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Rent-seeking for pure public goods: Wealth and group's size heterogeneity

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Abstract

In this paper, we study how between-group wealth and size heterogeneity affect aggregate rent-seeking efforts as well as success probabilities when two groups compete for the allocation of a pure public good. Unlike with previous analyses on between-group asymmetries, we measure the utility cost of rent-seeking in terms of the loss in private consumption an individual faces when contributing to this activity. This allows us to analyze both how asymmetries in either group's size or wealth affect aggregate rent-seeking efforts when group size is not neutral, and how the interaction between two dimension asymmetries affects aggregate rent-seeking efforts in this context. Our main general result is that fewer between-group asymmetries do not necessarily imply greater aggregate rent-seeking efforts. We describe the circumstances under which this happens. The result is at odds with the commonly held notion that the more homogeneous the contestants in a static rent-seeking model, the greater the aggregate rent-seeking efforts.

Key words: Rent-seeking, public goods, group size, wealth inequality, group asymmetries.
JEL classification: H41, D70, D72, D74, D31.

Cabildeo por bienes públicos: Heterogeneidad en el tamaño y la riqueza de grupo

Resumen

Este artículo estudia cómo las asimetrías en la riqueza y el tamaño de grupos que compiten por la asignación de un bien público afectan las probabilidades de éxito de cada grupo y el esfuerzo total invertido por estos en actividades de cabildeo. Diferente a estudios previos en este campo, nosotros medimos el costo de cabildeo en términos de la pérdida de consumo privado que los individuos perciben cuando contribuyen recursos en esta actividad. Esto nos permite analizar el efecto de las asimetrías en el tamaño de grupo y la riqueza sobre la cantidad agregada de esfuerzo en cabildeo cuando el tamaño de grupo es no-neutral, y entender cómo la interacción de estas asimetrías en estas dos dimensiones afecta dicho esfuerzo agregado. Nuestro principal resultado es que menos asimetrías entre grupos no necesariamente implica un mayor esfuerzo agregado de cabildeo. Las circunstancias bajo las cuales esto ocurre son descritas. Este resultado es opuesto a la noción común que existe en la literatura, es decir, entre más homogéneos son los competidores en un modelo de cabildeo estático, mayor es el esfuerzo agregado invertido en dicha actividad.

Palabras clave: Cabildeo, bienes públicos, tamaño de grupo, desigualdad de la riqueza, asimetría de grupos.
Clasificación JEL: D31, D70, D72, D74

1. Introduction

Many public goods or facilities are allocated in societies according to the efforts expended by different groups in trying to win these prizes. Some examples of this situation are cities or neighborhoods competing for different kinds of a public facility (hospitals, parks, libraries, etc.) or for a public project, industries struggling for government support, etc. Studies of allocations of this type are well represented in the rent-seeking literature.

The seminal contribution on rent-seeking for public goods comes from Katz et al. (1990). Their most important results can be summarized as follows. First, neither the total sum of rent-seekers nor their between-group distribution affect aggregate rent-seeking efforts. Moreover, regardless of the group-size, a richer group always invests more effort into rent-seeking than a poorer group. Consequently, richer groups are always more successful than poorer groups. This happens precisely because of the group size neutrality result. Finally, although they do not study the effect of wealth inequality on aggregate efforts, it can be inferred from their model that a redistribution of wealth from a richer to poorer group always increases the aggregate rent-seeking effort.

The first result noted above is quite surprising, and contradicts earlier works on collective action, wherein it has been suggested that group size matters, both with respect to group rent-seeking efforts and success probabilities (Olson, 1965; and McGuire, 1974). Actually, by expanding the individual consumption bundles—to include preferences not only with respect to the public good, but also to a private good—in the model developed by Katz et al. (1990), Riaz et al. (1995) demonstrate that group size does, in fact, affect aggregate rent-seeking efforts.[1]

On the other hand, the result regarding between group asymmetry in wealth is in line with previous findings in the rent-seeking literature—i.e., the more homogeneous the contestants

[1] Other authors have also obtained alternative results to this respect by introducing certain private characteristics to the contested rents (Nitzan, 1991; Katz and Tokatlidu, 1996; Esteban and Ray, 2001; Cheikbossian 2008).

in a static rent-seeking model, the 'greater' the aggregate rent-seeking efforts (Baye et al., 1993; Che and Gale, 1998; Szymanski and Valletti, 2005; Amegashie and Kutsoati, 2006; Epstein and Nitzan, 2006; and Fu, 2006). As we show, when the utility cost of rent-seeking is measured in terms of the loss in private consumption faced by an individual when he or she contributes to this activity (in the spirit of Riaz et al., 1995), this result does not necessarily hold.

The aim of this paper is to contribute to the literature in understanding how between-group asymmetries (in particular between-group wealth and size heterogeneity) affect rent-seeking efforts when groups compete for the allocation of a "pure" public good, and in a context in which group size is not neutral. As far as I know, no previous study has addressed this issue in this context. Our main general result is that a greater symmetry in one of the two dimensions (wealth or size) does not necessarily imply greater aggregate rent-seeking efforts. This result is at odds with the commonly held notion that the more homogeneous the contestants in a static rent-seeking model, the greater the aggregate rent-seeking efforts. Although expanding the individual consumption bundles to include preferences with respect to a private good drives many of our results, as we will see, the interaction between asymmetries in wealth and group size plays a key role in determining how between-group asymmetries affect aggregate rent-seeking efforts.

We consider a common type of rent-seeking situation wherein two groups formed by risk neutral individuals engage in lobbying activities to win a uniquely pure (within-group) public good. There are two dimensions by which the groups are differentiated in our framework—wealth and size (i.e., the number of members in each group). Following the literature on rent-seeking contests, we assume that each group's success probability depends on the relative amount of resources spent on rent-seeking by its members.

In the spirit of Riaz et al. (1995), we measure the utility cost of rent-seeking, not directly in terms of individual efforts, but in terms of the loss in private consumption an individual faces when he or she contributes to rent-seeking. This strategy introduces an interesting feature into the model—namely, that the marginal cost of rent-seeking changes with the level of private consumption. For tractability reasons, we restrict our analysis to the case wherein the marginal utility of private consumption does not depend on the consumption level of a public good, which unambiguously implies that aggregate rent-seeking efforts are positively affected by group-size.[2] This allows us to study how wealth and group size asymmetries affect aggregate rent-seeking when the latter is not neutral. Although we concentrate on this case, we claim that our main general results (i.e., that less between-group asymmetries does not necessarily imply more aggregate rent-seeking efforts, and that the interaction between asymmetries in wealth and group size plays a key role in determining how between-group asymmetries affect aggregate rent-seeking efforts) still holds under less restrictive assumptions on the marginal utility of private consumption vis-à-vis the consumption level of public goods.

For the purpose of exposition, we begin our analysis by showing that, under the circumstances described above, group size positively affects group rent-seeking efforts, aggregate rent-seeking efforts, and group success probabilities.[3] We also show that in our model, group average wealth positively affects these three outcomes.[4] Based on these results, we emphasize an interesting corollary that has not been previously stressed in the literature—namely that it is possible to observe a poor group being more successful that a richer group because of larger group size. We will exploit this result throughout our analysis.

We begin our study of between-group asymmetries by first analyzing the distribution of wealth across groups. Our study begins with the case where there are no asymmetries in group-size. Under these circumstances, we show that wealth asymmetries affect aggregate rent-seeking efforts, and that fewer asymmetries do not necessarily imply more aggregate efforts. The key element in this result is how the marginal cost of rent-seeking changes

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[2] This specific case is not explicitly analyzed in Riaz et al. (1995). They concentrate on the case wherein the marginal utility of private consumption increases as the consumption level of the related public good increases. As said above, under these circumstances they find that groups size in not neutral, although its effect on aggregate rent seeking may be positive or negative. As can be shown using both their framework and ours, aggregate rent-seeking efforts are positively affected by group-size when the marginal utility of private consumption does not depend on the consumption level of a public good.

[3] As noted above, this result can be obtained from the model proposed by Riaz et al. (1995), once our assumptions on individual preferences are imposed.

[4] This result is also reported by Katz et al. (1990), and Riaz et al. (1995).

across groups. If the change in the marginal cost of the rent-seeking of the poorer group is greater than the change in the marginal cost of that of the richer group, then aggregate rent-seeking efforts increases. However, aggregate rent-seeking efforts decrease when the opposite is true. This result demonstrates how the effect of wealth asymmetries on rent-seeking efforts depends on the between-group relative change in the marginal cost of rent-seeking.

Continuing with our analysis of wealth inequalities, we analyze the case wherein, not only are there between-group wealth asymmetries, there are also between-group size asymmetries. Here, our results depend not only on the between-group relative change in the marginal cost of rent-seeking but also on the relative group size. Relative group size (i.e., the asymmetries in group-size) matters with respect to relative wealth transfer. For instance, when the poorer group is smaller in size than the richer group, a progressive transfer of wealth implies that the increase in the average wealth of the poorer group will be relatively higher than the decrease in the average wealth of the richer group. The opposite occurs when the poorer group is larger than the richer one. Under these circumstances, we find that if the between-group relative change in the cost of rent-seeking is greater than the relative transfer, then less wealth inequality implies more aggregate rent-seeking efforts. The opposite happens when the between-group relative change in the cost of rent-seeking is smaller than the relative transfer. Once again, this result departs from the standard result—that less between-group asymmetries implies more aggregate rent-seeking efforts—and demonstrates the importance of group-size asymmetries when evaluating the effects of wealth asymmetries.

We conclude our wealth inequalities analysis by examining the case where the change in the marginal cost of rent-seeking always decreases as private consumption increases. Such an analysis is interesting because under these circumstances less wealth inequality will imply more rent-seeking aggregate efforts–which is actually the most standard result in the literature. Moreover, this allows us to link our results with the initial group success probability. We find that if the poorer group is smaller in size (and thus less successful) than the richer group, then less wealth asymmetry implies more aggregate rent-seeking efforts. However, when the poorer group is larger in size and more successful than the richer group, then less wealth asymmetry implies fewer aggregate rent-seeking efforts. This result is driven by the possibility that poorer but larger groups can be more successful than richer but smaller groups.

We next analyze the distribution of the population across groups. As before, our study begins with the case where there are no asymmetries in wealth—i.e., for the case where both groups have the same average wealth. We find that aggregate rent-seeking efforts increase as between-group size asymmetries decrease if the change in the marginal cost of the rent-seeking of the smaller group is greater than the change in the marginal cost of that of the larger group. However, unlike with the case of wealth redistribution, when the change in the smaller group's marginal cost is less than the change in the larger group's respective cost, we cannot guarantee that fewer group-size asymmetries implies less aggregate rent-seeking efforts.

We also analyze the case wherein, not only are there between-group size asymmetries, there are also between-group wealth asymmetries. Our general result in this case still holds—i.e., fewer between-group size asymmetries do not necessarily implies more aggregate rent-seeking efforts. In this case, the collective action productivity of each group plays an important role in determining the final result.

The remainder of the paper is as follows. Section 2 presents the model, and section 3 characterizes the respective equilibrium and presents the comparative static results. In Section 4, we present our analysis of between-group asymmetries. The conclusions are presented in the last section. The appendix contains all our proofs.

2. The model

Let us consider two groups ($g=1,2$), both of which are competing for the allocation of a public good. The good is indivisibly allocated in the sense that only the group that receives the allocation can enjoy it. We might think of this good as a public facility or a public project (a hospital, park, library, etc.), a special law or support that might favor a particular economic sector, and so forth. The two groups must engage in rent-seeking activities so as to influence the allocation of the good in their favor. This situation is referred to in the literature as rent-seeking for pure public goods.

Accordingly, only the group that is granted the prize receives utility from the allocation. We fix this gain at one. Thus, if the prize is allocated to group *g*, each individual *i* who belongs to this group receives an extra unit of utility, and each individual who belongs to the other group receives zero utility. Therefore, individual valuation of the public good is totally symmetric within and between groups.

The number of people in each group (group-size) is n_g. Each individual *i* has exogenous wealth w_i and spends a non-negative amount of resources r_i on rent-seeking so as to maximize his or her expected utility. We assume that individuals are risk neutral and cannot borrow, and that individual wealth is public information.

Let us define $c_i = w_i - r_i$. In our framework, c_i has at least two interpretations. On the one hand, it could be understood as the individual wealth net of contribution. On the other hand, it could also be understood as the individual consumption of a private good the price of which has been fixed at one. We use the second interpretation. As in the literature on public goods, in our framework, each individual derives utility from the consumption of both the public and the private good. The expected utility of an individual belonging to group *g* is assumed to be given by:

$$EU_i = p_g + f(c_i) \tag{1}$$

where p_g is the success probability of group *g*, and $f(.)$ is assumed to be a continuous, increasing and strictly concave function, with $\lim_{c_i \to 0} f'(c_i) = \infty$.[5] The concavity of $f(.)$ implies that the marginal utility of consumption is decreasing.

[5] A necessary condition for equilibrium existence in our framework is that $f''(.) \leq 0$ (See the appendix). However, as we will see later on, the most interesting case is when $f''(.) < 0$.

As anticipated in the introduction, for tractability reasons, we assume that individual preferences are such that private and public goods are unrelated in terms of consumption. Actually, this is a particular case of Riaz et al.'s model (1995), although they do not analyze this in their study. As we see below, the main implication of this assumption (apart from the standard income effect) is that group size will always positively affects group rent-seeking efforts, aggregate rent-seeking efforts, and group success probabilities.[6]

Our assumptions regarding the utility function also allow us to do an interesting comparison between that and more general utility functions such as are used in the standard literature. Standard rent-seeking models divide the individual payoff between the expected benefit of the prize and the rent-seeking costs. These costs are directly related to the individual efforts spent on lobbying (in our framework, r_i). Equation 1 also distinguishes between benefits and costs. However, contrary to the standard models, our pay-off function does not measure the utility cost in terms of individual efforts, but rather in terms of the loss in private consumption that individuals face when contributing to rent-seeking. Accordingly, we refer to $f(.)$ as the rent-seeking cost. Equation 1 also introduces an interesting feature to the model, namely that the marginal cost of rent-seeking can decrease with the level of private consumption if $f''(.) < 0$, as we assume.

Each group's success probability depends on the relative amount of resources spent on lobbying by its members. We assume the following, quite standard, functional form for success probability:

$$p_g = \frac{R_g}{R} \qquad (2)$$

for $g=1,2$, provided that $R>0$, and where R_g is the total amount of resources contributed by group g to rent-seeking (i.e., $R_g = \sum_{i \in g} r_i$), and R is the total amount of resources expended

[6] As shown in Riaz et al. (1995), this happens if the slope of the between-group reaction functions is greater than -1, which is actually the case here.

by the two groups on rent-seeking (i.e. $R = R_1 + R_2$). If $R=0$, then the respective success probabilities are given by an arbitrary vector, $\{\tilde{p}_1, \tilde{p}_2\}$, which is contained in the interior of the simplex. We refer to R as the aggregate rent-seeking effort.

3. Equilibrium and comparative static

In our framework, each individual in each group takes as a given the efforts contributed by everyone else in the society, and chooses $r_i \geq 0$ to maximize equation 1, subject to equation 2. The resources spent by individual i from group g is described by the following conditions:

$$\frac{1}{R}(1-p_g) = f'(c_i) \quad \text{if} \quad f'(w_i) < \frac{R_{-g}}{R_{-i}^2} \tag{3a}$$

$$r_i = 0 \quad \text{if} \quad f'(w_i) \geq \frac{R_{-g}}{R_{-i}^2} \tag{3b}$$

where $R_{-i} = R - r_i$ and $R_{-g} = R - R_g$. Equations 3a and 3b implicitly describe the Nash equilibrium contribution of each individual. Under an interior solution, equation 3a describes the usual equilibrium condition according whereby the marginal utility of the contribution must be equal to its marginal disutility.

It is possible to redefine the equilibrium based on the success probabilities and the aggregate rent-seeking efforts, rather than on personal efforts. Given that $f'(.)$ decreases monotonically, from equations 3a and 3b, the equilibrium condition can be written as:

$$r_i = Max\left\{0, w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right\} \tag{4}$$

Combining equations 2 and 4, we get:

$$p_g = \frac{1}{R}\sum_{i \in g} Max\left\{0, w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right\} \quad (5)$$

The equilibrium can now be interpreted as a vector, $\langle p_1, p_2 \rangle$, of the success probabilities (such that $p_g \geq 0 \ \forall g$, and $\Pi = p_1 + p_2 = 1$) and a positive scalar *R*, such that equation 5 is satisfied for every group. In the appendix, we show that an equilibrium always exists, and that this is unique.

From now on, let us assume that there exists an interior solution for every individual. If this is the case, equation 5 reduces to:

$$p_g = \frac{1}{R}\sum_{i \in g}\left(w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right) \quad (6)$$

Given the properties of $f'(.)$, after some algebraic manipulation, equation 6 can be written as:

$$\frac{1}{R}(1-p_g) = f'(c_g^*) \quad (7)$$

where $c_g^* = \overline{w}_g - p_g R / n_g$, and $\overline{w}_g$ is the average wealth of group *g*. Note that equations 7 and 3a represent the same condition. However, we now know that, at equilibrium, all individuals belonging to the same group have exactly the same level of private consumption.[7] Additionally, from equation 7, it follows that, at equilibrium, p_g and *R* are completely defined by $\overline{w}_g$ and n_g. Inasmuch then as, at equilibrium, individual wealth is irrelevant, we can conclude that within-group wealth inequality affects neither p_g nor *R* (this replicates the so called Neutrality theorem - War, 1983; Bergstrom et al., 1986). We will use equation 7 for our analysis.

[7] Actually, all individuals belonging to the same group obtain exactly the same level of utility. This characteristic has been analyzed by Itaya et al. (1997).

For the purpose of exposition, we state the comparative static result from our model in proposition 1. Doing this allows us to introduce the analysis strategy we use throughout the paper. Our strategy consists of examining how the sum of success probabilities change (i.e., how $\Pi = p_1(\overline{w}_1, n_1, R) + p_2(\overline{w}_2, n_2, R)$ changes, where $p_g(.)$ is the implicit function defined by equation 7) when either n_g or $\overline{w}_g$ changes, while keeping the level of aggregate rent-seeking efforts constant. Since Π decreases as R increases (see the appendix), once we know how Π changes, we can infer how R must move to recover the equilibrium (i.e., to recover $\Pi = 1$).

Proposition 1: Let us assume that for everyone there is an interior solution.

a) Both the aggregate rent-seeking effort and the success probability of group *g* strictly increase as the group-size of group *g* increases.

b) Both the aggregate rent-seeking effort and the success probability of group *g* strictly increase as the average wealth of group *g* increases.

As noted in the introduction, the result in proposition 1(a) can also be obtained from Riaz et al. (1995), once imposing our assumptions regarding individual preferences. It can be shown that there is individual free-riding in our model. However, the result in 1(a) implies that the reduction in an individual's own contribution when n_g increases is more than compensated by the contributions of new members in the group. As a result, both group rent-seeking efforts and aggregate rent-seeking efforts increase. Notice that if $f(.)$ is assumed to be linear, then n_g affects neither aggregate rent-seeking efforts nor success probabilities. (See the appendix.). Notice also that the result in proposition 1(a) implicitly requires that a group's average wealth remain unchanged as its size increases. Thus, if the addition of new members negatively affects a group's average wealth, the result can be different.

The result in 1(b) is more typical in the standard literature. However, we want to stress the manner in which the interaction between group size and group average wealth affects group success probabilities, which is actually an interesting feature that has not received enough attention in previous studies. Let us say that a group is poorer than another group if the average wealth of the former is smaller than the average wealth of the later.[8] From propositions 1, the following corollary can be obtained.

Corollary 1: It is possible to observe a poor group being more successful than a rich group because of a higher group size or vise versa.

The possibility suggested in corollary 1 can be illustrated through a simple example. Let $f(c_i) = \ln(c_i)$, $\overline{w}_b = 98.499$, $\overline{w}_s = 100$, and $n_s = 10$. When $n_b = 13$, then, at equilibrium, $\langle p_b, p_s \rangle = \langle 0.4976, 0.5024 \rangle$. When $n_b = 26$, then, $\langle p_b, p_s \rangle = \langle 0.5, 0.5 \rangle$. Finally, when $n_b = 57$, then, $\langle p_b, p_s \rangle = \langle 0.5013, 0.4987 \rangle$.

As indicated above, Katz et al. (1990) maintain that a richer group is always more successful that a poorer one, regardless of respective group sizes. This is due to their group size neutrality result. However, there are several situations–for instance, with respect to environmental legislation—where larger groups with low average wealth are more successful than smaller groups with relatively higher average wealth. This is exactly what our model is able to predict. The result in corollary 1 will be exploited in order to study between group asymmetries.

4. Between group asymmetries

We now analyze the effect of between-group asymmetries in wealth and size on aggregate rent-seeking efforts. As commented in the introduction, Katz et al. (1990) suggest that asymmetries in group-size are totally neutral and do not affect aggregate rent-seeking efforts. On the other hand, although they do not study between-group inequalities in wealth, it can be inferred from their model that fewer between-group asymmetries in wealth increases aggregate rent-seeking efforts. Against this, our model predicts that less between-

[8] This is exactly the same definition of poorer group used by Katz et al. (1990) and Riaz et al. (1995).

group asymmetries do not necessarily imply an increase in said efforts. We present our results in this section.

Before continuing with our analysis, we first state in lemma 1 an additional characteristic of our model that will be useful for understanding our subsequent results.

Lemma 1: If $p_1>p_2$ at equilibrium, then $f'(c_1^*) < f'(c_2^*)$. From the characteristics of $f(.)$, $p_1>p_2$ also implies that $c_1^* > c_2^*$.

Lemma 1 follows immediately from equilibrium condition 7. It says that, when the success probability of group 1 is larger than the success probability of group 2, then, at equilibrium, the marginal cost of rent-seeking for individuals belonging to group 1 is smaller than that for individuals belonging to group 2. This also implies that, at equilibrium, the private consumption of group 1 is larger than the private consumption of group 2.

Wealth asymmetries

Let us first analyze the effect of wealth inequality on the aggregate rent-seeking effort. More precisely, we wish to analyze the effect of a between-group progressive transfer of wealth (hereafter, BGPT) on the aggregate rent-seeking effort. By this we refer to a case wherein a richer group (group *h*) transfers part of its total wealth to a poorer group (group *l*) such that the sum of the total wealth of the two groups remains constant.[9] In our framework, group *h* is richer than group *l* if $\overline{w}_h > \overline{w}_l$.

The effect of a BGPT can be analyzed by looking at how success probabilities change over the cross-section of groups following the transfer of wealth, and assuming that *R* remains constant. Doing this, we can infer the manner in which *R* must change to adjust the sum of success probabilities to equal one. Notice that if we transfer one unit of money from the

[9] Because there is within-group neutrality, we do not restrict this transfer so as to maintain the within-group wealth distribution.

average wealth of group h ($\overline{w}_h$) to the average wealth of group l ($\overline{w}_l$), the latter increases by n_h / n_l. Taking this into account, we calculate the change in Π when there is a BGPT as:

$$\Delta\Pi^{wealth}\Big|_R = \frac{\partial p_l}{\partial \overline{w}_l}\Bigg|_R \frac{n_h}{n_l} - \frac{\partial p_h}{\partial \overline{w}_h}\Bigg|_R \qquad (8)$$

From the comparative static derived in proposition 1, we already know the derivatives implied in equation 8. Replacing these terms and manipulating the equation algebraically, we obtain:

$$\Delta\Pi\Big|_R = Rn_h\left(\frac{1}{R^2 - \Omega_l} - \frac{1}{R^2 - \Omega_h}\right) \qquad (9)$$

where $\Omega_g = n_g / f''(c_g^*) < 0$ for $g=h,l$. Thus, when $\Omega_l<\Omega_h$ ($\Omega_l>\Omega_h$), the transfer makes Π smaller (higher) than one; in order then to recover equilibrium conditions, R must decrease (increase) whenever p_g and R are negatively related. Notice that $\Omega_l<\Omega_h$ if and only if $\frac{n_l}{n_h} > \frac{f''(c_l^*)}{f''(c_h^*)}$. The opposite is true if $\Omega_l>\Omega_h$.

We start our analysis by studying the effect of a BGPT when there are only asymmetries in wealth but not in group size—i.e., when $n_l=n_h$.

Proposition 2. Let us assume that for everyone there is an interior solution, and $n_l=n_h$. A BGPT then will generate an increase in the aggregate rent-seeking effort if $f''(c_l^*) < f''(c_h^*)$. When this inequality is reversed, the BGPT generates a decrease in the aggregate rent-seeking effort. If both terms are equal, the BGPT does not have any effect on the aggregate rent-seeking effort.

The proof of proposition 2 follows immediately, once we replace $n_l / n_h = 1$ in the inequality involving Ω_l and Ω_h. Whether $f''(c_l^*)$ is larger, equal, or smaller than $f''(c_h^*)$ is crucial in determining the effect of a BGPT on aggregate rent-seeking effort. As we know, $f''(.)$ measures the change in the marginal cost of rent-seeking. Thus, the effect of a reduction in between-group wealth asymmetries will depend on the between-group relative change in the marginal cost of rent-seeking. Proposition 2 states that in order to observe an increase in aggregate rent-seeking efforts as between-group asymmetries in wealth are reduced, it is enough if the change in the marginal cost of the poorer group is greater than the change in that of the richer group.[10] However, when the change in the marginal cost of the poorer group is smaller than the change in that of the richer group, aggregate rent-seeking efforts will decrease. This result runs against the commonly held notion that fewer between-group asymmetries implies more aggregate rent-seeking efforts, and demonstrates that it depends on the between-group relative change in the marginal cost of rent-seeking.

Let us now consider the effect of a BGPT on the aggregate rent-seeking effort when there not only exist between-group wealth asymmetries, but also between-group size asymmetries. Some interesting results emerge.

Proposition 3: Let us assume that for everyone there is an interior solution. A BGPT then will generate an increase in the aggregate rent-seeking effort if $\frac{n_l}{n_h} < \frac{f''(c_l^*)}{f''(c_h^*)}$. When this inequality is reversed, the BGPT generates a decrease in the aggregate rent-seeking effort. If both terms are equal, the BGPT does not have any effect on the aggregate rent-seeking effort.

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[10] Based on proposition 1, it follows that $p_h > p_l$. From lemma 1 then, it must be the case that $c_h^* > c_l^*$. Therefore, to observe $f''(c_l^*) < f''(c_h^*)$, it is necessary that $f'''(.) > 0$. This property is satisfied by widespread concave utility functions like $f(c_i) = c_i^{\alpha}$ with $\alpha \in (0,1)$, and $f(c_i) = \ln c_i$.

There are two elements involved in the inequality of proposition 3, the relative group size (n_l / n_h) and the relative change in the marginal cost of rent-seeking ($f''(c_l^*) / f''(c_h^*)$). On the one hand, the relative group size will define the magnitude of the relative transfer. For instance, when the poorer group is smaller in size than the richer group, a progressive transfer will imply that the increase in $\overline{w}_l$ is relatively higher than the decrease in $\overline{w}_h$. Thus, the relative transfer becomes larger as n_l / n_h decreases. On the other hand, as we have already commented, the ratio of second derivatives measures the between-group relative change in the marginal cost of rent-seeking at equilibrium. Thus, the change in the marginal cost of the poorer group is greater than the change in that of the richer group as $f''(c_l^*) / f''(c_h^*)$ increases.

Therefore, proposition 3 states that if the relative change in the marginal cost of rent-seeking is greater than the relative transfer, then the decrease in the rent-seeking effort of the richer group will be dominated by the increase in the rent-seeking effort of the poorer group. As a result, aggregate rent-seeking efforts will increase. When the opposite happens, individuals in the poorer group will be better-off relatively increasing their private consumption. As a result, there will be a reduction in aggregate rent-seeking efforts. Once again, the result in proposition 3 shows that fewer between-group asymmetries do not necessarily imply more aggregate rent-seeking efforts. However, in this case, the result depends not only on the relative change in the marginal cost, but also on the magnitude of the relative transfer.

To conclude our analysis of wealth asymmetries, we concentrate on the case where the marginal cost of rent-seeking decreases more quickly for lower than for higher levels of private consumption—i.e., $f''' > 0$.[11] Such an analysis is interesting for at least three reasons. First, as is noted in footnote 7, this property is satisfied by widespread, strictly concave, utility functions. Second, from our discussion of proposition 2, we know that

[11] A similar analysis can be done when $f''' \leq 0$. For instance, when $f'''(.) = 0$, then $f''(c_l^*) / f''(c_h^*) = 1$. From proposition 3, it follows that: (1) if n_l=n_h, then a BGPT does not affect the aggregate rent-seeking effort; (2) if n_l<n_h, then a BGPT generates an increase in the aggregate rent-seeking effort, and; (3) if n_l>n_h, then a BGPT generates a decrease in the aggregate rent-seeking effort.

when n_l=n_h and $f''' > 0$, then less wealth inequality will imply more rent-seeking aggregate efforts–this is actually the most standard result in the literature. What we want then is to see how the introduction of between group-size asymmetries affects the result. Third, this analysis allows us to link our results to the initial group success probability.

Using the results obtained in proposition 1, we already know that when $n_l<n_h$, then $p_l<p_h$. However, when $n_l>n_h$, the relationship between the equilibrium probabilities is no longer clear. It might be that $p_l>p_h$ if the number of members in the poorer group is high enough to offset the negative effect due to its smaller average wealth. If this is not the case, then it must again be that $p_l<p_h$. Keeping in mind these facts, we can state our results in proposition 4.

Proposition 4: Let us assume that for everyone there is an interior solution, and that the marginal cost of rent-seeking decreases more quickly for lower than for higher levels of private consumption (i.e., $f'''(.) > 0$).

a) If $n_l<n_h$, then a BGPT will increase the aggregate rent-seeking effort.
b) If $n_l>n_h$ and $p_l \geq p_h$ (i.e., the number of members in the poorer group is high enough to compensate for the group's smaller average wealth), then a BGPT will reduce the aggregate rent-seeking effort.
c) If $n_l>n_h$ and $p_l<p_h$ (i.e., the number of members in the poorer group is not high enough to compensate for the group's smaller average wealth), then the effect of a BGPT on the aggregate rent-seeking effort will be ambiguous.

Proposition 4 demonstrates that wealth equality does not necessarily increase aggregate rent-seeking efforts, even when we assume $f'''(.) > 0$. For instance, when the poorer group has a higher success probability (i.e., when the group's size compensates for its small average wealth), wealth redistribution reduces the aggregate rent-seeking effort. This result shows how, when there are asymmetries not only in wealth but also in group-size, it is not necessarily the case that greater wealth equality increases aggregate rent-seeking efforts.

Group size asymmetries

Let us now analyze how between-group size asymmetries affect aggregate rent-seeking efforts. As above, the effect of redistributing people from a larger group (group *b*) to a smaller group (group *s*) – i.e. the effect of a reduction in group size asymmetries - can be analyzed by looking at how success probabilities change over the cross-section of groups following the transfer of people. The change in Π when there is a redistribution of people from group *b* to group *s* is given by:

$$\Delta\Pi^{size}\Big|_R = \frac{\partial p_s}{\partial n_s}\bigg|_R - \frac{\partial p_b}{\partial n_b}\bigg|_R \qquad (10)$$

Notice that equation 10 implicitly assumes that the redistribution of people affects neither the average wealth of group *b* nor the average wealth of group *s*. From the comparative static derived above, we already know the derivatives implied in equation 10. Replacing these terms, we obtain:

$$\Delta\Pi^{size}\Big|_R = R^2\left(\frac{f''\left(c_s^*\right)p_s}{n_s\left(R^2 f''\left(c_s^*\right)-n_s\right)} - \frac{f''\left(c_b^*\right)p_b}{n_b\left(R^2 f''\left(c_b^*\right)-n_b\right)}\right) \qquad (11)$$

If expression in equation 11 is positive (negative), the redistribution of people from group *b* to group *s* makes Π higher (smaller) than one; in order then to recover equilibrium conditions, *R* must increase (decrease) whenever p_g and *R* are negatively related.

Once again, whether $f''(c_s^*)$ is larger, equal, or smaller than $f''(c_b^*)$ is crucial in determining the sign of equation 11. As we know, $f''(.)$ measures the change in the marginal cost of rent-seeking. Thus, the effect of a reduction in group size asymmetries will depend on the between-group relative change in the marginal cost of rent-seeking. In order to isolate the effect of group size asymmetries on rent-seeking efforts, we first concentrate on the case where both groups have exactly the same average wealth.

Proposition 5: Let us assume that $\overline{w}_s = \overline{w}_b$. If $f''(c_s^*) \leq f''(c_b^*)$, then fewer between-group asymmetries in size will positively affect aggregate rent-seeking efforts. Otherwise the effect is ambiguous.

From our results in proposition 1 and lemma 1, we already know that before the redistribution of people, group *b*'s marginal cost for rent-seeking is smaller than that for group *s*. Fewer group size asymmetries imply an increase in the marginal cost of rent-seeking for group *b*, and a reduction in the respective marginal cost for group *s*. Proposition 5 states that, in order to observe an increase in aggregate rent-seeking efforts, it is enough if the change in the marginal cost for the smaller group is greater than the change in that for the larger group.[12] However, unlike what is the case in a redistribution of wealth, when the change in the marginal cost for the smaller group is less than that for the larger group, we cannot assume that fewer group-size asymmetries will imply less aggregate rent-seeking efforts.

Finally, let us consider the case wherein there exist asymmetries in wealth. We state the results in proposition 6.

Proposition 6: Let us assume that $\overline{w}_s \neq \overline{w}_b$.

a) If both $p_s/n_s > p_b/n_b$ and $f''(c_s^*) \leq f''(c_b^*)$, or both $p_s/n_s = p_b/n_b$ and $f''(c_s^*) < f''(c_b^*)$, then having fewer between-group asymmetries in size will positively affect aggregate rent-seeking efforts.

b) If both $p_s/n_s < p_b/n_b$ and $f''(c_s^*) \geq f''(c_b^*)$, or both $p_s/n_s = p_b/n_b$ and $f''(c_s^*) > f''(c_b^*)$, then having fewer between-group asymmetries in size will negatively affect aggregate rent-seeking efforts.

c) If $p_s/n_s = p_b/n_b$, and $f''(c_s^*) = f''(c_b^*)$, then between-group asymmetries in size will not affect aggregate rent-seeking efforts.

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[12] As commented in footnote 7, a sufficient condition for observing $f''(c_s^*) \leq f''(c_b^*)$ is that $f'''(.) \geq 0$.

d) Under any other circumstance, the effect of between-group asymmetries on aggregate rent-seeking efforts is ambiguous.

In the presence of between-group wealth asymmetries, the effect of a redistribution of people with respect to aggregate rent-seeking efforts will depend not only on how the change in the marginal cost of rent-seeking compares across groups, but also on how the initial ratio between group *g*'s success probability and its size (p_g / n_g) compares across groups. The combination of these two terms is critical for determining the final effect of a redistribution of people on aggregate rent-seeking efforts.

We have already discussed the intuition behind the conditions of the relative change in the marginal cost of rent-seeking stated in proposition 6. Let us study in more detail the role of the second key term, p_g / n_g . This term can be understood as the group-size productivity in collective action; the larger this ratio, the more productive is the group. Actually, as can be seen in the proof for proposition 1 (see the appendix), this ratio affects the magnitude of $\partial p_g / \partial n_g \big|_R$ via two channels. The first one is via the change in the marginal cost of rent-seeking, which we already discussed. The second one is via a direct positive effect on $\partial p_g / \partial n_g \big|_R$—i.e., the more productive the group, the greater the effect of n_g on p_g . This second channel is the one related to this second key term.

Therefore, roughly speaking, proposition 6 states that, when the smaller group is more productive than the larger group, and its change in the marginal cost of rent-seeking is greater than that for the bigger group, a reduction in group size asymmetries will increase the aggregate rent-seeking efforts. When the situation is just the opposite, a reduction in group-size asymmetries will decrease the aggregate rent-seeking efforts. Finally, when only one of these conditions is satisfied and the other is not, fewer asymmetries in group size has an ambiguous effect on rent-seeking efforts.

The conditions under which we are able to observe $p_s / n_s > p_b / n_b$ can be easily inferred from our analysis. For instance, if the average wealth of the smaller group is large enough vis-à-vis the average wealth of the larger group, such that $p_s > p_b$, then it immediately follows that, prior to the redistribution, the smaller group will be more productive than the larger group. It can be also observed that, when $p_b > p_s$, but $\overline{w}_b \le \overline{w}_s$—i.e., although the smaller group is richer on average than the larger group—it is not enough to compensate for group size. Finally, if $\overline{w}_b > \overline{w}_s$, it is possible to observe either $p_s/n_s > p_b/n_b$ or $p_s/n_s \le p_b/n_b$.

Although the results in proposition 6 indicate a large combination of possibilities and some cases wherein ambiguity emerges, they also demonstrate the most important result in our paper—i.e., that fewer between-group asymmetries does not necessarily imply more aggregate rent-seeking efforts.

5. Conclusions

This paper studies how group wealth and group size heterogeneity affects aggregate rent-seeking efforts when two groups are lobbying for a pure public good and group size is not neutral. In the spirit of Riaz et al. (1995), we avoid group size neutrality by measuring the utility cost of rent-seeking in terms of the loss in private consumption an individual faces when he or she contributes to rent-seeking. This strategy introduces an interesting feature into the model—namely, that the marginal cost of rent-seeking changes with the level of private consumption.

We perform an exhaustive study of the effect of between-group asymmetries on aggregate rent-seeking efforts. We begin by studying the case where there is only one between-group dimension asymmetry—either that related to group size or to group wealth. The general result is that fewer between-group asymmetries in size (wealth) implies more aggregate efforts, if the change in the marginal cost of rent-seeking of the smaller (poorer) group is larger than the change in that of the larger (richer) group. If the opposite happens, then greater homogeneity in wealth implies fewer aggregate rent-seeking efforts. This last result shows that fewer between-group asymmetries does not necessarily imply more aggregate rent-seeking efforts, if the marginal cost of rent-seeking changes with the level of private consumption.

We also study the effect of between-group asymmetries on aggregate rent-seeking efforts when there exist asymmetries in both dimensions—size and wealth. The most important result in this respect is that even when the change in the marginal cost of rent-seeking for the smaller (poorer) group is greater than that for the larger (richer) group, fewer between-group asymmetries does not necessarily imply more aggregate rent-seeking efforts. In the case of wealth, the new element that emerges in determining this effect is the magnitude of the relative transfer. In the case of group size, the new element that emerges in determining this effect is the relative productivity of each group. This result demonstrates how the existence of more than one asymmetry in a static rent-seeking model may imply that aggregate rent-seeking efforts decrease as between-group symmetry increases.

Our analysis concentrates on the case wherein the marginal utility of private consumption does not depend on the consumption level of a public good. A less restrictive assumption is to consider the case wherein the marginal utility of private consumption increases as the consumption level of the related public good increases (like in Riaz, et al. 1995). The main complication to study between-group asymmetries in this case is that group size might have an ambiguous effect of aggregate rent-seeking. However, it is still true in this case that the marginal cost of rent-seeking changes with the respective level of private consumption and that group size is not neutral. Thus, our prediction is that our main general results (i.e., that less between-group asymmetries does not necessarily imply more aggregate rent-seeking efforts, and that the interaction between asymmetries in wealth and group size plays a key role in determining how between-group asymmetries affect aggregate rent-seeking efforts) still holds under this less restrictive assumption. However, the final effect of between-group asymmetries on aggregate rent-seeking efforts might be importantly affected, especially in those cases in which group size has an ambiguous effect of aggregate rent-seeking.

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Appendix

Individual Optimal Contributions and equilibrium existence. Plugging equation 2 into equation 1, and taking as a given the contribution of the rest of individuals, each individual *i* in group *g* will maximize $EU_i = R_g / R + f(c_i)$ over r_i. It can be verified that EU_i is strictly concave in r_i. From the first order condition, we get $(R - R_g) / R^2 = f'(c_i)$. Reorganizing the terms and using the success probability function, we get equation 3a. Since $lim_{c_i \to 0} f'(c_i) = \infty$, then, at equilibrium, $r_i < w_i$. On the other hand, note that $\partial EU_i / \partial r_i \big|_{r_i=0} = R_{-g} / R_{-i}^2 - f'(w_i)$. This marginal utility is positive if and only if $f'(w_i) < R_{-g} / R_{-i}^2$. When this inequality holds, the total amount of resources spent on rent-seeking by individual *i* will be strictly positive and is implicitly described by equation 3a. When $f'(w_i) \geq R_{-g} / R_{-i}^2$, the marginal utility is not positive, and the individual *i*'s best response is $r_i = 0$.

Let us now prove existence and uniqueness of equilibrium**.** To do this, we use equation 5. Equation 5 implicitly defines p_g as a function of *R*. Moreover, it can be readily verified that p_g is a continuous function of *R*. Using the implicit function theorem, it can be shown that when $p_g>0$, it is strictly decreasing in *R*. From equation 5, we get the following:

$$\frac{\partial p_g}{\partial R} = -\frac{\frac{1}{R^2}\sum_{i\in g}\left(w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right) - \frac{n_g}{R^3}(f'^{-1})'\left(\frac{1-p_g}{R}\right)(1-p_g)}{1-\frac{n_g}{R^2}(f'^{-1})'\left(\frac{1-p_g}{R}\right)} \qquad (1A)$$

Since $f''(.) \leq 0$ and $\sum_{i\in g}\left(w_i - f'\left(\frac{1-p_g}{R}\right)\right) > 0$ under an interior solution, both the numerator and the denominator in 1A are positive. It then follows that $\partial p_g / \partial R < 0 \ \ \forall g$.

Consider the function $\Pi = \frac{1}{R}\sum_g \sum_{i\in g} Max\left\{0, w_i - f'^{-1}\left(\frac{1-p_g}{R}\right)\right\}$. At equilibrium, R must correspond to $\Pi=1$ and $p_g \geq 0 \ \forall g$. Note that Π strictly decreases in R, and approaches zero as R goes to infinity. On the other hand, when R approaches zero, then $p_g>0$ and Π approaches infinity. It follows then that there must be some R for which $\Pi=1$. Furthermore, it is unique.

Proof of proposition 1.

a) Assume that for every i, there is an interior solution. Keeping R constant in equation 7, and using the implicit function theorem, we get $\left.\frac{\partial p_g}{\partial n_g}\right|_R = \frac{R^2 f''(.) p_g}{n_g(R^2 f''(.) - n_g)} > 0$. Since $\left.\frac{\partial p_g}{\partial n_g}\right|_R = \left.\frac{\partial \Pi}{\partial n_g}\right|_R$, then an increase in p_g will make $\Pi > 1$. Since p_g and R are negatively related, it follows that R must increase to recover the equilibrium. This proves that R increases as the size of group g increases.

Let us now consider the complete effect of n_g on p_g. Until now, the change in n_g has not affected the success probability of group -g, and has affected p_g positively. Inasmuch as R increased, the success probabilities must go down to recover the equilibrium condition, $\Pi = 1$. To assure that this is the case, at the new equilibrium the final p_g must be larger than the initial p_g (likewise, p_{-g} will be smaller). This proves that the success probability of group g increases as the size of group g increases.

Notice that if $f(.)$ is assumed to be linear, then $\partial p_g / \partial n_g \big|_R = 0$; this implies that group-size affects neither p_g nor R.

b) Assume that for every i, there is an interior solution. As before, we keep R constant in equation 7. Again, using the implicit function theorem, we get $\left.\frac{\partial p_g}{\partial \overline{w}_g}\right|_R = \frac{n_g R f''(.)}{R^2 f''(.) - n_g} > 0$. Since $\left.\frac{\partial p_g}{\partial \overline{w}_g}\right|_R = \left.\frac{\partial \Pi}{\partial \overline{w}_g}\right|_R$, then an increase in p_g will make $\Pi > 1$.

Following the same argument as above, we can prove that both the success probabilities and the total rent-seeking effort increase as group size increases. Notice that if $f(.)$ is assumed to be linear, then $\partial p_g / \partial \overline{w}_g \big|_R = 0$; this implies that the average wealth of group g affects neither p_g nor R.

Proof of lemma 1. See the proof in the text.

Proof of proposition 2. See the proof in the text.

Proof of proposition 3. See the proof in the text.

Proof of proposition 4. Let us assume that for every i, there is an interior solution, and $f'''(.) > 0$.

a) When $n_l<n_h$, it must be the case that $p_l<p_h$. It follows from lemma 1 then that $f'(c_l^*) > f'(c_h^*)$; additionally that $c_l^* < c_h^*$, and $f''(c_l^*) / f''(c_h^*) > 1$. Thus, we conclude that $f''(c_l^*) / f''(c_h^*) > n_l / n_h$. Proposition 3 closes the proof.

b) When $n_l>n_h$ and $p_l \geq p_h$, it follows from lemma 1 that $f'(c_l^*) \leq f'(c_h^*)$, and so $c_l^* \geq c_h^*$, and $f''(c_l^*)/f''(c_h^*) \leq 1$. Thus, we conclude that $f''(c_l^*)/f''(c_h^*) < n_l/n_h$. Proposition 3 closes the proof.

c) When $n_l>n_h$ and $p_l < p_h$, it follows from lemma 1 that $f'(c_l^*) > f'(c_h^*)$; additionally, $c_l^* < c_h^*$, and $f''(c_l^*)/f''(c_h^*) > 1$. Since $n_l/n_h > 1$, the effect of a BGPT on the total rent-seeking effort is ambiguous.

Proof of proposition 5. From equation 11, it follows that $\Delta\Pi^{size}\Big|_R > 0$ if and only if $R^2 f''(c_s^*) f''(c_b^*)[n_b p_s - n_s p_b] + [n_s^2 p_b f''(c_b^*) - n_b^2 p_s f''(c_s^*)] > 0$. The first term on the left-hand side of this inequality is positive if $p_s/n_s > p_b/n_b$. Since $\overline{w}_s = \overline{w}_b$, then $p_s < p_b$. From lemma 1, it follows that $c_s^* < c_b^*$. From this inequality, it is easy to demonstrate that p_s/n_s is always greater than p_b/n_b. We might now consider the second term on the left-hand side of the first inequality equation. From our previous analysis, it follows that $n_s^2 p_b < n_b^2 p_s$. Thus, this term is always positive if $f''(c_s^*) \leq f''(c_b^*)$.

Proof of proposition 6. As in the proof of proposition 5, the results follow from the sign of $R^2 f''(c_s^*) f''(c_b^*)[n_b p_s - n_s p_b] + [n_s^2 p_b f''(c_b^*) - n_b^2 p_s f''(c_s^*)] > 0$.