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### Spatially asymmetric firms and the sustainability of a price agreement

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#### Abstract

We study collusion between price discriminating firms which are asymmetrically located in a linear city. We obtain that higher distance increases the sustainability of the collusive agreement for any degree of spatial asymmetry, and more spatial symmetry between firms increases collusion sustainability whatever is the location of the firms in the space, both assuming grim-trigger and optimal punishment.

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## 1. Introduction

Collusion sustainability in a context of discriminatory prices has received attention by spatial economists. By adopting the Hotelling linear city model, Liu and Serfes (2007) and Colombo (2010) consider the case of discriminating firms colluding on a uniform price. The rationale of this scheme is based on the difficulty to implement a collusive agreement where a significant number of prices have to be agreed on by the colluding firms. When collusion on discriminatory prices is too difficult to implement, firms may coordinate on much simpler collusive schemes, as in uniform price collusion. Examples of uniform price collusive agreements between price discriminating firms are *Austrian Banks* (D. Comm. June 12, 2002) and *Specialty Graphite* (D. Comm. Dec. 17, 2002).<sup>1</sup> In this article, we follow Liu and Serfes (2007) and Colombo (2010), and we study the conditions for the sustainability of a uniform price agreement between price discriminating firms. However, we introduce a relevant generalization: while Liu and Serfes (2007) assume maximally differentiated firms and Colombo (2010) assumes symmetric firms, we allow for any degree of spatial (a)symmetry between firms. The motivation is double. First, from casual observations, it is immediate to note that firms are usually not symmetrically localized in the space. Some firms are located near to the centre of the market, while some others are localized near to the periphery. Second, the assumption of symmetric localization is based on the idea that firms choose simultaneously where to locate. In this case (provided that firms are equal in any aspects), the most likely outcome is a symmetric equilibrium. However, it is far from being obvious that firms start to operate at the same time. Some firms may enter the market first, while some others may start to operate later. When the assumption of a simultaneous start of firms is removed, the assumption of symmetric localisation is no more obvious. In what follows, we do not investigate on the endogenous choice of localization when firms enter the market at different times, because this goes beyond the aim of the note. Instead, we assume that firms are (weakly) asymmetrically localized in the Hotelling market.

Having assumed spatial asymmetry between firms, we want to answer the following questions: *i)* How does firms' distance affect collusion sustainability, *given the degree of spatial symmetry between firms?* *ii)* how does spatial (a)symmetry affect collusion sustainability, *given the distance between firms?*<sup>2</sup> We obtain that, all else being equal, higher distance and higher spatial symmetry increases the sustainability of the collusive agreement.

The rest of this note proceeds as follows. In Section 2 we introduce the model. In Section 3 we study collusion sustainability. Section 4 concludes.

## 2. The model

Assume a linear city of length 1. Consumers are uniformly distributed over the line. Denote by  $x \in [0, 1]$  the location of each consumer. Each consumer buys no more than one unit of the good, and his reservation price is  $v$ . There are two firms,  $A$  and  $B$ . Fixed and marginal costs of both firms are constant and normalized to zero. Firm  $A$  is located at  $s - d$ , while firm  $B$  is located at  $s + d$ . Then, firms are symmetrically located in the space when  $s = 1/2$ . We restrict our attention to the case where firms' locations are (weakly) asymmetrically distorted to the left. That is,  $s \in (0, 1/2]$ . Parameter  $s$  measures the degree of symmetry between firms: for given  $d$ , the higher is  $s$ , the higher is symmetry. Parameter  $d$  instead measures the distance

<sup>1</sup> See also Colombo (2010) and the discussion therein.

<sup>2</sup> While the first question has been addressed by Chang (1991, 1992), Ross (1992) and Hackner (1995) in a uniform price context, and by Gupta and Venkatu (2002) and Colombo (2010) in a price discrimination context, the second issue has been unexplored in spatial frameworks.

between firms: for given  $s$ , the higher is  $d$ , the more the firms are distant. Since no firm can be located outside the market, it must be:  $d \in (0, s]$ . Firms pay the transportation costs to ship the good from the plant to the consumers' location. We assume linear transportation costs. Therefore, to ship one unit of the product from plant  $s-d$  (resp.  $s+d$ ) to consumer  $x$ , firm  $A$  (resp.  $B$ ) pays a transport cost equal to:  $t|x-s+d|$  (resp.  $t|x-s-d|$ ), where  $t$  is the (strictly positive) unit transport cost. We also assume that the reservation price is sufficiently high, so that the market is always covered in equilibrium. In particular:  $v \geq 2t$ . Finally, let  $p_J(x)$  denote the price schedule charged by firm  $J = A, B$ . We define a price schedule as follows: it refers to a positive valued function  $p_J(\cdot)$ , with  $J = A, B$ , defined on  $[0, 1]$  that specifies the price  $p_J(x)$  at which firm  $J$  is willing to sell one unit to consumer  $x$ .

Suppose that firms interact repeatedly in an infinite horizon setting. In supporting collusion, the firms may revert to competitive prices forever if a deviation occurs (*grim-trigger* punishment mechanism) or may adopt an optimal punishment scheme. Denote by  $\Pi_J^C$ ,  $\Pi_J^D$  and  $\Pi_J^P$  respectively the one-shot collusive, deviation and punishment profits of firm  $J = A, B$ , and with  $\delta$  the market discount factor. Collusion is sustainable as a sub-game perfect Nash equilibrium if and only if the discounted value of the profits that each firm obtains under collusion exceeds the discounted value of the profits that each firm obtains when deviates from the agreement. This amounts to require that the following incentive-compatibility constraint has to be satisfied:

$$\sum_{t=0}^{\infty} \delta^t \Pi_J^C \geq \Pi_J^D + \sum_{t=1}^{\infty} \delta^t \Pi_J^P, \text{ with } J = A, B \tag{1}$$

After rearranging, the incentive-compatibility constraints can be written as:

$$\delta_J \equiv \frac{\Pi_J^D - \Pi_J^C}{\Pi_J^D - \Pi_J^P}, \text{ with } J = A, B \tag{2}$$

Therefore, collusion is sustainable as a sub-game perfect equilibrium if and only if:  $\delta \geq \max[\delta_A, \delta_B]$ . In other words,  $\max[\delta_A, \delta_B]$  ("the critical discount factor") measures cartel sustainability: the higher is  $\max[\delta_A, \delta_B]$  the smaller is the set of market discount factors supporting collusion.

### 3. Collusion

We denote by  $p_J^N$  and  $\Pi_J^N$  respectively the equilibrium price schedule and the equilibrium profits of firm  $J = A, B$  when firms compete. While under collusion the firms charge uniform price, during the punishment stage they are free to use discriminatory prices. The following proposition defines the equilibrium competitive prices:

**Proposition 1:** *when the firms compete, the equilibrium price schedules are:*

$$p^N(x) = \max[|x-s+d|, |x-s-d|] \tag{3}$$

*Proof.* See Theorem 1 in Lederer and Hurter (1986). ■

The intuition of Proposition 1 is straightforward. For a given location  $x$ , Bertrand competition drives prices down to the level of the larger transportation costs, say  $|x - i|$ , where  $i$  is the location of the farthest firm.<sup>3</sup> Assuming that in case of equal prices each consumer buys from the nearer firm, the Nash profits of the two firms are:

$$\Pi_A^N = \int_0^s (p_A^N(x) - t|x - s + d|)dx = dt(2s - d) \quad (4)$$

$$\Pi_B^N = \int_s^1 (p_B^N(x) - t|x - s - d|)dx = td(2 - d - 2s) \quad (5)$$

In case of a *grim-trigger* punishment, the Nash profits coincide with the punishment profits. On the other hand, in case of optimal punishment, we state a result due to Espinosa (1992) which characterizes the equilibrium deviator's profits in case of optimal punishment:

**Proposition 2:** a credible punishment that minimizes the deviator's profits exists for any discount factor, and entails zero profits for the deviating firm (Espinosa, 1992).

The intuition of Proposition 2 is the following. We look for the mostly severe punishment. This occurs when the punishing firm, in case of a deviation of the other firm, starts a basing-point pricing where the location of the deviating firm serves as a base point. In this case the profits of the deviating firm are no larger than zero, whatever is the price schedule adopted by the deviating firm. Therefore, multiple price schedules equilibria exist, but they are all characterized by the fact that the deviating firm gets zero profits.<sup>4</sup> That is:  $\Pi^P = 0$ .

Next, we consider the collusive stage. We follow Hackner (1994) and we assume that firms collude on the uniform price which maximizes joint profits. Therefore, the firms set the uniform price in such a way to extract the whole consumer surplus. As the reservation price is  $v$ , the perfect collusive price is:  $p^{C*} = v$ . The collusive profits are:

$$\Pi_A^C = \int_0^s [p^{C*} - t|x - s + d|]dx = vs - t(d^2 + \frac{s^2}{2} - ds) \quad (6)$$

$$\Pi_B^C = \int_s^1 [p^{C*} - t|x - s - d|]dx = v(1 - s) - \frac{t}{2}[2d^2 - 2d(1 - s) + (1 - s)^2] \quad (7)$$

Suppose that firm  $A$  deviates. It undercuts firm  $B$  at the collusive price  $p^{C*}$  and obtains the whole market. Therefore, the deviation profits of firm  $A$  are:

<sup>3</sup> In case of positive marginal production costs, the equilibrium price at each location  $x$  would be represented by the maximum total (i.e. production plus transportation) marginal cost.

<sup>4</sup> The optimal punishment indicated in Proposition 2 implicitly excludes that some consumers closer to the deviating firm decides to purchase from the punishing firm instead than from the deviating firm ("consumers' deviation"). If this is possible, the basing-point strategy at the punishment stage may be risky for the punisher. In fact, the punishing firm may be damaged when it sets a below-cost price if some consumers buy from it. I thank an anonymous referee for this comment.

$$\Pi_A^D = \int_0^1 [p^C * -t|x-s+d|]dx = v - t\left(\frac{1}{2} + d^2 - s + s^2 + d(1-2s)\right) \quad (8)$$

Similarly, if firm  $B$  deviates, it undercuts firm  $A$  at the collusive price  $p^C *$  and obtains the whole market. Its deviation profits are:

$$\Pi_B^D = \int_0^1 [p^C * -t|x-s-d|]dx = v - t\left(\frac{1}{2} + d^2 - s + s^2 - d(1-2s)\right) \quad (9)$$

Inserting the collusive profits, the punishment profits and the deviation profits into the discount factor and after simplifying, we have:

$$\delta_A^{gt} = \frac{(1-s)[2v-t(1+2d-s)]}{2v-t(1+2d-2s+2s^2)} \quad (10)$$

$$\delta_B^{gt} = \frac{s[2v-t(s+2d)]}{2v-t[1+2d-2s+2s^2]} \quad (11)$$

in the case of *grim-trigger* punishment, and

$$\delta_A^{op} = \frac{(1-s)[2v-t(1+2d-s)]}{2v-t[1+2d^2+2d(1-2s)-2s+2s^2]} \quad (12)$$

$$\delta_B^{op} = \frac{s[2v-t(s+2d)]}{2v-t[1+2d^2-2d(1-2s)-2s+2s^2]} \quad (13)$$

in the case of optimal punishment. We can state the following result:

**Proposition 3.**  $\delta_A^{gt} \geq \delta_B^{gt}$  and  $\delta_A^{op} \geq \delta_B^{op}$ .

*Proof.* See the Appendix. ■

The discount factor of firm  $A$  is higher than the discount factor of firm  $B$ , both in the case of *grim-trigger* and optimal punishment. Hence, the critical discount factor is  $\delta_A^{gt}$  and  $\delta_A^{op}$ . Therefore, the firm with the greatest incentive to deviate is the firm with the worst location in the space. The intuition is the following. Recall that the discount factor of each firm decreases with the collusive profits (i.e. collusion is easier to sustain), while it increases with the deviation profits and the punishment profits (i.e. collusion is more difficult to sustain). As firm  $B$  is better positioned in the space than firm  $A$ , it has a natural advantage over firm  $A$ . Therefore, the collusive profits, the deviation profits and the Nash profits of firm  $B$  are higher than the correspondent profits of firm  $A$ . In the linear city model, the dominant effect is linked to the collusive profits: as firm  $A$  has lower collusive profits with respect to firm  $B$ , it is more prompt to deviate from the agreement than firm  $B$ .

The impact of symmetry and distance on collusion sustainability is described in the following proposition:

**Proposition 4.** Greater symmetry and greater firms' distance lower the critical discount factor.

*Proof.* See the Appendix. ■

Therefore, when any degree of spatial symmetry/asymmetry between firms is allowed, the relationship between firms' distance and collusion sustainability is positive. Similarly, the relationship between firms' symmetry and collusion sustainability, given the distance between firms, is positive. The intuition of Proposition 4 is the following. Consider first the effect of greater distance between firms. The punishment profits (in the case of the *grim-trigger* punishment) increase with  $d$ , as lower distance between firms implies less fierce competition during the non-cooperative stage. The deviation profits decrease with  $d$ . This is due to the fact that when the distance increases, each consumer is more "loyal" to the nearer firm, and it is more difficult for the cheating firm to steal consumers from the rival. Instead, the effect of  $d$  on the collusive profits is not monotonic and it can be positive or negative. In particular, as long as  $d$  approximates to  $s/2$ , collusive profits increase with  $d$ , and then decrease, as  $d = s/2$  allows firm  $A$  to minimize the transportation costs within its own market. The effect of  $d$  on the deviation profits is the dominant effect, whatever is the sign of the relationship between the firms' distance and the collusive profits. This determines the positive relationship between firms' distance and collusion sustainability. Consider now the impact of higher spatial symmetry on the sustainability of the collusive agreement. Higher  $s$  increases the punishment profits of firm  $A$  (in the case of the *grim-trigger* punishment), as it serves more consumers in equilibrium (recall that in equilibrium each firm serves only the nearer consumers). Higher  $s$  increases also the collusive profits, as the market area served by firm  $A$  is larger. The deviation profits increase with  $s$  too. In fact, during deviation, firm  $A$  keeps the collusive profits on locations  $x \leq s$ : as  $s$  increases, collusive profits increase, thus determining also higher deviation profits. The positive effect of more spatial symmetry on the collusive profits outweighs the increase of the punishment and the deviation profits, thus allowing for less stringent conditions on the market discount factor for the sustainability of the collusive agreement in equilibrium.

#### 4. Conclusions

A consistent number of studies, both in regional economics and industrial organization theory, analyse the impact of firms' locations on the sustainability of collusion in equilibrium.<sup>5</sup> As the location of firms can be easily observed, antitrust regulators should consider firms' localization as a relevant variable when assessing the likelihood of collusive behaviours by firms. This approach seems well established in U.S. antitrust, as the U.S. Department of Justice's Merger Guidelines (1984) explicitly suggest considering spatial localization of firms when analysing potential collusive phenomena.

In this note we study the case of price discriminating firms which collude on a uniform price within the context of the linear city model. We depart from literature by imposing no restriction on firms' location. That is, we allow the firms to be asymmetrically located in the space. In this way we are able to consider the impact of firms' distance on the sustainability of the collusive agreement in equilibrium maintaining as general as possible the degree of spatial (a)symmetry between firms. Similarly, we are able to analyse the impact of firms' spatial symmetry on collusion sustainability for any possible distance between firms. First, we obtain that the firm with the worst location in the space has always the greatest incentive

<sup>5</sup> See Colombo (2011) for a survey of the most relevant contributions in this area.

to deviate. Therefore, to assess the impact of spatial (a)symmetry and distance between firms on collusion sustainability, it is sufficient to consider the impact of these variables on the discount factor of the firm with the worst location. We obtain that both higher spatial symmetry and greater distance between firms lower the discount factor, thus making collusion easier to be sustained in equilibrium, both in the case of *grim-trigger* punishment and optimal punishment. This is due to the fact that higher distance between firms decreases the additional gains from deviation of the worst located firm, thus making collusion easier to be sustained in equilibrium. On the other hand, higher spatial symmetry between firms increases the collusive profits of the worst located firm, thus reducing its incentive to deviate from the agreement.

### Appendix

*Proof of Proposition 3.* The denominators of  $\delta_A^{gt}$  and  $\delta_B^{gt}$  are equal. Taking the difference between the numerator of  $\delta_A^{gt}$  and the numerator of  $\delta_B^{gt}$  we get:  $(1-2s)(2v-t-2td)$ , which is always non-negative, given the assumptions on  $v$ ,  $t$ ,  $s$  and  $d$ . Consider now  $\delta_A^{op}$  and  $\delta_B^{op}$ . The denominator of  $\delta_A^{op}$  is lower than the denominator of  $\delta_B^{op}$ . As the numerators of  $\delta_A^{op}$  (resp.  $\delta_B^{op}$ ) and  $\delta_A^{gt}$  (resp.  $\delta_B^{gt}$ ) are equal, it follows that  $\delta_A^{op}$  is larger than  $\delta_B^{op}$ . ■

*Proof of Proposition 4.* Consider the derivative of  $\delta_A^{gt}$  with respect to  $s$ :

$$\frac{\partial \delta_A^{gt}}{\partial s} = -2 \frac{[2t^2d^2 + td(t+2ts-2ts^2) + t^2(s-s^2)] + v[2v-4td-2ts+2ts^2-t]}{[2v-t(1+2d-2s+2s^2)]^2} \quad (14)$$

In order to prove that  $\partial \delta_A^{gt} / \partial s < 0$ , we need to show that the numerator is positive. First, note that the first term in the numerator is always positive. Therefore, it is sufficient to show that the second term is non-negative. Consider the second term. The derivative of the second term with respect to  $s$  is:  $-2t(1-2s) < 0$ . Therefore, the second term takes the highest value when  $s = 0$ . In this case, the second term becomes:  $2v-4td-t$ , which is always positive since  $d \leq 1/2$  and  $v \geq 2t$ . Consider the derivative of  $\delta_A^{gt}$  with respect to  $d$ , which is:

$$\frac{\partial \delta_A^{gt}}{\partial d} = - \frac{3t^2s(1-3s+2s^2)}{[2v-t(1+2d-2s+2s^2)]^2} < 0 \quad (15)$$

as  $s \leq 1/2$ . Consider now the derivative of  $\delta_A^{op}$  with respect to  $s$ :

$$\frac{\partial \delta_A^{op}}{\partial s} = -2 \frac{[2td^3 + t^2(2s-1) - 2td^2(v+ts) + (2v-t)(v-ts+t^2s)]}{[2v-t(1+2d^2+2d-4ds-2s+2s^2)]^2} \quad (16)$$

In order to prove that  $\partial \delta_A^{op} / \partial s < 0$ , we need to show that the numerator is positive. By taking the derivative of the numerator with respect to  $d$ , we get:  $t[t(6d^2-1+2s-4sd)-4vd] < 0$ . Therefore, the numerator takes the lowest value when  $d = s$ . By substituting into the numerator, we get:  $t^2s^2 + 2v^2 - vt(1-2s) > 0$ . Therefore, the numerator is always positive. Finally, let us consider the derivative of  $\delta_A^{op}$  with respect to  $d$ , which is:

$$\frac{\partial \delta_A^{op}}{\partial d} = -2 \frac{t(1-s)[2td^2 + s(4v-t) + 2d(2v+t(1-s))]}{[2v-t(1+2d^2 + 2d - 4ds - 2s + 2s^2)]^2} < 0 \quad (17)$$

as all the terms in the square brackets in the numerator are positive. ■

### References

- Chang, M.H., 1991, The effects of product differentiation on collusive pricing, *International Journal of Industrial Organization* 3: 453-470.
- Chang, M.H., 1992, Intertemporal product choice and its effects on collusive firm behaviour, *International Economic Review* 4: 773-793.
- Colombo, S. 2010, Product differentiation, price discrimination and collusion, *Research in Economics* 64: 18-27.
- Colombo, S. 2011, Collusion in two models of spatial competition with quantity-setting firms, *Annals of Regional Science*, forthcoming.
- Espinosa, M.P., 1992, Delivered pricing, FOB pricing and collusion in spatial markets, *RAND Journal of Economics* 32: 64-85.
- Gupta, B. and G. Venkatu, 2002, Tacit collusion in a spatial model with delivered pricing, *Journal of Economics* 76: 49-64.
- Hackner, J., 1994, Collusive pricing in markets for vertically differentiated products, *International Journal of Industrial Organization* 12: 155-177.
- Hackner, J., 1995, Endogenous product design in an infinitely repeated game, *International Journal of Industrial Organization* 13: 277-299.
- Lederer, P.J. and A.P. Hurter, 1986, Competition of firms: discriminatory pricing and location, *Econometrica* 54: 623-640.
- Liu, Q. and K. Serfes, 2007, Market segmentation and collusive behaviour, *International Journal of Industrial Organization* 25: 353-378.
- Ross, T., 1992, Cartel stability and product differentiation, *International Journal of Industrial Organization* 10: 1-13.