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## THE NEW KEYNESIAN PHILLIPS CURVE: EMPIRICAL RESULTS FOR LUXEMBOURG

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# **The New Keynesian Phillips Curve: Empirical Results for Luxembourg**

By Ieva Rubene\* and Paolo Guarda\*\*

## **Résumé**

La courbe de Phillips néo-keynésienne diffère de sa version traditionnelle en intégrant des anticipations prospectives et en liant l'inflation à une mesure du coût marginal et non pas au chômage ou à l'écart de production. Les modèles néo-keynésiens sous-jacents à cette nouvelle courbe de Phillips combinent les rigidités nominales avec des comportements optimisateurs et des anticipations rationnelles. Ainsi, à différence de la courbe de Phillips traditionnelle, la nouvelle courbe s'appuie sur des fondements issus de la théorie micro-économique. Ce qui explique sa robustesse vis-à-vis de certaines formes de la critique de Lucas, lui permettant ainsi d'analyser des changements structurels, tels que l'impact sur l'inflation dû à une augmentation de la flexibilité des prix.

Cette contribution présente des estimations pour le Luxembourg de la courbe de Phillips néo-keynésienne à l'aide d'une forme hybride (Galí et Gertler, 1999). Les résultats suggèrent que les entreprises ajustent leurs prix fréquemment, mais généralement sur la base de raccourcis rétrospectifs plutôt que sur des anticipations prospectives. Selon ces résultats, les prix au Luxembourg demeurent relativement flexibles, mais avec une prévalence d'un comportement rétrospectif dans la fixation des prix. Ce dernier peut être une source de persistance de l'inflation et ainsi augmenter le taux du sacrifice afférant à toute politique monétaire désinflationniste. Du point de vue des entreprises individuelles, un comportement rétrospectif dans la fixation des prix peut être qualifié de rationnel dans une petite économie très ouverte face à sa vulnérabilité aux chocs externes. Ces caractéristiques de l'économie luxembourgeoise peuvent impliquer un accroissement des coûts pour la collecte de l'information et une diminution des avantages d'un ajustement des prix vers leur niveau optimal.

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## **Abstract**

The New Keynesian Phillips curve (NPC) differs from the conventional expectations-augmented Phillips curve in that it is forward-looking and links inflation to a measure of marginal cost instead of unemployment or the output gap. More fundamentally, the NPC is derived from New Keynesian models that combine nominal rigidities with individual optimising behaviour and model-consistent (rational) expectations. Because the NPC is grounded in micro-theory (unlike the conventional expectations-augmented Phillips curve), it is robust to some forms of the Lucas critique and may serve to analyse the impact structural changes such as increased price flexibility may have on inflation.

New Keynesian Phillips curve estimates for Luxembourg using the Galí and Gertler (1999) hybrid form suggest that firms change prices often but tend to use backward-looking rules-of-thumb instead of resetting prices optimally using forward-looking expectations. In terms of policy implications, although the results suggest prices in Luxembourg are relatively flexible, the prevalence of backward-looking price setting implies greater inflation persistence and a higher sacrifice ratio attached to disinflationary monetary policy. From the perspective of individual firms, backward-looking price setting may be a rational response in a very small open economy because of its vulnerability to external shocks. Small size and openness plausibly imply higher costs of collecting information and lower benefits from optimal price setting.

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## 1. Introduction

Appropriate monetary policy must be based on an adequate understanding of inflation dynamics. In particular, policymakers need to assess the inflationary consequences of different shocks (to demand, productivity, import prices, etc.) and the speed at which they feed through the economy to affect prices. For example, in the late 1990s, the US Federal Reserve accurately judged that the acceleration of US productivity allowed output to grow more rapidly without generating pressure on prices. More recently, the European Central Bank has had to evaluate how the appreciation of the euro will affect inflation in the euro area through lower import prices and falling export activity. Finally, as the recovery strengthens in both major economies, central banks will have to decide when to increase interest rates in order to prevent inflationary pressure developing in their respective economies.

Inflation's response to different shocks will vary depending on the nominal rigidities affecting the price- and wage-setting mechanisms and any real rigidities in the markets for goods and labour. Traditionally, this response was embodied in the Phillips Curve relationship linking inflation and real activity. The original Phillips curve was "discovered" as an empirical regularity relating wage inflation to unemployment, but it has since been generalised to link price inflation to any cyclical indicator of real activity (i.e. the output gap or the capacity utilisation rate). However, this conventional Phillips curve remains a "reduced form" relationship as there is still no consensus regarding its theoretical underpinnings.

The New Keynesian Phillips Curve provided such micro-economic foundations by combining nominal rigidities with the forward-looking optimising behaviour of representative firms. The resulting specification differs from the traditional Phillips curve in that it links price developments to marginal cost rather than one of the usual activity variables. A more important difference is that the New Keynesian Phillips curve is generally forward-looking whereas the traditional Phillips curve is backward-looking. This has important consequences for the design of optimal monetary policy. In a forward-looking environment, the private sector will respond to the anticipated effects of monetary policy, so that small changes in monetary policy can have larger effects. On the other hand, in a backward-looking environment, shocks tend to be more persistent and the effects of different shocks will tend to accumulate. This could require a more active monetary policy in order to stabilise the economy.

This study presents estimates of a New Keynesian Phillips curve for Luxembourg, assessing the extent of price rigidity, as well as the degree of forward-looking behaviour in price setting. Empirical results suggest that while price rigidities may not be a problem in Luxembourg, price-setting behaviour is largely backward-looking. In terms of policy implications, backward-looking price setting could generate higher inflation persistence and increase the sacrifice ratio attached to any disinflationary monetary policy.

The rest of this study is structured as follows. The next section provides the theoretical framework, beginning with a survey of the literature from the origins of the Phillips curve to its more recent New Keynesian incarnation, and describing the hybrid version that combines forward- and backward-looking expectations, as well as the extension to an open economy context. We then review existing empirical results concerning the New Keynesian Phillips curve as well as some of the criticisms and alternative approaches it has stimulated. Section 4 presents our empirical results, including estimates of several specifications of the New Keynesian Phillips curve for Luxembourg. The final section concludes and indicates some possible directions for further research.

## 2. Theoretical Framework

Phillips (1958) found a negative relationship between wage inflation in the UK and the level of (and the change in) unemployment. Samuelson and Solow (1960) confirmed this relationship using US data and extended it to link price inflation directly to unemployment. The intuition behind this correlation is that employers will have to bid up wage rates more rapidly when there are fewer unemployed, raising production costs and therefore prices. On the other hand, when demand for labour is low and there are more unemployed, workers will moderate their wage demands, slowing wage and price inflation. The traditional Phillips curve can be represented as follows:

$$\pi_t = \alpha + \beta u_t, \quad (1)$$

where  $\pi_t$  is the period  $t$  inflation rate (either of prices or wages), calculated as the rate of change from period  $t-1$  to period  $t$ ,  $u_t$  is the unemployment rate,  $\alpha$  and  $\beta$  are the parameters to be estimated, and  $\beta$  is the slope of the Phillips curve. This relationship suggested a trade-off between unemployment and inflation, providing scope for aggregate demand management. In theory, it seemed possible to reduce unemployment if the policymaker was willing to tolerate the accompanying increase in inflation. As a result, the appearance of the Phillips curve bolstered the trend to policy activism encouraged by the growing popularity of Keynesian economics, which assumes that perfect competition is a poor approximation of the actual economy because nominal wages are sticky and markets do not clear immediately.

However, initial implementations of the Phillips curve neglected the role of expectations. Friedman (1968) and Phelps (1967) warned that the apparent trade-off would prove illusory since agents will adapt their inflation expectations to a new policy environment. In the long run, they argued, the economy cannot deviate from the "natural" rate of unemployment unless agents' inflation expectations are systematically wrong. In other words, the long-run equilibrium is still characterised by the money neutrality proposition of Classical economics, since policymakers would not be able to alter the long-run level of output or unemployment. As Friedman (1968, p. 11) declared: "...there is always a temporary trade-off between inflation and unemployment; there is no permanent trade-off. The temporary trade-off comes not from the inflation per se, but from unanticipated inflation... from a rising rate of inflation... A rising rate of inflation may reduce unemployment, a high rate will not." The long-run Phillips curve is vertical at the "natural rate of unemployment" which Friedman (1968, p. 8) defined as: "...the level that would be ground out by the Walrasian system of general equilibrium equations, provided there is imbedded in them the actual structural characteristics of the labour and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labour availabilities, the costs of mobility, and so on." Clarifying the important role of expectations, Friedman (1968) and Phelps (1967) made the crucial distinction between expected and unexpected changes in inflation. While they acknowledged that surprise factors can temporarily drive unemployment below the natural level, they insisted that surprise factors will necessarily disappear. As workers learn that the price level has increased and recognise that real wages have not changed, unemployment will return to its natural rate. The expectations-augmented Phillips curve derived from the work of Friedman and Phelps can be expressed as follows:

$$\pi_t = E_{t-1}\{\pi_t\} + \lambda \hat{y}_t, \quad (2)$$

where  $\pi_t$  denotes the period  $t$  inflation rate, defined as the rate of change of prices from the period  $t-1$  to period  $t$ ,  $E_{t-1}\{\pi_t\}$  is expected inflation in period  $t$  conditional on the information available at  $t-1$ , and the term  $\hat{y}_t$  denotes excess demand in period  $t$  (measured by the output gap between observed and potential output or by the unemployment gap between the observed and the natural rate of unemployment).

The theoretical criticism by Friedman and Phelps was reinforced in the 1970s by an apparent “breakdown” of the empirical Phillips curve for the United States which accumulated systematic forecast errors. The severe recession of 1974-75, during which high levels of unemployment coexisted with a high inflation rate, further reduced the credibility of the Keynesian theory. However, research continued on the expectations-augmented Phillips curve<sup>1</sup>.

A more fundamental criticism of the Phillips curve is that it originated as an empirical observation, with its theoretical justification provided ex post in a somewhat ad hoc manner. Along with Keynesian economic theory in general, the Phillips Curve was accused of lacking microeconomic foundations because it was not derived from a model of optimising behaviour by individual agents. Lucas (1972) provided such microeconomic foundations for the Phillips curve by introducing an imperfect information assumption in a general equilibrium model of rational optimising agents. However, he stressed that the Phillips Curve was only a reduced form relationship, meaning that the observed correlation between inflation and unemployment was inherently unstable and could not provide a useful guide for policy.

The work by Lucas was fundamental in extending macroeconomics to include the rational expectations hypothesis, the notion that inflation expectations could not systematically differ from actual inflation. The New Classical macroeconomics of the 1970s combined rational expectations with the assumption of flexible prices, often leading to the conclusion that policy was ineffective. More recently, the combination of rational expectations and flexible prices has been associated with real business cycle theory, which uses general equilibrium models of rational optimising agents to focus on productivity shocks as the source of business cycle fluctuations. This approach is theoretically appealing as it extends the Walrasian paradigm, the mostly widely understood model in economics, and provides a unified explanation for economic growth and economic fluctuations. However, as argued by Mankiw (1989), real business cycle theory is at odds with several features commonly observed in real economies. In particular, consumption and leisure generally move in opposite directions over the business cycle. To explain this observation in terms of relative prices, real business cycle theory would require procyclical movement in the real wage that contradicts observed data. Real business cycle theory also yields the counterfactual prediction that the price level should be countercyclical, contradicting the observed short-run Phillips curve relationship.

New Keynesian theory (see Ball, Mankiw and Romer 1988, Gordon 1990, Goodfriend and King 1997) attempts to inject more realism in general equilibrium models with rational optimising agents by introducing real and nominal rigidities. Early new Keynesian work by Fischer (1977),

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<sup>1</sup> See Gordon (1998), Lown and Rich (1997), Clark and Laxton (1997) and references therein.



Taylor (1980), Calvo (1983), and others sought to reconcile nominal rigidities with the rational expectations assumption and optimising behaviour. Fischer (1977) showed that given overlapping labour contracts, rational expectations did not imply that monetary policy was neutral in the short run. Taylor (1980) also used a model where wage contracts are “staggered” or overlapping to show that it was consistent with observed persistence in unemployment and inflation. If wage contracts last several periods but they are not all made at the same time, then firms must take account of wage rates set by other firms (in past and future periods) which will be in effect during their own contract period. Taylor derives an equation similar to the forward-looking expectations-augmented Phillips curve with the addition of a backward-looking or inertial component, since expectations incorporate both future and past wage decisions. One of Taylor’s main conclusions is that a rational expectations model with staggered wage contracts that last 3 to 4 quarters can generate unemployment persistence (and therefore business cycles) consistent with US evidence. In addition, his model also generates wage and price persistence that gives rise to a statistical Phillips curve relationship. These persistence features generate a policy trade-off between price stabilization and output stabilization (not between the level of inflation and unemployment). Calvo (1983) reached similar results but relaxed the assumption that all contracts had the same duration. In his model, firms need not be bound by nominal contracts, but are subject to rigidities that prevent them from making continuous price revisions. Instead, each period any firm has a constant probability of changing its price (possibly corresponding to the probability that it observes the “true” state of the economy).

Since prices reflect wage costs, the move from models with staggered wages to models with staggered prices may seem innocuous. In fact, it required another crucial innovation in New Keynesian economics: the introduction of monopolistic competition. This was necessary because perfectly competitive firms face an infinitely elastic demand and must all charge the same price, meaning that they cannot adjust prices independently. Price rigidities only make sense in a context where firms have some monopoly power that allows them to set their prices and therefore to choose when to adjust their price. Ball, Mankiw and Romer (1988) pointed out that the monopolistic competition framework had several additional advantages. First, it provided a microeconomic explanation for the failure of price setters to restore equilibrium. Second, under imperfect competition the level of aggregate output is constrained below that attained in a perfectly competitive equilibrium. This means that output is demand determined in the short run, explaining the real output effect of money, and allowing booms to raise welfare (as they cannot under perfect competition, where output in the absence of shocks is at an efficient level). Third, imperfect competition implies that firms fix prices as a markup on wage costs. If this markup is countercyclical, this allows the real wage to be acyclical or weakly procyclical as is actually observed, and not countercyclical as in some early models with nominal rigidities.

However, Gordon (1990) stresses that monopolistic competition itself does not generate price rigidities, since all producers are identical and relative prices must all equal unity. Instead, price rigidities are often justified by reference to “menu costs” associated with price changes. While such costs may be small, Mankiw (1985) and Akerlof and Yellen (1985) showed that in an imperfect competition setting their real effects can be significant. Because monopolistically competitive firms face a downward sloping demand curve, they have less incentive to cut their prices when the demand for their goods declines. Instead, under perfect competition, the gains from nominal adjustment are large so price rigidities would be unlikely. Blanchard and Kiyotaki

(1987) showed that imperfect competition generates an aggregate demand externality because firms leave prices unchanged even though the benefit to society from a price adjustment could be large and the benefit to the firm from keeping the price fixed is small. Ball, Mankiw and Romer (1988) emphasize that while the actual menu costs of physically changing prices may be small, the real source of price rigidities may be tied to the cost of collecting and processing information that makes continuously updating prices prohibitively expensive for most firms. Instead, firms simply take the convenient shortcut of infrequently reviewing and changing prices, which leads to a dynamic problem involving forward-looking expectations.

Ball, Mankiw and Romer (1988) point out that the gradual shift in New Keynesian economics from a focus on nominal rigidities in labour markets to also consider price stickiness in goods market brought important advantages. While there have been attempts to “explain away” nominal rigidities in wages by referring to long-term relationships, this is not possible for product prices which are often determined on spot markets. In addition, while the observed acyclical behaviour of real wages can be explained by the behaviour of the markup under imperfect competition, it is easier to explain if prices are rigid as well as wages. In this case, the response of real wages to a shock then depends on the relative speed of adjustment of prices and wages.

While New Keynesian economics established that nominal rigidities could produce substantial real effects, Ball and Romer (1990) pointed out that for plausible parameter values, small nominal rigidities produce only small real rigidities. They suggested that a more realistic approach would combine real rigidities and nominal rigidities, with the two reinforcing each other to produce significant effects for plausible parameter values. Ball and Romer considered several sources of real rigidity including models where imperfect information causes real rigidities in the goods market or in the labour market. They list several explanations for real rigidities: implicit contracts, customer markets, social customs, efficiency wages, inventory models, and models of countercyclical mark-ups.

Roberts (1995) showed that several New Keynesian models with rational expectations have a common New Keynesian Phillips Curve (NPC) representation, including models with costly price adjustment, staggered wage contracts or staggered price setting. While the NPC is related to the expectations-augmented Phillips curve, it explicitly links the output-inflation relationship to the extent of nominal rigidities in the economy. Galí and Gertler (1999) were the first to estimate a structural version of the new Keynesian Phillips curve, allowing them to recover the underlying parameters that quantify the degree of nominal rigidity. Galí and Gertler emphasized three distinctive features of their approach. First, instead of explaining inflation using the imprecisely measured output gap, it linked inflation to real marginal cost. Since marginal costs are the inverse of the mark-up, the cyclical behaviour of the latter plays an important role. Second, Galí and Gertler extended the baseline model to a hybrid version that allowed for a subset of firms using a backward-looking rule, making it possible to account for observed inflation persistence. Third, they identified and estimated all the structural parameters using conventional econometric techniques. This represents a major advantage of the NPC over the traditional Phillips curve, as the latter is only a reduced-form relationship. The theoretical foundation of the NPC provides it with a clear structural interpretation, meaning it is potentially useful in interpreting the impact on inflation of structural change.

## 2.1 The Baseline Model

The Galí and Gertler (1999) model is based on staggered pricing and monopolistic competition. It assumes a continuum of firms distributed uniformly on the unit interval. These are indexed by  $z \subseteq [0,1]$ , with each firm producing a differentiated good so that a typical firm,  $z$ , faces a downward-sloping demand curve for its product:

$$Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t. \quad (3)$$

The nominal price charged by the firm per unit of output is  $P_t(z)$ . The aggregate price,  $P_t$ , and aggregate output,  $Y_t$ , are represented:

$$P_t = \left[ \int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad Y_t = \left[ \int_0^1 Y_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}. \quad (4)$$

where the parameter  $\varepsilon$  is the constant price elasticity of demand facing an individual firm, as well as the Dixit-Stiglitz (1977) elasticity of substitution between differentiated goods. Each firm faces a constraint on the frequency of price adjustment that it can undertake. In the Calvo (1983) framework, each firm keeps its price fixed until it receives a random signal (possibly costly information) that allows it to change its price. Calvo assumes that each firm faces a constant probability,  $1-\theta$ , of adjusting its price in any given period, independent of the history of previous price adjustments. Thus, the expected duration for the period during which a firm's price remains unchanged is given by  $(1-\theta) \sum_{k=0}^{\infty} k \theta^{k-1} = 1/(1-\theta)$ .

The production side of the firm is characterized by a Cobb-Douglas<sup>2</sup> technology:

$$Y_t(z) = K_t(z)^\alpha (A_t N_t(z))^{1-\alpha}, \quad \alpha \in [0,1], \quad (5)$$

where  $K_t(z)$  and  $A_t N_t(z)$  represent a firm's capital and effective labour requirements to produce output. The variable  $A_t$  represents a common labour-augmenting technology shock.

At time  $t$ , a fraction  $1-\theta$  of firms reset their prices. The profit-maximization problem for every price-adjusting firm is identical since the firms are identical. Each firm chooses the same optimal price,  $P_t^*$ , by maximizing the expected discounted profits conditional on the expected future technology and demand conditions, and the possibility of price stickiness in future periods. Letting lower case letters indicate variables expressed as percent deviations from their level in a zero-inflation flexible-price equilibrium, a firm's optimal pricing rule<sup>3</sup> can be written:

$$p_t^* = \log \mu + (1-\theta\beta) \sum_{j=0}^{\infty} (\theta\beta)^j E_t \left( mc_{t,t+j}^n(z) \right), \quad (6)$$

<sup>2</sup> Other production function specifications are possible, for example, constant elasticity of substitution between imported and domestic goods, but they will be discussed later.

<sup>3</sup> Equation (6) is the optimal pricing rule as reported by Galí and Gertler (1999). A detailed derivation is available, though with a slightly different approach, in Gagnon and Kahn (2001).

where  $p_t^*$  is the optimal price set at time  $t$  by an optimising firm,  $\beta$  is a constant subjective discount factor,  $mc_{t,t+j}^n(z)$  is the firm's nominal marginal cost in period  $t$  (as a percentage deviation from the steady state), and  $\mu = \varepsilon/(\varepsilon-1)$  is the firm's desired gross mark-up over costs under flexible prices. Intuitively, a firm sets its price as a mark-up over a discounted stream of expected future nominal marginal costs. In the limiting case of complete price flexibility ( $\theta=0$ ), price is just a fixed mark-up over the current marginal cost. As the degree of price rigidity measured by  $\theta$  increases, so does the expected time the price is likely to be fixed, so the firm places more weight on expected future marginal costs in choosing current price. The aggregate price level evolves according to:

$$P_t = \left[ (1-\theta)(P_t^*)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (7)$$

and its log-linear approximation is:

$$p_t = (1-\theta)p_t^* + \theta p_{t-1}. \quad (8)$$

Substituting for  $p_t^*$  in equation (6) yields the New Keynesian Phillips curve:

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \lambda \xi \hat{m}c_t, \quad (9)$$

where  $\lambda$  and  $\xi$  are functions of the structural parameters of the underlying model:  $\lambda = (1-\theta)(1-\theta\beta)/\theta$ , and  $\xi = (1-\alpha)/[1+\alpha(\varepsilon-1)]$  (assuming constant returns to labour, marginal cost will be identical across firms and  $\xi=1$ ). Otherwise this parameter captures non-constant marginal costs across the firms, see section 2.3). Equation (9) is the baseline New Keynesian Phillips curve (NPC) relating inflation to current expectations of future inflation and a cyclical measure of real activity – real marginal costs. Expressed in terms of  $\lambda$  and  $\xi$  it is a reduced form, as these two parameters depend on the underlying structural parameters  $\theta$ ,  $\beta$ ,  $\alpha$ , and  $\varepsilon$ . The structural form can be estimated conditional on calibrated values of the technology parameter  $\alpha$  and the demand elasticity  $\varepsilon$ .

Iterating equation (9) forward yields an expression for current inflation in terms of a discounted sum of expected future marginal costs:

$$\pi_t = \lambda \xi \sum_{k=0}^{\infty} \beta^k E_t \{ \hat{m}c_{t+k} \}. \quad (10)$$

Galí and Gertler call this discounted sum “fundamental” inflation and compare it to the actual inflation to provide an additional goodness-of-fit measure.

## 2.2 Output gap measure of real marginal cost

Traditional Phillips curve theory emphasizes some output gap measure as the proxy of real economic activity as opposed to real marginal costs. However, under certain assumptions, there is an approximate log-linear relationship between the two variables. Rotemberg and Woodford (1997) used a standard sticky-price framework to show there is an approximate proportional relation between marginal costs and output in the neighbourhood of the steady state. Denoting the output gap  $x_t \equiv y_t - y_t^*$ , where  $y_t$  is the log of output and  $y_t^*$  the log of the ‘natural’ level of

output (under perfectly flexible prices), this proportional relationship with marginal costs can be written:

$$mc_t = \kappa x_t, \quad (11)$$

where  $\kappa$  is the output elasticity of marginal cost and  $x_t$  is the output gap. Combining equation (11) with (9) yields a Phillips curve relationship:

$$\pi_t = \lambda \kappa x_t + \beta E_t \{ \pi_{t+1} \}, \quad (12)$$

with a key difference in the expectation term. In this case the expectations term  $E_t \{ \pi_{t+1} \}$  is forward-looking, in contrast to the conventional expectations-augmented Phillips curve where the expectations term is  $E_{t-1} \{ \pi_t \}$  and is often replaced with lagged inflation  $\pi_{t-1}$  by assuming adaptive expectations.

However, there are a few drawbacks when using the output gap as a measure of real marginal costs. Fuhrer and Moore (1995) point out that the new Keynesian Phillips curve implies no short-run trade-off between output and inflation. Iterating equation (12) forwards yields:

$$\pi_t = \lambda \kappa \sum_{k=0}^{\infty} \beta^k E_t (x_{t+k}), \quad (13)$$

which implies that inflation depends entirely on the discounted sequence of future output gaps. Thus, if a central bank could credibly commit to disinflation policy there should be no reduction in output. Unfortunately, as pointed out by Ball (1995), this result contradicts actual disinflation experiences. In addition, a new Keynesian Phillips curve including the output gap implies that inflation should lead the output gap over the cycle. In other words, a rise in current inflation should signal a future rise in the output gap. However, Galí and Gertler show that exactly the opposite can be observed in the data – the current output gap co-moves positively with future inflation and negatively with lagged inflation.

Roberts (2001) argues that the importance of lagged inflation might be an indication that inflation expectations deviate from full rationality. For example, inflation expectations may be a weighted average of rational expectations and a simple univariate forecasting equation. In earlier work, Roberts (1998) analysed survey measures of inflation expectations and found that 40% of respondents may be using a simple univariate rule for the forecasting inflation while the rest of population has rational expectations. Roberts concluded that the presence of lagged inflation in empirical estimates of New Keynesian Phillips curve may reflect imperfectly rational expectations rather than the underlying structure of the economy.

Use of the output gap to proxy real marginal cost is also problematic because it is generally acknowledged that output gap estimates are subject to considerable measurement error. This reflects the unobservable nature of underlying concepts such as potential output and the natural rate of unemployment. It is common to adopt a univariate approach, estimating the output gap using a deterministic trend or the Hodrick-Prescott filter. This ignores evidence that the natural rate of unemployment varies over the time (Gordon, 1998) and could be influenced by policy changes. The resulting measurement error in the output gap variable (Lown and Rich, 1997) could bias estimates of the Phillips curve parameters.

### 2.3 Unit labour cost measure of real marginal cost

To find an expression for inflation in terms of an observable measure of aggregate marginal costs, Galí and Gertler assume that factor markets are competitive so that cost minimization implies that firms' real marginal cost equals the ratio of the real wage to the marginal product of labour (unit labour cost). With Cobb-Douglas technology (equation (5)), a firm that optimally sets its price in  $t$  will face real marginal cost in period  $t+k$  given by:

$$MC_{t,t+k} = \frac{(W_{t+k} / P_{t+k})}{(1-\alpha)(Y_{t,t+k} / N_{t,t+k})}, \quad (14)$$

where  $Y_{t,t+k}$  and  $N_{t,t+k}$  are output and employment for a firm that last adjusted its price to the optimal level  $P^*$  at time  $t$ . Since individual firm-level marginal costs are not observable, Galí and Gertler define the observable 'average' marginal cost, which depends only on aggregates:

$$MC_t = \frac{(W_t / P_t)}{(1-\alpha)(Y_t / N_t)} \quad (15) \quad \text{or} \quad MC_t = \frac{S_t}{1-\alpha}, \quad (16)$$

where  $S_t$  is the unit labour cost measured by the labour share of income and is defined  $S_t = (W_t/P_t)/(Y_t/N_t) = (W_t N_t)/(Y_t P_t)$ , with  $W_t N_t$  denoting the aggregate wage bill of the economy,  $N_t$  aggregate employment, and  $Y_t$  real GDP.

Galí, Gertler, and López-Salido (2001) obtain the following log-linear relation between individual marginal cost,  $MC_{t,t+k}$ , and aggregate marginal cost,  $MC_t$ :

$$\hat{m}c_{t,t+k} = \hat{m}c_{t+k} - \frac{\varepsilon\alpha}{1-\alpha}(p_t^* - p_{t+k}), \quad (17)$$

where  $\hat{m}c_{t,t+k}$  and  $\hat{m}c_{t+k}$  are the log deviations of  $MC_{t,t+k}$  and  $MC_{t+k}$  from their respective steady-state values. According to equation (17), given the concave production function, firms that maintain a high relative price will face a lower marginal cost than the norm. In the case of linear technology ( $\alpha=0$ ) all firms will be facing a common marginal cost (in equation (9) this would be consistent with  $\xi=1$ ). However, in reality  $\alpha \neq 0$  and relative prices are different across firms. The parameter  $\xi$  is meant to capture the fact that the unobservable firm-level marginal cost may deviate from the observable average marginal cost. The value of  $\xi$  is calibrated according to the underlying characteristics of the economy (details of calibration are given later).

Substitution of the equation (17) into equation (6) yields equation (9) but in the form:

$$\pi_t = \beta E_t(\pi_{t+1}) + \lambda \xi \hat{s}_t, \quad (18)$$

where  $\hat{m}c_t$  is approximated by  $\hat{s}_t$ , the log deviation of the labour income share ( $S_t$ ) from its steady-state value, and  $\xi = \frac{1}{1 + [\varepsilon\alpha/(1-\alpha)]} = \frac{(1-\alpha)}{[1 + \alpha(\varepsilon-1)]}$ .

Equation (17) relating firm-level marginal costs and the average marginal cost is based on the underlying Cobb-Douglas (CD) production function. Gagnon and Khan (2001) provide explicit derivations of alternative marginal cost measures using other production technologies: Cobb-Douglas with overhead labour (CDOL), constant elasticity of substitution (CES) and CES with

overhead labour (CESOL). They conclude that for the US, UK, and Canada the measures of marginal costs based on CES and CESOL yield better (more precise) estimates of the NPC relative to the CD measure.

## 2.4 Hybrid New Keynesian Phillips Curve

The original baseline NPC described in the previous section is not consistent with observed persistence in inflation (see equation (9)), so Galí and Gertler (1999) extend it to allow for a subset of firms that set prices using a backward-looking rule-of-thumb.

As in the baseline model, each firm adjusts its price in any given period with a fixed probability  $1-\theta$  that is independent of the time it last changed its price. However, Galí and Gertler (1999) depart from the original Calvo setting by modelling two types of firms. A fraction  $1-\omega$  of the firms are 'forward-looking' and behave as in baseline model – i.e. set prices optimally, given the time constraints on adjustment and using all available information to forecast future marginal costs. The remaining fraction of firms  $\omega$  are 'backward-looking' and use a simple rule-of-thumb based on the recent history of aggregate price behaviour. The aggregate price level in Hybrid NPC evolves according to:

$$p_t = \theta p_{t-1} + (1-\theta) \bar{p}_t^*, \quad (19)$$

where  $\bar{p}_t^*$  is an index of prices newly set in period  $t$ . Let  $p_t^f$  denote the price set by a forward-looking firm and  $p_t^b$  the price set by a backward-looking firm at time  $t$ , then the index for newly set prices can be written:

$$\bar{p}_t^* = (1-\omega) p_t^f + \omega p_t^b. \quad (20)$$

Backward-looking firms choose to reset prices  $p_t^b$  according to the simple rule of thumb:

$$p_t^b = p_{t-1}^* + \pi_{t-1}, \quad (21)$$

where  $p_{t-1}^*$  is the average reset price in time  $t-1$  across both backward- and forward-looking firms. Backward-looking firms observe how firms set their price last period and then make a correction for inflation, using lagged inflation as a predictor. Note that backward-looking firms do take account of optimal prices set by those forward-looking firms that adjusted in the previous period. As in the baseline model, when relaxing the assumption of constant marginal costs ( $\xi \neq 1$ ), the hybrid version of the marginal-cost based Phillips curve can be written as follows:

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \{ \pi_{t+1} \} + \tilde{\lambda} \hat{m}c_t, \quad (22)$$

where  $\tilde{\lambda} \equiv \frac{(1-\omega)(1-\theta)(1-\beta\theta)(1-\alpha)}{\phi[1+\alpha(\varepsilon-1)]}$ ,  $\gamma_b \equiv \omega\phi^{-1}$ ,  $\gamma_f = \beta\theta\phi^{-1}$ , and

$\phi \equiv \theta + \omega[1-\theta(1-\beta)]$ . Equation (22) is the reduced form of the Hybrid NPC where the coeffi-

coefficients  $\gamma_b$ ,  $\gamma_f$  and  $\tilde{\lambda}$  are functions of the structural parameters of the model. In the limiting case of no backward-looking firms ( $\omega=0$ ) this specification reduces to the baseline model of NPC. Thus, since the hybrid model nests the pure forward-looking model, one can check for backward-looking behaviour by testing whether parameter  $\omega$  is equal to zero.

The fundamental inflation measure presented earlier provides an indication of the goodness-of-fit of the model. For the hybrid NPC this is obtained from the solution to the first-order difference equation (22) given by:

$$\pi_t = \delta_1 \pi_{t-1} + \frac{\lambda}{\delta_2 \gamma_f} \sum_{k=0}^{\infty} \left( \frac{1}{\delta_2} \right)^k E_t(mc_{t+k}) + \varepsilon_t, \quad (23)$$

where  $\delta_1$  and  $\delta_2$  are, respectively, the stable and unstable roots of the difference equation. These characteristic roots are given by:

$$\delta_1 = \frac{1 - \sqrt{1 - 4\gamma_b \gamma_f}}{2\gamma_f} \quad \text{and} \quad \delta_2 = \frac{1 + \sqrt{1 - 4\gamma_b \gamma_f}}{2\gamma_f},$$

where  $\gamma_b$  and  $\gamma_f$  are obtained from the estimated structural parameters of the NPC as presented with equation (22), (Galí, Gertler, López-Salido, 2003).

## 2.5 Open-economy Phillips Curve

The original NPC described in sections 2.1 and 2.4 has been extended to the open economy using two different approaches in the recent literature. First, several studies augment real marginal costs by some measure of the relative price of domestic and foreign inputs (Galí and López-Salido, 2001; Balakrishnan and López-Salido, 2002; Leith and Malley, 2002; Gagnon and Khan, 2001). The second approach is to augment real marginal costs by the real effective exchange rate (Guender, 2003). However, the second approach has a slightly different monopolistic competition framework: firms aim to minimize menu costs weighed against the cost of being away from the optimal price they would charge in the absence of those menu costs. Since this approach lacks an underlying structural model of the economy, the emphasis below will be on the first approach.

To account for the openness of the economy and to provide a channel for import prices to affect marginal costs and inflation dynamics, the production function is characterized by CES technology as follows:

$$Y_t = F(N, M) = \left[ \alpha_N (Z_t N_t)^{\frac{1}{\sigma}} + \alpha_M (M_t)^{\frac{1}{\sigma}} \right]^{\sigma}, \quad (24)$$

where  $M_t$  represents imported materials (i.e. intermediate goods), and  $\sigma$  is the elasticity of substitution between domestic and imported inputs. From cost minimization, the following equilibrium condition holds:

$$\frac{N_t}{M_t} = \left( \frac{P_{M_t}}{W_t} \right)^{\sigma}, \quad (25)$$

where  $P_{M_t}$  is the price of imported materials, and  $W_t$  is the nominal wage. The expression for real marginal cost<sup>4</sup> is:



$$mc_t = \frac{s_t}{1 - k \left( \frac{Y_t}{M_t} \right)^{\frac{1}{\sigma-1}}}. \quad (26)$$

Substituting expression (25) into (26), and log-linearising the equation, yields the following specification for real marginal costs:

$$\hat{m}c_t = \hat{s}_t + \phi(p_{M_t} - w_t), \quad (27)$$

where  $\phi = \left( \frac{1 - \mu s}{\mu s} \right) (\sigma - 1)$ .

In the open economy specification real marginal costs depend on real unit labour costs and an additional term - the relative price of domestic and foreign inputs. This allows movements in the exchange rate to affect inflation dynamics as the price of imported materials  $p_M$  affects the price of output. The parameter  $\phi$  determines how changes in the ratio of relative prices affect movements of marginal costs and therefore inflation. Note that  $\phi$  can take either sign as  $0 < \mu s < 1$ . If  $\sigma > 1$  then  $\phi$  is positive and if  $\sigma < 1$  then  $\phi$  is negative.

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<sup>4</sup> Derivation available in the appendix of Balakrishnan and López-Salido (2002).

### 3. Empirical Evidence on the New Keynesian Phillips Curve

The structural NPC was first estimated by Galí and Gertler (1999) using quarterly US data. Galí and Gertler use the GDP deflator to measure inflation, make no adjustment for non-constant marginal costs across firms, and ignore open economy considerations. To allow for forward-looking expectations, the equation is estimated by generalized method of moments (GMM). The main conclusions are:

- Real marginal costs, approximated by real unit labour costs, are a statistically significant and quantitatively important determinant of inflation, as predicted by theory. The coefficient  $\lambda$  on marginal costs is positive and statistically significant;
- Forward-looking price-setting behaviour is important. Model estimates suggest that 60-80% of firms exhibit forward-looking price-setting behaviour;
- Backward-looking behaviour is statistically significant (using hybrid NPC specification), however, Galí and Gertler consider that approximately 20% of firms using backward-looking price setting behaviour is not quantitatively important;
- Prices are fixed on average for 5 quarters;
- The output gap is not a good approximation of real aggregate marginal costs;
- A measure of fundamental inflation reflects observed inflation dynamics quite well;

This article was followed by Galí, Gertler, and López-Salido (2001) who estimated the NPC for the US and the euro area using quarterly data for the time period 1970-1998. Main conclusions were that the NPC explains inflation dynamics quite well in both economies. Forward-looking behaviour is important, but more dominant in the euro area. Price stickiness appears to be smaller in the US – the average period prices are set is now estimated at 2 quarters compared to 3-5 quarters in the euro area. For the euro area, the measure of fundamental inflation based on the baseline model seems to track actual inflation quite closely. The assumption of constant marginal costs across the firms suggests an overestimation of the degree of price stickiness, thus an aggregation adjustment  $\xi$  for variable marginal costs (equation (9)) is necessary. The authors also conclude that the sluggish behaviour of marginal costs is an important factor accounting for the high degree of persistence observed in inflation, and that a future analysis of the behaviour of wage mark-up could help to explain the dynamics of marginal costs.

Jondeau and Le Bihan (2001) examined the importance of the forward-looking component in the inflation dynamics of four European countries (Germany, France, Italy and the UK), the euro area, and the US. They used both maximum likelihood and GMM estimation procedures, tried both the output gap and unit labour costs as the driving variable, and also accounted for more than just one lag and lead of inflation. They concluded that the backward-looking component is significant in all cases as well as the forward-looking component. The two have roughly equal weights; however, the backward-looking component for the euro area appears to be too small compared to the results for individual countries.

Galí and López-Salido (2001) find support for the NPC using Spanish data. They conclude that the labour share explains inflation dynamics quite well as implied by the theory. In addition, they find substantial backward-looking price-setting behaviour in Spain, with the weights for forward- and backward-looking behaviour being almost equal. The degree of price stickiness is estimated at 3 to 4 quarters when allowing for non-constant marginal costs ( $\xi \neq 1$ ). Relative prices of imported materials are significant, but the structural parameter estimates are not very different from those of the closed economy specification.

Gagnon and Khan (2001) estimate the NPC model for Canada, the US, and the euro area. They find the proportion of forward-looking behaviour in the euro area ( $\approx 70\%$ ) close to that in Galí, Gertler, and López-Salido (2001), lower in Canada ( $\approx 50\%$ ) and higher in the US ( $\approx 57\%$ ). Leith and Malley (2002) estimate the NPC model for G7 countries. They conclude that the results are different across the countries and that open economy considerations do not significantly alter the parameter estimates from those in the closed economy specification. Benigno and López-Salido (2002) also estimate the NPC for several euro area countries (Germany, France, Italy, Spain and the Netherlands) and find that there is a significant backward-looking component in inflation in four of them, although inflation in Germany is dominated by the forward-looking component. Balakrishnan and López-Salido (2002) find for the UK that adjusting the labour share for public sector employment and incorporating relative import prices yields a better fit of fundamental inflation to actual inflation, but some overprediction and underprediction problems remain.

To summarize, it is clear that the estimates of the NPC yield different empirical results across countries: the degrees of forward- and backward-looking price-setting behaviour differ substantially across the EU, the US, and other countries, as does the degree of price stickiness. There is still no consensus as to whether unit labour costs, as a proxy of real marginal cost, should be preferred to the output gap. Most of the studies conclude that the measures of real marginal costs derived from different production functions do not yield substantially different results compared to the baseline Cobb-Douglas production function. Relaxing the assumption of constant individual marginal costs ( $\xi=1$ ) yields more precise estimates of the duration of price stickiness. Furthermore, almost all studies (except for Galí and Gertler, 1999, and Galí, Gertler, and López-Salido, 2001) reject the pure forward-looking NPC, and the hybrid version of the NPC dominates. Thus, there is a clear indication that not all agents form purely rational expectations, a result consistent with evidence from surveys of inflation expectations (Roberts, 1997; Mankiw, Reis and Wolfers, 2003). This is an indication that further research on expectations formation is crucial to better understand the behaviour of the economy.

### 3.1 Criticisms of the New Keynesian Phillips Curve

Fuhrer and Moore (1995) criticised the New Keynesian staggered pricing approach, claiming that it could not generate a realistic degree of inflation persistence. They suggested a model in which agents care about relative rather than nominal wages, and found that this explains quarterly US data better than the standard Taylor model. While the standard staggered contracts model relates a two-sided average of the price level to an excess demand term, the relative contracting model relates a two-sided average of inflation to the excess demand term:

$$\pi_t = \delta x_t + (1 - \phi) E_t \{ \pi_{t+1} \} + \phi \pi_{t-1} \quad (28)$$

with  $0 < \phi < 1$ . This specification focuses the discussion on the relative size of the weights attached to forward-looking expectations and lagged inflation, where the latter helps to capture inflation persistence missing in the purely forward-looking NPC. Although this formulation may display more realistic dynamics, its motivation is largely empirical and it has no micro foundations in individual optimisation.

Several other authors also criticised the New Keynesian Phillips curve for neglecting the lagged inflation terms that have proved important in empirical implementations. Fuhrer (1997) sought to assess the empirical relevance of forward-looking inflation expectations in the Phillips curve using quarterly US data. He estimated a specification that nests the purely forward-looking Phillips curve, the purely backward-looking Phillips curve and a two-sided version that mixes both forward-looking and backward-looking inflation expectations. Fuhrer estimated that 80% of firms set prices according to a backward-looking rule while the remaining 20% are forward looking. However, since the latter proportion is not statistically significant, he concluded that forward-looking expectations are empirically unimportant in explaining inflation. Fuhrer did stress, however, that the dynamic properties of the model including both forward-looking and backward-looking expectations seemed more plausible than those of the purely backward-looking specification. A similar point was made by Roberts (2001), who also found that the purely forward-looking New Keynesian Phillips curve fits US data poorly and that performance was improved by including additional lags of inflation not implied by the model under rational expectations.

Rudd and Whelan (2001) also criticised the New Keynesian Phillips curve for omitting lagged inflation. They begin by noting that small specification errors can produce misleading results regarding the relative weight of forward- and backward-looking expectations in the hybrid specification. In fact, they show that the relatively large weight estimated on the forward-looking component could be due to omitted variable bias and is actually consistent with a wholly backward-looking mechanism. Observing that Galí and Gertler's tests have very low power against the alternative, non-nested, backward-looking specification, Rudd and Whelan propose a new test of the New Keynesian Phillips curve based on the expected present discounted value of the future driving variables. On this basis, they find little support for the claim that forward-looking expectations are an important determinant of inflation dynamics.

In a second paper, Rudd and Whelan (2002) criticise the New Keynesian Phillips curve from a different angle. Here they focus on the model's implication that inflation should reflect the net present value of future levels of the driving variable (i.e. the fundamental inflation measure):

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k E_t x_{t+k}, \quad (29)$$

where  $x_{t+k}$  may stand for either the output gap or the labour share. Equation (29) resembles fundamental inflation as defined in equation (10). Rudd and Whelan use a vector autoregressive (VAR) system to generate expected future values of the driving variable  $x_t$  in terms of currently observed variables. They find that the resulting fundamental inflation fits observed inflation poorly and that the fit is sensitive to variations in the specification of the VAR. In addition, when they augment equation (29) with lagged inflation, the relevant coefficient appears to be significant, contradicting the model. Thus, Rudd and Whelan conclude that the NPC model fails to account for observed inflation persistence. Finally, they find no evidence that inflation Granger-causes the labour share of income as would be implied by the model (in which inflation summarizes agents expectations regarding future values of the driving variable). In defense of the New Keynesian Phillips curve, one could argue that Rudd and Whelan augment the fundamental inflation measure instead of the reduced form of the structural NPC itself, and that Galí and Gertler (2001) found no evidence that lagged inflation was significant beyond the one lag included in the hybrid specification of the NPC model.

In a third paper, Rudd and Whelan (2003) focus on the hybrid model of the new Keynesian Phillips curve. They point out that this version has a clear implication for the change (rather than the level) of inflation and find that this is rejected by the data both using their present-value approach based on VAR forecasts of the driving variable and using GMM estimates. Their results confirm their earlier conclusion that the discounted sum of future labour shares or output gaps explains very little of the inflation process. They conjecture that the problem with the New Keynesian Phillips curve is that it relies too heavily on a strict form of the rational expectations assumption. They conclude that: "it may well be that  $E_t \pi_{t+1}$  is a key influence on current inflation. But... the evidence indicates that this expectation is not determined in the manner that the current generation of rational sticky-price models would predict."

Galí, Gertler and López-Salido (2003) defend their findings with respect to the significance of the labour share and the importance of the forward-looking behaviour. They claim that their results are robust to a variety of estimation procedures, including GMM estimation of the reduced form and non-linear instrumental variable estimation. They also quote work on sticky-price models of the business cycle that find similar results using maximum likelihood procedures. Galí, Gertler and López-Salido do acknowledge that the incorporation of the lagged inflation term in the NPC needs to have more rationale and that the simple Calvo price-setting might be unrealistic.

More specifically, the Calvo assumption that firms face a constant probability of changing prices is not realistic. The conventional wisdom is that higher inflation should induce firms to change prices more often, because it implies that firms with fixed nominal prices will see their markups eroding faster. Ball, Mankiw and Romer (1988) used cross-country data to explore the implication that the Phillips curves are flatter in low-inflation environments. Bakhshi, Burriel-Llombart, Khan and Rudolf (2003) showed that the standard Calvo assumption of a constant probability of price changes places an upper bound on the trend inflation for which sticky-price models can be solved. Under this assumption, firms would stop production completely if annualised trend inflation rises above 5.5 percent. Instead, Bakhshi et al. developed a model of endogenous price stickiness which allows firms to choose the frequency of their price adjustment, so that the probability that firms keep their price unchanged falls when trend inflation rises. Ascari (2003) noted that trend inflation matters a lot in models with staggered prices, not only for the steady-state properties of the model but also for its dynamic properties. If trend inflation is not considered then the model outcomes might be misleading.

Mankiw (2001) criticises the New Keynesian Phillips curve on three different points. Beyond the costless disinflation issue raised by Ball (1995) and the inflation persistence problem identified by Fuhrer and Moore (1995), Mankiw argues that the NPC generates implausible impulse response functions to monetary policy shocks. Because the NPC links inflation to unemployment with certain leads or lags, it implies a restriction across the impulse response function for inflation and that for unemployment. Mankiw shows that under the NPC shocks to monetary policy fail to generate realistic effects on unemployment and also fail to generate the typical delayed and gradual effect on inflation.

Neiss and Nelson (2002) question Galí and Gertler's (1999) criticism of the output gap as a poor proxy for marginal cost in New Keynesian Phillips curves. Neiss and Nelson argue that the output gap deserves reconsideration in the NPC, provided it is derived in a manner consistent

with theory. They stress that “potential output” is defined differently in dynamic stochastic general equilibrium (DSGE) models from how the concept is typically used in empirical work. In DSGE models, potential output corresponds to the output level that would prevail if there were no nominal rigidities in the economy, whereas in practice it is often estimated by extracting the trend from observed output. Neiss and Nelson emphasize two implications of the DSGE definition. First, the correct notion of potential output responds to real shocks over the business cycle, so it does not follow a smooth path as implied by filtering or detrending methods. Second, this output gap concept captures that portion of the movement in output that can be attributed solely to the existence of nominal rigidities, so the output gap properly defined is not a measure of the business cycle. If prices are fully flexible, the output gap should be zero even if there is variation in output at business-cycle frequencies. Neiss and Nelson obtain an output gap measure from a DSGE model with capital accumulation and habit formation. Simulating the model under a policy reaction function estimated from the data, they find that their ‘correct’ output gap has a negative correlation (-0.68) with detrended output. They conclude that estimates of the NPC using output gap measures based on detrending methods do not provide valid tests of the output-gap-based NPC. If theory-consistent output gap series are used, NPC estimates feature the expected positive coefficients on the output gap. Neiss and Nelson also find some support for the hypothesis that gap-based Phillips curves are more empirically robust than cost-based Phillips curves.

Bårdsen, Jansen and Nymoer (2002a,b) argue that existing evidence in favour of the NPC is weak. They point out that the emphasis on goodness-of-fit is misplaced because despite the NPC’s economic content, its empirical implementations are usually close to random walk models of inflation. They also point out that tests of the significance of the forward term in the hybrid NPC model may be invalidated by the model misspecification implicit in the common practice of pre-whitening residuals. Bårdsen et al. argue that a proper evaluation of the backward- or forward-looking nature of the solution for inflation can only be established within a system that includes an equation for the forcing process (marginal cost or output gap) in addition to the NPC. In contrast, single-equation estimates of the NPC implicitly assume the forcing process is strongly exogenous, excluding a possibly stabilising interaction between price setting and product (or labour) markets. Finally, Bårdsen et al use the encompassing principle to show that existing results in modelling wage and price formation provide a clear rejection of the NPC in a rational expectations framework. They conclude that if forward expectations are to play a significant role in explaining inflation it must be in other better specified models.

McAdam and Willman (2003) argued that the poor empirical performance of New Keynesian Phillips curve may be due to the common practice of calibrating the production function parameters. In particular, they argue that the observed shifts in the labour share of income in Europe are inconsistent with the practice of specifying a standard Cobb-Douglas production function and then calibrating using the observed labour income share. They prefer to estimate a full supply-side model in a framework that allows for changing factor shares. This produces more plausible parameter estimates and a New Keynesian Phillips curve that is robust to re-estimation using the Net Present Value approach advocated by Rudd and Whelan (2003).

### 3.2 An Alternative New Keynesian Approach to Modelling Inflation Dynamics

So far we have discussed the New Keynesian Phillips curve linking inflation and real marginal cost, whether marginal cost is approximated by the output gap or unit labour costs (the labour share of income). However, Mankiw and Reis (2002) propose a different approach to explain the dynamic effects of aggregate demand on output and the price level. The main idea of their model is that information about macroeconomic conditions diffuses slowly through the economy due to costs of acquiring information or costs of reoptimization. It follows that although prices are always changing, price decisions are not always based on current information. Mankiw and Reis name this the sticky-information model and contrast it to the standard sticky-price model on which the New Keynesian Phillips curve is based. The sticky-information model combines elements of the Calvo (1983) model of random adjustment with elements of the Lucas (1973) model of imperfect information. In some ways, the dynamic response in the sticky-information model resembles a Phillips curve with backward-looking expectations, but there is an important difference – in this model expectations are rational and credibility matters, in particular, the earlier a disinflationary policy is anticipated, the smaller is the resulting recession.

#### A Sticky-Information Model (SIM)

In a SIM every firm sets its price every period, but firms only gradually gather information and recompute optimal prices. In each period, a fraction  $\lambda$  of firms obtains new information about the state of the economy and computes a new path of optimal prices, whereas the other firms continue to set prices based on old plans and outdated information. Mankiw and Reis make an assumption analogous to the adjustment assumption in Calvo model: each firm has the same probability of updating pricing plans, regardless of how long it has been since its last update. A firm's optimal price is:

$$p_t^* = p_t + \alpha y_t, \quad (30)$$

where firm's desired price  $p_t^*$  depends on the overall price level  $p_t$  and output  $y_t$  (potential output is normalized to zero, so  $y_t$  should be interpreted as the output gap). A firm's desired relative price,  $p_t^* - p_t$ , rises in booms and falls in recessions. Mankiw and Reis note that this expression can be derived from a firm's profit-maximization problem within a framework of identical monopolistically competitive firms (Blanchard and Kiyotaki, 1987).

A firm that last updated its plans  $j$  periods ago sets the price:

$$x_t^j = E_{t-j} p_t^*. \quad (31)$$

The aggregate price level is the average of the prices of all firms in the economy:

$$p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j x_t^j. \quad (32)$$

Substituting (30) and (31) into (32) yields the following equation for the price level:

$$p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-j}(p_t + \alpha y_t). \quad (33)$$

Equation (33) indicates that output is positively associated with surprise movements in the price level. With some algebra one can obtain the following equation for the inflation rate:

$$\pi_t = \left( \frac{\alpha\lambda}{1-\lambda} \right) y_t + \lambda \sum_{j=0}^{\infty} (1-\lambda)^j E_{t-1-j}(\pi_t + \Delta y_t) \quad (34),$$

where  $\Delta y_t = y_t - y_{t-1}$  is the growth rate of output. In this model inflation depends on output, expectations of inflation, and expectations of output growth. Mankiw and Reis call equation (34) the sticky-information Phillips curve (SIPC).

In the NPC, expectations of future economic conditions play an important role in determining the inflation rate, whereas in the SIPC expectations are important as well, but the relevant expectations are past expectations of current economic conditions. This yields large differences in the dynamic pattern of prices and output in response to monetary policy: according to the NPC, inflation responds immediately and substantially to monetary shocks (unless backward-looking behaviour dominates), whereas according to the SIPC, the effect of monetary shocks on inflation is delayed and gradual (Mankiw 2001, 2002).

To examine dynamic properties of SIPC, Mankiw and Reis complete the model by specifying an aggregate demand equation as follows:

$$m_t = p_t + y_t, \quad (35)$$

where  $m_t$  is the logarithm of nominal GDP. This equation resembles a quantity theory approach to aggregate demand, where  $m_t$  is interpreted as money supply and log velocity is assumed constant or zero. Alternatively,  $m_t$  can be viewed more broadly as incorporating the many other variables that shift aggregate demand<sup>5</sup>. Now a firm's desired nominal price can be written as:

$$p_t^* = (1-\alpha)p_t + \alpha m_t. \quad (36)$$

If  $\alpha$  is small then each firm gives more weight to the prices other firms are charging than to the level of aggregate demand. The inertial behaviour of inflation in the SIPC model requires  $\alpha$  to be less than one. If  $\alpha = 1$ , then the desired price moves only with the money supply, and firms adjust their prices immediately upon learning of the change in policy, so that inflation responds quickly. If  $\alpha < 1$  then firms also care about the overall price level and therefore need to consider what information other firms have at their disposal. Mankiw and Reis explain that a small value of  $\alpha$  can be interpreted as a high degree of real rigidity. In the SIPC this real rigidity is a source of inflation inertia.

Mankiw and Reis treat  $m_t$  as exogenous and simulate the model to examine how output and inflation respond to changes in the path of  $m_t$  for a SIPC, a NPC in the form  $\pi_t = \beta y_t + E_t \pi_{t+1}$  and a traditional backward-looking Phillips curve in the form  $\pi_t = \beta y_t + \pi_{t-1}$ . They find that the SIPC

<sup>5</sup> Mankiw and Reis note that it is possible to model aggregate demand more realistically, for example, by adding an IS equation together with an interest-rate policy rule.



yields more realistic responses of the economic variables (inflation and output) whether to a drop in the level of aggregate demand, to sudden inflation, to an anticipated inflation, or to monetary shocks. In a way, the results based on the SIM combine the elements of the other two models. Like the conventional backward-looking Phillips curve (but unlike the NPC), disinflations consistently cause recessions rather than booms. Like the NPC (but unlike the conventional backward-looking Phillips curve), expectations, announcements, and credibility matter for the path of inflation and output. Examining the response of inflation to a monetary shock, Mankiw and Reis conclude that the SIM can explain a long lag between the monetary policy shocks and inflation adjustment, whereas the standard sticky-price model cannot.

Khan and Zhu (2002) present an empirical implementation of the SIPC for both closed and open economies. They also provide a derivation of the model based on utility maximization by individuals and profit maximization by firms. Khan and Zhu estimate the key structural parameter of the SIPC - the average duration of information stickiness for the United States, Canada and the United Kingdom. The main results indicate that the average frequency of information updates is 4 quarters in the US, between 4 and 5 quarters in Canada, and over 7 quarters in the UK. Estimates for Canada and the UK in an open economy framework are similar to those in the closed economy framework.

Mankiw, Reis and Wolfers (2003) analyse disagreement about inflation expectations among respondents to different surveys. They reach three main conclusions:

- Not everyone has the same expectations: the amount of disagreement is substantial;
- The amount of disagreement varies over time together with other economic aggregates;
- The sticky-information model, according to which some people form expectations based on outdated information, seems capable of explaining many features of the observed evolution of both the central tendency and the dispersion of inflation expectations over past 50 years in the US economy.

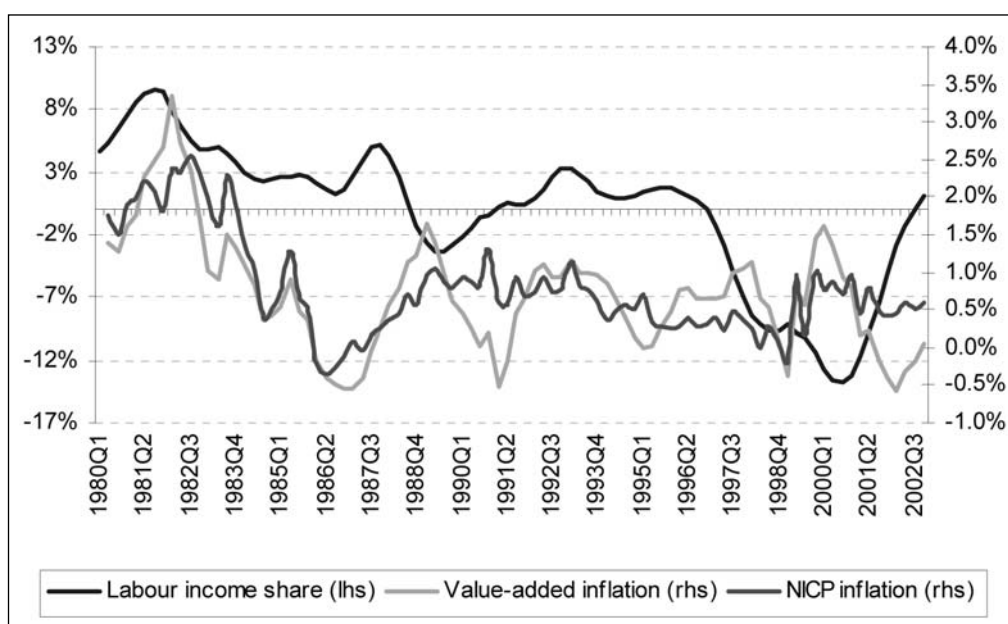
Mankiw and Reis (2002) admit that the choice between the SIPC and the NPC is mostly an empirical issue, but they claim that there are three findings in favour of the SIPC:

- In the SIPC disinflations are always contractionary (although announced disinflations are less costly than surprise ones);
- In the SIPC monetary shocks have their maximum effect on inflation with a substantial delay;
- The SIPC can explain the acceleration phenomenon associated with the positive correlation between the change in inflation and the level of economic activity.

It is not possible to directly compare estimates of the NPC and the SIPC because the two models capture quite different mechanisms. The NPC is a model based on nominal rigidity, whereas the SIPC is a model based on real rigidity. However, one can compare the different estimates of the slope of the Phillips curve, i.e. the size of the short-run output-inflation trade-off. The crucial difference between the two models comes from the role of expectations. The NPC gives a major role to current expectations of future inflation, these expectations adjust quickly in response to the changes in monetary policy. Instead, the SIPC gives more weight to past expectations of current inflation: these expectations are predetermined and therefore cannot change quickly.

#### 4. NPC for Luxembourg – Empirical Results

This section reports estimates of a NPC for Luxembourg using quarterly data obtained by interpolating annual national accounts series for the period 1980-2002. The analysis is restricted to the private sector in Luxembourg as the price markup behaviour implicit in the New Keynesian Phillips curve reflects a monopolistic competition assumption that is inappropriate in the public sector. In the following, the private sector is defined as NACE sectors A-K<sup>6</sup>. Inflation is measured by the quarterly change in the private sector value added deflator. Real marginal cost is measured by the labour income share as obtained from the remuneration of employees and nominal value added in private sector (roughly, the GDP deflator). In contrast to findings for the UK reported by Balakrishnan and López-Salido (2002), results obtained with the whole-economy labour income share (not reported) are not fundamentally different. Figure 1 plots this measure of marginal cost and two alternative measures of inflation.



**Figure 1 Labour income share (log deviation from sample mean) and inflation (value-added deflator or National Index of Consumer Prices)**

The figure reveals no clear relationship between the labour income share and inflation as measured by the value-added deflator. However, the linear correlation coefficient between the two series is 0.28, which is statistically significant. To check whether our interpolation procedure could be smoothing away valuable information, the figure also plots inflation as measured by Luxembourg's National Index of Consumer Prices (NICP). This measure is available monthly, so it does not rely on our interpolation procedure, and it is also of interest because of its link to the ECB definition of price stability. According to the NICP measure, inflation does have a higher correlation coefficient with the labour income share in the private sector (0.44). However, from the perspective of the theoretical model, the NICP is a less satisfactory measure of inflation due to Luxembourg's high degree of openness. In theory, the appropriate inflation

<sup>6</sup> Galí Gertler (1999) concentrate on the US non-farm business sector.

measure should reflect pricing decisions taken by Luxembourg firms. However the NICP includes many imported goods and services and excludes many exported goods and services produced in Luxembourg. Furthermore, many imported goods whose prices that affect the NICP reflect exchange rate movements and foreign price changes rather than marginal costs faced by Luxembourg firms. Therefore, results using NICP inflation are only presented for comparison purposes.

The baseline reduced form NPC, equation (9), can be rewritten with the labour income share replacing real marginal cost:

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \xi \hat{s}_t,$$

where  $\pi_t = p_t - p_{t-1}$  denotes inflation from period  $t-1$  to  $t$ ,  $E_t\{\pi_{t+1}\}$  denotes expected inflation conditional on information in period  $t$ ,  $\hat{s}_t$  denotes the log deviation of  $S_t$ , the labour income share (equivalently, the real unit labour cost), from its steady state level (as approximated by its sample mean) and  $\beta$ ,  $\lambda$ , and  $\xi$  are parameters. Under rational expectations, the forecast error  $E_t\{\pi_{t+1}\} - \pi_{t+1}$  is uncorrelated with the information set dated  $t$ . It follows that:

$$E_t \{(\pi_t - \beta \pi_{t+1} - \lambda \xi \hat{s}_t) z_t\} = 0, \quad (37)$$

where  $z_t$  is a vector of instrumental variables dated  $t$  or earlier and is therefore orthogonal to the inflation surprise in period  $t+1$ . This orthogonality condition forms the basis for the estimation of the reduced form NPC by the Hansen (1982) Generalised Method of Moments (GMM). The instruments selected include up to 3 lags of inflation, the labour income share, the output gap (obtained by applying the Hodrick-Prescott filter to interpolated quarterly real value added), changes in the labour income share, and the spread between long-term and short-term interest rates. Lag 1 of the output gap and the level and changes of the labour income share are not included because it is likely that information about the previous period is unavailable at time  $t$  when people form expectations about inflation in period  $t+1$ . Lag 1 of inflation and the interest rate are included because these series are released in a more timely fashion and are usually not subject to revision.

The parameter  $\xi$  denotes an aggregation factor that intervenes when averaging across individual firms' marginal costs. Gagnon and Kahn (2000) have shown that ignoring this aggregation factor can distort results. For example, in the Cobb-Douglas case  $\xi = (1-\alpha)/(1+\alpha(\varepsilon-1))$ , which may well differ from unity. Following the approach in Galí, Gertler, and López-Salido (2001), we calibrate  $\xi$  from information on the labour share of income and assumptions concerning the markup. By definition the average markup equals the inverse of average real marginal costs. From the assumption of Cobb-Douglas technology it follows that<sup>7</sup>:

$$\alpha = 1 - \frac{S_t}{\mu_t}.$$

<sup>7</sup> In a recent unpublished note, Galí, Gertler and López-Salido provided a correction to this expression. This has the effect of slightly increasing duration in our results.

Thus the calibrated value of  $\alpha$  can be obtained using the sample mean of the labour income share to estimate its steady-state value and an assumption about the markup. One can estimate the value of the price elasticity of demand,  $\epsilon$ , by observing that the steady-state markup should correspond to the desired or frictionless markup, implying the relationship:

$$\epsilon = \frac{\mu}{\mu - 1}.$$

For the average labour share  $S$  we follow Galí, Gertler, and López-Salido (2001) and use the euro area average 0.75 (which yields  $\alpha = 0.32$ ), and the average steady-state markup is assumed to be  $\mu = 1.1$ , which corresponds to a 10% markup over marginal costs (Rotemberg and Woodford, 1995). Galí, Gertler, López-Salido (2001, footnote 24) and Galí and López-Salido (2001, figure 4b) found that results were not affected substantially within a range of plausible values of the steady-state markup. The selected values for  $\mu$  and  $\alpha$  yield an aggregation factor  $\xi = 0.22$  which is assumed in the estimation of the structural form NPC.

#### 4.1 Conventional Phillips Curve

Before presenting the NPC estimation results, we provide estimates of the conventional expectations-augmented Phillips curve.

$$\pi_t = \beta E_{t-1} \{\pi_t\} + \lambda \hat{y}_t. \quad (38)$$

According to theory,  $\beta$  should be just below one and  $\lambda$  should be positive. Table 1 presents the GMM estimates using the instrument set described above. Robust standard errors reported in parentheses are based on the Newey-West heteroscedastic- and autocorrelation-consistent covariance matrix estimator.

**Table 1 Conventional Phillips Curve Estimates: equation (38)<sup>8</sup>**

$\beta$ (s.e.)	$\lambda$ (s.e.)	Adjusted R <sup>2</sup>	J-statistic (p-value)
0.852** (0.0052)	-0.054* (0.021)	0.79	5.31 (0.15)

The significantly negative estimate for  $\lambda$  contradicts the conventional Phillips curve prediction that a positive output gap produces upward pressure on the inflation rate. However, the negative sign is consistent with the NPC interpretation that the output gap is an approximate measure of real marginal costs. To see this, lag equation (12) one period and assume  $\beta \approx 1$  to obtain:

$$\pi_t = -\lambda k x_{t-1} + \pi_{t-1} + \epsilon_t,$$

where  $\epsilon_t = \pi_t - E_{t-1} \pi_t$ . Galí and Gertler (1999) had found a positive output gap coefficient in the US Phillips curve and concluded that the output gap is a poor approximation for real marginal

<sup>8</sup> \*\*, \*, denotes significance at 1% and 5% significance levels respectively.

costs. Paloviita (2002) estimated a similar specification for eleven euro area countries using annual data over 1983-2000 and also found a positive (but insignificant) output gap coefficient for Luxembourg. Instead, the negative coefficient estimate for Luxembourg reported here is consistent with the new Keynesian theory of Phillips curve. The contrast with Paloviita's results for Luxembourg may not only be due to our use of interpolated quarterly series but also because our data has been revised and covers a slightly longer sample.

## 4.2 Pure Forward-Looking NPC

Turning to the NPC with unit labour costs as a proxy for real marginal costs (equation (9)), Table 2 presents *reduced form* estimates of the pure forward-looking NPC.

**Table 2 Reduced form estimates of the forward-looking NPC: Equation (9)**

	$\beta$ (s.e.)	$\lambda\xi$ (s.e.)	Adjusted R <sup>2</sup>	J-statistic (p-value)
$\mu = 1.1,$ $\alpha = 0.32$	1.000** (0.022)	0.007** (0.001)	0.792	11.707 (0.305)

The discount rate  $\beta$  is estimated practically equal to one and the marginal cost coefficient is significantly positive as is implied by theory. The Hansen (1982) test of over-identifying restrictions (J-statistic) cannot reject the validity of the instruments.

The results of the *structural form* NPC are more interesting since they provide an indication of the level of price stickiness. As with the reduced form, the structural coefficients are estimated by GMM but using the following orthogonality condition:

$$E_t \left\{ \left( \pi_t - \beta\pi_{t+1} - \frac{(1-\theta)(1-\theta\beta)}{\theta} \xi s_t \right) z_t \right\} = 0.$$

**Table 3 Structural estimates of the pure forward-looking NPC**

$\mu = 1.1, \alpha = 0.32$	$\beta$	$\theta$	$\tilde{\lambda} = \lambda*\xi$	Duration	J-statistic (p-value)
Unrestricted $\beta$	1.065** (0.002)	0.768** (0.008)	0.012	4.3	11.71 (0.31)
$\beta$ restricted to 0.99	0.99	0.791** (0.015)	0.012	4.8	10.82 (0.46)

The results reported in Table 3 suggest that the proportion of firms in Luxembourg that keep prices constant any given quarter is  $\theta = 77\%$ , which implies an average duration of prices in Luxembourg above four quarters. The estimated discount factor for the unrestricted equation is slightly higher than one, which is contrary to theory. Therefore, the second line reports estimation results with  $\beta$  restricted to 0.99, which yields similar results. The J-statistic still cannot reject instrument validity. Estimates of the parameters  $\beta$  and  $\theta$  are statistically significant at the 1% significance level.

### 4.3 Hybrid NPC

The results of the pure forward-looking NPC are conditional on the rational expectations assumption. To test whether expectations are purely forward-looking, the baseline NPC can be compared to the hybrid NPC that incorporates both forward-looking and backward-looking components. For the hybrid NPC, the orthogonality condition for the reduced form is:

$$E_t \left\{ \left( \pi_t - \gamma_b \pi_{t-1} - \gamma_f \pi_{t+1} - \tilde{\lambda} s_t \right) z_t \right\} = 0. \quad (39)$$

For the structural form, two different normalisations of the orthogonality condition were estimated:

$$E_t \left\{ \left( \pi_t - \omega \phi^{-1} \pi_{t-1} - \beta \theta \phi^{-1} \pi_{t+1} - \phi^{-1} (1-\omega)(1-\theta)(1-\beta\theta) \xi s_t \right) z_t \right\} = 0, \quad (40)$$

$$E_t \left\{ \left( \phi \pi_t - \omega \pi_{t-1} - \beta \theta \pi_{t+1} - (1-\omega)(1-\theta)(1-\beta\theta) \xi s_t \right) z_t \right\} = 0. \quad (41)$$

Results were generally similar so only those for the first normalisation are reported.

**Table 4 Estimates of the Hybrid NPC: equation (22)**

	$\beta$	$\omega$	$\theta$	$\gamma_b$	$\gamma_f$	$\tilde{\lambda}$	Duration 1/(1- $\theta$ )	J-statistic (p-value)
Reduced form				0.495** (0.017)	0.498** (0.022)	0.005** (0.002)		5.76 (0.76)
Structural form <sup>9</sup>	0.958** (0.046)	0.678** (0.034)	0.711** (0.037)	0.495	0.498	0.005	3.46	5.76 (0.76)
Structural ( $\beta=0.99$ )	0.99	0.707** (0.055)	0.683** (0.067)	0.511	0.488	0.005	3.15	6.77 (0.747)

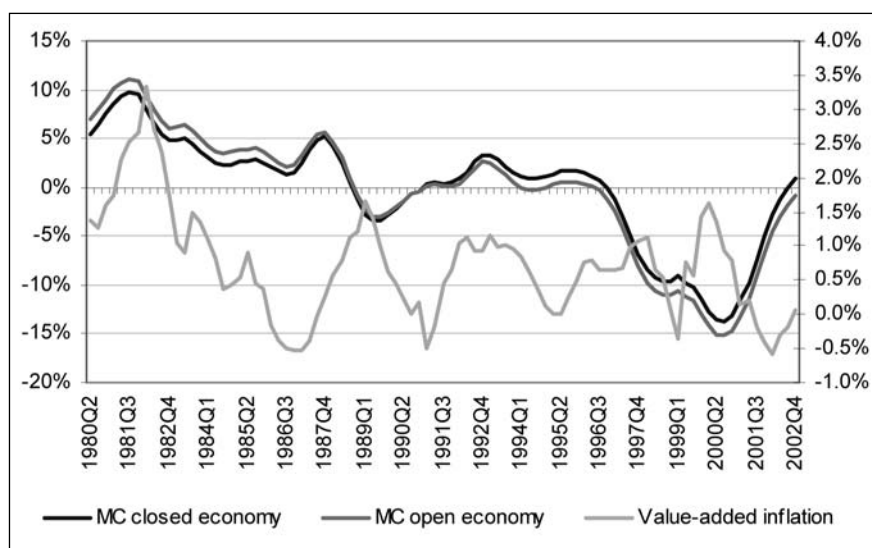
The reduced form estimates of the hybrid model reported in Table 4 reject pure forward-looking price-setting behaviour since  $\gamma_b$  is significantly different from zero. The estimated discount factor  $\beta$  in the structural form is reasonably close to unity. The parameter  $\theta$  determining price rigidity is lower at about 70%, implying a lower average duration (length of time prices remain fixed) of 3.15 to 3.46 quarters. The parameter  $\omega$  indicating the proportion of firms that are backward-looking is about 70%, so only 30% of firms reset prices optimally using forward-looking expectations. All estimates of the parameters are significant at 1% significance level and the Hansen J-statistic is consistent with the validity of the instruments. Restricting the discount factor  $\beta$  to 0.99 does not produce very different estimates of the other parameters. Results obtained using NICP as a measure of inflation (not reported) are similar but with a slightly higher duration of price stickiness.

<sup>9</sup> The structural parameters  $\beta$ ,  $\omega$ ,  $\theta$  were estimated using the restrictions following equation 22 and then used to obtain the parameters  $\gamma_b$ ,  $\gamma_f$  and  $\tilde{\lambda}$ .

#### 4.4 Open economy NPC

Luxembourg is a very open economy, a feature ignored by both baseline and hybrid NPC models. As explained in section 2.5, Galí and López-Salido (2001) suggested an extension of the model to an open economy by using a production function with constant elasticity of substitution between domestic and imported inputs. In the open economy case, real marginal costs are given by equation (27), which includes not only real unit labour costs, but also a term in the relative prices of domestic and imported inputs as well as  $\sigma$ , the elasticity of substitution between the domestic and imported inputs. In the empirical implementation below, the price of imported inputs is measured using unit values of imported goods, which are available on a monthly basis in Luxembourg and so do not require interpolation. The price of domestic inputs is approximated using the quarterly interpolation of compensation per employee in the private sector.

Setting the CES elasticity of substitution at  $\sigma = 1.2$ , open economy real marginal costs are not very different from those calculated in the standard framework. Figure 2 plots these as log deviations from their sample means along with the private sector value added inflation measure.



**Figure 2 Luxembourg open and closed economy real marginal cost measure (lhs) and value-added inflation (rhs)**

The correlation between inflation and real marginal costs is slightly higher for the open economy measure (0.31 compared to 0.28 for the closed economy), but the difference is marginal. Since estimates of the hybrid NPC reject pure forward-looking price-setting behaviour, only the hybrid NPC is estimated in the open economy framework. Results are reported in the Table 5. Following Balakrishnan and López-Salido (2002), estimates are reported for a variety of possible values of  $\sigma$ . The open economy hybrid NPC is also estimated for two alternative measures of inflation: the interpolated private-sector value-added deflator and the observed NICP.

**Table 5 Estimates of the open economy hybrid NPC**

	$\beta$	$\omega$	$\theta$	$\gamma_b$	$\gamma_f$	$\tilde{\lambda}$	Duration 1/(1- $\theta$ )	J-statistic (p-value)
<b>Labour share as MC, GDP Deflator, <math>\mu = 1.1</math></b>								
$\sigma = 0.9$	0.945** (0.059)	0.732** (0.029)	0.607** (0.032)	0.557	0.436	0.006	2.55	6.83 (0.87)
$\sigma = 1.1$	0.968** (0.054)	0.730** (0.030)	0.630** (0.042)	0.543	0.453	0.005	2.70	6.82 (0.87)
$\sigma = 1.2$	0.980** (0.053)	0.731** (0.031)	0.640** (0.028)	0.537	0.460	0.004	2.78	6.83 (0.87)
$\sigma = 1.5$	1.007** (0.053)	0.738** (0.033)	0.663** (0.027)	0.525	0.476	0.003	2.97	6.86 (0.87)
<b>Labour share as MC, NICP<sup>10</sup>, <math>\mu = 1.1</math></b>								
$\sigma = 0.9$	0.910** (0.017)	0.717** (0.021)	0.708** (0.012)	0.520	0.467	0.003	3.42	7.65 (0.81)
$\sigma = 1.1$	0.944** (0.018)	0.699** (0.027)	0.766** (0.013)	0.487	0.504	0.002	4.27	9.60 (0.65)
$\sigma = 1.2$	0.969** (0.016)	0.672** (0.033)	0.811** (0.014)	0.458	0.536	0.001	5.30	9.65 (0.65)
$\sigma = 1.5$	0.987** (0.020)	0.678** (0.049)	0.819** (0.016)	0.455	0.542	0.001	5.51	9.72 (0.64)
<b>Gap as MC, GDP deflator, <math>\mu = 1.1</math></b>								
$\sigma = 0.9$	0.992** (0.024)	-0.041 (0.095)	0.428** (0.031)	-0.105	1.096	0.144	1.75	10.45 (0.58)
$\sigma = 1.1$	1.086** (0.035)	0.437** (0.113)	0.536** (0.047)	0.440	0.585	0.018	2.16	10.80 (0.55)
$\sigma = 1.1,$ $\beta = 0.99$	0.99	0.484** (0.090)	0.596** (0.049)	0.449	0.548	0.013	2.47	11.93 (0.53)
$\sigma = 1.2$	1.094** (0.039)	0.471** (0.093)	0.541** (0.039)	0.455	0.571	0.016	2.18	10.46 (0.58)
$\sigma = 1.2,$ $\beta = 0.99$	0.99	0.603** (0.167)	0.689** (0.122)	0.468	0.530	0.005	3.22	11.48 (0.57)
$\sigma = 1.5$	1.120** (0.059)	0.580** (0.078)	0.578** (0.038)	0.484	0.540	0.008	2.37	10.20 (0.60)
$\sigma = 1.5,$ $\beta = 0.99$	0.99	0.600** (0.084)	0.666** (0.057)	0.475	0.523	0.006	3.00	11.75 (0.55)

Using the private-sector value-added deflator measure of inflation, the estimated value of the parameter  $\theta$  suggests that between 60% and 66% of firms keep prices fixed any given period, depending on the level of the parameter  $\sigma$ , representing the elasticity of substitution between domestic and imported inputs. This lower value of  $\theta$  implies that the average period over which

<sup>10</sup> For the NICP measure of inflation, estimations using the first orthogonality condition (40) did not converge, so reported results rely on the second normalisation (41).



prices remain fixed is shorter than using the standard hybrid specification, falling to between 2.55 and 2.97 quarters for  $\sigma$  values of 0.9 and 1.5. The discount factor  $\beta$  is estimated in a range reasonably close to unity, as is consistent with theory. The proportion of firms  $\omega$  that set prices suboptimally using a backward-looking rule of thumb is quite large – approximately 70% according to both measures of inflation. When the NPC is based on NICP inflation instead of the private-sector value-added deflator, the parameter  $\omega$ , the share of firms using backward-looking rules of thumb, takes a somewhat lower value, but  $\theta$ , the share of firms that keep prices fixed any given period, is slightly higher. Overall, the open economy estimates of the hybrid NPC suggest less price rigidity but more backward-looking behaviour.

The slope of the Phillips curve given by  $\lambda$ , the estimated coefficient on real marginal costs, is very small for Luxembourg compared to results for selected EU countries reported in Benigno and López-Salido (2002). For Luxembourg it is between 0.003 and 0.006 (private sector value added deflator) while Benigno and López-Salido (2002) find the smallest coefficients for Spain (in the range 0.01 – 0.02) and the highest for Germany (in the range 0.096 – 0.135). Another noteworthy finding is that with a higher  $\sigma$ , the Phillips curve slope is flatter. In other words, the easier it is to substitute domestic inputs for foreign inputs, the less sensitive is inflation to movements in real marginal costs. In addition, higher values of  $\sigma$  also systematically increase the degree of price stickiness given by the parameter  $\theta$ .

When comparing the results for Luxembourg with those of Benigno and López-Salido (2002), the parameter  $\theta$  indicating the fraction of firms that keep prices fixed is similar in Luxembourg to that in Germany, France, Italy, Spain and Netherlands. However, there is a much higher proportion of firms that do not set prices optimally, preferring to use a backward-looking rule-of-thumb (we find  $\omega = 70\%$  for Luxembourg, Benigno and López-Salido only find similar results for Spain at  $\omega = 66.7\%$  and Italy at  $\omega = 59.4\%$ ).

Overall, these results tend to suggest that given a small (although significant) slope estimate, real marginal costs measured by the labour income share do not play a very significant role in the determination of inflation in Luxembourg, in contrast to the evidence for other countries found by Galí and Gertler (1999), Galí and López-Salido (2001), and Balakrishnan and López-Salido (2002) and Benigno and López-Salido (2002). However, this finding does not represent a rejection of the New Keynesian Phillips curve model itself, which still provides insight into the level of nominal price stickiness and backward-looking price-setting behaviour in Luxembourg.

#### 4.5 Fundamental inflation

Galí and Gertler (1999) suggest that ‘fundamental inflation’ can provide an alternative measure of the model’s goodness-of-fit. This can be obtained by iterating the estimated NPC forward. The equation for fundamental inflation was given in the section 2 (equation (23)) as follows:

$$\pi_t = \delta_1 \pi_{t-1} + \frac{\lambda}{\delta_2 \gamma_f} \sum_{k=0}^{\infty} \left( \frac{1}{\delta_2} \right)^k E_t \{ mc_{t+k} \} + \varepsilon_t, \quad (42)$$

where  $\delta_1$  and  $\delta_2$  are the characteristic roots of the difference equation and are functions of  $\gamma_b$  and  $\gamma_f$ , which are calculated from the estimates of the structural parameters in the hybrid NPC.

Since the inflation expectations  $E_t\{mc_{t+k}\}$  are not directly observable, Galí and Gertler (1999) propose the following procedure. Under the hypothesis that the model holds, one can construct an estimate of  $E_t\{mc_{t+k}\}$  given information available at time  $t$ , and use a VAR to recursively forecast future values of marginal costs given only the information available to firms at the time of the forecast<sup>11</sup>.

Figure 3 plots the fundamental inflation measure for the hybrid NPC using the open economy estimates of the structural parameters and setting  $\mu = 1.1$  and  $\sigma = 1.1$ .



**Figure 3 Actual and fundamental inflation for the Hybrid open economy NPC**

For the pure forward-looking NPC the fundamental inflation measure performs very poorly and is not reported here. The hybrid model’s measure of fundamental inflation tracks observed inflation dynamics much more closely, as one might expect from previous results - pure forward-looking price-setting behaviour was rejected by the hybrid model estimates. However, the very close fit should be interpreted with caution, because the hybrid model’s fundamental inflation  $\gamma_0$  and  $\gamma_f$  estimates imply that the coefficient for the lagged inflation – root  $\delta_1$  – is close to one (0.96 for the Figure 3), suggesting a random walk forecast would do just as well. The coefficient on the sum of future expected marginal costs  $\lambda/\delta_2\gamma_f$  is very small, in Figure 3 it is only 0.008, suggesting the economic content of the NPC is hardly adding to the fit. Most of the improvement in fit using the hybrid NPC comes from the term in lagged inflation and the additional explanatory power of the sum of future marginal costs is negligible. Nevertheless, this is not surprising as it was already suggested by the small estimated value of the  $\tilde{\lambda}$  parameter, and the very large proportion of firms using backward-looking price-setting behaviour. Finally, it should be noted that expected future marginal costs were proxied by forecasts based on a fairly rudimentary expectations-formation mechanism. As is often observed, poor model fit may reflect a misspecified expectations formation mechanism rather than misspecification in the model.

<sup>11</sup> For more detailed description see Galí and Gertler (1999).

## 5. Conclusions and Future Research

The NPC aims to explain inflation dynamics. It is appealing because it incorporates model-consistent (rational) expectations and is derived from explicit microfoundations. Despite evidence of empirical success for some other countries, the NPC has only qualified success in Luxembourg. The labour share of income does not seem to act as the main driving force of inflation, which is largely dependent on its past value. However, structural estimates of the New Keynesian Phillips curve do yield valuable insight into firms' price-setting behaviour in Luxembourg. On average, prices are unchanged between two and four quarters. When firms do update prices, the proportion using a backward-looking rule of thumb is large (70%) compared to levels estimated in other EU countries. This may be because in a very small very open economy subject to external shocks, optimal forward-looking price setting implies higher costs to update the required information set and calculate expected inflation.

Another possible reason why the proportion of backward-looking firms is so high in Luxembourg could be low inflation over the time period considered. Average quarterly inflation was 0.7% and its standard deviation was 0.8%. Perhaps firms find it cheaper to use past information (rule-of-thumb behaviour) since changes in inflation are relatively small, limiting the size of deviations from optimal prices. This conjecture could be explored in future work using the Mankiw and Reis (2002) Sticky Information Phillips curve (SIPC) outlined in section 3.2. This model focuses on the assumption that it is costly and time-consuming for the firms to gather and process information. Estimation of the SIPC would provide additional insight into the level of real rigidities in Luxembourg.

In the framework of the hybrid New Keynesian Phillips curve, we conjecture that in low inflation periods (countries), the proportion of backward-looking firms may be higher than in high inflation periods (countries). This adapts the NPC to the findings of Ball, Mankiw and Romer (1988), and Bakhshi, Burriel-Llombart, Khan and Rudolf (2003) that the slope of the Phillips curve is affected by average (trend) inflation. It would also seem likely that the probability of adjusting prices is not constant over time and rises with the rate of inflation. Further, firms' behaviour may switch between backward- and forward-looking price setting depending on the level of inflation. Estimation of the NPC with a time-varying proportion of backward-looking firms would yield insight into this proposition.

We are also disturbed by the similarity between measures of marginal costs for the closed and open economy NPC. We believe that the high degree of openness in Luxembourg should result in a larger discrepancy between these two measures. One possible alternative is suggested by the work of Batini, Jackson and Nickell (2000). It may also be that the unit values of imported goods used here are a poor measure of the price of imported inputs. The import deflator in national accounts may be preferable because it also takes account of imported services. In addition, the publication of quarterly national accounts may lead to improved estimates by eliminating the measurement error due to data interpolation.

Finally, future work should apply the encompassing tests in Paloviita (2002) and Bårdsen et al. (2002) to compare the predictive accuracy of the New Keynesian Phillips curve with that of the conventional backward-looking Phillips curve or other benchmarks such as the random walk.

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## Appendix A

Table A.1 presents reduced and structural form estimates of the NPC: 1) using the NICP measure of inflation and the labour income share as a measure of real marginal costs, and 2) using the private-sector value-added deflator to measure inflation and the output gap as proxy for real marginal costs.

	$\beta$	$\omega$	$\theta$	$\gamma_b$	$\gamma_f$	$\tilde{\lambda}$	Duration 1/(1- $\theta$ )	J-statistic (p-value)
<b>NICP and labour income share, closed economy, <math>\mu=1.1</math></b>								
Structural pure forward-looking NPC	0.887** (0.014)		0.550** (0.011)			0.035	2.22	7.84 (0.64)
Hybrid NPC	0.986** (0.046)	0.612** (0.126)	0.798** (0.123)	0.436	0.561	0.001	4.95	4.32 (0.89)
<b>Private sector GDP deflator and output gap, closed economy, <math>\mu=1.1</math></b>								
Structural pure forward-looking NPC	1.031** (0.021)		0.397** (0.018)			0.146	1.66	8.95 (0.54)
Hybrid NPC (1)	1.101** (0.129)	0.628 (0.420)	0.634** (0.214)	0.482	0.536	0.005	2.73	8.36 (0.50)
Hybrid NPC (2)	1.023** (0.021)	0.000 (0.114)	0.382** (0.030)	-0.001	1.024	0.161	1.62	7.98 (0.54)

Note: The instruments used are lags 1 to 3 of inflation, lags 2 and 3 of the labour income share, of the output gap (obtained by applying the Hodrick-Prescott filter to interpolated private sector quarterly real GDP), and of wage inflation, and lags 1 and 2 of the spread between long-term and short-term interest rates. (1) and (2) refer to the two orthogonality conditions (equations (40) and (41)).

