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Ranking alternatives by a fair bidding rule: a theoretical and experimental analysis

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Abstract

We introduce a procedurally fair rule to study a situation where people disagree about the value of three alternatives in the way captured by the voting paradox. The rule allows people to select a final collective ranking by submitting a bid vector with six components (the six possible rankings of the three alternatives). In a laboratory experiment we test the robustness of the rule to the introduction of subsidies and taxes. We have two main results. First, in all treatments, the most frequently chosen ranking is the socially efficient one. Second, subsidies slightly enhance overbidding. Furthermore, an analysis of individual bid vectors reveals interesting behavioral regularities.

JEL Classification: C92, D02, D71

Keywords: Bidding behavior, Procedural fairness, Voting paradox

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1 Introduction

Voters often are not only interested in who is ranked highest, but also in the entire collective ranking of candidates. Examples are university hiring committees and tournaments in sports. To introduce the specific scenario that we shall analyze, think of three individuals (the voters) who will attend the performance of three musicians (the candidates or alternatives). The performance is a collective good from which all voters will benefit. The three voters disagree about the value of the musicians in the way captured by the well-known Condorcet's voting paradox. Nonetheless, they must agree on a collective ranking which establishes how long each musician will perform in the show that the voters will attend.

More generally, we have in mind any situation where a collectivity has to settle for a common ranking of alternatives which are evaluated differently by different members and in such a way that collective preferences are cyclic (i.e., not transitive), even if individual preferences are not. The alternatives are open to a variety of interpretations. For example, they can be thought of as different waste disposal methods (e.g., incineration, recycling, landfill) or as different energy production technologies (e.g., nuclear, fossil fuels, renewable). The ranking of the alternatives would then be the relative importance given to each method/technology within the waste/energy management plan.

While in the traditional literature collectivity members are required to cast votes for the several alternatives in order to obtain a final collective ranking (see, e.g., Kelly 1974a, 1974b; for more recent work see Brams et al., 2008, and references therein), we allow each member to bid for each feasible ranking. Bids express how much a member is willing to pay for implementing a certain ranking. By reporting such a bid vector, each collectivity member can influence which ranking is chosen. Our idea, here, is that as there is a value that each member places on the selected ranking, he should pay some money in order to implement it. On the one hand, this is reminiscent of how private goods are allocated by markets. On the other hand, letting everyone pay for the selected ranking (besides solving the paradox endogenously) may help contain potential inefficiencies arising from the "resource-wasting struggles"[1] to bring about the ranking that each member prefers the most.

The first contribution of this paper is to derive axiomatically a procedurally fair bidding rule for determining the selected ranking and the resulting individual payments. In our setting, fairness is a property of the selection mechanism (or game form), not of the selected ranking. Our procedurally fair mechanism guarantees indeed that all parties are treated equally according to an objective criterion (namely their bids) even if, ex post, the selected ranking is not valued equally by all collectivity members. In this respect, our approach differs from previous models that define fairness with respect to final outcomes (so-called allocative fairness).[2] Additionally, we are interested in legal or constitutional mechanisms where fairness is defined in terms of observables rather than in terms of idiosyncratic (and usually privately known) characteristics such as the bidders' true valuations of all rankings.

The second objective of our paper is to explore, via an experiment, whether and how the selected ranking and the bidding behavior are affected by changes in some features of the bidding contest whose game form is procedurally fair. Obviously, this requires a proper bidding game which can be implemented in the laboratory.[3] We consider a simple game with three alternatives and three individuals. As in the introductory examples above, each ranking of the three alternatives can be viewed as a collective good: none of the three individuals is excluded from enjoying the selected ranking, even though each distinct individual may assign a different value to it. Assuming cardinal utilities for each alternative (Güth and Selten, 1991), we obtain the individual true valuations of each ranking by weighting and summing up the utilities associated with the three alternatives. Additionally, we suppose that the true valuations are common knowledge.[4] The sum of individual true val-

[1]This term is used by Buchanan (1983) in relation to noncompensated transfers and the emergence of rent-seeking behavior.

[2]A recent work highlighting the distinction between procedural and allocative fairness is by Chassang and Zehnder (2011).

[3]Most of the experimental literature on the voting paradox has tested whether people vote sincerely or strategically, and focused on the effect of information about the distribution of preferences on voting strategies (see, e.g., Tyszler and Schram, 2011; for an overview of experimental results see Palfrey, 2009).

[4]It may be argued that much of the social choice literature is concerned with situations where people have incomplete information about others' utilities. However, the Condorcet

uations of a certain ranking identifies the welfare that society (i.e., the group of three individuals) derives from that ranking.

The proper game features the three individuals as heterogeneous with respect to the (cardinal) utility they derive from the alternatives. This implies that the feasible rankings differ in the level of social welfare they generate, allowing us to study the effects of procedural fairness in situations where collective preferences are cyclic and the feasible rankings can be ordered with respect to their social welfare.

The key design feature of our experiment is that we vary the sum of individual payments associated to the selected ranking, keeping the game form procedurally fair. In one treatment individual payments add up to zero. In another they add up to a positive amount, resembling a situation in which the three individuals pay a "tax" for bringing about the selected ranking. Finally, there is a treatment where individual payments add up to a negative amount, corresponding to a situation in which the individuals receive a "subsidy" for implementing the selected ranking. Through these treatments, we can examine whether (and if so how) variations in the required payments impinge on the selected ranking and the bidders' stated preferences for the six possible rankings.

We observe slightly more overbidding (i.e., bids above the true value) in the treatment with the subsidy than in the other two. Yet, we find that the introduction of a tax or a subsidy does not affect the relative frequencies of selected rankings: in all three treatments, the most frequently chosen ranking is the one generating the highest social welfare.

The remainder of the paper is organized as follows. Section 2 presents the axiomatic characterization of the procedurally fair bidding rule. Section 3 defines the experimentally implemented proper game. In Section 4 we describe the experimental protocol, and in Section 5 we present the findings. Finally, Section 6 concludes.

paradox is viewed as a phenomenon arising from cyclic collective preferences, and not as a problem of incomplete information. Additionally, the assumption of complete information may not be too unrealistic if collectivity members know each other well.

2 The procedurally fair bidding rule

Let $N = \{1, \ldots, n\}$, with $n \geq 2$, be a group of evaluators/bidders $i = 1, \ldots, n$, facing a finite set $\mathcal{A} = \{A_1, A_2, \ldots, A_m\}$ of m (≥ 2) alternatives. Denote by σ a linear ranking of the m alternatives, such as $A_1 \succ A_2 \succ \ldots \succ A_m$, and by Σ the set of all linear rankings of $\mathcal{A}$ (we shall write "$(A_1, A_2, \ldots)$" for rankings). Thus the rankings correspond to all possible permutations of $\mathcal{A}$ and the set Σ contains $m!$ elements. A superscript index in parentheses will be sometimes used to refer to a specific permutation; for example, the notation $\sigma^{(r)}$, with $(r) = 1, 2, 3, \ldots, |\Sigma|$, will indicate any permutation of the alternatives to which we assign the rth position in the rankings sequence.

As explained in the Introduction, each evaluator $i \in N$ specifies a monetary bid $b_i(\sigma)$ for each $\sigma \in \Sigma$, i.e., he reports how much each ranking is worth to him. Hence each i submits a bid vector $\boldsymbol{b}_i = (b_i(\sigma) \in \mathbb{R} : \sigma \in \Sigma)$. The bid vectors of all n evaluators result in a bid profile $\boldsymbol{b} = (\boldsymbol{b}_1, \ldots, \boldsymbol{b}_n)$.

For each profile $\boldsymbol{b}$, a bidding rule must specify, first, which collective ranking is selected, and, second, which amount should be paid by each evaluator. To uniquely derive such a rule, we impose three ethical or procedural fairness requirements.[5] Note that the analysis is in objective terms, namely in monetary bids. Nothing is said about the subjective perceptions and valuations of the rankings. The reason is that we want to stay in the tradition of legal or constitutional mechanisms, which deal with game forms (constitutions) and define fairness by observables rather than by idiosyncratic true values. Hence, we define fairness with respect to bids and derive the procedurally fair bidding rule from the following three axioms.

(O) "Optimality with respect to bids" means that the selected collective ranking, denoted by σ^*, satisfies

$$\sigma^* = \sigma^*(\boldsymbol{b}) := \underset{\sigma \in \Sigma}{\operatorname{argmax}} \sum_{i=1}^{n} b_i(\sigma),$$

i.e., the selected ranking attains the maximal sum of bids.

[5] See Güth and Kliemt (2011) for a more elaborate discussion of these requirements.

(C) "Cost balancing" requires that the individual payments, denoted by $c_i(\sigma^*, \boldsymbol{b})$, add up to $K \in \mathbb{R}$; formally:

$$\sum_{i=1}^{n} c_i(\sigma^*, \boldsymbol{b}) = K.$$

(E) "Equal net benefits with respect to bids" affirms that if $\sigma^* \in \Sigma$ is selected, then

$$b_i(\sigma^*) - c_i(\sigma^*, \boldsymbol{b}) = b_j(\sigma^*) - c_j(\sigma^*, \boldsymbol{b}) \qquad \forall i, j, \text{ and } \boldsymbol{b}.$$

Due to axiom (E), we can write

$$b_i(\sigma^*) - c_i(\sigma^*, \boldsymbol{b}) = \Delta \qquad \forall i \in N. \tag{1}$$

Aggregating over all n evaluators yields

$$\sum_{i=1}^{n} b_i(\sigma^*) - \sum_{i=1}^{n} c_i(\sigma^*, \boldsymbol{b}) = n\Delta,$$

which, using (C), can be written as

$$\sum_{i=1}^{n} b_i(\sigma^*) = n\Delta + K.$$

Substituting Δ into Eq. (1), we obtain

$$c_i(\sigma^*, \boldsymbol{b}) = b_i(\sigma^*) - \Delta = b_i(\sigma^*) - \frac{\sum_{j=1}^{n} b_j(\sigma^*) - K}{n} \tag{2}$$

for all $i \in N$. Hence, the procedurally fair rule for collectively ranking alternatives selects the ranking with the highest sum of bids, and imposes the payment given in (2) on each bidder. Note that if the sum of bids for the selected ranking is at least K, nobody has to pay more than his bid.

We have so far derived a game form. In the next section we describe the experimentally implemented proper game. As mentioned in the Introduction, we assume commonly known true valuations of all rankings.

3 The experimental bidding game

There are three bidders, $N = \{1, 2, 3\}$, and three alternatives, $\mathcal{A} = \{A, B, C\}$. Each bidder i has a true preference ordering (denoted by $\succ_i$) over the alternatives. Without loss of generality, a bidder's utilities for his least and most preferred alternatives can be normalized to 0 and 1, respectively. We use the symbol m_i for player i's utility of his second most preferred (midle) alternative with $m_i \in (0, 1)$ for $i = 1, 2, 3$. To obtain the voting paradox, we assume the following true individual preference orderings of the alternatives:

$$B \succ_1 A \succ_1 C, \quad C \succ_2 B \succ_2 A, \quad A \succ_3 C \succ_3 B. \tag{3}$$

Thus, for instance, bidder 2 prefers C to B, B to A, and (because of transitivity) C to A. Since $A \succ_{\{1,3\}} C$ (to be read: "the majority composed by 1 and 3 prefers A to C"), $C \succ_{\{2,3\}} B$ but $B \succ_{\{1,2\}} A$, no majority ranking emerges; instead, we have a cycle.

Given our utility specification and the preference orderings shown in (3), the cardinal utilities attached to each alternative by the three bidders are

$$\begin{aligned}
&u_1(B) = 1, \quad u_1(A) = m_1, \quad u_1(C) = 0; \\
&u_2(C) = 1, \quad u_2(B) = m_2, \quad u_2(A) = 0; \\
&u_3(A) = 1, \quad u_3(C) = m_3, \quad u_3(B) = 0.
\end{aligned}$$

There are six possible rankings of the three alternatives:

$$\begin{aligned}
&\sigma^{(1)} = (A, B, C), \quad \sigma^{(2)} = (A, C, B), \quad \sigma^{(3)} = (B, A, C), \\
&\sigma^{(4)} = (B, C, A), \quad \sigma^{(5)} = (C, A, B), \quad \sigma^{(6)} = (C, B, A).^{6}
\end{aligned}$$

Each bidder $i = 1, 2, 3$ must therefore submit a bid vector $\boldsymbol{b}_i$ containing six bids, one bid for each of the above rankings. The combined bid vector of all

[6] The rankings denoted by $\sigma^{(3)}$, $\sigma^{(6)}$, and $\sigma^{(2)}$ reflect the true preference orderings of, respectively, bidders 1, 2, and 3 (see (3)).

Table 1: Bidders' true valuations $U_i(\sigma^{(r)})$ of the six rankings ($i = 1, 2, 3$; $r = 1, 2, \ldots, 6$)

Bidders	$\sigma^{(1)}$	$\sigma^{(2)}$	$\sigma^{(3)}$	$\sigma^{(4)}$	$\sigma^{(5)}$	$\sigma^{(6)}$
1	$\frac{m_1}{2} + \frac{1}{3}$	$\frac{m_1}{2} + \frac{1}{6}$	$\frac{m_1}{3} + \frac{1}{2}$	$\frac{m_1}{6} + \frac{1}{2}$	$\frac{m_1}{3} + \frac{1}{6}$	$\frac{m_1}{6} + \frac{1}{3}$
2	$\frac{m_2}{3} + \frac{1}{6}$	$\frac{m_2}{6} + \frac{1}{3}$	$\frac{m_2}{2} + \frac{1}{6}$	$\frac{m_2}{2} + \frac{1}{3}$	$\frac{m_2}{6} + \frac{1}{2}$	$\frac{m_2}{3} + \frac{1}{2}$
3	$\frac{m_3}{6} + \frac{1}{2}$	$\frac{m_3}{3} + \frac{1}{2}$	$\frac{m_3}{6} + \frac{1}{3}$	$\frac{m_3}{3} + \frac{1}{6}$	$\frac{m_3}{2} + \frac{1}{3}$	$\frac{m_3}{2} + \frac{1}{6}$

bidders $\boldsymbol{b} = (\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3)$ determines the selected collective ranking, σ^*, and the payments of the three bidders according to the bidding rule presented in the previous section. If two or more rankings tie for first place (i.e., they receive the same maximal bid sum), the tie is broken by selecting the ranking that generates the highest social welfare.[7] The resulting monetary payoff of each bidder $i \in N$ is

$$\pi_i(\boldsymbol{b}) = U_i(\sigma^*) - b_i(\sigma^*) + \frac{\sum_{j=1}^{3} b_j(\sigma^*) - K}{3} \tag{4}$$

where $U_i(\sigma^*)$ is bidder i's true valuation of the selected ranking.

To derive $U_i(\sigma^{(r)})$ from the cardinal utilities that i attaches to each alternative, we consider the weighted sum of the utilities where the weights depend on the relative position of the alternative in the ranking. More specifically, given a generic ranking $\sigma^{(r)} \in \Sigma$, we weight the utility for the alternative ranked first, second, and last in $\sigma^{(r)}$ by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$, respectively. This implies that for each bidder i the value of his most preferred ranking is $U_i(\sigma^{(r)}) = \frac{1}{2} + \frac{m_i}{3}$. In the same way one can determine the individual true valuations of all rankings in Σ (see Table 1).

The sum of all bidders' true valuations of a ranking defines that ranking's social welfare

$$W(\sigma^{(r)}) := \sum_{i=1}^{3} U_i(\sigma^{(r)}) \quad \text{for } r = 1, 2, \ldots, 5, 6.$$

[7]This tie-breaking rule is not required by our axioms, but used only for facilitating the experimental procedures.

In non-degenerate games, i.e., in games with $m_i \neq m_j$ $(i, j \in N,\ i \neq j)$, the social welfare generated by each feasible ranking varies. Specifically, the larger the difference between m_i and m_j, the larger the difference in social welfare across rankings.

Assuming that the true valuations are commonly known, we have a well-defined game with strategies $\boldsymbol{b}_i$ and payoffs $\pi_i(\boldsymbol{b})$ specified in Eq. (4). This game has an abundance of pure strategy equilibria (the derivation of one such equilibria is given in Appendix A).

4 The experimental design

4.1 Treatments

One of our main goals is to determine whether and to what extent bidding behavior under our procedurally fair rule is affected by variations in the required total payments. To this end, we distinguish three treatments differing only in the value assumed by K in payoff function (4).

- In one treatment we fix $K = 0$. In this case, each bidder's payment (i.e., his own bid) is reduced by 1/3 of the sum of bids for the selected ranking.
- In another treatment we fix $K > 0$. In this case, each bidder's payment is reduced by less than 1/3 of the sum of bids, resembling a situation in which the bidders have to pay a tax for bringing about the selected ranking. We refer to this treatment as the T treatment (for "taxation").
- In a third treatment, we fix $K < 0$. In this case, each bidder's payment is reduced by more than 1/3 of the sum of bids. This resembles a situation in which the bidders receive a "subsidy" for implementing the ranking. We refer to this treatment as the S treatment (for "subsidy").

The treatment with $K = 0$ is used as baseline (B) treatment, whereas treatments T (with $K > 0$) and S (with $K < 0$) allow checking how behavior is affected by the introduction of, respectively, a tax and a subsidy. We expect more underbidding and less overbidding in treatment T than in treatment

Table 2: Bidders' true valuations of the rankings in the implemented proper game with $(m_1, m_2, m_3) = (0.4, 0.5, 0.6)$

Bidders	$\sigma^{(1)}$	$\sigma^{(2)}$	$\sigma^{(3)}$	$\sigma^{(4)}$	$\sigma^{(5)}$	$\sigma^{(6)}$
1 (low-utility)	533	367	633	567	300	400
2 (interm.-utility)	333	417	417	583	583	667
3 (high-utility)	600	700	433	367	633	467
$W(\sigma^{(r)})$	1466	1484	1483	1517	1516	1534
$I(\sigma^{(r)})$	138.78	179.76	120.57	120.57	179.76	138.78

B. The rationale behind this is that bidders face an additional cost (of $K/3$) for each of the rankings. We expect the opposite to happen in treatment S (namely less underbidding and more overbidding relative to B) as bidders receive a kind of subsidy.

4.2 Experimental parameters

We wanted the basic game to be non-degenerate so that concerns for social welfare or efficiency (defined as the sum of the bidders' true valuations of a ranking) and for equality (defined as the dispersion of the bidders' true valuations within a ranking) might play a role in the bidders' decisions.[8] At the same time, however, we wanted to have the "fairest" possible non-degenerate proper game (in the sense of not favoring extremely one player over the other). Thus, we set $m_1 = 0.4$, $m_2 = 0.5$, and $m_3 = 0.6$. Bidder 1 is, therefore, the low-utility bidder; bidder 2 is the intermediate-utility bidder; and bidder 3 is the high-utility bidder.

The resulting individual true valuations of the six rankings and the corresponding welfare levels are reported in Table 2, together with the standard deviations of the valuations (which we take as a measure of the inequality

[8]If $m_1 = m_2 = m_3$, bidders would only differ in their true preference orderings of the alternatives, whereas the welfare attainable under the six rankings as well as the inequalities among the individuals generated by each ranking would stay constant.

associated with each ranking and denote by $I(\sigma^{(r)}))$. The true valuations are obtained from Table 1 multiplying each entry by 1000. The socially efficient ranking is $\sigma^{(6)}$, whereas there are two rankings minimizing inequality among bidders: $\sigma^{(3)}$ and $\sigma^{(4)}$. Yet, $\sigma^{(4)}$ guarantees a higher social welfare than $\sigma^{(3)}$.

The valuations and the bids were expressed in terms of ECU (Experimental Currency Unit), with 100 ECU = €1. Bids could be any integer number between 0 and 1000 ECU. The parameter K was 150 ECU, implying that in treatment T (S) each bidder paid his bid for the selected ranking and received one third of the sum of the group's bids minus (plus) 150.

4.3 Procedures

The experiment was programmed in z-Tree (Fischbacher, 2007) and conducted in the experimental laboratory of the Max Planck Institute of Economics (Jena, Germany). The subjects were undergraduates students from the Friedrich Schiller University of Jena. Considering the complexity of the experimental procedures, only students with relatively high analytical skills were invited, i.e., students majoring in subjects such as mathematics, physics, engineering, economics, and business administration. They were recruited using the ORSEE software (Greiner, 2004). Once entering the laboratory, the participants were randomly assigned to visually isolated computer terminals.

The three treatments were run one-shot in a within-subject design, i.e., participants played each treatment exactly once within a given session.[9] At the beginning of each session, each participant was randomly assigned one of three roles: bidder 1 (the low-utility bidder with $m_1 = 0.4$), bidder 2 (the intermediate-utility bidder with $m_2 = 0.5$), or bidder 3 (the high-utility bidder with $m_3 = 0.6$).[10] The role was retained throughout the session. We implemented a "perfect stranger" protocol, which ensures that nobody meets the same person in more than one treatment.

[9] One-shot games eliminate the possibility of strategic behavior that may exist in early periods of finitely repeated games.

[10] Since we numbered the rankings for which participants had to bid from 1 to 6, in the instructions we preferred to refer to the three group members as X, Y, and Z (rather than 1, 2, and 3).

Each of the three treatments was presented separately in a different part of the experiment. Instructions (reproduced in Appendix B) were distributed and read aloud in each of the three parts, and participants had the chance to go through a series of control questions and four (two) practice periods in the first (second and third) treatment.[11] Once the experimenter ensured that everyone understood the game, the corresponding treatment started and subjects submitted their bid vector. Only after all participants made their decisions in one treatment, they received the instructions for the following treatment.

Participants were informed about each bidder's true valuations of the rankings by a table similar to Table 2. They could sort the six rankings according to several attributes (namely each of the three bidders' true valuations; minimum, maximum, average valuations; sum of valuations). Additionally, participants were equipped with a profit calculator to simulate the earnings of each group member in different scenarios.

To minimize path dependence (i.e., dependence of current bids on previous outcomes), subjects did not receive any feedback until the end of the session. At the end of the session, one treatment was chosen randomly and subjects were paid according to their decisions in that treatment. Subjects knew about these procedures in advance.

Instead of considering all possible permutations of our treatments, we concentrate on treatment sequences where the baseline treatment is played at the very beginning. We wanted the participants to interact in the simplest scenario before adding taxes and subsidies. We will refer to the two implemented sequences as BTS and BST.

We ran two sessions per sequence. Each session involved 30 participants, matched in groups of three. With the bidders' roles remaining constant throughout a session, we had 20 players of each type (low-, intermediate-, and high-utility) for each treatment in any sequence. Sessions lasted about 2 hours. Each session was composed of two independent parts. The first part

[11] The practice periods did not involve any interaction (the others' decisions were selected randomly by the computer). Their sole aim was to familiarize participants with the game and its incentives (no payments were associated with them).

is presented in this paper. The second part refers to a different experiment, which was completely unrelated to bidding. Average earnings in the first part of the experiment were €9.60 (inclusive of a €5.50 show-up fee), ranging from a minimum of €6 to to a maximum of €14.40.

5 Results

First, we evaluate the data at the aggregate level by examining (i) how often each ranking is collectively selected in each treatment and (ii) the average bidding behavior. Then, we analyze individual data so as to find regularities in the submitted bid vectors. Given the focus of the paper, we will be primarily interested in the ordinal aspects of submitted bids, i.e., in the bidders' stated orderings of the rankings, rather than in the cardinal and absolute bid levels. This implies that we will neither report statistics of bid levels nor compare treatments with respect to the players' bids.

5.1 The selected ranking

Let us first check for the presence of order effects, i.e., investigate whether the frequency of selection of a ranking in a certain treatment varies across the two treatment sequences that we consider. In both sequences, the first treatment that subjects played was the baseline B. If recruitment was unbiased, we should observe no significant difference in the distribution of rankings selected in B between the two sequences. A Kolmogorov-Smirnov test indicates that this is indeed the case (p-value=0.739).[12] We can therefore conclude that randomization worked (i.e., the participants were sufficiently similar). Running the same test for the other two treatments reveals that the distributions of selected rankings in both T and S are not affected by the order in which the treatments are played (p-value = 0.739 for T; p-value = 1.000 for S). Thus, in analyzing how often each ranking is selected, we can pool the data from the BTS and BST sequences, which gives us a total of 40 groups for each treatment.

[12]Unless otherwise stated all statistical tests are two-sided.

Table 3: Percentage of groups selecting each ranking in the three treatments.

Treatment	$\sigma^{(1)}$	$\sigma^{(2)}$	$\sigma^{(3)}$	$\sigma^{(4)}$	$\sigma^{(5)}$	$\sigma^{(6)}$
B (baseline)	10.0%	17.5%	15.0%	27.5%	2.5%	27.5%
S (subsidy)	10.0%	7.5%	17.5%	22.5%	17.5%	25.0%
T (taxation)	7.5%	17.5%	17.5%	17.5%	15.0%	25.0%

Note: 40 groups per treatment.

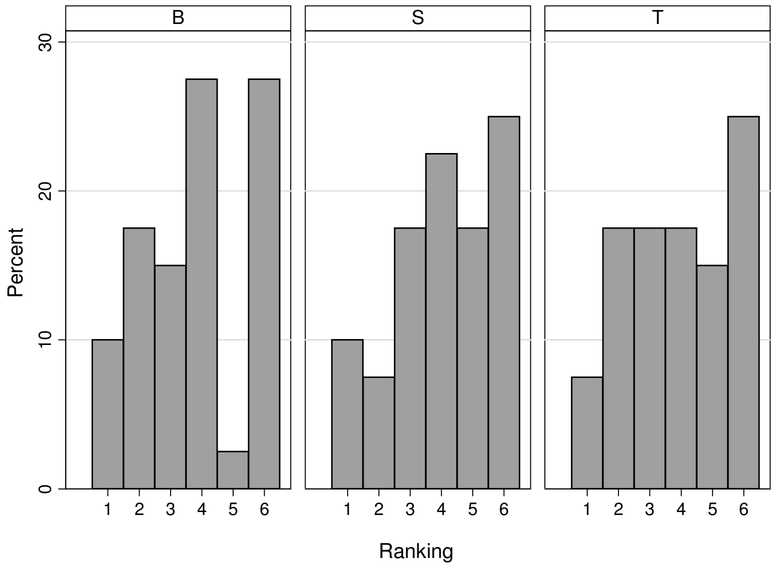

Figure 1: Histograms of the selected rankings. Rankings are ordered on the horizontal axes from $\sigma^{(1)}$ to $\sigma^{(6)}$.

Table 3 and Figure 1 illustrate how often each ranking attains the highest sum of group bids separately for each treatment.[13] In the baseline treatment, rankings $\sigma^{(4)}$ and $\sigma^{(6)}$ have the highest relative frequency (27.5%). They represent, respectively, the ranking that minimizes inequality among the group members and the socially efficient ranking. In both the other treatments, $\sigma^{(6)}$ is chosen relatively more often than all the other rankings (by 25% of

[13]Only in few instances is a ranking selected because of the tie-breaking rule. More specifically, in B the rule is applied to 2 groups out of 40 (in both cases $\sigma^{(6)}$ is selected); in S it is applied to 3 groups out of 40 (the favored rankings are $\sigma^{(5)}$, $\sigma^{(3)}$, and $\sigma^{(6)}$); in T it is applied to 4 groups out of 40 (favoring $\sigma^{(6)}$ three times and $\sigma^{(2)}$ once).

the groups in each treatment). The second most frequently chosen ranking is $\sigma^{(4)}$ in treatment S (22.5%), whereas the distribution appears to be flatter in treatment T. On the basis of Wilcoxon signed-rank tests (accounting for dependence of observations within groups across treatments), we cannot reject the null hypothesis that the distributions of selected rankings in the three treatments are identical (p-value=0.713 for B vs S as well as for B vs T; p-value=0.727 for S vs T).

For testing the robustness of our results, we constructed a larger dataset by randomly matching our 120 participants in three-person groups for 100/ 1,000/10,000/100,000 times. In this way, we generated 4,000/40,000/400,000 and 4,000,000 groups for each treatment. Using the bid vector submitted by each player in each treatment, we obtained the ranking selected by each randomly generated group.

The results of these simulations are given in Table 4. They are consistent with those observed with the actual data: in all four simulations and in all three treatments, the ranking selected with the highest frequency is $\sigma^{(6)}$. In treatments B and T the second most frequently chosen ranking is $\sigma^{(4)}$ (whatever the simulated sample size), whereas in treatment S $\sigma^{(3)}$ is slightly more common than $\sigma^{(4)}$. From this evidence we conclude that our procedurally fair bidding rule is robust to the introduction of taxes and subsidies: even with simulated data, the frequency in which a ranking is collectively selected does not vary with the treatments. In particular, the socially efficient ranking remains the most often chosen ranking.

5.2 Average bidding behavior

In this section we present results on the average bidding behavior across treatments focusing on underbidding, overbidding, and truthful bidding. Our analysis will consider the occurrence (rather than the magnitude) of each specific behavior.

We proceed in two steps. First, for each of the six bids $b_i(\sigma^{(r)})$, with $r = 1, 2, \ldots, 6$, submitted by each subject in each treatment, we create three dummies. Each dummy corresponds to a specific bidding behavior:

Table 4: Simulated data: percentage of groups selecting each ranking in the three treatments.

Treatment	$\sigma^{(1)}$	$\sigma^{(2)}$	$\sigma^{(3)}$	$\sigma^{(4)}$	$\sigma^{(5)}$	$\sigma^{(6)}$
			4,000 groups			
B (baseline)	10.5%	10.1%	20.4%	21.8%	11.6%	25.6%
S (subsidy)	9.4%	10.0%	20.8%	20.0%	11.5%	28.3%
T (taxation)	8.3%	17.5%	17.3%	20.4%	13.6%	22.9%
			40,000 groups			
B (baseline)	10.3%	9.3%	19.4%	22.2%	13.3%	25.5%
S (subsidy)	9.1%	10.9%	21.0%	19.9%	11.4%	27.7%
T (taxation)	7.7%	18.0%	17.1%	20.1%	13.4%	23.8%
			400,000 groups			
B (baseline)	10.3%	9.3%	19.5%	22.4%	13.0%	25.5%
S (subsidy)	9.1%	11.0%	20.8%	20.2%	11.1%	27.7%
T (taxation)	7.8%	17.7%	17.2%	20.0%	13.7%	23.6%
			4,000,000 groups			
B (baseline)	10.3%	9.4%	19.5%	22.3%	12.9%	25.5%
S (subsidy)	9.1%	10.9%	20.8%	20.2%	11.2%	27.7%
T (taxation)	7.8%	17.7%	17.3%	20.0%	13.7%	23.6%

un(derbidding), ov(erbidding), and tr(uthful bidding). Therefore, for each individual bid

- underbidding is captured by the dummy $un_i(\sigma^{(r)})$ which takes value 1 if $b_i(\sigma^{(r)}) < U_i(\sigma^{(r)})$ and 0 otherwise;
- overbidding is captured by the dummy $ov_i(\sigma^{(r)})$ which takes value 1 if $b_i(\sigma^{(r)}) > U_i(\sigma^{(r)})$ and 0 otherwise;
- truthful bidding is captured by the dummy $tr_i(\sigma^{(r)})$ which takes value

1 if $b_i(\sigma^{(r)}) = U_i(\sigma^{(r)})$ and 0 otherwise.

Once we have coded (as 0 or 1) each individual bid, we proceed to the second step. For each subject we construct three new variables, counting how often each of the above dummies equals 1. Thus, the number of times in which the six individual components of a bid vector are below, above, and equal to the true values is captured, respectively, by the variables

$$UN_i = \sum_{\sigma=1}^{6} un_i(\sigma^{(r)}), \quad OV_i = \sum_{\sigma=1}^{6} ov_i(\sigma^{(r)}), \quad TR_i = \sum_{\sigma=1}^{6} tr_i(\sigma^{(r)}).$$

In each treatment and for each bidder, UN_i, OV_i and TR_i take values from 0 to 6, depending on how many times underbidding, overbidding and truthful bidding is observed in the submitted bid vector. Considering the distribution of each of these variables over our 120 participants, we gain an idea of the relevance of underbidding, overbidding, and truthful bidding within a given treatment.

Before presenting our results and in analogy with the analysis of the previous section, we test for the presence of order effects. This test checks whether the relevance of a given bidding behavior within a given treatment varies with the considered sequence. According to a series of Kolmogorov-Smirnov tests, there is no significant difference in the distribution of occurrences of underbidding, overbidding, and truthful bidding in the BST and the BTS sequences for both treatment S (all p-values are greater than 0.586) and treatment T (all p-values exceed 0.998).[14] On the basis of these findings, we pool the data on UN_i, OV_i, and TR_i from the two sequences.

Table 5 reports, for each treatment, the distribution of the occurrences of underbidding, overbidding and truthful bidding. The table shows that underbidding is the most common behavior: in each treatment about 90% of the participants underbid for all six rankings. Truthful bidding as well as overbidding are observed less often. By means of a set of Wilcoxon signed-

[14]We compared as well the three distributions of interest in the B treatment across the two sequences with the aim of finding out whether recruitment was unbiased also under this respect. The p-value of all three tests equals 1.00, confirming that the participants were sufficiently similar.

Table 5: Frequencies of underbidding, overbidding and truthful bidding in each treatment.

Treatment	0/6	1/6	2/6	3/6	4/6	5/6	6/6
				underbidding			
B	1.67%	0.83%	0.83%	3.33%	0.83%	1.67%	90.83%
S	1.67%	1.67%	0.83%	3.33%	1.67%	2.50%	88.33%
T	1.67%	0.83%	0%	4.17%	1.67%	0.83%	90.33%
				overbidding			
B	95.83%	0.83%	0.83%	1.67%	0.83%	0%	0%
S	92.50%	0.83%	0.83%	2.50%	0.83%	1.67%	0.83%
T	95.00%	0%	2.50%	0.83%	0.83%	0%	0.83%
				truthful bidding			
B	93.33%	1.67%	1.67%	1.67%	0%	0%	1.67%
S	95.83%	1.67%	0.83%	0.83%	0%	0%	0.83%
T	93.33%	3.33%	0.83%	1.67%	10%	0%	0.83%

Note: 120 observations per treatment.

ranks tests we can compare each given behavior across treatments. We begin with underbidding. Contrary to our expectations of more (less) underbidding in T (S) relative to B, we do not observe any significant difference in the occurrences of underbidding between B and S (p-value = 0.202) as well as between B and T (p-value = 0.995). A significant difference emerges only when comparing the distribution of underbidding in S and T (p-value = 0.038).

Focus now on overbidding. Although this behavior is not the most common, we observe the expected significant differences: the test is slightly above (i) the conventional 5% level when we compare the distribution of overbidding in B and S (p-value = 0.055); (ii) the 1% level when we compare the distribution of overbidding in S and T (p-value= 0.012). No significant difference is evident in the distribution of overbidding between B and T.

Finally, considering truthful bidding we find no significant differences in distributions between treatments (B vs S: p-value = 0.155; B vs T: p-value = 0.985; S vs T: p-value = 0.178).

5.3 Behavioral regularities

In this section we analyze the individual bid vectors in order to identify possible behavioral patterns in the bidders' stated orderings of the rankings. Such an analysis will shed light on i) how our procedurally fair bidding rule translates into actual behavior, and ii) whether the introduction of a tax or a subsidy causes a change in stated orderings.

We begin by checking whether and to what extent subjects submit bid vectors which reflect their true preference ordering of the rankings. From Table 2, this implies that

- low-utility bidder 1 should submit a bid vector $\boldsymbol{b}_1$ such that $b_1(\sigma^{(3)}) > b_1(\sigma^{(4)}) > b_1(\sigma^{(1)}) > b_1(\sigma^{(6)}) > b_1(\sigma^{(2)}) > b_1(\sigma^{(5)})$;
- intermediate-utility bidder 2 should submit a bid vector $\boldsymbol{b}_2$ such that $b_2(\sigma^{(6)}) > b_2(\sigma^{(5)}) = b_2(\sigma^{(4)}) > b_2(\sigma^{(3)}) = b_2(\sigma^{(2)}) > b_2(\sigma^{(1)})$;
- high-utility bidder 3 should submit a bid vector $\boldsymbol{b}_3$ such that $b_3(\sigma^{(2)}) > b_3(\sigma^{(5)}) > b_3(\sigma^{(1)}) > b_3(\sigma^{(6)}) > b_3(\sigma^{(3)}) > b_3(\sigma^{(4)})$.

Only a few bid vectors conform to these patterns (see Table 6), and they are mainly submitted by the high-utility bidders 3 in treatment T (25%).

A further ordering of the rankings which may capture actual bidding behavior involves (i) preserving strict monotonicity between the most and the second most truly preferred ranking, and (ii) requiring weak monotonicity between the other rankings. This criterion implies that

- low-utility bidder 1 should submit a bid vector $\boldsymbol{b}_1$ such that $b_1(\sigma^{(3)}) > b_1(\sigma^{(4)}) \geq b_1(\sigma^{(1)}) \geq b_1(\sigma^{(6)}) \geq b_1(\sigma^{(2)}) \geq b_1(\sigma^{(5)})$;
- intermediate-utility bidder 2 should submit a bid vector $\boldsymbol{b}_2$ such that $b_2(\sigma^{(6)}) > b_2(\sigma^{(5)}) = b_2(\sigma^{(4)}) \geq b_2(\sigma^{(3)}) = b_2(\sigma^{(2)}) \geq b_2(\sigma^{(1)})$;

Table 6: Percentage of each player type who reports bid vectors reflecting their true preference ordering of the rankings. The last column reports the percentage of consistent bidders who submit vectors in line with their true preference ordering in all treatments.

Bidders	Treatments			Over all
	B	*S*	*T*	treatments
1 (low-utility)	7.5%	10.0%	7.5%	2.5%
2 (interm.-utility)	17.5%	17.5%	7.5%	10.0%
3 (high-utility)	15.0%	15.0%	25.0%	10.0%
ALL	13.33%	14.17%	13.33%	4.17%

Note: 40 observations per treatment (10 subjects × 4 sessions);
120 observations over all treatments/bidder types.
Bottom right number: overall percentage of consistent subjects.

- high-utility bidder 3 should submit a bid vector $\boldsymbol{b}_3$ such that $b_3(\sigma^{(2)}) > b_3(\sigma^{(5)}) \geq b_3(\sigma^{(1)}) \geq b_3(\sigma^{(6)}) \geq b_3(\sigma^{(3)}) \geq b_3(\sigma^{(4)})$.

We name this criterion "weak ordering" and regard the bid vectors complying with it as showing *selfishness.*

Averaging over all treatments and player types, this criterion is able to accommodate 49.7% (179/360) of the submitted bid vectors. Table 7 shows the experimentally observed frequencies of "weak ordering" per player type and treatment as well as over all players and treatments. There are a couple of observations which are immediate from inspecting the table: (1) in each treatment, the low-utility bidders are those who most often comply with the criterion (especially when a subsidy is introduced), whereas the high-utility bidders are those who least often abide by it (especially in the baseline); (2) the high-utility bidders appear to be the most consistent type as 35% of them submit a selfish bid vector in all three treatments (vs 27.5% of both low- and intermediate-utility bidders).[15]

[15]If we relax the strong monotonicity between the first two preferred rankings, and check how often bidders state a bid vector whose component bids weakly monotonically decrease with decreasing true preferences over the rankings, we find that 68.06% (245/360)

Table 7: Percentage of each player type who reports bid vectors reflecting weak ordering of the rankings. The last column reports the percentage of consistent bidders who submit vectors in line with the weak ordering criterion in all treatments.

Bidders	Treatments B	S	T	Over all treatments
1 (low-utility)	52.5%	62.5%	52.5%	27.5%
2 (interm.-utility)	52.5%	55.0%	47.5%	27.5%
3 (high-utility)	35.0%	42.5%	47.5%	35.0%
ALL	56.67%	53.33%	49.17%	30.0%

Note: 40 observations per treatment (10 subjects $\times$ 4 sessions);
120 observations over all treatments/bidder types.
Bottom right number: overall percentage of consistent subjects.

Can we detect further regularities in the bid vectors that do not fall into the selfish category? To address this question, we focus on two kinds of other-regarding preferences (inequality aversion and efficiency) as well as on preferences for abstention (from stating discriminatory preferences). Abstention means that the six individual components of the bid vector have the same value (often equal to zero). We refer to a bid vector displaying this pattern as showing *indifference*.

Turning to a classification based on other-regarding preferences, we say that a bid vector reveals preferences for *inequality aversion* (IA) if it assigns to either $\sigma^{(3)}$ or $\sigma^{(4)}$ (namely the two rankings minimizing inequality within the group) a position higher than the position these rankings have in the bidder's true preference ordering. Then, we distinguish two kinds of inequality aversion, depending on whether the bidder with an IA vector favors his own or the group's interest.

- If between $\sigma^{(3)}$ and $\sigma^{(4)}$ the bidder reveals a preference for the ranking that he truly values the most, e.g., $b_i(\sigma^{(3)}) > b_i(\sigma^{(4)})$ if $U_i(\sigma^{(3)}) >$

of the submitted bid vectors can be accounted for by this simple criterion.

$U_i(\sigma^{(4)})$, we say that his bid vector exhibits IA plus *selfishness* (IA-SEL).

- If between $\sigma^{(3)}$ and $\sigma^{(4)}$ the bidder reveals a preference for the more socially efficient ranking, e.g., $b_i(\sigma^{(3)}) > b_i(\sigma^{(4)})$ if $W(\sigma^{(3)}) > W(\sigma^{(4)})$, we say that his bid vector exhibits IA plus *efficiency concern* (IA-EF).

Finally, we say that a bid vector reveals preferences for *efficiency* (EF) if it assigns to $\sigma^{(6)}$ (i.e., the socially efficient ranking) a position higher than the position this ranking has in the bidder's true preference ordering.

If a vector does not fall in any of the above classifications, it is included in the category *other*.

Since the three player types differ in the trade-off between selfishness and other-regarding concerns, we evaluate the data separately for each type.

5.3.1 Low-utility bidders

The ranking that a low-utility bidder truly values the most is $\sigma^{(3)}$, which coincides with one of the rankings minimizing inequality among bidders. Hence, this type of bidder cannot assign $\sigma^{(3)}$ to a higher position than the one it has in the true ordering. From the above definition, it follows that no bid vector submitted by bidder 1 is classifiable as IA-SEL.

Figure 2 displays, for each treatment, how many observed bid vectors can be classified as showing selfishness, other-regarding preferences, and indifference. Unclassifiable vectors are into the category "other". The classification of the vectors according to other-regarding preferences is reported in Table 8.

As noted above, selfishness (i.e., the weak ordering criterion) accommodates most of the data. Yet, the fraction of bid vectors consistent with other-regarding preferences is not negligible: it ranges from 20% in T to 30% in B. Except for the B treatment, where the two kinds of other-regarding preferences are equally common, in both the other treatments IA-EF slightly outperforms EF (see Table 8).

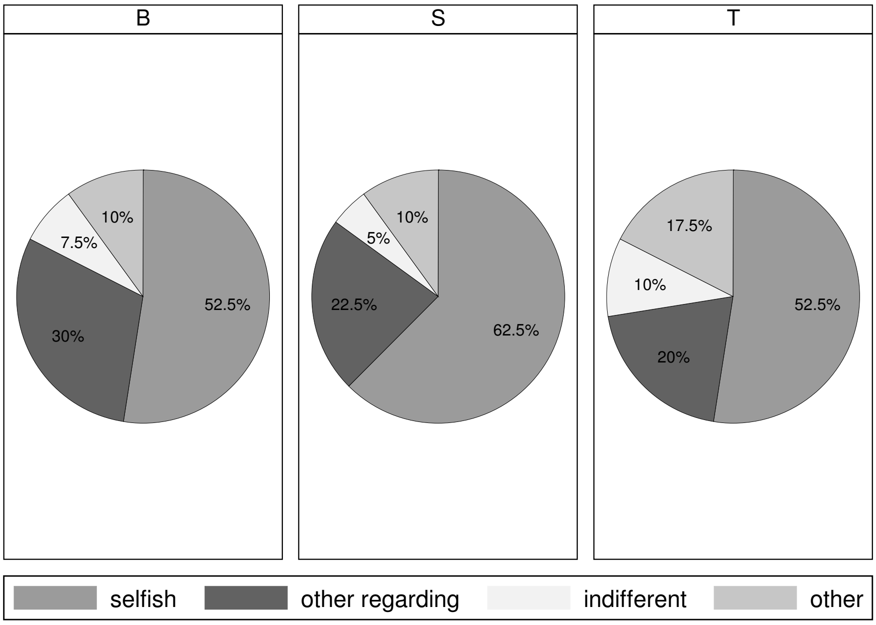

Figure 2: Classification of bid vectors submitted by the low-utility type in each treatment.

Table 8: Classification of the low-utility type's bid vectors exhibiting other-regarding preferences (numbers in parentheses refer to the entire population of low-utility bidders).

	B	*S*	*T*
IA-EF	50.0% (15%)	55.6% (18.52%)	75% (15%)
EF	50.0% (15%)	44.4% (14.81%)	25% (5%)
N	12 (40)	9 (40)	8 (40)

Note: N denotes the number of observations per treatment.

5.3.2 Intermediate-utility bidders

For an intermediate-utility bidder there is no trade-off between selfishness and efficiency as his true preference ordering assigns $\sigma^{(6)}$ to the highest position. Additionally, this type of bidder does not face any trade-off between the two kinds of inequality aversion: both IA with a selfish aspect and IA with an

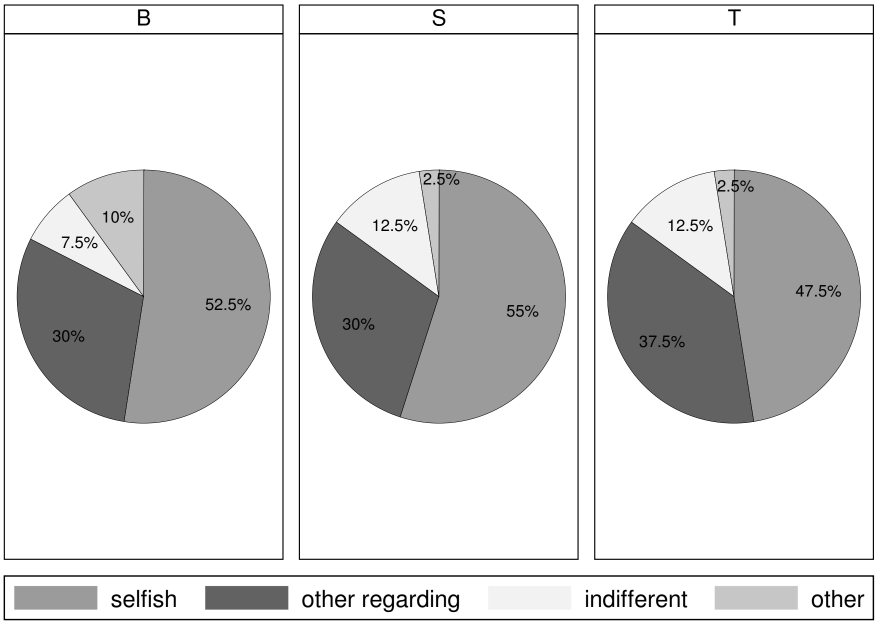

Figure 3: Classification of bid vectors submitted by the intermediate-utility type in each treatment.

efficiency aspect require him to submit a bid vector where $\sigma^{(4)}$ occupies the first position in the ordering. Thus, the other-regarding bid vectors submitted by the intermediate-utility bidders can just display inequality aversion.

The classification of observed bid vectors is reported in Figure 3. The frequency of vectors consistent with our definition of other-regarding preferences is higher for this type than for low-utility bidders in the S and T treatments.

5.3.3 High-utility bidders

The least favorite rankings of a high-utility bidder are exactly $\sigma^{(4)}$, $\sigma^{(3)}$, and $\sigma^{(6)}$. Thus, differently from what pertains to the other types, for bidders 3 the trade-off between selfishness and other-regarding concerns is more substantial. Yet, the percentage of bid vectors classifiable as other-regarding is not insignificant (see Figure 4). The bid vector of a high-utility bidder must value $\sigma^{(3)}$ more than $\sigma^{(4)}$ to be classified as IA-SEL, and the vice versa must hold to be classified as IA-EF. The classification of the vectors by other-regarding

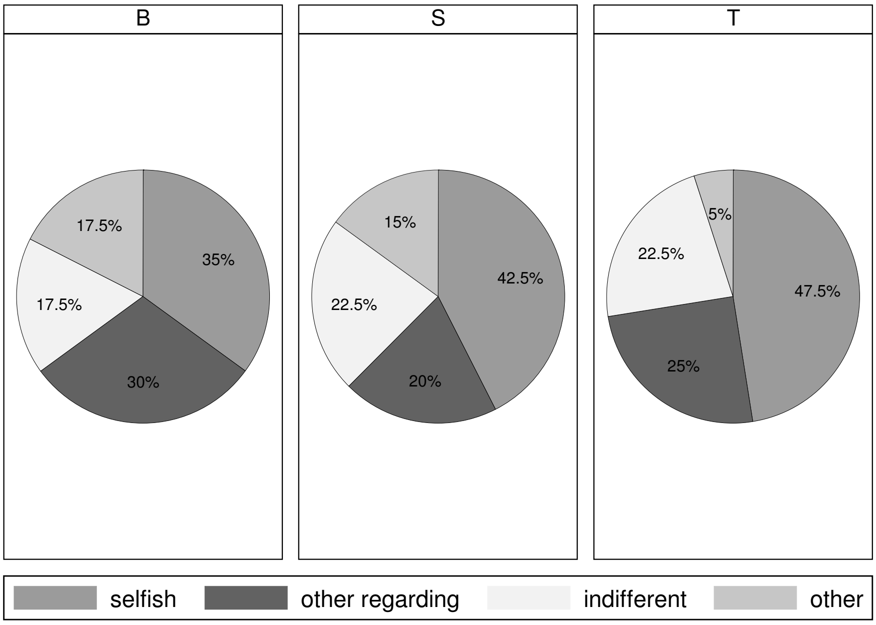

Figure 4: Classification of bid vectors submitted by the high-utility type in each treatment.

Table 9: Classification of the high-utility type's bid vectors exhibiting other-regarding preferences (numbers in parentheses refer to the entire population of high-utility bidders).

	B	*S*	*T*
IA-SEL	8.33% (2.5%)	37.5% (7.5%)	10.0% (2.5%)
IA-EF	33.33% (10%)	25.0% (18.52%)	40.0% (15%)
EF	58.33% (17.5%)	37.5% (7.5%)	50.0% (12.5%)
N	12 (40)	8 (40)	10 (40)

Note: N denotes the number of observations per treatment.

preferences is reported in Table 9.

Considering Figure 4, we find that selfish bid vectors are observed less often than for the other types. However, the occurrence of other-regarding preferences is not higher than for the low-utility type. Rather, indifferent bid vectors increase in frequency. From Table 9, we see that preferences for efficiency (either alone or together with inequality aversion) accommodate most bid vectors. Jointly, these observations suggest that being the most favored with respect to utility somehow discourages selfishness, and supports indifference and concerns for social welfare.

6 Conclusions

We are used to assign monetary values to the things we want. For private goods this happens in markets, where goods are exchanged for money. In this paper, we suggest an extension of such money principle to settings in which a collectivity has to agree on a common ranking of alternatives. The relative positions of the alternatives in the ranking determines how much this ranking is worth to each collectivity member whose preferences for the alternatives differ from those of the others in the way captured by the voting paradox. The idea, used here, of allowing each member to place a monetary bid on each feasible ranking represents an endogenous solution to the paradox and is akin to the ways in which private goods are allocated by markets.

The bidding rule that we propose is derived from three axioms so as to be procedurally fair. Procedural fairness is ensured by our basic equality axiom requiring that the individual members of the collectivity should receive equal net benefits with respect to their bids, namely they should be treated equally according to their bids. The other two axioms simply demand optimality with respect to bids, and some sort of cost balancing. The latter, together with the equality axiom, is used to derive the individual payments. One might wonder why we define fairness in terms of bids, rather than (what is more common) in terms of payoffs. The answer is twofold. First, we want fairness to be a property of the selection mechanism (i.e., of the game form), not of the selected ranking. Second, and related to the first, we are interested in legal or constitutional mechanisms, which deal with game forms (constitutions) and define fairness by observables rather than by idiosyncratic true values.

Having a bidding rule that satisfies our procedural fairness requirements is certainly important. Yet, of equal importance is to assess which outcomes it finally induces. Exploration of this issue entails defining a proper bidding game which can be implemented in the laboratory. Here, we focus on a simple game with a group of three individuals facing three alternatives, and thus six possible rankings. The main aim of our experiment is to examine whether and how, keeping the game form procedurally fair, the selected ranking and the individuals' stated preferences for the six rankings are affected by variations in the required individual payments.

The experimental results provide clean evidence that changing the due payments, introducing a tax and a subsidy, does not significantly alter the relative frequencies of selected rankings: with both actual and simulated data, the ranking chosen most often is the one that generates the highest social welfare. Moreover, the treatments do not differ significantly in the occurrence of overbidding, underbidding, and truthful bidding, with underbidding always being the most common behavior. Only the presence of a subsidy appears to slightly raise overbidding.

Finally, we observe that submitted bid vectors are quite heterogenous. Most of them can be classified as selfish in the sense of reflecting, in a weakly monotonic way, the true preference ordering of the rankings. Yet, the fraction of bid vectors consistent with our definition of other-regarding preferences (namely inequality aversion and efficiency) is not negligible. Indifferent bid vectors (in which all six individual components have the same value) are present as well, especially for the most favored group member.

In sum, our experimental findings indicate that the axiomatically derived and procedurally fair bidding rule is not only implementable, but also functional. More research is necessary for the generalization of our findings. But the experimental evidence garnered here suggests that the proposed bidding rule for collectively selecting a ranking of alternatives tends to maximize social welfare and is robust to slight changes in the required payments.

Appendix A. Equilibrium analysis

In this appendix, we will focus on the equilibrium that implements the socially efficient ranking, based on truthful bidding for all other non-selected rankings.

Denote by σ^{ef} the socially efficient ranking and by σ^{sb} the second most efficient ranking. Suppose that in case of equal maximal bid sums for two or more rankings, the ranking that generates the maximum social welfare is selected. Then the bid profile in which all three players (i) strategically underbid for σ^{ef}, and (ii) bid truthfully for all rankings but σ^{ef} can be an equilibrium. More formally, the following bid profile can be an equilibrium:

$$b_i^*(\sigma^{\text{ef}}) < U_i(\sigma^{\text{ef}}) \ \ \forall\, i \in N, \ \ \sum_{i=1}^{3} b_i^*(\sigma^{\text{ef}}) = \sum_{i=1}^{3} U_i(\sigma^{\text{sb}}),$$
$$\text{and} \ \ \sum_{i=1}^{3} b_i^*(\sigma) = \sum_{i=1}^{3} U_i(\sigma) \ \ \forall\, \sigma \neq \sigma^{\text{ef}}, \ \sigma \in \Sigma.$$

We show that no bidder $i = 1, 2, 3$ has an incentive to unilaterally deviate from this bid profile. If all bidders underbid for σ^{ef} and bid truthfully for σ^{sb}, as prescribed by the equilibrium, bidder i earns (assuming $K = 0$ in Eq. (4))

$$\pi_i(\boldsymbol{b}^*) = U_i(\sigma^{\text{ef}}) - b_i^*(\sigma^{\text{ef}}) + \frac{\sum_{j=1}^{3} U_j(\sigma^{\text{sb}})}{3}. \tag{A-1}$$

If, instead, bidder i unilaterally increases his bid for σ^{sb} to $U_i(\sigma^{\text{sb}}) + \delta$, he changes the selected ranking to σ^{sb}. In this case, his payoff would be

$$\pi_i(\boldsymbol{b}) = \frac{\sum_{j=1}^{3} U_j(\sigma^{\text{sb}})}{3} - \frac{2}{3}\delta,$$

which is lower than the equilibrium payoff in (A-1) due to $b_i^*(\sigma^{\text{ef}}) < U_i(\sigma^{\text{ef}})$. On the other hand, setting $b_i(\sigma^{\text{sb}}) < U_i(\sigma^{\text{sb}})$ does not change the outcome, σ^{ef}, and thus cannot pay.

Lowering $b_i(\sigma^{\text{ef}})$ below $b_i^*(\sigma^{\text{ef}})$, for $i = 1, 2, 3$, so as to induce the second most efficient ranking σ^{sb} does not pay either: bidder i would earn

$\frac{\sum_{j=1}^{3} U_j(\sigma^{\text{sb}})}{3}$, which is lower than $\pi_i(\boldsymbol{b}^*)$ in (A-1).

Positive linear transformations of the payoff function do not influence the solution. Hence, the above outcome remains an equilibrium in a game where $K \neq 0$ holds.

Appendix B. Experimental instructions

This appendix reports the instructions (originally in German) that we used for the sequence BST. The instructions for the other sequence were adapted accordingly. We include only the instructions for part 1, which pertain to the experiment described in this paper.

Welcome! You are about to participate in an experiment funded by the Max Planck Institute of Economics. Please switch off your mobile(s) and remain silent. It is strictly forbidden to talk to other participants. Please raise your hand whenever you have a question; one of the experimenters will come to your aid.

You will receive €5.50 for showing up on time. Besides this, you can earn more. But there is also a small possibility of ending up with a loss that you will compensate by using your show-up fee. The show-up fee and any additional amounts of money you may earn will be paid to you in cash at the end of the experiment. Payments are carried out privately, i.e., the others will not see your earnings.

In the course of the experiment, we shall speak of ECUs (Experimental Currency Unit) rather than euros. The conversion rate is 100 ECUs per 1 €.

The experiment consists of two parts. The instructions for the first part follow below. The instructions for the second part will be distributed after all participants have completed the first part.

Detailed Information on Phase 1 (Part 1)

Part 1 of the experiment consists of 3 phases. You have received the instructions for the first phase. The instructions for the second phase will be distributed after all participants have completed the first phase and the instructions for the third phase will be distributed after all participants have completed the second phase.

Group formation

You will be placed in a group of three persons. The three group members will interact with each other just once. You will never be informed of the identity of the two participants in your group.

Each group member will be identified by a letter: X, Y, or Z. Specifically, when placed in a group:

- with 1/3 probability you will be identified by the letter “X”;
- with 1/3 probability you will be identified by the letter “Y”;
- with 1/3 probability you will be identified by the letter “Z”.

You will learn your identifying letter at the beginning of the experiment.

The situation you will face

You and the other two members of your group will face a situation involving six possible alternatives, namely alternatives A_1, A_2, A_3, A_4, A_5, and A_6. The three of you will decide which of the six alternatives will be selected. (The rules determining which alternative will be selected are described below).

Individual gain from each alternative

The three group members X, Y, and Z have different individual gains from the different alternatives. Table B-1 shows the gain of each group member from each alternative. Consider, for example, alternative A_1. If you are group member X, you obtain 533 ECUs from A_1. If you are group member Y, you obtain 333 ECUs. If you are group member Z, you obtain 600 ECUs. More generally, if you are X, the *first* row of Table B-1 shows your gains from the various alternatives; if you are Y, the *second* row of Table B-1 shows your gains from the various alternatives; if you are Z, the *third* row of Table B-1 shows your gains from the various alternatives.

Please, note that the order in which the six alternatives will appear on your screen during the experiment can be different from that given in Table B-1.

Table B-1: Individual Gains from the Six Alternatives

Group member	A_1	A_2	A_3	A_4	A_5	A_6
X	533	367	633	567	300	400
Y	333	417	417	583	583	667
Z	600	700	433	367	633	467

Your screen will show you additional information on each of the six alternatives. More specifically, you will see four additional rows:

- one row will display the “average gain” from each alternative (namely the sum of individual gains divided by 3);

- another row will display the minimum gain associated with each alternative and (in parentheses) who – among the three group members (X, Y, or Z) – gets this minimum;
- a third row will display the maximum gain associated with each alternative and (in parentheses) who – among the three group members (X, Y, or Z) – gets this maximum;
- finally, a fourth row will display the sum of individual gains from each alternative.

Your decision

Having learned if you are group member X, Y, or Z, you will need to place a bid for each alternative A_1, A_2, A_3, A_4, A_5, and A_6, so that you will have to submit six bids. Regardless of your individual gains from the alternatives, your bids can be any integer number between 0 and 1000 ECUs (i.e., $0, 1, 2, \ldots, 998, 999, 1000$).

Rules for the selection of an alternative

Which of the six alternatives will be selected depends on the total number of ECUs that you and the other two members of your group bid for each alternative.

Specifically: the alternative with the highest bid sum will be selected.

If two or more alternatives tie for first place (i.e., they receive the same highest bid sum), the tie is broken by selecting the alternative with the highest sum of individual gains. For example, if alternatives A_1 and A_4 tie, then A_4 will be selected because the sum of individual gains for A_4 ($567 + 583 + 367 = 1517$) is greater than that for A_1 ($533 + 333 + 600 = 1466$).

Your experimental earnings

Your earnings depend on the selected alternative, and the bids submitted by you and your group members for the selected alternative. Call the selected alternative A^*. Then,

– you are paid your gain from A^* (as reported in Table 1),
– you pay your bid for A^*,
– you receive one third of the sum of your group's bids for A^*.

Thus, your earnings summarized in a formula are

$$\boxed{\text{your gain from } A^* \quad - \quad \text{your bid for } A^* \quad + \quad \text{one third of the bid sum for } A^*}$$

For example, if A_1 is the selected alternative and you are group member X, then your earnings are:

your gain from A_1	$-$	your bid for A_1	$+$	one third of the bid sum for A_1
(533)	$-$	(integer within 0 to 1000)	$+$	$1/3 \times$ (bid sum for A_1)

Note that if your bid for the selected alternative exceeds your gain from that alternative, then your earnings could be negative, i.e., you may suffer a loss.

The following examples should help you better understand the calculation of your earnings. Notice that the numbers in the examples are just for illustrative purposes. They DO NOT intend to suggest how you may choose your bids.

Example 1

Suppose that A_5 is the alternative selected by your group (this means that A_5 receives the highest bid sum in your group). Suppose further that the bids of each group member for A_5 are the following:

- X's bid = 300;
- Y's bid = 583;
- Z's bid = 633.

Therefore each group member's earnings are:

- X's Earnings: $300 - 300 + \frac{300+583+633}{3} = \frac{1516}{3} = 505$ ECUs.
- Y's Earnings: $583 - 583 + \frac{300+583+633}{3} = \frac{1516}{3} = 505$ ECUs.
- Z's Earnings: $633 - 633 + \frac{300+583+633}{3} = \frac{1516}{3} = 505$ ECUs.

Note that in this case each group member has submitted a bid for the selected alternative which *equals* his/her own gain.

Example 2

Suppose that A_1 is the alternative selected by your group. Suppose further that the bids of each group member for A_1 are the following:

- X's bid = 233;
- Y's bid = 0;
- Z's bid = 550.

Therefore each group member's earnings are:

- X's Earnings: $533 - 233 + \frac{233+0+550}{3} = 200 + \frac{783}{3} = 461$ ECUs.
- Y's Earnings: $333 - 0 + \frac{233+0+550}{3} = 333 + \frac{783}{3} = 594$ ECUs.

- Z's Earnings: $600 - 550 + \frac{233+0+550}{3} = 50 + \frac{783}{3} = 311$ ECUs.

Note that in this case each group member has submitted a bid for the selected alternative which is *lower* than his/her own gain.

Example 3

Suppose that A_3 is the alternative selected by your group. Suppose further that the bids of each group member for A_3 are the following:

- X's bid = 633;
- Y's bid = 1000;
- Z's bid = 0.

Therefore each group member's earnings are:

- X's Earnings: $633 - 633 + \frac{633+1000+0}{3} = 0 + \frac{1633}{3} = 544$ ECUs.
- Y's Earnings: $417 - 1000 + \frac{633+1000+0}{3} = -583 + \frac{1633}{3} = -583 + 544 = -39$ ECUs.
- Z's Earnings: $433 - 0 + \frac{633+1000+0}{3} = 433 + \frac{1633}{3} = 433 + 544 = 977$ ECUs.

In this example, group member Y suffers a loss because his/her bid for A_3 exceeds his/her gain from A_3 plus 1/3 of the bid sum for A_3 (i.e., $1000 > 417 + 544$).

Timing of provided information

You will be informed about your group members' choices in phase 1 of part 1 only after the end of the session. Thus, you will learn (a) your group members' bids in the first phase of part 1, (b) which alternative is selected, and (c) your experimental earnings in this phase on completion of part 2 of the experiment.

Your final payoff

Only one of the three phases of part 1 will end up affecting your final payoff, but you do not know in advance which phase will be used. After part 2 is over, we will randomly select one participant by drawing a card from a deck that contains as many cards as the number of participants. This participant will in his/her turn randomly select one of the three phases of part 1 by drawing a ball from an urn containing three balls numbered 1 to 3. The experimental earnings corresponding to this phase will be converted to euros and paid out in cash. The outcome of the draw will apply to all the participants.

Since you do not know which phase will determine your payoff, think carefully when choosing your bids!

Summary for Phase 1 (Part 1)

- You will be matched with two other participants; each of you will be identified by a letter (X, Y, or Z) which is assigned randomly.
- You will face six alternatives A_1, A_2, A_3, A_4, A_5 and A_6.
- For each of the alternatives you will have a different gain, as shown in Table 1.
- You will have to decide how much to bid for each alternative. Your bids must be integers between 0 and 1000 ECUs and are submitted exactly once.
- The alternative with the highest bid sum is selected (ties are broken by selecting the alternative with the highest sum of individual gains).
- If we call A^* the selected alternative, your experimental earnings are: your gain from A^* $-$ your bid for A^* + one-third of the bid sum for A^*.
- You will be informed about phase 1's results and earnings once the experiment is over.

Control questions and practice rounds

Before starting you will have to answer some control questions which will ensure your understanding of these rules. Once everybody has answered all questions correctly, four practice rounds will help you familiarize yourself with the experiment. In these rounds you will not be matched with other people in this room, but the computer will randomly select the others' bids. The result of these rounds will not be relevant to your final payoff.

On the screen you will have an earnings calculator that you can use to simulate your earnings in different scenarios. You can start the calculator by pressing the corresponding button on your screen. If you do so, a window will appear on your screen. Into this window you must enter your six bids, and the six bids that you expect from each of your group members. Given these figures, if you press the apposite button, you will know which alternative is selected and your corresponding earnings.

Please remain quietly seated during the whole experiment. If you have any questions, please raise your hand now. Please click "ok" on your computer screen when you have finished reading the instructions of this phase of the experiment.

Detailed Information on Phase 2 (Part 1)

In this phase you will face a situation similar to that encountered in the first phase. As before:

- you will be matched with two other participants;
- the three group members will be identified by a letter (X, Y, or Z); your identifying letter is the same as in phase 1 (i.e., you will be X, Y, or Z, if you previously were, respectively, X, Y, or Z);
- you and the other two members of your group will face six alternatives;
- your individual gains from the six alternatives are those shown in phase 1's Table 1;
- you will submit a bid for each alternative (your bids can be any integer number between 0 and 1000 ECUs);
- the six bids have to be submitted just once;
- the alternative with the highest bid sum will be selected;
- if two or more alternatives tie for first place, the tie is broken by selecting the alternative with the highest sum of individual gains.

But now

▷ you will be placed in a **new** group of three persons (i.e., the two participants you will be matched with are different ones);

▷ your experimental earnings will include an **extra constant term.** Specifically, if A^* is the alternative selected by your group, your earnings are:

$$\boxed{\text{your gain from } A^* \quad - \quad \text{your bid for } A^* \quad + \quad 1/3 \text{ of the bid sum for } A^* \quad + \quad \mathbf{150}}$$

That is, in this phase:

- you are paid your gain from the selected alternative A^*,
- you pay your bid for A^*,
- you receive one third of the sum of your group's bids for A^* **plus 150**.

Please note that you may once again suffer a loss if your bid for the selected alternative exceeds your gain from that alternative.

The following example should help you better understand the calculation of your earnings in this phase.

Example

Suppose that A_1 is the alternative selected by your group (this means that A_1 receives the highest bid sum in your group). Suppose further that the bids of each group member for A_1 are the following:

- X's bid = 233;
- Y's bid = 0;
- Z's bid = 550.

Therefore each group member's earnings are:

- X's Earnings: $533 - 233 + \frac{233+550}{3} + 150 = 200 + \frac{783}{3} + 150 = 611$ ECUs.
- Y's Earnings: $333 - 0 + \frac{233+550}{3} + 150 = 333 + \frac{783}{3} + 150 = 744$ ECUs.
- Z's Earnings: $600 - 550 + \frac{233+550}{3} + 150 = 50 + \frac{783}{3} + 150 = 461$ ECUs.

As with the previous phase:

- feedback on 1) your group members' bids, 2) which alternative is selected, and 3) your experimental earnings will be provided after the end of the session (i.e., after part 2).
- control questions and practice rounds will help you familiarize yourself with the rules of this phase of the experiment (the structure of the practice rounds remains the same: the computer determines randomly the other's decisions and the result are not relevant to your final payoff).

Please click "ok" if you have finished reading the instructions for the present phase and have no further questions.

Detailed Information on Phase 3 (Part 1)

The third phase of part 1 of the experiment resembles the previous two phases. Specifically:

- you will be matched with two other participants;
- the three group members will be identified by a letter (X, Y, or Z); your identifying letter is the same as in the previous two phases;
- you and the other two members of your group will face six alternatives;
- your individual gains from the six alternatives are those shown in phase 1's Table 1;
- you will submit a bid for each alternative (yours bids can be any integer number between 0 and 1000 ECUs);
- the six bids have to be submitted just once;
- the alternative with the highest bid sum will be selected;
- if two or more alternatives tie for first place, the tie is broken by selecting the alternative with the highest sum of individual gains.

But now:

▷ you will be placed in a **new** group of three persons (i.e., the other two members of your group are participants you have never before interacted with);

▷ the 150 ECUs will be **subtracted** (rather than added) to compute your experimental earnings. Specifically, if A^* is the alternative selected by your group, your earnings in this phase are:

$$\boxed{\text{your gain from } A^* \quad - \quad \text{your bid for } A^* \quad + \quad 1/3 \text{ of the bid sum for } A^* \quad - \quad \mathbf{150}}$$

That is, in this phase:

- you are paid your gain from the selected alternative A^*,
- you pay your bid for A^*,
- you receive one third of the sum of your group's bids for A^* **minus 150**.

Be aware that you may once again suffer a loss if your bid for the selected alternative exceeds your gain from that alternative.

The following example should help you better understand the calculation of your earning in this phase.

Example

Suppose that A_1 is the alternative selected by your group (this means that A_1 receives the highest bid sum in your group). Suppose further that the bids of each group member for A_1 are the following:

- X's bid $= 233$;
- Y's bid $= 0$;
- Z's bid $= 550$.

Therefore each group member's earnings are:

- X's Earnings: $533 - 233 + \frac{233+550}{3} - 150 = 200 + \frac{783}{3} - 150 = 311$ ECUs.
- Y's Earnings: $333 - 0 + \frac{233+550}{3} - 150 = 333 + \frac{783}{3} - 150 = 444$ ECUs.
- Z's Earnings: $600 - 550 + \frac{233+550}{3} - 150 = 50 + \frac{783}{3} - 150 = 161$ ECUs.

As before:

- feedback on this phase's bids and earnings will be provided after the end of the session (i.e., after part 2).
- control questions and practice rounds will help you familiarize yourself with the rules of this phase of the experiment (the structure of the practice rounds remains the same: the computer determines randomly the other's decisions and the result are not relevant to your final payoff).

Please click "ok" if you have finished reading the instructions for the present phase and have no further questions.

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