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Abstract

We propose a method to decompose the logarithmic change of the Divisia price index into the pure price effect, the preference effect and the substitution effect. Our empirical results in the Philippines shows the effects of preference change on the Divisia price index are heterogeneous but positive across all regions and income deciles. However, they are dominated by the pure price effect.

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1 Introduction

The Divisia price and quantity indices named after Divisia (1925) have a number of desirable properties (Richter, 1966). One aspect of the Divisia index that has received little attention is how preference change affects the Divisia price index. Balk (2005) shows that, under the assumption of rational economic behavior and continuously changing preferences, the Divisia price index coincides with both the Laspeyres- and Paasche-perspective cost of living. However, no study has looked at the impact of preference changes on the Divisia index. We propose a method to decompose the changes in the Divisia price index into the changes due to pure price change, preference change, and substitution, though the latter two are difficult to distinguish in practice. Our empirical results indicate that the pure price effect dominates other effects in the Philippines.

2 Divisia price index under preference change

Suppose that, at time $t \in \mathbb{R}_+$, a consumer with preference parameters $\alpha(t) \in \mathbb{R}^L$ faces a price vector $p(t) \in \mathbb{R}_{++}^K$, and has disposable income $y(t) \in \mathbb{R}_{++}$ for current consumption with L and K being the number of preference parameters and consumption goods in the economy respectively. We assume y , α , and p change smoothly over time, and the consumer solves the following problem at each t :

$$V(p(t), y(t), \alpha(t)) \equiv \max_{q \in \mathbb{R}_{++}^K} U(q, \alpha(t)) \quad \text{s.t.} \quad \sum_k p^k(t) q^k \leq y(t),$$

where $U : \mathbb{R}_{++}^K \times \mathbb{R}^L \rightarrow \mathbb{R}_{++}$ is a utility function that is strictly quasiconcave, once continuously differentiable and strictly increasing in q^k for all k . We write the demand function derived from above by $x(p, y, \alpha) \in \mathbb{R}^K$, and the share function by $s^k(p, y, \alpha) \equiv x^k(p, y, \alpha) p^k y^{-1}$, where the superscript k refers to the k -th component of the vector. Unless otherwise indicated, the subscripts are used for partial derivatives, and the dot notation for time derivatives (*e.g.* $V_y \equiv \frac{\partial V}{\partial y}$ and $\dot{V} \equiv \frac{dV}{dt}$). We denote the income elasticity of utility by $\theta \equiv \frac{\partial V}{\partial y} \frac{y}{V}$. Using the chain rule and the identity $y = \sum_k p^k x^k$, and normalizing the prices and quantities to unity at $t = 0$, the Divisia price index $P(t) \in \mathbb{R}_{++}$ and the Divisia quantity index are defined in the following

manner:

$$\frac{\dot{y}}{y} = \sum_k s^k \frac{\dot{p}^k}{p^k} + \sum_k s^k \frac{\dot{x}^k}{x^k} \equiv \frac{\dot{P}}{P} + \frac{\dot{Q}}{Q}.$$

The following lemma, which follows from Roy's identity and the chain rule, is useful for subsequent discussion:

Lemma 1 *For all t , we have:*

$$\frac{d \ln V(p(t), y(t), \alpha(t))}{dt} = \theta(t) \left[\frac{\dot{y}}{y} - \frac{\dot{P}}{P} \right] + \sum_l \frac{V_{\alpha^l}}{V} \dot{\alpha}^l. \quad (1)$$

The right-hand-side of Eq(1) shows the utility of a consumer changes due to the changes in the income, the Divisia price index, and the direct effect of preference changes. Hereafter, we shall say that the Divisia price index is *almost* exact when its logarithmic change is proportionate to (but not necessarily equal to) the logarithmic changes in the utility under fixed income and prices. The following proposition, which is a straightforward extension of the Homogeneity Price Theorem (HPT) by Samuelson and Swamy (1974), shows that the Divisia price index is almost exact if and only if the preference is homothetic.¹

Proposition 1 *Let c be a positive constant. Then,*

$$\ln \frac{V(p(t^2), \bar{y}, \bar{\alpha})}{V(p(t^1), \bar{y}, \bar{\alpha})} = c \frac{\ln P_{|\alpha=\bar{\alpha}}(t^1)}{\ln P_{|\alpha=\bar{\alpha}}(t^2)} \quad \text{for any } \bar{y}, \bar{\alpha}, t^1, t^2, p(t^1), \text{ and } p(t^2) \quad (2)$$

$$\Leftrightarrow V(p(t), y(t), \bar{\alpha}) = A(\bar{\alpha}) \left[\frac{y(t)}{P_{|\alpha=\bar{\alpha}}(t)} \right]^c \quad \text{for some } c \text{ and } A(\alpha(t)) \in \mathbb{R}_{++}. \quad (3)$$

Four remarks are in order. First, the HPT requires $c = 1$ so that the aggregate price index is exact. On the other hand, Proposition 1 only requires that the change in the logarithmic price index is *proportionate* to the change in the logarithmic utility. Second, in general, $P(t)$ is dependent on the path of α . However, since the indirect utility function is a function of the current price, income and parameters, it must have the form $V(p(t), y(t), \alpha(t)) = A(\alpha(t))(y(t)/T(p(t), \alpha(t)))^c$ by Eq(3), where T is the preference-adjusted price index. Note that T is in general not unique. This can be easily seen since we have $V = \tilde{A}(y/\tilde{T})^c$ if we set $\tilde{A} = 1$ and $\tilde{T} = TA^{-1/c}$. Third, it is straightforward to verify $\frac{\dot{P}}{P} = \frac{\dot{T}}{T} - \sum_l \frac{T_{\alpha^l}}{T} \dot{\alpha}^l$. Therefore, we have $\frac{\dot{P}}{P} = \frac{\dot{T}}{T}$ whenever α is held fixed.

¹The proof is provided upon request.

Fourth, by the preceding argument as well as Lemma 1 and Proposition 1, the following holds when the Divisia price index is almost exact:

$$\frac{\dot{V}}{V} = c \left[\frac{\dot{y}}{y} - \frac{\dot{P}}{P} \right] + \sum_l \left[\frac{A_{\alpha^l}}{A} - c \frac{T_{\alpha^l}}{T} \right] \dot{\alpha}^l.$$

This shows that, if the preferences are homothetic, the Divisia price index measures the changes in logarithmic utility due to price changes up to a scalar multiplier, after controlling for the income change and the direct effects of preference change. While homothetic preference may appear very restrictive, one should note V is actually quite flexible because the preference parameter is allowed to change at each time t .

3 Decomposition of the Changes in the Divisia Price Index

We can identify the effects of preference change through the changes in the Divisia price index by decomposing the changes in s^k . Using a short-hand notation $s^k(t) \equiv s^k(p(t), \alpha(t))$ and similar notations for partial derivatives, we have:

$$\begin{aligned} s^k(t) &= s^k(r) + \int_r^t \sum_l s_{\alpha^l}^k(\tau) \dot{\alpha}^l(\tau) d\tau + \int_r^t \sum_j s_{p^j}^k(\tau) \dot{p}^j(\tau) d\tau \\ &\equiv s^k(r) + \psi^k(t, r) + \pi^k(t, r) \end{aligned}$$

The first term $s^k(r)$ is simply the share function at the reference time period. $\psi^k(t, r)$ and $\pi^k(t, r)$ are the changes in the share function from the reference time period at time t due to the preferences changes and price changes respectively. Plugging these into the definition of P , we can decompose the logarithmic change $D(t^1, t^2) \equiv \ln P(t^2) - \ln P(t^1)$ in the Divisia price index between t^1 and t^2 in the following manner:

$$\begin{aligned} D(t^1, t^2) &= \int_{t^1}^{t^2} \sum_k s^k(r) \frac{\dot{p}^k}{p^k} dt + \int_{t^1}^{t^2} \sum_k \psi^k(t, r) \frac{\dot{p}^k}{p^k} dt + \int_{t^1}^{t^2} \sum_k \pi^k(t, r) \frac{\dot{p}^k}{p^k} dt \\ &\equiv \Delta_p(t^1, t^2, r) + \Delta_\psi(t^1, t^2, r) + \Delta_\pi(t^1, t^2, r) \end{aligned} \quad (4)$$

We call Δ_p the pure price effect, since it captures the changes in $\ln P$ with the shares fixed at the reference period. The preference effect and substitution effect are captured by Δ_ψ and Δ_π respectively.

One obvious question is how to choose the reference period. We can choose $r = t^1$ or $r = t^2$, which respectively provides Laspeyres-perspective and Paasche-perspective decomposition. We can also take the average of Eq(4) over $r \in [t^1, t^2]$. That is, by defining $\Delta_z(t^1, t^2) \equiv (t^2 - t^1)^{-1} \int_{t^1}^{t^2} \Delta_z(r) dr$ for $z \in \{p, \psi, \pi\}$, we have $D(t^1, t^2) = \bar{\Delta}_p(t^1, t^2) + \bar{\Delta}_\psi(t^1, t^2) + \bar{\Delta}_\pi(t^1, t^2)$. The following proposition holds.

Proposition 2 *Let the average share and the logarithmic price change of good k between t^1 and t^2 be $\bar{s}^k \equiv (t^2 - t^1)^{-1} \int_{t^1}^{t^2} s^k(t) dt$ and $\bar{\phi}^k \equiv (t^2 - t^1)^{-1} (\ln p^k(t^2) - \ln p^k(t^1))$, and define $\delta_\phi^k(t) = \frac{\dot{p}^k}{p^k} - \bar{\phi}^k$.*

$$\bar{\Delta}_p(t^1, t^2) = (t^2 - t^1) \sum_k \bar{s}^k \bar{\phi}^k \quad (5)$$

$$\bar{\Delta}_z(t^1, t^2) = (t^2 - t^1)^{-1} \sum_k \int_{t^1}^{t^2} \int_{t^1}^{t^2} z^k(t, r) \delta_\phi^k(t) dt dr \quad \text{for } z \in \{\psi, \pi\}, \quad (6)$$

Proof of Proposition 2: $\bar{\Delta}_p(t^1, t^2) = (t^2 - t^1)^{-1} \int_{t^1}^{t^2} \int_{t^1}^{t^2} \sum_k s^k(r) \frac{\dot{p}^k}{p^k} dt dr = \sum_k \bar{\phi}^k \int_{t^1}^{t^2} s^k(r) dr = (t^2 - t^1) \sum_k \bar{s}^k \bar{\phi}^k$, proving Eq(5). Eq(6) follows from the fact that $\int_{t^1}^{t^2} \int_{t^1}^{t^2} \left[\int_r^t f(\tau) d\tau \right] dt dr = 0$ for any integrable function f . \square

There are four points worth making here. First, if all the prices change at a constant rate between t^1 and t^2 so that $\delta_\phi^k(t) = 0$ for all k and $t \in [t^1, t^2]$, then we have $\bar{\Delta}_\psi = \bar{\Delta}_\pi = 0$. Second, Eq(6) shows that $\bar{\Delta}_z$ has a covariance-like structure. For example, $\bar{\Delta}_\psi$ is larger when the deviation of the rate of price change from the mean and the deviation of the share from the reference level due to preference change tend to move together.

Third, the averaging-out approach is useful only if we have frequent observations of prices. To highlight this point, consider the case when we have observations of the price of each good only for two periods. Then, we may have no option but to assume the prices change at a constant rate between the two periods, so that we have $\bar{\Delta}_\psi = \bar{\Delta}_\pi = 0$. The decomposition with a specific reference period may still be meaningful.

Fourth, since we generally don't know the functional form of s , it is difficult to distinguish between the preference effect and the substitution effect in practice. If the utility function is misspecified so that the assumed preference parameters are actually a function of prices, the estimated preference effect captures the substitution effect. Because of this issue, we do not try to distinguish between the preference and substitution effects in the next section.

4 Application to the Inflation in the Philippines

We apply the decomposition method to the Philippines, using seven rounds of the Family Income Expenditure Survey (FIES) between 1988 and 2006, and the annual averages of the monthly Consumer Price Index (CPI) with the base year of year 2000. Both FIES and CPI are collected by the National Statistics Office of the Philippines. We have aggregated goods to match the definitions of the consumption goods between FIES and CPI, and we have $K = 19$ as a result. In this section, we let the unit time length be a year, and $t = 0$ be the year 1988. We obtained from the FIES data the observation of the share \hat{s}_t^k at $t = 0, n, 2n, \dots, mn$ where $m = 6$ and $n = 3$. The CPI data contain an observation of the price \hat{p}_t^k at each $t = 0, 1, 2, \dots, mn$. The data allow us to conduct the analysis at the level of seventeen regions.

We assume the logarithmic price linearly changes between observations so that $\ln p^k(t) = ([t+1] - t) \ln \hat{p}_{[t]}^k + (t - [t]) \ln \hat{p}_{[t+1]}^k$, where $[t]$ is the largest integer not exceeding $t \in [0, mn]$. We also assume the consumer has (time-varying) log-linear preferences so that $\ln T(\alpha(t), p(t)) = \sum_k \alpha^k(t) \ln p^k(t)$, where $\sum_k \alpha^k(t) = 1$ for all t . Assuming the preference parameters also change linearly, we have $\alpha^k(t) = ([t/n + 1] - t/n) \hat{s}_{n[t/n]}^k + (t/n - [t/n]) \hat{s}_{n[t/n+1]}^k$. With these assumptions and observations, we can calculate $\bar{\Delta}_p$ and $\bar{\Delta}_\psi$ in the following manner:

$$\begin{aligned} \bar{\Delta}_p &= \frac{1}{2} \sum_k \sum_{\tau=0}^{m-1} \left(\hat{s}_{n\tau}^k + \hat{s}_{n(\tau+1)}^k \right) \ln \frac{\hat{p}_{mn}^k}{\hat{p}_0^k} \\ \bar{\Delta}_\psi &= \sum_k \sum_{\tau=0}^{m-1} \sum_{v=0}^{n-1} \left(\frac{2v+1}{2n} \hat{s}_{n(\tau+1)}^k + \frac{2n-2v-1}{2n} \hat{s}_{n\tau}^k \right) \ln \frac{\hat{p}_{\tau n+v+1}^k}{\hat{p}_{\tau n+v}^k} - \bar{\Delta}_p \end{aligned}$$

There are two points to note here. First, under the assumption of log-linear preferences, we have $s_{p_l}^k(t) = 0$ for all t, k , and l , so that we have $\bar{\Delta}_\pi(0, mn) = 0$. Thus, we have $D(0, mn) = \bar{\Delta}_p(0, mn) + \bar{\Delta}_\psi(0, mn)$, so that the estimated preference effect captures the substitution effect that would exist under the correct specification. Second, the advantage of the log-linear preferences is that the data requirement is minimal since we only need one observation of the share and the price for each good at each point in time. Hence, the framework of analysis we chose in this paper is applicable to many countries.

[Table 1 about here.]

Table 1 shows the results of our calculation. The second and third columns show the logarithmic change in the CPI and the Divisia price index between 1988 and 2006. They are very

close in most regions. The fourth and fifth columns are the pure price effect and the preference effect based on the averaging-out method. Both are positive in all regions, and the order of magnitude of the former is much larger than the latter. This should not be surprising because (i) the changes in the preference parameters have a first-order effect only on the current rate of change (and not the current level) of the Divisia price index, and (ii) the value shares remained quite stable between 1988 and 2006.

One should note that the contributing factors (goods) are different between $\bar{\Delta}_p$ and $\bar{\Delta}_\psi$. Factors that positively contribute to more than 20 percent of $\bar{\Delta}_\psi$ include cereal and cereal preparation, fish, cloth, rental of occupying dwelling, and transportation and communications. Education is the only factor whose effect is negative and more than 20 percent of $\bar{\Delta}_\psi$ in absolute value. No single good accounted for more than 20 percent of $\bar{\Delta}_p$, either positive or negative.

In the sixth and seventh columns, we report the Laspeyres-perspective pure price effect ($\Delta_p^{LAS} \equiv \Delta_p(0, mn, 0)$), and Paasche-perspective pure price effect ($\Delta_p^{PAA} \equiv \Delta_p(0, mn, mn)$). Both are close to $\bar{\Delta}_p$, and the order of magnitude of the estimated pure price effect and the preference effect are similar regardless of the choice of the reference time period.

The eighth column reports the change Δ_y in logarithmic disposable income. The ninth column reports its difference from D , and indicates the existence of substantial heterogeneity in terms of the changes in the standards of living across the Philippines. We also observe a moderate correlation ($\rho = 0.4$) between $\bar{\Delta}_\psi$ and $\Delta_y - D$. People in those regions which are getting rich more quickly tend to increase the share of the goods that are getting expensive more quickly.

We also looked at the inflation in the Philippines by decile. We first calculated the value shares for each income decile in each round of the FIES. We then conducted the decomposition analysis using the national average prices in order to control for the changes in the regional composition of each decile over time. The qualitative nature of the results does not change even when we use the regional prices instead of national average prices.

As the second column of Table 2 shows, richer people have experienced higher inflation. The third column shows that the inflation due to the preference effect is positive for all deciles, and tends to be higher for richer people. This may be because rich people have stronger preferences for the goods that are getting expensive fast, or because rich people don't need to substitute away from the goods that are getting expensive fast. Despite this, the improvement in the

standards of living for bottom deciles has not kept pace with that for top deciles, because the income increases of top deciles generally outpaced those of bottom deciles.

[Table 2 about here.]

5 Discussion

As we have shown, the changes in the Divisia price index can be decomposed into the pure price effect, the preference effect and the substitution effect. If (and only if) the preference is homothetic at each point in time, each effect has a clear relationship with the logarithmic utility change. Even when the homotheticity is violated, the decomposition exercise is still valid since the derivation does not rely on homotheticity. The interpretation of the decomposition, however, become ambiguous in the absence of homotheticity.

In the Philippines, the preference effect is small relative to the pure price effect. Also, the inflation due to the price effect is smaller for poor people both in absolute and relative terms. One possible objection to these findings would be that the consumption goods are so aggregated that it underestimates the effects of preference change. Due to the data limitations, we cannot check whether such objection is valid. One should note, however, that the prices of close substitutes are likely to move together closely provided there are no market distortions. If this is indeed the case, there should be no serious underestimation.

References

- Balk, B., 2005, Divisia price and quantity indexes: 80 years after, *Statistica Neerlandica* 59, 119–158.
- Divisia, F., 1925, L'indice monétaire et la théorie de la monnaie, *Revue d'Economie Politique* 39, 980–1008.
- Richter, M., 1966, Invariance axioms and economic indexes, *Econometrica* 34, 739–755.
- Samuelson, P. and S. Swamy, 1974, Invariant economic index numbers and canonical duality: Survey and synthesis, *American Economic Review* 64, 566–593.

Appendix for reviewers

Proof of Proposition 1: \Leftarrow is obvious. Let us now prove \Rightarrow . Since t^1 and t^2 are arbitrary, let $t^2 = t^1 + \Delta_t$, and consider a change between t^1 and t^2 such that $y(t^1) = y(t^2) = \bar{y}$ and $\alpha(t^1) = \alpha(t^2) = \bar{\alpha}$. Then, $\dot{y}(t^1) \rightarrow 0$ and $\dot{\alpha}(t^1) \rightarrow 0$ as $\Delta_t \rightarrow 0$. Hence, using this and Lemma 1, we have $\frac{d \ln V(p(t^1), y(t^1), \alpha(t^1))}{dt} = -\theta(t^1) \frac{d \ln P(t^1)}{dt}$ as $\Delta_t \rightarrow 0$. Also, dividing both sides of Eq(2) by Δ_t and letting $\Delta_t \rightarrow 0$, we have $\frac{d \ln V(p(t^1), y(t^1), \bar{\alpha})}{dt} = -c \frac{d \ln P|_{\alpha=\bar{\alpha}}(t^1)}{dt}$, so that we must have $\theta(t^1) = c$. Since t^1 is arbitrary, $\theta(t) = \frac{\partial V}{\partial y} \frac{y}{V} = c$ for all t . Rearranging the terms, integrating over y , and exponentiating both sides of the equality, we have $V(p, y, \bar{\alpha}) = y^c B(p, \bar{\alpha})$ for some function B . Plugging this in Eq(2), dividing both sides of the equality by Δ_t and letting $\Delta_t \rightarrow 0$ in Eq(2), we have $\frac{\partial \ln B}{\partial \ln P}|_{\alpha=\bar{\alpha}} = -c$. Integrating over $\ln P$ with α fixed at $\bar{\alpha}$ and exponentiating both sides of the equality, and defining $A(\bar{\alpha})$ appropriately, we have the desired result. \square

Table 1: The decomposition of inflation in the Philippines between 1988 and 2006.

Region	Δ_{CPI}	D	$\bar{\Delta}_p$	$\bar{\Delta}_\psi$	Δ_p^{LAS}	Δ_p^{PAA}	Δ_y	$\Delta_y - D$
NCR	1.444	1.479	1.462	0.017	1.484	1.464	1.449	-0.030
Reg. I	1.350	1.371	1.360	0.011	1.341	1.378	1.478	0.107
Reg. II	1.269	1.281	1.279	0.002	1.254	1.301	1.505	0.224
Reg. III	1.263	1.282	1.267	0.015	1.250	1.297	1.477	0.194
Reg. IV-A	1.337	1.349	1.330	0.019	1.290	1.363	1.643	0.294
Reg. IV-B	1.207	1.225	1.216	0.009	1.206	1.225	1.343	0.118
Reg. V	1.355	1.367	1.361	0.006	1.342	1.388	1.521	0.154
Reg. VI	1.269	1.277	1.266	0.011	1.257	1.277	1.416	0.139
Reg. VII	1.457	1.462	1.458	0.004	1.429	1.478	1.682	0.220
Reg. VIII	1.300	1.298	1.283	0.015	1.274	1.292	1.558	0.260
Reg. IX	1.308	1.308	1.305	0.003	1.283	1.320	1.304	-0.004
Reg. X	1.338	1.339	1.328	0.011	1.306	1.358	1.295	-0.044
Reg. XI	1.235	1.230	1.221	0.009	1.215	1.245	1.322	0.092
Reg. XII	1.225	1.166	1.158	0.009	1.138	1.169	1.168	0.001
Reg. XIII	1.278	1.281	1.270	0.011	1.244	1.289	1.261	-0.020
CAR	1.362	1.377	1.362	0.016	1.313	1.389	1.599	0.222
ARMM	1.487	1.534	1.533	0.001	1.505	1.549	1.164	-0.370
Philippines	1.352	1.358	1.349	0.010	1.339	1.365	1.477	0.119

Table 2: Inflation between 1988 and 2006 by deciles. The third column ($\bar{\Delta}_\psi/D$) is expressed in percentage.

Decile	D	$\bar{\Delta}_\psi/D$	Δ_y	$\Delta_y - D$
1	1.413	0.995	1.642	0.230
2	1.357	0.836	1.624	0.267
3	1.339	0.678	1.594	0.256
4	1.328	0.608	1.573	0.245
5	1.319	0.518	1.551	0.232
6	1.314	0.509	1.520	0.206
7	1.311	0.462	1.489	0.178
8	1.307	0.396	1.463	0.156
9	1.303	0.306	1.428	0.125
10	1.301	0.236	1.431	0.129