

SELECTION OF OPTIMAL PORTFOLIO BY USE OF RISK DIVERSIFICATION METHOD¹

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Summary

The paper will discuss how securities investors can protect themselves from risk through diversification. There will be proposals how investors should structure their portfolio, i.e. proposals of investment percentages for particular shares, in order to achieve stable solid returns at a low level of risk. The paper will analyze three types of stock: INA – Oil Industry Plc., IGH – Croatian Institute of Civil Engineering Plc. and Viro Sugar Factory Plc., which can be used to gain a better understanding of the investment business. We shall describe the basic tenets of modern portfolio theory so as to explicate some fundamental issues of securities investment and portfolio creation. The paper will provide an analysis of Markowitz' theory as the origin of modern portfolio optimization theory, which in turn represents the starting point for securities investments.

Keywords: risk, diversification, Markowitz' theory, decision making, securities analysis, programming

1. Introduction

Contemporary market economies imply the existence of the securities market. The investor can deposit surplus financial assets either in depositary institutions or in securities. The securities market entails a higher level of risk (than investing into depositary institutions); nevertheless, it provides the opportunity of higher returns. The success of the investment business should be observed within the limits of the return gained in relation to the risk taken.

Risk is defined as insecurity in the accomplishment of expected results, i.e. the danger of undesired events. At the capital market, risk is the likelihood of variation of the actual rate of return from the expected (planned) rate of return.

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Investors who invest their free assets into the securities market strive to achieve the maximum possible return by accepting the defined risk. The relationship between the risk and the potential return depends on the type of financial instrument. The length of investment is a highly important factor when investing into securities. It is essential to balance the amount of risk with the length of investment, which increases the gain of a potentially higher rate of return, and, on the other hand, reduces the danger of loss. In other words, short-term investment (up to one year) should be an investment into safe and stable securities, while long-term investments could be investments into securities of a higher risk level, and at the same time, investments with potentially higher returns. It is well known that the securities market has a long-term tendency of growth, whereas in short terms it is subject to considerable oscillations. The study focuses on the investment in shares, and the time period of investment was ignored.

Diversification as means of risk reduction

Average investors are risk averse. Therefore, they will be ready to invest into securities under the presumption of an adequate compensation for risk taking. The compensation for the risk taken should be in the form of minimal rate of return for the invested financial assets, and the rate is named the *required rate of return*. It has two components:

- Delayed consumption compensation (investors could have purchased goods and services with the assets they are to invest) and
- Risk acceptance compensation.

Diversification is used to stabilise the potential return, and thus increase the value of the investment. Diversification stands for the investment of capital into several different securities or projects, all together called the *portfolio*. Each security or project entails certain risk; however, the only thing that matters to the investors who diversify their investments is the total risk (portfolio risk) and the portfolio return. There are two types of risk:

- *Systemic* – risk that can be diversified and
- *Non-systemic* - risk that can not be diversified.

It is a known fact that companies with a high level of portfolio diversification have more stable and higher returns in comparison to companies that do not diversify their portfolio. It should be mentioned that investments into securities entail both the systemic and non-systemic risk.

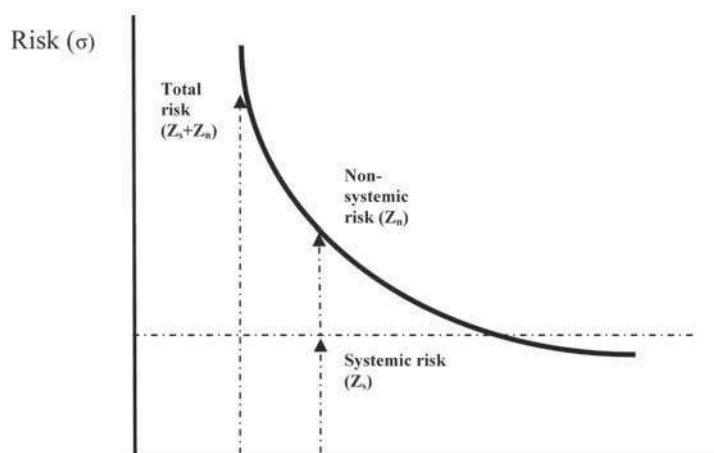
Systemic risk cannot be suspended, i.e. it is always present, and at the securities market it is manifested as the threat of recession, inflation, political turmoil, rise

in interest rates etc. Therefore, no matter how much investors may diversify their investments, the systemic risk hazard is always present. The key to protection from non-systemic risk lies in the diversification, and the investor must pay attention to the fact that the security returns are in as little correlation as possible.

By analysing Graph 1 the conclusion can be drawn that the increase the number of different shares in the portfolio decreases the non-systemic risk, thereby the total risk as well, while the systemic risk remains unchanged. On the other hand, the total risk can never be completely eliminated.

For example, take an investor who has evenly arranged the investment of his financial assets into 25 shares of different companies from different branches (business activities) in order for the return on securities to be in as little correlation as possible. One of the companies from his/her portfolio reports a poor financial result which causes the company share to plunge by 50 %. It will be assumed that other shares from his/her portfolio that day did not change significantly. Since the investor evenly arranged the investment among 25 companies, the share of each company is 4 %, including the company the value of which fell by 50 %. The investor has lost 2 % of his/her investment.

The above example shows that diversification is desirable; however, when investing, investors face the dilemma of how to diversify their assets, i.e. according to which ratio to invest the total available financial assets into individual securities and/or projects. The answer to the identified problem is given by the Markowitz theory of portfolio optimisation.



Graph 1: Relation between total, systemic and non-systemic risk:

3. Markowitz' theory of portfolio optimisation

Harry Markowitz developed a theory according to which we can balance our investment by combining different securities, illustrating how well selected shares portfolio can result in maximum profit with minimum risk. He proved that investors who take a higher risk can also achieve higher profit (see Graph 2). The central measure of success or failure is the relative portfolio gain, i.e. gain compared to the selected benchmark.

Markowitz portfolio theory is based on three assumptions about the behaviour of investors who:

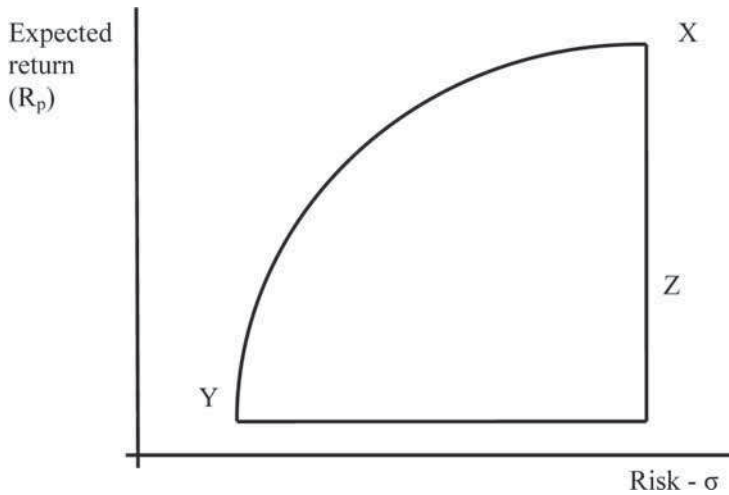
- wish to maximise their utility function and who are risk averse,
- choose their portfolio based on the mean value and return variance,
- have a single-period time horizon.

By using risk (standard deviation - σ) and the expected return (R_p) in a two-dimensional space, Graph 2 presents portfolio combinations available to the investor. Thus, each point within the space enclosed by points XYZ, represents a certain portfolio. By analysing Graph 2 the conclusion can be drawn that in a new combination of securities the portfolio can be moved:

- upwards – which would imply higher returns with the same level of risk or
- to the left – this implies higher returns with less risk.

It can be noticed that the portfolios below the XY curve, unlike the portfolios on the curve, offer the investor the same return with a higher level of risk or a higher risk with less return, which is not acceptable to the investor. Investors tend to select the combination of shares that would position their portfolio on the XY curve, called the *efficient frontier*. If the portfolio does not belong to the frontier, the investor can improve the situation by changing the structure of the portfolio, i.e. by changing its content.

Investors will opt for the portfolio that best corresponds to their risk attitude. Those who are more risk inclined will select the portfolio on the efficient frontier, closer to point X, whereas the more risk averse will select the portfolio closer to point Y. It can be said that the Markowitz portfolio theory helps investors in the selection of the set of shares that will ensure a higher portfolio return with the desired level of risk (*the tendency is to minimise risk and maximise return on investment*).



Graph 2: *Efficient frontier*

Markowitz presumes that the identification of the optimal portfolio is based on the maximising of investor's utility. The utility function U is higher the higher the portfolio returns (R_p), and it is lower the lower the portfolio risk (σ), which can be presented by the following formula:

$$U = A \times R_p - \frac{1}{2} \sigma^2$$

Variable A shows the amount of risk the investor is willing to take, as well as the number of return units the investor wishes to have per risk unit.

The expected portfolio return (R_p) equals the sum of individual share returns of a portfolio multiplied by their participations (x_i) in the portfolio, and can be defined by the following equation²:

$$R_p = \sum_{i=1}^m x_i R_i$$

The standard deviation is used for the risk of individual shares, and the portfolio risk is given by the equation:

$$\sigma_p^2 = \sum_{i=1}^m x_i^2 \delta_i^2 + 2 \sum_{\substack{i,j=1 \\ i \neq j}}^m x_i x_j \delta_{i,j}^2$$

² According to: Latković, M.: STREETWISE, Zagreb, 2001., p. 8, source: www.phy.hr/~laci/art/streetwise.pdf [21. 2. 2008.]

The sum of all share participations in the portfolio must equal 1, namely:

$$\sum_{i=1}^m x_i = 1$$

In order to solve the Markowitz theory mathematically, it is necessary to identify the x_i share participations that satisfy the last equation and maximise the utility function U . Investors have to define all the required coefficients for the optimisation of the portfolio, and the solution is arrived at by solving a set of linear equations using quadratic programming.

4. Example of a portfolio model with shares from three different companies

The postulated investment problem discusses the way in which the decision maker, the investor, should invest his/her assets in order to maximise his/her expected utility. The fact that the investor is risk averse and that he/she selects the portfolio based on the mean value and return variance was taken into account. The investor tends to select the portfolio somewhere along the efficient frontier field.

For the purpose of this example the trend in the price of shares of the following three companies were monitored over a period of 12 months:

- INA – Oil Industry Plc.,
- IGH - Croatian Civil Engineering Institute Plc. and
- Viro Sugar Factory Plc.

Table 1 contains the average prices for individual months per each share. The zero month price is the initial value of the share.

Table 1: *Company share prices over 12 months*

	Share A INA – Oil Industry Plc.	Share B IGH - Croatian Civil Engineering Institute Plc.	Share C Viro Sugar Factory Plc.
Month	Monthly price	Monthly price	Monthly price
0	2400.00	3905.01	650.00
1	2965.00	4219.99	700.00
2	2861.00	4699.00	800.00
3	3365.00	6149.99	1116.00
4	2611.00	11010.00	1240.99
5	2805.00	12600.00	1280.02
6	2869.00	11280.00	1288.00

7	2979.00	11697.50	1335.00
8	3350.00	10000.00	1351.30
9	2599.00	12694.98	1440.00
10	2660.00	13490.00	1500.00
11	2800.00	12940.00	1685.00
12	2670.00	14512.00	1687.99

The expected return value should be calculated for each share. For this purpose, the monthly return for each share (Table 2) will be calculated first. This will be the percentage that the investor will gain by buying the share at the end of each month $t-1$ and by selling it at the end of the following month. The R_{At} monthly return for t month for share A is determined as follows:

$$R_{At} = \frac{P_{At} - P_{At-1}}{P_{At-1}}, \text{ where:}$$

- R_{At} – means share A return at t moment
- P_{At} – means share A price at t moment
- P_{At-1} – means share A price at $t-1$ moment

Table 2: *Monthly return per each share*

Month	MONTHLY RETURN - R		
	Share A	Share B	Share C
1	0.2354166667	0.0806604849	0.0769230769
2	-0.0350758853	0.1135097477	0.1428571429
3	0.1761621811	0.3087869760	0.3950000000
4	-0.2240713224	0.7902468134	0.1119982079
5	0.0743010341	0.1444141689	0.0314506966
6	0.0228163993	-0.1047619048	0.0062342776
7	0.0383408853	0.0370124113	0.0364906832
8	0.1245384357	-0.1451164779	0.0122097378
9	-0.2241791045	0.2694980000	0.0656404943
10	0.0234705656	0.0626247540	0.0416666667
11	0.0526315789	-0.0407709414	0.1233333333
12	-0.0464285714	0.1214837713	0.0017744807
ΣR	0.2179228630	1.6375878034	1.0455787980
$(\Sigma R)^2$	0.0474903742	2.6816938138	1.0932350228

Assuming that the return data for 12 months represent the return distribution for the following month, namely that the past can provide some information on how the returns will behave in the future, then the conclusion could be that the mean value of the historical data would represent the expected monthly gain for each share and that the future return variance could be learned from the historical data. The risk measure is generally expressed by the mean variance or standard deviation, as its second square root, and this must certainly be known in order to reach a quality decision.

In the given example, the expected return values are as follows:

$$E(R_A) = \frac{\sum(R_{At})}{12} = \frac{0.2179228630}{12} = 0.0181602386$$

$$E(R_B) = \frac{\sum(R_{Bt})}{12} = \frac{1.6375878034}{12} = 0.1364656503$$

$$E(R_C) = \frac{\sum(R_{Ct})}{12} = \frac{1.0455787980}{12} = 0.0871315665$$

The future return variances can be calculated by using the following formula:

$$\sigma^2(R_A) = \frac{n \sum R^2 - (\sum R)^2}{n(n-1)}$$

The calculation requires the square values of the monthly return value for each share given in Table 3:

Table 3: *Squared values of monthly returns per each share*

Month	$(R_{At})^2$	$(R_{Bt})^2$	$(R_{Ct})^2$
1	0.0554210069	0.0065061138	0.0059171598
2	0.0012303177	0.0128844628	0.0204081633
3	0.0310331140	0.0953493965	0.1560250000
4	0.0502079575	0.6244900261	0.0125435986
5	0.0055206437	0.0208554522	0.0009891463
6	0.0005205881	0.0109750567	0.0000388662
7	0.0014700235	0.0013699186	0.0013315700
8	0.0155098220	0.0210587922	0.0001490777
9	0.0502562709	0.0726291720	0.0043086745
10	0.0005508674	0.0039218598	0.0017361111
11	0.0027700831	0.0016622697	0.0152111111
12	0.0021556122	0.0147583067	0.0000031488
Σ	0.2166463071	0.8864608270	0.2186616273

Now, the future return variances can be calculated:

$$\sigma^2(R_A) = \frac{n \sum R^2 - (\sum R)^2}{n(n-1)} = \frac{(12 * 0.2166463071) - 0.0474903742}{12 * 11} = 0.0193353433$$

$$\sigma^2(R_B) = \frac{n \sum R^2 - (\sum R)^2}{n(n-1)} = \frac{(12 * 0.8864608270) - 2.6816938138}{12 * 11} = 0.0602714857$$

$$\sigma^2(R_C) = \frac{n \sum R^2 - (\sum R)^2}{n(n-1)} = \frac{(12 * 0.2186616273) - 1.0932350228}{12 * 11} = 0.0115962462$$

Table 4: *Expected monthly return and variance per share*

	Share A	Share B	Share C
Expected return	0.0181602386	0.1364656503	0.0871315665
Variance	0.0193353433	0.0602714857	0.0115962462

The portfolio risk will be affected by the level of correlation of the securities in the portfolio. Therefore, the portfolio return variance must include a covariance between the portfolio securities. The covariance (and the correlation coefficient calculated from it) determines to which level the returns by the two shares will move in a common manner.

The definition is,

$$\text{Cov}(R_A, R_B) = \frac{1}{M} \sum [R_{At} - E(R_A)] * [R_{Bt} - E(R_B)],$$

in which M is the number of distribution points (in the given example – months), namely M=12.

Table 5: *Difference between monthly return and expected return value per share*

Month	(R _{At}) - E(R _A)	(R _{Bt}) - E(R _B)	(R _{Ct}) - E(R _C)
1	0.2172564281	-0.0558051654	-0.0102084896
2	-0.0532361239	-0.0229559026	0.0557255764
3	0.1580019425	0.1723213257	0.3078684335
4	-0.2422315610	0.6537811631	0.0248666414
5	0.0561407955	0.0079485187	-0.0556808699
6	0.0046561607	-0.2412275550	-0.0808972889
7	0.0201806467	-0.0994532389	-0.0506408833
8	0.1063781971	-0.2815821282	-0.0749218287

9	-0.2423393431	0.1330323497	-0.0214910722
10	0.0053103270	-0.0738408962	-0.0454648998
11	0.0344713404	-0.1772365917	0.0362017668
12	-0.0645888100	-0.0149818790	-0.0853570858

Table 6: *Product of differences between the monthly return and the expected return value of two shares*

Month	$((R_{A_t}) - E(R_A)) * ((R_{B_t}) - E(R_B))$	$((R_{A_t}) - E(R_A)) * ((R_{C_t}) - E(R_C))$	$((R_{B_t}) - E(R_B)) * ((R_{C_t}) - E(R_C))$
1	-0.0121240309	-0.0022178600	0.0005696864
2	0.0012220833	-0.0029666137	-0.0012792309
3	0.0272271042	0.0486438105	0.0530522966
4	-0.1583664317	-0.0060234854	0.0162573417
5	0.0004462362	-0.0031259683	-0.0004425804
6	-0.0011231943	-0.0003766708	0.0195146552
7	-0.0020070307	-0.0010219658	0.0050363999
8	-0.0299541991	-0.0079700491	0.0210966480
9	-0.0322389722	0.0052081323	-0.0028590078
10	-0.0003921193	-0.0002414335	0.0033571690
11	-0.0061095829	0.0012479234	-0.0064162778
12	0.0009676617	0.0055131126	0.0012788095
Σ	-0.2124524758	0.0366689324	0.1091659094

$$Cov(R_A, R_B) = \frac{1}{M} \sum [R_{A_t} - E(R_A)] * [R_{B_t} - E(R_B)] = \frac{1}{12} * (-0.2124524758) = -0.0177043730$$

$$Cov(R_A, R_C) = \frac{1}{M} \sum [R_{A_t} - E(R_A)] * [R_{C_t} - E(R_C)] = \frac{1}{12} * 0.0366689324 = 0.0030557444$$

$$Cov(R_B, R_C) = \frac{1}{M} \sum [R_{B_t} - E(R_B)] * [R_{C_t} - E(R_C)] = \frac{1}{12} * 0.1091659094 = 0.0090971591$$

The covariance between share A and share B is – 0.017704. This number is difficult to explain as its size depends on the return measurement units (if percentages were calculated, the covariance would be 177.04 which is 10 000 times higher than the one calculated).

Therefore, the correlation coefficient³ will be calculated as well by using the following formula:

$$\rho_{AB} = \frac{\text{cov}(A, B)}{\sigma_A \sigma_B}$$

For the purpose of the above calculation, the standard deviation values are required and they are obtained by taking the second square root of the mean variance:

$$\sigma_A = \sqrt{\sigma_A^2} = \sqrt{0.0193353433} = 0.1390515849$$

$$\sigma_B = \sqrt{\sigma_B^2} = \sqrt{0.0602714857} = 0.2455025167$$

$$\sigma_C = \sqrt{\sigma_C^2} = \sqrt{0.0115962462} = 0.1076858684$$

Now it is possible to calculate the correlation coefficient:

$$\rho_{A,B} = \frac{\text{cov}(A, B)}{\sigma_A \sigma_B} = \frac{-0.0177043730}{0.1390515849 * 0.2455025167} = -0.5186192808$$

$$\rho_{A,C} = \frac{\text{cov}(A, C)}{\sigma_A \sigma_C} = \frac{0.0030557444}{0.1390515849 * 0.1076858684} = 0.2040715024$$

$$\rho_{B,C} = \frac{\text{cov}(B, C)}{\sigma_B \sigma_C} = \frac{0.0090971591}{0.2455025167 * 0.1076858684} = 0.3441051146$$

This coefficient measures the linear relationship between the shares and its value can be from -1 to $+1$.

Provided the relationship between the shares is strong, the correlation coefficient value is closer to 1, and if there is no relationship between them or if it is very weak the coefficient relation value is closer to 0. The algebraic sign before the correlation coefficient indicates whether the relationship between the shares is positive or negative. If the relationship between the shares is positive, it would be expected that positively correlated securities behave in a similar way. The more positive the correlation, the more it is certain that positively correlated securities behave in the same way. Therefore, the more positive the correlation between the securities, the higher is the risk of the portfolio. Assuming the relationship between the shares is negative, each variance of one share will be followed by the opposite variance of the other share.

³ The correlation coefficient of i and j securities returns.

In the given example, the correlation coefficient between share A and share C, same as the correlation coefficient between share B and share C, is positive. Since it amounts to 0.204 or 0.344, the relationship between the shares can be considered insignificant. The correlation coefficient between share A and share B is negative and amounts to $-0,519$. The conclusion is that there is a relationship between these two shares, and that it has a certain cause and result effect. For instance, the rise in the price of share A will be linked to the decrease in the value of share B, and vice versa.

The efficient portfolio, i.e. the portfolio that entails minimum risk at the given expected return value, can be determined by the following mathematical model:⁴

$$\begin{aligned} \min \delta_p^2 &= \sum_{i=1}^m \sum_{j=1}^m x_i x_j \delta_i \delta_j \rho_{ij} \\ E(R_p) &= \sum_{i=1}^m x_i E(R_i) \\ \sum_{i=1}^m x_i &= 1 \\ x_i &\geq 0, \quad i = 1, \dots, m \end{aligned}$$

Lagrange function for the problem reads as follows:⁵

$$L(x_1; \dots; x_m; \lambda_1; \lambda_2) = \sum_{i=1}^m \sum_{j=1}^m x_i x_j \delta_i \delta_j \rho_{ij} + \lambda_1 \left(E(R_p) - \sum_{i=1}^m x_i E(R_i) \right) + \lambda_2 \left(1 - \sum_{i=1}^m x_i \right).$$

To determine the efficient portfolio structure, the Lagrange function has to be derived partially.

The result is a linear equation system:⁶

$$\begin{pmatrix} 2\delta_1^2 & 2\text{cov}_{12} & K & 2\text{cov}_{1m} & E(R_1) & 1 \\ 2\text{cov}_{21} & 2\delta_2^2 & K & 2\text{cov}_{2m} & E(R_2) & 1 \\ M & M & M & M & M & M \\ 2\text{cov}_{m1} & 2\text{cov}_{m2} & K & 2\delta_m^2 & E(R_m) & 1 \\ E(R_1) & E(R_2) & K & E(R_m) & 0 & 0 \\ 1 & 1 & K & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ M \\ x_m \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ M \\ 0 \\ E(R_p) \\ 1 \end{pmatrix}$$

⁴Barković, D.: OPERACIJSKA ISTRAŽIVANJA U INVESTICIJSKOM ODLUČIVANJU, Faculty of Economics in Osijek, Osijek, 2004, p. 204.

⁵Ibidem

⁶Ibidem, p. 205.

Following the insertion of values obtained for the expected share returns, and of the variance and covariance between the shares, the following system of equations is obtained:

$$\begin{pmatrix} 0.03867 & -0.03541 & 0.00611 & 0.01816 & 1 \\ -0.03541 & 0.12054 & 0.01819 & 0.13647 & 1 \\ 0.00611 & 0.01819 & 0.02319 & 0.08713 & 1 \\ 0.01816 & 0.13647 & 0.08713 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.08713 \\ 0 \end{pmatrix}$$

The efficient portfolio structure is calculated by using one of the mathematical programmes, MATLAB for instance (Figure 1).

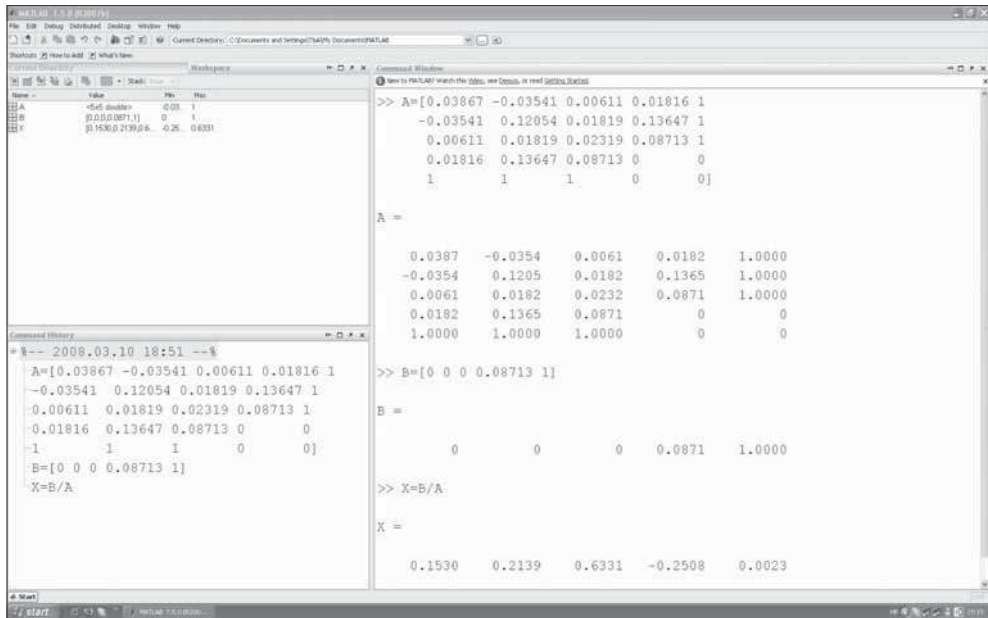


Figure 1: Efficient portfolio structure calculation in MATLAB (mathematical programme)

According to the calculation, the portfolio with the lowest level of risk at the expected rate of return $E(R) = 0.08713$ or 8.71% has the following structure: $x_1 = 0.1535$; $x_2 = 0.2139$ and $x_3 = 0.6327$.

This means that the investor should invest 15.35% of his/her financial assets in the shares of INA – Oil Industry Plc., 21.39% in the shares of IGH – Croatian

Institute of Civil Engineering Plc. and 63.27% of assets in the shares of Viro Sugar Factory Plc.

5. Conclusion

Investors reach their investment decisions by applying the Markowitz theory of portfolio optimisation, which requires the understanding of expected returns, securities correlation, risk level and the investor's risk attitude. Each investment implies a certain level of risk, and the main factor that determines a favourable investment for the investor is the compatibility of the investor's risk attitude with the expected return of a given security. By diversifying the portfolio into numerous securities, investors can reduce the level of risk in the portfolio, nevertheless, they must pay attention to the fact that the securities should have a negative correlation. The correlation indicates the cause and result link between the securities. If the link is negative, the consequence of the decrease in the value of one share will be the rise in the value of the other share.

Investors tend to select a portfolio structure that would with minimum risk maximise their utility, i.e. expand the return as much as possible.

In order to assist the investor in constructing the portfolio, the assumption is that the security values from a previous period could assist in the forecasting of the portfolio return level in the future, and be the basis for the decision to invest into a certain portfolio. The category of risk is always present.

The analysis of the obtained results indicated that the efficient portfolio is a portfolio with the lowest return variance among the portfolios with the same expected return, i.e. the portfolio with the highest expected return among portfolios with the same variance.

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