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WAGE RIGIDITY AND CAPITAL MOBILITY IN AN OPTIMIZING MODEL OF A SMALL OPEN ECONOMY**

BY

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1 INTRODUCTION

Exchange-rate overshooting models of small open economies with highly integrated financial markets, based on Dornbusch (1976), have become very popular in both the theoretical and empirical literature. For example, Buiter and Miller (1982) use them to discuss monetary disinflation and the Medium Term Financial Strategy of the Thatcher experiment. The models typically assume nominal wage rigidity, which is not too unreasonable for the U.S. For the European economies real wage rigidity may be more relevant (*e.g.*, Branson and Rotemberg (1983)). Real wage rigidity can reverse the effectiveness of fiscal and monetary policy (*e.g.*, Casas (1975); Argy and Salop (1977); Sachs (1980)). In particular, under real wage rigidity monetary policy has no real effects and a fiscal expansion leads to an appreciation of the real exchange rate, reduces the wedge between the producers' wage and the consumers' wage and thus increases output and employment. Real wage rigidity also gives a more interesting role to supply-side policies. Another important aspect of most of these models is that they assume uncovered interest parity, which is not a good empirical approximation. Some authors do allow for imperfect capital mobility in overshooting models (*e.g.*, Frenkel and Rodriquez (1982)) and find that the exchange rate may undershoot for low degrees of capital mobility. These extensions of the basic overshooting model are based on the flow approach to the balance of payments and are not based on a portfolio model of international asset holdings. Furthermore, they do not allow for current-account and wealth dynamics (along the lines of Dornbusch and Fischer (1980)).

Another problem with this literature is that the models are *ad hoc* and are not based on satisfactory micro foundations. This means that they suffer from the Lucas (1976) critique of econometric policy evaluation and that the welfare gains or losses, based on gross consumers' surplus, of policies such as monetary

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disinflation cannot be obtained. Also, it is difficult to analyse fiscal and monetary policy mixes within these *ad hoc* (linear) models as the intertemporal budget constraints for the government and private sector have not been carefully specified.

The objective of this paper is to formulate an optimizing model of a small open economy, which extends Turnovsky (1985) to allow for imperfect substitutability between home and foreign assets and to allow for unemployment caused by nominal or real wage rigidity, and use it to discuss the effects of monetary disinflation, fiscal contraction and an increase in the world interest rate. It also gives a micro foundation of Mundell-Fleming models and extends these models to allow for the intemporal government budget constraint and current account dynamics. Section 2 presents the basic version of the model, which assumes purchasing power parity and discusses its steady-state and transient properties. It is thus based on Turnovsky (1985) and incorporates a stock approach to the balance of payments. The asset menu consists of home currency, traded bonds and non-traded bonds. Section 3 extends the basic model to allow for imperfect substitution between home and foreign goods. Section 4 uses a 'multiple shooting' algorithm to obtain numerical simulations of a monetary disinflation, fiscal contraction and an increase in the world interest rate under nominal wage rigidity, real wage rigidity or labour market equilibrium and under purchasing power parity or imperfect substitution between home and foreign goods. Section 5 concludes the paper.

2 A SMALL OPEN ECONOMY WITH PURCHASING POWER PARITY

2.1 *Structure of the Economy under Full Employment*

The economy consists of a representative household, a representative firm and the government. Agents have perfect foresight, although the variant of nominal wage rigidity presumes sluggish adjustment of the core inflation rate. It is assumed that the economy is small, so that it can take the foreign interest rate and foreign price level as given on the world markets. The asset menu consists of domestic currency, non-traded bonds issued by the home government and traded bonds issued abroad. Domestic agents do not hold foreign currency, so that there is no currency substitution. The government finances its public sector borrowing requirement either by printing cash or by issuing non-traded bonds. Households demand goods, traded bonds, non-traded bonds and money and supply labour. Firms demand labour and supply goods. Firms do not invest. Although the goods and all asset markets are assumed to clear instantaneously, the labour market does not clear immediately either due to nominal wage rigidity or due to real wage rigidity. Households are therefore rationed in their supply of labour. The notional supply of labour is assumed to be inelastic. The full-employment version of the present model with purchasing power parity and an elastic supply of labour is discussed in Turnovsky (1985).

The household maximizes discounted utility,

$$\operatorname*{Max}_{c,m,b,a} \int_0^\infty e^{-\delta t}[u(c,l,g)+v(m)]\,\mathrm{d}t,\ u_c,\ u_g>0,\ u_l<0,\ v''<0, \tag{2.1}$$

subject to a certainty-equivalent version of the household's intertemporal budget constraint,

$$\dot{m}+\dot{a}+\dot{b}=(r-p)a+(r^*-p+e)b+wl+z-c-\tau-pm-(\tfrac{1}{2}\alpha b^2), \tag{2.2}$$

and the initial conditions, $m(0)=M_0/P(0)$, $a(0)=A_0/P(0)$ and $b(0)=E(0)B_0^*/P(0)$ where:

c = real consumption (C/P); C = nominal consumption,
g = real government spending,
τ = real lump-sum taxes,
m = real money balances (M/P); M = nominal money balances,
a = real stock of non-traded bonds (A/P); A = nominal stock of non-traded bonds,
b = real stock of traded bonds (EB^*/P),
B^* = nominal stock of traded bonds denominated in foreign currency,
l = labour; $1-l$ = leisure,
w = real wage rate (W/P), W = nominal wage rate,
z = real profits,
r = home nominal interest rate,
r^* = foreign nominal interest rate,
p = home inflation rate $(\dot{P}/P)$; P = home price level,
p^* = foreign inflation rate $(\dot{P}^*/P^*)$; P^* = foreign price level,
e = anticipated (and actual) rate of exchange depreciation $(\dot{E}/E)$,
E = nominal exchange rate,
δ = home rate of time preference.

For completeness the other symbols used later in the paper are

μ = marginal utility of wealth,
u_n = natural rate of unemployment,
ϑ = growth in the nominal money supply,
π = core or trend rate of inflation,
w^d = desired long-run target for the real wage,
γ = real liquidity (m/w),
p^c = consumers' price index (CPI),
c' = consumption deflated by the CPI (cP/P^c),
m' = nominal money deflated by the CPI (M/P^c),
c_d = household's consumption of home goods,
c_m = household's consumption of foreign goods (imports),
λ = real exchange rate or terms of trade (P^*E/P),
w^c = real consumption wage (wP/P^c),
β = share of labour in output.

The utility function is separable in real money balances and concave in c, l, g and m. The household's budget constraint says that disposable income, that is, profit and labour income plus interest income on traded and non-traded bonds minus lump-sum and inflation taxes, minus consumption equals real savings. These real savings are invested in cash, traded bonds and non-traded bonds. The certainty-equivalent version of the household's budget constraint also includes a risk premium, $\frac{1}{2}\alpha b^2$ where α depends on the degree of risk aversion and the variance of exchange risk. This avoids a full stochastic specification. The supply of labour is assumed to be inelastic and is typically rationed by the demand for labour (see section 2.2). It is assumed that the household's decision problem has an interior solution, which is ensured when $\lim_{c \to 0} u_c = \lim_{m \to 0} v' = \infty$. The first-order conditions follow from $\partial H/\partial c = 0$, $\partial H/\partial m = \partial H/\partial a = \partial H/\partial b = \delta\mu - \dot{\mu}$, where $H = u(c,l,g) + v(m) + \mu[(r-p)a + (r^* - p- + e)b + wl + z - c - \tau - pm - \frac{1}{2}\alpha b^2]$ denotes the Hamiltonian and μ denotes the shadow price of household's wealth. It follows that the marginal utility of consumption equals the marginal utility of wealth, $u_c = \mu$, that the marginal rate of substitution between consumption and real money balances equals the nominal interest rate, $v'(m)/u_c = r$, that the holdings of traded bonds are given by

$$b = (r^* + e - r)/\alpha \tag{2.3}$$

and that the marginal utility of wealth satisfies

$$\dot{\mu} = [\delta - (r-p)]\mu, \; \lim_{t \to \infty} e^{-\delta t}\mu(t)i(t) = 0, \; i = m, a, b. \tag{2.4}$$

The first two conditions give rise to the consumption function,

$$c = c(\mu, l, g), \; c_\mu = u_{cc}^{-1} < 0, \; c_i = -u_{ci}u_{cc}^{-1}, \; i = l, g, \tag{2.5}$$

and the demand-for-money function,

$$m = m(r\mu), \; m' = 1/v'' < 0. \tag{2.6}$$

Equation (2.3) shows that holdings of traded bonds increase when the differential between the expected return on foreign assets and the return on home assets increases. The limiting case of risk neutrality, $\alpha = 0$, corresponds to perfect capital mobility and yields the uncovered interest parity condition, $r = r^* + e$. An infinite risk premium on foreign assets, $\alpha \to \infty$, leads to zero capital mobility, $b \to 0$. This simple portfolio specification seems more satisfactory than an approach based on capital flows (*e.g.*, Frenkel and Rodriquez (1982)) and has been postulated before (Driskill and McCafferty (1980); Turnovsky and Bhandari (1982)). Equation (2.4) shows that, when the rate of time preference exceeds the real interest rate, then *ceteris paribus* the marginal utility of wealth increases and consumption falls over the life cycle. For example, when utility is logarithmic, it follows that $\dot{c}/c = (r - p) - \delta$ and consumption is linear in wealth, that is, $c + rm = \delta(m + a + b + h)$ where $h(t) = \int_t^\infty (y - \tau)\exp(-\int_t^s (r - p)\mathrm{d}s')\mathrm{d}s$ defines human wealth. In general, utility need not to be separable in c and g. For example, if $u_{cg} < 0$, an increase in public spending means that, for a given marginal utility of wealth, private consumption falls, which is an example of direct crowding out. Similarly, if $u_{cl} > 0$, an increase in employment leads to an increase in private consumption, $c_l > 0$. Even if $u(\cdot)$ is weakly separable and therefore $u_{ci} = 0$, $i = l, g$, it is possible that l and g have direct effects on economic welfare.

The firm maximizes profits, $z = f(l) - wl$ where $f(l)$ is a neoclassical production function, so that $f'(l) = w$ and the demand for labour is given by

$$l = l(w),\ l' = 1/f'' < 0. \tag{2.7}$$

Labour market equilibrium (LME) gives

$$l = 1 - u_n \tag{2.8}$$

where u_n denotes the natural rate of unemployment. Money market equilibrium gives

$$\dot{m} = m(\vartheta - p) \tag{2.9}$$

where $\vartheta \equiv \dot{M}/M$ denotes the exogenous growth in the nominal supply of money. There is no commercial banking system. Goods market equilibrium gives

$$\dot{b} = f(l) - c - g + (r^* - p + e)b, \tag{2.10}$$

so that the current account of the balance of payments equals the balance of trade, *i.e.* the excess of home production over home absorption, plus real interest payments on holdings of foreign debt. Upon integration of (2.10), one obtains the economy's intertemporal budget constraint:

$$b(t) = \int_t^\infty [c + g - f(l)]\,exp(-\int_t^s (r^* - p + e)\mathrm{d}s')\mathrm{d}s$$
$$+ \lim_{s\to\infty} [b(s)\,exp(-\int_t^s (r^* - p + e)\mathrm{d}s')]. \tag{2.10'}$$

The economy cannot run up an indefinite debt or credit abroad, so the second term in (2.10') must be zero. Thus today's debt is the discounted stream of the excess of domestic production over future domestic absorption. The government's budget constraint is given by

$$\dot{a} = g + (r - p)a - \tau - \vartheta m, \; a(0) = m(0)A_0/M_0, \tag{2.11}$$

so that government spending plus interest payments on the domestic debt minus lump-sum taxes and seignorage revenues has to be financed by issuing bonds. Alternatively, the current stock of government debt plus the discounted stream of future government spending has to be paid off by the discounted stream of future taxes and seignorage revenues. Finally, relative purchasing power parity (PPP) yields

$$p = p^* + e. \tag{2.12}$$

Since the household's budget constraint can be derived from the goods market equilibrium condition and the government's budget constraint, the model can be summarized by equations (2.3)–(2.12). It is assumed that the government maintains a constant stock of real bonds, $a = \bar{a}$, by adjusting lump-sum taxes to balance its budget, $\tau = g + (r - p)\bar{a} - \vartheta m$. This assumption avoids the problem of an ever-diverging interest-bearing government debt. The alternative of exogenous τ and endogenous a typically requires feedback policy rules for g, τ or ϑ to stabilize the government debt (*e.g.* Buiter, 1986). Both these assumptions imply that the initial jumps in a and m satisfy $a(0) = m(0)A_0/M_0$. Another assumption is that the government can engage in an initial open market operation to ensure that the economy does not run up an infinite debt, which means that both $a(0)$ and $m(0)$ are free to jump. Note that, as debt neutrality holds, equation (2.11) is independent of the rest of the model, so that it does not really matter which of the above three alternatives is chosen. The assumption of debt neutrality is of course debatable and in Van de Klundert and Van der Ploeg (1987) we attempt to relax this when it is possible to shift the burden of higher taxation onto future (yet unborn) generations.

Note that the current account can be rewritten as

$$\dot{b} = [(r - p)a + (r^* - p + e)b + wl + z - c - \tau - pm] \\ - [g - \tau + (r - p)a - pm], \tag{2.10'}$$

that is, the difference between private sector savings and the public sector deficit. Even though there is savings and no investment, it is quite possible to have a current-account deficit when the public sector deficit is large enough (witness the U.S. in the eighties).

2.2 *Nominal and Real Inertia in the Labour Market*

Two alternatives to full employment (LME) are considered: nominal wage rigidity (NWR) and real wage rigidity (RWR). Nominal wage rigidity (NWR) corresponds to

$$\frac{\dot{w}}{w} + p = \emptyset(1 - l) + \pi, \; \emptyset' < 0, \; \emptyset'' > 0, \; \lim_{u \to 0} \emptyset(u) = \infty, \; \emptyset(u_n) = 0, \tag{2.8'}$$

where the core or trend rate of inflation, π, is either given by the monetary growth rate, $\pi = \vartheta$, or is a weighted average of past rates of inflation, $\dot{\pi} = \xi(p - \pi)$, $\xi > 0$. Linear versions of these augmented Phillips curves have been used in Buiter and Miller (1982). Here $\emptyset(.)$ is nonlinear, so that households are always rationed when they sell labour and firms are never rationed. Real wage rigidity (RWR) corresponds to

$$\frac{\dot{w}}{w} = \emptyset(1 - l) - \eta \, log[w/w^d], \, \eta > 0, \tag{2.8''}$$

where w^d denotes the desired long-run target for the real wage. Real wage rigidity implies complete indexation to increases in the cost of living. Equation (2.8″) corresponds to an error-correction mechanism of the type estimated by Sargan (1964) and Layard and Nickell (1985). This ensures that the real wage recovers from past disequilibria in the labour market and gradually adjusts towards its long-run target value:

$$log[w(t)/w^d] = log[w(0)/w^d]exp(-\eta t) + \int_0^t \emptyset(1 - l(s))exp[\eta(s - t)]ds.$$

The extreme case of a completely rigid real wage, $w = w^d$, corresponds to $\eta \to \infty$. The case $\eta = 0$ is identical to the case of NWR with 'rational expectations' or immediate adjustment to increases in the cost of living ($\xi \to \infty$ in the equation for the core rate of inflation, $\dot{\pi} = \xi(p - \pi)$). In general, real wage rigidity implies a long-run relationship between the real wage and employment, *viz.* $w = w^d exp[\emptyset(1 - l)/\eta]$, and therefore in the long run the real wage only reaches its target value when the unemployment rate happens to be at its natural rate. Note that the target value of the real wage need not be consistent with the natural rate of unemployment and profit-maximizing behaviour. Indeed, if the desired wage is excessive, $w^d > f'(1 - u_n)$, the equilibrium unemployment rate (or the non-accelerating inflation rate of unemployment, NAIRU) exceeds the natural rate of unemployment. Branson and Rotemberg (1980), Bruno and Sachs (1985) and Van der Ploeg (1987) demonstrate empirically that nominal wage rigidity is more relevant for the U.S. but that real wage rigidity is more relevant for the European countries.

2.3 *Steady-state Properties*

The steady state of the model does not depend on whether full employment (LME) or nominal wage rigidity prevails. In the long run the real wage adjusts until employment is at its equilibrium level, $\bar{w} = f'(1 - \bar{u})$ and $\bar{l} = 1 - \bar{u}$, where with LME or NWR equilibrium unemployment is given by its natural rate $\bar{u} = u_n$, and with RWR $\bar{u} = \bar{u}(w^d, \eta)$ follows from $f'(1 - \bar{u}) = w^d \exp[\emptyset(\bar{u})/\eta]$. Note that with real wage rigidity an increase in the desired real wage or the flexibility of the real wage ($\eta\downarrow$) increases equilibrium unemployment ($\bar{u}_{w^d} > 0$, $\bar{u}_\eta < 0$). Only when there is no long-run conflict between the aspirations of workers and firms, $w^d = f'(1 - u_n)$, will equilibrium unemployment be at its natural rate.

Long-run inflation is entirely a monetary event, $\bar{p} = \bar{\pi} = \vartheta$. The steady-state rate of exchange depreciation equals the difference between home monetary growth and foreign inflation, $\bar{e} = \vartheta - p^*$. The steady-state real interest equals the pure rate of time preference, $\bar{r} - \bar{p} = \delta$. It is assumed, following Turnovsky (1985), that

$$\delta = (r^* - p^* + \alpha\bar{\delta})/(1 + \alpha)$$

where $\bar{\delta}$ is a constant, so that $\delta = r^* - p^*$ under perfect capital mobility ($\alpha = 0$) and $\delta = \bar{\delta}$ under zero capital mobility ($\alpha \to \infty$). This device ensures that steady-state consumption is stationary under perfect capital mobility ($\dot{\mu}/\mu = \delta - (r - p) \to 0$ as $t \to \infty$)[1] and that the real interest rate is independent of the rate abroad under zero capital mobility. If $r^* - p^* = \delta^*$, the economy has in the long run a debt (credit) with the rest of the world when its pure rate of time preference is greater (less) than the foreign discount rate, $\bar{b} = (\delta^* - \delta)/\alpha = (\delta^* - \bar{\delta})/(1 + \alpha)$. Steady-state consumption is given by $\bar{c} = f(1 - \bar{u}) + (r^* - p^*)(r^* - p^* - \bar{\delta})\ (1 + \alpha)^{-1} - g$ and holdings of real money balances by $\bar{m} = m((\delta + \vartheta)\ u_c(\bar{c},\ 1 - \bar{u},\ g))$. Hence, an increase in the natural unemployment rate or the level of public spending and a reduction in the world real interest rate mean less scope for private sector consumption and therefore a higher marginal utility of wealth and less holdings of real money balances. The net effect of an increase in the world real interest rate on money balances is ambiguous, because the increase in interest-income from abroad permits more

TABLE 1 - STEADY-STATE PROPERTIES[a]

	ϑ	a	g	$\bar{u}_n$	$r^* - p^* = \delta^*$
π, p	1	0	0	0	0
$b, r-p=\delta$	0	0	0	0	$(1+\alpha)^{-1}>0$
c	0	0	-1	$-\bar{w}$	$(2\delta^*-\bar{\delta})(1+\alpha)^{-1}(>0)$
l	0	0	0	-1	0
w	0	0	0	$1/f''>0$	0
μ	0	0	$u_{cg}-u_{cc}(>0)$	$-u_{cl}-\bar{w}u_{cc}(>0)$	$u_{cc}(2\delta^*-\bar{\delta})(1+\alpha)^{-1}(<0)$
m	$u_c/v''<0$	0	$\frac{\bar{r}}{v''}(u_{cg}-u_{cc})\ (<0)$	$\frac{\bar{r}}{v''}(u_{cl}+\bar{w}u_{cc})(<0)$	$\frac{\mu + \bar{r}\,u_{cc}(2\delta^*-\bar{\delta})}{(1+\alpha)v''} \gtrless 0$
τ	$-\bar{m}$	δ	$1-\frac{\vartheta\bar{r}}{v''}(u_{cg}-u_{cc})(>0)$	$\frac{\vartheta\bar{r}}{v''}(u_{cl}+\bar{w}u_{cc})(>0)$	$\frac{\bar{a}}{1+a} - \vartheta\frac{\partial m}{\partial\delta^*} \gtrless 0$

[a] When the sign is indicated in parentheses, it is derived under the assumption of weak separability of $u(c,l,g)$.

1 Interest parity is ensured since (when $\alpha = 0$), $r = r^* + e$ and $e = \vartheta - p^*$ yields $r - \vartheta = r^* - p^*$ in the long run. This device is important, because without it the steady-state value of private consumption is not well defined.

consumption, reduces the marginal utility of consumption and thus increases real money balances, yet when there is some capital mobility the home interest rate rises via (2.13) and therefore real money balances fall. An increase in monetary growth raises the nominal interest rate and therefore reduces real money balances. The comparative statics are summarized in Table 1. Note that, in contrast to Turnovsky (1985), an increase in public spending has no long-run effect on employment and output and is fully off-set by a reduction in consumption. Similarly, a decrease in the foreign real interest rate has no long-run effect on employment and output but reduces income from abroad (provided $2\delta^* > \bar{\delta}$ and thus b is not too negative) and diminishes consumption. These results are due to the assumption of an inelastic labour supply.

2.4 *Transient Properties*

The reduced-form of the model with purchasing power parity and labour market equilibrium can be written as:

$$\dot{m} = m[\vartheta - \alpha b - (v'(m)/\mu) + r^* - p^*],\ m(0) = \text{free}, \tag{2.13}$$

$$\dot{b} = f(l(w)) + (r^* - p^*)b - c(\mu, l(w), g) - g,\ b(0) = B_0^*/P_0^* \tag{2.14}$$

$$\dot{\mu} = \mu[\delta + \alpha b - (r^* - p^*)],\ \mu(0) = \text{free} \tag{2.15}$$

and

$$w = f'(1 - u_n). \tag{2.16}$$

Equations (2.13), (2.14) and (2.15) describe the dynamics of inflation, the current account and consumption, respectively. With nominal wage rigidity the LME-condition is replaced by

$$\dot{y} = y[\vartheta - \pi - \emptyset(1 - l(m/y))],\ y(0) = y_0 \tag{2.16'}$$

and either $\pi = \vartheta$ or

$$\dot{\pi} = \xi[\alpha b + (v'(m)/\mu) - \pi - (r^* - p^*)], \tag{2.17}$$

where the initial jump in π is given by $\pi(0) - \pi(0_-) = \xi log[m(0_-)/m(0)]$ and the real wage is given by $w = m/y$. With NWR m (the price level) and μ are forward-looking variables, that is, they depend on expectations about future changes in exogenous variables as well as the past, whilst y (*c.q.* the nominal wage) and π are backward-looking variables, that is, they only depend on past history. Even though π is a backward-looking variable, it is a non-predetermined variable as it jumps at time zero. Hence the perfect-foresight model with PPP and NWR has to have two unstable eigenvalues in order to satisfy the saddlepoint property (*e.g.*, Buiter (1984)). With real wage rigidity (2.16) is replaced by

$$\dot{w} = w[\emptyset(1 - l(w)) - \eta\, log(w/w^d)],\ w(0) = w_0, \tag{2.16''}$$

where the real wage is assumed to be predetermined. The saddlepoint property for this version of the model also requires two stable eigenvalues associated with b and w and two unstable eigenvalues associated with m and μ. The analysis of perfect-foresight solutions under NWR is cumbersome, but numerical simulations are discussed in section 4. An analytical discussion of the dynamic behaviour of the model with PPP and LME or with PPP and RWR is feasible and follows below.

Linearization around the steady state of equations (2.13)–(2.15) yields:

$$\begin{bmatrix} \dot{m} \\ \dot{b} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} -mv''/\mu & -\alpha m & mv'/\mu^2 \\ 0 & r^* - p^* & -\mu \\ 0 & \alpha\mu & 0 \end{bmatrix} \begin{bmatrix} m - \bar{m} \\ b - \bar{b} \\ \mu - \bar{\mu} \end{bmatrix} = \mathbf{J} \begin{bmatrix} m - \bar{m} \\ b - \bar{b} \\ \mu - \mu \end{bmatrix} \quad (2.18)$$

The economy is locally unstable, since trace $(\mathbf{J}) = r^* - p^* - mv''/\mu > 0$. Because $\det(\mathbf{J}) = -\alpha v'' m\mu > 0$, the product of the eigenvalues is positive and therefore there are two unstable and one stable eigenvalues. These are associated with the forward-looking variables, m and μ, and the backward-looking variable b, respectively. Under full employment the wage is constant, $w = f'(1 - u_n)$, and under real wage rigidity the wage is unaffected by monetary or fiscal policies or foreign events and tends to $f'(1 - \bar{u})$. With PPP there is therefore no essential difference between LME and RWR. However, when there is imperfect substitution between home and foreign goods, a wedge can be driven between the producers' and consumers' real wage, policy is non-neutral and RWR differs from LME in an interesting way (see section 3).

The case of zero capital mobility, say ZCM, occurs when there is an infinite risk premium on foreign assets ($\alpha \to \infty$) and leads to no net holdings of foreign assets ($b = 0$), so that (2.13)–(2.16) becomes

$$\dot{m} = m[\delta + \vartheta - v'(m)/\mu(g, u_n)], \; m(0) = \text{free}, \quad (2.19)$$

where $\mu(.)$ follows from $f(1 - u_n) = c(\mu, 1 - u_n, g) + g$ (so that $\mu_g = u_{cg} - u_{cc}$ (> 0) and $\mu_{u_n} = -u_{cc}f' - u_{cl}$ (> 0)). Since $\partial\dot{m}/\partial m = -mv''/\mu > 0$ (evaluated at the equilibrium), m immediately jumps to its new equilibrium value unless there are anticipated shocks. Figure 1 shows the effects when there is an announcement about a future change in policy, say a monetary disinflation, a cut in government spending or a reduction in the natural rate of unemployment. The anticipated excess supply of goods leads to an immediate fall in the price level, an associated increase in real liquidity and an appreciation of the real exchange rate ($E \to A$). This leads to less inflation and therefore to further increases in real liquidity ($A' \to B$). At the time of implementation (B) no jump in real money balances is required, since the economy has already discounted this news.

Under perfect capital mobility ($\alpha = 0$), (2.3) and (2.15) give rise to uncovered

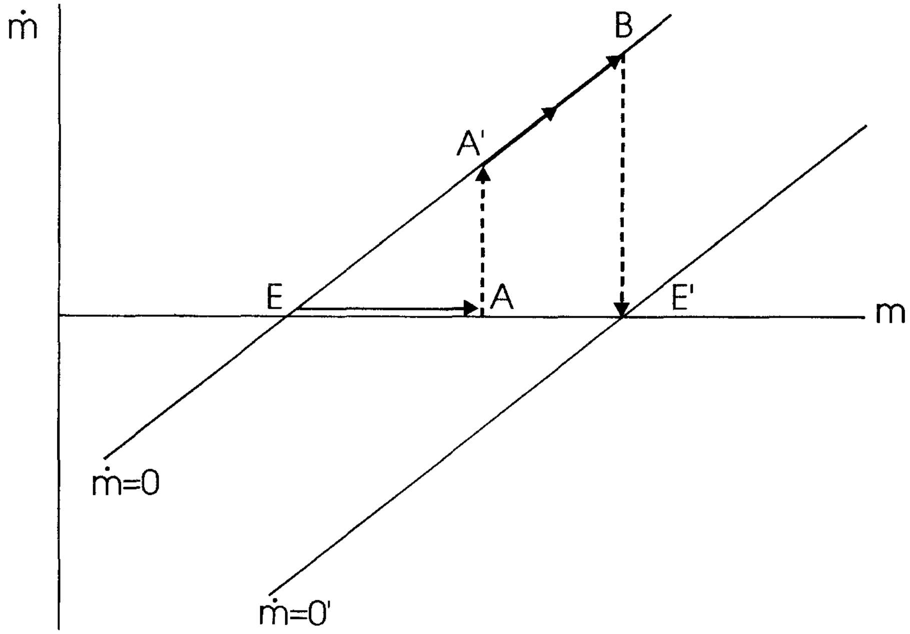

Figure 1 - Anticipated policy in a model with PPP, LME and ZCM

interest parity, $r^* + e = r$, equalization of real interest rates, $r - p = r^* - p^* = \delta$, and consequently to a flat consumption profile, $\mu = \bar{\mu}$ where $\bar{\mu}$ follows from (2.10′) and the initial debt, b_0. Note that $\bar{\mu}$ now depends negatively on the initial level of the public debt and positively on the entire path of present and future government spending levels and natural rates of unemployment: $b_0 = \int_0^\infty [c(\bar{\mu}, 1 - u_n, g) + g - f(1 - u_n)] exp(-\delta t) dt$. Under zero capital mobility (ZCM) the economy cannot borrow against future income, so μ and consumption depend only on the current levels of g and u_n. It follows that (2.14) reduces to $\dot{m} = m[\delta + \vartheta - v'(m)/\bar{\mu}]$, which has the same properties as (2.19). Turnovsky (1985) discusses the analytical properties of a model with PPP, LME and general degrees of capital mobility.

3 IMPERFECT SUBSTITUTION BETWEEN HOME AND FOREIGN GOODS

3.1 *Two-stage Decision Problem of the Representative Household*

Section 2 assumed the 'law of one price,' but here the model is extended to allow for imperfect substitution between home and foreign goods. This is essential for the discussion of issues such as real-exchange-rate overshooting (Dornbusch, 1976). Also, with real wage rigidity it allows demand-side policies to have a long-run effect on employment and output through the wedge between the producers' real wage and the consumers' real wage. The household's decision problem is split up in two stages. In the first stage it decides on its aggregate consumption, savings and portfolio decisions and in the second stage it decides how to divide aggregate consumption into consumption of home goods and consumption of foreign goods. Denote real consumption and liquidity when deflated with the consumers' price index, say P^c, by $c' \equiv cP/P^c$ and $m' \equiv M/P^c$, respectively, and when deflated with the producers' price index by c and m, respectively. Instantaneous utility is now given by $u(c', l, g) + v(m')$ and the household's budget constraint is as before.

Stage I yields $u_{c'} = \mu P^c/P$, $\delta\mu - \dot{\mu} = v'(m')(P/P^c) - p\mu = (r - p)\mu = (r^* - p + e - \alpha b)\mu$ and the usual transversality condition, so that $c' = c(\mu P^c/P, l, g)$ and $m' = m(r\mu P^c/P)$. Equations (2.3) and (2.4) are as before. In stage II the household maximizes $U(c_d, c_m)$ subject to the static budget constraint, $Pc_d + P^*Ec_m = P^c c' = Pc$ or $c_d + \lambda c_m = c$, where c_d and c_m denote the household's consumption of home goods and of foreign goods, respectively, and $\lambda \equiv P^*E/P$. The sub-utility function, $U(.)$, is assumed to be homogeneous of degree one. The household sets the marginal rate of substitution between home and foreign goods equal to the real exchange rate, $U_{c_m}/U_{c_d} = \lambda$. This yields $c_m = \Phi(\lambda)c$, $\Phi' < 0$ and $c_d = [1 - \lambda\Phi(\lambda)]c$. It follows from the indirect utility function that the ideal definition of the cost-of-living index is given by[2]

$$P^c/P = 1/U(1 - v\Phi(\lambda), \Phi(\lambda)) \equiv \Omega(\lambda), \Omega' > 0. \tag{3.1}$$

For example, if the sub-utility function is Cobb-Douglas, say $U(\cdot) = (c_d/\gamma)^\gamma (c_m/(1-\gamma))^{1-\gamma}$, then $\Phi(\lambda) = (1-\gamma)/\lambda, \Omega(\lambda) = \lambda^{1-\gamma}$ and $P^c = P^\gamma(P^*E)^{1-\gamma}$. Exports are given by $c_m^* = \Phi^*(\lambda^{-1})c^*$, where foreign aggregate consumption, c^*, is treated as exogenous in a small open economy. A real appreciation of the exchange rate reduces the relative price of imported goods and therefore reduces the cost-of-living index. It also increases imports and reduces exports.

3.2 *Structure of the Economy*

The goods market equilibrium condition can be written as

$$f(l) = c_d + g + c_m^* = c + g + c_m^* - \lambda c_m. \tag{3.2}$$

Upon substitution of (3.2) into (2.10), one obtains

$$\dot{b} = (c_m^* - vc_m) + (r^* + e - p)b, \tag{3.3}$$

so that the current account, *i.e.* the balance of trade plus current transfers, gives the increase in foreign assets. This holds only for floating exchange rates, because then the balance of payments is always in equilibrium.

The complete model with imperfect substitution between home and foreign goods can now be summarized by:

2 Strictly speaking this index should also include the cost of holding real money balances, but for simplicity this is ignored.

$$b = (r^* + e - r)/\alpha, \tag{2.3}$$

$$\dot{\mu} = [\delta - (r - p)]\mu, \ \mu(0) = \text{free}, \tag{2.4}$$

$$\dot{m} = m(\vartheta - p), \ m(0) = \text{free}, \tag{2.9}$$

$$\tau = g + (r - p)\bar{a} - \vartheta m \tag{2.11}$$

$$f(l(w)) = [1 - v\Phi(\lambda)]c + g + \Phi^*(-\lambda^{-1})c^*, \tag{3.2'}$$

$$\dot{b} = \Phi^*(\lambda^{-1})c^* - v\Phi(\lambda)c + (r^* - p + e)b, \tag{3.3'}$$

$$c = \Omega(\lambda)c(\mu\Omega(\lambda), l(w), g), \tag{3.4}$$

$$m = \Omega(\lambda)m(r\mu\Omega(\lambda)), \tag{3.5}$$

$$\dot{\lambda}/\lambda = p^* + e - p, \tag{3.6}$$

$$p^c = p + [\Omega'(\lambda)\dot{\lambda}/\Omega(\lambda)] \tag{3.7}$$

and *either* $l(w) = 1 - u_n$ for the case of full employment *or*

$$\dot{w}/w + p = \emptyset(1 - l(w)) + \pi \tag{2.8'}$$

with $\pi = \vartheta$ or $\dot{\pi} = \xi(p^c - \pi)$, $\xi > 0$ for the case of nominal wage rigidity *or*

$$\dot{w}^c/w^c \equiv \dot{w}/w + p - p^c = \emptyset(1 - l(w)) - \eta \, log(w^c/w^d), \ w^c(0) = w_0^c, \tag{3.8}$$

where $w^c \equiv wP/P^c$, for the case of real wage rigidity. These 11 (12) equations can be solved for the 11 (or 12) endogenous variables, *viz.* c, m, b, μ, r, e, p, p^c, λ, w (π or w^c), and τ given the exogenous variables, *viz.* g, ϑ, $\bar{a}$, u_n, $\bar{\delta}$, r^*, p^* and c^*.

Upon substitution of (3.4) into (3.2), one can solve for the real exchange rate that clears the goods market:

$$\lambda = V(\mu, g, c^*, w). \tag{3.9}$$

An increase in public spending, foreign demand or the wage and a reduction in the marginal value of wealth induce an excess demand for goods, which is choked off by an appreciation of the real exchange rate ($V_\mu > 0$, $V_i < 0$, $i = g, c^*, w$). Under real wage rigidity the real consumers' wage is given at any point of time, so that employment, $l(w_c\Omega(\lambda))$, and output increase when an appreciation of the real exchange cuts the wedge between the producers' and consumers' wage.

3.3 *Transient and Steady-state Properties*

The LME version has two forward-looking state variables, m and μ, and one backward-looking state variable, b, hence requires two unstable and one stable eigenvalues for the saddlepoint property to be satisfied. The NWR version or the RWR version with high elasticities of the demand for labour with respect to the wage has two forward-looking variables, m and μ, and two backward-looking variables, b and γ (for NWR) or w^c (for RWR), hence requires two unstable and two stable eigenvalues. If the RWR system has three unstable eigenvalues and one stable eigenvalue, then m, μ and λ are forward-looking variables and (b/λ) is a backward-looking variable.

The steady-state properties of the monetary side of the imperfect-substitution model are the same as under purchasing power parity, that is, $\bar{p}^c = \bar{p} = \pi = \vartheta$, $\bar{e} = \vartheta - p^*$, $\bar{r} - \bar{p} = \delta$ and $\bar{b} = (\delta^* - \bar{\delta})/(1 + \alpha)$. In the long run demand-side policies have no real effects for the cases of full employment and nominal wage ridigity, hence $\bar{l} = 1 - u_n$ and $\bar{w} = f'(1 - u_n)$. However, for the case of real wage rigidity fiscal policy is non-neutral in the long run (*cf.* Van de Klundert, 1982). This can be seen from the long-run aggregate supply schedule,

$$\bar{w}^c = w^d exp[\emptyset(1 - \bar{l})/\eta] = f'(\bar{l})/\Omega(\bar{\lambda}), \tag{3.10}$$

which is drawn in Figure 2(b). It is upward-sloping in $(P/P^*E) - l$ space, because an appreciation of the real exchange rate reduces the relative price of foreign goods, increases the real income of consumers, diminishes the wedge between the consumers' and producers' real wage, and therefore increases employment ($\bar{P}^c/\bar{P}\downarrow$, $\bar{w}^c\uparrow$, $\bar{l}\uparrow$, $\bar{w}\downarrow$). Obviously, for LME or NWR the long-run aggregate supply schedule is vertical (see Fig. 2(a)). The long-run aggregate demand schedule is given by

$$\begin{aligned}\lambda &= V(u_c, (\bar{c}/\Omega(\bar{\lambda}), \bar{l}, g)/\Omega(\bar{\lambda}), g, c^*, f'(\bar{l})) \\ &\equiv \bar{V}(\bar{l}, g, r^* - p^*, c^*),\end{aligned} \tag{3.11}$$

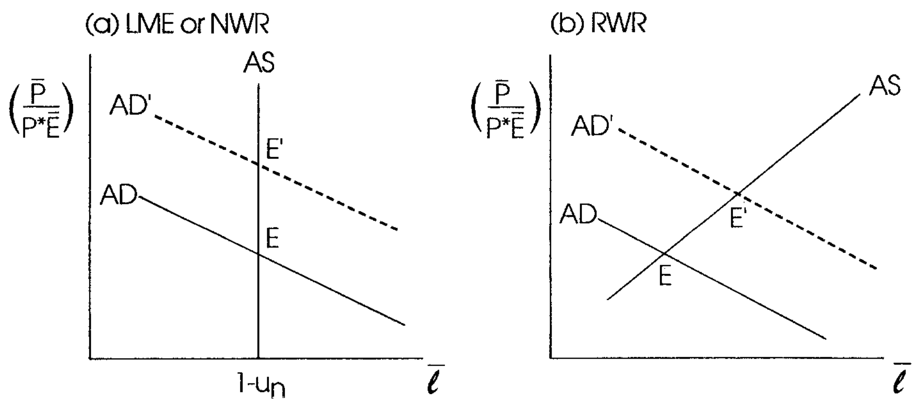

Figure 2 - Effects of an increase in public spending, foreign consumption or the foreign real interest rate on the steady state of the imperfect-substitution model

since the long-run level of consumption that ensures current-account equilibrium is given by $\bar{c} = f(\bar{l}) + (r^* - p^*)(r^* - p^* - \bar{\delta})(1 + \alpha)^{-1} - g$. The AD-schedule is downward-sloping ($\bar{V}_l > 0$), because a reduction in the relative price of home goods increases exports and aggregate demand given that the marginal propensity to consume out of income from production, $1 - \lambda\Phi(\lambda)$, is less than unity. An increase in public spending or foreign consumption increases aggregate demand and shifts out the AD-schedule ($\bar{V}_i < 0$, $i = g, c^*$). An increase in the foreign real interest rate or a foreign-exchange increment to wealth means the economy can sustain a higher level of aggregate consumption without suffering a loss in foreign reserves, hence the AD-schedule shifts out ($\bar{V}_i < 0$, $i = r^* - p^*$).

Intersection of the long-run AD- and AS-schedules gives $\bar{l} = 1 - u_n$ for full employment or nominal wage rigidity and

$$\bar{l} = 1 - \bar{\bar{u}}(\overset{-}{g}, \overset{-}{c}^*, r^* \overset{-}{-} p^*, \overset{-}{w}^d, \overset{+}{\eta}, \overset{+}{u}_n) \tag{3.20}$$

for real wage rigidity. It follows that with RWR there is no unique equilibrium rate of unemployment, since fiscal policy, foreign events and incomes policy affect long-run (or natural) levels of employment and output. Fiscal expansion is effective in the long run under RWR, because it requires an appreciation of the real exchange rate (in order to ensure equilibrium in the goods market) which in turn reduces the wedge between the consumers' and producers' real wage and therefore increases employment.

4 NUMERICAL SIMULATIONS

The transient properties of the full model are difficult to discuss analytically, so that we decided to use numerical simulation instead. A Cobb-Douglas production function is assumed, say $f(l) = \varepsilon l^{\beta}$, $0 < \beta < 1$, so that the labour-demand curve is given by $l(w) \equiv (\varepsilon\beta/w)^{1/(1-\beta)}$. This gives rather high wage elasticities in the labour demand equation, $-(1 - \beta)^{-1}$, hence we also considered a CES production function, $f(l) = [(1 - \beta)\varepsilon^{\varrho/1-\beta} + \beta l^{\varrho}]^{1/\varrho}$. This has an approximate wage elasticity of $-(1 - \beta)^{-1}(1 - \varrho)^{-1}$, which is low for low elasticities of substitution between labour and the fixed factor and consequently supply shocks will have less severe effects. The excess demand for labour term is assumed to be logarithmic, say $\emptyset(u) = \emptyset_0 log(u_n/u)$, so that $\emptyset(u) \rightarrow \infty$ as $u \rightarrow 0$. Cobb-Douglas utility functions are assumed, so that $u(c', l, g) = \gamma_1 log(c') + \gamma_2 log(1 - l) + \gamma_3 \log(g)$, $\gamma_i \geq 0$, $\sum_{i=1}^{3} \gamma_i = 1$ and $U(c_d, c_m) = (c_d/\gamma)^{\gamma} (c_m/(1 - \gamma))^{1-\gamma}$. This implies that $c = \gamma_1/\mu$, $c_d = \gamma c$, $\Phi(\lambda) = (1 - \gamma)/\lambda$ and $\Omega(\lambda) = \lambda^{1-\gamma}$. Money demand follows from a CES utility function, say $v(m') = \gamma_4 m'^{\,1-\gamma_5}/(1 - \gamma_5)$, where $0 < \gamma_5 < 1$, so that $m = (r\mu/\gamma_4)^{-1/\gamma_5}\lambda^{(1-\gamma)(\gamma_5 - 1)/\gamma_5}$. The CES specification has been chosen, because it allows for a non-unitary elasticity of the demand for money with respect to the interest rate.

The numerical values of the parameters, exogenous variables and initial con-

TABLE 2 - VALUES OF PARAMETERS, EXOGENOUS VARIABLES AND INITIAL CONDITIONS

PARAMETERS

$\varepsilon = 1.0$, $\beta = 0.75$, $\phi_0 = 0.005$, $\xi = 1.0$, $\eta = 0$ or 2, supply-side parameters

$\delta = 0.05$, $\gamma_1 = 0.5$, $\gamma_2 = 0.3$, $\gamma_3 = 0.2$, $\gamma = 0.75$, $\gamma^* = 0.975$, $\gamma_4 = 0.05$, $\gamma_5 = 2.0$, demand-side parameters

$\alpha = 0.1$ or 1.0, high or low degree of capital mobility

EXOGENOUS VARIABLES

$u_n = 0.05$, $w^d = 0.76$, supply-side policies

$g = 0.2$, $\vartheta = 0.1$, demand-side policies

$r^* = 0.15$, $p^* = 0.1$, $c^* = 10.0$, foreign variables

INITIAL CONDITIONS

$B_0^* = b_0 = 0.$

$\gamma_o = 1.066$, for the case of NWR

$w_0^c = w_0$, for the case of RWR

ditions are presented in Table 2. It would be possible to experiment and investigate the robustness of the results with respect to a wide range of plausible parameter values in a systematic manner along the lines of Camilleri *et al.* (1984), but this is left as a matter for further research. Instead, attention is focussed on a number of key alternative specifications. We can distinguish between (i) PPP and imperfect substitution or, (ii) LME, NWR and RWR, which gives 6 variants of the model. Obviously, the two variants with imperfect substitution between home and foreign goods and NWR (for the U.S.) and RWR (for Europe) are the most interesting ones. A multiple shooting algorithm (Lipton *et al.* (1982)) based on routine DDØ3 of the Culham Laboratory is used to solve the model (England (1976)).

4.1 *Monetary Disinflation*

Under labour market equilibrium (LME) a change in the monetary growth rate (ϑ) has no effect on real variables such as production, consumption, employment and international trade (*cf.* Turnovsky (1985)). There will be an instantaneous jump towards the new long-run equilibrium value of real cash balances. In our numerical examples a decline in the monetary growth rate from 10% to 8% will lead to an instantaneous increase of real money balances by 7.4%. Such an increase is caused by a discrete appreciation of the nominal exchange rate and fall in the price level. The domestic nominal rate of interest falls two percentage points. Since inflation in domestic prices corresponds to monetary growth ($p = \vartheta$), the real rate of interest remains unchanged. To maintain portfolio equilibrium there must be a permanent rate of appreciation of the nominal exchange rate of two percent.

In the case of RWR, the same results apply irrespective of whether the foreign good is a perfect or an imperfect substitute for the domestic good. Nominal wages adapt without friction to the price level. The nominal wage rate acts as a jump variable. Relative prices can therefore stay at their equilibrium level. A change in monetary growth does not affect the real exchange rate. It therefore does not affect the wedge between consumers' and producers' prices either, so that output and employment remain unaffected.

Things are different under NWR as the nominal wage rate is predetermined in this case. The results of a monetary disinflation of two percentage points are presented in Table 3. The model specification now matters. The outcomes depend on whether goods are imperfect or perfect substitutes. There is also some influence of the specification with regard to inflation expectations. This latter variant is only presented for the PPP case. The models exhibit saddlepoint stability. The long-run results are the same as under LME and RWR. In the numerical examples these results are approximated up to period $t = 100$, because a longer horizon does not significantly affect the results for the first 10 periods. On impact of a monetary disinflation there is unemployment caused by an increase in the real wage rate. This leads to a reduction of real wealth, which induces households to consume less. As time passes nominal wages gradually adjust to the lower price level and unemployment declines. Long-run equilibrium is attained eventually.

Let us now turn to the model-specific results of a monetary disinflation. Focussing first on the PPP models it appears from Table 3 that the case of adaptive expectations with regard to core inflation (π) exhibits a faster adjustment towards the long-run equilibrium. This is what should be expected. Moreover, it can be observed that the impact at $t = 0$ is somewhat larger in this case too. Turning to the balance of payments we see in both variants that consumption declines less than output at first, implying a deficit on the current account. It pays to borrow abroad, because the appreciation of the exchange rate exceeds the fall of the domestic interest rate. However, in the course of time debt accumulates and consumption must be pushed downward to service debt and repay the loans. As observed above, in the long run there are no effects on real variables such as consumption and output.

Shifting attention towards the case of imperfect substitution between the domestic and the foreign good we observe no changes in the foreign account. In this model a fall in output implies that consumption and exports of the domestic good are reduced. With total demand abroad given, there will be an improvement of the terms of trade of the domestic economy. In our Cobb-Douglas example the change in the terms of trade just matches the decline in total consumption. As a result the volume of imports will not change and there

TABLE 3 – MONETARY DISINFLATION ($\vartheta = 0.08$); NOMINAL WAGE RIGIDITY (NWR)

Model	PPP; π adaptive								PPP; $\pi = \vartheta$								Imperfect Substitution; $\pi = \vartheta$							
Period Variable	0	1	2	3	4	5	10	100	0	1	2	3	4	5	10	100	0	1	2	3	4	5	10	100
m	3.7	3.7	4.0	4.3	4.7	5.0	6.6	7.4	3.0	3.0	3.1	3.3	3.5	3.7	4.8	7.4	2.0	2.3	2.6	2.9	3.2	3.5	4.6	7.4
w	3.7	1.1	1.5	1.1	0.8	0.6	0.1	0	3.0	2.4	2.1	1.8	1.6	1.5	1.0	0	1.9	1.8	1.7	1.6	1.5	1.4	1.0	0
$c(c_d)$	−5.8	−5.1	−4.6	−4.1	−3.6	−3.0	−1.0	0	−7.0	−6.9	−6.7	−6.3	−6.0	−5.6	−4.0	0	−7.2	−6.7	−6.3	−5.9	−5.4	−5.2	−3.6	0
b	0	−3.8	−5.1	−5.6	−5.6	−5.3	−2.8	0	0	−2.1	−3.2	−3.7	−3.9	−3.9	−3.1	0	0	0	0	0	0	0	0	0
l	−1.5	−4.6	−5.9	−4.4	−3.4	−2.5	−0.2	0	−12.1	−9.8	−8.2	−7.2	−6.4	−5.8	−3.9	0	−7.5	−7.0	−6.6	−6.2	−5.8	−5.4	−3.8	0
$\lambda(c_m^*)$																	−7.2	−6.7	−6.3	−5.9	−5.4	−5.2	−3.6	0

Percentage deviation (except b, for which it is 100 times the absolute deviation).

will be no effect on the current account. It should be noted that the impact (at $t = 0$) on employment is less in this model than in the PPP versions. The real appreciation of the exchange rate implies that there is a wedge between the consumer price index (P^c) and the producers' price (P). The jump in λ leads to a fall in P^c exceeding the decline in P. Real cash balances in terms of consumption goods may rise to the same extent in both models. This implies a smaller jump in terms of output for the case of imperfect substitution, because of the wedge referred to above.

It is interesting to observe the percentage changes in the nominal exchange rate. From Table 3 one can see that for all three variants the nominal exchange rate appreciates by 7.4% in the long run. The jump appreciation on impact with imperfect substitution is 9.2%. (This result is obtained by subtracting λ from m.) As may be expected the nominal exchange rate overshoots its long-run value in this case (*cf.* Dornbusch (1976); Dornbusch (1980, Chapter 12); Corden (1981); Buiter and Miller (1982)).

The results of this section are consistent with Casas (1975), Argy and Salop (1977), and Sachs (1980), who find that monetary policy under perfect capital mobility and flexible exchange rates is ineffective (effective) when there is real (nominal) wage rigidity.

4.2 *Fiscal Contraction*

A decline in government expenditure leads of course to an instantaneous complete crowding in of private consumption when there is full employment (LME). The conclusion is also true for real wage rigidity (RWR) and purchasing power parity (PPP), but does not apply to RWR with imperfect substitution between home goods and foreign goods as shown in section 3.3. The results for the latter case are presented in Table 4, last panel. A decline in government expenditure induces a deterioration of the terms of trade of the domestic economy, because the value of exports must increase. Even if the nominal wage rate adjusts immediately to the change in the consumption price level there would be unemployment, because the producers' price declines by more. To restore full employment a gradual fall of the consumption wage rate (w^c) would be necessary. However, our model has the feature that the desired long-run target for the real consumption wage (w^d) is fixed at the initial level. This prevents a sufficient decline of real wages, which leads to a sustained higher level of unemployment, as can be observed from Table 4 (see also section 2.2). Unemployment increases by almost 6 percentage points ($u = 0.108$ in the new steady state). It appears that by $t = 4$ the economy is within 1% of its long-run equilibrium, which is of course related to our choice of the error-correction parameter ($\eta = 2$).

Under the regime of nominal wage rigidity (NWR) real wages increase at the impact of a deflationary cut in government expenditure. This results of course in unemployment. As nominal wages gradually adjust to the lower price level full employment is eventually restored. The situation with regard to the current

TABLE 4 - FISCAL CONTRACTION ($g = 0.1$)

Model	PPP; NWR $\pi = \vartheta$								Imperfect Substitution; NWR $\pi = \vartheta$								Imperfect Substitution; RWR							
Period / Variable	0	1	2	3	4	5	10	100	0	1	2	3	4	5	10	100	0	1	2	3	4	5	10	100
m	2.8	2.8	2.9	3.0	3.2	3.4	4.4	6.4	2.3	2.7	3.0	3.3	3.6	3.9	5.2	8.0	4.2	4.5	4.5	4.5	4.5	4.5	4.5	4.5
w	2.8	2.3	1.9	1.6	1.5	1.3	0.8	0	2.2	2.1	1.9	1.8	1.7	1.6	1.1	0	1.6	1.5	1.5	1.5	1.5	1.5	1.5	1.5
$c(c_d)$	6.6	6.7	6.9	7.3	7.7	8.0	9.7	13.1	4.7	5.2	5.7	6.2	6.6	7.1	9.0	13.1	6.9	7.2	7.3	7.3	7.3	7.3	7.3	7.3
b	0	−2.0	−2.9	−3.4	−3.5	−3.5	−2.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
l	−11.2	−9.0	−7.6	−6.6	−5.8	−5.2	−3.3	0	−8.8	−8.3	−7.8	−7.3	−6.8	−6.3	−4.4	0	−6.5	−6.2	−6.1	−6.1	−6.1	−6.1	−6.1	−6.1
$\lambda(c_m^*)$									4.7	5.2	5.7	6.2	6.6	7.1	9.0	13.1	6.9	7.2	7.3	7.3	7.3	7.3	7.3	7.3

account is qualitatively similar to the case of a monetary disinflation, discussed in section 4.1. In the PPP model total absorption initially exceeds output, which leads to a deficit on current account. Interest payments and repayment of debt necessitate an excess of output over total absorption in later periods. At first it pays to borrow abroad because the appreciation of the exchange rate makes foreign capital comparatively cheaper despite a fall in the domestic nominal rate of interest.

With imperfect substitution between the domestic and the foreign good and NWR, there will be again no change in the current account. An increase in the demand for the foreign good (imports) is choked off by a deterioration of the terms of trade. Exports rise and on balance there will be no change in the trade balance in our numerical example. The deterioration of the terms of trade or real exchange rate depreciation explains the difference in outcomes with respect to real cash balances (m) in both models under NWR. However, there are several causes which must be taken into account as appears from the specification of equation (3.5):

$$m = \sqrt{\frac{0.1\,c\lambda^{0.25}}{r}}$$

On impact m is lower for the case of imperfect substitution mainly because the volume of consumption increases less compared with PPP. The long-run difference can be attributed to the real exchange depreciation in case of imperfect substitution leading to a relatively higher value of m. In other words: real cash balances in terms of consumption goods are the same in both models in the long run. However, under imperfect substitution the consumption price falls less than the output price. Real cash balances in terms of domestic output should therefore be higher.

One can see from Table 4 that with NWR fiscal policy has no long-run effects on employment and output whilst with RWR fiscal policy can affect real variables (via the wedge between the consumers' and producers' wage induced by changes in the real exchange rate). Again, for the case of perfect capital mobility and flexible exchange rates, these results are consistent with the findings of Casas (1975), Argy and Salop (1977) and Sachs (1980).

4.3 *Foreign Disturbances*

Foreign disturbances relate to changes in the nominal interest rate and the inflation rate in the rest of the world. Given the symmetry with which both foreign variables impinge on the domestic economy, it will suffice to consider changes in the nominal rate of interest. The results of an increase in the nominal interest rate abroad (r^*) from 15% to 16% are presented in Table 5a (for the PPP versions) and Table 5b (for the case of imperfect substitution between domestic and foreign goods). As shown in section 2.3 a rise in the foreign interest rate leads to an increase in the stock of traded bonds or foreign assets (b). In our numerical example the volume of the stock of foreign assets in-

TABLE 5a – FOREIGN DISTURBANCE ($r^* = 0.16$): PPP versions

Model	NWR; π adaptive								NWR; $\pi = \vartheta$								RWR and LME							
Period Variable	0	1	2	3	4	5	10	100	0	1	2	3	4	5	10	100	0	1	2	3	4	5	10	100
m	−2.0	−1.5	−1.2	−1.0	−0.8	−0.6	0.1	0.4	−1.6	−1.0	−0.7	−0.5	−0.4	−0.3	−0.0	0.4	−2.5	−1.9	−1.4	−1.0	−0.7	−0.5	0.2	0.4
w	−2.0	−0.2	0.3	0.3	0.2	0.2	0.0	0	−1.6	−0.6	−0.1	0.1	0.1	0.2	0.2	0	0	0	0	0	0	0	0	0
c	−2.6	−1.9	−1.5	−1.1	−0.8	−0.5	0.4	0.8	−2.0	−1.3	−0.9	−0.6	−0.4	−0.3	0.1	0.8	−3.2	−2.4	−1.7	−1.1	−0.7	−0.4	0.4	0.8
b	0	4.7	6.0	6.5	6.9	7.2	8.9	10.0	0	4.7	6.8	7.9	8.5	8.8	9.4	10.0	0	2.2	3.9	5.2	6.3	7.1	9.2	10.0
l	8.1	0.6	−1.0	−1.2	−0.9	−0.6	0.1	0	6.4	2.5	0.6	−0.2	−0.6	−0.7	−0.6	0	0	0	0	0	0	0	0	0

creases from 0 to 0.1 in the new steady state. To accumulate these assets consumption must be reduced in the first instance. However, as interest accrues in the course of time consumption can be increased again. In the long-run equilibrium consumption rises by 0.8%, as shown in Table 5a and Table 5b for $t = 100$. Therefore, whatever model specification applies the situation can be characterised by the French saying: 'reculer pour mieux sauter.'[3]

Developments in the labour market and changes in the terms of trade interfere with the process of accumulation referred to above. Let us start with the PPP model. The results for labour market equilibrium (LME) are given in the last panel of Table 5a. As can be seen the accumulation of foreign assets requires a substantial fall in consumption over the first few periods of the time path considered. On impact of the change in the foreign rate of interest the domestic currency depreciates instantaneously, which is reflected in a decline of real cash balances at $t = 0$. Thereafter, the domestic currency appreciates gradually until in the new steady state the real cash balances are increased by 0.4%. The outcomes under real wage rigidity and PPP are the same as under LME. The nominal wage rate adjusts without delay to the price level in this specification of the model.

Under NWR and PPP the situation differs because a rise in the price level now leads to an instantaneous fall in real wages, as can be seen from Table 5a. Employment and output increase, which allows for a faster accumulation of foreign assets while mitigating the decline in consumption compared with LME and RWR. As time passes the nominal wage rate adjusts to the price level and overemployment is reduced. As can be observed the adjustment process is cyclical. Here again, adaptive expectation with regard to core inflation imply a faster response than fixed expectation ($\pi = \vartheta$).

Under imperfect substitution account must be taken of changes in the terms of trade. Focussing on the case of labour market equilibrium (LME, Table 5b, last panel) the terms-of-trade effect can be traced most clearly. An increase in the foreign rate of interest induces an instantaneous real exchange rate depreciation, *i.e.* the terms of trade deteriorate. As a result the decline in consumption is less up to period $t = 4$ and the accumulation of foreign assets is retarded in comparison with LME under PPP (Table 5a). The long-run results for consumption and the stock of traded bonds are equal under perfect and imperfect substitution between goods. In the new steady state the terms of trade improve by 2.4%.

With nominal wage rigidity (Table 5b, first panel) there is in this case also an instantaneous fall of the real wage and a corresponding increase in employment, but the quantitative effects are much smaller than under PPP. These results relate of course to the deterioration of the terms of trade. It should be observed that there is no influence on the accumulation of traded bonds. In-

3 In the extreme case of a large initial foreign debt ($b(0) \ll 0$), the long-run consumption level may decline as r^* rises, because the burden of interest payments may be heavier.

TABLE 5b – FOREIGN DISTURBANCE ($r^* = 0.16$): IMPERFECT SUBSTITUTION

el	NWR; $\pi = \vartheta$								RWR; $\eta = 0$								LME							
od able	0	1	2	3	4	5	10	100	0	1	2	3	4	5	10	100	0	1	2	3	4	5	10	100
	−0.2	−0.2	−0.2	−0.2	−0.2	−0.2	−0.2	0.1	−2.5	−2.0	−1.5	−1.1	−0.7	−0.5	0.1	0.1	−0.7	−0.6	−0.6	−0.5	−0.4	−0.4	−0.2	0.1
	−0.2	−0.1	−0.1	−0.1	−0.1	−0.0	+0.0	0	0.6	0.4	0.3	0.1	0.1	+0.0	−0.1	0	0	0	0	0	0	0	0	0
	−0.8	−0.7	−0.6	−0.5	−0.5	−0.4	−0.0	0.8	−3.6	−2.7	−2.0	−1.4	−0.9	−0.5	0.5	0.8	−1.5	−1.3	−1.0	−0.8	−0.7	−0.5	+0.0	0.8
	0	1.1	2.0	2.9	3.6	4.3	6.7	10.0	0	1.1	2.0	2.9	3.6	4.3	6.7	10.0	0	1.1	2.0	2.9	3.6	4.3	6.7	10.0
	0.7	0.6	0.4	0.3	0.2	0.2	−0.1	0	−2.2	−1.6	−1.0	−0.6	−0.3	−0.0	0.5	0	0	0	0	0	0	0	0	0
	5.2	4.3	3.5	2.8	2.2	1.7	−0.2	−2.4	2.3	2.2	2.1	1.9	1.8	1.5	0.4	−2.4	4.5	3.8	3.1	2.5	2.0	1.6	−0.1	−2.4
									0	−0.2	−0.3	−0.3	−0.4	−0.4	−0.2	0.6								

deed, the outcome with respect to b is the same irrespective of the functioning of the labour market. This is in line with our earlier finding that changes in the terms of trade are offset by opposite changes in the volume of trade, given the specification of the model equations. The terms of trade deteriorate further at $t = 0$ in case of NWR compared with LME, because the additional output requires a larger volume of exports.

The situation with real wage rigidity without an error-correction mechanism (RWR with $\eta = 0$) is to a certain extent the mirror image of the NWR variant. On impact the real wage rate in terms of output rises and employment declines correspondingly. The real wage in terms of the composite consumption good remains the same. The rise in nominal wages therefore exceeds the increase in the domestic price level. The terms of trade deteriorate less as the potential for exports is reduced. The process of adjustment towards a full employment equilibrium follows a cyclical pattern in this case too.

4.4 *Sensitivity to Degree of Capital Mobility*

With lower capital mobility the results of different shocks are *qualitatively* the same. The *quantitative* effects of a change in α depend on the importance of the current account in the various exercises. If capital is relatively less mobile, there are higher risks involved and therefore international lending and borrowing will be more restricted. The mutations of the current account will be smaller, mitigating the impact of shocks on the system.

To give an impression of the quantitative effects of capital mobility, the results of a demand shock ($\Delta g = -0.1$) and of a foreign disturbance ($\Delta r^* = 0.01$) for the case of PPP and NWR ($\pi = \vartheta$) are presented in Table 6. The parameter α is increased from 0.1 to 1. The outcomes in Table 6 should be compared with those in Tables 4 and 5a, respectively. With the shock to fiscal policy intermediate foreign borrowing is substantially lower when there is relatively low capital mobility. However, the effect on the other variables is attenuated and spread out over a longer period. In the long run there is no change at all. This is different for an increase in the foreign interest rate, which now increases the long-run value of consumption, real cash balances and foreign debt by less (a factor of ten) than with high capital mobility. As a consequence the impact effects are also reduced substantially. The foreign disturbance shock is extreme in the sense that the quantitative effect of a change in capital mobility is relatively strong. On the other hand there are cases in which changes in capital mobility have no effect whatsoever. This relates to all cases in which the current account is not affected by a shock, which happens in the imperfect substitution case. This is due to the assumption of Cobb-Douglas sub-utility functions, which implies that the balance of trade is always in equilibrium. With a CES sub-utility function, the balance of trade can depend on the real exchange rate. In that case, fiscal policy may have larger real effects in the short run with low degrees of capital mobility (*cf.* the standard Mundell-Fleming analysis).

TABLE 6 – LOW CAPITAL MOBILITY ($\alpha = 1$); NWR/PPP-MODEL, $\pi = \vartheta$

Shock	Fiscal contraction ($\Delta g = -0.1$)								Foreign disturbance ($\Delta r^* = 0.01$)							
Period Variable	0	1	2	3	4	5	10	100	0	1	2	3	4	5	10	100
m	2.3	2.4	2.6	2.9	3.2	3.4	4.4	6.4	−0.5	−0.1	−0.1	−0.0	−0.0	−0.0	0.0	0.04
w	2.3	1.9	1.8	1.6	1.5	1.4	0.9	0	−0.5	−0.0	0.0	0.0	0.0	0.0	0.0	0
c	6.0	6.2	6.7	7.2	7.6	8.1	9.9	13.1	−0.6	−0.1	−0.1	−0.1	−0.0	−0.0	0.0	0.08
b	0	−0.4	−0.5	−0.4	−0.4	−0.4	−0.3	0	0	0.9	1.0	1.0	1.0	1.0	1.0	1.0
l	−9.2	−7.7	−7.0	−6.5	−6.0	−5.5	−3.6	0	2.1	0.1	−0.1	−0.1	−0.1	−0.1	−0.1	0

5 CONCLUDING REMARKS

A perfect-foresight model with micro foundations and intertemporal budget constraints for the government and private sector of a small open economy has been analyzed. It therefore extends Turnovsky (1985) to allow for imperfect substitution between home and foreign goods and to allow for unemployment caused by nominal or real wage rigidity. It also provides a micro foundation of Mundell-Fleming models and extends these models to allow for intertemporal budget constraints for the government and the private sector. The various versions of the models allow for purchasing power parity or imperfect substitution between home and foreign goods, nominal or real wage rigidity, and high or low degrees of capital mobility. The transient and steady-state consequences of a monetary disinflation, a fiscal contraction and an increase in the world interest rate have been analyzed for the various versions of the model.

In future research, it may be interesting to extend the model to allow for capital accumulation, for imported raw materials, for non-tradeable and tradeable sectors, and for more than one country. It may also be useful to use such optimizing models to discuss optimal government policy, as they give a natural definition of a social welfare function, but an interesting analysis of distributional issues requires one to relax the assumption of a representative household and a representative firm. For example, the government might choose fiscal and monetary policies to maximize the gross consumers surplus which is the problem of optimal dynamic taxation and the optimal provision of public goods. Such optimal policies are typically time inconsistent (Kydland and Prescott, 1977). For instance, a government might renege and impose a 'surprise' inflation tax (Calvo, 1978). Eaton (1985) has discussed the time inconsistency of optimal exchange rate management in an overlapping generations economy, but obviously an interesting area for future research is to analyze the time inconsistency of optimal government policy (and the inefficiency of time-consistent government policy) within the context of a model of a small open economy with micro foundations.

REFERENCES

Argy, V. and J. Salop (1977), 'Price and Output Effects of Monetary and Fiscal Policy under Flexible Exchange Rates,' *IMF Staff Papers*, 24, pp. 224–256.

Branson, W.H. and J.J. Rotemberg (1980), 'International Adjustment with Wage Rigidity,' *European Economic Review*, 13, pp. 309–332.

Bruno, M. and J.D. Sachs (1985), *Economics of Worldwide Stagflation*, Oxford.

Buiter, W.H. (1984), 'Saddlepoint Problems in Continuous Time Rational Expectations Models: A General Method and Some Examples,' *Econometrica*, 52, pp. 665–680.

Buiter, W.H. (1986), 'Fiscal Policy in Open, Interdependent Economies,' in: A. Razin (ed.), *Economic Policy in Theory and Practice*, Oxford.

Buiter, W.H. and M. Miller (1982), 'Real Exchange Rate Overshooting and the Output Cost of Bringing Down Inflation,' *European Economic Review*, 18, pp. 85–123.

Calvo, G.A. (1978), 'On the Time Consistency of Optimal Policy in a Monetary Economy,' *Econometrica*, 46, pp. 1411-1428.

Camilleri, A.P., D.T. Nguyen and R.B. Campbell (1984), 'Policy Changes and External Disturbances in a Small Open Economy: Stability and Dynamic Responses,' *International Economic Review*, 25, pp. 123-158.

Casas, F.R. (1975), 'Efficient Macroeconomic Stabilization Policies under Floating Exchange Rates,' *International Economic Review*, 16, pp. 682-698.

Corden, W.M. (1981), 'The Exchange Rate, Monetary Policy and North Sea Oil: The Economic Theory of the Squeeze on Tradeables,' in: W.A. Eltis and P.J.N. Sinclair (eds.), *The Money Supply and the Exchange Rate*, Oxford.

Dornbusch, R. (1976), 'Expectations and Exchange Rate Dynamics,' *Journal of Political Economy*, 84, pp. 1161-1176.

Dornbusch, R. and S. Fischer (1980), 'Exchange Rates and the Current Account,' *American Economic Review*, 70, pp. 960-971.

Driskill, R. and S.A. McCafferty (1980), 'Exchange Rate Variability, Real and Monetary Shocks, and the Degree of Capital Mobility under Rational Expectations,' *Quarterly Journal of Economics*, 95, pp. 577-586.

Eaton, J. (1985), 'Optimal and Time Consistent Exchange Rate Management in an Overlapping Generations Economy,' *Journal of International Money and Finance*, 1, pp. 83-100.

England, R. (1976), 'A Program for the Solution of Boundary Value Problems for Systems of Ordinary Differential Equations,' report PPN 3/73, Culham Laboratory.

Frenkel, J.A. and C.A. Rodriquez (1982), 'Exchange Rate Dynamics and the Overshooting Hypothesis,' *IMF Staff Papers*, 29, pp. 1-30.

Klundert, Th. van de (1982), 'Distribution, Taxation and Employment in an Open Economy,' *De Economist*, 130, pp. 9-37.

Klundert, Th. van de and F. van der Ploeg (1987), 'Fiscal Policy and Finite Lives in Interdependent Economies with Real and Nominal Wage Rigidity,' mimeo, Tilburg University.

Kydland, F.E. and E. Prescott (1977), 'Rules rather than Discretion: The Time Inconsistency of Optimal Plans,' *Journal of Political Economy*, 85, pp. 473-491.

Layard, R. and S. Nickell (1985), 'The Causes of British Unemployment,' *National Institute Economic Review*, pp. 62-85.

Lipton, D., J.M. Poterba, J. Sachs and L.H. Summers (1982), 'Multiple Shooting in Rational Expectations Models,' *Econometrica*, 50, pp. 1329-1333.

Lucas, R.E. (1976), 'Econometric Policy Evaluation: A Critique,' in: K. Brunner and A.H. Meltzer (eds.), *The Phillips Curve and Labor Markets*, Amsterdam.

Ploeg, F. van der (1987), 'International Interdependence and Policy Coordination in Economies with Real and Nominal Wage Rigidity,' Discussion paper no. 217, Centre for Economic Policy Research, London.

Sachs, J. (1980), 'Wages, Flexible Exchange Rates, and Macroeconomic Policy,' *Quarterly Journal of Economics*, 95, pp. 731-747.

Sargan, J.D. (1964), 'Wages and Prices in the U.K.,' in: P.E. Hart, G. Mills and J.K. Whittaker (eds.), *Econometric Analysis for National Economic Planning*, New York.

Turnovsky, S.J. (1985), 'Domestic and Foreign Disturbances in an Optimizing Model of Exchange-rate Determination,' *Journal of International Money and Finance*, 1, pp. 151-171.

Turnovsky, S.J. and J.S. Bhandari (1982), 'The Degree of Capital Mobility and the Stability of an Open Economy under Rational Expectations,' *Journal of Money, Credit and Banking*, 13, pp. 303-326.

Summary

WAGE RIGIDITY AND CAPITAL MOBILITY IN AN OPTIMIZING MODEL OF A SMALL OPEN ECONOMY

This paper formulates an optimizing model of a small open economy with a representative (immortal) household, a firm and a government. The asset menu consists of domestic currency, non-traded bonds and traded bonds. There is a risk premium on traded bonds, which leads to deviations from perfect capital mobility and uncovered interest parity. Taxes are lump-sum, so that finance by bonds and by taxation are equivalent. The model allows for current-account and wealth dynamics. The model assumes either purchasing power parity or imperfect substitution between home and foreign goods and either labour market equilibrium, nominal wage rigidity or real wage rigidity. The steady-state effects of a fiscal contraction, a monetary disinflation and an increase in the world interest rate are discussed. The transient effects of these policies are analysed with the aid of a 'multiple shooting' algorithm.