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ECONOMIC GROWTH AND INDUCED TECHNICAL PROGRESS

BY

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1 INTRODUCTION

It is notorious that technical progress contributes to economic growth. In most studies on economic growth technical progress is treated as an autonomous factor. This is not satisfactory because in this way the long run growth of the economy remains partly unexplained. Technical progress will therefore be considered as endogenous in this study. Consequently we will consider the possibility that economic growth influences technical progress.

Technical progress can be characterized by its bias and by its magnitude as well. We will only consider the last characteristic by assuming that technical progress is purely labour augmenting. The main reason for this is that we will make use of the concept of steady growth. It is well-known, that steady growth paths are relatively easy to construct when only labour augmenting technical progress is involved.[1]

The above mentioned idea of interdependence between economic growth and technical progress is not new, it was introduced by Verdoorn as early as 1949.[2] He presented the idea in the form of two laws, of which the second should follow logically from the first, while the first is drawn from empirical evidence of industrial production in a number of countries. When, as the first law states, there exists a log-linear relationship between the productivity of labour (y/a) and cumulated production:

$$log\left(\frac{y}{a}\right)_t = b + c\,log \int_0^t y_\theta \mathrm{d}\theta \tag{1.1}$$

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1 Besides when the bias of technical progress is endogenously determined according to the well-known theory developed by C. Kennedy, the economic system itself generates a situation in which technical progress is purely labour augmenting. See for instance P. A. Samuelson, 'A theory of Induced Innovation along Kennedy-Weizsäcker lines,' *Review of Economics and Statistics*, XLVII (1965), pp. 343–356.

2 P. J. Verdoorn, 'Fattori che regolano lo sviluppo della produttivita del lavoro,' *L'industria*, 1949.

then, according to Verdoorn, there also has to exist a log-linear relationship between the productivity of labour and production, so

$$log\left(\frac{y}{a}\right)_t = d + e\,log\,y_t, \quad e < 1 \tag{1.2}$$

The second law, represented by equation (1.2), establishes a relationship between labour augmenting technical progress and the growth of production.[3] It should be noticed that steady growth is involved whenever equation (1.2) has to be considered as being derived from equation (1.1).[4]

In the case of steady growth induced technical progress along the lines set out by Verdoorn implies, that the growth of production ($\mathring{y}$) is limited by the rate of growth of labour supply (π). For as can be shown, steady growth implies[5]

$$\mathring{y} = \pi + \left(\frac{\mathring{y}}{a}\right) \tag{1.3}$$

which, in combination with equation (1.2) rewritten as

$$\left(\frac{\mathring{y}}{a}\right)_t = e\,\mathring{y}_t \tag{1.4}$$

gives

$$\mathring{y} = \frac{\pi}{1 - e} \tag{1.5}$$

This could be the reason why Kaldor, in his inaugural lecture in 1966 imputed the relatively poor performance of Britain with respect to economic growth, to a chronic shortage of labour.[6] A limited supply of labour, according to Kaldor, is typical for a mature economy in which the hidden reserves, formed by disguised unemployment in agriculture and services, are exhausted.

3 The identification of technical progress with the increase in the productivity of labour is a simplification made by us. Other possible sources of increases in the labour productivity of labour are sufficiently well-known.

4 This is proved in R. J. de Groof, *Geïnduceerde technische ontwikkeling*, unpublished doctoral thesis, Tilburg, 1977.

5 A small circle above a variable indicates the relative change of that variable. A list of symbols is added at the end of the article.

6 N. Kaldor, *Causes of the Slow Rate of Growth of the United Kingdom: An Inaugural Lecture*, Cambridge University Press, 1966.

In a recent discussion with Rowthorn on this subject, Kaldor took the opportunity to state that he had abandoned the standpoint that the low rate of growth of the British economy was due to labour scarcity resulting from 'economic maturity.'[7] Kaldor now believes that the United Kingdom was far from economic maturity. As a consequence of that he thinks British economic growth to be 'demand-induced' instead of being 'resource-constrained.'

This shift from a neo-classical to a Keynesian point of view has no repercussions for the explanation of induced technical progress as introduced by Verdoorn. In neo-classical theory the growth of actual production and the growth of capacity or potential production come down to the same. The rate of growth is then ultimately determined by the rate of growth of labour supply and the rate of technical progress. In this case the second Verdoorn law can be inserted in the neo-classical equation of steady growth as shown in the equations (1.3) to (1.5). This is not the right thing to do in the Keynesian case, in which economic expansion is less determined by capacity growth than by the rate of increase of effective demand. The determinants of the latter should then be specified. In this case only employment is influenced by technical progress unless it is assumed that the latter has consequences for effective demand as well. This may be true with respect to investment.

A complete theory of economic growth should contain factors determining effective demand as well as capacity growth. Combined with endogenous technical progress such a theory might become rather complex. In order to simplify the analysis we will assume in this study that actual and potential production coincide.

The arguments used by Verdoorn to rationalize his laws come down to what is usually called 'learning by doing,' of which Arrow's relationship between labour productivity and cumulated investments is another example.[8]

Learning by doing can be considered as a kind of technical progress occurring from inside the production process itself. It is a purely technical event, not an economic feature. As will be shown however, it is possible to generate the second Verdoorn law by means of economic reasoning. In that case one has to deal with factors that influence the 'state of arts' from outside the production sphere. These factors have to do with activities aimed at inventions that may lead to

7 R. Rowthorn, 'What Remains of Kaldor's Law?,' *Economic Journal*, LXXXV (1975), pp. 10–19; N. Kaldor, 'Economic Growth and the Verdoorn Law – A Comment on Mr. Rowthorn's Article,' *Economic Journal*, LXXXV (1975), pp. 891–896; R. Rowthorn, 'A Reply to Lord Kaldor's Comment,' *Economic Journal*, LXXXV (1975), pp. 897–901.

8 K. Arrow, 'The Economic Implications of Learning by Doing,' *Review of Economic Studies*, XXIX (1962), p. 155–173.

innovations. The activities we are referring to, consist of research and development efforts.

In this connection at least two questions have to be answered. First, what are the results of the expenditures for research and development? Second, what determines the level of those expenditures? The first question deals with the production of new technical knowledge, leading to new and cost saving production techniques as well as new final goods. We will confine ourselves to novelties of the first kind, *i.e.* cost saving technical progress. The second question has to do with consequences of the inventor's behaviour. With respect to this we initially assume profit maximization. In the last two sections however we will proceed from a more behaviouristic assumption.

The present study will be conducted as follows. In the next section a theory of induced technical progress will be presented, in which the micro-economic behaviour of the innovating firms is explicitly stated. Next the macro-economic consequences of induced technical progress are further explored. This will be done with the help of a model which will be introduced in section 3. Steady growth paths based on the model are studied in section 4. In the final section we will investigate the dynamic properties of the model. For this purpose a simulation experiment will be carried out, the result of which will be presented in graphical form.

2 THEORY OF INDUCED TECHNICAL PROGRESS

In this section we will present a model of induced technical change, based on some ideas revealed in Nordhaus' pioneering article on the subject.[9] As revealed in the first section technical progress is assumed to be purely labour augmenting. So, except for pinning down the type of technical progress, we abstract from product innovations. The model contains the following essential features of the innovation process in a capitalistic economy.

In the first place research projects have to be considered as investment projects, of which the expected costs and revenues determine whether these projects can be carried out or not. The revenues consist of the resulting cost savings. The cost consist of the research and development expenditures.

Secondly, the knowledge embodied in innovations is in essence a public good. The free use of that knowledge by others can only be impeded for a time – thanks to the technical complexity of the innovation – or be prevented by legal arrangements like a patent system.

9 W. Nordhaus, 'Theory of Innovation, An Economic Theory of Technological Change,' *American Economic Review*, LIX (1969), pp. 18–28. This theory is elaborated in R. J. de Groof, *op. cit.*

Thirdly, a necessary condition for the existence of private research is that the inventor has the possibility to the exclusive exploitation of his invention for some period. We assume that the inventor enjoys a period of T years in which he is protected against imitation.

The specification of the production function with respect to output can be postponed to the next section. The production function with respect to technology however is crucial for the description of technical progress. The rate of technical progress (μ_t) is assumed to be positively related with the amount of research and development expenditures (o_t) as an indicator of inventive input. Moreover it is assumed that the effect of a given amount of inventive input on the rate of technical progress will be lower, the more technology has advanced already. It seems plausible that the most promising labour saving projects will be chosen first.[10]

$$\mu_t \equiv \overset{\circ}{h}_t = o_t^{\omega} h_t^{-\gamma}, \qquad 0 < \omega < 1, \quad \gamma > 0 \tag{2.1}$$

where h_t is the productivity index of labour in the production function of commodities. The reduction of the cost price (p) caused by technical progress is given by:

$$\mathrm{d}p_t = -\lambda_t \mu_t p_t \tag{2.2}$$

The symbol λ stands for the share of labour income.

Setting $p = 1$ and considering equations (2.1) and (2.2), the profits gained from research and development (R&D) efforts, discounted back by a constant rate of interest, can be calculated as:

$$W_t = \int_t^{t+T} -\mathrm{d}p_\theta y_\theta e^{-r(\theta-t)} \mathrm{d}\theta - o_t = o_t^{\omega} h_t^{-\gamma} \int_t^{t+T} \lambda_\theta y_\theta e^{-r(\theta-t)} \mathrm{d}\theta - o_t \tag{2.3}$$

When factor shares are expected to stay constant and production is expected to grow at a constant rate ($\mathring{y}^*$) for the next T years, equation (2.3) can be rewritten as:

$$W_t = \lambda_t o_t^{\omega} h_t^{-\gamma} y_t \phi - o_t,$$

10 This assumption is quite the opposite from what is assumed by E. Phelps, *Golden Rules of Economic Growth*, Amsterdam, 1967, p. 139. In his view researchers become more able as technology advances. A considerable part of this effect will be swallowed up however by the fact that at the same time technology becomes more complex. The matter is, as Phelps admits, controversial.

where

$$\phi = \frac{1 - e^{(\mathring{y}^* - r)T}}{r - \mathring{y}^*} \tag{2.4}$$

The first order condition for profit maximization is $\partial W_t/\partial o_t = 0$, so the optimal amount of research and development expenditures equals[11]:

$$o_t = (\lambda_t \omega h_t^{-\gamma} y_t \phi)^{\frac{1}{1-\omega}} \tag{2.5}$$

Assuming constant expectations equation (2.5) can be transformed into:

$$\mathring{o}_t = \frac{1}{1 - \omega}(\mathring{\lambda}_t - \gamma \mathring{h}_t + \mathring{y}_t) \tag{2.6}$$

The relative change of the rate of technical progress can be derived from equation (2.1):

$$\mathring{\mu}_t = \omega \mathring{o}_t - \gamma \mathring{h}_t \tag{2.7}$$

In the case of steady growth we can write:

$$\mathring{\mu}_t = 0, \tag{2.8}$$

$$\mathring{\lambda}_t = 0. \tag{2.9}$$

Putting (2.8) into equation (2.7) yields:

$$\mathring{o}_t = \frac{\gamma}{\omega} \mathring{h}_t \tag{2.10}$$

Substituting equations (2.9) and (2.10) in (2.6) and taking into account $\mu_t \equiv \mathring{h}_t$, we get:

$$\mu_t = \frac{\omega}{\gamma} \mathring{y}_t \tag{2.11}$$

As will be shown in the following section, the situation of steady growth implies, that the growth of production equals the sum of the rates of population growth

11 The second order condition $\frac{\partial^2 W_t}{\partial o_t^2} < 0$ is fulfilled because of the assumption $0 < \omega < 1$.

(π) and technical progress:

$$\mathring{y}_t = \pi + \mu_t \tag{2.12}$$

Equation (2.12) combined with (2.11) then gives the constant rate of technical progress in a steadily growing economy:

$$\mu = \frac{\omega}{\gamma - \omega}\pi, \qquad \gamma > \omega \tag{2.13}$$

Equation (2.11) represents the second law of Verdoorn. It appears as a special case of the theory developed by Nordhaus. In this theory a positive rate of growth of output induces an acceleration of technical progress ($\mathring{\mu}_t > 0$). Profit maximization implies amongst other things a relation between the rate of technical progress and the *level* of output. On the other hand there is an inverse relationship between the productivity index of labour and the rate of labour augmenting technical progress. Now if the effect of $\mathring{y}_t$ on μ_t is counterbalanced by the effect of $\mathring{h}_t$ on μ_t (so that $\mathring{\mu}_t = 0$), relation (2.13) which was postulated by Verdoorn, is obtained.

For our purpose, *i.e.* an analysis of the macro-economic consequences of induced technical progress, we think of entrepeneurs assigning a certain fraction of total output to expenditures on research and development. We assume that this fraction is adapted with a certain time lag to relative changes in the real wage rate and eventually also to changes in the rate of output. The underlying idea is that extrepeneurs increase the R&D efforts if real wages rise in order to maintain the rate of profit. Besides, the rate of growth of output may have some influence on the fraction of R&D expenditures, because innovations become more lucrative at a higher growth rate of output. So, instead of straightforward profit maximization we introduce a more routine-like behaviour pattern, which may be more realistic in a dynamic and uncertain world. Assuming a similar geometrically distributed lag with regard to both explanatory variables and applying the Koyck transformation the level of expenditure for R&D follows from[12]:

$$\frac{o_t}{y_t} = \psi_2 \frac{o_{t-1}}{y_{t-1}} + \delta_1(1-\psi_2)\mathring{w}_t + \delta_2(1-\psi_2)\mathring{y}_t + (1-\psi_2)\delta_3 \tag{2.14}$$

The production function with respect to technology is written as:

$$\mu_t = \psi_1 \mu_{t-1} + \varepsilon_1(1-\psi_1)\frac{o_t}{y_t} + \varepsilon_2(1-\psi_1) \tag{2.15}$$

12 Because of the simulation experiments to be carried out later we pass to discrete time.

A distributed lag is introduced because the fruits of R&D efforts will not immediately be available.

In the steadily growing economy the following results hold:

$$\mu = \mu_t = \mu_{t-1}, \quad \frac{o}{y} = \frac{o_t}{y_t} = \frac{o_{t-1}}{y_{t-1}}$$

and, as will be shown in the next section, $\mathring{w}_t = \mu$, $\mathring{y}_t = \mu + \pi$. Substitution of these results in (2.14) and (2.15) yields after some rearrangements:

$$\mu = \frac{\varepsilon_1(\delta_2\pi + \delta_3) + \varepsilon_2}{1 - \varepsilon_1(\delta_1 + \delta_2)}, \quad \delta_1 + \delta_2 < \frac{1}{\varepsilon_1} \tag{2.16}$$

and

$$\frac{o}{y} = (\delta_1 + \delta_2)\mu + \delta_2\pi + \delta_3 \tag{2.17}$$

The next section will be devoted to problems of economic growth taking account of induced technical progress as described in the equations (2.14)–(2.17).

3 A MODEL OF ECONOMIC GROWTH

In order to investigate the consequences of induced technical progress we have to construct a model of economic growth. Such a model based upon a vintage approach is presented in this section.

It is assumed that the level of output and the demand for labour can be explained by a vintage model of the well-known 'clay-clay' type.[13] The implication of this assumption is that there are no substitution possibilities for entrepreneurs. When investments (i_t) are made entrepreneurs apply the best technique available at that time. After installation of the capital goods production coefficients of labour and capital remain constant over the entire lifetime of these capital goods. Technical progress takes the form of *embodied* labour augmenting innovations.

Total output capacity (y_t) is obtained by summing production capacities of vintages still in use:

$$y_t = \frac{1}{\kappa} \sum_{\tau=t-m_t}^{t-1} i_\tau \tag{3.1}$$

13 See for an analysis along similar lines S. K. Kuipers, 'A Vintage Model of Growth, Employment and Inflation,' *De Economist*, CXXIII (1975), pp. 531–558. The main difference with the approach of Kuipers is that we introduce induced labour savings.

where $t - m_t$ stands for the year of construction of the oldest vintage still in operation. The capital coefficient (κ) is a constant.

The number of jobs in each vintage depends upon the corresponding volume of investment and the capital intensity of the particular vintage. More recent vintages are characterized by a higher capital intensity, because labour augmenting innovations imply a rise of the productivity of labour from one vintage to the next. Total employment (a_t) can therefore be expressed by the following formula:

$$a_t = \frac{\alpha_0}{\kappa} \sum_{\tau=t-m_t}^{t-1} \frac{i_\tau}{\prod_{\theta=1}^{\tau} (1 + \mu_\theta)}$$

As explained in section 2, the rate of embodied labour augmenting technical progress (μ_t) is a variable. Labour productivity on vintage t is then the compound result of changes that took place in the past:

$$\frac{1}{\alpha_t} = \frac{1}{\alpha_0} \prod_{\theta=1}^{t} (1 + \mu_\theta)$$

Entrepreneurs will keep vintages in operation until the quasi-rent on them becomes negative, at least if we assume perfect competition on the market for final products. The age of the oldest vintage in use ($t - m_t$) follows under these circumstances from the condition that the wage rate (w_t) equals the productivity of labour on the marginal vintage:

$$w_t = \frac{1}{\alpha_0} \prod_{\theta=1}^{t-m_t} (1 + \mu_\theta) \tag{3.3}$$

The economic lifespan of capital goods is equal to: $(t - 1) - (t - m_t) + 1 = m_t$ periods. As said in section 1 we assume that total production always equals output capacity. So, there are no problems of effective demand. However, there may be unemployment of labour, because the real wage level may deviate from the equilibrium or full employment level, as will be explained below. It is also possible that the demand for labour derived from equation (3.2) exceeds the supply of labour (a_t^s). In this case total employment equals the supply of labour. The age of the marginal vintage is then determined by equation (3.2) substituting a^s for a on the left hand side of the equation. Equation (3.3) should then of course be skipped. As a consequence vintages which still earn a positive quasi rent are scrapped. There is not enough labour available to realize the profit foregone.

With regard to investment we introduce the following assumption. A fraction β

of total real profits (z_t) is available for investment and R&D outlays (o_t). The investment function can than be written as:

$$i_t = \beta z_t - o_t + i_t^{au} \tag{3.4}$$

where i_t^{au} stands for autonomous investment. The parameter β will be referred to as the propensity to invest out of profits. Total output is divided between consumption (not explicitly distinguished), investment and outlays for research and development.

The development of real wages over time follows from the rate of change of nominal wages (l_t) on the one hand, of the price level (p_t) on the other. The relative change of the nominal wage rate is explained by relative change of the price level, the relative rise of labour productivity,[14] the rate of unemployment (u_t) and an autonomous factor.

$$\mathring{l}_t = v_1\mathring{p}_{t-1} + v_2(\mathring{y}_{t-1} - \mathring{a}_{t-1}) - v_3 u_{t-1} + \mathring{l}_t^{au} \quad 0 < v_1 < 1;\ v_2, v_3 > 0 \tag{3.5}$$

The relative change of the price level depends on the development of wage cost per unit of output and also on an autonomous factor:

$$\mathring{p}_t = \eta_1 \mathring{l}_{t-1} - \eta_2(\mathring{y}_{t-1} - \mathring{a}_{t-1}) + \mathring{p}_t^{au} \quad 0 < \eta_1 < 1;\ \eta_2 > 0 \tag{3.6}$$

The model has to be completed by the following definitions:

$$\mathring{w} = \mathring{l}_t - \mathring{p}_t \tag{3.7}$$

$$w_t = w_{t-1}(1 + \mathring{w}_t) \tag{3.8}$$

$$z_t = y_t - a_t w_t \tag{3.9}$$

$$\mathring{y}_t = \frac{y_t - y_{t-1}}{y_{t-1}} \tag{3.10}$$

$$\mathring{a}_t = \frac{a_t - a_{t-1}}{a_{t-1}} \tag{3.11}$$

$$u_t = \frac{a_t^s - a_t}{a_t^s} \tag{3.12}$$

14 The relative change of labour productivity is equal to $\frac{(y/a)_t - (y/a)_{t-1}}{(y/a)_{t-1}}$. Instead we compute $(\mathring{y} - \mathring{a}_t)$ which is an acceptable approximation. The error involved is very small indeed. A similar remark should be made with regard to the determination of the relative change of the real wage rate according to equation (3.7).

The equations (2.14), (2.15) and (3.1)–(3.12) determine the 14 unknown variables of the model: $\mathring{l}_t$, $\mathring{p}_t$, $\mathring{w}_t$, w_t, m_t, y_t, a_t, $\mathring{y}_t$, $\mathring{a}_t$, o_t, μ_t, z_t, i_t and u_t. The model is made recursive to avoid numerical noise in the simulation experiments. The variables can be solved in the order given above.

4 STEADY GROWTH PATHS

A steady growth solution of the model implies a constant rate of technological change. The values of μ and o/y in a steadily growing economy are given by the equations (2.16) and (2.17).

Production capacity (y_t) grows at a constant rate if the volume of investment increases exponentially over time and if m_t is constant. Writing g for the constant growth rate of investment it follows from equation (3.1):

$$y_t = \frac{i_{t-1}}{\kappa} \cdot \frac{1 - \left(\dfrac{1}{1+g}\right)^m}{1 - \dfrac{1}{1+g}} \tag{4.1}$$

The investment ratio can be defined as $\sigma_t = i_t/y_t$. Substitution of this ratio in (4.1) gives:

$$\sigma = \kappa(1+g)\frac{1 - \dfrac{1}{1+g}}{1 - \left(\dfrac{1}{1+g}\right)^m} \tag{4.2}$$

As appears from equation (4.2) the investment ratio is a constant on a path of steady growth.

Application of the same procedure to equation (3.2) yields:

$$a_t = \frac{\alpha_0}{\kappa} \cdot \frac{i_{t-1}}{(1+\mu)^{t-1}} \cdot \frac{1 - \left(\dfrac{1+\mu}{1+g}\right)^m}{1 - \dfrac{1+\mu}{1+g}} \tag{4.3}$$

For the share of labour in income we can write $\lambda_t = a_t w_t / y_t$. After substitution of equations (3.3), (4.1) and (4.3) we then get the following expression for the share of labour:

$$\lambda = \left(\frac{1}{1+\mu}\right)^{m-1} \cdot \frac{1-\left(\frac{1+\mu}{1+g}\right)^m}{1-\frac{1+\mu}{1+g}} \cdot \frac{1-\frac{1}{1+g}}{1-\left(\frac{1}{1+g}\right)^m} \tag{4.4}$$

According to equation (4.4) the share of labour is also constant in a steadily growing economy.

Next we turn to the wage-price system. As can be seen from equations (4.1) and (4.3) labour productivity increases at the rate of labour augmenting technical progress. Steady growth of the wage rate and the price level implies: $\mathring{l}_t = \mathring{l}_{t-1}$ and $\mathring{p}_t = \mathring{p}_{t-1}$. In addition we put $\mathring{l}_t^{au} = v_4$ and $\mathring{p}_t^{au} = \eta_3$. Under these assumptions equations (3.5), (3.6) and (3.7) can be used to derive the following result for the relative change of the real wage rate:

$$\mathring{w} = \frac{\eta_2(1-v_1)+v_2(1-\eta_1)}{1-v_1\eta_1}\mu - \frac{v_3(1-\eta_1)}{1-v_1\eta_1}u_{t-1} + \frac{v_4(1-\eta_1)-\eta_3(1-v_1)}{1-v_1\eta_1} \tag{4.5}$$

With m and μ being constant it follows from equation (3.3) that the wage rate increases with the rate of labour augmenting technical progress: $\mathring{w} = \mu$. The rate of unemployment in the case of steady growth can now be found using equation (4.5):

$$u = \frac{\eta_2(1-v_1)+v_2(1-\eta_1)-(1-v_1\eta_1)}{v_3(1-\eta_1)}\mu + \frac{v_4(1-\eta_1)-\eta_3(1-v_1)}{v_3(1-\eta_1)} \tag{4.6}$$

Economically meaningful solutions to the equations (4.5) and (4.6) are obtained under the additional condition $v_1 < 1/\eta_1$.

The rate of unemployment remains constant over time if the demand for labour according to equation (4.3) grows as the same rate as the supply of labour. The constant growth rate of labour supply is symbolized by π. The growth rate of investment and output is therefore equal to:

$$g = \pi + \mu \tag{4.7}$$

Division of all terms in equation (3.4) by y_t gives an expression for σ_t. Substitution of (2.17) into this expression and disregarding autonomous investment (i_t^{au}) results in:

$$\sigma = \beta(1-\lambda) - \{(\delta_1+\delta_2)\mu + \delta_2\pi + \delta_3\} \tag{4.8}$$

The formulas derived for the case of steady growth may be used to analyse the comparative dynamic properties of the system. A rigorous analysis of this type will not be given here. Instead we will discuss briefly the results of some numerical experiments.[15] What we like to call the *basic* path of steady growth is determined by the following set of more or less realistic values for the relevant parameters.

$\alpha_0 = 1$	$\varepsilon_1 = 0.8$	$\delta_1 = 1$	$v_1 = 1$	$\eta_1 = 0.5$
$\kappa = 2.5$	$\varepsilon_2 = 0.008$	$\delta_2 = 0$	$v_2 = 1.2$	$\eta_2 = 0.5$
$\beta = 0.6820$		$\delta_3 = 0$	$v_3 = 0.5$	$\eta_3 = 0$
			$v_4 = 0$	

The supply of labour is held constant at: $a^s = 4065.04065$. As a consequence of the implicit assumption $\pi = 0$, the rate of growth of output is equal to μ. Equations (4.3) and (4.4) have to be adapted using De L'Hopital's rule:

$$a = \frac{\alpha_0}{\kappa} \cdot m \cdot i_0 \tag{4.3a}$$

$$\lambda = m\left(\frac{1}{1+\mu}\right)^{m-1} \cdot \frac{1 - \dfrac{1}{1+\mu}}{1 - \left(\dfrac{1}{1+\mu}\right)^m} \tag{4.4a}$$

The following *variants* will be considered.

a) A change in the rate of autonomous technical progress (ε_2) or alternatively a change in autonomous expenditure for research and development (δ_3).
b) An alteration of the propensity to invest out of profits (β).
c) A different rate of autonomous wage increase (v_4).

The results of the computations for the basic path and the variants mentioned above are given in table 1. The presentation is restricted to a few variables of special interest.

In the case of slightly higher autonomous labour savings the inducement mechanism yields a rate of technical progress that is substantially higher than on the basic path. Labour is comparatively more abundant in this case. For this reason the share of labour is lower than on the reference or basic path of economic growth. With a higher share of profits the macro-economic investment ratio (σ) is also higher. As a consequence the economic lifetime of capital goods is shorter than before. The rate of unemployment is higher because the rate of change of labour productivity that appears in the equation for the wage rate is multiplied

15 We are indebted to Mr. T. van den Aker for invaluable programming assistance.

TABLE 1 – STEADY GROWTH SOLUTIONS

Variant / variable	*basic*	$\Delta\varepsilon_2 = 0.002$	$\Delta\delta_3 = 0.0025$	$\Delta\beta = 0.118$	$\Delta\nu_4 = 0.01$
μ	0.04	0.05	0.05	0.04	0.04
σ	0.1840	0.2072	0.2063	0.1979	0.1840
λ	0.6716	0.6229	0.6206	0.7027	0.6716
m	20.00	18.95	19.09	17.95	20.00
u	0.016	0.02	0.02	0.016	0.036

by a factor greater than one ($\nu_2 = 1.2$). This is realistic if there is a wage leader with a rate of change of productivity above average.

The results in the case of an increase in autonomous expenditure for research and development (R&D) are similar as is shown in table 1 (column 3). The investment ratio is slightly lower, because a little bit more is spent on research and development in order to get the same increase in μ. The differences with regard to the other variables correspond to this minor deviation.

An increase in the propensity to invest out of profits leads to a higher macro-economic investment ratio, although the share of profits in income declines. Compared with the basic path the relative scarcity of labour is greater. This is also reflected in a higher level of the real wage rate. For that reason the economic lifetime of capital goods m is shorter than on the basic path.

Although the *level* of real wages is higher now, the rate of *change* ($\mathring{w}_t$) is still the same as before. This implies that the rate of unemployment also keeps the same value as on the basic path.

The last variant, an autonomous increase in the nominal wage rate (ν_4), has a simple solution. There is no change except for an increase in the rate of unemployment in comparison with the basic path. According to equation (4.3) the real wage rate increases as before with $100\mu\%$ per period. The autonomous change must therefore be compensated by a decline in the induced part. The Phillips curve mechanism can do the job at the expense of a substantially higher unemployment rate.

The numerical outcomes presented in table 1 (and corresponding results that can be computed) can be seen as the *asymptotic* values of the time path of the variables which emerge when one (or more) of the parameters or one (or more) exogenous variable(s) is (are) changed. It is the exploration of such time paths that stands central in the following section.

5 GROWTH DYNAMICS

The time path of the variables can be calculated if the lagged variables which appear in the equations of the model are specified. We start with the assumption that the economy moves along the steady growth path which we have labelled as *basic*. The lagged variables can then be calculated easily with the help of the formulas derived in section 4. If nothing happens the economy will remain on the basic path of steady growth forever.

Now suppose that beginning with period $t = 1$ one of the parameters is changed. Then the economy will leave the basic path and move towards a new path of steady growth if the system meets the stability conditions. In order to compute the movement over time we have to specify two additional parameters: $\psi_1 = 0.5$ and $\psi_2 = 0.5$. In this section two non-steady growth paths will be studied. First, we will discuss the path emerging from a change in autonomous expenditure for research and development ($\Delta\delta_3 = 0.0025$). Secondly, we will comment on the effects of change in the propensity to invest out of profits ($\Delta\beta = 0.118$). Computations are made for the period from $t = 1$ to $t = 100$.

Some results of a change in autonomous expenditure for R&D are presented in figure 1. The new *asymptotic* values of the variables considered are indicated by broken lines. As can be observed, the asymptotic values of the different variables are approached in a cyclical manner. However, the 5% growth figure is still not realized at the end of the entire time span of 100 periods.

The cycles are explained by echo-effects that are typical for vintage models. For instance, the unemployment rate declines sharply after approximately 20 periods. At that time the marginal vintage reflects the increase in μ at the beginning of the simulation period, whereas the real wage rate is about the same as on the basic path. As a result of these developments the scrappage of unprofitable capital goods is retarded.

The resulting lower unemployment causes with a time lag of one period an acceleration in the rate of change of the real wage. This in turn leads to a sharp rise in the rate of labour augmenting technical progress (μ). As a result of the rise in μ the cycle is repeated after another 22 periods. The length of a cycle is determined by the economic lifetime of capital goods and the lags in the equations explaining the development of w_t and μ_t. The path followed by the unemployment rate shows local maxima at $t = 20$, $t = 42$, $t = 66$ and $t = 89$.

The cyclical behaviour is manifest throughout the system. The growth rate of production for instance shows a similar behaviour. However, more important is the rising *trend* of the growth rate of y_t indicated by the dotted line in figure 1.[16]

16 Regression of $\hat{y}_t$ on t gives the following result: $\hat{y}_t = 0.000081t + 0.040101$.

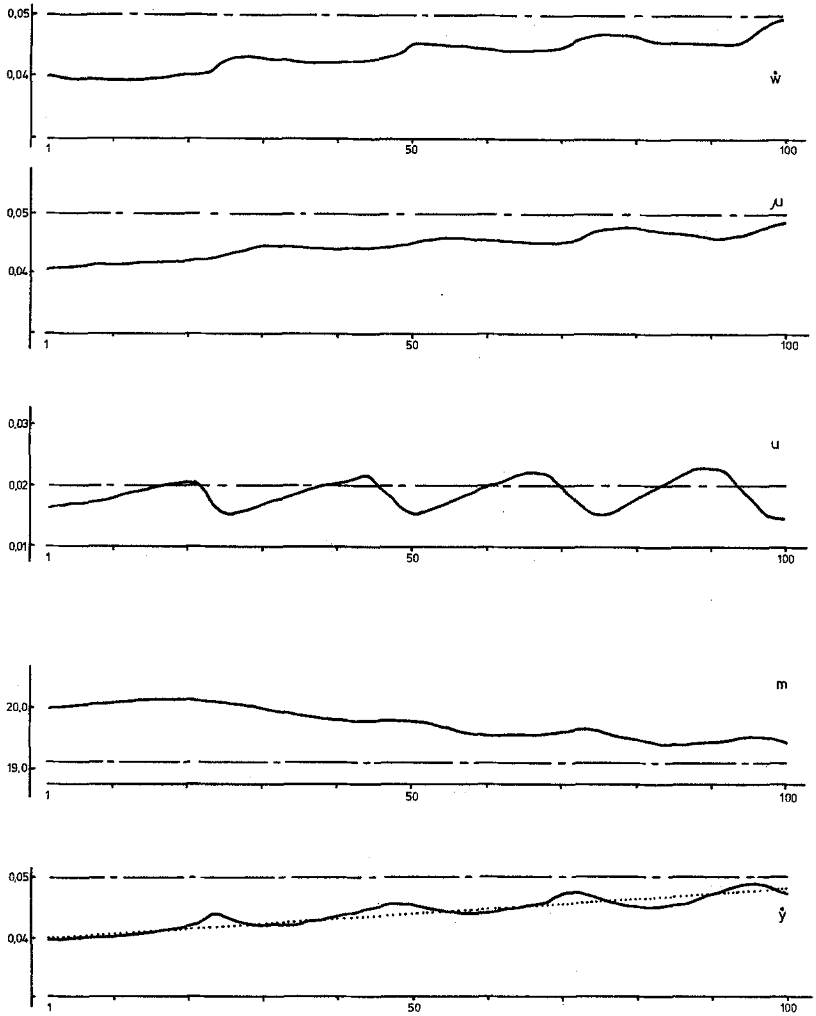

Figure 1 Increase in autonomous expenditure for R&D ($\Delta\delta_3 = 0.0025$).

It shows that a slightly higher autonomous expenditure for R&D or – what comes to the same thing – a small positive change in autonomous technical progress can induce a gradual increase in the growth rate of the economy extending over a substantial time span.

In the case of an increase in the propensity to invest out of profits ($\Delta\beta = 0.118$) echo-effects dominate the picture as can be seen from figure 2. The rate of unemployment shows a sharp rise after period $t = 17$. At this time the economic lifetime of capital goods has come down to about 18 periods. The marginal

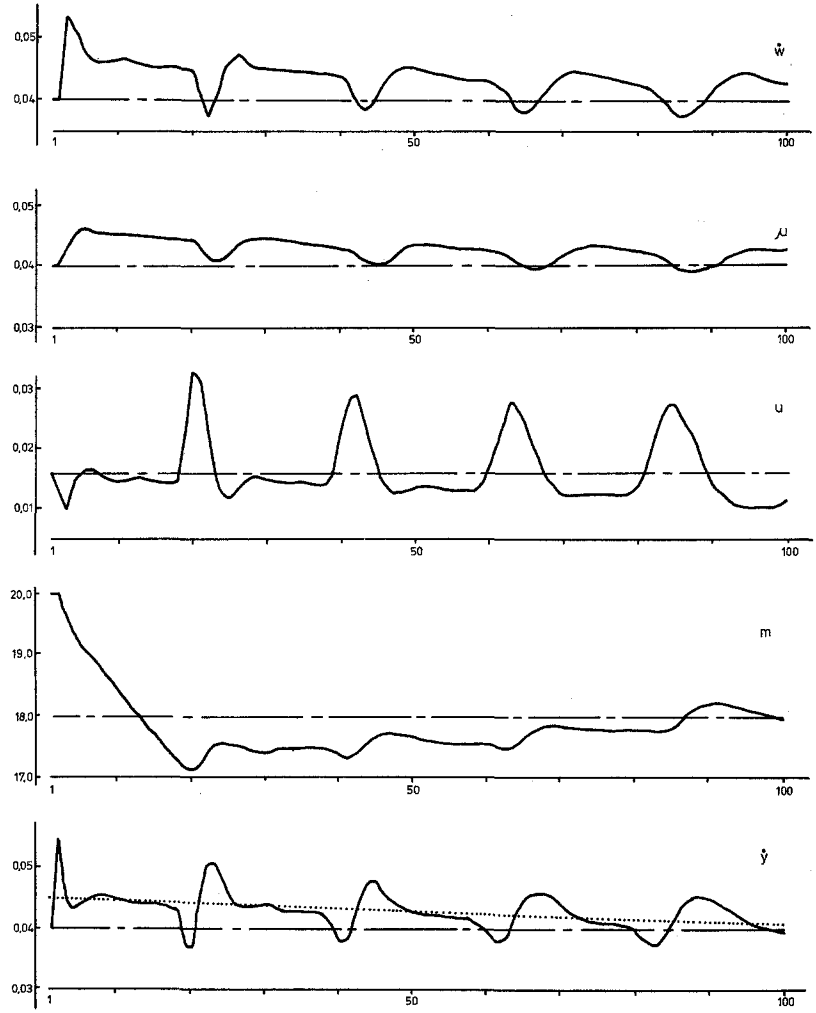

Figure 2 Increase in the propensity to invest out of profits ($\Delta\beta = 0.118$).

vintage is then relatively large, because the increase of the level of investment at the beginning of the simulation period. If such voluminous vintages are scrapped, unemployment must increase.

The cycle is sustained by the resulting behaviour of $\mathring{w}_t$ and μ_t in the same way as is explained above. The movement however is reinforced by humps in the level of investment resulting from a shift in the distribution of income. A lower rate of employment and a retardation in the rise of real wages induce a lower share of labour and therefore a higher share of profits. Because of the time lags involved, the second and following investment humps come after the turning point of the employment rate. This explains the 'widening' of the peaks with regard to the cyclical pattern of μ_t.

As is indicated by the dotted line in figure 2 the growth rate of production shows a declining *trend*.[17] Nevertheless, the *trend* values are higher over the entire period than the *asymptotic* value of $\mathring{y}_t$, which has not changed (see table 1). An increase in the propensity to invest leads to a temporarily higher growth as is well known from neoclassical growth theory. However, in our model the effect of a rise in the propensity to invest on economic growth is reinforced by the increase in μ_t along the trend. In reality such an increase in the propensity to invest may be induced by a higher rate of technical progress. The effect on economic growth of an autonomous change in innovating activities is then amplified.

A similar story can be told in the case of a *decline* in the autonomous rate of technical progress or a slower growth rate of the supply of labour [see equation (2.16)]. We may even go a step further by assuming – as is often done in the literature[18] – some kind of clustering of basic innovations. It may then be possible to simulate growth cycles of some duration that are less esoteric than the echo-determined cycles that appear in our results. But then the question rises what determines the clustering of innovation over time. The answer to this question may be found in the exhaustion and regeneration of the innovating potential. However, we do not want to go so far. We only intended to show that the mechanism of induced technical progress contributes to the explanation of an acceleration or a deceleration in the rate of economic growth over a substantial period of time.

LIST OF SYMBOLS

Relative changes of variables are indicated by a small circle above the letter. The suffix τ attached to a variable indicates that the variable belongs to vintage τ.

Variables

a	employment
a^s	supply of labour
h	productivity index of labour
i	volume of investment
l	nominal wage rate
m	economic lifetime of capital goods

17 The relevant regression equation takes the form: $\mathring{y}_t = -0.000039t + 0.044936$.

18 See for instance the contribution of J. J. van Duijn elsewhere in this issue.

o	expenditures for research and development
p	price level
r	rate of interest, discount rate
u	rate of unemployment
w	real wage rate
W	discounted profits of research and development efforts
y	volume of production
z	real profits
α	labour requirements per unit of production of a certain vintage
λ	share of labour in income
μ	rate of technical progress
σ	investment ratio

Constants

g	growth rate of investment
T	period during which the innovator is protected against imitation
β	investment ratio out of profits
γ	elasticity of the level of technology with respect to technical progress
$\delta_1, \delta_2, \delta_3$	parameters in the function explaining research expenditures as a fraction of total output
$\varepsilon_1, \varepsilon_2$	parameters in the production function with respect to technology
η_1, η_2, η_3	parameters in the price equation
κ	coefficient of capital
$\nu_1, \nu_2, \nu_3, \nu_4$	parameters in the equation for the nominal wage rate
ϕ	capitalization factor in the profit function with respect to research expenditures
ω	elasticity of research expenditures with respect to technical progress.

Summary

ECONOMIC GROWTH AND INDUCED TECHNICAL PROGRESS

In this article some consequences of induced technical progress for economic growth are discussed. Technical progress is assumed to be purely labour augmenting. First the rate of technical progress is explained along lines as set out by Nordhaus. Secondly we introduce a more routine-like behaviour pattern of inventors showing a relation between expenditures for R&D on the one hand and the rate of wage increase and the growth of output on the other.

The emphasis of this study lies on the exploration of the macro-economic consequences of induced technical progress. To this end a vintage model with induced technical progress is constructed, on the basis of which some steady growth variants are studied. Finally simulation experiments are carried out to investigate the dynamic properties of the model.