TRAFFIC OFFERED BEHAVIOUR REGARDING TARGET QOS PARAMETERS IN NETWORK DIMENSIONING

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Abstract: We consider a model of overall telecommunication network with virtual circuits switching, in stationary state, with Poisson input flow, repeated calls, limited number of homogeneous terminals and 8 types of losses. One of the main problems of network dimensioning/redimensioning is estimation of traffic offered in network because it reflects on finding of necessary number of circuit switching lines on the basis of the consideration of detailed users manners and target Quality of Service (QoS). In this paper we investigate the behaviour of the traffic offered in a network regarding QoS variables: “probability of blocked switching” and “probability of finding B-terminals busy”. Numerical dependencies are shown graphically. A network dimensioning task (NDT) is formulated, solvability of the NDT and the necessary conditions for analytical solution are researched as well.
The received results make the network dimensioning/redimensioning, based on QoS requirements easily, due to clearer understanding of important variables behaviour.

The described approach is applicable directly for every (virtual) circuit switching telecommunication system e.g. GSM, PSTN, ISDN and BISDN. For packet-switching networks, at various layers, proposed approach may be used as a comparison basis and when they work in circuit switching mode (e.g. VoIP).

**Keywords:** Overall Network Traffic, Offered Traffic, Virtual Circuits Switching.

**ACM Classification Keywords:** C.2.1 Network Architecture and Design; C.2.3 Network Operations; C.4 Performance of Systems.

1. **Introduction**

   We consider a model of telecommunication system with virtual circuits switching, in stationary state, with Poisson input flow, repeated calls, limited number of homogeneous terminals and losses due to abandoned and interrupted dialing, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing and abandoned conversation.

   One of the main problems of network dimensioning/redimensioning is estimation of traffic offered in network because it reflects on finding of necessary number of circuit switching lines. There are many different factors that we need to take into account when analyzing traffic. QoS parameters are administratively specified in Service Level Agreement (SLA) between users and operators [1].

   Based on the ITU definitions [2] (E.600- 4.1, 4.2, 2.8 and 2.11), as QoS parameters, we use the follow two parameters, dependable from the network macro-state (Yab): probability $P_{bs}$ (blocked switching) due to lack of resources and probability $P_{br}$ of finding B-terminals busy. We denote the target value of blocked switching by $trg.P_{bs}$.

   A network dimensioning task (NDT) is formulated, solvability of the NDT and the necessary conditions for analytical solution are researched as well [3].

   In this paper we investigate the behaviour of the traffic offered ($ofr.Ys$) in a network regarding QoS variables $P_{bs}$ and $P_{br}$. Numerical dependencies are shown graphically.

   The results are useful for finding the range of $ofr.Ys$ variability in every concrete case and developing of the best suitable numerical method for finding of the necessary number of equivalent internal switching lines ($Ns$) in dimensioning and redimensioning tasks.

2. **Conceptual model and analytical models**

   The conceptual model [4] of the telecommunication system includes the paths of the call attempts, generated from (and occupying) the A-terminals in the considered network. All assumptions made are described and the base general equations are explained in [4]. A system of equations based on the conceptual model and some dependencies [5] between parameters of the researched telecommunication system, is derived.

3. **Network Dimensioning Task**

   3.1 **Formulation of a network dimensioning task (NDT):**

   To determine the volume of telecommunication resources, based on previous experience in other telecommunication networks, that is enough for serving expected input flow of demands, with prescribed characteristics of QoS, is one of the main problems that often have to be solved by operators. It includes the following two tasks:

   1. Finding the values of the designed parameters, describing the designed system state. For example, a system parameter, describing offered traffic intensity of the switching system ($dsn.ofr.Ys$), designed probability to find B terminal “busy” ($dsn.P_{br}$), etc…
2. Dimensioned a network. It means to be found the number of internal switching lines, necessary to satisfy a level of QoS that has been administratively pre-determined, and for which the values of known parameters are measured and/or calculated (in the case of any operational network).

For solving of these tasks we have to determine the dependencies of designed offered traffic intensity with respect to QoS – parameters and users’ behaviour characteristics in telecommunication system.

3.2 Parameters and aims in the Network Dimensioning Task (NDT):

Given parameters:

- Administrative determined parameters: \( \text{trg}.\text{Pbs}, \text{Nab} = \text{adm}.\text{Nab} \)
- Parameters with empirical values: \( F_0, S_1, S_2, S_3, R_1, R_2, R_3, S_{1z}, S_{2z}, Tb \)

Aim: To determine the number of equivalent internal switching lines \( N_s \); and the values of following

Designed unknown parameters: \( \text{dsn}.\text{Pbr}, \text{dsn}.\text{ofr}.\text{Ys} \)

3.3 Analytical solution of the NDT:

In the denoted papers [3], [4] and [5] the follow facts, in different theorems, are proved and the conditions are researched:

1. \( Yab \) is the intensity of the terminal traffic [4] in the system and in NDT is derived as the function of dynamic parameters with empirical values \( F_0, S_1, S_2, S_3, R_1, R_2, R_3, S_{1z}, S_{2z} \):

\[
Yab = \frac{F_0(1 - \text{Pbr})\{S_1 - S_2\text{Pbr} - (S_1 - S_2\text{Pbr})\text{Pbs}\}}{F_0(1 + \text{MPbr})\{S_1 - S_2\text{Pbr} - (S_1 - S_2\text{Pbr})\text{Pbs}\} - \text{Pbr}\{1 - R_1 - R_2 \{1 - \text{Pbr}\} - R_3\text{Pbs}\}}
\] (3.1)

\( Yab \) is the intensity of the terminal traffic \( \neq 0 \) and \( \text{Pbr} \neq 0 \).[5]

For calling rate of call demand exist in NDT an expression [3]

\[
dem.\text{Fa} = F_0\{(\text{Nab} - 1)(1 + \text{MPbr}) + 1 + M\}.
\] (3.2)

2. If \( \text{Pbr} \neq 0 \) and \( \text{Pbs} \neq \frac{S_1 - S_2\text{Pbr}}{S_3 - S_2\text{Pbr}} \) in the NDT, then for \( \text{rep.Fa} \) [3] and \( \text{ofr.Ys} \) [3], an analytical expression exist for its evaluation

\[
\text{rep.Fa} = \frac{Yab\{R_1 + R_2\{(1 - \text{Pbs})\text{Pbr} + R_3\text{Pbs}\}\}}{S_1 - S_2\{(1 - \text{Pbr})\text{Pbs} - S_3\text{Pbs}\}}
\] (3.3)

\[
\text{ofr.Ys} = \frac{Yab\{(1 - \text{Pad})(1 - \text{Pid})\{(S_1z - S_2z\text{Pbr}\}\}}{S_1 - S_2\{(1 - \text{Pbs})\text{Pbr} - S_3\text{Pbs}\}}.
\] (3.4)

3. In NDT equation [3]

\[
A\text{Pbr}^2 + B\text{Pbr} + C = 0, \quad \text{where}
\]

\[
A = R_2\{(1 - \text{trg}.\text{Pbs})\text{Nab} - 1\}
\]

\[
B = \{(1 - \text{trg}.\text{Pbs})\{R_2 + \text{dem.Fa} S_1\}\} + (1 - R_1 + R_3\text{trg}.\text{Pbs})(\text{Nab} - 1)
\]

\[
C = 1 - R_1 + R_3\text{trg}.\text{Pbs} - \text{dem.Fa}(S_1 - S_3\text{trg}.\text{Pbs}),
\]

when \( F_0 \neq 0 \) and \( \text{trg}.\text{Pbs} \neq \frac{S_1 - S_2}{S_3 - S_2} \), has at least one solution \( \text{Pbr}^* \in (0; 1) \).
If the value of $Pbr$ is determined thereby, we say that $Pbr$ is determined on the base of the NDT and we denote it $dsn.Pbr$. Based on it (when we know $trg.Pbs$ and $dsn.Pbr$), in NDT, we may evaluate design values of unknown parameters $dsn.Yab$, $dsn.ofr.Ys$ e.t.c. For example, the expression for $ofr.Ys$ is

$$dsn.ofr.Ys = \frac{(1 + dsn.Pbr(\text{Nab} - 1))(1 - Pad)(1 - Pid)(S_{1z} - S_{2z} dsn.Pbr)}{S_1 + S_3(1 - trg.Pbs)dsn.Pbr + S_3 trg.Pbs}.$$  

(3.6)

In the NDT, only one solution of the equation

$$Erl_b(Ns,dsn.ofr.Ys) = trg.Pbs$$

regarding the number of switching lines $Ns$ exists.

The expression $Erl_b(Ns,dsn.ofr.Ys)$ is the famous Erlang B-formula.

$trg.Pbs \in (0; 1]$ is in advance administratively determined target value of blocking probability, providing of GoS [3].

It is proved [3] that only one solution of $Ns$ exists, fulfilling the equation (4.3.1) and corresponding to the determined administratively in advance value of the blocking probability $trg.Pbs \in (0; 1]$.

The number of internal switching lines $Ns$ and the values of $dsn.ofr.Ys$ are calculated on the conditions of the theorems in [3], [4] and [5]. Algorithm and computer program for calculating the values of the NDT parameters are worked out.

4 Parameters dependency in NDT

4.1. Dependency – used definitions.

As mathematical approach are used partial derivatives of researched parameters [3] and following definitions:

Let $P(x_1, x_2, ..., x_n)$ is tuple (ordered set) consists of variables $x_1, x_2, ..., x_n$.

Tuple $P_0$ of empirical or evaluated parameters’ values is $P_0(x_1^0, x_2^0, ..., x_n^0)$, where the parameters’ values are $x_1^0, x_2^0, ..., x_n^0$.

Let parameter A depends on tuple $P$ then $A(P) = f(x_1, x_2, ..., x_n)$ where $x_1 \in D_1$, $x_2 \in D_2$, ..., $x_n \in D_n$. The value of A in $P_0$ is $A(P_0) = f(x_1^0, x_2^0, ..., x_n^0)$.

Range of parameters’ values of A according parameter x is amplitude of it $Max A(x_i^*) - Min A(x_i^*)$, where $Max A(x_i^*)$ and $Min A(x_i^*)$ are absolute maximum resp. minimum received in points $x_i^* \in D_k$ and $x_i^* \in D_k$.

$$Range(A | x_i) = Max A(x_i^*) - Min A(x_i^*)$$

4.2. Research of parameters dependency in NDT regarding GoS parameters probability of blocking due to $Pbs$ and $Pbr$. Knowing functional dependencies from a parameter, we may estimate the parameters’ importance and necessary accuracy of its measurement.

We consider $ofr.Ys$ - dependency regarding $Pbs$ and $Pbr$ because $Ns$ is direct dependent on it.

We denote the tuple of lost probabilities $L = (Pad, Pid, Pis, Pns, Par, Pac)$.

Then $ofr.Ys = ofr.Ys(Fo, L, Pbs, Pbr)$. We assume that in NDT the users behaviour is ordinary and then $Fo$ and $L = (Pad, Pid, Pis, Pns, Par, Pac)$ have fixed mean values, empirical received from measurements in operational system.

Theorem 1: Function $ofr.Ys$ is increasing regarding $Pbs$ and has not extremum regarding $Pbs$ in NDT.

Proof: From equations (3.4) and $Yab = 1 + Pbr(\text{Nab} - 1)$
\[ \text{ofr.Ys}(Fo, L, dsn.Pbr, Pbs) = \frac{(1 + Pbr(Nab - 1)) (1 - Pad)(1 - Pid)(S_{1z} - S_{2z} Pbr)}{S_1 + S_2 (1 - Pbs) Pbr + S_3 Pbs} \] \hspace{1cm} (4.1)

follows

\[ \frac{\partial \text{ofr.Ys}}{\partial Pbs \mid_{dsn.Pbr}} = (1 - Pad)(1 - Pid)(S_{1z} - S_{2z} Pbr) \frac{(S_3 - S_2 Pbr)(1 + Pbr(Nab - 1))}{(S_1 - S_2 (1 - Pbs) Pbr - S_3 Pbs)} \]

From \( S_3 = (1 - Pad)(1 - Pid)[PisTis - Tbs + (1 - Pis)[PnsTns + (1 - Pns)[Tcs + 2Tb]]] = \)

\[ = (1 - Pad)(1 - Pid)[PisTis - Tbs + (1 - Pis)[PnsTns + (1 - Pns)[Tcs + Tbr]]] + S_2 \]

follows \( S_3 > S_2 \) therefore \( S_3 - S_2 Pbr > 0 \).

Therefore \( \text{ofr.Ys} \) increases concerning \( Pbs \) for each \( dsn.Pbr \) and has not extremum in NDT.

**Fig. 1.** and **Fig. 2.** Dependencies of offered traffic \( \text{ofr.Ys/Nab} \) and \( Ns/Nab \) on target blocking probability due to insufficient lines value (trg. \( Pbs \)).

Let \( \text{trg.Pbs} \) is determined in advance and fixed. Then \( \text{ofr.Ys} = \text{ofr.Ys}(Fo, L, Pbs, Pbr) \) is function regarding \( Pbr \) only.

**Theorem 2:** If \( \text{ofr.Ys} \) has extremum regarding \( Pbr \) in NDT, then the equation \( (4.2) \) is in force:

\[ L Pbr^2 + M Pbr + N = 0, \text{ where} \]

\[ L = S_{2z} S_3 (1 - \text{trg.Pbs})(Nab - 1) \]

\[ M = 2 S_{2z} (S_3 \text{trg.Pbs} - S_1)(Nab - 1) \]

\[ N = S_{1z}[(Nab - 1)(S_1 - S_3 \text{trg.Pbs}) + S_2(1 - \text{trg.Pbs})] - S_{2z} (S_1 - S_3 \text{trg.Pbs}) \]

Proof: From equations (3.1.7) and (3.1.8) in analytical model [3] follows

\[ \text{ofr.Ys} = (1 - Pad)(1 - Pid)(S_{1z} - S_{2z} Pbr) Fa \]

\[ \frac{\partial \text{ofr.Ys}}{\partial Pbr \mid_{\text{avg.Pbs}}} = (1 - Pad)(1 - Pid) \left( \frac{\partial Fa}{\partial Pbr}(S_{1z} - S_{2z} Pbr) - S_{2z} Fa \right) \]

\hspace{1cm} (4.3)
where
\[ Fa = \frac{1 + Pbr(\text{Nab}-1)}{S_1 - S_2(1 - \text{Pbs})Pbr - S_3\text{Pbs}} \]  
\[ \frac{\partial Fa}{\partial Pbr} = \frac{(\text{Nab}-1)(S_1 - S_2\text{Pbs}) + S_2(1 - \text{Pbs})}{(S_1 - S_2(1 - \text{Pbs})Pbr - S_3\text{Pbs})^2} . \]

Based on the analytical condition for existing of local extremum of \( \text{ofr.Ys} \) regarding \( Pbr \) follow the equation
\[ S_{zz}Fa = \frac{\partial Fa}{\partial Pbr}(S_{iz} - S_{zz}Pbr) . \]  
We may substitute (4.5) to (4.6) and after algebraic transform we receive regarding \( Pbr \) equation (4.2).

With this we proved that if \( \text{ofr.Ys} \) has local extremum regarding \( Pbr \) in NDT then \( Pbr \) is a solution of eq. (4.2). We will prove that exist at least one solution of eq. (4.2) in conditions of NDT.

**Theorem 3:** For equation (4.2) exist a solution \( Pbr^* \in (0;1) \) in NDT with analytical conditions (4.7)-(4.8).

**Proof:** 1) If in eq. (4.2) coefficient \( L = 0 \) and \( M \neq 0 \) (i.e. \( Tb \neq Tbr \)), then exist only one solution \( Pbr^* \in (0;1) \) on the conditions, following from the system:
\[
0 < S_{iz}[(\text{Nab}-1)(S_1 - S_3\text{trg.Pbs}) + S_2(1 - \text{trg.Pbs})] - S_{zz}(S_1 - S_3\text{trg.Pbs}) < 1 \\
S_{zz}S_2(1 - \text{trg.Pbs})(\text{Nab}-1) = 0 \\
Tb \neq Tbr
\]

2) When coefficient \( L \neq 0 \) in eq. (4.2) and discriminant \( D \geq 0 \)
\[
D = [-S_{zz}(S_1 - S_3\text{trg.Pbs})(\text{Nab}-1)]^2 - S_{zz}S_2(1 - \text{trg.Pbs})(\text{Nab}-1) \\
[S_{iz}[(\text{Nab}-1)(S_1 - S_3\text{trg.Pbs}) + S_2(1 - \text{trg.Pbs})] - S_{zz}(S_1 - S_3\text{trg.Pbs})] \\
[S_2(1 - \text{trg.Pbs})]S_{zz}(\text{Nab}-1) + S_{iz} + (S_{iz} - 2S_{zz})(\text{Nab}-1)(S_1 - S_3\text{trg.Pbs}) \geq 0 \\
S_{iz}[(\text{Nab}-1)(S_1 - S_3\text{trg.Pbs}) + S_2(1 - \text{trg.Pbs})] - S_{zz}(S_1 - S_3\text{trg.Pbs}) \neq 0 \\
S_{zz}S_2(1 - \text{trg.Pbs})(\text{Nab}-1) \neq 0
\]  
then exist at least one solution of eq. (4.2).

The condition \( Pbr^* \in (0;1) \) is equivalent to \( N(L+M+N) \neq 0 \) or to the follow system:
\[
[-2S_{zz}(S_2 - S_3\text{trg.Pbs})(\text{Nab}-1)]^2 - S_{zz}S_2(1 - \text{trg.Pbs})(\text{Nab}-1) \\
[S_{iz}[(\text{Nab}-1)(S_1 - S_3\text{trg.Pbs}) + S_2(1 - \text{trg.Pbs})] - S_{zz}(S_1 - S_3\text{trg.Pbs})] \geq 0 \\
S_{iz}[(\text{Nab}-1)(S_1 - S_3\text{trg.Pbs}) + S_2(1 - \text{trg.Pbs})] - S_{zz}(S_1 - S_3\text{trg.Pbs}) \neq 0 \\
S_{zz}S_2(1 - \text{trg.Pbs})(\text{Nab}-1) \neq 0
\]  
Numerical analysis shows that the bigger root in this case, fulfills the conditions above.

**Theorem 4:** \( \text{ofr.Ys} \) has a local maximum regarding \( Pbr \) if \( Pbr^* = \frac{S_1 - S_3\text{trg.Pbs}}{S_2(1 - \text{trg.Pbs})} \) when \( Tb > Tbr \) in NDT.

**Proof:** Investigation of equation (4.2) shows that when \( S_{zz} < 0 \) (if coefficient \( L < 0 \)), respective \( Tb > Tbr \), then \( \text{ofr.Ys} \) has local maximum when \( Pbr^* = \frac{S_1 - S_3\text{trg.Pbs}}{S_2(1 - \text{trg.Pbs})} \) in NDT. Note that \( Tb > Tbr \) fulfills in all real situations.

**Consequence 1:** The value of local maximum of \( \text{ofr.Ys} \) regarding \( Pbr \) is
\[
\max_{\text{org.} \text{Pbs}} \text{offr.} \text{Ys}(Pbr) = \text{offr.} \text{Ys}(Pbr^*) = \left[ 1 + Pbr^* (Nab - 1) \right] (1 - Pad) (1 - Pid) Ys / S_1 - S_3
\]

when target \( \text{org.} \text{Pbs} \) is determined administratively in advance.

**Consequence 2:** In NDT the absolute maximum of \( \text{offr.} \text{Ys} \) regarding \( Pbr \), coincides with relative maximum in \( Pbr^* \).

The absolute minimum of \( \text{offr.} \text{Ys} \) is
\[
\min_{\text{org.} \text{Pbs}} \text{offr.} \text{Ys}(Pbr) = \lim_{Pbr\rightarrow0} \text{offr.} \text{Ys}(Pbr) = (1 - Pad) (1 - Pid) S_{iz} / S_1.
\]

It is researched and numerical results are shown graphically on Fig.3 and Fig.4.

**Fig. 3. and Fig. 4.** Dependency of offered traffic \( \text{offr.} \text{Ys}/\text{Nab} \) and \( \text{Ns}/\text{Nab} \) on blocking probability of finding B – terminal busy \( Pbr \).

The range of \( \text{offr.} \text{Ys} \) is
\[
\text{Range}(\text{offr.} \text{Ys})_{\text{org.} \text{Pbs}} = \left[ \text{offr.} \text{Ys}(Pbr^*) - \text{offr.} \text{Ys}(0) \right]_{\text{org.} \text{Pbs}} =
\]
\[
(1 - Pad) (1 - Pid) \left( \frac{(Nab - 1) (1 + Pbr^* (Nab - 1))}{S_1 - S_3} (S_{iz} - S_{zz} Pbr^*) - \frac{S_{iz}}{S_1} \right)_{\text{org.} \text{Pbs}}
\]

5. **Conclusions**

1. Detailed normalized conceptual model, of an overall (virtual) circuit switching telecommunication system is considered.
2. The target blocking probability \( \text{org.} \text{Pbs} \) and probability of finding B – terminal busy \( Pbr \) as GoS – parameters in network dimensioning task, are used.
3. The behaviour of the traffic offered regarding QoS variables \( Pbs \) and \( Pbr \) is investigated. Function \( \text{offr.} \text{Ys} \) is increasing regarding \( Pbs \) and has not extremum regarding \( Pbs \) in NDT. \( \text{offr.} \text{Ys} \) has a local maximum regarding \( Pbr \) which coincides with its absolute maximum. The conditions of these facts are investigated.
4. The results are useful for finding the range of \( \text{offr.} \text{Ys} \) in every concrete case and developing of the best suitable numerical method for finding of the necessary number of equivalent internal switching lines (\( \text{Ns} \)) in dimensioning and redimensioning tasks.
5. The received results make the network dimensioning/redimensioning, based on QoS requirements easily, due to clearer behaviour of the important variables.

6. Numerical experiments are made and the results are graphically shown.

7. The described approach is applicable directly for every (virtual) circuit switching telecommunication system (like GSM and PSTN) and may help considerably for ISDN, BISDN and most of core and access networks dimensioning. For packet switching systems, like Internet, proposed approach may be used as a comparison basis and when they work in circuit switching mode.

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