

## Depreciation, Deterioration and Obsolescence when there is Embodied or Disembodied Technical Change

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### Abstract

The paper considers how to measure capital in a model where technical progress is either embodied in new units of capital or it is “disembodied” and simply causes the price of capital services to fall. The disembodied case is considered in sections 2-4. Sections 2 and 3 set out standard vintage capital aggregation models when there is no embodied technical progress. Section 4 discusses disembodied obsolescence in more detail. Section 5 introduces new (more efficient) models of the capital good so that technical progress is embodied in the new models. Section 6 shows how the parameters in the Jorgenson model of capital services could be estimated by statistical agencies if their investment surveys covered sales and retirements of used assets as well as purchases of new assets. Section 7 concludes.

### Journal of Economic Literature Classification Numbers

C43, C81, D24, D92, E22, M4.

### Keywords

Aggregation of capital, embodiment of technical progress, depreciation, deterioration, obsolescence, index number theory.

### 1. Introduction

The chapter considers how to measure capital (both as stock and as a flow) in a model where technical progress is either *embodied* in new units of capital or it is *disembodied*

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and simply causes the price of capital services to fall in real terms over time. The disembodied case is considered first in sections 2-4.

Section 2 sets out the standard vintage capital aggregation model when there is no embodied technical progress<sup>2</sup> that has been developed by Jorgenson (1989), Hulten (1990), Diewert and Lawrence (2000) and Diewert (2005a). Section 3 specializes this model to a more restrictive model due to Jorgenson (1973). This more restrictive model assumes that the different vintages of the capital good provide differing amounts of capital services, but those services can be measured in a common unit and the services of the various vintages are perfect substitutes.<sup>3</sup> This Jorgensonian model will prove to be very useful when new models of the capital good that embody technical progress are introduced in section 5.

Section 4 draws on the theory developed in the previous sections and interprets *disembodied obsolescence* as a decline in the real price of capital services. This real decline could be caused by new technologies that render the existing capital services obsolete (e.g., motor vehicles replaced horses) or it could be caused by changes in tastes for goods and services produced by the capital services under consideration (e.g., the demand for cigarettes has declined as educational campaigns changed tastes and hence the demand for capital equipment for making cigarettes has also declined).<sup>4</sup>

Section 5 introduces new (more efficient) models of the capital good so that technical progress is *embodied* in the new models. The Jorgenson (1973) model proves to be very useful in forming capital aggregates in this case.

Section 6 shows how the parameters in the Jorgensonian model of capital services could be estimated by statistical agencies if they changed their investment surveys to cover sales and retirements of used assets as well as purchases of new assets.

Section 7 concludes.

## **2. Asset Prices, Rental Prices and Depreciation when there is no Embodied Technical Change**

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<sup>2</sup> In sections 2-4, there is only one capital good that we are considering and there are no new models being introduced to the marketplace and hence, there is no direct quality adjustment problem in these sections. However, indirectly, quality changes may be taking place in related markets and taste changes may be taking place as well, so that the anticipated price of the capital services under consideration may be changing in real terms. Our initial problem in sections 2-4 is to form capital stock aggregates and capital service aggregates where we aggregate over vintages of the single capital good. We also consider how to define depreciation in this framework. The problems involved in aggregating over many types of capital is a “straightforward” index number problem, which we do not consider in this paper. The problems involved in dealing with direct quality changes in the type of capital services under consideration are postponed until section 5.

<sup>3</sup> On the other hand, the model developed by Diewert and Lawrence (2000) and Diewert (2005a) is more flexible and allows for varying degrees of substitution between the services of the various vintages; i.e., these authors suggested the use of superlative indexes to aggregate over the different vintages of the asset.

<sup>4</sup> Economic progress and nonunitary income elasticities of demand can also lead to declines in prices; e.g., as households become richer, the demand for “basic” bicycles generally falls.

The relationship between the asset value of a capital stock component that is used in production and the value of its contribution to production in any period is a complex matter. In this section, we will lay out some assumptions that will allow us to quantify the relationship between the asset value and the service flow that the asset contributes over its life. Following Böhm-Bawerk (1891; 342), the value of an asset at the beginning of an accounting period is equal to the discounted stream of future rental payments that the asset is expected to yield. Thus the *stock value* of the asset is equal to the discounted future *service flows* that the asset is expected to yield in future periods. Let the price of a new capital input purchased at the beginning of period  $t$  be  $P_0^t$ . In an inflationary environment, it is necessary to distinguish between the (potentially) observable rental prices for the asset at different ages at the beginning of period  $t$  and future *expected* rental prices for assets of various ages. Thus let  $c_0^t$  be the rental price of a new asset at the beginning of period  $t$ , let  $c_1^t$  be the rental price of a one period old asset at the beginning of period  $t$ , let  $c_2^t$  be the rental price of a 2 period old asset at the beginning of period  $t$ , etc. Then *the fundamental equation* relating the *stock value of a new asset* at the beginning of period  $t$ ,  $P_0^t$ , to the sequence of *cross sectional rental prices for assets of age  $n$*  prevailing at the beginning of period  $t$ ,  $\{c_n^t : n = 0, 1, 2, \dots\}$  is<sup>5</sup>:

$$(1) P_0^t = c_0^t + [(1+i_1^t)/(1+r_1^t)] c_1^t + [(1+i_1^t)(1+i_2^t)/(1+r_1^t)(1+r_2^t)] c_2^t + \dots$$

In the above equation,  $1+i_1^t$  is the *rental price escalation factor* that is *expected* to apply to a one period old asset going from the beginning of period  $t$  to the end of period  $t$  (or equivalently, to the beginning of period  $t+1$ ),  $(1+i_1^t)(1+i_2^t)$  is *the rental price escalation factor* that is *expected* to apply to a 2 period old asset going from the beginning of period  $t$  to the beginning of period  $t+2$ , etc. Thus the  $i_n^t$  are *expected rates of price change for used assets of varying ages  $n$*  that are formed at the beginning of period  $t$ .<sup>6</sup> The term  $1+r_1^t$  is the discount factor that makes a dollar received at the beginning of period  $t$  equivalent to a dollar received at the beginning of period  $t+1$ , the term  $(1+r_1^t)(1+r_2^t)$  is the discount factor that makes a dollar received at the beginning of period  $t$  equivalent to a dollar received at the beginning of period  $t+2$ , etc. Thus the  $r_n^t$  are one period *nominal interest rates* that represent the *term structure of interest rates* at the beginning of period  $t$ .

We now generalize equation (1) to relate the *stock value of an  $n$  period old asset* at the beginning of period  $t$ ,  $P_n^t$ , to the sequence of *cross sectional vintage rental prices* prevailing at the beginning of period  $t$ ,  $\{c_n^t\}$ ; thus for  $n = 0, 1, 2, \dots$ , we assume:

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<sup>5</sup> The period  $t$  sequence of (cross sectional) rental prices by age  $\{c_n^t\}$  is called the *age-efficiency profile* of the asset.

<sup>6</sup> Note that the expected (nominal) rental price for the asset next period is  $(1+i_1^t)c_1^t$ , which we could define as a separate expected variable, suppressing the expected rental price escalation factor,  $(1+i_1^t)$ . A similar comment applies to the other expected future period rental prices. However, we have set up the notation in equation (1) and subsequent equations in anticipation of a simplification that we will make later, namely we will later assume that all of the future period rental price escalation factors  $i_n^t$  are equal to each other; see (3) below. This simplification will make the analysis of disembodied obsolescence much more transparent.

$$(2) P_n^t = c_n^t + [(1+i_1^t)/(1+r_1^t)] c_{n+1}^t + [(1+i_1^t)(1+i_2^t)/(1+r_1^t)(1+r_2^t)] c_{n+2}^t + \dots$$

Thus older assets discount fewer terms in the above sum; i.e., as  $n$  increases by one, we have one less term on the right hand side of (2). However, note that we are applying the same price escalation factors  $(1+i_1^t)$ ,  $(1+i_1^t)(1+i_2^t)$ , ..., to escalate the cross sectional rental prices prevailing at the beginning of period  $t$ ,  $c_1^t$ ,  $c_2^t, \dots$ , and to form estimates of future expected rental prices for each vintage of the capital stock that is in use at the beginning of period  $t$ .

The rental prices prevailing at the beginning of period  $t$  for assets of various ages,  $c_0^t$ ,  $c_1^t$ , ... are potentially observable. These cross section rental prices reflect the relative efficiency of the various vintages of the capital good that are still in use at the beginning of period  $t$ . It is assumed that these rentals are paid (explicitly or implicitly) by the users at the beginning of period  $t$ . Note that the sequence of asset stock prices for various ages at the beginning of period  $t$ ,  $P_0^t$ ,  $P_1^t$ , ... is not affected by general inflation provided that the general inflation affects the expected asset rates of price change  $i_n^t$  and the nominal interest rates  $r_n^t$  in a proportional manner. We will return to this point later.

The physical productivity characteristics of a unit of capital of each age are determined by the sequence of cross sectional rental prices. Thus a brand new asset is characterized by the vector of current rental prices for assets of various ages,  $c_0^t$ ,  $c_1^t$ ,  $c_2^t$ , ... , which are interpreted as “physical” contributions to output that the new asset is expected to yield during the current period  $t$  (this is  $c_0^t$ ), the next period (this is  $c_1^t$ ), and so on. An asset which is one period old at the start of period  $t$  is characterized by the vector  $c_1^t$ ,  $c_2^t$ , ..., etc.

At this point, we make some simplifying assumptions about the expected rates of rental price change for future periods  $i_n^t$  and the interest rates  $r_n^t$ . We assume that these anticipated specific price change escalation factors at the beginning of each period  $t$  are all equal; i.e., we assume:

$$(3) i_n^t = i^t; \quad n = 1, 2, \dots$$

We also assume that the term structure of (nominal) interest rates at the beginning of each period  $t$  is constant; i.e., we assume:

$$(4) r_n^t = r^t; \quad n = 1, 2, \dots$$

However, note that as the period  $t$  changes,  $r^t$  and  $i^t$  can change.

Using assumptions (3) and (4), we can rewrite the system of equations (2), which relate the sequence or profile of *stock prices* of age  $n$  at the beginning of period  $t$   $\{P_n^t\}$  to the sequence or profile of (cross sectional) *rental prices* for assets of age  $n$  at the beginning of period  $t$   $\{c_n^t\}$ , as follows; for  $n = 0, 1, 2, \dots$  :

$$(5) P_n^t = c_n^t + [(1+i^t)/(1+r^t)] c_{n+1}^t + [(1+i^t)/(1+r^t)]^2 c_{n+2}^t + [(1+i^t)/(1+r^t)]^3 c_{n+3}^t + \dots$$

On the left hand side of equations (5), we have the sequence of period  $t$  asset prices by age starting with the price of a new asset,  $P_0^t$  (when  $n = 0$ ), moving to the price of an asset that is one period old at the start of period  $t$ ,  $P_1^t$  (when  $n = 1$ ), then moving to the price of an asset that is 2 periods old at the start of period  $t$ ,  $P_2^t$ , and so on. On the right hand side of equations (5), the first term in each equation is a member of the sequence of rental prices by age of asset that prevails in the market (if such markets exist) at the beginning of period  $t$ . Thus  $c_0^t$  is the rent for a new asset,  $c_1^t$  is the rent for an asset that is one period old at the beginning of period  $t$ ,  $c_2^t$  is the rent for an asset that is 2 periods old, and so on. This sequence of current market rental prices for the assets of various vintages is then extrapolated out into the future using the anticipated price escalation rates  $(1+i^t)$ ,  $(1+i^t)^2$ ,  $(1+i^t)^3$ , etc. and then these future expected rentals are discounted back to the beginning of period  $t$  using the nominal discount factors  $(1+r^t)$ ,  $(1+r^t)^2$ ,  $(1+r^t)^3$ , etc. Note that given the period  $t$  expected asset inflation rate  $i^t$  and the period  $t$  nominal discount rate  $r^t$ , we can go from the (cross sectional) sequence of vintage rental prices  $\{c_n^t\}$  to the (cross sectional) sequence of vintage asset prices  $\{P_n^t\}$  using equations (5). Following Jorgenson (1989; 10), Hulten (1990; 128), Diewert and Lawrence (2000; 276) and Diewert (2005a; 483-485), we shall show below how this procedure can be reversed; i.e., we shall show how given the sequence of cross sectional asset prices, we can construct estimates for the sequence of cross sectional rental prices.

Note that equations (5) can be rewritten as follows:<sup>7</sup>

$$(6) P_n^t = c_n^t + [(1+i^t)/(1+r^t)] P_{n+1}^t; \quad n = 0, 1, 2, \dots$$

The first equation in (6) (when  $n = 0$ ) says that the value of a new asset at the start of period  $t$ ,  $P_0^t$ , is equal to the rental that the asset can earn in period  $t$ ,  $c_0^t$ ,<sup>8</sup> plus the expected asset value of the capital good at the end of period  $t$ ,  $(1+i^t) P_1^t$ , but this expected asset value must be divided by the discount factor,  $(1+r^t)$ , in order to convert this future value into an equivalent beginning of period  $t$  value.

Now it is straightforward to solve equations (6) for the sequence of period  $t$  cross sectional rental prices,  $\{c_n^t\}$ , in terms of the cross sectional asset prices,  $\{P_n^t\}$ :

$$(7) c_n^t = P_n^t - [(1+i^t)/(1+r^t)] P_{n+1}^t = (1+r^t)^{-1} [P_n^t (1+r^t) - (1+i^t) P_{n+1}^t]; \quad n = 0, 1, 2, \dots$$

Thus equations (5) allow us to go from the sequence of rental prices by age  $n$   $\{c_n^t\}$  to the sequence of asset prices by age  $n$   $\{P_n^t\}$  while equations (7) allow us to reverse the process.

<sup>7</sup> Christensen and Jorgenson (1969; 302) do this for the geometric depreciation model except that they assume (implicitly) that the rental is paid at the end of the period rather than the beginning. Variants of the system of equations (6) were derived by Christensen and Jorgenson (1973), Jorgenson (1989; 10), Hulten (1990; 128), Diewert and Lawrence (2000; 276) and Diewert (2005a; 482). Irving Fisher (1908; 32-33) also derived these equations in words.

<sup>8</sup> Note that we are implicitly assuming that the rental is paid to the owner at the beginning of period  $t$ .

Equations (7) can be derived from elementary economic considerations. Consider the first equation in (7). Think of a production unit as purchasing a unit of the new capital asset at the beginning of period  $t$  at a cost of  $P_0^t$  and then using the asset throughout period  $t$ . However, at the end of period  $t$ , the producer will have a depreciated asset that is expected to be worth  $(1+i^t) P_1^t$ . Since this offset to the initial cost of the asset will only be received at the end of period  $t$ , it must be divided by  $(1+r^t)$  to express the benefit in terms of beginning of period  $t$  dollars. Thus the expected net cost of *using* the new asset for period  $t$ <sup>9</sup> is  $P_0^t - [(1+i^t)/(1+r^t)] P_1^t$ .

The above equations enable us to convert assumptions about the pattern of cross sectional rental prices (or relative efficiencies of the different vintages of capital that are being used at any point of time) into assumptions about the pattern of asset prices by age at any point in time. It is convenient to develop additional sets of equations that enable us to relate the rental price and asset price profiles to depreciation profiles and we now do this.

Recall that  $P_n^t$  was defined to be the price of an asset that was  $n$  periods old at the beginning of period  $t$ . Generally, the decline in asset value as we go from one vintage to the next oldest (at a single point in time) is called *deterioration*. More precisely, we define the *cross sectional depreciation* or *deterioration*  $D_n^t$ <sup>10</sup> of an asset that is  $n$  periods old at the beginning of period  $t$  as

$$(8) D_n^t \equiv P_n^t - P_{n+1}^t ; \quad n = 0, 1, 2, \dots$$

Thus  $D_n^t$  is the value of an asset that is  $n$  periods old at the beginning of period  $t$ ,  $P_n^t$ , minus the value of an asset that is  $n+1$  periods old at the beginning of period  $t$ ,  $P_{n+1}^t$ .

Obviously, given the sequence of period  $t$  cross section asset prices  $\{P_n^t\}$ , we can use equations (8) to determine the period  $t$  sequence of declines in asset values by age,  $\{D_n^t\}$ . Conversely, given the period  $t$  cross section deterioration sequence or *profile*,  $\{D_n^t\}$ , we can determine the period  $t$  asset prices by age  $n$  by adding up amounts of deterioration:

$$(9) P_n^t = D_n^t + D_{n+1}^t + D_{n+2}^t + \dots ; \quad n = 0, 1, 2, \dots$$

Rather than working with first differences of asset prices by age, it is more convenient to reparameterize the pattern of cross sectional deterioration by defining the *period  $t$  deterioration rate*  $\delta_n^t$  for an asset that is  $n$  periods old at the start of period  $t$  as follows:

<sup>9</sup> This explains why the rental prices  $c_n^t$  are sometimes called *user costs*. This derivation of a user cost was used by Diewert (1974; 504), (1980; 472-473), (1992a; 194) and by Hulten (1996; 155).

<sup>10</sup> Hulten and Wykoff (1981a) (1981b) used the term *deterioration* for this form of depreciation. Hill (1999) called the decline in second hand asset values due to aging *cross sectional depreciation* and called the decline in an asset value from the beginning to the end of an accounting period *time series depreciation*. Triplett (1996; 98-99) also used the term *deterioration* and showed that it is equal to the concept of capital consumption in the national accounts under the assumption of no expected real asset price changes and no obsolescence. This deterioration definition of depreciation dates back to Hicks (1939; 176) at least and was used extensively by Edwards and Bell (1961; 175), Hulten and Wykoff (1981a) (1981b), Diewert (1974; 504) and Hulten (1990; 128) (1996; 155).

$$(10) \delta_n^t \equiv 1 - [P_{n+1}^t/P_n^t] = D_n^t / P_n^t ; \quad n = 0,1,2,\dots$$

In the above definitions, we require  $n$  to be such that  $P_n^t$  is positive.

Obviously, given the sequence of period  $t$  asset prices by age  $n$ ,  $\{P_n^t\}$ , we can use equations (10) to determine the period  $t$  sequence of *cross sectional depreciation or deterioration rates*,  $\{\delta_n^t\}$ . Conversely, given the cross sectional sequence of period  $t$  deterioration rates,  $\{\delta_n^t\}$ , as well as the price of a new asset in period  $t$ ,  $P_0^t$ , we can determine the period  $t$  asset prices by age as follows:

$$(11) P_n^t = (1 - \delta_0^t)(1 - \delta_1^t)\dots(1 - \delta_{n-1}^t) P_0^t ; \quad n = 1,2, \dots$$

The interpretation of equations (11) is straightforward. At the beginning of period  $t$ , a new capital good is worth  $P_0^t$ . An asset of the same type but which is one period older at the beginning of period  $t$  is less valuable by the amount of depreciation  $\delta_0^t P_0^t$  and hence is worth  $(1 - \delta_0^t) P_0^t$ , which is equal to  $P_1^t$ . An asset which is two periods old at the beginning of period  $t$  is less valuable than a one period old asset by the amount of depreciation  $\delta_1^t P_1^t$  and hence is worth  $P_2^t = (1 - \delta_1^t) P_1^t$  which is equal to  $(1 - \delta_1^t)(1 - \delta_0^t) P_0^t$  using the first equation in (11) and so on. Suppose  $L - 1$  is the first integer which is such that  $\delta_{L-1}^t$  is equal to one. Then  $P_n^t$  equals zero for all  $n \geq L$ ; i.e., at the end of  $L$  periods of use, the asset no longer has a positive rental value. If  $L = 1$ , then a new asset of this type delivers all of its services in the first period of use and the asset is in fact a nondurable asset.

In the following section, we will make a further simplification to the above algebra, which will prove to be helpful in dealing with new models of the capital good.

### 3. Aggregation over Assets and the Constant Relative Efficiency Hypothesis

Jorgenson (1973; 190) proposed a simplification of the model presented in the previous section; namely, he proposed that the relative efficiency of an asset of age  $n$  compared to a newly purchased asset was constant; i.e., the relative efficiencies of a capital input by age were constant over time. In terms of the notation used in the previous section, this simplifying assumption means that the sequence of period  $t$  rental prices by age  $n$ ,  $\{c_n^t\}$ , should satisfy the following equations in a competitive market situation:

$$(12) c_n^t = c_0^t \varphi_n ; \quad t = 0,1, \dots ; n = 0,1, \dots$$

where the  $\varphi_n$  are the *relative efficiencies of a capital input by age*. They are nonnegative constants with  $\varphi_0 = 1$ .<sup>11</sup> Thus given the period  $t$  rental price or user cost for a new unit of the capital input,  $c_0^t$ , each period  $t$  user cost for an older unit of capital is proportional to  $c_0^t$  with the constant relative efficiency factors  $\varphi_n$  giving us the factors of proportionality.

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<sup>11</sup> Jorgenson (1973; 190) also assumes that the  $\varphi_n$  do not increase as age  $n$  increases but this (very reasonable) assumption will not be required for our purposes.

A sufficient condition that will ensure that (12) holds is that the different vintages of the capital good are *perfect substitutes in production* once an adjustment is made for their relative efficiencies. We follow the example of Jorgenson (1973; 191) and make this assumption. Let  $K_n^t$  denote the number of units of an age  $n$  asset that the production unit under consideration has available for use at the beginning of period  $t$ . Then under the perfect substitutes assumption, the aggregate amount of capital services in constant efficiency units<sup>12</sup> that the production unit has available at the beginning of period  $t$  is

$$(13) K^t \equiv K_0^t + \varphi_1 K_1^t + \varphi_2 K_2^t + \dots$$

The aggregate value of capital services,  $V_{SER}^t$  used by the production unit in period  $t$  is the sum of the value of capital services over all ages in use during period  $t$ :

$$\begin{aligned} (14) V_{SER}^t &\equiv c_0^t K_0^t + c_1^t K_1^t + c_2^t K_2^t + \dots \\ &= c_0^t K_0^t + c_0^t \varphi_1 K_1^t + c_0^t \varphi_2 K_2^t + \dots && \text{using (12)} \\ &= c_0^t K^t && \text{using (13).} \end{aligned}$$

Thus under the Jorgenson assumptions, the aggregate value of capital services used by the production unit under consideration is equal to the user cost of a new unit of capital  $c_0^t$  times the quantity of the capital services aggregate  $K^t$  where  $K^t$  is the sum of the efficiency adjusted units of capital available to the production unit at the beginning of period  $t$ ; i.e., see definition (13).<sup>13</sup>

We now trace out the implications of assumptions (12) on the structure of asset prices by age (the  $P_n^t$ ) and on deterioration rates by age (the  $\delta_n^t$ ). By substituting (12) into (5), we find that the asset prices by age can be defined in terms of the user cost of a new unit of capital  $c_0^t$ , the relative efficiency factors  $\varphi_n$ , the anticipated rate of growth in rental prices  $i^t$  and the nominal interest rate  $r^t$  as follows: for  $n = 0, 1, 2, \dots$

$$(15) P_n^t = c_0^t \{ \varphi_n + [(1+i^t)/(1+r^t)] \varphi_{n+1} + [(1+i^t)/(1+r^t)]^2 \varphi_{n+2} + [(1+i^t)/(1+r^t)]^3 \varphi_{n+3} + \dots \}.$$

Once the period  $t$  sequence of asset prices by age has been determined, the sequence of period  $t$  deterioration rates can be determined by equations (10); i.e., we have

$$(16) \delta_n^t \equiv 1 - [P_{n+1}^t/P_n^t]; \quad n = 0, 1, 2, \dots$$

For some purposes, it is useful to replace the nominal period  $t$  interest rate  $r^t$  and the period  $t$  nominal expected asset rental price inflation rate  $i^t$  by corresponding real rates,  $r^{*t}$

<sup>12</sup> In national income accounting circles, this is known as the *productive capital stock*; e.g., see Schreyer, Diewert and Harrison (2005).

<sup>13</sup> Diewert and Lawrence (2000) show that it is not necessary to make the strong perfect substitutes assumption to form a capital services aggregate. They show that normal index number theory can be used to aggregate over vintages of capital and so in particular, it is not necessary to make the perfect substitutes assumption. However, in the present paper, when we introduce new models of a capital input that have varying efficiency factors, we will find that the Jorgensonian framework is able to deal with this situation whereas the Diewert Lawrence framework cannot readily do so.



and  $i^{*t}$  respectively. Let  $\rho^t$  be the rate of general inflation that is anticipated at the beginning of period  $t$ . Then the period  $t$  *real* interest rate  $r^{*t}$  can be defined in terms of the nominal interest rate  $r^t$  and the expected inflation rate  $\rho^t$  as follows:

$$(17) 1 + r^{*t} \equiv (1 + r^t)/(1 + \rho^t).$$

In a similar manner, the period  $t$  anticipated rate of *real* rental price change  $i^{*t}$  as follows:

$$(18) 1 + i^{*t} \equiv (1 + i^t)/(1 + \rho^t).$$

Upon substituting (17) and (18) into (15), we find that the asset prices by age can be defined in terms of the user cost of a new unit of capital  $c_0^t$ , the relative efficiency factors  $\varphi_n$ , the anticipated rate of growth in real rental prices  $i^{*t}$  and the real interest rate  $r^{*t}$  as follows:

$$(19) P_n^t = c_0^t \{ \varphi_n + [(1+i^{*t})/(1+r^{*t})] \varphi_{n+1} + [(1+i^{*t})/(1+r^{*t})]^2 \varphi_{n+2} \\ + [(1+i^{*t})/(1+r^{*t})]^3 \varphi_{n+3} + \dots \}; \quad n = 0, 1, 2, \dots$$

Once the period  $t$  sequence of asset prices by age has been determined by (19), the sequence of period  $t$  deterioration rates can be determined by equations (10).

Conversely, given the sequence of period  $t$  deterioration rates,  $\delta_n^t$ , and the stock price of a new asset at the beginning of period  $t$ ,  $P_0^t$ , we can determine the sequence of period  $t$  asset prices by age,  $P_n^t$  for  $n = 1, 2, \dots$ , by using equations (11). Then we can combine equations (7) and (12) in order to solve for the sequence of efficiency factors,  $\varphi_n$ , as follows:

$$(20) \varphi_n = \{ P_n^t - [(1+i^t)/(1+r^t)] P_{n+1}^t \} / c_0^t = \{ P_n^t - [(1+i^{*t})/(1+r^{*t})] P_{n+1}^t \} / c_0^t; \quad n = 1, 2, \dots$$

where  $c_0^t \equiv P_0^t - [(1+i^t)/(1+r^t)] P_1^t = P_0^t - [(1+i^{*t})/(1+r^{*t})] P_1^t$ . Looking at (19), it can be seen that if the anticipated real rental inflation rate  $i^{*t}$  and the real interest rate  $r^{*t}$  are constant over time, then asset prices by age will vary in fixed proportion over time, with the factor of proportionality being the rental price of a new unit of capital,  $c_0^t$ .<sup>14</sup> However, in general,  $i^{*t}$  and  $r^{*t}$  will vary over time. In this case, the aggregate stock value<sup>15</sup> at the beginning of period  $t$  for the production unit under consideration is:

$$(21) V_{STO}^t \equiv P_0^t K_0^t + P_1^t K_1^t + P_2^t K_2^t + \dots$$

<sup>14</sup> It is more likely that real interest rates and real rental price inflation rates be constant over time than nominal interest rates and nominal inflation rates, which explains why we have introduced (19) in addition to (15). Under the assumption that either  $(1+i^t)/(1+r^t)$  or  $(1+i^{*t})/(1+r^{*t})$  is constant over time, then asset prices by age will vary in fixed proportion over time and the deterioration rates defined by (10) will be constant over time.

<sup>15</sup> This is the *net capital stock* or *wealth stock* in national income accounting terminology; see Schreyer, Diewert and Harrison (2005). To decompose this value aggregate into price and quantity (or volume) components, use normal index number theory. For the special case of geometric depreciation, the volume measure for the net capital stock coincides with the volume measure for capital services; see (27) below.

where the asset prices  $P_n^t$  are defined by (15) or (19). In the general case where  $r^t$  or  $r^{*t}$  and  $i^t$  or  $i^{*t}$  vary over time, index number theory will have to be used in order to decompose the asset value aggregate defined by (21) into price and quantity components.

We conclude this section by looking at a special case of the above model.<sup>16</sup> Suppose that the relative efficiency factors satisfy the following restrictions:

$$(22) \varphi_n \equiv (1-\delta)^n ; 0 < \delta < 1 ; \quad n = 0, 1, 2, \dots$$

where the parameter  $\delta$  can be interpreted as a constant geometric deterioration rate. Under assumptions (22), we find, using (19), that period  $t$  asset prices by age have the following form:

$$(23) P_0^t = c_0^t [1 + [(1+i^{*t})(1-\delta)/(1+r^{*t})] + [(1+i^{*t})(1-\delta)/(1+r^{*t})]^2 + \dots \\ = c_0^t [1 + r^{*t}] / [1+r^{*t} - (1+i^{*t})(1-\delta)] ;^{17}$$

$$(24) P_n^t = (1-\delta)^n P_0^t ; \quad n = 1, 2, \dots$$

Using equations (23), (24) and (16), we find that the sequence of deterioration rates by age is independent of time and these rates are all equal to  $\delta$ ; i.e., we have

$$(25) \delta_n^t \equiv 1 - [P_{n+1}^t/P_n^t] = 1 - [1-\delta] = \delta ; \quad n = 0, 1, 2, \dots$$

It can be shown that for the general Jorgenson model, the capital services quantity aggregate coincides with the capital stock quantity aggregate. We verify this for the geometric model, (22). To verify this, substitute (22) into (13), which defined the period  $t$  capital services aggregate  $K^t$ :

$$(26) K^t \equiv K_0^t + \varphi_1 K_1^t + \varphi_2 K_2^t + \dots \\ = K_0^t + (1-\delta) K_1^t + (1-\delta)^2 K_2^t + \dots \quad \text{using (22).}$$

Now substitute (24) into (21) in order to obtain the following decomposition for the value of the capital stock aggregate:

$$(27) V_{STO}^t \equiv P_0^t K_0^t + P_1^t K_1^t + P_2^t K_2^t + \dots \\ = P_0^t [K_0^t + (1-\delta) K_1^t + (1-\delta)^2 K_2^t + \dots] \quad \text{using (24)} \\ = P_0^t K^t \quad \text{using (26).}$$

Thus the capital stock value aggregate decomposes into the price of a new unit of the capital asset,  $P_0^t$ , times the quantity aggregate for capital services defined by the second

<sup>16</sup> This is the geometric depreciation model that is favored by Jorgenson and his coworkers; e.g., see Jorgenson and Griliches (1967), Christensen and Jorgenson (1969) (1973) and Jorgenson (1973) (1989) (1996).

<sup>17</sup> In order to derive this second equality, we require that the magnitude of  $(1+i^{*t})(1-\delta)/(1+r^{*t})$  be less than one.

line of (26),  $K^t$ . Comparing (27) with (14) shows that the quantity aggregate for capital services equals the quantity aggregate for the capital stock when we have constant geometric depreciation rates.

Now that we have covered the algebra involved in aggregating over vintages of the same capital good when there is no technical progress, we can turn our attention to the problems involved in defining depreciation when there is embodied or disembodied technical progress. In the following section, we look at the disembodied case and then in the subsequent section, we study the embodied case.

#### 4. The Revaluation Term and Disembodied Obsolescence

The material in the previous sections did not deal explicitly with the interaction of obsolescence and deterioration to form an overall depreciation charge. The recognition that obsolescence charges on the use of a capital input are similar to deterioration or normal wear and tear depreciation charges dates back at least 170 years:

“Machinery for producing any commodity in great demand, seldom actually wears out; new improvements, by which the same operations can be executed either more quickly or better, generally superceding it long before that period arrives: indeed, to make such an improved machine profitable, it is usually reckoned that in five years it ought to have paid itself, and in ten to be superceded by a better.” Charles Babbage (1835; 285).

“The possibility of New Inventions, processes, or machines coming into use, which may supercede or render an existing plant Obsolete, is a contingency that presses on most manufacturing trades, principally those which have long established, but sometimes also in new concerns where old methods have been adopted or imitated just as they were being superceded elsewhere.” Ewing Matheson (1910; 38).

“A reserve beyond the ordinary depreciation above described may then become necessary, because the original plant, when once superceded by such inventions, may prove unsaleable as second-hand plant, except in so far as it may have a piecemeal or scrap value. ... This risk sometimes arises, not from improvements in the machinery, but from alterations in the kind of product, rendering new machines necessary to suit new patterns or types. Contingencies such as these should encourage an ample reduction of nominal value in the early years of working, so as to bring down the book value of the plant to a point which will allow even of dismantling without serious loss. In such trades, profits should be large enough to allow for a liberal and rapid writing off of capital value, which is in effect the establishment of a reserve-fund as distinct from depreciation for wear and tear.” Ewing Matheson (1910; 39-40).

“Even though a machine is used fairly and uniformly as contemplated when the rate of depreciation was fixed there is another influence that may shorten its period of usefulness in an unexpected way. The progress of the technical art in which it is employed may develop more efficient machines for doing the same work, so that it becomes advisable to scrap it long before it is worn out. The machine becomes obsolete and the loss of value from this cause is called ‘obsolescence’. Again, unless the machine is of a very generalized type, such as an engineer’s lathe, another type of misfortune may overtake it. If it is a machine that can only be used for certain definite kinds of work or some special article, as for example many of the machines used in automobile and bicycle manufacture, it may happen that changes in demand, or in style, make the manufacture of that special article no longer profitable. In this case, unless the machine can be transformed for another use, it is a dead loss.” A. Hamilton Church (1917; 192-193).

“Allowance must be made for such part of capital depletion as may fairly be called ‘normal’; and the practical test of normality is that the depletion is sufficiently regular to be foreseen, if not in detail, at least

in the large. This test brings under the head of depreciation all ordinary forms of wear and tear, whether due to the actual working of machines or to mere passage of time— rust, rodents and so on— and all ordinary obsolescence, whether due to technical advance or to changes of taste. It brings in too the consequences of all *ordinary* accidents, such as shipwreck and fire, in short of all accidents against which it is customary to insure. But it leaves out capital depletion that springs from the act of God or the King's enemies, or from such a miracle as a decision tomorrow to forbid the manufacture of whisky or beer. These sorts of capital depletion constitute, not depreciation to be made good before current net income is reckoned, but capital losses that are irrelevant to current net income.” A.C. Pigou (1935; 240-241).

Note that Matheson, Church and Pigou all noted that obsolescence could arise not only from new inventions but also from shifts in demand. Thus a downward shift in demand for some product will generally lead to a downward shift in the demand for capital services in the industry that produces the declining demand product. If some of these types of capital equipment or structures have few alternative uses, the downward shift in final demand will lead to a downward shift in the *price* of these specialized capital services. If the downward shift in future demand is foreseen, then under the above conditions, we will have an expected real decline in the price of future period capital services; i.e., in the context of our Jorgensonian model, the anticipated real capital services inflation rate,  $i^{*t}$ , will be negative. The case of a negative anticipated real capital services inflation rate can be interpreted as an obsolescence charge on income that is analogous to wear and tear depreciation or deterioration. This type of obsolescence charge could be termed a *disembodied obsolescence charge* since it can occur even if no new, improved models of the capital input appear on the market.<sup>18</sup> In the following section, we will consider the more complex problems associated with the introduction of a new, improved model of the capital input into the marketplace.

In order to quantify the effects of disembodied obsolescence, substitute (17) and (18) (which defined the real interest rate  $r^{*t}$  and the real anticipated capital services inflation rate  $i^{*t}$  in terms of the corresponding nominal rates  $r^t$  and  $i^t$ ) into the first equation in (7), which defined the period  $t$  user cost  $c_0^t$  for a newly purchased unit of capital at the beginning of period  $t$ :

$$\begin{aligned}
 (28) \quad c_0^t &= P_0^t - [(1+i^t)/(1+r^t)] P_1^t \\
 &= P_0^t - [(1+i^{*t})/(1+r^{*t})] P_1^t && \text{using (17) and (18)} \\
 &= P_0^t - [(1+i^{*t})/(1+r^{*t})](1 - \delta_0^t) P_0^t && \text{using the first equation in (11)} \\
 &= (1+r^{*t})^{-1} [r^{*t} - i^{*t} + (1+i^{*t})\delta_0^t] P_0^t && \text{rearranging terms} \\
 &= (1+r^{*t})^{-1} [r^{*t} + \delta_0^t - i^{*t}(1-\delta_0^t)] P_0^t.
 \end{aligned}$$

Thus the period  $t$  user cost of capital for a newly purchased unit of capital,  $c_0^t$ , decomposes into the sum of three terms.<sup>19</sup> Neglecting the multiplicative factor

<sup>18</sup> Of course, the anticipated introduction of a new improved model that is cheaper and highly substitutable with the older models will also lead to a negative anticipated real capital services inflation rate for the older models. For example, everyone anticipates that a new computer will be introduced next period at a much lower price (in constant quality units) than the competing model in this period. This “fact” must be taken into account when calculating the rental rate for an old computer for this period.

<sup>19</sup> The analysis is similar when end of period user costs (to be introduced shortly) are used instead of our present beginning of period user costs. The end of period counterpart to the beginning of the period user cost defined by (28) is  $(1+r^t)c_0^t = (1+r^{*t})(1+\rho^t)c_0^t = (1+\rho^t) [r^{*t} + \delta_0^t - i^{*t}(1-\delta_0^t)] P_0^t$ .

$(1+r^{*t})^{-1}P_0^t$  for each of these terms, the first term is the real interest rate  $r^{*t}$ , which obviously corresponds to a real interest charge for the use of the capital in period  $t$ . The second term is  $\delta_0^t$ , which corresponds to a wear and tear depreciation or deterioration charge for the use of the capital input during period  $t$ . The final term,  $-i^{*t}(1-\delta_0^t)$ , is the negative of the period  $t$  anticipated inflation rate  $i^{*t}$  for the type of capital service under consideration times one minus the period  $t$  cross sectional depreciation rate for a new unit of capital  $\delta_0^t$ . If  $i^{*t}$  is negative, then this last term can be interpreted as an *obsolescence charge* and if we are attempting to construct a period  $t$  estimate of the net income of the production unit under consideration, this (disembodied) obsolescence charge should be subtracted from gross income along with wear and tear depreciation in order to obtain an estimate of (ex ante) net income for the production unit.<sup>20</sup>

If the anticipated capital services real inflation rate  $i^{*t}$  is negative, it seems reasonable to us to treat the (positive) revaluation term  $-(1+r^{*t})^{-1} i^{*t}(1-\delta_0^t)P_0^t$  times the quantity of newly purchased capital  $K_0^t$  as charge against gross income when forming a net income measure for the production unit in period  $t$ . But suppose  $i^{*t}$  is positive? Should we then add the term  $(1+r^{*t})^{-1} i^{*t}(1-\delta_0^t)P_0^t K_0^t$  to the gross operating income of the production unit when calculating a net income aggregate for period  $t$ ? We argue that the answer to this question is yes<sup>21</sup> but we concede that reasonable economists and national income accountants could have differing opinions on the answer to this question.

At this point, it is useful to make the role of expectations more explicit and to also distinguish between beginning and end of period user costs. Thus in the first line of (28), we assume that the price of a new asset,  $P_0^t$ , can be observed at the beginning of period  $t$  and the relevant period  $t$  opportunity cost of capital  $r^t$  can also be observed. However, the anticipated end of period  $t$  price of the used asset, which is defined as  $(1+i^t)P_1^t$  in equation (28), cannot be observed at the beginning of period  $t$ : we can only form an expectation for this price. We now denote this expected price by  $P_1^{t+1}(t)$ , where the notation  $(t)$  means that this expectation for the price of a 1 period old asset at the beginning of period  $t+1$  is formed at the beginning of period  $t$ .<sup>22</sup> Using this new notation, the *expected user cost of a new unit of capital purchased at the beginning of period  $t$* ,  $c_0^t(t)$ , is defined as follows:

$$(29) \ c_0^t(t) \equiv P_0^t - (1+r^t)^{-1} P_1^{t+1}(t) .$$

Note that we have changed  $c_0^t$  to  $c_0^t(t)$  to indicate that this user cost is an anticipated one. However, once the end of period  $t$  occurs, we can observe (in principle) the price of a one

<sup>20</sup> In the case where  $i^{*t}$  is negative, Diewert (2005a; 501) identified the sum of the deterioration rate and the obsolescence charge,  $\delta_0^t - i^{*t}(1-\delta_0^t)$ , as the *real time series depreciation rate*. The general concept of real time series depreciation is due to Hill (2000; 6) and Hill and Hill (2003; 617). The above material follows Diewert's (2005a; 494-502) algebraic implementation of the concept.

<sup>21</sup> This means that anticipated real capital gains and losses (which we have interpreted as a disembodied obsolescence charge) would be treated in a symmetric manner when constructing estimates of net income for the production unit.

<sup>22</sup> Thus  $P_n^{t+s}(t)$  denotes the expectation of the price of an asset that is  $n$  periods old at the beginning of period  $t+s$ , where the expectation is formed at the beginning of period  $t$ .

period old asset at the beginning of period  $t+1$ , which we denote by  $P_1^{t+1}$ . Moreover, once the end of period  $t$  occurs, we can define the *ex post period  $t$  user cost for a new unit of capital purchased at the beginning of period  $t$*  as

$$(30) c_0^t \equiv P_0^t - (1+r^t)^{-1} P_1^{t+1} .$$

The user cost  $c_0^t(t)$  defined by (29) above is an anticipated cost for using one unit of a newly purchased capital good at the beginning of period  $t$  and this cost is charged at the beginning of period  $t$  and hence is termed a *beginning of the period anticipated user cost* by Diewert (2005a; 485). However, for accounting purposes, it is preferable to work with the corresponding *end of the period anticipated user cost*,<sup>23</sup>  $C_0^t(t)$ , defined as  $1+r^t$  times the corresponding beginning of the period anticipated user cost  $c_0^t(t)$ ; i.e., we have

$$(31) C_0^t(t) \equiv (1+r^t)c_0^t(t) \\ = (1+r^t)[P_0^t - (1+r^t)^{-1} P_1^{t+1}(t)] \\ = r^t P_0^t + P_0^t - P_1^{t+1}(t) .$$

The corresponding *end of the period ex post user cost*,  $C_0^t$ , is defined as  $1+r^t$  times the corresponding beginning of the period ex post user cost  $c_0^t$  defined by (30) i.e., we have

$$(32) C_0^t \equiv (1+r^t)c_0^t \\ = r^t P_0^t + P_0^t - P_1^{t+1} .$$

The first term on the last lines of (31) and (32),  $r^t P_0^t$ , is easy to interpret: it is simply the actual or imputed nominal interest payments that must be made to the owners of the asset as a payment for tying up financial capital during period  $t$ . The second term in (31),  $P_0^t - P_1^{t+1}(t)$ , is the *anticipated total deterioration and revaluation charge* and the second term in (32),  $P_0^t - P_1^{t+1}$ , is the *ex post deterioration and revaluation charge* for using the asset during period  $t$ .

Note that two factors are at work when we evaluate the ex post difference,  $P_0^t - P_1^{t+1}$ :

- The asset is aging one period (the change from 0 to 1) and
- The asset is priced at different price levels (the change from  $t$  to  $t+1$ ).

It is useful to separate out these two effects: the first effect is a *deterioration effect* and the second effect is a *revaluation effect*. We will define these two effects more carefully below.

We first define two *ex ante deterioration concepts* (or cross sectional depreciation concepts) for a newly purchased asset at the beginning of period  $t$ . We have two natural choices:  $D_0^t(t)$  uses cross sectional asset prices at the beginning of period  $t$  and assumes that this pattern of price decline will characterize the cross sectional depreciation of the

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<sup>23</sup> See Diewert (2005a; 485) who stressed the distinction between the beginning and end of period user cost concepts.

newly purchased asset or  $D_0^{t+1}(t)$  uses expected cross sectional asset prices at the end of period  $t$ :

$$(33) \quad D_0^t(t) \equiv P_0^t - P_1^t ;$$

$$(34) \quad D_0^{t+1}(t) \equiv P_0^{t+1}(t) - P_1^{t+1}(t) .$$

At the end of period  $t$ , we will know more and we can define the observable *ex post counterparts* to the *ex ante* formulae (33) and (34) as follows:

$$(35) \quad D_0^t \equiv P_0^t - P_1^t ;$$

$$(36) \quad D_0^{t+1} \equiv P_0^{t+1} - P_1^{t+1} .$$

Note that definitions (33) and (35) coincide; i.e., since we can observe (in principle) the prices of a new asset and a one period old asset at the beginning of period  $t$ ,  $P_0^t$  and  $P_1^t$ , these observable values coincide with their expectations at the beginning of period  $t$ . In the above definitions, time  $t$  is held constant and we calculate the expected or actual decline in asset price due to the effects of aging over one period. This is what Hill (1999) and Diewert (2005a; 487) called *cross sectional depreciation* and what others, including Jorgenson (1973) (1996), Hulten and Wykoff (1981a) (1981b) (1996) and Triplett (1996), called *deterioration*.

Now we define *ex ante* and *ex post* revaluation or capital gains terms for a newly purchased capital asset at the beginning of period  $t$ . The definition of the *ex ante capital gain term*,  $G_0^t(t)$ , is straightforward: we simply take the difference between the end of period  $t$  expected price of a new asset and the beginning of period  $t$  (observable) price for a new asset. The definition of the *ex ante capital gain term*,  $G_1^t(t)$ , is similar: we take the difference between the end of period  $t$  expected price of a one period old asset and the beginning of period  $t$  (observable) price for a one period old asset:

$$(37) \quad G_0^t(t) \equiv P_0^{t+1}(t) - P_0^t ;$$

$$(38) \quad G_1^t(t) \equiv P_1^{t+1}(t) - P_1^t ;$$

The *ex post counterparts* to (37) and (38) are defined as follows:

$$(39) \quad G_0^t \equiv P_0^{t+1} - P_0^t ;$$

$$(40) \quad G_1^t \equiv P_1^{t+1} - P_1^t ;$$

Note that (39) defines the *ex post* capital gain over the duration of period  $t$  on a new unit of capital purchased at the beginning of period  $t$  whereas (40) defines the *ex post* capital gain on a second hand asset that is one period old at the beginning of period  $t$ .

Recall the end of period *ex ante* user cost formula defined by (31) above,  $C_0^t(t) = r^t P_0^t + P_0^t - P_1^{t+1}(t)$ . Using the above definitions (33) and (34) of *ex ante* depreciation and (37) and (38) of *ex ante* capital gains, it can be seen that we have the following *two exact decompositions for the expected change in the value of a newly purchased asset due to use and revaluation over period t*:

$$(41) P_0^t - P_1^{t+1}(t) = D_0^t(t) - G_1^t(t) = D_0^t - G_1^t(t);$$

$$(42) P_0^t - P_1^{t+1}(t) = D_0^{t+1}(t) - G_0^t(t);$$

An interpretation of the first decomposition given by (41) is that we undertake the following sequence of transactions:<sup>24</sup>

- Buy a unit of the new asset at the beginning of period t;
- Immediately depreciate it using the beginning of period t depreciation schedule  $D_0^t(t)$ , which is equal to  $D_0^t \equiv P_0^t - P_1^t$ ;
- Use the depreciated asset for the duration of period t and finally
- Subtract the anticipated capital gain on the depreciated unit of capital at the end of period t; i.e., subtract  $G_1^t(t) \equiv P_1^{t+1}(t) - P_1^t$  from the above amount of depreciation.

An interpretation of the second decomposition given by (42) is that we undertake the following sequence of transactions:<sup>25</sup>

- Buy a unit of the new asset at the beginning of period t;
- Immediately calculate the expected capital gain on holding a unit of the new asset over period t,  $G_0^t(t) \equiv P_0^{t+1}(t) - P_0^t$ , and treat this change as an offset to the depreciation that will be charged in the next step;
- Depreciate the new asset using the anticipated end of period t or beginning of period t+1 depreciation schedule  $D_0^{t+1}(t) \equiv P_0^{t+1}(t) - P_1^{t+1}(t)$ .

What are the relative merits of formula (41) versus (42)? Using formula (41), we calculate depreciation using beginning of period t prices of used assets and so there is no problem in forming expectations about the pattern of end of period t prices, which is an initial advantage of this formula. A disadvantage of this formula is that we calculate the expected capital gain term not for a new unit of capital but for a one period old unit of capital purchased at the beginning of period t. This form of the capital gains term will be unfamiliar to users. Formula (42) has the advantage of having a “traditional” Jorgensonian capital gains or revaluation term (that applies to a newly purchased unit of capital) but it has the disadvantage of having to calculate deterioration using expected end of period used asset prices and thus a revaluation term has crept into the deterioration term.

In addition to the exact decompositions (41) and (42), we could take any average of these decompositions.<sup>26</sup> The most convenient symmetric average in the present context is the

<sup>24</sup> Note the correspondence of this sequence of transactions with the following beginning of the period user cost formula taken from (28):  $c_0^t(t) = (1+r^{*t})^{-1}[r^{*t} + \delta_0^t - i^{*t}(1-\delta_0^t)]P_0^t$ .

<sup>25</sup> Note the correspondence of this sequence of transactions to the following form of the beginning of the period user cost formula (28):  $c_0^t(t) = (1+r^{*t})^{-1}[r^{*t} - i^{*t} + (1+i^{*t})\delta_0^t]P_0^t$ .

<sup>26</sup> Balk and Bergen (2006) were the first to suggest taking the arithmetic average of the two polar decompositions; see their equation (5c). Ahmad, Aspden and Schreyer (2004) suggested the less familiar decomposition (41).



arithmetic average and so we have our *third decomposition of the ex ante change in value of the asset due to use and revaluation*:

$$(43) P_0^t - P_1^{t+1}(t) = (1/2)[D_0^t(t) + D_0^{t+1}(t)] - (1/2)[G_1^t(t) + G_0^t(t)] .$$

An advantage of the decomposition (43) over (41) and (42) is that national income accountants will like the fact that cross sectional depreciation is calculated at the average of the beginning and (anticipated) end of period prices.<sup>27</sup>

The reader can readily repeat the above analysis for alternative decompositions of the ex post difference,  $P_0^t - P_1^{t+1}$ , that appears in the end of the period ex post user cost,  $C_0^t$ , that was defined by (32), which was equal to  $r^t P_0^t + P_0^t - P_1^{t+1}$ . Using the above definitions (35) and (36) of ex post depreciation and (39) and (40) of ex post capital gains, it can be seen that we have the *following two exact decompositions for the ex post change in the value of a newly purchased asset due to use and revaluation over period t*:

$$(44) P_0^t - P_1^{t+1} = D_0^t - G_1^t .^{28}$$

$$(45) P_0^t - P_1^{t+1} = D_0^{t+1} - G_0^t .$$

We note that the decomposition given by (45) corresponds to the ex post user cost formula used by Jorgenson and his coworkers; i.e., using (45), one can obtain the following exact ex post end of period user cost formula that is favored by Jorgenson and his coworkers:

$$(46) C_0^t = r^t P_0^t + P_0^t - P_1^{t+1} = r^t P_0^t + D_0^{t+1} - G_0^t .$$

Unfortunately, the ex post revaluation term  $G_0^t$  can be quite large and very variable and this can cause the ex post user cost  $C_0^t$  to become negative. If we want our user cost to approximate a market rental rate for the asset, then a negative user cost is not plausible. Hence for many purposes, an anticipated user cost will be more suitable, since the anticipated capital gains term will be smoother than the actual ex post term. Our preferred anticipated user cost concept is  $C_0^t(t)$  equal to nominal interest payments  $r^t P_0^t$  plus the difference  $P_0^t - P_1^{t+1}(t)$ , where the difference is defined by (41) or (42).<sup>29</sup>

The important point to notice in all of this is that anticipated revaluation terms enter in a natural way into user cost formulae. In high or moderate inflation countries, these revaluation terms cannot be neglected. Even in low inflation countries, if the real price of the asset is expected to change, then expected revaluation terms can be important. In

<sup>27</sup> National income accountants prefer to calculate wear and tear depreciation at the average prices of the period; see Balk and Bergen (2006) on this point.

<sup>28</sup> This decomposition was first derived by Ahmad, Aspden and Schreyer (2004; 5); see their equation (3).

<sup>29</sup> If formula (41) is used in the ex ante user cost formula, then the ex post user cost defined by (32),  $C_0^t$ , is equal to the ex ante user cost,  $C_0^t(t)$ , plus  $G_1^t(t) - G_1^t$ , which is anticipated capital gains for a one period old asset over period t less the corresponding actual gains. Following System of National Accounts conventions, this last term should go into the revaluation accounts in order to reconcile the anticipated user cost with the actual ex post user cost.

particular, we have shown how a negative expected real asset price change can be interpreted as an obsolescence charge that should be added to wear and tear deterioration in order to obtain an overall measure of depreciation.

As the reader may have noticed, we feel that anticipated revaluation terms belong in the user cost formula and hence in the System of National Accounts when a user cost of capital concept is introduced into the accounts. However, national income accountants, following the example of Pigou, have an aversion to introducing any type of revaluation term into the production accounts. To conclude this section, we review some of the early accounting literature on the topic of capital gains.

Pigou and Clark gave the national income accountant's case against including any kind of revaluation term in estimates of net income as follows:

“The concrete content of the dividend is, indeed, unambiguous— the inventory of things made and (double counting being eliminated) and services rendered, *minus*, as a negative element, the inventory of things worn out during the year. But how are we to *value* this negative element? For example, if a machine originally costing £1000 wears out and, owing to a rise in the general price level, can only be replaced at a cost of £1500, is £1000 or £1500 the proper allowance? Nor is this the only, or, indeed, the principle difficulty. For depreciation is measured not merely by the physical process of wearing out, and capital is not therefore maintained intact when provision has been made to replace what is thus worn out. Machinery that has become obsolete because of the development of improved forms is not really left intact, however excellent its physical condition; and the same thing is true of machinery for whose products popular taste has declined. If, however, in deference to these considerations, we decide to make an allowance for obsolescence, this concession implies that the value, and not the physical efficiency, of instrumental goods [i.e., durable capital inputs] is the object to be maintained intact. But, it is then argued, the value of instrumental goods, being the present value of the services which they are expected to render in the future, necessarily varies with variations in the rate of interest. Is it really a rational procedure to evaluate the national dividend by a method which makes its value in relation to that of the aggregated net product of the country's industry depend on an incident of that kind? If that method is adopted, and a great war, by raising the rate of interest, depreciates greatly the value of existing capital, we shall probably be compelled to put, for the value of the national dividend in the first year of that war, a very large *negative* figure. This absurdity must be avoided at all costs, and we are therefore compelled, when we are engaged in evaluating the national dividend, to leave out of account any change in the value of the country's capital equipment that may have been brought about by broad general causes. This decision is arbitrary and unsatisfactory, but it is one which it is impossible to avoid. During the period of the war, a similar difficulty was created by the general rise, for many businesses, in the value of the normal and necessary holding of materials and stocks, which was associated with the general rise of prices. On our principles, this increase of value ought not to be reckoned as an addition to the income of the firms affected, or, consequently, to the value of the national dividend.” A.C. Pigou (1924; 39-41).

“The appreciation in value of capital assets and land must not be treated as an element in national income. Depreciation due to physical wear and tear and obsolescence must be treated as a charge against current income, but not the depreciation of the money value of an asset which has remained physically unchanged. Appreciation and depreciation of capital were included in the American statistics of national income prior to 1929, but now virtually the same convention has been adopted in all countries.” Colin Clark (1940; 31).

Thus Pigou argued that depreciation should be measured relative to a concept that maintained capital intact from a physical point of view and hence only wear and tear depreciation should be deducted from gross product when forming a net income concept that was based on a physical maintenance of capital concept. He later elaborated on his position as follows:

“I accept too the view that, if maintaining capital intact has to be defined in such a way that capital need not be maintained intact even though every item in its physical inventory is unaltered, the concept is worthless. But the inference I draw is, not that we should abandon the concept; rather that we should try to define it in such a way that, when the physical inventory of goods in the capital stock is unaltered, capital *is* maintained intact; more generally, in such a way that, not indeed the quantity of capital—which, with heterogeneous items, can only a conventionalised number—is independent of the equilibrating process, but changes in its quantity are independent of changes in that process.” A.C. Pigou (1941; 273).

Pigou (1941; 274) went on to suggest that the Paasche quantity index for capital could be used to determine whether capital was maintained intact between two points in time; i.e., the price weights of the second point in time should be used to value the two capital stocks. Hence if the two capital stocks were unchanged in each component, the Paasche quantity index would be equal to unity, correctly indicating that there was no physical change in the capital stock between the two points in time. However, Hayek responded, correctly, that Pigou’s concept of maintaining capital intact would neglect *foreseen obsolescence*:

“Professor Pigou’s answer to the question of what is meant by ‘maintaining capital intact’ consists in effect of the suggestion that for this purpose we should disregard obsolescence and require merely that such losses of value of the existing stock of capital goods be made good as are due to physical wear and tear. ... If Professor Pigou’s criterion is to be of any help, it would have to mean that we have to disregard *all* obsolescence, whether it is due to foreseen or foreseeable causes, or whether it is brought about by entirely unpredictable causes, such as the ‘acts of God or the King’s enemy’, which alone he wanted to exclude in an earlier discussion of this problem.” F.A. v. Hayek (1941; 276).

Hayek went on to give a clear example of where Pigou’s point of view would lead to a mismeasurement of depreciation and income:

“Assume three entrepreneurs, X, Y, and Z, to invest at the same time in equipment of different kinds but of the same cost and the same potential physical duration, say ten years. X expects to be able to use his machine continuously throughout the period of its physical ‘life’. Y, who produces some fashion article, knows that at the end of one year his machine will have no more than its scrap value. Z undertakes a very risky venture in which the changes of employing the machine continuously so long as it lasts and having to scrap it almost as soon as it starts to produce are about even. According to Professor Pigou the three entrepreneurs will have to order their investments in such a way that during the first year they can expect to earn the same gross receipts: since the wear and tear of their respective machines during the first years will be the same, the amount they will have to put aside during the first year to ‘maintain their capital intact’ will also be the same, and this procedure will therefore lead to their earning during that year the same ‘net’ income from the same amount of capital. Yet it is clear that the foreseen result of such dispositions would be that at the end of the year X would still possess the original capital, Y one tenth of it, while Z would have an even chance of either having lost it all or just having preserved it. ... To treat all receipts except what is required to make good physical wear and tear as net income for income tax purposes would evidently discriminate heavily against industries where the rate of obsolescence is high and reduce investment in these industries below what is desirable.” F.A. v. Hayek (1941; 276-277).<sup>30</sup>

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<sup>30</sup> The accounting literature has been wrestling with the appropriate treatment of expected obsolescence for a long time as well: “In a number of industries development has been so rapid and revolutionary over a period of years that the capitalization of losses due to so-called premature retirements would have led to an absurd inflation of asset values. In such situations, the use of higher depreciation rates, rather than capitalization of losses, is indicated.” W.A. Paton (1931; 93). This issue was important in Paton’s time with respect to the regulation of public utilities and it is still important today.

Thus Hayek advocated a maintenance of real financial capital approach to constructing estimates of net income. Our approach to forming net income estimates is consistent with Hayek's position as opposed to Pigou's maintenance of real physical capital position.<sup>31</sup>

## 5. Obsolescence and Embodied Technical Change

The advantages for constructing capital services aggregates using the Jorgensonian constant relative physical efficiency assumption become apparent when a new, improved capital model is introduced into the marketplace. Assume that the new more efficient model is introduced at the beginning of period  $t$  and it is characterized by the sequence of *physical productivities*,  $\{\varphi_n^\circ ; n = 0, 1, 2, \dots\}$ , where  $\varphi_n^\circ$  is the number of units of capital services the new model can produce if it is  $n$  periods old. We assume that the capital services produced by the new model are perfect substitutes for the capital services produced by the older existing model, which is characterized by the sequence of physical productivities,  $\{\varphi_n ; n = 0, 1, 2, \dots\}$ .<sup>32</sup> We assume that the capital services produced by a unit of the new model are generally larger than the services produced by the old model and in particular, we assume that a new unit of capital in its first period of use has at least the productivity of a unit of old capital in its first period of use; i.e., we assume that:

$$(47) \varphi_0^\circ \geq \varphi_0 = 1.$$

Let the period  $t$  price of capital services for a unit of *old model capital* that is 0 years old be  $c_0^t$  and let the period  $t$  price of capital services for a unit of *new model capital* that is 0 years old be  $c_0^{t^\circ}$ . Under our perfect substitutes assumption, these prices should be equal in a competitive market; i.e., the following relationship should hold:

$$(48) c_0^{t^\circ} = c_0^t.$$

Define the period  $t$  asset price of a new model unit of capital that is  $n$  periods old at the beginning of period  $t$  as  $P_n^{t^\circ}$  for  $n = 0, 1, \dots$ .<sup>33</sup> These new model asset prices can be defined in terms of the user cost of a new unit of capital  $c_0^t$ , the relative efficiency factors  $\varphi_n^\circ$ , the anticipated rate of growth in real rental prices  $i^{*t}$  and the real interest rate  $r^{*t}$  by modifying equations (19) as follows:

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<sup>31</sup> For further discussion on these two approaches to the measurement of income, see Diewert (2006). We note that Paul Schreyer has suggested an ingenious method for justifying Pigou's position: "Hayek's argument rests on the notion of a physical, engineering service life and he is absolutely right about the strange outcome of Pigou's method under these circumstances. But as soon as one admits that service lives are economic in nature, the three entrepreneurs would actually have assets with different service lives, and consequently with different (cross section) depreciation rates. This may at least go down some way the road of adjusting for obsolescence."

<sup>32</sup> Recall that  $\varphi_0$  was assumed to equal 1.

<sup>33</sup> These new model asset prices  $P_n^{t^\circ}$  are hypothetical (except for the case  $n = 0$ ) but if we know (or can estimate) the sequence of new model physical productivities  $\varphi_n^\circ$ , then these asset prices can be defined using equations (49).

$$(49) P_n^{to} = c_0^t \{ \varphi_n^o + [(1+i^{*t})/(1+r^{*t})] \varphi_{n+1}^o + [(1+i^{*t})/(1+r^{*t})]^2 \varphi_{n+2}^o + [(1+i^{*t})/(1+r^{*t})]^3 \varphi_{n+3}^o + \dots \}; \quad n = 0, 1, 2, \dots$$

Once the period  $t$  sequence of new asset prices by age has been determined by (49), the sequence of period  $t$  deterioration rates can be determined by using the following counterparts to equations (10):

$$(50) \delta_n^{to} \equiv 1 - [P_{n+1}^{to}/P_n^{to}]; \quad n = 0, 1, 2, \dots$$

Note that a counterpart to equation (28) will hold for the user cost of a new unit of new model capital services,  $c_0^{to}$ . Using this counterpart and equation (48) means that the period  $t$  asset prices for new and old model units of capital,  $P_0^{to}$  and  $P_0^t$ , and the corresponding depreciation rates,  $\delta_0^{to}$  and  $\delta_0^t$ , will satisfy the following equations:

$$(51) c_0^t = (1+r^{*t})^{-1} [r^{*t} + \delta_0^t - i^{*t}(1-\delta_0^t)] P_0^t = (1+r^{*t})^{-1} [r^{*t} + \delta_0^{to} - i^{*t}(1-\delta_0^{to})] P_0^{to} = c_0^{to}.$$

The above algebra shows that provided that we can form estimates of the sequence of physical productivities by age for the new model, there is no particular difficulty in working out the sequence of user costs and depreciation rates by age for the new model.

We now consider how capital can be aggregated over vintages for both models. It is at this point that the Jorgenson (1973; 191) perfect substitutes hypothesis leads to a very simple aggregation procedure for capital services. As in section 3, let  $K_n^t$  denote the number of units of an old model age  $n$  asset that the production unit under consideration has available for use at the beginning of period  $t$ . Let  $K_0^{to}$  denote the purchases of new model capital at the beginning of period  $t$ . Then under the perfect substitutes assumption, the aggregate amount of capital services in constant efficiency units that the production unit has available at the beginning of period  $t$  is

$$(52) K^t \equiv \varphi_0^o K_0^{to} + K_0^t + \varphi_1 K_1^t + \varphi_2 K_2^t + \dots$$

The aggregate value of capital services,  $V_{SER}^t$  used by the production unit in period  $t$  is the sum of the value of capital services over all ages and models in use during period  $t$ :

$$(53) V_{SER}^t \equiv c_0^t \varphi_0^o K_0^{to} + c_0^t K_0^t + c_1^t K_1^t + c_2^t K_2^t + \dots \\ = c_0^t \varphi_0^o K_0^{to} + c_0^t K_0^t + c_0^t \varphi_1 K_1^t + c_0^t \varphi_2 K_2^t + \dots \\ = c_0^t K^t \quad \text{using (52).}$$

Thus under the Jorgenson perfect substitutes assumption, the aggregate value of capital services used by the production unit under consideration is equal to the user cost of a new unit of capital  $c_0^t$  times the quantity of the capital services aggregate  $K^t$  where  $K^t$  is the sum of the efficiency adjusted units of capital available to the production unit at the beginning of period  $t$ ; i.e., see definition (52).

It is easy to see how the above definitions can be adjusted for forming capital services aggregates in subsequent periods. For example, using the obvious notation, the counterpart to (52) for period  $t+1$  is:

$$(54) K^{t+1} \equiv \varphi_0^\circ K_0^{t+1\circ} + \varphi_1^\circ K_1^{t+1\circ} + K_0^{t+1} + \varphi_1 K_1^{t+1} + \varphi_2 K_2^{t+1} + \dots$$

We conclude this section by considering some special cases of the above framework.

For our first special case, consider the geometric depreciation model, which was defined by equations (22)-(26) for the old model. Recall that the geometric deterioration rate for this model was  $\delta$  where  $0 < \delta < 1$ . Suppose that the new model delivers the same type of capital services as the old model but it is more durable; i.e., the new model geometric deterioration rate  $\delta^\circ$  is less than  $\delta$  so that

$$(55) 0 < \delta^\circ < \delta < 1.$$

The sequence of physical productivities for the new model is given by:

$$(56) \varphi_n^\circ \equiv (1-\delta^\circ)^n; \quad n = 0, 1, 2, \dots$$

where the parameter  $\delta$  can be interpreted as a constant geometric deterioration rate. Under assumptions (56), the period  $t$  asset prices by age for the new model,  $P_n^{t\circ}$ , can be obtained by using equations (23) and (24) where  $\delta^\circ$  replaces  $\delta$  and  $P_n^{t\circ}$  replaces  $P_n^t$ . Equation (51) can be used to relate the period  $t$  asset prices for a newly purchased new and old model,  $P_0^{t\circ}$  and  $P_0^t$ , respectively. If the anticipated capital services real inflation rate  $i^{*t}$  is zero, then (51) tells us that the ratio of  $P_0^{t\circ}$  to  $P_0^t$  is equal to:

$$(57) P_0^{t\circ} / P_0^t = [r^{*t} + \delta_0^t] / [r^{*t} + \delta_0^{t\circ}] > 0 \quad \text{assuming } r^{*t} > 0 \text{ and } \delta_0^t > \delta_0^{t\circ} > 0$$

so that the lower deterioration rate asset is more valuable than the higher rate asset (but note that both assets will earn the same period  $t$  rental rate). The capital services aggregate for period  $t+1$  will be:

$$(58) K^{t+1} \equiv \varphi_0^\circ K_0^{t+1\circ} + \varphi_1^\circ K_1^{t+1\circ} + K_0^{t+1} + \varphi_1 K_1^{t+1} + \varphi_2 K_2^{t+1} + \dots \\ = K_0^{t+1\circ} + (1-\delta^\circ) K_1^{t+1\circ} + K_0^{t+1} + (1-\delta) K_1^{t+1} + (1-\delta)^2 K_2^{t+1} + \dots$$

For our second special case, we suppose that the physical productivities of the new model are uniformly greater than the corresponding physical productivities of the old model; i.e., we assume that

$$(59) \varphi_n^\circ = (1+\theta) \varphi_n; \quad n = 0, 1, 2, \dots$$

where  $\theta > 0$  is the positive rate of productivity increase. When one works through equations (48)-(53) for this special case of our general model, we find that this case is particularly simple. There is no need to set up a separate set of computations for the new

model; all we have to do is multiply the quantities of new models in use in and period  $t$ ,  $K_n^{to}$ , by  $1+\theta$  and then add this quality adjusted number of units,  $(1+\theta)K_n^{to}$ , to the corresponding number of old model units of age  $n$  in use at the beginning of period  $t$ ,  $K_n^t$ , in order to obtain a quality adjusted total quantity equal to  $(1+\theta)K_n^{to} + K_n^t$ , which will have the old model user cost for an age  $n$  asset,  $c_n^t = c_0^t \varphi_n$ . Thus this special case of our general framework is particularly simple. Put another way, the model represented by (59) corresponds to the type of quality adjustment of price indexes that statistical agencies typically undertake.<sup>34</sup> With this type of quality change and with typical statistical agency quality adjustment procedures so that the price index for new investment goods is quality adjusted, outside observers could use this quality adjusted price index and apply the basic model that was explained in sections 2 and 3 above. However, if the type of quality change is *not* of the proportional type as represented by (59) above, then the statistical agency will have to use the more general procedures explained at the beginning of this section. Unfortunately, this more general procedure will require estimates of the new and old efficiency profiles,  $\{\varphi_n^o\}$  and  $\{\varphi_n\}$  respectively. Thus in the following section, we will examine the practical problems involved in estimating the sequence of relative efficiency factors  $\varphi_n$  in the general Jorgenson capital services model defined by assumptions (12) using data on sales and retirements of used assets.

## 6. Investment Surveys and the Estimation of Deterioration Rates and Relative Efficiencies

Hall (1971), Beidelman (1973) (1976) and Hulten and Wykoff (1981a) (1981b) (1996) have been pioneers in using data on the sales of used assets in order to estimate depreciation rates for various asset classes. In the present section, we will use the same type of methodology but in the context of a statistical agency investment survey that also asks questions about the sale or disposal of assets as it collects information on purchases of investment goods. Canada,<sup>35</sup> the Netherlands<sup>36</sup> and New Zealand ask such questions on retirements in their investment surveys and Japan is about to follow suit.<sup>37</sup> In this section, we will indicate how such survey information can be used to estimate depreciation rates and relative efficiency factors for the various ages of a particular capital input.

We assume that the statistical agency has an augmented investment survey that also asks the following questions about the *disposal of assets* during the survey period:<sup>38</sup>

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<sup>34</sup> See also Hulten (1992) on this model.

<sup>35</sup> For a description and further references to the Canadian program on estimating depreciation rates, see Baldwin, Gellatly, Tanguay and Patry (2005).

<sup>36</sup> Since 1991, the Dutch have a separate (mail) survey for enterprises with more than 100 employees to collect information on discards and retirements: The Survey on Discards; see Bergen, Haan, Heij and Horsten (2005; 8) for a description of the Dutch methods.

<sup>37</sup> The Economic and Social Research Institute (ESRI), Cabinet Office of Japan, with the help of Koji Nomura, is preparing a new survey to be implemented as of the end of 2006.

<sup>38</sup> The listed set of questions is a minimal set of questions that assumes that no unusual renovations were made to it during the period when it was used. However, the simple case that we consider can be generalized to deal with more complex cases.

- What was the age of the asset when it was sold or scrapped?
- Was the asset new when it was purchased? If not, what was its age when it was purchased?
- What was the value of the asset when it was purchased?
- If the asset was sold, what was the disposal price?

We assume that in period  $t$ , the statistical agency has collected information on asset disposals during the period for a certain class of assets pertaining to a set of production units. In what follows, we will focus on how to process the information collected on disposals of assets that are  $n$  periods old when they are sold or scrapped. We assume that the agency has collected information on  $J$  disposals of age  $n$  assets.

Let  $V_{nj}^t$  be the *disposal value* of the asset of age  $n$  that corresponds to observation  $j$  in the period  $t$  sample collected by the statistical agency in the survey<sup>39</sup> and let  $A_{nj}^t$  be the *acquisition value* of the same asset in period  $t-n$  for  $j = 1, 2, \dots, J$ .<sup>40</sup> We need to convert the historical cost asset values  $A_{nj}^t$  into period  $t$  current values so we assume that the statistical agency has an appropriate (investment) price index  $\pi^t$  for new units of the type of capital under consideration. Using this investment price index, we can calculate an *imputed value*  $V_{0j}^t$  for what asset  $j$  would cost in period  $t$  if it were new:

$$(60) V_{0j}^t \equiv A_{nj}^t \pi^t / \pi^{t-n}; \quad j = 1, 2, \dots, J.$$

We now need to consider our theoretical model. We rearrange equations (11) in order to define the *n period value survival rates*  $\sigma_n^t$  that pertain to period  $t$ :

$$(61) P_n^t / P_0^t = (1 - \delta_0^t)(1 - \delta_1^t) \dots (1 - \delta_{n-1}^t) \equiv \sigma_n^t; \quad n = 1, 2, \dots$$

where  $P_0^t$  is the period  $t$  price of a new asset in the class of assets under consideration and  $P_n^t$  is the period  $t$  price of the corresponding asset that is  $n$  periods old. The period  $t$  survival rates  $\sigma_n^t$  give the proportion of period  $t$  value for an  $n$  period old asset compared to a newly purchased asset of the same type; i.e., these survival rates summarize the effects of deterioration on initial asset value as a function of the age of the asset, using the structure of used asset prices that prevails at the beginning of period  $t$ .

Given the sequence of period  $t$  survival rates  $\sigma_n^t$  for ages  $n = 1, 2, \dots$ , we can use equations (61) in order to obtain the sequence of period  $t$  deterioration rates  $\delta_n^t$ . Then, as noted in section 3, given the sequence of period  $t$  deterioration rates,  $\delta_n^t$ , and the stock price of a new asset at the beginning of period  $t$ ,  $P_0^t$ , we can determine the sequence of period  $t$  asset prices by age,  $P_n^t$  for  $n = 1, 2, \dots$ , by using equations (11). Then we can use equations (20), along with a knowledge of either  $(1+i^t)/(1+r^t)$  or  $(1+i^t)/(1+r^t)$ , in order to

<sup>39</sup> If the asset was scrapped, we set  $V_{nj}^t$  equal to zero. We assume  $V_{0j}^t$  is always greater than zero so that the  $s_{nj}^t$  defined by (62) are well defined nonnegative numbers.

<sup>40</sup> If several assets of the same type were acquired at the same time and then disposed of at the same time, then they can be bundled together into the same values  $V_{nj}^t$  and  $A_{nj}^t$ .



solve for the sequence of efficiency factors,  $\varphi_n$ . Thus our problem now is to use the sample of capital asset disposal information collected by the statistical agency to estimate the survival rates  $\sigma_n^t$ .

It can be seen that the ratio of the disposal value  $V_{nj}^t$  to the corresponding imputed value for a newly purchased asset  $V_{0j}^t$  is an estimator for the survival rate  $\sigma_n^t$  defined by (61) if our model assumptions are approximately correct. Thus we define the following  $J$  estimators for  $\sigma_n^t$ :

$$(62) s_{nj}^t \equiv V_{nj}^t / V_{0j}^t ; \quad j = 1, 2, \dots, J.$$

Obviously, we need to average the above estimators for the survival rate  $\sigma_n^t$  but what type of average should be used? We suggest that the following average is a natural one:

$$(63) s_n^t \equiv \frac{\sum_{j=1}^J V_{nj}^t}{\sum_{j=1}^J V_{0j}^t} = \frac{\sum_{j=1}^J w_{nj}^t s_{nj}^t}{\sum_{j=1}^J w_{nj}^t} \quad \text{using (62) and (63)}$$

where the weights  $w_{nj}^t$  is the share of observation  $j$  in the total imputed value of the period  $t$  sample when the disposal values are converted into corresponding period  $t$  new purchase values; i.e.,

$$(64) w_{nj}^t \equiv V_{0j}^t / \sum_{k=1}^J V_{0k}^t ; \quad j = 1, 2, \dots, J.$$

Thus  $s_n^t$ , our overall estimator for  $\sigma_n^t$ , is a share weighted average of the individual estimators  $s_{nj}^t$ .

The definition in (63) tells us that our period  $t$  estimator of the value survival rate for assets of age  $n$  is simply equal to the sum of the disposal values in our sample,  $\sum_{j=1}^J V_{nj}^t$ , divided by the sum of our estimated values for the same assets if they were newly purchased in period  $t$ ,  $\sum_{j=1}^J V_{0j}^t$ . Note that this definition can deal with zero disposal values and thus there is no need to undertake the type of adjustments that Hulten and Wykoff (1981a) (1981b) had to make to their auction data to account for the fact that not all assets are scrapped at the same time.<sup>41</sup> The relative simplicity of the above approach is due to the comprehensive nature of the statistical agency survey. As the sample size  $J$  becomes large, we would expect that  $s_n^t$  would become close to the population parameter  $\sigma_n^t$ .

In practice, statistical agency investment and disposal surveys are not likely to be very large in scope and hence the survival rate estimates  $s_n^t$  for a particular asset class may not be very reliable and for many ages  $n$ , there may be no estimator at all. In this case, two strategies could be followed:

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<sup>41</sup> On this issue, see also Beidleman (1973) (1976) and Hulten and Wykoff (1996; 22). Schmalenbach (1959; 91) was perhaps the first to note that neglect of the survival problem leads to serious errors in the estimation of depreciation rates.

- The estimates for the survival rates  $s_n^t$  could be used as the dependent variables in a regression where these rates are smoothed or
- We could assume that the survival rates  $s_n^t$  are independent of time and then the results of many surveys could be combined.

With respect to the first strategy, nonparametric smoothing methods could be used or we could assume a simpler model, such as the geometric depreciation model. In this case, for ages  $n$  where the statistical agency collected information on asset disposals for this age of asset for a set of ages  $S$  say, we could run a nonlinear regression of the following form in order to estimate the constant deterioration rate  $\delta$ :<sup>42</sup>

$$(65) s_n^t = (1-\delta)^n + \text{error}; \quad n \in S.$$

In order to justify the second strategy, which would combine information over several surveys, we would have to assume that either  $(1+i^t)/(1+r^t)$  or  $(1+i^{*t})/(1+r^{*t})$  is at least approximately constant over the time periods included in the combined survey in order to be consistent with our theoretical model.<sup>43</sup>

## 7. Conclusion

The treatment of obsolescence in the context of vintage capital accounts is rather complicated. We distinguished two types of obsolescence:

- *Disembodied obsolescence* where there are no new and improved models introduced for the type of capital under consideration but the real price of the underlying capital service declines over time due to shifts in demand or other exogenous factors and
- *Embodied obsolescence* where new and improved models of the capital good are produced over time.

We modeled the first type of obsolescence in sections 2-4 above. Although our analysis is reasonably straightforward, it may be rather controversial in national income accounting circles because we argue for the inclusion of *expected* capital gains and losses in the production accounts of the SNA.<sup>44</sup> The second type of obsolescence was modeled in section 5 above using vintage production accounts that were originally developed by Jorgenson (1973). Section 6 suggested that statistical agencies should modify their investment surveys so that in addition to collecting information on new investments,

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<sup>42</sup> Alternatively, the individual observation estimates  $s_{nj}^t$  could be used in place of the aggregate  $s_n^t$ . This latter procedure would give more regression weight to ages  $n$  where there were more observations.

<sup>43</sup> Obviously, an infinite sequence of distinct deterioration rates  $\delta_n$  could not be estimated so that it would be necessary to assume that  $\delta_{n^*}$  equals zero for some age  $n^*$ , which would truncate the infinite series.

<sup>44</sup> This sentence requires a bit of careful interpretation. We view a user cost as an approximation to a market rental price, which we would use in the accounts, if we could observe it. Since we cannot observe the market rental price in many situations, we approximate it by an ex ante user cost and one element of this user cost is the ex ante capital gain or loss that we expect will occur over the course of the period. These expected gains or losses are in the present System of National Accounts but they appear only as part of Gross Operating Surplus.

information on asset retirements and disposals is also collected as is the case with some statistical agencies. Section 6 showed how this asset retirement information could be used in order to estimate relative efficiencies and deterioration rates in the Jorgenson model of vintage production.

We conclude by observing a potential limitation of our treatment of the obsolescence problem. The models presented in this paper seemingly ignore the fact that capital asset lives and service prices can be affected by changes in output and input prices. For example, our models assume that the length of life of the asset is fixed and is independent of other prices. For models that relax this assumption, the reader is referred to Solow (1960), Solow, Tobin, von Weizsäcker and Yaari (1966), Harper (2004) and Diewert (2005b), where the length of life for each vintage is endogenously determined. In principle, the Jorgenson type model introduced in section 5 can handle this situation since the pattern of efficiency factors for the new model, the  $\varphi_n^\circ$ , can have a larger or smaller number of nonzero terms than the old model efficiency factors, the  $\varphi_n$ . However, the endogenous life models can provide some practical guidance to the statistician on how to actually estimate the new model efficiency factors under certain circumstances.<sup>45</sup>

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<sup>45</sup> See Harper (2004) and Diewert (2005b) on this point.

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