an average*. For example, suppose, as we do in this paper, we work by quartiles of score in each type. We first compute, for each quartile the investment policy that equalizes (expected) scores, across the various types. This would give us 3 different investment policies. And we declare the EOp policy to be the average of these 3 policies.

3.2. EOp and quantile regressions

Roemer (1998) emphasizes that, as the distribution of effort of a type is a characteristic of the type and not of any individual, it is a circumstance for a particular individual. For example, if an individual's effort is low in *absolute* terms because he belongs to a type whose mean effort is low, this individual should not be held responsible. He claims that we should turn our attention to *relative* levels of effort within given types, and that relative effort is best captured by the rank of an individual in the effort distribution of his type, or at least by the quantile to which he belongs in such distribution.

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[10] For more details, see appendix.

With regards to an empirical application of Roemer's theory to education, we have to bear in mind that the impact of school spending on score for a given type of pupil may vary with the pupil's effort (i.e., the quantile to which he belongs) in the score distribution. Quantile regressions (Koenker & Bassett, 1978), estimated separately for each type of pupils, constitute the most appropriate technique for the application of Roemer's EOp theory. The set of coefficients obtained from quantile regressions performed for each type of pupils allows for non-linearities in the relation between score and spending per pupil.

3.3. Score-EOp results

We have estimated equation [1] three times (π=0.25, 0.5, 075) for each of our five types of pupils (1-5). Vector *Z* includes current level of peer quality (*PEER*) and private school dummy (*PRIVD*).

Using the set of estimated coefficients *a, b, c, d* and rewriting equation [1] by type as:

$$lnS(X^t, \pi) = A_\pi + b_\pi . X^t_\pi + \varepsilon \qquad [4]$$

with $A_\pi = a_\pi + c_\pi PEER^t_\pi X^t_\pi + d_\pi PRIVD$

we get 5 linear functions of X^t_π providing the expected score for each type of pupils. Exploiting the idea that EOp basically means equalizing expected score across types, and using the budget constraint, we develop a system of *T+ 1* equations, which is resolved incrementally:

- X^T_π as a function of *R* and $X^{T-1}_{\pi},\dots, X^1_\pi$
- X^{T-1}_π as a function of *R* and $X^{T-2}_{\pi},\dots, X^1_\pi$
- ...
- X^1_π as a function of *R* and the set of known (*p*) or estimated parameters (*A(a,c,d), b*)

In table 2, panel A, we report the current level of per-pupil spending for each type, and the distribution of scores for each type and quartile. We also report in table 2 the allocation of inputs and distributions of scores obtained as a result of two redistributions of inputs (as our benchmark policies): (i) Panel A: assuming an input-egalitarian allocation of per-pupil spending (*Xi=R*, for all i), and (ii) Panel C: the score-EOp distribution.

Table 2 – Reallocation required for achieving score-EOp with no minimal per-pupil spending restriction

Quantile (π)	Type 1	Type 2	Type 3	Type 4	Type 5	Average score* (*AS*)	Efficiency ratio** (*V*)
Frequencies (%)	*6.78*	*25.88*	*22.10*	*26.07*	*19.17*	-	-
PANEL A: CURRENT	**Current allocation of per pupil spending (X_C)**						
All	0.121	0.131	0.143	0.158	0.200	-	-
	Current distribution of score (S_C)						
q25	184.69	197.53	206.18	229.38	252.84	217.48	-
q50	213.88	226.75	237.50	260.31	283.13	247.81	-
q75	241.67	255.86	265.87	288.26	310.44	276.02	-

PANEL B: EQUAL-RESOURCE	**Equal resource policy (X_{ER})**						
All	0.150	0.150	0.150	0.150	0.150	-	-
	Distribution of score under equal resource policy (S_{ER})						
q25	185.50	198.28	206.50	229.24	251.44	217.49	1.0001
q50	214.56	227.38	237.86	260.14	281.69	247.78	0.9999
q75	242.20	256.55	266.27	288.08	309.16	276.03	1.0000

PANEL C: EOP	**Allocation of X necessary to achieve score-EOp (X_{EOP})**						
q25	1.376	0.727	0.540	-0.200	-1.017	-	-
q50	1.593	0.813	0.415	-0.209	-1.053	-	-
q75	2.071	0.749	0.388	-0.175	-1.151	-	-
Average	*1.680*	*0.763*	*0.448*	*-0.195*	*-1.074*	-	-
1=current national average (i.e., R=0.15)	***11.201***	*5.087*	*2.984*	*-1.300*	***-7.157***	-	-
	Distribution of score under EOp (S_{EOP})						
q25	218.58	218.58	218.58	218.58	218.58	218.58	1.0051
q50	246.88	246.88	246.88	246.88	246.88	246.88	0.9962
q75	275.45	275.45	275.45	275.45	275.45	275.45	0.9979

* Weighted, across-types, score average.
** Ratio of average achievement under the policy considered with respect to current average achievement.

The distribution of scores that results from the input-egalitarian allocation of the education input (S_{ER}) is not extremely different from the current distribution of scores (S_C). In fact, for any pair type-quantile, the changes would be less than 1%, in absolute value. For example, for type 1, quantile 0.25, while current average score is 184.69, under an egalitarian allocation of spending per pupil, average score would increase to 185.50, a positive variation of 0.44%. For type 5, quantile 0.75, there would be a negative oscillation of 0.41%, with average scores decreasing from 310.44 to 309.16. So, moving from current allocation of inputs into an input-egalitarian allocation of inputs would involve a certain amount of redistribution of inputs (especially from type 5 individuals), with quite modest impacts on scores. More importantly, such a policy would not be in line with EOp ethics, since it would not provide sufficient compensation for types whose circumstances are not favorable.

In contrast, when we compare the current distribution of scores (S_C) with the distribution of scores under EOp allocation (S_{EOP}), variations are substantial. For the same types and quantiles mentioned in the previous paragraph, the variations of average scores would be, respectively, of 18.35% (from 184.69 to 218.58) and of –11.27% (from 310.44 to 275.45).

If we focus on the values of *X* securing score-EOp (X_{EOP}), we immediately note the dramatic and immense reallocation that is required by the EOp agenda. If the aim of the social planner is really to equalize expected scores across types, it will be necessary to multiply the current level of spending of the lowest achieving type of pupils by a factor 11, while the level of spending on the highest achiever should actually be multiplied by a factor –7 (thus become negative). Combining information presented on tables 1 and 2, we can observe that a considerable fraction of the pupils' population would have to make efforts in order for the score-EOp allocation to be achieved. Ex ante well-off pupils (types 4 and 5), those who would face a decrease in their relative input allocation, represent around 45% of the pupils' population, while the ex ante worse-off groups (types 1, 2 and 3) represent about 55%.

The reader should take good note of the fact that this result is based on a *conservative interpretation* of the score to spending elasticities (i.e. *b*). We have indeed assumed that these coefficients capture the sensitivity of scores to the average (or cumulated) per pupil spending since entrance in the education system. But the measure of spending we used (*X*) is purely cross-sectional and 8^{th}-grade-specific. Should our *b* thus be interpreted as the effect of current (and not average or cumulated) spending on score? If so, EOp could be achieved gradually, and not on one shot. Table 2 suggests for example that raising type 1's score to the EOp target means spending 1.680 instead of the current 0.121. Assuming constant elasticity (i.e., *b*s are of similar magnitude across grades) and constant return to scale (i.e., repeated small increments of *X* produce similar score improvements as a single big one), the same EOp type-1 score could be achieved over a period of 9 years (1 preprimary grade + 8 primary school grades) and require an increment of 0.173 per year[11]. This could appear as a redistribution of much lower magnitude and possibly be perceived as more politically acceptable.

Yet, from a purely econometric point of view, we believe it makes more sense to stick to our initial and rather conservative interpretation where the *b*s capture the relationship between score and average (or cumulated) spending. It is well know, from the production function literature, that coefficients estimated using cross-sectional data are generally *upward biased*. The intuitive reason, put into the context of our data, is that 8th grade per-pupil spending is most likely highly correlated with per-pupil spending during all preceding grades. Hence, our assumption that *X* could be nothing more than a proxy for average or cumulated spending[12].

Table 3 contains the results of a similar exercise, but where we impose that each type should get at least a fraction, α, of the current national average per-pupil spending, *R* – in line with Roemer's idea that EOp should be more about maximizing the achievement of the worst-off types (maximin). We arbitrarily set α=1/3. An interesting result of this exercise is to be found in the distribution of scores (S'_{EOP}). While there are large gains for some type-quantile pairs (e.g., average score increases by 14.01% for type 1, quantile 0.25), there are not important losses for any pair type-quantile. The type-quantile pair which is mostly penalized is type 5, quantile 0.25, whose loss is only about 1.78%.

But in this case, roughly 2/3 of the population (types 3, 4 and 5) would be penalized in order to make sure that roughly 1/3 of the pupils (types 1 and 2) achieve average scores that would be in line with this weaker version of score-EOp allocation. And, more importantly, even in this less ambitious context, our results suggest that per-pupil spending on the low achieving type should be multiplied by the still large factor of 8.6.

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[11] (1.680-0.121)/9 = 0.173.

[12] We tend to apply the same reasoning to cross-sectional peer quality (*PEER)* and private school attendance likelihood (*PRIV).* The values they take for 8th grade are likely to be proxies for average values since the beginning of schooling.

Table 3 – Reallocation required for achieving score-EOp, with each type receiving at least 1/3 of *R*

Quantile (**π**) *Frequencies (%)*	**Type 1** *6.78*	**Type 2** *25.88*	**Type 3** *22.10*	**Type 4** *26.07*	**Type 5** *19.17*	**Average score*** (***AS***) -	**Efficiency ratio**** (***V***) -
	Current allocation of per pupil spending (X_C)						
All	0.121	0.131	0.143	0.158	0.200	-	-
	Allocation of X to achieve score-EOp, with α=1/3 (X'_{EOP})						
q25	1.098	0.507	0.050	0.050	0.050	-	-
q50	1.176	0.487	0.050	0.050	0.050	-	-
q75	1.468	0.410	0.050	0.050	0.050	-	-
Average	*1.283*	*0.459*	*0.050*	*0.050*	*0.050*	-	-
1=current national average (i.e., R=0.15)	***8.551***	*3.057*	*0.333*	*0.333*	*0.333*	-	-
	Distribution of score under EOp, with α=1/3 (S'_{EOP})						
q25	210.56	210.56	203.37	226.06	248.34	220.26	1.0128
q50	237.03	237.03	234.37	256.28	278.52	249.41	1.0065
q75	264.52	264.52	262.31	284.04	306.34	277.14	1.0041

* Weighted, across-types, score average.
** Ratio of average achievement under the policy considered with respect to current average achievement.

4. Per-pupil spending, non-monetary inputs and score-EOp

Following the intuition we exposed in section 2, we now turn to the case where the EOp algorithm is applied after some reallocation of non-monetary inputs has taken place. Algebraically, this means that we redefine *A* in equation [4] to become.

$$A^*_\pi = a_\pi + c_\pi PEER^* X^t_\pi + d_\pi PRIVD^* \quad [5]$$

where $PEER^*$ is the national average of the peer quality endowment and $PRIVD^*$ is the national average attendance at private schools. Using parameters (A^*, b) we then identify the EOp solution, following the same logic as the one exposed in section 3.3.

Table 4 shows that the simultaneous redistribution of monetary and *non-monetary inputs,* like peer group quality and school effectiveness, considerably reduces the magnitude of financial redistribution required. We show that per-pupil spending on the low achieving type should now be multiplied only by a factor 4.8, quite lower than the 8.6 obtained previously.

In this case, the burden of the policy would be bore by 45% of the population (types 4 and 5). The variations of scores would range from –12.54% to +15.08%.

Table 4 – Reallocation required for achieving score-EOp, with each type receiving at least 1/3 of *R* and with ex-ante redistribution of non-monetary inputs: peer quality (full de-segregation) and school effectiveness (equal probability of attending a private school)

Quantile (π)	**Type 1**	**Type 2**	**Type 3**	**Type 4**	**Type 5**	**Average score* (*AS*)**	**Efficiency ratio** (*V*)**
Frequencies	*6.78*	*25.88*	*22.10*	*26.07*	*19.17*		
	Current allocation of per pupil spending (X_C)						
All	0.121	0.131	0.143	0.158	0.200		
	Allocation of X to achieve score-EOp, with α=1/3 and redistribution of non-monetary inputs (X''_{EOP})						
q25	0.653	0.222	0.132	0.050	0.050		
q50	0.617	0.253	0.108	0.050	0.050		
q75	0.903	0.185	0.099	0.050	0.050		
Average	*0.724*	*0.220*	*0.113*	*0.050*	*0.050*		
1=current national average (i.e., R=0.15)	*4.828*	*1.466*	*0.754*	*0.333*	*0.333*		
	Distribution of score under EOp, with α=1/3 (S''_{EOP})						
q25	212.55	212.55	212.55	217.81	221.14	215.57	0.9912
q50	243.15	243.15	243.15	248.36	252.51	246.30	0.9939
q75	270.78	270.78	270.78	276.78	285.42	275.15	0.9969

* Weighted, across-types, score average.
** Ratio of average achievement under the policy considered with respect to current average achievement.

5. Efficiency issues

As pointed by Betts & Romer (2004), policies aimed at equalizing achievement – be it in a strictly egalitarian or in an EOp perspective – are often criticized for being 'inefficient', that is, for decreasing average (or total) output. It is indeed possible that when one reallocates per-pupil spending (*X*), the overall score (*S*) will diminish. That would typically the case if relative elasticities (i.e., *b*) were higher for types 4 or 5 from which resources are being removed.

Therefore we also calculate an efficiency indicator (*V*), defined as the ratio between the average score (AS_{EOP}) predicted to occur under an EOp scenario and the current average score (AS_C). Our calculations are based on the assumptions that type frequencies (first line of Tables 2, 3 and 4) computed from our dataset adequately represent the Brazilian reality.

Results appear in the last two columns of Tables 2, 3, 4. Efficiency ratios values all are very close to 1, which suggests that implementing EOp would not come at any particular cost (or benefit) in terms of efficiency. A closer look at the results shows however that not all EOp policies are efficient-equivalent. An EOp agenda, with no lower bound limit to per-pupil spending, could lead to a *reduction* of average score of up to 0.5 percent (last column of Table 2). The policy that simultaneously redistributes monetary and non-monetary resources could also mean a reduction of average score of up to 0.8 percent (Table 4). By contrast, the variant of EOp in which each type receives at least 1/3 of *R* could lead to an *increase* of the average score of 0.5 to 1 percent (Table 3).

6. Conclusions

We applied Roemer's EOp theory to education policy to Brazilian data, calculating the reallocations of educational expenditure required to equalize test scores opportunities for pupils of different socio-economic background.

In Brazil, current educational spending pattern is miles away from one which would consist of providing each pupil with an equal amount of spending (egalitarian allocation of inputs). Thus even the most conservative reallocation scheme mentioned here – which would not sufficiently compensate people for their circumstances and which would lead to modest results in terms of shifts in scores – would already require important policy changes, certainly with considerable practical difficulties, let alone political resistance. For example, how expenditure could be reallocated across types if pupils from rich families (typically from type 5) usually study in *private schools*, where teachers earn higher wages, while poor pupils go to *public schools*, where wages are lower?[13]

When we turn to EOp policies, practical and political difficulties would arguably be even greater, because these policies would involve substantial reallocations of resources between types. In a more sensible setting – namely, one which ensures that nobody receives less that 1/3 of the current national average per-pupil spending – still it would be necessary to multiply by 8.6 the current level

[13] It is useful so mention some summary statistics concerning teachers' wages:

Type of school	Fraction	Average teacher's wage (in *SM*)	Std. Dev.	Min.	Max.
Private	36.72%	6.03	3.68	0.5	15
Public	63.28%	4.50	2.68	0.5	15
All	100.00%	5.06	3.17	0.5	15

of spending on the lowest achieving pupils. Such result is driven by the extremely low elasticity of scores to per-pupil spending, but also by the great across-type dispersion of inputs and scores in Brazil. Such large reallocations would probably remain politically unfeasible.

Betts & Roemer (2004) finished their paper suggesting that "money alone will not suffice to equalize educational opportunity", and urging for "finding complementary means of improving outcomes for the disadvantaged". We tried to contribute in this sense, and we showed that the simultaneous redistribution of monetary and non-monetary inputs (like peer and school quality) would considerably reduce – by almost 50% – the magnitude of the financial redistribution needed. We believe this is an important result of our paper, which should be taken into account by EOp proponents and by policymakers.

Of course, this set of policies (i.e., simultaneous redistribution of monetary and non-monetary inputs) would not be easily implemented in Brazil, nor anywhere else, since practical and political problems related to the financial redistribution, although downgraded, would still exist. Moreover, policymakers would still have to find appropriate ways for redistributing peer group quality and school effectiveness. Reducing segregation (that is, redistributing peer group quality) and increasing the probability of disadvantaged pupils to attend private schools (best-run schools in Brazil) are goals which can be linked to each other. Ideally, some sort of SES-sensitive formula funding would have to be coupled with an equity-sensitive voucher scheme, in such a way that good public schools (through formula funding), and especially good private schools (through voucher scheme), face strong incentives to enroll disadvantaged kids and to mix them with advantaged kids[14]. If these non-monetary redistributions were actually implemented, it is likely that a monetary redistribution would come as a by-product, which makes this set of non-monetary policies even more attractive.

Brazilian schooling system has its own particularities, but it is similar in many respects, to those of other developing countries (highly unequal distribution of inputs and outcomes; coexistence of private and public schools etc.). To a lesser extent, even schooling systems of developed countries share characteristics of the Brazilian one (nowhere can we find a system where all pupils are allocated the same amount of resources, socio-economic background typically has a strong influence on schooling outcomes etc.). So, we believe our results are not limited to observers who

[14] Formula-funding and voucher systems experiences from countries such as Belgium (Vandenberghe, 1996) or Chile (Gradstein et al, 2004) could inspire Brazilian educational authorities.

are interested in the Brazilian context, or in developing countries. In any case, in our future research, we plan to extend our application of EOp theory to education policy of other countries.

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Appendix

Roemer's EOp algorithm requires that, for each quantile, we find an allocation $X(t,л) = X^1, \ldots X^T$, such that:

$$Argmax_X\ [Min_T\ (a^{t,л} + b^{t,л}X^t)] \qquad [1]$$

$$st \sum p_t X_t = R$$

where variables are defined as in section 3.

Assuming types $t=1, 2$, and assuming that type 1 will be the worst-off at the solution, [1] can be restated as:

$$Max_X\ (a^{1,л} + b^{1,л}X^1\ ) \qquad [2]$$

st

$$a^{2,л} + b^{2,л}X^2 \geq a^{1,л} + b^{1,л}X^1 \qquad [3]$$

$$p_1 X^1 + p_2 X^2 = R \qquad [4]$$

By rewriting and combining [3] and [4], we obtain:

$$X^1 \leq (R\ b^{2,л} + p_2(a^{2,л} - a^{1,л}))/(p_1\ b^{2,л} + p_2\ b^{1,л}) \qquad [5]$$

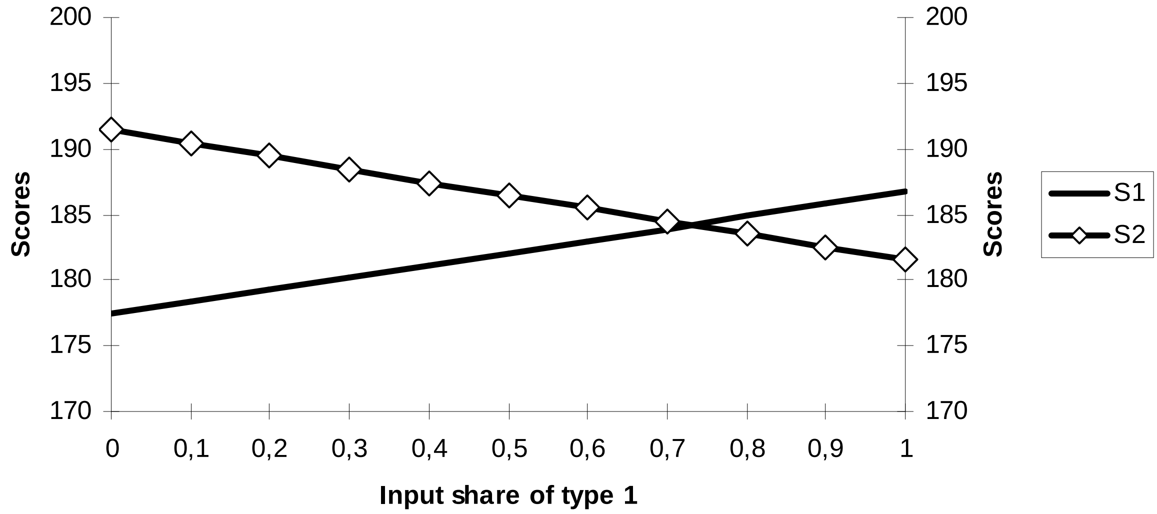
$$X^2 = (R- p_1 X^1)/\ p_2 \qquad [6]$$

Expression [5] sets an upper bound for the amount of resources to be allocated to individuals of type 1 at quantile *л*. As we are interested in maximizing the expected score of the worst-off individuals (type 1) at quantile *л*, we would like to set X^1 as large as possible, that is, such that [5] holds with equality. Assuming objective function [2] is rising with X^1 (i.e., elasticity of score to spending is significantly positive) we have that [5] at equality and [6] define a solution candidate.

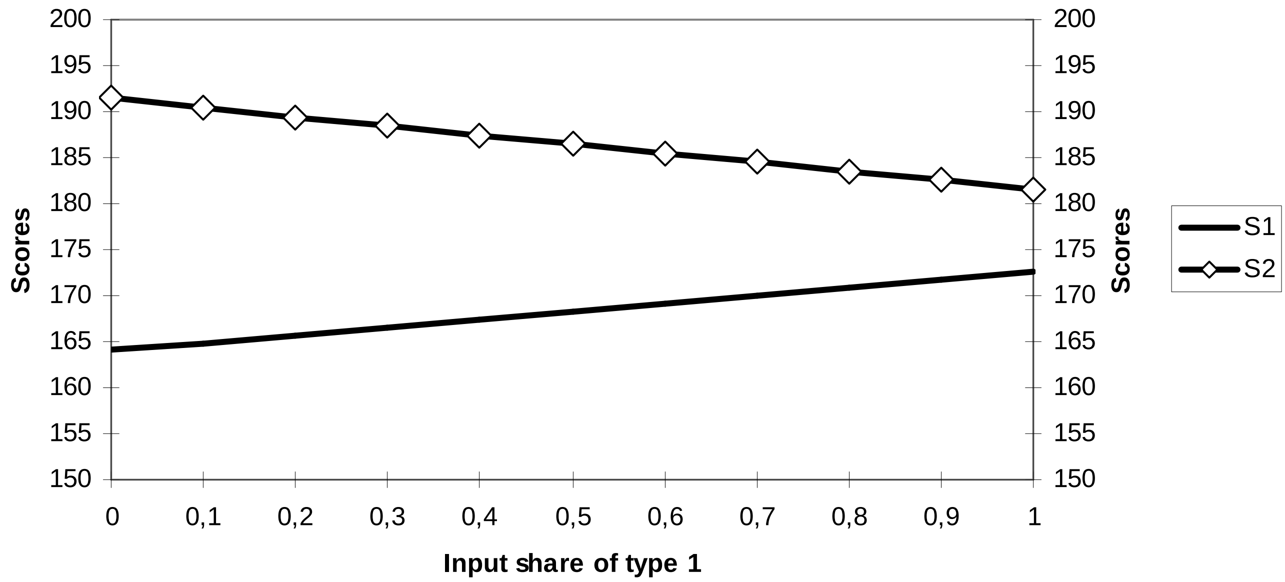
Generally, we would have to solve the program assuming each type to be the worst-off at the solution, but in the two-type case, this is not necessary. It is in fact redundant, because by combining the expression that defines the upper bound for the resources allocated to individual 2 and the budget constraint, we obtain an optimal allocation which is equivalent to the one provided by [5] and [6]. The intuition can be understood in the graphical representation of the “education technologies” below.

Figure 1. EOp solution with 2 types, at a given quantile.

Panel A: “Interior solution”.



Panel B: “Corner solution”.



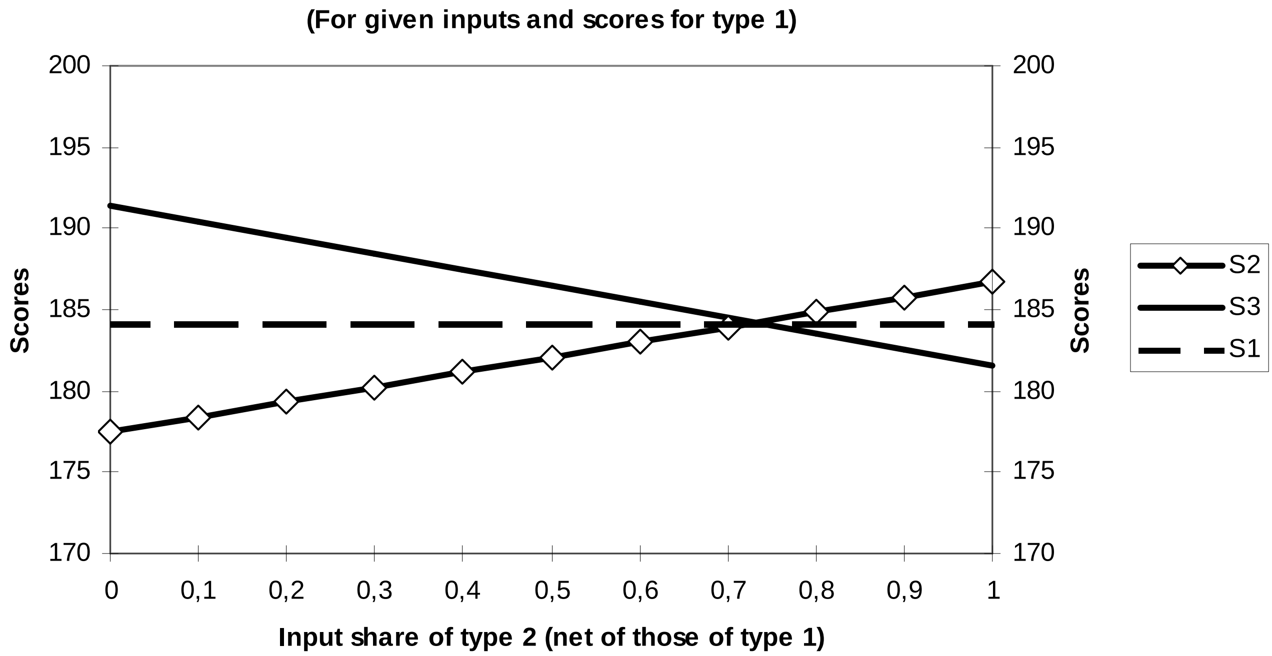
In Panel A of Figure 1, the EOp solution is given by the point where the two lines cross, that is, where $S^1 = S^2$, which is exactly the outcome-egalitarian solution. In Panel B, the EOp solution differs from the outcome-egalitarian solution. The latter would require, either the allocation of negative amounts of the educational resource to type-1 individuals (such that their scores are driven down to reach those of type-2 individuals), or the violation of the budget constraint (such that a larger amount of input is allocated to type-2 individuals to improve their scores).

If we work with three types, we have to assume each type is the worst-off, and solve the program defined by [2], [3], [4] and an additional constraint that guarantees that a given type is really the worst-off at the solution. For example, if type 1 is assumed to be the worst-off, inequality [7] would also have to be respected:

$$a^{3,\pi} + b^{3,\pi}X^3 \geq a^{1,\pi} + b^{1,\pi}X^1 \qquad [7]$$

If, for a given quantile and for all pair of types, there were "interior" solutions (such as the one shown in Panel A of Figure 1, or in Figure 2), then the egalitarian solution would obtain, and there would be no need to solve program [2]-[4] and [7] assuming each type to be the worst-off.

Figure 2. EOp solution with 3 types, at a given quantile.



Beyond equity considerations, a pragmatic advantage of Roemer's algorithm would be that it doesn't harm efficiency very much, since it consists of a maximin objective function, instead of an egalitarian one. But that difference is meaningful only when the technology is such that an equal outcome is not feasible, even under a very generous redistribution (cf. Panel B in Figure 1). When technologies are linear, the elasticities are positive (estimated $b > 0$), and the differences between the parameters (*a*s and *b*s) across types are not spectacular, the optimal distribution of the educational resources will be such that the expected scores are equal across types (i.e., outcome-egalitarian solutions will obtain).

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