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INFERRING AGENT BEHAVIOR AND ECONOMIC INFORMATION, WITH FREE ENTRY AND EXIT OF FIRMS

Hernán Vallejo Miguel Espinosa

Abstract

This article proposes an identity regarding economic outcomes when producers maximize profits, with free entry and exit of firms. The identity links consumer and producer theory and leads to several results that contribute to understand what should -and should not- be expected under the assumptions made, from the behavior of firms and households, and from the technology of a firm. Given that unit prices are usually known, the identity also allows to infer the value of a range of economic variables, when reasonable information is available on the price elasticity of the residual demand, the marginal revenue associated to the residual demand, the marginal cost or the elasticity of scale.

Keywords: Price elasticity of demand, elasticity of scale, free entry and exit of firms, homogeneous production function.

JEL Classification: D20, D21, D24, D40

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INFIRIENDO COMPORTAMIENTO DE AGENTES E INFORMACIÓN ECONÓMICA, CON LIBRE ENTRADA Y SALIDA DE FIRMAS

Resumen

Este artículo propone una identidad que relaciona resultados económicos cuando los productores maximizan ganancias, con libre entrada y salida de firmas. La identidad relaciona la teoría del hogar y la teoría de la firma y arroja resultados que contribuyen a entender bajo los supuestos, lo que se debería –y lo que no se debería- esperar del comportamiento de las firmas y de los hogares, y de la tecnología de las firmas. Dado que los precios unitarios usualmente se conocen, la identidad también permite inferir el valor de una serie de variables económicas, cuando se dispone de información suficientemente precisa sobre la elasticidad de la demanda residual, el ingreso marginal asociado a la demanda residual, el costo marginal o la elasticidad de escala.

Palabras clave: elasticidad precio de la demanda, elasticidad de escala, libre entrada y salida de firmas y función de producción homogénea.

Clasificación JEL: D20, D21, D24 y D40.

1 Introduction

This article links standard consumer and producer theory results, to propose an identity regarding economic outcomes when producers maximize profits with free entry and exit of firms, within any goods market between natural monopoly and perfect competition. It is argued that the identity allows to restrict firm and household behavior, and firm technologies, that are compatible with the assumptions. It is also argued that the identity allows to infer a range of economic outcomes when fairly accurate information is available regarding some economic variables.

2 Assumptions

The assumptions on which the results of this article are based are:

Assumption 1 *Input markets are competitive, firms have identical technologies and thus, identical cost functions.*

Assumption 2 *The cost functions are, C^2, convex and are strictly increasing continuous functions in the level of output. That is, $\frac{\partial c(\mathbf{w},y)}{\partial y} = mc\left(y,\mathbf{w}\right) > 0$ and $\frac{\partial^2 c(\mathbf{w},y)}{\partial y^2} < 0$.*

Assumption 3 *The household inverse demand functions of a firm's output $p\left(m,\mathbf{y}\right)$ are C^2 and are non increasing functions in the level of output y. Thus, $\frac{\partial p(m,\mathbf{y})}{\partial y} = p_y\left(m,\mathbf{y}\right) \leq 0$.*

Assumption 4 *There is a continuum of firms with free entry and exit.*

3 Market Equilibrium Identity

Proposition *Given the assumptions, if firms maximize profits, the following identity holds for any goods market between natural monopoly and perfect competition*

$$\frac{\eta}{\eta - 1} = \frac{p}{mr\left(m,\mathbf{y}\right)} = \frac{ac\left(y,\mathbf{w}\right)}{mc\left(y,\mathbf{w}\right)} = \epsilon$$

where η is the price elasticity of the residual demand $\left(\eta = -\frac{1}{p_y(m,\mathbf{y})}\frac{p(m,\mathbf{y})}{y}\right)$, $mr\left(m,\mathbf{y}\right)$, $ac\left(y,\mathbf{w}\right)$ and ϵ are marginal revenue of the residual demand, average cost and elasticity of scale, respectively.

Proof. Since the general representation for the marginal revenue of the residual demand is $mr\left(m,\mathbf{y}\right) = p\left[1 - \frac{1}{\eta}\right]$, then, $\frac{\eta}{\eta-1} = \frac{p(m,\mathbf{y})}{mr(m,\mathbf{y})}$ holds. Second, free entry and exit of firms in the long run implies that $p\left(m,\mathbf{y}\right) = ac\left(y,\mathbf{w}\right)$, while profit maximization implies $mr\left(m,\mathbf{y}\right) = mc\left(y,\mathbf{w}\right)$. Therefore, $\frac{p}{mr(m,\mathbf{y})} = \frac{ac(y,\mathbf{w})}{mc(y,\mathbf{w})}$. Third, $\frac{ac(y,\mathbf{w})}{mc(y,\mathbf{w})} = \epsilon$, as shown in Frisch (1965 p. 167). The identity is well defined since $mc\left(y,\mathbf{w}\right) > 0$ by assumption 2. Thus, the proposition holds. ■

Remark 1 *If furthermore the technology is homogeneous of degree* $\gamma, \epsilon = \gamma$, *and* $\frac{\eta}{\eta-1} = \frac{p}{mr(m,\mathbf{y})} = \frac{ac(y,\mathbf{w})}{mc(y,\mathbf{w})} = \epsilon = \gamma$.

Remark 2 *Given* $mr(m,\mathbf{y}) > 0, 1 - \frac{1}{\eta} > 0$, *and thus* $\eta \in (1,\infty)$.

Remark 3 *Given the proposition and remark 2:*

$p = ac(y,\mathbf{w}) \in (mr(m,\mathbf{y}) = mc(y,\mathbf{w}), \infty)$

$mr(m,\mathbf{y}) = mc(y,\mathbf{w}) \in (0, p = ac(y,\mathbf{w}))$

$\epsilon \in (1,\infty)$

and if the production function is homogeneous of degree γ,

$\gamma \in (1,\infty)$.

Given assumption 2, and remark 3, there are no free goods $(p > 0)$. Also, profit maximizing firms under free entry and exit, do not produce where the elasticity of scale is less than or equal to one. Furthermore, if the technology is homogeneous, production functions with degrees of homogeneity less than or equal to one, are not compatible with profit maximization under free entry and exit of firms.

Note for example that if $\eta \to \infty$, then $mr(m,\mathbf{y}) \to p; mc(y,\mathbf{w}) \to ac(y,\mathbf{w})$; and $\epsilon \to 1$. And if the technology is homogeneous, $\gamma \to 1$. Besides, if $\eta \to 1$, then $p >> mr(m,\mathbf{y}); ac(y,\mathbf{w}) >> mc(y,\mathbf{w})$; and $\epsilon \to \infty$. In this later case, if the technology is homogeneous, then $\gamma \to \infty$.

Lemma *Given the assumptions, the average cost curves will not be tangent to the demand curves where such demand curves have price elasticities less than or equal to one.*

Proof. By contradiction, assume $\eta \leq 1$. Let y^* be a production level such that there is tangency between the demand curve and the average cost curve. Tangency implies that at y^*, $p(m,\mathbf{y}^*) = ac(y^*,\mathbf{w})$ and that $\frac{\partial p(m,\mathbf{y})}{\partial y} = \frac{\partial ac(y,\mathbf{w})}{\partial y}$. Given assumption 3, the residual inverse demand curves can only be tangent to the non increasing segments of the average cost curves. Free entry and exit of firms in the long run implies that $p(m,\mathbf{y}) \leq ac(y,\mathbf{w}),\ \forall\ y$. So at y^* with free entry and exit of firms, $mr(m,\mathbf{y}) = mc(y,\mathbf{w}) > 0$ given assumption 2, and since $p(m,\mathbf{y})\left[1 - \frac{1}{\eta}\right] > 0$, $\eta > 1$, which is a contradiction. Thus, the lemma holds. ■

Remark 4 *Given the proposition, unknown variables of firms and market outcomes under equilibrium can be found as follows*[1]*:*

	η	$p = ac(\cdot)$	$mr(\cdot) = mc(\cdot)$	$\epsilon\ (= \gamma)$
η		$\frac{\eta mr(m,\mathbf{y})}{\eta-1}$	$\frac{(\eta-1)p}{\eta}$	$\frac{\eta}{\eta-1}$
$p = ac(\cdot)$	$\frac{p}{p-mr(m,\mathbf{y})}$		$\frac{(\eta-1)p}{\eta}$	$\frac{p}{mr(m,\mathbf{y})}$
$mr(\cdot) = mc(\cdot)$	$\frac{p}{p-mr(m,\mathbf{y})}$	$\frac{\eta mr(m,\mathbf{y})}{\eta-1}$		$\frac{p}{mr(m,\mathbf{y})}$
$\epsilon\ (= \gamma)$	$\frac{\epsilon}{1-\epsilon}$	$\epsilon mr(m,\mathbf{y})$	$\frac{p}{\epsilon}$	

[1] Note that Lerner´s index $\frac{p-mc(y,\mathbf{w})}{p} = \frac{1}{\eta}$. Recall as well that if the production function is homogeneous of degree γ, $\epsilon = \gamma$.

Since p is usually known, having a reasonable estimate of η, $mr(m, \mathbf{y}) = mc(y, \mathbf{w})$ or ϵ, will allow us to obtain estimates of the rest of the variables in the table.

Remark 5 *If firms have differentiated cost structures, then in the long run for any firm in the market,* $\frac{\eta}{\eta-1} = \frac{p}{mr(m,\mathbf{y})} \geq \frac{ac(y,\mathbf{w})}{mc(y,\mathbf{w})} = \epsilon$. *Note that for the marginal firm with zero profits, the proposition holds. And for the other firms with lower costs,* $p > ac(y, \mathbf{w})$ *and* $\epsilon \in (0, \infty)$.

Remark 6 *If firms have differentiated cost structures and the market is competitive, then* $p = mr(m, \mathbf{y}) = mc(y, \mathbf{w}) \geq ac(y, \mathbf{w})$ *and thus,* $\epsilon \in (0, 1]$, *as in Hanoch (1975, p. 495).*

4 Conclusions

This article has presented an identity that links standard results from household and firm theory for any goods market between natural monopoly and perfect competition. The identity allows to restrict firm technologies and households and firm behavior under the assumptions. Given that unit prices are commonly observed, the identity also allows to infer economic behaviour and economic information, when reasonable estimates of the price elasticity of the residual demand, the marginal revenue associated to the residual demand, the marginal costs or the elasticity of scale, are available.

References

[1] Frisch, R. (1965) Theory of Production. Holland: D. Reidel Publishing Co.

[2] Hanoch, G (1975) The Elasticity of Scale and the Shape of Average Costs, The American Economic Review, Vol 65 No. 3, June, pp. 492-496