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Introduction

In many distribution problems of daily life, inequality is intuitively associated with injustice. If the individuals involved in a distribution problem are equals in terms of the personal characteristics relevant to the problem,[1] then there is no reason for giving a *particular* individual more than another *particular* individual since any valid reason has to refer to differences in the personal characteristics of the given individuals. In other words, every inequality is in a sense ethically unjustifiable. It seems evident, therefore, that equality of ethically relevant personal characteristics should translate ideally into an equal division of the 'thing' to be distributed.[2]

In the economic literature on income distribution[3]—with which this dissertation is concerned—it is assumed that individuals are identical with respect to all ethically relevant personal characteristics, or, alternatively, that the 'income' concept has been defined such that it corrects for differences in these personal characteristics.[4] In accordance with the argument presented above, an appeal to equality of incomes is clearly present in the literature. It should be stressed that the argument does not imply, however, that any equal division should be considered better than any unequal one. Each theory of distributive justice specifies additional concerns that may override the preference for equality. A popular such overriding concern is that of Pareto efficiency, which says that moves towards more inequality are allowed if everyone benefits. For another example, note that it is conventional in the literature on income distribution to assume that welfare depends only on two variables, i.e., mean income and the level of income inequality. The additional concern of mean income is hence adopted in terms of a trade-off—only sufficiently significant increases of mean income can justify a given increase in inequality.

The outlined perspective places the main topics of this dissertation—the measurement of inequality and the measurement of inequality aversion—at the centre

[1]Needs, effort levels and rights are examples of such characteristics that could be deemed relevant.

[2]See Kolm (1997) for an especially thorough exposition of this argument.

[3]See Cowell (2000) and Lambert (2001) for overviews.

[4]Besides the term 'income,' we will sometimes use the terms 'utility' or 'well-being' throughout this dissertation.

of the problem of distributive justice. For choices between income distributions which are undistinguishable with respect to additional concerns (in the example of the size-distribution representation of welfare, choices between income distributions with the same mean incomes) the problem of distributive justice reduces to an exercise of inequality minimization, thus giving rise to the need for a theory of inequality measurement. For choices between income distributions which differ in terms of the additional concerns (in the example of the size-distribution representation, choices between income distributions with different mean incomes) it becomes important to assess how important these concerns are relative to the inequality concern, i.e., to determine the degree of inequality aversion.

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The social welfare functions conventionally used to evaluate income distributions are natural analogues of models studied in the literature on decision under risk, viz., the expected utility (EU) model, the Yaari (1987) model and, generalizing the previous two, the rank-dependent expected utility (RDEU) model.[5] Without a doubt, the EU social welfare function has received the most attention in the literature on income distribution. The key axiom underlying the EU social welfare function is separability, which says, roughly speaking, that the income level of any individual with the same income in two income distributions should not influence the evaluative ranking of the given income distributions. The Yaari and RDEU social welfare functions do not in general satisfy separability. In Part I of this dissertation, we critically examine how well these three social welfare functions describe the opinions of questionnaire respondents, and to what extent the inequality measures corresponding to them are sufficiently flexible to allow for all plausible inequality judgements with respect to the specific question of how the relative numbers of poor people and rich people should affect inequality.

In Chapter 1, we present the results of a questionnaire study that takes as a starting point Harsanyi's (1953, 1955) impartial observer theory, which links preferences under risk with social preferences. Harsanyi claims that income distributions ought to be ranked as they would be ranked by a rational individual behind the so-called 'veil of ignorance.' Behind this veil of ignorance, an individual is confronted with the income distributions reinterpreted as lotteries: for every income distribution, the individual has an equal probability of ending up with the income of each of the members of society. The supporters of this approach argue that an individual behind the veil of ignorance has the correct perspective to make choices from a moral point of view since she will be both impartial and sympathetic towards the members of society. The questionnaire used in Chapter 1

[5]See the overview by Gajdos (2001).

confronts respondents with eight choices over pairs of income distributions. In the first version, respondents are asked to state their direct ethical preference over each of the pairs. In the second and third versions, the income distributions in the eight pairs are reinterpreted as lotteries, and respondent are put behind a veil of ignorance (choices affect the incomes of everyone in society) or in a purely individual risk situation (choices affect only the own income), respectively. The results reveal that, although there are important similarities between the three types of preferences, the direct ethical preferences and the purely individual risk preferences of the respondents form two extremes, while the veil of ignorance preferences lie in between the other two. An important question is whether the responses are consistent with the EU, Yaari and RDEU social welfare functions. For all three questionnaire versions, the results reveal many violations of the separability axiom, and hence inconsistencies with the EU social welfare function. Moreover, the direction in which separability is violated by the respondents does not correspond to the direction prescribed by the Yaari and RDEU social welfare functions. By contrast, respondents' choices turn out to be well described by certain simple concepts from the literature on decision under risk that have not been studied in the context of the evaluation of income distributions. The most important example in this respect is that the respondents reveal a positive attitude to mixing income distributions—an income distribution is a mixture of two given income distributions if it is obtained by convexly combining the relative frequencies (at the same income level) of those given income distributions, e.g., the income distribution of a country is a mixture of the income distributions of its regions—whereas the EU social welfare function implies neutrality to mixing, and the Yaari and RDEU social welfare functions imply averseness to mixing.

Taking as an inspiration the importance of attitudes to mixing income distributions in describing respondent's choices, we provide in Chapter 2 a formal examination of the behaviour of inequality measures with respect to mixing. It is shown that virtually all inequality measures commonly considered in the literature on income distribution—i.e., the inequality measures corresponding to a social welfare function of the EU, Yaari or RDEU form, as well as the class of decomposable inequality measures—satisfy a quasi-concavity property, which implies, roughly speaking, that mixing income distributions increases inequality. The relevancy of this quasi-concavity property lies in the fact that it has direct consequences for the question of how inequality judgements ought to depend on the relative group sizes of the poor and rich. In order to understand this, note that an income distribution, say A, in which the number of poor and the number of rich are equal, is a mixture of (i) an income distribution B in which there are many poor and few rich, and (ii) an income distribution C in which there are few poor and many rich. By consequence, all common inequality measures tend to indicate higher inequality for A than for B or C since they all satisfy the quasi-concavity property. However, it has been argued by Fields (1987, 1993), among others, that it would be at least as plausible to consider *A* less unequal than *B* and *C*. It can be shown, moreover, that such an alternative view is not inconsistent with basic inequality axioms as those underlying the well known Lorenz inequality criterion. Again, a possible conclusion is that the standard approach in the literature on income distribution is too restrictive.

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To make comparisons of social welfare orderings with respect to inequality aversion, the literature on income distribution traditionally relies on the well known Arrow-Pratt concept, which is borrowed from the literature on decision under risk. In its most straightforward formulation, the Arrow-Pratt concept says the following: if a given social welfare ordering prefers a perfectly equal income distribution to an unequal income distribution in all cases in which another social welfare ordering does, then the given social welfare ordering is (weakly) more inequality averse than the other one. The application of this concept of inequality aversion has been mostly restricted to EU social welfare functions, i.e., to social welfare orderings satisfying the separability axiom. In our analysis of inequality aversion in Part II, we move beyond the standard practice in two respects. First, in line with the limitations of the separability axiom that were noted in the outline of Part I above, we study inequality aversion without the imposition of separability. Second, we consider alternatives to the Arrow-Pratt concept itself. Throughout Part II, special attention is paid to the problem of reconciling the ideals of extreme inequality aversion and monotonicity, i.e, the question of how to implement the egalitarian ideal of always choosing for less inequality, except in cases in which no individual would gain by doing so. We note that the qualification ‘extreme’ should not be interpreted as a label for this egalitarian ideal itself: the term simply stresses the technical point that we are dealing with the most inequality averse position one can take.

Hammond (1975), Meyer (1975) and Lambert (2001) have studied extreme inequality aversion in accordance with the standard practice, i.e., using the Arrow-Pratt concept and focusing on separable social welfare orderings. Their result says that the leximin social welfare ordering can be obtained as the limit case, in terms of inequality aversion, of the class of EU social welfare functions: each choice between a pair of income distributions implied by leximin coincides with the choices implied by all of the most inequality averse EU social welfare functions. In Chapter 3, we establish an analogous result using the Arrow-Pratt concept, but focusing on the class of social welfare orderings not necessarily satisfying separability. It is established that, for this broader class of social welfare orderings, the limit case is not a single social welfare ordering but a class, viz., the class of weakly maximin social welfare orderings, which give priority to the worst off individual in all cases in which the worst off does not have the same income in the two alternatives under comparison. While the weakly maximin class has leximin as one of its members, it also includes social welfare orderings very different from leximin.

The Arrow-Pratt concept has the drawback of being based on a rather primitive inequality criterion: since it compares social welfare orderings only with respect to the choices they imply over pairs of income distributions of which one is perfectly equal and one is not, the Arrow-Pratt concept implicitly supposes that all there is to say about inequality comparisons is that unequal income distributions are more unequal than perfectly equal ones. Because there are broadly accepted inequality criteria in the literature on income distribution that do allow to make comparisons between unequal income distributions, the Arrow-Pratt approach seems unduly narrow. In Chapter 4, we consider, therefore, two alternative concepts of inequality aversion. The first alternative concept is based on the Lorenz inequality criterion and says the following: if a given social welfare ordering prefers a Lorenz dominating income distribution to a Lorenz dominated income distribution in all cases in which another social welfare ordering does, then the given social welfare ordering is (weakly) more inequality averse than the other one. The second alternative concept is defined similarly but using, instead of the Lorenz criterion, the so-called relative differentials inequality criterion, which is intermediate in strength between the Lorenz criterion and the primitive inequality criterion underlying the Arrow-Pratt concept. It is shown that although the relative differentials based concept of inequality aversion is not equivalent to the Arrow-Pratt concept in general, it is equivalent to it for separable social welfare orderings and thus supports important results in the literature that were established using the Arrow-Pratt concept. The Lorenz concept of inequality aversion is more profoundly inconsistent with the Arrow-Pratt concept: the two concepts are not equivalent for the class of separable social welfare orderings and not even for its popular constant elasticity of substitution subclass. With respect to the problem of reconciling monotonicity and extreme inequality aversion, the relative differentials concept is once again consistent with the Arrow-Pratt concept, i.e., it identifies as extremely inequality averse the class of weakly maximin social welfare orderings (and thus also confirms the result of Chapter 3). The Lorenz concept, on the other hand, concludes that the ideals of extreme inequality aversion and monotonicity are incompatible.

Not all approaches to implementing the idea of extreme inequality aversion have taken as a starting point a concept of inequality aversion. An alternative approach that has been explored is to impose the so-called Hammond equity axiom. Hammond equity requires that reducing the inequality between any two individuals—by increasing the income of the poorer one by some amount and decreasing that of the richer one by some (possibly different) amount—leads to an improvement in social welfare. Hammond (1976) has characterized leximin using Hammond equity and the basic axioms anonymity and strong Pareto (if one individual gets more without anyone getting less, then social welfare increases). In Chapter 5, we provide an analogous characterization of the maximin social welfare ordering using Hammond equity, anonymity, continuity and weak Pareto (if all individuals get more, then social welfare increases). A shortcoming of Hammond equity is also pointed out: the transformations it recommends do not always decrease inequality according to the Lorenz criterion, and sometimes even only decrease inequality according to rather exotic inequality criteria (and increase it according to others). We introduce a modified Hammond equity axiom that does not have this shortcoming, and show that an alternative characterization of maximin is obtained by simply replacing Hammond equity by this modified version in the characterization mentioned above: that is, maximin is also characterized by modified Hammond equity, anonymity, continuity and weak Pareto. Using the modified Hammond equity axiom, we present, moreover, another characterization of the weakly maximin class.

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In contrast to Parts I and II of this dissertation, Part III is concerned with distribution problems in which the individuals involved differ with respect to ethically relevant characteristics. We assume that the amount that each individuals should ideally receive is known, and refer to it as the 'claim' of the individual. The ethically relevant differences between individuals are, thus, captured completely by differences in their claims: if the amount available for distributing between the individuals is equal to the sum of their claims, then the just division is that in which each individual receives her claim. We focus on the problem of how to divide an amount that falls short of the sum of the claims among the individuals. The literature on the so-called claims problem deals with the axiomatic examination of 'rules,' which are mappings that associate with each claims problem—given by the claims vector and the amount available for distribution—a division among the individuals involved.[6] The most prominent and also most straightforward division rules are the proportional rule, the constrained equal awards rule, and the constrained equal losses rule. Several more complex rules have been studied as well in the literature. In Part III, we apply concepts from the theory of inequality measurement to the literature on claims problems.

In Chapter 6, we examine how nine well known rules—the proportional rule, the constrained equal awards rule, the constrained equal losses rule, the Talmud rule, Piniles' rule, the constrained egalitarian rule, the adjusted proportional rule,

[6]See Thomson (2003) for an overview.

the random arrival rule, and the minimal overlap rule—compare in terms of inequality aversion. The relevance of inequality aversion comparisons of rules, better referred to as progressivity comparisons in this framework, can be illustrated using Aristotle's well known adage: "Justice is equality among equals and inequality among unequals." The degree to which individuals have to be seen as 'unequals,' and hence the degree to which they ideally ought to be treated unequally, depends on the actual economic context of the claims problem because this determines the ethical status of the claims—e.g., differences between claims may possibly be considered more relevant ethically if they reflect differences in needs, than if they reflect differences in talents. Rules, being functions only of the claims vector and the amount to be distributed, are, however, invariant with respect to the economic context. In order to determine which rule ought to be used in which economic context, it is therefore important to be able to compare the different rules with respect to how progressive they are, i.e., how unequally they treat 'unequals.' In the analysis of Chapter 6, it is shown that the constrained equal awards rule, the constrained equal losses rule, the Talmud rule, Piniles' rule and the constrained egalitarian rule can be characterized as most or least progressive among a class of rules of which the members satisfy particular basic axioms. We also provide several results involving the remaining rules in order to uncover the complete set of Lorenz dominance relationships that hold between the nine rules.

Chapter 7 describes the results of a questionnaire in which the respondents are confronted with nine specific claims problems, which are presented in two different economic contexts. In the first version of the questionnaire, respondents have to divide revenue among the owners of a firm who contribute to the activities of the firm in different degrees, and, in the second version, they have to divide tax money among pensioners who have paid different contributions during their active career. The results reveal that, for both questionnaire versions, the proportional rule performs very well in describing the choices of the respondents. By contrast, other prominent division rules—in particular the constrained equal awards rule and the constrained equal losses rule—fail to capture the basic intuitions of the respondents. We also find that responses in the pensions version of the questionnaire are significantly more egalitarian than those in the firm version. Finally, the results show that the way in which respondents' choices vary with respect to basic changes in the characteristics of the claims problem appear to be well described in terms of inequality. More precisely, a substantial part of the respondents tend to become more progressive as the amount to be distributed decreases other things equal, and tend to become more progressive as the inequality in the distribution of claims becomes more unequal other things equal. The fact that inequality considerations seem to be useful in organizing respondents' choices is encouraging for the inequality perspective on the theoretical study of claims problems.

Part I

Social Welfare and Inequality

Chapter 1

Social Welfare, the Veil of Ignorance, and Purely Individual Risk: An Empirical Examination

This chapter is based on Bosmans and Schokkaert (2004).

1.1 Introduction

The central problem of distributive justice is that of finding an ethical ranking of income distributions. It is generally accepted that such an ethical ranking should reflect in a certain sense the preferences of an impartial and sympathetic observer (henceforth referred to as 'ISO preferences')—"... a person taking a positive sympathetic interest in the welfare of *each* participant but having no partial bias in favor of *any* participant" (Harsanyi, 1977, p. 49). ISO preferences have been analysed in the literature in many different ways, but a particularly influential approach has been the exploration of the formal links between inequality and risk (Cowell and Schokkaert, 2001). This link has been put forward in its most explicit form in Harsanyi's (1953, 1955) approach of the veil of ignorance.[1]

Harsanyi rephrases the problem of distributive justice as a problem of individual decision making under risk: income distributions should be ranked according to the preferences of a rational individual behind the veil of ignorance (henceforth, 'VOI preferences'). VOI preferences are the preferences over income distributions of a rational individual who does not know her own position in each income distribution (nor the position of the other members of society) and has (like these other

[1] See also Vickrey (1945, 1960) and Rawls (1971). The latter coined the term 'veil of ignorance.' Harsanyi used the approach to justify utilitarianism while Rawls used it to justify his deontological theory which couples a respect for basic liberties with maximin in 'primary goods.'

members), for each income distribution, an equal probability of ending up with the income of any member of society. Harsanyi argues that rationality requires that VOI preferences be consistent with expected utility (EU) theory. Hence, the social welfare function, which represents ethical preferences, inherits the properties of the EU model and is of the mean utilitarian type.[2] This approach is often seen as providing a justification for the most frequently used social welfare function in the income distribution literature, which is of the mean utilitarian form with utility a function exclusively of own income and an identical utility function for each individual.[3] However, this approach raises two sets of questions.

First, it is not obvious that VOI preferences and ISO preferences indeed coincide. The idea of the veil of ignorance is only one among many approaches to the problem of finding an ethical ranking of income distributions. Moreover, the assumption that utility is a function exclusively of own income does not follow directly from Harsanyi's conditions. Indeed, VOI preferences are defined over lotteries that have complete income distributions as outcomes, not over lotteries with individual incomes as outcomes. We refer to the latter type of preferences as purely individual risk preferences (henceforth, 'PIR preferences'). The assumption that utility is a function exclusively of own income can be justified if VOI preferences are identical to PIR preferences. Differences between VOI preferences and PIR preferences can result from the fact that the individuals do not care only about their own incomes, but also for instance about overall equality or about their own relative income position. A comparison of ISO and VOI preferences with PIR preferences therefore could give some insight into the importance of externalities. What is the relationship between ISO, VOI and PIR preferences?

Second, the risk literature has provided ample empirical evidence of systematic violations of EU theory (the Allais paradox being the most famous example). A theoretical literature on non-expected utility (non-EU) models has developed mainly to accommodate these empirical violations.[4] It seems interesting to check whether these violations of EU theory for PIR preferences are also relevant for the ethical ranking of income distributions, that is, for ISO and VOI preferences. In fact, one of the most popular concepts from the non-EU literature, i.e., rank-dependent expected utility (RDEU), has in its simplified form (Yaari, 1987) received considerable attention in the income distribution literature because it provides a normative basis for an important subclass of the class of generalized Gini inequality indices.[5] Recent contributions have explored further links between the RDEU model in its general form and the measurement of inequality (Gajdos,

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[2] Harsanyi's claim that mean utilitarianism follows from his assumptions has been criticized on several accounts. See Mongin (2001) for a thorough overview of the literature.

[3] See Cowell (2000) and Lambert (2001) for recent overviews of this literature.

[4] For overviews, see Camerer (1995) and Starmer (2000).

[5] More precisely, the subclass that satisfies Dalton's (1920) Population Principle.

2001). How attractive are these non-EU approaches from an ethical point of view?

The present chapter examines both issues through a questionnaire approach with Belgian students. In order to benefit from the accumulated knowledge in the risk literature, the setup of our questionnaire will be analogous to the conventional approach used in that literature. We put respondents into three different choice contexts allowing revelation of ISO, VOI and PIR preferences, respectively. In each of these cases we test whether we discover any violations of the standard properties of the EU model. Such violations can also raise doubts about some of the standard assumptions in the literature on income distribution. Moreover, we will also check the empirical relevancy of the Yaari and RDEU models as well as that of some more basic non-EU concepts.

The questionnaire approach has recently become more popular in the economic literature on distributive justice. It has been used extensively for testing the acceptance of the crucial axioms from the literature on income distribution.[6] Recent work has explicitly compared the acceptability of these axioms for the income inequality and the risk setting (Amiel et al., 2001; Amiel and Cowell, 2002). Camacho-Cuena et al. (2003) and Traub et al. (2005) have run experiments in which subjects get material incentives to rank either income distributions or risky prospects. The close relationship between social welfare judgements and choice under risk and the theoretical suppositions of the EU approach are far from evident for large groups of respondents. Closest related to our work is a questionnaire study by Bernasconi (2002). He also checks the relevance of EU axioms for ISO, VOI and PIR preferences. The formulation of our questions is very different, however, and we go further in testing explicitly some non-EU alternatives. Despite these differences, some of our results turn out to be similar to those of Bernasconi.

The chapter is organized as follows. Section 1.2 gives an overview of relevant findings from EU theory and non-EU theory and links these to the evaluation of income distributions. The actual questionnaire study is presented in Section 1.3. In Section 1.4 we present the results. Section 1.5 concludes.

1.2 (Non-)Expected Utility Theory and the Evaluation of Income Distributions

We first consider expected utility (EU) theory (Subsection 1.2.1) and some basic concepts from non-EU theory (Subsection 1.2.2). In Subsection 1.2.3 we summarize the basic characteristics of the RDEU model and of Yaari's theory. Finally, in

[6]The most influential work is by Amiel and Cowell, who summarize their most important findings in Amiel and Cowell (1999). See also Harrison and Seidl (1994a, 1994b).

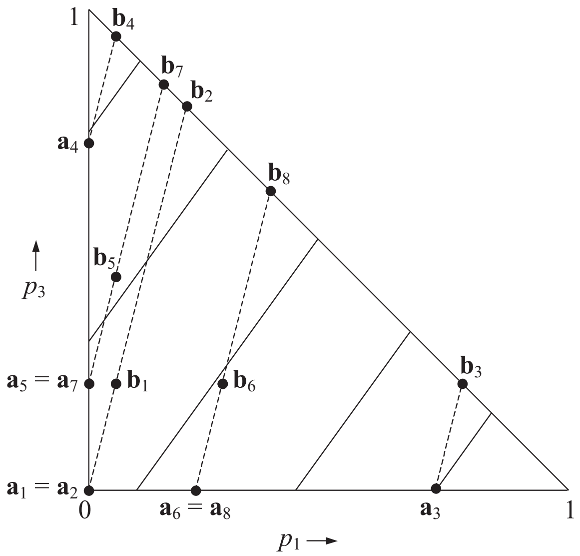

Figure 1.1. EU Indifference Curves in the Marschak-Machina Triangle

Subsection 1.2.4, we return to the evaluation of income distributions.

We use the following notation. The set of incomes is $X = \{x_1, x_2, \ldots, x_n\}$, where the incomes are indexed such that $x_1 \leq \cdots \leq x_n$. An income distribution is a vector $\mathbf{p} = (p_1, \ldots, p_n)$ with $p_i \in [0,1]$ for all i and $\sum_{i=1}^{n} p_i = 1$, where p_i is the proportion of the population with income x_i. The set of all income distributions is denoted by Δ. In the case of individual decision under risk, income distributions have to be interpreted as lotteries, where p_i is the probability of outcome x_i. Preferences over alternatives, either income distributions or lotteries, are captured by a binary relation $\succeq$ ('is at least as good as'). The relation has an asymmetric factor $\succ$ ('is better than'), and a symmetric factor $\sim$ ('is equally good as'). Under certain conditions, a function, F, can be used to represent preferences. The function F has to be interpreted either as a social welfare function or as an individual utility function, depending on the given choice situation.

A convenient representation device to compare the implications of EU theory with the implications of various non-EU theories is the so-called Marschak-Machina triangle[7] (see Figure 1.1). Focusing on lotteries with only three possible outcomes (or income distributions with only three income levels) $x_1 < x_2 < x_3$, each alternative can be written as a pair (p_1, p_3), with p_2 determined implicitly

[7] This graphical device was introduced into the literature by Marschak (1950) and popularized by Machina (1982). It has since been used in many empirical studies concerning individual decision under risk.

Table 1.1. The Choice Pairs, (p_1, p_2, p_3)

Question	**a**	**b**
1	(0, 1, 0)	(0.05, 0.75, 0.2)
2	(0, 1, 0)	(0.2, 0, 0.8)
3	(0.75, 0.25, 0)	(0.8, 0, 0.2)
4	(0, 0.25, 0.75)	(0.05, 0, 0.95)
5	(0, 0.8, 0.2)	(0.05, 0.55, 0.4)
6	(0.2, 0.8, 0)	(0.25, 0.55, 0.2)
7	(0, 0.8, 0.2)	(0.16, 0, 0.84)
8	(0.2, 0.8, 0)	(0.36, 0, 0.64)

as $p_2 = 1 - p_1 - p_3$. Since, furthermore, for $i = 1,2,3$, we have $p_i \in [0,1]$, all these alternatives are points in the triangle $\{(p_1, p_3) \in \mathbb{R}^2_+ \mid p_1 + p_3 \leq 1\}$. In the Marschak-Machina triangle of Figure 1.1 the different points represent thirteen possible alternatives. Our questionnaire study will focus on eight pairwise choices: each choice problem, $j = 1,\ldots,8$, involves a choice among a pair of alternative lotteries or income distributions $(\mathbf{a}_j, \mathbf{b}_j)$. Note that the dashed lines connecting each of these pairs of alternatives have the same slope equal to four. The probabilities corresponding to the specific options represented in Figure 1.1 are shown in Table 1.1.

1.2.1 Expected Utility Theory

Let us first summarize in a loose way the basic idea of EU theory. Suppose that all the alternatives can be ordered (implying that the preference relation is reflexive, transitive and complete) and that this ordering is continuous and monotonic. Suppose moreover that the following condition is satisfied:

Independence. For any alternatives $\mathbf{p}, \mathbf{q}, \mathbf{r} \in \Delta$ and any scalar $\alpha \in (0,1)$, we have $\mathbf{p} \succeq \mathbf{q}$ if and only if $\alpha\mathbf{p} + (1-\alpha)\mathbf{r} \succeq \alpha\mathbf{q} + (1-\alpha)\mathbf{r}$.[8]

Under these assumptions, preferences over alternatives can be represented by

$$F(\mathbf{p}) = \sum_{i=1}^{n} p_i u(x_i) \quad \text{for all } \mathbf{p} \in \Delta, \tag{1.1}$$

where u is a strictly increasing function. This condition on u ensures monotonicity, which means that (first order) stochastically dominating alternatives are preferred. Strong risk aversion, implying that mean preserving spreads are disapproved, requires that u be strictly concave.

[8]This condition is very similar to the separability axiom considered in Chapters 3 and 4.

Expression (1.1) has very strong implications for alternatives in the triangle diagram. In fact, the slope of the implied indifference curves is

$$\left.\frac{dp_3}{dp_1}\right|_{F=\bar{F}} = \frac{u(x_2)-u(x_1)}{u(x_3)-u(x_2)}, \tag{1.2}$$

which is constant (since the incomes x_1, x_2 and x_3 are given for all points in the triangle) and positive (since under monotonicity $u(x_3) > u(x_2) > u(x_1)$).

Positivity of the slope of indifference curves is a general property of preference theories that respect monotonicity. Note that monotonicity also implies that indifference curves lying more to the northwest correspond to higher preference. For any point **p** in the triangle, the set of points strictly to the northwest of **p** (that is, all points **q** such that $q_1 \leq p_1$ and $q_3 \geq p_3$, with at least one of the inequalities strict) constitutes the set of points strictly stochastically dominating **p**.

The most important implication of EU theory, however, is the fact that the slope of these indifference curves is constant.[9] Thus, in EU theory, indifference curves are parallel straight lines. The connected lines in Figure 1.1 represent such a set of EU indifference curves. One number, the value of the constant slope, determines the preferences over the entire triangle diagram. The figure shows that this feature severely restricts the number of response patterns allowed. In fact, EU theory implies that respondents choose consistently either **a** or **b** or are indifferent in each of the eight choice pairs, depending on the relative values of the slopes of the indifference curves (i.e, the connected lines) and the dashed lines. With the indifference curves drawn in the figure, this choice should be **b**. With a larger value for the slope it could be indifference or **a**. Note that in EU theory the slope can be seen as a kind of measure for the degree of risk aversion—in a choice between a riskless lottery and a risky one, such as in pairs 1 and 2 in the figure, the riskless one is chosen only for sufficiently high values of the slope.[10]

1.2.2 Some Basic Concepts from Non-Expected Utility Theory

The well known problems discovered by Allais (1953) offer an important challenge to the restrictive implications of EU theory. The first three choice pairs in Figure 1.1 illustrate these problems. Allais' 'common consequence effect' (also known as the Allais paradox) suggests a tendency for choosing **a** in choice pair 1 and **b** in choice pair 3, thus violating EU theory. Allais' 'common ratio effect' concerns a tendency for choosing **a** and **b**, respectively, in choice pairs such as 2 and 3, again violating EU theory. There is by now overwhelming experimental

[9] The slope is not required to be equal across different triangles, i.e., for different sets X.

[10] Indeed, Machina (1982) has shown that the slope given in expression (1.2) is related to the Arrow-Pratt measure of risk aversion. See Chapters 3 and 4 for more on the Arrow-Pratt concept.

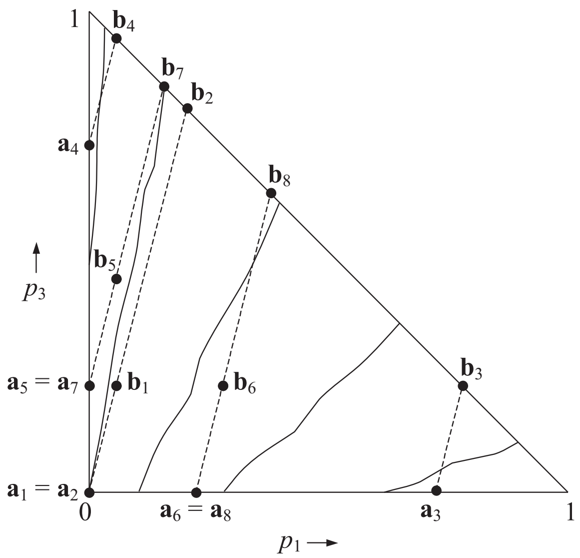

Figure 1.2. Fanning-out

evidence for the empirical relevancy of both predictions (Camerer, 1995; Starmer, 2000).

One solution for 'explaining' the Allais problems is to drop the assumption of parallel indifference curves. In fact, Machina (1982) introduced for that purpose the notion of *fanning-out*. In its pure form, fanning-out represents a monotonic increase in the slope of indifference curves as one moves northwest in the triangle. This can be seen in Figure 1.2, which also reveals that fanning-out does not require linearity of indifference curves. Concretely, fanning-out says that, given any two points **p** and **q** in the triangle, such that **q** lies to the northwest of **p** (that is, **q** stochastically dominates **p**), the slope in point **q** has to be at least as high as that in **p**. For the choice pairs in the figures, fanning-out has the following implications: given any two choice pairs k and ℓ, if $\mathbf{a}_\ell$ stochastically dominates $\mathbf{a}_k$ and $\mathbf{b}_\ell$ stochastically dominates $\mathbf{b}_k$, then the choice of alternative **a** from pair k implies that alternative **a** has to be chosen from pair ℓ as well, and indifference in pair k implies that either alternative **a** has to be chosen from pair ℓ or that one has to be indifferent between the alternatives of ℓ. Fanning-out therefore accounts for the dominant behaviour in situations such as those suggested by Allais.

Empirical research, however, sometimes reveals the opposite pattern: that of *fanning-in* (see, e.g., Battalio et al., 1990). In that case, the slope of the indifference curves becomes smaller as one moves to stochastically dominating alternatives. For the choice pairs in the figures, fanning-in has the following implications: given any two choice pairs k and ℓ, if $\mathbf{a}_\ell$ stochastically dominates $\mathbf{a}_k$ and $\mathbf{b}_\ell$ stochastically dominates $\mathbf{b}_k$, then the choice of alternative **b** in k implies that alternative **b** has to be chosen in ℓ as well, and indifference in k implies that either alternative **b** has to be chosen in ℓ or that one has to be indifferent between the alternatives of ℓ.

Both fanning-out and fanning-in deal with a change in slope as one moves to different indifference curves (at least when preferences satisfy monotonicity). The research on extensions of EU theory has also focused on the relevancy of the linearity of the indifference curves implied by expressions (1.1) and (1.2). Three different assumptions have been proposed:

Betweenness. For any alternatives $\mathbf{p},\mathbf{q} \in \Delta$ and any scalar $\alpha \in (0,1)$, we have $\mathbf{p} \succeq \mathbf{q}$ if and only if $\mathbf{p} \succeq \alpha\mathbf{p} + (1-\alpha)\mathbf{q} \succeq \mathbf{q}$.

Quasi-convexity. For any alternatives $\mathbf{p},\mathbf{q} \in \Delta$ and any scalar $\alpha \in (0,1)$, we have $F(\alpha\mathbf{p} + (1-\alpha)\mathbf{q}) \leq \max\{F(\mathbf{p}),F(\mathbf{q})\}$.

Quasi-concavity. For any alternatives $\mathbf{p},\mathbf{q} \in \Delta$ and any scalar $\alpha \in (0,1)$, we have $F(\alpha\mathbf{p} + (1-\alpha)\mathbf{q}) \geq \min\{F(\mathbf{p}),F(\mathbf{q})\}$.

Betweenness obviously is an implication of independence. It implies that, if $\mathbf{p} \sim \mathbf{q}$, then for any scalar $\alpha \in (0,1)$ we have $\mathbf{p} \sim \alpha\mathbf{p} + (1-\alpha)\mathbf{q} \sim \mathbf{q}$, which means that indifference curves are straight lines—but not necessarily parallel. Betweenness implies neutrality to mixtures of alternatives on the same indifference curve. Straightforward extensions are concave indifference curves (corresponding to the assumption of quasi-convexity), describing mixture aversion, and convex indifference curves (corresponding to the assumption of quasi-concavity), describing mixture proneness. The latter case is illustrated in Figure 1.3. Betweenness, quasi-convexity and quasi-concavity have implications for the combinations of choice pairs (1, 2), (5, 7), and (6, 8) in the figures. In each of those combinations, the only response patterns consistent with betweenness are **aa**, **bb** and $\sim\sim$. Quasi-convexity allows, in addition to the betweenness patterns, **ab**, **a**$\sim$ and $\sim$**b**. Quasi-concavity, on the other hand, allows, in addition to the betweenness patterns, **ba**, **b**$\sim$ and $\sim$**a**.

1.2.3 Rank-dependent Expected Utility and Yaari's Dual Theory

The most popular alternative to the EU model is Quiggin's (1982) rank-dependent expected utility (RDEU) model (see, e.g., Starmer, 2000). Most popular within the income distribution literature is Yaari's (1987) dual theory, which is a special

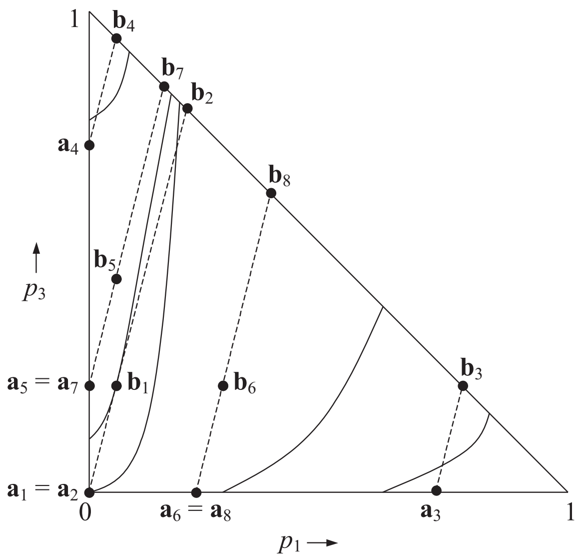

Figure 1.3. Quasi-concavity

case of the RDEU model. We will first summarize Yaari's model and then return to the more general RDEU approach.

Preferences consistent with Yaari's theory can be represented using

$$F(\mathbf{p}) = \sum_{i=1}^{n} w(p_i, p_1 + \cdots + p_i)x_i \quad \text{for all } \mathbf{p} \in \Delta, \tag{1.3}$$

where, for any $i \neq n$,

$$w(p_i, p_1 + \cdots + p_i) = f(p_i + \cdots + p_n) - f(p_{i+1} + \cdots + p_n),$$

$w(p_n, p_1 + \cdots + p_n) = f(p_n)$ and $f : [0,1] \to [0,1]$ is a strictly increasing and continuous function for which $f(0) = 0$ and $f(1) = 1$. Given the conditions on f, preferences are monotonic. Strong risk aversion requires that f be strictly convex (Yaari, 1987). Note that while in the EU approach a change in an income is evaluated in function of the size of the income, in the Yaari approach it is evaluated as a function of its rank position (defined as $p_1 + \cdots + p_i$ for an income x_i).

For the alternatives in the triangle diagram, Yaari's theory implies that

$$F(\mathbf{p}) = [1 - f(1-p_1)]x_1 + [f(1-p_1) - f(p_3)]x_2 + f(p_3)x_3, \tag{1.4}$$

which yields for the slope of the indifference curves

$$\left.\frac{dp_3}{dp_1}\right|_{F=\bar{F}} = \frac{f'(1-p_1)}{f'(p_3)}\frac{x_2 - x_1}{x_3 - x_2}. \tag{1.5}$$

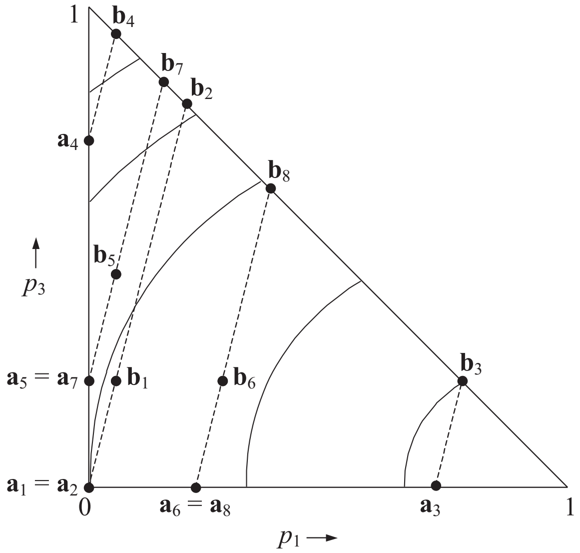

Figure 1.4. RDEU (Yaari) Indifference Curves

Again, indifference curves are positively sloped (since $f'(p) > 0$ for all p). If f is strictly convex, the slope decreases as p_1 increases, ceteris paribus, and also as p_3 increases, ceteris paribus. If p_1 decreases and p_3 increases, the slope does not necessarily go up or down. This means that indifference curves strictly fan out horizontally—that is, the slope becomes strictly higher moving horizontally west in the triangle diagram—and strictly fan in vertically—that is, the slope becomes strictly smaller moving vertically north in the triangle diagram. Moving diagonally northwest, however, the slope can go up or down. This pattern is illustrated in Figure 1.4. By consequence, for the choices in the figures, fanning-out is satisfied for combinations of the choice pairs 1, 3 and 6 (a horizontal move in the triangle), while fanning-in is satisfied for combinations of the choice pairs 1, 4 and 5 (a vertical move). There are no implications concerning fanning-out or fanning-in for combinations of choices 2, 3, 4, 7 and 8.

Another important property is that, whenever f is strictly convex, the slope of an indifference curve decreases as one moves to the northeast. Therefore, under the assumption of strong risk aversion, preferences are strictly quasi-convex.[11]

The Yaari model is a special case of the RDEU model, which is given by

$$F(\mathbf{p}) = \sum_{i=1}^{n} w(p_i, p_1 + \cdots + p_i)u(x_i) \quad \text{for all } \mathbf{p} \in \Delta, \tag{1.6}$$

[11] For a formal statement and proof, see Lemma 2.2 in Chapter 2.

where, for any $i \neq n$,

$$w(p_i, p_1 + \cdots + p_i) = f(p_i + \cdots + p_n) - f(p_{i+1} + \cdots + p_n),$$

$w(p_n, p_1 + \cdots + p_n) = f(p_n)$, $f : [0,1] \to [0,1]$ is a strictly increasing and continuous function for which $f(0) = 0$ and $f(1) = 1$ and u is a strictly increasing function. Again the conditions required for monotonicity are satisfied. Strong risk aversion requires that the function f be convex and that the function u be concave and, furthermore, that either f be strictly convex or u be strictly concave or both (Chew et al., 1987). When u is the identity function, the RDEU model (1.6) reduces to the Yaari model. When f is the identity function, it reduces to the EU model.

The slope of an indifference curve in the diagram for the RDEU model is

$$\left.\frac{dp_3}{dp_1}\right|_{F=\bar{F}} = \frac{f'(1-p_1)}{f'(p_3)} \frac{u(x_2) - u(x_1)}{u(x_3) - u(x_2)}. \tag{1.7}$$

Clearly, the indifference curves of the RDEU social welfare function have (more or less) the same properties as those of the Yaari model. That is, indifference curves are concave, fan out horizontally and fan in vertically.

1.2.4 Evaluating Income Distributions

There is a close formal relationship between the literature on income distribution and the theory of decision making under risk. With the Gini index as a prominent exception, the most common inequality measures (including the Atkinson-Kolm and the generalized entropy measures) can all be interpreted in a social welfare framework formally equivalent to the EU model as given in expression (1.1).[12] This means that they can be interpreted as reflecting VOI-preferences, i.e., the preferences of a rational individual behind the veil of ignorance.[13] Of course one can also defend EU-type assumptions without explicitly referring to the idea of the veil of ignorance. One then has to justify the independence condition for ISO preferences directly on ethical grounds rather than as a requirement of rationality behind the veil of ignorance.

A strong competitor of the Atkinson-Kolm and the generalized entropy measures is the class of generalized Gini indices. These are based on a social welfare function of the form of the Yaari model (1.3) (or, at least, an important subclass is) and therefore do not satisfy the independence axiom. The most popular social welfare function of the form (1.3) is the S-Gini social welfare function, where

[12] Accordingly, all these inequality measures satisfy the decomposability axiom, which is closely related to the independence axiom. See Chapter 2.

[13] Dahlby (1987) explicitly works out this interpretation.

$f(p) = p^{\rho}$ with $\rho > 1$ (Donaldson and Weymark, 1980; Yitzhaki, 1983). The parameter ρ can be seen as a measure of inequality aversion. Note that the popular Gini index is based on the S-Gini social welfare function with $\rho = 2$. A few studies such as Ebert (1988) and more recently Chateauneuf (1996) and Chateauneuf et al. (2002), have considered the evident extension to the Yaari model which is to base the evaluation of income distributions on the RDEU model.[14]

The idea of strong risk aversion is interpreted within the income distribution literature as the Pigou-Dalton transfer principle, i.e., the notion that a rank preserving transfer from a richer to a poorer person increases social welfare. As we have seen, the transfer principle requires in the EU model that the function u be strictly concave. As can be seen from expression (1.2) the restriction to linear parallel indifference curves does not depend on the concavity of u and a test of this restriction can be seen as a direct test of the independence assumption without any need to make assumptions about risk aversion.

On the other hand, imposing the transfer principle has stronger consequences for the Yaari and the RDEU models within the triangle. As we have seen, it requires, for instance, in both cases that the indifference curves be strictly concave. Since the transfer principle occupies such a dominant position in the income distribution literature, we will use in the empirical part the terms Yaari model and RDEU model for expressions (1.3) and (1.6), respectively, with the assumption of concave indifference curves imposed.

However, we know from previous empirical work that the transfer principle is violated consistently by respondents.[15] Let us therefore define the weaker principle of 'weak inequality aversion:' given a fixed population, a completely equal income distribution is better than any unequal income distribution with the same total income. This principle seems absolutely essential for an egalitarian social welfare function. It gives additional support for the transfer principle with an EU social welfare function, i.e., a social welfare function satisfying independence, because such a welfare function will only satisfy weak inequality aversion if it satisfies the transfer principle. However, in the Yaari (and RDEU) framework, weak inequality aversion does not imply the transfer principle. It has been shown (Chateauneuf, 1996 and Chateauneuf and Moyes, 2004) that the Yaari social welfare function (1.3) satisfies weak inequality aversion if and only if $f(p) < p$ for all $p \in (0,1)$. This condition is strictly weaker than strict convexity (since $f(0) = 0$ and $f(1) = 1$). The RDEU social welfare function satisfies weak inequality aversion if $f(p) \leq p$ for all $p \in (0,1)$ and u is concave, with at least one of the condi-

[14] All the inequality measures that are mentioned in this subsection are defined formally in Chapter 2.

[15] This is found especially in the context of inequality comparisons (see for instance Amiel and Cowell, 1992, 1998, Ballano and Ruiz-Castillo, 1993, and Harrison and Seidl, 1994a, 1994b), but also in the context of social welfare comparisons (Amiel and Cowell, 1994a).

tions holding strictly.[16] In our empirical work we will consider these extensions as well and label them Yaari′ and RDEU′.

In the risk literature, forms of the RDEU weighting function f that do not satisfy the condition relating to weak inequality aversion have been considered and sometimes offer a better explanation of observed choice patterns (see, e.g., Gonzalez and Wu, 1999). We do not consider these forms in our empirical analysis because in our view it does not make sense to base the evaluation of income distributions on a welfare function which does not even satisfy the principle of weak inequality aversion.

1.3 The Setup of the Questionnaire

The target group of the questionnaire consisted of first year business students of the K.U.Leuven (Catholic University Leuven, Belgium). The students had not yet been exposed to any lectures on the evaluation of income distributions or on decision making under risk, which ensured that the respondents were not prejudiced. The questionnaires were distributed and filled in in the classroom, after the teacher had given a short and non-suggestive oral introduction. The survey was organized twice (with different respondents in two subsequent academic years): in April 2002 and in November 2002. The results were stable over time. In order to test for the differences between ISO, VOI and PIR preferences, there were three different versions of the questionnaire. Accordingly, the group of students was divided into three subgroups. Each subgroup participated in only one version of the questionnaire and respondents did not know that there were three different versions. For the ISO version, the VOI version and the PIR version, there are 93, 92 and 94 respondents, respectively.

Each questionnaire version consists of the same eight questions, where in each question, the respondent is asked to make a choice between two alternatives, which are either income distributions or lotteries, depending on the given choice situation. The eight choice pairs correspond to the alternatives shown in Figure 1.1 (with the probabilities as given in Table 1.1). Throughout the questionnaire, the same set of three incomes $X = \{x_1 = €500, x_2 = €1500, x_3 = €2500\}$

[16] In fact, Chateauneuf (1996) has shown that these conditions for the Yaari and RDEU models imply consistency with the ‘absolute differentials ordering,’ which is a stronger requirement than the one of weak inequality aversion (see also Chateauneuf and Moyes, 2004). This stronger principle can be formulated as follows. Suppose that we have two income distributions with the same population and total income, and in the first income distribution the absolute income difference for each income pair is greater than, or equally great as, in the second distribution while for at least one pair the absolute income difference is greater, then the first income distribution is more unequal than the second. It seems natural to extend the principle to the social welfare context by stating that the second income distribution should be evaluated as better than the first.

is used. In line with the Allais problems described earlier, we refer to questions 1, 3, 4, 5 and 6 as the 'common consequence questions,' and to questions 2, 3, 4, 7 and 8 as the 'common ratio questions.'

Although the same choice pairs are used, the background stories are different for the three versions of the questionnaire.[17] Each of the three versions deals with recently graduated students that are going to be employed in one of two firms. Each firm offers three types of jobs which are identical in every respect except for the income that is earned: the first job pays €2500, the second €1500 and the third €500. For the ISO and VOI versions, a firm corresponds to an income distribution, for the PIR version it corresponds to a lottery.

In the ISO version, the respondent is asked to consider the situation of 100 recently graduated students that will all be employed in either of two firms, which are different only with respect to the number of positions that are available for each of the jobs. The respondent is then asked to reveal, for the eight cases, which of the two firms he or she thinks offers the largest social welfare.

The VOI version also asks the respondent to consider the situation of 100 recently graduated students, but this time the respondent has to picture himself or herself as being one of them. Again, the firms are different only with respect to the number of positions that are available for each of the jobs. The respondent and the 99 other graduated students will all be employed in the same firm and each has an equal chance of ending up in any of the 100 positions available in the firm. The respondent is then asked to state, for the eight cases, which firm he or she prefers.

In the PIR version, the respondent is asked to picture himself or herself as being a recently graduated student who will be employed in either of two firms. The firms are identical except with respect to the probabilities of ending up with each of the jobs. The respondent is then, again, asked to state, for each of the eight cases, which firm he or she prefers.

As mentioned already in the introduction, the setup of our questionnaire is similar to the one used by Bernasconi (2002). There are three main differences, however. First, we use different and more income distributions (and therefore test some axioms which could not be tested by him). Second, he represents the different income distributions in the questionnaires with pie charts, while we simply give the relevant sets of numbers. Third, he formulates the ISO, VOI and PIR cases in a more abstract form, while we tried to formulate a question which was closer to the everyday experience of our respondents. The comparison of his results with ours will therefore give some insight into the importance of framing effects (for which, again, see Camerer, 1995).

[17] The precise formulation of the background stories in each of the versions is given in Appendix 1.A. For each background story there were two variants of the questionnaire with the questions ordered differently. Since the results show that there is only a slight indication of order effects, we simply pooled the answers for these different variants.

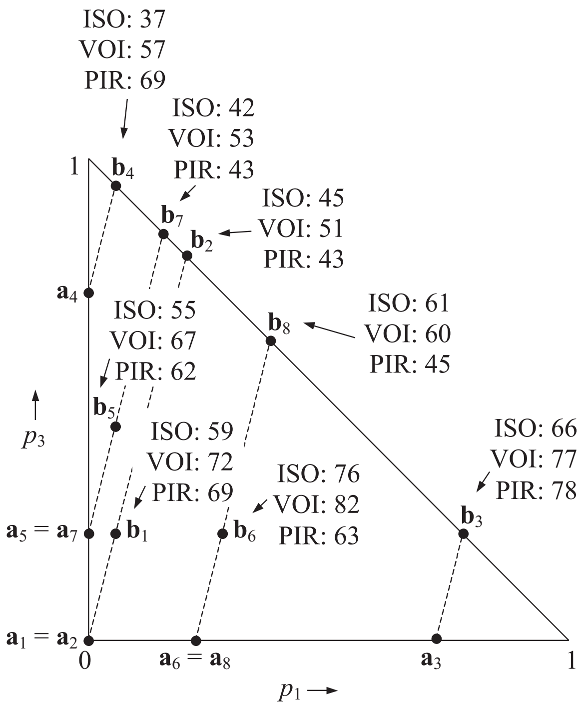

Figure 1.5. Overview of the Results, **b** Responses (in %)

1.4 Results

Our discussion of the results focuses on the two general issues raised before: the comparison of the ISO, VOI and PIR versions of the questionnaire, and the degree of consistency with the preference theories presented in Section 1.2. In Subsection 1.4.1 we have a first look at the question of how the three versions of the questionnaire compare through an analysis of the responses for separate questions. Combining the answers on different questions makes it possible to test also the relevancy of the different basic axioms of choice theory (Subsection 1.4.2). In Subsection 1.4.3 we conclude the discussion by focusing on the different theories which have been proposed in the income distribution literature.

1.4.1 A First Look

Figure 1.5 and Table 1.2 give the results for the separate questions. The chi-square test statistics reported in Table 1.3 test for each question separately the null hypothesis that population proportions for categories **a** and **b**, respectively, are equal for the two versions under comparison (ISO-VOI, VOI-PIR or ISO-PIR) (there

Table 1.2. Results for Separate Questions (in %)

	ISO			VOI			PIR		
Question	**a**	**b**	∼	**a**	**b**	∼	**a**	**b**	∼
1	37	59	4	26	72	2	27	69	4
2	50	45	5	45	51	4	58	43	0
3	30	66	4	20	77	3	17	78	5
4	61	37	2	38	57	5	28	69	3
5	44	55	1	29	67	3	35	62	3
6	19	76	4	16	82	2	35	63	2
7	56	42	2	41	53	5	51	43	6
8	30	61	9	36	60	4	48	45	8

Table 1.3. Chi-square Tests for Homogeneity for Separate Questions

Question	ISO-VOI	VOI-PIR	ISO-PIR
1	2.72 (0.099)	0.03 (0.867)	2.20 (0.138)
2	0.57 (0.451)	2.15 (0.143)	0.49 (0.483)
3	2.93 (0.087)	0.15 (0.703)	4.35 (0.037)
4	8.94 (0.003)	2.68 (0.101)	21.29 (0.000)
5	3.91 (0.048)	0.71 (0.399)	1.31 (0.253)
6	0.38 (0.539)	8.64 (0.003)	5.47 (0.019)
7	3.23 (0.073)	2.07 (0.151)	0.12 (0.726)
8	0.39 (0.530)	3.58 (0.058)	6.21 (0.013)

Note: *p*-values in brackets.

is one degree of freedom).[18] To some extent Table 1.3 suggests that the results for the ISO and PIR versions are furthest removed from each other while the results for the VOI version lie in between. This is exactly what one would expect a priori: ISO preferences deal exclusively with uninvolved common interest, PIR preferences deal exclusively with involved self interest and VOI preferences deal with involved common interest (that is, the common interest is at stake). We will see that this pattern is confirmed in more detailed analyses.

Table 1.2 shows that, overall, alternative **b** is more popular than the other two alternatives. In the risk literature **b** alternatives are usually seen as more risky than the corresponding **a** alternatives. Analogously, we could say that they are more unequal in the income distribution context. The popularity of the **b** answers can be explained by the choice of the set of incomes in our questionnaire design. Consider as a benchmark the case of a respondent who has preferences consistent with the Atkinson social welfare function: for the given income amounts, such a respondent only prefers **a** over **b** if she has a relatively high value of approximately 1.75 or more for the parameter of inequality aversion.

[18]We ignore the category of indifference (∼) in the tests because it usually has frequencies lower than five, which would make the chi-square test less appropriate.

Table 1.4. Results for Pairs of Common Consequence Questions (in %)

Questions	Version	EU (**aa**, **bb**, ∼∼)	Fanning-out (**ba**, **b**∼, ∼**a**)	Fanning-in (**ab**, **a**∼, ∼**b**)
3, 1	ISO	61	23 (0.203)	16
	VOI	63	22 (0.196)	15
	PIR	63	23 (0.088)	14
1, 4	ISO	53	35 (0.001)	12
	VOI	60	26 (0.049)	14
	PIR	55	22 (0.562)	22
3, 6	ISO	61	14	25 (0.066)
	VOI	73	12	15 (0.345)
	PIR	62	27 (0.014)	12
6, 1	ISO	55	31 (0.010)	14
	VOI	67	22 (0.049)	11
	PIR	63	14	23 (0.088)
1, 5	ISO	67	19 (0.237)	14
	VOI	72	16 (0.279)	12
	PIR	66	21 (0.108)	13
5, 4	ISO	62	28 (0.003)	10
	VOI	54	27 (0.140)	18
	PIR	63	15	22 (0.155)

Note: *p*-values in brackets.

1.4.2 Testing Some Concrete Hypotheses of Choice Theory

More interesting insights can be gained by analysing the response patterns for different choice pairs together. We will first look at combinations of two questions and then analyse the overall response patterns for the eight questions. We focus, again, on the two main issues. In the first place, we test the empirical relevancy of the concrete hypotheses of choice theory. In the second place, we check for the possible differences between ISO, VOI and PIR preferences.

Pairs of Questions

(a) Tables 1.4 and 1.5 show the results for combinations of several pairs of common consequence questions and common ratio questions, respectively. For each combination of two choice pairs (described in the first column) we give separately the results for the three versions of the questionnaire. As shown in Subsection 1.2.1, only three of the nine possible response patterns are consistent with EU theory for each of the combinations of two questions included in Tables 1.4 and 1.5: the respondent can prefer **a** in both choice pairs, she can prefer **b** in both pairs or she can be indifferent (∼) in both choice situations. We call these patterns, (**aa**,

Table 1.5. Results for Pairs of Common Ratio Questions (in %)

Questions	Version	EU (**aa**, **bb**, ∼∼)	Fanning-out (**ba**, **b**∼, ∼**a**)	Fanning-in (**ab**, **a**∼, ∼**b**)
3, 2	ISO	55	33 (0.001)	12
	VOI	57	36 (0.000)	8
	PIR	51	45 (0.000)	4
2, 4	ISO	56	27 (0.106)	17
	VOI	62	15	23 (0.155)
	PIR	57	6	36 (0.000)
3, 8	ISO	62	20 (0.368)	17
	VOI	60	29 (0.004)	11
	PIR	53	41 (0.000)	5
8, 2	ISO	61	29 (0.002)	10
	VOI	52	29 (0.087)	18
	PIR	64	21 (0.196)	15
2, 7	ISO	68	17 (0.428)	15
	VOI	62	17	21 (0.368)
	PIR	67	16	17 (0.500)
7, 4	ISO	60	23 (0.256)	17
	VOI	62	17	21 (0.368)
	PIR	56	9	35 (0.000)

Note: *p*-values in brackets.

bb, ∼∼), therefore 'EU consistent' and the percentage of respondents with one of these three response patterns is given in the third column of Tables 1.4 and 1.5. Analogously we can say that the response patterns (**ba**, **b**∼, ∼**a**) and (**ab**, **a**∼, ∼**b**) are consistent with indifference curves that fan out and fan in, respectively. In both cases we exclude EU consistent patterns from the categories fanning-out and fanning-in. The percentages of respondents with these patterns are given in the last two columns of the tables.

Clearly, EU consistent responses dominate. One should be aware that this does not necessarily imply that our respondents follow the axioms of EU theory, as it is quite possible for an individual to be consistent with EU theory over two questions but not over three or more. We will return to this issue in the next section. At this stage it is more interesting to consider whether the violations of EU theory for each of the question pairs are systematic, that is, whether the percentage of observed patterns consistent with fanning-out (or fanning-in) is significantly higher than the percentage that would be observed if the response patterns of the respondents that violate EU theory were completely random. The null hypothesis is that the population frequency of fanning-out (or fanning-in) violations relative to the total population frequency of violations is equal to 50%. The tables report *p*-values for the one sided exact test based on the binomial distribution.

Table 1.6. Chi-square Tests for Homogeneity for Pairs of Questions

Questions	ISO-VOI	VOI-PIR	ISO-PIR
3, 1	0.06 (0.969)	0.12 (0.942)	0.20 (0.907)
1, 4	1.93 (0.381)	2.15 (0.342)	5.88 (0.053)
3, 6	3.16 (0.206)	6.43 (0.040)	8.03 (0.018)
6, 1	3.11 (0.211)	6.04 (0.049)	8.99 (0.011)
1, 5	0.56 (0.756)	0.86 (0.650)	0.14 (0.932)
5, 4	3.07 (0.216)	4.25 (0.120)	8.40 (0.015)
3, 2	0.96 (0.620)	2.04 (0.361)	5.01 (0.082)
2, 4	4.00 (0.135)	6.33 (0.042)	18.16 (0.000)
3, 8	2.85 (0.241)	4.07 (0.131)	13.25 (0.001)
8, 2	3.23 (0.199)	2.65 (0.267)	2.20 (0.333)
2, 7	1.05 (0.591)	0.57 (0.753)	0.16 (0.923)
7, 4	0.94 (0.626)	6.56 (0.038)	11.80 (0.003)

Note: *p*-values in brackets.

The first combinations of choice pairs in Tables 1.4 and 1.5, the combinations (3, 1) and (3, 2), are of particular interest, as they are similar to the original examples used by Allais for introducing the common consequence and common ratio effects, respectively. In both cases the predicted fanning-out patterns are more popular than the fanning-in patterns. The statistical significance of fanning-out is much weaker for Allais' common consequence effect (questions 3 and 1) than for Allais' common ratio effect (questions 3 and 2).

The overall picture shows some interesting differences between the ISO, VOI and PIR versions of the questionnaire. A mixed pattern of fanning-out and fanning-in is observed in the PIR version. This is in line with the experimental research on decision making under risk. However, with only one exception, fanning-out is always dominating in the ISO version. The VOI version is between the other two, but with a relatively strong presence of fanning-out. Table 1.6 presents the chi-square test statistics for the hypothesis of homogeneity of two versions with respect to the categories EU, fanning-out and fanning-in between versions (there are two degrees of freedom).[19] The hypothesis formulated on the basis of Table 1.3 is corroborated by the results in Table 1.6: the results for the ISO and PIR versions form the extremes while the results for the VOI version are situated in between.

The question pairs in Table 1.4 also allow to test for some aspects of the Yaari and RDEU models (with the Pigou-Dalton transfer principle imposed). As we have seen (Subsection 1.2.3), these models imply that fanning-out is satisfied hor-

[19]Note the difference with Table 1.3, in which we tested for homogeneity of the three versions with respect to the responses (**a** or **b**) for the eight separate questions. Table 1.6 tests for homogeneity of the three versions with respect to response patterns (EU consistent, fanning-out or fanning-in) for combinations of two questions.

Table 1.7. Results for Pairs of Questions (in %)

Questions	Version	EU (**aa**, **bb**, ∼∼)	Quasi-convexity (**ab**, **a**∼, ∼**b**)	Quasi-concavity (**ba**, **b**∼, ∼**a**)
6, 8	ISO	57	15	28 (0.040)
	VOI	61	9	30 (0.001)
	PIR	54	14	32 (0.007)
1, 2	ISO	70	8	23 (0.006)
	VOI	61	9	30 (0.001)
	PIR	55	7	37 (0.000)
5, 7	ISO	63	13	24 (0.061)
	VOI	57	14	29 (0.019)
	PIR	53	14	33 (0.005)

Note: *p*-values in brackets.

izontally, that is, for the question pairs (3, 1), (3, 6) and (6, 1), while fanning-in is satisfied vertically, and thus for the question pairs (1, 4), (1, 5) and (5, 4) (of course, the EU patterns for these pairs are also consistent with the models). This pattern is not supported by the results for the ISO and VOI versions, especially where the Yaari and RDEU models imply fanning-in.

(b) Table 1.7 presents the results for the question pairs (6,8), (1,2) and (5,7). These combinations allow to test betweenness, i.e., the linearity of indifference curves (which is EU consistent) against quasi-convexity (excluding EU consistent patterns) and quasi-concavity (again, excluding EU consistent patterns). The corresponding response patterns have already been described in Section 1.2.2. The results in Table 1.7 are striking. There is a clear and significant domination of quasi-concavity, i.e., convex indifference curves. This mixture proneness is found in all three versions of the questionnaire.[20] Quasi-concavity has also been found in experimental work on decision making under risk (see, e.g., Camerer and Ho, 1994). We will return to the implications of these findings in Subsection 1.4.3.

The Total Pattern of Answers

In Table 1.8 we summarize the results for a more ambitious approach in which the eight questions are considered jointly. Each column refers to a specific hypothesis of choice theory. We first give, for each hypothesis, as a reference point the proportion of the 256 ($= 2^8$) possible patterns that is actually consistent with the given hypothesis.[21] If individual response patterns were completely random, we

[20] Chi-square tests show that the null hypothesis of homogeneity over the versions cannot be rejected.

[21] For convenience, we have neglected patterns with indifferences. There are only very few cases of indifference in the answers of our respondents.

Table 1.8. Results for the Combination of All Eight Questions (in %)

	EU	Fanning-out	Fanning-in	Between-ness	Quasi-convexity	Quasi-concavity
Reference	0.8	6.3	6.3	12.5	42.2	42.2
ISO	10	30	13	32	46	68
Test 1	(0.000)	(0.000)	(0.013)	(0.000)	(0.246)	(0.000)
Test 2		(0.000)	(0.889)	(0.002)	(0.854)	(0.001)
VOI	13	29	18	26	40	70
Test 1	(0.000)	(0.000)	(0.000)	(0.000)	(0.686)	(0.000)
Test 2		(0.000)	(0.570)	(0.393)	(0.998)	(0.003)
PIR	11	24	15	21	37	72
Test 1	(0.000)	(0.000)	(0.002)	(0.012)	(0.859)	(0.000)
Test 2		(0.002)	(0.762)	(0.675)	(0.999)	(0.000)

Note: *p*-values in brackets.

would expect to find the ‘reference’ degree of support for the various hypotheses. We then test whether the actual number of consistent response patterns in the data is significantly larger than what would be expected for random responses.[22] This test is labeled ‘Test 1’ in Table 1.8.

For all three versions, all hypotheses except quasi-convexity pass Test 1. Note that about 10% to 13% of the observed patterns are consistent with EU theory—which is significantly more than the 0.8% which would be found with a completely random response pattern. An explanation of the success of EU theory could be that respondents use the expected value rule. At the same time it should be mentioned that 10% to 13% is far from overwhelming considering the focal role of EU theory in the risk and in the income distribution literature.

Since all the other hypotheses generalize EU theory, they all benefit from the relatively good performance of that theory. It is more revealing therefore to test whether they ‘add’ something to EU theory. We do this by removing from the sample all EU consistent patterns. For the remaining (non-EU consistent) responses we follow an analogous procedure as described before. For each hypothesis (each column) we first compute, with respect to the set of all possible patterns excluding the EU consistent patterns, the proportion of consistent responses to be expected if individual response patterns were completely random. We then test whether the proportion of consistent responses in the (reduced) sample is significantly larger than what would be expected in the random case. The resulting *p*-values are summarized in Table 1.8 under the label ‘Test 2.’[23]

[22]More specifically, we test the null hypothesis that the population proportion in support of the given hypothesis of choice theory is equal to the population proportion in support of the same hypothesis if choices were completely random against the alternative hypothesis that it is greater.

[23]More specifically, ‘Test 2’ considers the null hypothesis that the population proportion in

Table 1.9. Results for the Combination of All Eight Questions (in %)

	EU	S-Gini	Yaari	RDEU	Yaari′	RDEU′
Reference	0.8	2.7	15.2	16.4	65.6	79.3
ISO	10	13	23	23	69	78
Test 1	(0.000)	(0.000)	(0.039)	(0.075)	(0.298)	(0.633)
Test 2		(0.273)	(0.708)	(0.805)	(0.895)	(0.990)
VOI	13	16	25	25	78	88
Test 1	(0.000)	(0.000)	(0.010)	(0.023)	(0.006)	(0.021)
Test 2		(0.268)	(0.794)	(0.869)	(0.518)	(0.829)
PIR	11	15	26	26	74	90
Test 1	(0.000)	(0.000)	(0.007)	(0.016)	(0.042)	(0.003)
Test 2		(0.113)	(0.496)	(0.621)	(0.627)	(0.440)

Note: *p*-values in brackets.

For all three versions, fanning-out adds significantly to EU theory, while the fanning-in hypothesis does not. Looking at the shape of the indifference curves, betweenness adds explanatory power to EU theory for the ISO version, but not for the other versions. An approach with linear but non-parallel indifference curves in the Marschak-Machina triangle seems to have some relevance to describe the preferences of an impartial and sympathetic observer. However, more striking is the significance of quasi-concavity for all three versions. The global response patterns therefore confirm what we found already by analysing the combinations of choice pairs two by two.

1.4.3 The Fate of Different Theories of Income Distribution Evaluation

The importance of quasi-concavity and fanning-out already suggests that the most popular approaches in the income distribution literature will not get much support in our data. Table 1.9, which is constructed in a similar way as Table 1.8, summarizes the results in a more structured way. We repeat the results for the EU model as a benchmark. Remember that the EU approach performs significantly better than what would be predicted if the answers were random. As shown by the results for 'Test 1,' the same is true for the S-Gini, the Yaari, the RDEU, the Yaari′ and the RDEU′ models (for the latter three only in the VOI and PIR versions).

support of a specific non-EU hypothesis, excluding the part of the population that is in support of EU theory as well, is equal to what would be the population proportion in support of that non-EU hypothesis, excluding the part of the population that is in support of EU theory as well, if choices were random. The alternative hypothesis is that the former population proportion is greater than the latter.

However, in our setup all these alternative theories are less restrictive than EU theory. In fact, each of them can also rationalize each pattern that is EU consistent.[24] We therefore want to test whether any of these theories adds some explanatory power to the EU model. Analogously to the previous section, we therefore computed again the 'Test 2' results. For none of the versions, Yaari's theory or the (more restricted) S-Gini model passes this stricter test. Nor does the RDEU model. To repeat: this implies that the proportion of observed response patterns in the subsample of non-EU consistent responses which is consistent with these models is not significantly larger than what would be expected if the answers of the respondents were completely random. It is important to remember that we imposed the Pigou-Dalton transfer principle in the Yaari and the RDEU model, i.e., convexity of the weighting function f, and that our results can only be seen as a test of this restricted model. Yet relaxation of this convexity condition does not seem to help very much, given the fate of the Yaari′ and the RDEU′ models, which only impose the property of weak inequality aversion. It is difficult to see how one could construct an attractive egalitarian theory of social welfare which does not satisfy this very weak property. Both models (Yaari′ and RDEU′) are quite flexible and it is therefore not surprising that the proportion of response patterns compatible with them is very high. Again, however, the models do not add significantly to EU, in the sense that randomly chosen patterns would have performed equally well.

These results seem to suggest that it is worthwhile to work out alternatives for the EU-type social welfare functions, i.e., to try and elaborate an alternative which does not embody the independence assumption. At the same time, however, the Yaari- and RDEU-type extensions with weak inequality aversion imposed do not seem to be very promising, at least when one wants to rationalize the preferences of our respondents (and they appear to be even less successful for the ISO than for the VOI or PIR version). Comparing Tables 1.8 and 1.9 it is striking how much better is the performance of other alternatives to the EU model like fanning-out and quasi-concavity. It remains to be seen whether these ideas can be integrated in an attractive theory of income distribution.

1.5 Conclusion

With our questionnaire study we wanted to test whether the veil of ignorance approach captures in an adequate way the preferences of an impartial and sympathetic observer. Moreover, we wanted to check whether the answers of our

[24] This is not a general property—but it is true for our set of specific questions within the Marschak-Machina triangle.

respondents satisfy the independence axiom—underlying EU theory and most approaches to inequality measurement—and its most popular alternatives. These two questions are related but different. One can accept the VOI approach and at the same time argue in favour of a non-EU model behind the veil of ignorance. And one can defend the independence assumption for inequality measurement without the detour of the veil of ignorance.

As to the first question, the results for the three questionnaire versions (ISO, VOI and PIR) are to a certain degree similar: both of Allais' problems are present, there is quite a lot of systematic fanning-out or fanning-in, and quasi-concavity is an important systematic violation of EU (or betweenness). However, there are differences and it appears that the ISO and PIR versions are at both extremes. The identification of ISO preferences with VOI preferences is not evident. Note that the results for the PIR version are reassuringly comparable to the results encountered in empirical studies from the literature on decision under risk: e.g., Allais' problems, a complex fanning pattern, and systematic violations of betweenness.

The EU model yields a significant contribution to the explanation of the response patterns. At the same time, however, there are clear indications of the relevancy of fanning-out and quasi-concavity, also in the ISO version. Fanning-out and quasi-concavity do not characterize the most popular alternatives to the EU model—the RDEU model with as a special case the Yaari model, which provides the normative basis for an important subclass of the family of generalized Ginis. It is therefore not surprising that they do not add much explanatory power.

These are the results of only one limited study. However, they are in the line of much previous research on the empirical acceptance of the most popular inequality axioms. Moreover, despite the differences in the concrete formulation of the questionnaires and in the general setup of the empirical study, some of our results are strikingly similar to those of Bernasconi (2002): he also finds that the equivalence of VOI and ISO preferences cannot be taken for granted and that quasi-concavity, i.e., mixture proneness, is important to explain the empirical results.

The conclusion that the traditional inequality literature does not adequately capture the intuitions of our respondents seems clear. Even if we take the Yaari and the EU model together, only a quarter of the students has a response pattern which is in line with one of them. Of course, one can reasonably argue that the normative relevancy of this kind of questionnaire results is limited, as they can never substitute for critical reflection and thorough assessment of the ethical argumentation. We do not go into that debate here. However, a conditional conclusion seems possible. If one wants to construct a theory of income distribution which is more attuned to the intuitions of lay respondents, the RDEU model with imposition of weak inequality aversion does not seem to be the most promising starting point.

Appendix 1.A: The Questionnaire Versions

ISO Version

Consider the situation of two firms, A and B, that each plan to employ 100 recently graduated students. Assume that in each firm there are three types of jobs that are identical in all respects but yield a different monthly net income. The first job yields €2500, the second €1500 and the third €500. The firms differ however with respect to the numbers of positions they have available for each of the three jobs.

Evidently, due to the different distribution of incomes, the global welfare of the 100 employees can be different in the firms A and B. We are interested in your personal judgement of these welfare differences.

Indicate in each of the eight questions below which firm leads to the highest welfare according to you by marking A or B. So, the marked letter corresponds to the firm that you prefer from a welfare perspective. If you consider both firms to be equally good, then mark both letters. Of course each question needs to be treated separately and a different answer can be given in each case.

	A:	B:
Question 1	100 earn €1500 each	20 earn €2500 each 75 earn €1500 each 5 earn €500 each
Question 2	100 earn €1500 each	80 earn €2500 each 20 earn €500 each
Question 3	25 earn €1500 each 75 earn €500 each	20 earn €2500 each 80 earn €500 each
Question 4	75 earn €2500 each 25 earn €1500 each	95 earn €2500 each 5 earn €500 each
Question 5	20 earn €2500 each 80 earn €1500 each	40 earn €2500 each 55 earn €1500 each 5 earn €500 each
Question 6	80 earn €1500 each 20 earn €500 each	20 earn €2500 each 55 earn €1500 each 25 earn €500 each
Question 7	20 earn €2500 each 80 earn €1500 each	84 earn €2500 each 16 earn €500 each

Question 8	80 earn €1500 each	64 earn €2500 each
	20 earn €500 each	36 earn €500 each

VOI Version

Try to put yourself in the position of a recently graduated student who has to choose, just as 99 other recently graduated students, between accepting a job in firm A or in firm B. Assume that in each firm there are three types of jobs that are identical in all respects but yield a different monthly net income. The first job yields €2500, the second €1500 and the third €500. The firms differ however with respect to the numbers of positions they have available for each of the three jobs.

You and the 99 other recently graduated students either all end up in firm A or all in firm B. Each of the 100 of you has an equal probability of ending up in each of the 100 positions. So, it is unknown beforehand which job you will get.

Indicate in each of the eight questions below which firm you would prefer by marking A or B. So, the marked letter corresponds to the firm that would be preferred by you in this situation. If you consider both firms to be equally good, then mark both letters. Of course each question needs to be treated separately and a different answer can be given in each case.

Note: The formulation of the questions is identical to that of the ISO version. The questions are therefore omitted.

PIR Version

Try to put yourself in the position of a recently graduated student who has to choose between accepting a job in firm A or in firm B. Assume that in each firm there are three types of jobs that are identical in all respects but yield a different monthly net income. The first job yields €2500, the second €1500 and the third €500. The firms differ however with respect to the numbers of positions they have available for each of the three jobs. Beforehand it is not known with certainty which of the three possible jobs you will eventually get. Your chances are different in both firms.

Indicate in each of the eight questions below which firm you would prefer by marking A or B. So, the marked letter corresponds to the firm that would be preferred by you in this situation. If you consider both firms to be equally good, then mark both letters. Of course each question needs to be treated separately and a different answer can be given in each case.

	A:	B:
Question 1	100% chance to earn €1500	20% chance to earn €2500 75% chance to earn €1500 5% chance to earn €500
Question 2	100% chance to earn €1500	80% chance to earn €2500 20% chance to earn €500
Question 3	25% chance to earn €1500 75% chance to earn €500	20% chance to earn €2500 80% chance to earn €500
Question 4	75% chance to earn €2500 25% chance to earn €1500	95% chance to earn €2500 5% chance to earn €500
Question 5	20% chance to earn €2500 80% chance to earn €1500	40% chance to earn €2500 55% chance to earn €1500 5% chance to earn €500
Question 6	80% chance to earn €1500 20% chance to earn €500	20% chance to earn €2500 55% chance to earn €1500 25% chance to earn €500
Question 7	20% chance to earn €2500 80% chance to earn €1500	84% change to earn €2500 16% chance to earn €500
Question 8	80% chance to earn €1500 20% chance to earn €500	64% chance to earn €2500 36% chance to earn €500

Connection between Chapters 1 and 2: One of the more striking results of Chapter 1 was that the questionnaire responses reveal a positive attitude to mixtures. This finding is inconsistent with the conventional social welfare functions, which either imply a neutral attitude to mixtures (in the EU case), or aversion to mixtures (in the Yaari and RDEU cases). In Chapter 2 it is shown, moreover, that all common inequality measures tend to indicate a higher level of inequality for mixtures—for the relation between the mixture properties of the EU, Yaari and RDEU social welfare functions and the mixture properties of the inequality measures based on them, see the discussion of Lemma 2.2 and Proposition 2.2 in Section 2.4. Section 2.5 mentions a study by Amiel and Cowell (1994b) which shows that the opposite view, i.e., that mixtures tend to be less unequal, is quite popular among questionnaire respondents. This result is consistent with the findings concerning mixture attitudes of Chapter 1: perhaps respondents prefer mixtures because they consider them to be less unequal. It is advisable to remain cautious in drawing this conclusion, though, since the alternative income distributions in the questionnaire of Chapter 1 do not only differ in terms of inequality, but also in terms of mean income.

Chapter 2

Income Inequality, Quasi-Concavity, and Gradual Population Shifts

2.1 Introduction

In the analysis of income inequality, it is often useful to view the income distribution of interest as being composed of several constituent income distributions, e.g., the income distributions corresponding to different regions, sectors, or genders. The question of how inequality in the overall income distribution is affected if the constituent income distributions change, has received considerable attention in the form of decomposability analysis.[1] By contrast, the complementary question of how overall inequality changes if the population shares corresponding to the constituent income distributions change, has not been studied much. Nevertheless, the latter question is interesting both from the empirical and the theoretical perspective.

There are several empirical phenomena that involve a shift of the population from one constituent income distribution to another. Take as an example the phenomenon of demographic ageing. In this case, the overall income distribution changes over time because population gradually shifts from the income distribution of working consumers to the income distribution of retired consumers. Another, particularly straightforward, example is that of a country with two regions that have different population growth rates: here also, population shifts from the income distribution corresponding to the region with the lower growth rate to that corresponding to the region with the higher growth rate. As a final example, consider the development process studied by Kuznets (1955), which involves a gradual population shift from the income distribution of the agricultural sector to that of the industrial sector.

[1] See, e.g., the overview of the literature on inequality measurement by Cowell (2000).

Besides being of empirical interest, the gradual population shift process is relevant theoretically. In order to see this, assume that the overall income distribution is constituted of two perfectly equal income distributions: one in which everyone has income 10 and another in which everyone has income 50. Now, suppose that we start off with the entire population in the former income distribution, and that over time population gradually shifts to the latter. The income distribution will take, among others, the following three forms at various stages of this simple process:

$$A = \begin{cases} 90\% \text{ has } 10 \\ 10\% \text{ has } 50 \end{cases}, \quad B = \begin{cases} 50\% \text{ has } 10 \\ 50\% \text{ has } 50 \end{cases}, \quad C = \begin{cases} 10\% \text{ has } 10 \\ 90\% \text{ has } 50 \end{cases}.$$

Thinking about how inequality evolves as the income distribution changes from A to B and from B to C obviously means thinking about how inequality judgements are influenced by the relative population sizes of the 'rich' and 'poor.' For this reason, this simple case of the gradual population shift process has been considered of importance for the theoretical question of how inequality comparisons ought to be made in the first place. It has been studied in this way by Fields (1987, 1993), among others.

The key to tackling the question of how inequality evolves during a gradual population shift lies in the behaviour of inequality measures with respect to mixing income distributions. Let us first explain what we mean with mixing income distributions. Assuming that income distributions are defined in terms of relative frequencies, any income distribution can be defined as a mixture, i.e., a convex combination, of its constituent income distributions. As an illustration, consider a country with two regions: 'region P' and 'region Q,' representing population shares of α and $1-\alpha$, respectively. Indeed, if p_x and q_x are the proportions of the population with income x in regions P and Q, respectively, then the proportion of the population with income x in the country is equal to $\alpha p_x + (1-\alpha)q_x$. Now, during a gradual population shift process, the income distribution at any stage is a mixture of the income distribution at any earlier stage and the income distribution at any later stage—as an illustration, note that income distribution B, in the example of the simple case of the process above, is indeed a fifty-fifty mixture of income distributions A and C. In order to describe the evolution of inequality during a gradual population shift process, the important question is whether income inequality in a mixture is greater than, smaller than, or equal to, income inequality in each of its constituent income distributions. Moreover, can a general answer even be given to this question, or does the answer depend on the specifics of the constituent income distributions and on the particular inequality measure that is used?

In this chapter, we show that a general answer can indeed be given to the question of how inequality measures behave with respect to mixing income distributions. It is demonstrated that virtually all inequality measures that are studied in the literature on inequality measurement—viz., the class of decomposable inequality measures and the class of normative inequality measures based on the general social welfare function of the rank-dependent expected utility form—satisfy quasi-concavity properties, which entail a positive response to mixing income distributions. To be concrete, assuming that inequality is equal in the two constituent income distributions that are mixed, the following is shown to be true: inequality in the mixture is at least as great as that in each of its constituent income distributions, and if the mean incomes of the constituent income distributions are not equal, then inequality in the mixture is *strictly* greater than that in each of its constituent income distributions. We emphasize that while all well known inequality measures satisfy these quasi-concavity properties, the properties are not implied by the fundamental Lorenz type axioms on their own. With respect to the problem of how inequality evolves during a gradual population shift process, the quasi-concavity properties are shown to reduce the possible patterns describing the evolution of inequality to only three: (i) an inverted-U pattern in which inequality increases in the first stages of the process and decreases afterwards, (ii) an increasing pattern in which inequality increases during the entire process, and (iii) a decreasing pattern in which inequality decreases during the entire process. This result generalizes some results of Kakwani (1988) and Anand and Kanbur (1993) in this context.

The chapter is structured as follows. Section 2.2 deals with notation and basic concepts. In Section 2.3, we show axiomatically that the quasi-concavity properties are satisfied by all inequality quasi-orderings satisfying the transfer principle, a weak invariance axiom and decomposability. Instead of focusing exclusively on relative inequality concepts, as is common in the literature, we consider the weak invariance axiom of Bossert and Pfingsten (1990) which allows for relative and absolute inequality concepts as well as intermediate ones. While the result of Section 2.3 applies to, among others, the inequality measures based on a social welfare function of the expected utility form, it does not apply to its rank-based alternatives, the generalized Gini indices, as these are not decomposable. Therefore, we consider in Section 2.4 the class of inequality measures (absolute, relative as well as intermediate cases) based on a social welfare function of the rank-dependent expected utility form, which generalizes both the class of expected utility inequality measures and the class of generalized Gini indices. Benefiting from functional representability of the given inequality orderings, it is shown that the quasi-concavity properties are also satisfied by all members of this general class of normative inequality measures. In Section 2.5 we spell out the implications of the results of Sections 2.3 and 2.4 for the question of how inequality evolves during a gradual population shift process. Section 2.6 concludes. All the proofs are contained in Appendix 2.A.

2.2 Preliminaries

2.2.1 Basic Notation and Axioms

An income distribution is an ordered pair (p,x) where $p = (p_1, p_2, \ldots, p_n)$ is a vector (of finite length) of relative frequencies with $p_i > 0$ for all $i = 1,2,\ldots,n$ and $p_1 + p_2 + \cdots + p_n = 1$, and where $x = (x_1, x_2, \ldots, x_n)$ is the corresponding vector of income levels that are elements of $\mathbb{R}_{++}$. So, for all $i = 1,2,\ldots,n$, the proportion of the population with income x_i is equal to p_i, which we sometimes write as p_{x_i}. We assume that the components of x are ordered such that $0 < x_1 < x_2 < \cdots < x_n$. The set $\mathscr{P}$ collects all income distributions. For any $(p,x) \in \mathscr{P}$, the set $\{x_1, x_2, \ldots, x_n\}$ is referred to as the support of the income distribution (p,x), and the mean income $p_1x_1 + p_2x_2 + \cdots + p_nx_n$ is denoted by $\mu(p,x)$. Any income distribution $(p,x) \in \mathscr{P}$ is said to be perfectly equal if there exists an income level e such that $p_e = 1$ and is said to be unequal otherwise. Inequality comparisons of income distributions are captured by a binary relation $\preceq$ ('is at most as unequal as') on $\mathscr{P}$. The relation's asymmetric and symmetric factors are denoted by $\prec$ ('is less unequal than') and $\sim$ ('is equally unequal as'), respectively. We assume that the relation $\preceq$ is a quasi-ordering, i.e., is reflexive and transitive. A quasi-ordering that is complete is an ordering. An inequality measure is defined as a function $I : \mathscr{P} \to \mathbb{R}$ that represents some inequality ordering.

Throughout this chapter, we are often required to view the overall income distribution as a mixture, i.e., a convex combination, of its constituent income distributions. Suppose $(r,z) \in \mathscr{P}$ is the overall income distribution, constituted of the income distributions $(p,x) \in \mathscr{P}$ and $(q,y) \in \mathscr{P}$ with population shares $\alpha \in (0,1)$ and $(1-\alpha) \in (0,1)$, respectively. Then the support of (r,z) is the union of the supports of (p,x) and (q,y) and, for all elements z_i in the support of (r,z), we have

$$r_i = \begin{cases} \alpha p_{z_i} & \text{if } z_i \text{ occurs in } x \text{ and not in } y; \\ (1-\alpha) q_{z_i} & \text{if } z_i \text{ occurs in } y \text{ and not in } x; \\ \alpha p_{z_i} + (1-\alpha) q_{z_i} & \text{if } z_i \text{ occurs in } x \text{ and in } y. \end{cases}$$

This mixture (r,z) of (p,x) and (q,y) is denoted by $\alpha(p,x) + (1-\alpha)(q,y)$.

We now consider three basic axioms. To define the well known transfer principle, we require the concept of the mean preserving spread. Consider any $(p,x) \in \mathscr{P}$ and let $0 < z_1 < z_2 \leq z_3 < z_4$ be any four income levels with z_2 and z_3 belonging to the support of (p,x). The income distribution (q,y) is obtained from (p,x) by a mean preserving spread if and only if there exists a scalar $\delta > 0$ such that

$$q_{z_1} = p_{z_1} + \delta > 0,\ \ q_{z_2} = p_{z_2} - \delta \geq 0,\ \ q_{z_3} = p_{z_3} - \delta \geq 0,\ \ q_{z_4} = p_{z_4} + \delta > 0,$$

(if z_1 (respectively, z_4) does not belong to the support of (p,x), we set p_{z_1} (respectively, p_{z_4}) equal to 0), $q_{x_i} = p_{x_i}$ for all other elements x_i in the support of (p,x), and $\mu(p,x) = \mu(q,y)$. Informally, whenever (q,y) is obtained from (p,x) by a mean preserving spread, this means that (q,y) is obtained from (p,x) by a series of poorer-to-richer transfers. The transfer principle demands that such transfers increase inequality.

Axiom 2.1 (TP). For any $(p,x) \in \mathscr{P}$, if (q,y) is obtained from (p,x) by a mean preserving spread, then we have $(p,x) \prec (q,y)$.

The second axiom we consider is an invariance condition, i.e., it defines a transformation, by which all incomes are changed in the same direction, that leaves inequality invariant. For any transformation $f : \mathbb{R}_{++} \to \mathbb{R}$ and any $(p,x) \in \mathscr{P}$, we denote the transformed vector of income levels $(f(x_1), f(x_2), \ldots, f(x_n))$ by $f(x)$. So, for instance, $(p, \tau x)$ denotes the income distribution obtained from (p,x) by multiplying each individual's income by τ. The β-invariance axiom is a general, linear, invariance condition first proposed by Bossert and Pfingsten (1990).

Axiom 2.2 (βINV). For the scalar $\beta \in [0,1]$, the following is true. For any $(p,x) \in \mathscr{P}$ and any scalar λ such that $(p, x + \lambda(\beta x + 1 - \beta)) \in \mathscr{P}$, we have $(p,x) \sim (p, x + \lambda(\beta x + 1 - \beta))$.

The axiom βINV encompasses both the popular relative ($\beta = 1$) case, which says that multiplication of all incomes by the same scalar leaves inequality invariant, and the absolute ($\beta = 0$) case, which says that addition to all incomes of the same scalar leaves inequality invariant. Inequality relations satisfying βINV for $\beta \in (0,1)$ are referred to as intermediate inequality relations. In line with the literature, we consider TP and βINV to be the two fundamental axioms. Accordingly, both axioms are satisfied by all concepts of inequality comparisons considered in this chapter.

Decomposability, finally, is a popular axiom, but is usually interpreted as being less compelling than TP and βINV.[2] Roughly speaking, decomposability says that any transformation of the overall income distribution that changes only one of its constituent income distributions and leaves population shares and mean income unaffected, should affect inequality in the overall income distribution in the same direction as it affects inequality in the given constituent income distribution.

Axiom 2.3 (DEC). For any $(p,x), (q,y), (r,z) \in \mathscr{P}$ with $\mu(p,x) = \mu(q,y)$, we have

$$(p,x) \preceq (q,y) \Leftrightarrow \alpha(p,x) + (1-\alpha)(r,z) \preceq \alpha(q,y) + (1-\alpha)(r,z)$$

for any $\alpha \in (0,1)$.

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[2]For a critique of decomposability, see Sen and Foster (1997, pp. 149-163). The axiom they refer to as subgroup consistency is similar to our definition of the concept.

2.2.2 Properties Concerning Mixtures

The main focus of this chapter are properties that describe how inequality relations behave with respect to mixing income distributions, i.e., how a mixture compares in terms of inequality to its constituent income distributions. We emphasize that we are interested in examining which of these properties are satisfied by the inequality relations proposed in the literature and in what the implications of the properties are. By consequence, we do not want to impose them as a priori desirable properties on inequality relations—accordingly, we refer to them as 'properties' and not as 'axioms' because the latter term would suggest otherwise.

Quasi-concavity and strict quasi-concavity describe a positive inequality attitude to mixing income distributions. Loosely speaking, the properties say that mixing tends to increase inequality. To give an example, (strict) quasi-concavity implies that a mixture of two equally unequal income distributions is at least as unequal as (is more unequal than) the given two income distributions.

Property 2.1 (QC). For any $(p,x),(q,y) \in \mathscr{P}$, we have

$$(p,x) \preceq (q,y) \Rightarrow (p,x) \preceq \alpha(p,x) + (1-\alpha)(q,y) \quad \text{for any } \alpha \in (0,1). \tag{2.1}$$

Property 2.2 (SQC). For any $(p,x),(q,y) \in \mathscr{P}$ with $(p,x) \neq (q,y)$, we have

$$(p,x) \preceq (q,y) \Rightarrow (p,x) \prec \alpha(p,x) + (1-\alpha)(q,y) \quad \text{for any } \alpha \in (0,1). \tag{2.2}$$

More relevant than SQC, however, will turn out to be the following conditional strict quasi-concavity property, which says that (2.2) has to be satisfied only if the means of the two constituent income distributions are not equal.

Property 2.3 (CSQC). For any $(p,x),(q,y) \in \mathscr{P}$ with $\mu(p,x) \neq \mu(q,y)$, (2.2) is true.

Note that SQC implies both QC and CSQC, while the latter two properties are independent.

Quasi-convexity and strict quasi-convexity describe negative inequality attitudes to mixing income distributions and, thus, are the natural counterparts of QC and SQC. Loosely speaking, these properties say that mixing tends to decrease inequality. For instance, (strict) quasi-convexity implies that a mixture of two equally unequal income distributions is at most as unequal as (is less unequal than) each of the given two income distributions.

Property 2.4 (QV). For any $(p,x),(q,y) \in \mathscr{P}$ with neither (p,x) nor (q,y) perfectly equal, we have

$$(p,x) \preceq (q,y) \Rightarrow \alpha(p,x) + (1-\alpha)(q,y) \preceq (q,y) \quad \text{for any } \alpha \in (0,1). \tag{2.3}$$

Property 2.5 (SQV). For any $(p,x),(q,y) \in \mathscr{P}$ with $(p,x) \neq (q,y)$ and neither (p,x) nor (q,y) perfectly equal, we have

$$(p,x) \preceq (q,y) \Rightarrow \alpha(p,x)+(1-\alpha)(q,y) \prec (q,y) \quad \text{for any } \alpha \in (0,1). \tag{2.4}$$

The reason why the perfectly equal income distributions are excluded from the set of income distributions over which (2.3) and (2.4) are required to hold, is that the properties QV and SQV would otherwise be incompatible with the commonsense requirement that any unequal income distribution is strictly more unequal than any perfectly equal one.[3]

Using the minimal framework of inequality quasi-orderings, we demonstrate in Section 2.3 that the three axioms TP, βINV and DEC are sufficient for the properties QC and CSQC to be satisfied. In Section 2.4, similar results are shown to be true for the members of an important class of inequality orderings consistent with TP and βINV but not (necessarily) with DEC. We remark that although QC and CSQC turn out to be satisfied very generally, this does not necessarily imply that these are *desirable* properties for inequality relations—indeed, in Section 2.5 we discuss a critique of some of the implications of these properties that has been put forward in the literature.

2.3 Inequality Quasi-Orderings

In this section, we examine the implications of the three basic axioms, TP, βINV and DEC, for the behaviour of inequality quasi-orderings with respect to mixing income distributions.

First, the two fundamental axioms TP and βINV are sufficient to rule out the two quasi-convexity properties, but are not sufficient to imply any of the three quasi-concavity properties. To see this, consider the following lemma.

Lemma 2.1. *Let $\preceq$ be any inequality quasi-ordering satisfying TP and βINV. For any $(p,x) \in \mathscr{P}$ and any scalar λ such that $(p,x+\lambda(\beta x+1-\beta)) \in \mathscr{P}$ and $(p,x) \neq (p,x+\lambda(\beta x+1-\beta))$, we have*

$$(p,x) \prec \alpha(p,x)+(1-\alpha)(p,x+\lambda(\beta x+1-\beta)) \quad \textit{for any } \alpha \in (0,1).$$

Note that βINV requires, moreover, that $(p,x) \sim (p,x+\lambda(\beta x+1-\beta))$ in Lemma 2.1. By letting $(q,y) = (p,x+\lambda(\beta x+1-\beta))$ in conditions (2.3) and (2.4), it thus follows from the lemma that TP and βINV imply violations of these

[3] This can be seen by letting (p,x) and (q,y) in (2.3) or (2.4) both be perfectly equal income distributions (with $(p,x) \neq (q,y)$). Note, furthermore, that the 'commonsense requirement' is implied by TP and βINV jointly.

conditions and, hence, of QV and SQV. In a similar way, it is established that TP and βINV imply (2.1) and (2.2) in cases where $(q,y) = (p, x+\lambda(\beta x + 1 - \beta))$. Although the latter reveals that TP and βINV imply instances of QC, SQC and CSQC, the two axioms are not sufficient for any of the three quasi-concavity properties to be satisfied in general. The following example provides an illustration of this point.

Example 2.1. Consider the inequality measure $I : \mathscr{P} \to \mathbb{R} : (p,x) \mapsto \kappa I^{10}_{GE}(p,x) + I^{-9}_{GE}(p,x)$ where $\kappa > 0$ is a scalar and where I^{θ}_{GE} is the generalized entropy inequality measure, given by

$$I^{\theta}_{GE} : \mathscr{P} \to \mathbb{R} : (p,x) \mapsto \frac{1}{\theta^2 - \theta} \sum_{i=1}^{n} p_i \left[\left(\frac{x_i}{\mu(p,x)} \right)^{\theta} - 1 \right],$$

with θ a scalar. Since the generalized entropy inequality measure satisfies TP and βINV (for $\beta = 1$), I obviously satisfies these axioms as well. By contrast, I^{θ}_{GE} satisfies DEC, while I does not. Now consider the following three income distributions: $(p,x) = ((0.4, 0.6), (10, 50))$, $(q,y) = ((0.9, 0.1), (10, 50))$ and $(r,z) = ((0.65, 0.35), (10, 50))$. Let, moreover, $\kappa = \frac{I^{-9}_{GE}(p,x) - I^{-9}_{GE}(q,y)}{I^{10}_{GE}(p,y) - I^{10}_{GE}(p,x)} \approx 0.719$. We have

$$23.370 \approx I(r,z) = I(0.5(p,x) + 0.5(q,y)) < I(p,x) = I(q,y) \approx 270.061,$$

which implies that I violates QC, SQC and CSQC. Furthermore, it can be shown that (2.4) is true for the chosen (p,x) and (q,y).

Second, on its own DEC implies a bias neither to quasi-concavity, nor to quasi-convexity: DEC implies instances of the weak versions of both quasi-concavity and quasi-convexity, and is (typically) incompatible with the strict versions of both. In order to see this, consider any income distributions $(p,x), (q,y) \in \mathscr{P}$ with $\mu(p,x) = \mu(q,y)$ and $(p,x) \preceq (q,y)$. Then, DEC implies

$$(p,x) \preceq \alpha(p,x) + (1-\alpha)(q,y) \preceq (q,y) \quad \text{for any } \alpha \in (0,1).^4 \qquad (2.5)$$

In other words, DEC implies instances of QC or QV in those cases in which the income distributions in the mixture have equal means. If $(p,x) \sim (q,y)$ (still with $\mu(p,x) = \mu(q,y)$), then DEC implies that the inequality relations in (2.5) hold with equivalence ($\sim$), thus giving rise to violations of both (2.2) and (2.4). Hence, given the weak assumption—which would follow, e.g., from completeness and continuity—that at least one pair of income distributions $(p,x), (q,y) \in \mathscr{P}$ such

[4] This is obtained by letting (r,z) in the definition of DEC equal in turn (p,x) and (q,y).

that $\mu(p,x) = \mu(q,y)$ and $(p,x) \sim (q,y)$ exists, DEC is incompatible with both SQC and SQV.[5]

To summarize, we have seen that TP and βINV are not sufficient for QC, SQC or CSQC to be satisfied, and also that DEC typically rules out SQC. The following result says that any inequality quasi-ordering satisfying TP, βINV and DEC must satisfy QC as well as CSQC.

Proposition 2.1. *Any inequality quasi-ordering satisfying TP, βINV and DEC satisfies QC and CSQC.*

Proposition 2.1 has implications that are relevant in the context of the study of the evolution of inequality during a process in which population gradually shifts from one constituent income distribution to another. We postpone the discussion of these implications until Section 2.5, but consider here relevant results concerning this context by Kakwani (1988) and Anand and Kanbur (1993) that are generalized in Proposition 2.1. Anand and Kanbur present results that imply that the inequality orderings represented by the following relative inequality measures satisfy CSQC: the first and second Theil inequality measures, the coefficient of variation, the entire class of Atkinson inequality measures, and the Gini index in the case of non-overlapping income distributions.[6] The same has been shown by Kakwani for the entire class of generalized entropy inequality measures, thus generalizing the results pertaining to all measures considered by Anand and Kanbur except the Gini index.[7] Proposition 2.1 demonstrates that neither the demand that inequality be a relative concept, nor even completeness or continuity are essential in obtaining the result. Examples of absolute inequality measures covered by Proposition 2.1 are the variance and the entire class of Kolm inequality measures. Notable inequality measures that Proposition 2.1 does not deal with—because they do not satisfy DEC—are the Gini index in the general, possibly overlapping, case, as well as its rank-based generalizations. In the next section, we show that a similar result as Proposition 2.1 holds for a class of normative inequality measures that encompasses both the well known classes of decomposable normative

[5]In a sense, CSQC is as far as one can go in the direction of SQC while still satisfying DEC. To see this, consider any income distributions $(p,x),(q,y) \in \mathscr{P}$ with $\mu(p,x) = \mu(q,y)$ and $(p,x) \preceq (q,y)$, i.e., any income distributions for which SQC implies (2.2), while CSQC does not. Now, if $(p,x) \sim (q,y)$, then DEC is inconsistent with (2.2) which means that SQC goes too far, whereas if $(p,x) \prec (q,y)$, then DEC already implies (2.2) on its own.

[6]Moreover, the logarithmic variance, also considered by Anand and Kanbur, can be added to the list. Anand and Kanbur borrow the result concerning this inequality measure from Robinson (1976). We do not consider the logarithmic variance as it does not satisfy TP.

[7]Kakwani (1988, pp. 210-213) mistakenly believes to have proven the result only for the generalized entropy inequality measures for which $\theta \geq 1$ and $\theta = 0$. However, he proves the result also for the entire Atkinson class, which is ordinally equivalent to the generalized entropy class in the case where $\theta < 1$. Therefore, the ordinal nature of the property CSQC implies that the result applies to the entire generalized entropy class.

inequality measures (the Atkinson and Kolm inequality measures) and the generalized Gini indices.

2.4 Normative Inequality Orderings

Normative inequality measures are based on some conception of social ethics, captured by a social welfare function $W : \mathscr{P} \to \mathbb{R}$.[8] Define the equally distributed equivalent income for any income distribution $(x, p) \in \mathscr{P}$, denoted by $\xi(p,x)$, as the per capita income which, if distributed equally, yields the same level of social welfare as (p,x). That is, $\xi(p,x)$ for any income distribution $(p,x) \in \mathscr{P}$ is defined as $\xi(p,x) = e$, where $e \in \mathbb{R}_{++}$ is such that there is a $(q,y) \in \mathscr{P}$ for which $q_e = 1$ and $W(p,x) = W(q,y)$. It is common to define relative normative inequality measures using

$$I : \mathscr{P} \to \mathbb{R} : (p,x) \mapsto 1 - \frac{\xi(p,x)}{\mu(p,x)}, \tag{2.6}$$

and absolute normative inequality measures using

$$I : \mathscr{P} \to \mathbb{R} : (p,x) \mapsto \mu(p,x) - \xi(p,x). \tag{2.7}$$

The literature on inequality measurement has focused mainly on two particular social welfare functions: the social welfare function of the expected utility (EU) form on which among others the Atkinson and Kolm inequality measures are based, and the social welfare function of the Yaari (1987) form on which the generalized Gini indices are based. Both are special cases of the social welfare function of the rank-dependent expected utility (RDEU) form,

$$W : \mathscr{P} \to \mathbb{R} : (p,x) \mapsto \sum_{i=1}^{n} \pi_i(p)u(x_i), \tag{2.8}$$

with, for all $i = 1,2,\ldots,n-1$, $\pi_i(p) = \phi(p_i + p_{i+1} + \cdots + p_n) - \phi(p_{i+1} + p_{i+2} + \cdots + p_n)$, and $\pi_n(p) = \phi(p_n)$. Furthermore, $\phi : [0,1] \to [0,1]$ is a continuous and strictly increasing function with $\phi(0) = 0$ and $\phi(1) = 1$, and $u : \mathbb{R} \to \mathbb{R}$ is a continuous and strictly increasing function. In the case where ϕ coincides with the identity function, we have $\pi_i(p) = p_i$ for all $i = 1,2,\ldots,n$, and (2.8) reduces to the EU social welfare function. In the case where u coincides with the identity function, (2.8) reduces to the Yaari social welfare function.

Relative RDEU inequality measures, which we denote by $I_{RDEU}^{1,\varepsilon,\phi}$, are given by (2.6) with W as in (2.8), with

$$u : \mathbb{R} \to \mathbb{R} : t \mapsto \frac{1}{1-\varepsilon} t^{1-\varepsilon}, \quad \varepsilon \geq 0, \tag{2.9}$$

[8] For an overview of the normative approach to inequality measurement, see Gajdos (2001).

and with ϕ a convex function. In order for TP to be satisfied, we assume, moreover, that either u is strictly concave (i.e., $\varepsilon > 0$), ϕ is strictly convex, or both.[9] Following Bossert and Pfingsten (1990), we obtain the relative, intermediate and part of the absolute RDEU inequality measures as $I^{\beta,\varepsilon,\phi}_{RDEU}(p,x) = \frac{1}{\beta} I^{1,\varepsilon,\phi}_{RDEU}(p, x + \frac{1-\beta}{\beta})$ for all $(p,x) \in \mathscr{P}$. Hence, we have

$$I^{\beta,\varepsilon,\phi}_{RDEU} : \mathscr{P} \to \mathbb{R} : (p,x) \mapsto \frac{1}{\beta} \left[1 - \frac{\left(\sum_{i=1}^{n} \pi_i(p) \left(x_i + \frac{1-\beta}{\beta} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}}{\mu(p,x) + \frac{1-\beta}{\beta}} \right], \quad (2.10)$$

where $0 < \beta \leq 1$ if $\varepsilon > 0$ and $0 \leq \beta \leq 1$ if $\varepsilon = 0$, π is defined as above, and ϕ is strictly convex whenever $\varepsilon = 0$ and convex otherwise. The absolute RDEU inequality measures not given by $I^{\beta,\varepsilon,\phi}_{RDEU}$ are those for which u does not coincide with the identity function. These are obtained as (2.7) with W as in (2.8), and with

$$u : \mathbb{R} \to \mathbb{R} : t \mapsto -\exp(-\gamma t), \quad \gamma > 0, \quad (2.11)$$

and are denoted by $I^{0,\gamma,\phi}_{RDEU}$. Hence, we have

$$I^{0,\gamma,\phi}_{RDEU} : \mathscr{P} \to \mathbb{R} : (p,x) \mapsto \mu(p,x) + \frac{1}{\gamma} \ln\Big(\sum_{i=1}^{n} \pi_i(p) \exp(-\gamma x_i) \Big), \quad (2.12)$$

where $\gamma > 0$, π is defined as above, and ϕ is convex.

Several well known inequality measures belong to the class of RDEU inequality measures. For ϕ coinciding with the identity function, we obtain the class of EU inequality measures, with as special cases the Atkinson class (by letting, furthermore, $\beta = 1$ in $I^{\beta,\varepsilon,\phi}_{RDEU}$) and the Kolm class (given by $I^{0,\gamma,\phi}_{RDEU}$). For u coinciding with the identity function (i.e., $\varepsilon = 0$), we obtain the Yaari, or generalized Gini, indices.[10] A well known subclass of the generalized Gini indices is that of the S-Gini indices, for which $\phi : t \mapsto t^\rho$, with $\rho > 1$, which has as a notable special case the Gini index ($\rho = 2$). RDEU inequality measures for which neither ϕ nor u coincide with the identity function, and which, consequently, belong to neither the EU class nor the Yaari class, have been studied by Ebert (1988) and Chateauneuf et al. (2002), among others.

All RDEU inequality measures are consistent with TP and βINV. However, while all RDEU inequality measures also incorporate a weak decomposability

[9] See Chew et al. (1987).

[10] The absolute subclass of the generalized Gini indices is obtained by, furthermore, taking the limit $\beta \to 0$ in $I^{\beta,\varepsilon,\phi}_{RDEU}$.

idea as shown by Ebert (1988), only the inequality orderings corresponding to members of the EU subclass satisfy DEC. By consequence, the EU inequality measures are the only members of the RDEU class that are covered by Proposition 2.1. To prove that the inequality orderings representable by any of the remaining RDEU inequality measures also all satisfy the quasi-concavity properties of Proposition 2.1, we require a result which relates the convexity of the weighting function ϕ to the convexity of the RDEU social welfare function. A social welfare function W is said to be convex if and only if, for any $(p,x),(q,y) \in \mathscr{P}$, we have

$$W(\alpha(p,x)+(1-\alpha)(q,y)) \leq \alpha W(p,x)+(1-\alpha)W(q,y) \quad \text{for any } \alpha \in (0,1). \tag{2.13}$$

The following lemma summarizes the required relationship.

Lemma 2.2. *Let W be any social welfare function of the RDEU form, given by* (2.8)*.*

(i) *If ϕ is linear, then W is linear, i.e.,* (2.13) *holds with equality for any (p,x), $(q,y) \in \mathscr{P}$.*

(ii) *If ϕ is convex, then W is convex, i.e.,* (2.13) *holds for any $(p,x),(q,y) \in \mathscr{P}$.*

(iii) *If ϕ is strictly convex, then W is strictly convex, i.e.,* (2.13) *holds with strict inequality for any $(p,x),(q,y) \in \mathscr{P}$ with $(p,x) \neq (q,y)$.*

Note that we have $\alpha W(p,x)+(1-\alpha)W(q,y) \leq \min\{W(p,x),W(q,y)\}$ for any $(p,x),(q,y) \in \mathscr{P}$ and any $\alpha \in (0,1)$. Using this observation together with (2.13), we see that Lemma 2.2 has implications for the behaviour of RDEU social welfare functions with respect to mixing income distributions: a linear weighting function ϕ, as that of the EU social welfare function, corresponds to a neutral social welfare attitude to mixing, whereas a (strictly) convex weighting function ϕ, as that of the RDEU or Yaari social welfare functions, corresponds to a (strictly) negative attitude to mixing. Now, note that Proposition 2.1 can be interpreted as revealing that inequality orderings based on social welfare function with a neutral attitude to mixing (the EU social welfare function) have a positive attitude to mixing as expressed by the properties QC and CSQC. Since social welfare and inequality are negatively related concepts (see expressions (2.6) and (2.7)), we would expect that if a neutral social welfare attitude to mixing translates into a positive inequality attitude to mixing, then a negative social welfare attitude to mixing should definitely translate into a positive inequality attitude to mixing. In other words, if inequality orderings based on a EU social welfare function satisfy the properties QC and CSQC, then this should be true a fortiori for inequality orderings based on an RDEU social welfare function. Proposition 2.2 confirms this intuition.

Proposition 2.2. *Any RDEU inequality ordering $\preceq$, i.e., any inequality ordering representable by* (2.10) *or* (2.12)*, satisfies QC and CSQC. If the weighting function ϕ corresponding to $\preceq$ is in addition strictly convex, then $\preceq$ also satisfies SQC.*

Note that, since all generalized Gini indices have strictly convex weighting functions ϕ, the inequality orderings represented by these inequality measures all satisfy the strongest quasi-concavity property SQC.

2.5 Inequality and Gradual Population Shifts

We now examine the implications of the properties QC and CSQC for the question of how inequality evolves during an adjustment process in which the population gradually shifts from one constituent income distribution to another over time. As discussed in Section 2.1, several empirical phenomena involve such an adjustment process. Suppose that the constituent income distributions are $(p,x) \in \mathscr{P}$ and $(q,y) \in \mathscr{P}$, and that population shifts from (q,y) to (p,x). Then, the overall income distribution is $\alpha(p,x) + (1-\alpha)(q,y)$ and α gradually rises over some interval $(\underline{\alpha}, \overline{\alpha}) \subseteq (0,1)$. The question we are interested in is how inequality in the overall income distribution evolves as α rises over $(\underline{\alpha}, \overline{\alpha})$.

In the previous sections, we saw that all well known inequality concepts satisfy the properties QC and CSQC. As the following proposition shows, these properties reduce the number of allowed patterns, describing inequality evolution during the considered adjustment process, to only three: (i) an inverted-U pattern in which inequality increases in the early stages of the process and decreases afterwards, (ii) an increasing pattern in which inequality increases during the entire process, and (iii) a decreasing pattern in which inequality decreases during the entire process. The proposition focuses on CSQC, which has the stronger implications of the two properties (given that $\mu(p,x) \neq \mu(q,y)$), and, for convenience, restricts attention to inequality orderings.

Proposition 2.3. *Let $\preceq$ be any inequality ordering satisfying CSQC. Consider, moreover, any $(q,y),(p,x) \in \mathscr{P}$ with $\mu(p,x) \neq \mu(q,y)$. Only the following three patterns, describing the evolution of inequality in $\alpha(p,x) + (1-\alpha)(q,y)$ during a process in which α rises over the interval $(\underline{\alpha}, \overline{\alpha}) \subseteq (0,1)$, are possible.*

(i) *An inverted-U pattern, i.e., there exists an $\alpha^* \in (\underline{\alpha}, \overline{\alpha})$ such that, for any $\alpha, \alpha' \in (\underline{\alpha}, \alpha^*]$, if $\alpha > \alpha'$ then*

$$\alpha'(p,x) + (1-\alpha')(q,y) \prec \alpha(p,x) + (1-\alpha)(q,y),$$

and, for any $\alpha, \alpha' \in [\alpha^*, \overline{\alpha})$, if $\alpha > \alpha'$ then

$$\alpha(p,x) + (1-\alpha)(q,y) \prec \alpha'(p,x) + (1-\alpha')(q,y).$$

(ii) *A strictly increasing pattern, i.e., for any* $\alpha, \alpha' \in (\underline{\alpha}, \overline{\alpha})$, *if* $\alpha > \alpha'$ *then*

$$\alpha'(p,x) + (1-\alpha')(q,y) \prec \alpha(p,x) + (1-\alpha)(q,y).$$

(iii) *A strictly decreasing pattern, i.e., for any* $\alpha, \alpha' \in (\underline{\alpha}, \overline{\alpha})$, *if* $\alpha > \alpha'$ *then*

$$\alpha(p,x) + (1-\alpha)(q,y) \prec \alpha'(p,x) + (1-\alpha')(q,y).$$

A case of the gradual population shift process of theoretical interest is the simple one in which the two constituent income distributions (p,x) and (q,y) are both perfectly equal. If in (p,x) everyone has the income $\hat{x}$ and in (q,y) everyone has the income $\hat{y}$, then the overall income distribution can be written as $((\alpha, 1-\alpha), (\hat{x}, \hat{y}))$. We assume, furthermore, that $\hat{x} > \hat{y}$ and that α rises over $(0,1)$. During this simple process, the relative group sizes of the 'rich' and 'poor'—those with incomes $\hat{x}$ and $\hat{y}$, respectively—change continuously. For this reason, this case has been regarded by Fields (1987, 1993), among others, as interesting for the theoretical question of how to define the concept of inequality comparisons. Fields criticizes the popular inequality measures—by which he means those studied by Anand and Kanbur (1993) (see Section 2.3 of this chapter)—because they all imply an inverted-U pattern of inequality in the simple gradual population shift process, while there are other patterns that would be at least as plausible in his opinion. The inverted-U pattern implies, loosely speaking, that income distributions with equally sized poor and rich groups are more unequal than income distributions with a small number of poor and a large number of rich or with a large number of poor and a small number of rich. In his own work, Fields defends the opposite view, the U pattern (inequality rises in the early stages of the process and decreases afterwards), which implies that situations with few poor and many rich or few rich and many poor are considered particularly unequal.[11]

It follows from the results of this chapter that Fields' critique applies not only to the inequality measures dealt with by Anand and Kanbur, but to all inequality measures commonly considered in the literature. Propositions 2.1 and 2.2 imply

[11] Fields (1993) provides a justification for the U pattern on the basis of the notions 'elitism of the rich' and 'isolation of the poor.' Loosely speaking, elitism of the rich says that, for relatively low values of α, decreases in α lead to greater inequality because the 'rich' then attain a more elite position. Similarly, isolation of the poor says that, for relatively high values of α, increases in α cause inequality to increase because the 'poor' then become more isolated. The simple case of the gradual population shift process has also been considered by Temkin (1986) and by Amiel and Cowell (1994b). Using his own framework for inequality measurement, the philosopher Temkin gives justifications for the three patterns dealt with in Proposition 2.3 as well as for a pattern of constant inequality during the entire process. Amiel and Cowell provide questionnaire results showing that respondents support several patterns among which the U pattern proposed by Fields is quite popular. See also the discussion by Kolm (1998, pp. 36-38).

that the inequality orderings corresponding to all these inequality measures satisfy CSQC, and Proposition 2.3 implies that all continuous inequality orderings satisfying CSQC imply an inverted-U pattern in the simple gradual population shift process. The latter follows from the fact that if the two constituent income distributions are equally unequal, as is the case in the simple gradual population shift process,[12] then the patterns (ii) and (iii) in Proposition 2.3 are only possible for noncontinuous inequality orderings since these patterns involve a discontinuity at $\alpha = 0$ or at $\alpha = 1$. Note, finally, that, as Example 2.1 shows, the fundamental axioms TP and βINV do not exclude the occurrence of a U pattern over part of the gradual shift process for some constituent income distributions.

2.6 Conclusion

The literature on inequality measurement has focused exclusively on the specific strategy of supplementing the fundamental axioms, TP and βINV, with decomposability ideas, i.e., ideas concerning how changes in the inequality of constituent income distributions have to relate to changes in overall inequality—directly, in the form of the DEC axiom, or, indirectly, by basing inequality measures on a(n) (RDEU) social welfare function that incorporates a weak decomposability condition. It was demonstrated in this chapter that all inequality measures considered in the literature satisfy the quasi-concavity properties QC and CSQC. Moreover, it was shown that the latter property allows only three patterns describing how inequality evolves during a process in which population gradually shifts from one constituent income distribution to another.

On the one hand, the latter result reveals an attractive feature of CSQC: the property facilitates the study of empirical phenomena in which gradual population shifts occur. On the other hand, it may be argued that the three patterns allowed by CSQC are not the only plausible ones. If it is concluded that the other—non CSQC consistent—inequality views should also be expressible within a theory of inequality measurement, then our results show that one should focus on supplementing the fundamental axioms, TP and βINV, in alternative ways, rather than with decomposability ideas.

[12] At least, this would follow from βINV or from the commonsense assumption that all perfectly equal income distributions are equally equal.

Appendix 2.A: Proofs

In the proofs, we usually abbreviate, for any $(p,x),(q,y) \in \mathscr{P}$ and any scalar α, the expression $\alpha(p,x)+(1-\alpha)(q,y)$ with $(\alpha;p,x;q,y)$.

Proof of Lemma 2.1. Consider any inequality quasi-ordering $\preceq$ that satisfies TP and βINV. Consider, moreover, any $(p,x) \in \mathscr{P}$, any scalar λ such that $(p,y) = (p,x+\lambda(\beta x+1-\beta)) \in \mathscr{P}$ and $(p,x) \neq (p,y)$, and any $\alpha \in (0,1)$. Note that we have $\lambda = \frac{\mu(p,y)-\mu(p,x)}{\beta\mu(p,x)+1-\beta}$ by definition. What has to be shown is that $(p,x) \prec (\alpha;p,x;p,y)$.

Consider $(p,z) = (p,x+\lambda'(\beta x+1-\beta))$, where $\lambda' = \frac{\mu(\alpha;p,x;p,y)-\mu(p,x)}{\beta\mu(p,x)+1-\beta}$. The choice of λ' ensures that $\mu(p,z) = \mu(\alpha;p,x;p,y)$. Since either $0 < x_i < z_i < y_i$ for all $i = 1,2,\ldots,n$, or $x_i > z_i > y_i > 0$ for all $i = 1,2,\ldots,n$, we have, furthermore, $(p,z) \in \mathscr{P}$. We now prove the claim that TP implies $(p,z) \prec (\alpha;p,x;p,y)$. Note that the supports of (p,x), (p,y) and (p,z) have the same number of elements. Now, clearly, to any element in the support of (p,x), say income level t, there corresponds one element in the support of (p,y) equal to $t+\lambda(\beta t+1-\beta)$. The frequency with which t appears in (p,x), say frequency s, is equal to the frequency with which $t+\lambda(\beta t+1-\beta)$ appears in (p,y). By consequence, in $(\alpha;p,x;p,y)$, there is, for any element in the support of (p,x), a pair of incomes such that the sum of frequencies is s and the mean income for the group of individuals with any of these two incomes is $\alpha t+(1-\alpha)(t+\lambda(\beta t+1-\beta))$. Similarly, to any element in the support of (p,x), say t occurring with frequency s, there corresponds one income in the support of (p,z) equal to $t+\lambda'(\beta t+1-\beta)$ and occurring with frequency s. Now, we have $t+\lambda'(\beta t+1-\beta) = \alpha t+(1-\alpha)(t+\lambda(\beta t+1-\beta))$. Therefore, $(\alpha;p,x;p,y)$ can be obtained from (p,z) by a sequence of mean preserving spreads and, hence, TP implies $(p,z) \prec (\alpha;p,x;p,y)$.

Since $(p,z) \prec (\alpha;p,x;p,y)$ by TP and $(p,x) \sim (p,z)$ by βINV, we obtain $(p,x) \prec (\alpha;p,x;p,y)$ using transitivity. ■

Proof of Proposition 2.1. Consider any inequality quasi-ordering $\preceq$ that satisfies TP, βINV and DEC. Consider, moreover, any $(p,x),(q,y) \in \mathscr{P}$ such that $(p,x) \preceq (q,y)$ and any $\alpha \in (0,1)$. In the case where $\mu(p,x) = \mu(q,y)$, DEC already implies $(p,x) \preceq (\alpha;p,x;q,y)$. Therefore, we assume $\mu(p,x) \neq \mu(q,y)$ in what follows. What has to be shown is that $(p,x) \prec (\alpha;p,x;q,y)$.

Consider $(p,z) = (p,x+\lambda(\beta x+1-\beta))$, where $\lambda = \frac{\mu(q,y)-\mu(p,x)}{\beta\mu(p,x)+1-\beta}$. The choice of λ ensures that $\mu(p,z) = \mu(q,y)$. Two cases are possible: either (a) $(p,z) \in \mathscr{P}$, or (b) $(p,z) \notin \mathscr{P}$.

In case (a), we have $(p,x) \sim (p,z)$ by βINV. Using transitivity, we have $(p,z) \preceq (q,y)$ and, hence, $(\alpha;p,x;p,z) \preceq (\alpha;p,x;q,y)$ by DEC. Lemma 2.1 implies $(p,x) \prec (\alpha;p,x;p,z)$, and we obtain $(p,x) \prec (\alpha;p,x;q,y)$ using transitivity.

Case (b) occurs if and only if λ is such that in going from (p,x) to (p,z), nonpositive incomes get nonzero frequency (which is only possible if $\mu(p,x) > \mu(q,y)$). Consider $(p,x') = (p, x + \lambda'(\beta x + 1 - \beta))$ and $(q,y') = (q, y + \lambda'(\beta y + 1 - \beta))$ where λ' is any scalar such that $[x_1 + \lambda'(\beta x_1 + 1 - \beta)] + \lambda(\beta[x_1 + \lambda'(\beta x_1 + 1 - \beta)] + 1 - \beta) > 0$. We can then return to the beginning of this proof and prove the result for (p,x') and (q,y') without getting case (b). If the result is true for (p,x') and (q,y'), then it must be true for (p,x) and (q,y) as well by βINV and transitivity. ∎

Proof of Lemma 2.2. First note that (2.8) can be rewritten as

$$W : \mathscr{P} \to \mathbb{R} : (p,x) \mapsto u(x_1) + \sum_{i=2}^{n} \phi\Big(\sum_{j=i}^{n} p_j\Big)\,(u(x_i) - u(x_{i-1})).$$

Consider any $(p,x),(q,y) \in \mathscr{P}$ and any scalar $\alpha \in (0,1)$. Define, furthermore, the ordered pairs (p',z) and (q',z) with $z = (z_1, z_2, \ldots, z_m)$ the vector that contains the components of both x and y ordered such that $0 < z_1 < z_2 < \cdots < z_m$. Moreover, $p' = (p'_1, p'_2, \ldots, p'_m)$ is a vector where, for all $i = 1,2,\ldots,m$, $p'_i = p_{z_i}$ if z_i occurs in x and $p'_i = 0$ otherwise, and, similarly, $q' = (q'_1, q'_2, \ldots, q'_m)$ is a vector where, for all $i = 1,2,\ldots,m$, $q'_i = q_{z_i}$ if z_i occurs in y and $q'_i = 0$ otherwise. We then have

$$\begin{aligned}
W(\alpha(p,x) + (1-\alpha)(q,y)) \\
&= u(z_1) + \sum_{i=2}^{m} \phi\Big(\sum_{j=i}^{m} \alpha p'_j + (1-\alpha) q'_j\Big)\,(u(z_i) - u(z_{i-1})) \\
&\le \alpha\left[u(z_1) + \sum_{i=2}^{m} \phi\Big(\sum_{j=i}^{m} p'_j\Big)\,(u(z_i) - u(z_{i-1}))\right] \\
&\quad + (1-\alpha)\left[u(z_1) + \sum_{i=2}^{m} \phi\Big(\sum_{j=i}^{m} q'_j\Big)\,(u(z_i) - u(z_{i-1}))\right] \\
&= \alpha W(p,x) + (1-\alpha) W(q,y),
\end{aligned}$$

where the inequality follows from the convexity of ϕ. The inequality holds with equality if ϕ is linear, and holds strictly if ϕ is strictly convex and $(p,x) \neq (q,y)$ since the latter implies $p' \neq q'$. ∎

Proof of Proposition 2.2. Consider any $(p,x),(q,y) \in \mathscr{P}$ such that $(p,x) \preceq (q,y)$, and any scalar $\alpha \in (0,1)$. Since the case where $(p,x) = (q,y)$ is trivial, we assume $(p,x) \neq (q,y)$ in what follows.

We first consider the case where $\preceq$ is representable by (2.10). Defining the function W^β as $W^\beta(p,x) = W(p, x + \frac{1-\beta}{\beta})$ for all $(p,x) \in \mathscr{P}$ with W as in (2.8) and u as in (2.9), we have

$$I^{\beta,\varepsilon,\phi}_{RDEU}(\alpha;p,x;q,y) = \frac{1}{\beta}\left[1-\frac{\Big((1-\varepsilon)W^{\beta}(\alpha;p,x;q,y)\Big)^{\frac{1}{1-\varepsilon}}}{\mu(\alpha;p,x;q,y)+\frac{1-\beta}{\beta}}\right]. \qquad (2.14)$$

We have to show the following: (a) expression (2.14) is at least as great as (strictly greater than) $I^{\beta,\varepsilon,\phi}_{RDEU}(p,x)$ whenever ϕ is (strictly) convex, and (b) expression (2.14) is strictly greater than $I^{\beta,\varepsilon,\phi}_{RDEU}(p,x)$ whenever $\mu(p,x) \neq \mu(q,y)$.

First, consider

$$\frac{1}{\beta}\left[1-\frac{\Big((1-\varepsilon)\Big[\alpha W^{\beta}(p,x)+(1-\alpha)W^{\beta}(q,y)\Big]\Big)^{\frac{1}{1-\varepsilon}}}{\mu(\alpha;p,x;q,y)+\frac{1-\beta}{\beta}}\right] \qquad (2.15)$$

$$= \frac{1}{\beta}\left[1-\frac{1}{\mu(\alpha;p,x;q,y)+\frac{1-\beta}{\beta}}\left[\alpha\left(\mu(p,x)+\frac{1-\beta}{\beta}\right)^{1-\varepsilon}\left(\frac{\left[(1-\varepsilon)W^{\beta}(p,x)\right]^{\frac{1}{1-\varepsilon}}}{\mu(p,x)+\frac{1-\beta}{\beta}}\right)^{1-\varepsilon}\right.\right.$$

$$\left.\left. +(1-\alpha)\left(\mu(q,y)+\frac{1-\beta}{\beta}\right)^{1-\varepsilon}\left(\frac{\left[(1-\varepsilon)W^{\beta}(q,y)\right]^{\frac{1}{1-\varepsilon}}}{\mu(q,y)+\frac{1-\beta}{\beta}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}\right]$$

$$= \frac{1}{\beta}\left[1-\left(1-\beta I^{\beta,\varepsilon,\phi}_{RDEU}(p,x)\right)A\right],$$

where

$$A = \frac{\left[\alpha\left(\mu(p,x)+\frac{1-\beta}{\beta}\right)^{1-\varepsilon}+(1-\alpha)\left(\mu(q,y)+\frac{1-\beta}{\beta}\right)^{1-\varepsilon}\left(\frac{1-\beta I^{\beta,\varepsilon,\phi}_{RDEU}(q,y)}{1-\beta I^{\beta,\varepsilon,\phi}_{RDEU}(p,x)}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\mu(\alpha;p,x;q,y)+\frac{1-\beta}{\beta}}$$

$$= B\times C,$$

where

$$B = \frac{\alpha\left(\mu(p,x)+\frac{1-\beta}{\beta}\right)+(1-\alpha)\left(\mu(q,y)+\frac{1-\beta}{\beta}\right)\left(\frac{1-\beta I^{\beta,\varepsilon,\phi}_{RDEU}(q,y)}{1-\beta I^{\beta,\varepsilon,\phi}_{RDEU}(p,x)}\right)}{\alpha\mu(p,x)+(1-\alpha)\mu(q,y)+\frac{1-\beta}{\beta}}$$

and

$$C = \frac{\left[\alpha\left(\mu(p,x)+\frac{1-\beta}{\beta}\right)^{1-\varepsilon}+(1-\alpha)\left(\mu(q,y)+\frac{1-\beta}{\beta}\right)^{1-\varepsilon}\left(\frac{1-\beta I^{\beta,\varepsilon,\phi}_{RDEU}(q,y)}{1-\beta I^{\beta,\varepsilon,\phi}_{RDEU}(p,x)}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}{\alpha\left(\mu(p,x)+\frac{1-\beta}{\beta}\right)+(1-\alpha)\left(\mu(q,y)+\frac{1-\beta}{\beta}\right)\left(\frac{1-\beta I^{\beta,\varepsilon,\phi}_{RDEU}(q,y)}{1-\beta I^{\beta,\varepsilon,\phi}_{RDEU}(p,x)}\right)}.$$

It is readily checked that $0 < B \leq 1$. Furthermore, if $\varepsilon = 0$, then $C = 1$, while if $\varepsilon > 0$, then

$$C = 1 - I_{RDEU}^{1,\varepsilon,\iota}\left((\alpha, 1-\alpha), \left[\mu(p,x) + \tfrac{1-\beta}{\beta}, \left(\mu(q,y) + \tfrac{1-\beta}{\beta}\right)\left(\tfrac{1-\beta I_{RDEU}^{\beta,\varepsilon,\phi}(q,y)}{1-\beta I_{RDEU}^{\beta,\varepsilon,\phi}(p,x)}\right)\right]\right)$$

where ι is the identity function. By consequence, we have $0 < C \leq 1$.

Second, notice that $\frac{1}{\beta}\left[1 - \left(1 - \beta I_{RDEU}^{\beta,\varepsilon,\phi}(p,x)\right)BC\right] \geq I_{RDEU}^{\beta,\varepsilon,\phi}(p,x)$ since $0 < B \leq 1$ and $0 < C \leq 1$. Because, moreover, it follows from Lemma 2.2 that whenever ϕ is (strictly) convex, expression (2.14) is at least as great as (is strictly greater than) expression (2.15), (a) follows. The case in which ϕ is strictly convex has been dealt with, and hence we assume $\varepsilon > 0$ in what follows. Notice that whenever $\mu(p,x) \neq \mu(q,y)$ and $I_{RDEU}^{\beta,\varepsilon,\phi}(p,x) = I_{RDEU}^{\beta,\varepsilon,\phi}(q,y)$, we have $B = 1$ but $C < 1$ since $\varepsilon > 0$, so that $\frac{1}{\beta}\left[1 - \left(1 - \beta I_{RDEU}^{\beta,\varepsilon,\phi}(p,x)\right)BC\right] > I_{RDEU}^{\beta,\varepsilon,\phi}(p,x)$, and whenever $\mu(p,x) \neq \mu(q,y)$ and $I_{RDEU}^{\beta,\varepsilon,\phi}(p,x) < I_{RDEU}^{\beta,\varepsilon,\phi}(q,y)$, we have $B < 1$, so that, again, $\frac{1}{\beta}\left[1 - \left(1 - \beta I_{RDEU}^{\beta,\varepsilon,\phi}(p,x)\right)BC\right] > I_{RDEU}^{\beta,\varepsilon,\phi}(p,x)$. Combining this with the fact that convexity of ϕ implies that expression (2.14) is at least as great as expression (2.15), we find that (b) follows.

We now consider the second case where $\preceq$ is representable by (2.12). Using W as in (2.8) and u as in (2.11), we have

$$I_{RDEU}^{0,\gamma,\phi}(\alpha; p,x; q,y) = \mu(\alpha; p,x; q,y) + \frac{1}{\gamma}\ln(-W(\alpha; p,x; q,y)). \qquad (2.16)$$

The following has to be shown: (c) expression (2.16) is at least as great as (strictly greater than) $I_{RDEU}^{0,\gamma,\phi}(p,x)$ whenever ϕ is (strictly) convex, and (d) expression (2.16) is strictly greater than $I_{RDEU}^{0,\gamma,\phi}(p,x)$ whenever $\mu(p,x) \neq \mu(q,y)$.

Consider

$$\mu(\alpha; p,x; q,y) + \frac{1}{\gamma}\ln(-[\alpha W(p,x) + (1-\alpha) W(q,y)]), \qquad (2.17)$$

and

$$\alpha\mu(p,x) + (1-\alpha)\mu(q,y) + \frac{1}{\gamma}\alpha\ln(-W(p,x)) + (1-\alpha)\ln(-W(q,y)) \qquad (2.18)$$

$$= \alpha I_{RDEU}^{0,\gamma,\phi}(p,x) + (1-\alpha) I_{RDEU}^{0,\gamma,\phi}(q,y). \qquad (2.19)$$

It follows from Lemma 2.2 that whenever ϕ is (strictly) convex, expression (2.16) is at least as great as (is strictly greater than) expression (2.17). Since, moreover, expression (2.17) is at least as great as expression (2.18) by concavity of the ln function, we have (c). In the case where $\mu(p,x) \neq \mu(q,y)$ and $I^{0,\gamma,\phi}_{RDEU}(p,x) = I^{0,\gamma,\phi}_{RDEU}(q,y)$, we have $W(p,x) \neq W(q,y)$ and, hence, expression (2.17) is strictly greater than expression (2.18) by strict concavity of the ln function. If $\mu(p,x) \neq \mu(q,y)$ and $I^{0,\gamma,\phi}_{RDEU}(p,x) \neq I^{0,\gamma,\phi}_{RDEU}(q,y)$, then expression (2.19) is strictly greater than $I^{0,\gamma,\phi}_{RDEU}(p,x)$. Hence, (d) follows. ∎

Proof of Proposition 2.3. Consider any inequality ordering $\preceq$ that satisfies CSQC. Consider, moreover, any $(p,x),(q,y) \in \mathscr{P}$ with $\mu(p,x) \neq \mu(q,y)$.

Consider the following two subpatterns, both of which describe how inequality evolves as α rises over some subinterval $(\underline{\tilde{\alpha}},\overline{\tilde{\alpha}}) \subseteq (\underline{\alpha},\overline{\alpha})$ in $(\alpha;p,x;q,y)$:

(a) A constant pattern over $(\underline{\tilde{\alpha}},\overline{\tilde{\alpha}})$, i.e., for any $\alpha,\alpha' \in (\underline{\tilde{\alpha}},\overline{\tilde{\alpha}})$, $(\alpha;p,x;q,y) \sim (\alpha';p,x;q,y)$.

(b) A U pattern over $(\underline{\tilde{\alpha}},\overline{\tilde{\alpha}})$, i.e., there exists an $\alpha^* \in (\underline{\tilde{\alpha}},\overline{\tilde{\alpha}})$ such that, for any $\alpha,\alpha' \in (\underline{\tilde{\alpha}},\alpha^*]$, if $\alpha > \alpha'$, then $(\alpha;p,x;q,y) \prec (\alpha';p,x;q,y)$, and, for any $\alpha,\alpha' \in [\alpha^*,\overline{\tilde{\alpha}})$, if $\alpha > \alpha'$, then $(\alpha';p,x;q,y) \prec (\alpha;p,x;q,y)$.

We first show by contradiction that neither subpattern can be the case for any subinterval $(\underline{\tilde{\alpha}},\overline{\tilde{\alpha}})$. Suppose, therefore, that (a) or (b) holds over some subinterval $(\underline{\tilde{\alpha}},\overline{\tilde{\alpha}}) \subseteq (\underline{\alpha},\overline{\alpha})$. Both subpatterns imply that there exist some $\alpha,\alpha',\alpha'' \in (\underline{\tilde{\alpha}},\overline{\tilde{\alpha}})$ where $\alpha > \alpha' > \alpha''$ such that $(\alpha';p,x;q,y) \preceq (\alpha;p,x;q,y)$ and $(\alpha';p,x;q,y) \preceq (\alpha'';p,x;q,y)$. This is obvious in the case of (a), while in the case of (b) this can be seen by letting α' equal the α^* in the definition of (b). Note now that, for $\alpha''' = \frac{\alpha'-\alpha''}{\alpha-\alpha''}$, we have $(\alpha''';(\alpha;p,x;q,y);(\alpha'';p,x;q,y)) = (\alpha';p,x;q,y)$. By consequence, we obtain both $(\alpha''';(\alpha;p,x;q,y);(\alpha'';p,x;q,y)) \preceq (\alpha;p,x;q,y)$ and $(\alpha''';(\alpha;p,x;q,y);(\alpha'';p,x;q,y)) \preceq (\alpha'';p,x;q,y)$. Since, moreover, $\alpha''' \in (0,1)$ and $\mu(\alpha;p,x;q,y) \neq \mu(\alpha'';p,x;q,y)$, we have a violation of CSQC.

Now, any pattern for which subpattern (a) is not the case for any $(\underline{\tilde{\alpha}},\overline{\tilde{\alpha}})$, is either pattern (i), (ii), (iii), or a pattern for which pattern (b) is the case for some $(\underline{\tilde{\alpha}},\overline{\tilde{\alpha}})$. However, as we have seen, the latter is impossible. ∎

Part II

Inequality Aversion

Chapter 3

Extreme Inequality Aversion without Separability

3.1 Introduction

For social preferences that can be represented by social welfare functions of the expected utility form, it is broadly accepted that leximin constitutes the case of extreme inequality aversion. As far as we know, the only formal justification for this connection between leximin and the concept of extreme inequality aversion is a result that can be attributed to Hammond (1975), Meyer (1975) and Lambert (2001). Loosely speaking, their result says that any pairwise choice implied by leximin is *unanimously* approved by *the most inequality averse* members (in the Arrow-Pratt sense) of the class of expected utility type social preferences.

Our contribution in this chapter is to study the concept of inequality aversion for more general social preferences, viz., for social preferences that do not necessarily satisfy the separability axiom that underlies expected utility theory. This more general outlook is very common in the social choice literature: see for instance the standard overviews of Bossert and Weymark (2004) and d'Aspremont and Gevers (2002).[1] We show that, for these more general social preferences, the case of extreme inequality aversion is covered by the class of *weakly maximin* social preferences—these are all social preferences that have in common the property that a given alternative is strictly preferred over another if the worst off is strictly better off in the given alternative.[2] In order to establish this result, we prove an analogue of the Hammond-Meyer-Lambert result mentioned above.

[1] As we saw in Chapter 1, the literature on individual decision under risk has likewise studied several non-separable alternatives to expected utility.

[2] In their analysis of extreme inequality aversion, Tungodden and Vallentyne (2005) also arrive at the class of weakly maximin social preferences. The same holds for our analysis of Chapter 4. However, in both cases approaches are used that deviate from the standard Arrow-Pratt approach.

3.2 Preliminaries

A social alternative is a vector $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n_{++}$ where n is the number of individuals in society and x_i measures the well-being of individual i. The set of individuals is N and the set of social alternatives is X. We use the symbol 1_n to denote an n-dimensional vector of which all components are equal to 1. For any $x \in X$, $\hat{x} = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)$ denotes a permutation of x such that $\hat{x}_1 \leq \hat{x}_2 \leq \cdots \leq \hat{x}_n$. For any $x, y \in X$, we write $x > y$ if and only if $x_i \geq y_i$ for all $i \in N$ with at least one inequality holding strictly. Social preferences are represented by a relation $\succeq$ ('is at least as good as') on X. It is assumed throughout that social preference relations are orderings.[3] We denote the asymmetric and symmetric parts of $\succeq$ by $\succ$ and $\sim$, respectively.

The Hammond-Meyer-Lambert result applies to social preferences that are formally similar to the preferences of expected utility maximizers. We shall say that $\succeq$ is a member of this class, which we denote by $\mathscr{W}_{EU}$, if and only if it satisfies the following four well known properties:[4]

(i) *Anonymity:* $\forall x \in X,\ [x' \text{ is any permutation of } x] \Rightarrow x \sim x'$.

(ii) *Continuity:* $\forall x \in X,\ \{y \,|\, y \in X, y \succeq x\}$ and $\{y \,|\, y \in X, x \succeq y\}$ are closed in X.

(iii) *Strong Pareto:* $\forall x, y \in X,\ x > y \Rightarrow x \succ y$.

(iv) *Separability:* $\forall \tilde{N} \subset N,\ \forall x, y, x', y' \in X$,

$$[\forall i \in \tilde{N},\ x_i = y_i,\ x'_i = y'_i, \text{ and, } \forall i \in N \setminus \tilde{N},\ x_i = x'_i,\ y_i = y'_i] \Rightarrow\ [x \succeq y \Leftrightarrow x' \succeq y'].$$

The result we shall present deals with more general social preferences that satisfy the basic properties (i) to (iii), but not necessarily separability. We use the symbol $\mathscr{W}$ to indicate this general class of social preferences.

The Arrow-Pratt concept was designed originally to compare members of $\mathscr{W}_{EU}$ with respect to inequality aversion. However, one of the several equivalent formulations of the concept—that based on the 'equally distributed equivalent well-being'—can be applied to the general class $\mathscr{W}$ as well. The equally distributed equivalent well-being, $\xi(\succeq; x)$, for any $x \in X$ and any $\succeq \in \mathscr{W}$, is the per capita level of well-being that, when equally distributed, yields the same level of welfare according to $\succeq$ as x. Formally, for all $\succeq \in \mathscr{W}$ and for all $x \in X$, $\xi(\succeq; x) = e$ if and only if $e1_n \sim x$. For each $\succeq \in \mathscr{W}$, the function $\xi(\succeq; \cdot)$ on X is defined and is, moreover, a representation of $\succeq$ on X. The Arrow-Pratt concept says the following: for

[3] An ordering is a reflexive, transitive and complete relation.

[4] See Chapter 4 or for instance Bossert and Weymark (2004) for a discussion of these properties. In Chapter 4, strong Pareto is referred to as monotonicity.

any $\succeq, \succeq' \in \mathscr{W}$, $\succeq$ is at least as inequality averse as $\succeq'$, written as $\succeq R_{AP} \succeq'$, if and only if, for all $x \in X$, $\xi(\succeq; x) \leq \xi(\succeq'; x)$.

The Hammond-Meyer-Lambert result shows that leximin—which gives priority to the worst off in a lexicographical fashion—can be interpreted as being extremely inequality averse with respect to the class $\mathscr{W}_{EU}$. Leximin, which we denote by $\succeq_{\text{lex}}$, is defined as follows: $\forall x, y \in X$,

$$x \succeq_{\text{lex}} y \;\Leftrightarrow\; [x \text{ is a permutation of } y, \text{ or,} \\ \exists k \in N,\ \forall i < k,\ \hat{x}_i = \hat{y}_i \text{ and } \hat{x}_k > \hat{y}_k]\,.$$

Leximin is a member of the class of weakly maximin social preferences. The members of this class give priority to the worst off in all cases in which the worst off is not indifferent. Formally, a social preference relation $\succeq$ is weakly maximin if and only if, for all $x, y \in X$,

$$\hat{x}_1 > \hat{y}_1 \Rightarrow x \succ y.$$

It can be shown that leximin is the only social preference relation that is both weakly maximin and separable.[5] We denote the class of anonymous and strongly Paretian weakly maximin social preferences by $\mathscr{M}$. Since $\mathscr{M}$ does not contain any continuous members, the classes $\mathscr{M}$ and $\mathscr{W}$ are disjoint.[6] Clearly, leximin is a member of $\mathscr{M}$. We wish to emphasize, however, that $\mathscr{M}$ contains as well social preferences that are very different from leximin. To see this, note that for any anonymous and strongly Paretian social preference relation $\succeq'$ there exists a $\succeq$ belonging to $\mathscr{M}$ such that, for all $x, y \in X$,

$$\hat{x}_1 = \hat{y}_1 \;\Rightarrow\; [x \succeq y \Leftrightarrow x \succeq' y].$$

Loosely speaking, this means that in all cases in which the worst off are equally well off, any social preferences are allowed as long as they satisfy anonymity and strong Pareto.

In the next section we provide a result, analogous to the Hammond-Meyer-Lambert result, which shows that if we broaden our attention from only the separable members of $\mathscr{W}$ (i.e., the members of $\mathscr{W}_{EU}$) to all members of $\mathscr{W}$, then the corresponding class of extremely inequality averse social preferences broadens from only the separable members of $\mathscr{M}$ (i.e., the single member of $\{\succeq_{\text{lex}}\}$) to all members of $\mathscr{M}$.

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[5] See Chapter 5.

[6] Each member of $\mathscr{M}$ is a positional dictatorship and positional dictatorships cannot satisfy both continuity and strong Pareto (see Bossert and Weymark, 2004, p. 1114). In relation to this, note that maximin—which implies social indifference for all pairs for which the worst off individuals are equally well off—is the only continuous member of the class of weakly maximin social preferences (see Chapter 5), but that it is not strongly Paretian and hence does not belong to $\mathscr{M}$.

3.3 Result

Relying heavily on Hammond (1975) and Meyer (1975), Lambert (2001, Theorem 4.4) presents a result that justifies the interpretation of leximin as extremely inequality averse, provided that separability is demanded.[7]

Theorem 3.1. *For all $x,y \in X$ such that $\hat{x} \neq \hat{y}$,*

$$x \succ_{\text{lex}} y \;\Leftrightarrow\; [\exists \succeq \in \mathscr{W}_{EU},\, \forall \succeq' \in \mathscr{W}_{EU},\, \succeq' R_{AP} \succeq \Rightarrow x \succeq' y].$$

Theorem 3.1 says that (i) whenever leximin implies a strict preference over a pair of social alternatives, then the most inequality averse social preferences in the class $\mathscr{W}_{EU}$ unanimously agree weakly with that preference, and, conversely, (ii) whenever all most inequality averse members of $\mathscr{W}_{EU}$ weakly prefer one social alternative over another, then leximin strictly agrees with this preference.

Our contribution is to provide the following corresponding result for the case where separability is not demanded.

Theorem 3.2. *For all $x,y \in X$ such that $\hat{x} \neq \hat{y}$,*

$$[\forall \succeq \in \mathscr{M},\, x \succ y] \;\Leftrightarrow\; [\exists \succeq \in \mathscr{W},\, \forall \succeq' \in \mathscr{W},\, \succeq' R_{AP} \succeq \Rightarrow x \succeq' y].^{8}$$

Proof. $(\Rightarrow)$ Take any $x,y \in X$ such that $\hat{x} \neq \hat{y}$, for which

$$\forall \succeq \in \mathscr{M},\, x \succ y,$$

which is equivalent to $\hat{x}_1 > \hat{y}_1$. We have to show that there exists a $\succeq \in \mathscr{W}$ such that

$$\forall \succeq' \in \mathscr{W},\, \succeq' R_{AP} \succeq \Rightarrow x \succeq' y. \tag{3.1}$$

Consider any $\succeq'' \in \mathscr{W}$ such that $\xi(\succeq''; x) \leq \xi(\succeq''; y)$. If such a $\succeq''$ does not exist, then any member of $\mathscr{W}$ serves as a $\succeq$ for which (3.1) holds. If, on the other hand, such a $\succeq''$ exists, then we define $\succeq$ such that, for all $w,z \in X$,

$$w \succeq z \;\Leftrightarrow\; \alpha \hat{w}_1 + (1-\alpha)\xi(\succeq''; w) \geq \alpha \hat{z}_1 + (1-\alpha)\xi(\succeq''; z),$$

[7] Hammond (1975) proves the result only for a proper subclass of $\mathscr{W}_{EU}$, viz., for social preferences of the CES type. Lambert (2001) uses the analysis of Meyer (1975) to extend Hammond's result to the entire class $\mathscr{W}_{EU}$.

[8] An alternative to Theorem 3.2 which is closer to Hammond's (1975) original formulation can as well be obtained. This requires a somewhat stronger concept of inequality aversion: $\succeq$ is strongly more inequality averse than $\succeq'$, which is written as $\succeq \hat{P}_{AP} \succeq'$, if and only if, for all $x \in X$ with at least two distinct components, $\xi(\succeq; x) < \xi(\succeq'; x)$ (note that this version of the Arrow-Pratt concept is more demanding than the asymmetric part of R_{AP}). The alternative to Theorem 3.2 is: $\forall x,y \in X$, $[\forall \succeq \in \mathscr{M},\, x \succ y] \;\Leftrightarrow\; [\exists \succeq \in \mathscr{W},\, \forall \succeq' \in \mathscr{W},\, \succeq' \hat{P}_{AP} \succeq \Rightarrow x \succ' y]$. We omit the proof because it is very similar to that of Theorem 3.2.

where $\alpha \in [0,1)$ is such that

$$\alpha \hat{y}_1 + (1-\alpha)\xi(\succeq'';y) = \hat{x}_1.$$

By strong Pareto and reflexivity, $\xi(\succeq'';x) \geq \hat{x}_1$, and, hence, we have $\xi(\succeq'';y) \geq \hat{x}_1$. The latter, combined with the fact that $\hat{x}_1 > \hat{y}_1$, ensures that $\succeq$ can be defined in the above way. It can be readily checked that $\succeq \in \mathscr{W}$. Since $\xi(\succeq'';\hat{x}_1 1_n) = \hat{x}_1$, we have $\hat{x}_1 1_n \sim y$ and, consequently, $\xi(\succeq;y) = \hat{x}_1$.

What remains to be shown is that (3.1) holds for the constructed $\succeq$. For any $\succeq'$ such that $\succeq' R_{AP} \succeq$, we have

$$\hat{x}_1 = \xi(\succeq;y) \geq \xi(\succeq';y),$$

and, by strong Pareto and reflexivity, $\xi(\succeq';x) \geq \hat{x}_1$. By consequence, we have $\xi(\succeq';x) \geq \xi(\succeq';y)$ and so $x \succeq' y$.

($\Leftarrow$) Seeking a contradiction, we assume there exists a pair $x, y \in X$ such that $\hat{x} \neq \hat{y}$, for which

$$\exists \succeq \in \mathscr{W},\ \forall \succeq' \in \mathscr{W},\ \succeq' R_{AP} \succeq \Rightarrow x \succeq' y, \tag{3.2}$$

while there exists a $\succeq'' \in \mathscr{M}$ such that $y \succeq'' x$. Note that from $y \succeq'' x$ it follows that $x > y$ does not hold by strong Pareto, and also that $\hat{x}_1 \leq \hat{y}_1$.

Define $\succeq''' \in \mathscr{W}$ such that, for all $w, z \in X$,

$$w \succeq''' z \ \Leftrightarrow\ \frac{\sum_{i=1}^n \phi_i \hat{w}_i}{\sum_{i=1}^n \phi_i} \geq \frac{\sum_{i=1}^n \phi_i \hat{z}_i}{\sum_{i=1}^n \phi_i},$$

where $\phi_1, \phi_2, \ldots, \phi_n > 0$. The weights are determined in two steps. First, we choose any $\phi_1, \phi_2, \ldots, \phi_n > 0$ such that $y \succ''' x$. A choice of the weights such that $y \succ''' x$ is possible since $x > y$ does not hold. Second, we increase the weight ϕ_1 while holding all other weights fixed until, for all $w \in X$, $\xi(\succeq''';w) \leq \xi(\succeq;w)$. This is possible, since by choosing ϕ_1 sufficiently high,

$$\xi(\succeq''';w) = \frac{\sum_{i=1}^n \phi_i \hat{w}_i}{\sum_{i=1}^n \phi_i}$$

can be chosen as close to $\hat{w}_1$ as necessary for all $w \in X$. Note that increasing ϕ_1 while holding all the other weights fixed preserves the ranking $y \succ''' x$ since $\hat{x}_1 \leq \hat{y}_1$.

Now, since $\succeq''' \in \mathscr{W}$ is such that, for all $w \in X$, $\xi(\succeq''';w) \leq \xi(\succeq;w)$, and, moreover, $y \succ''' x$, we have that (3.2) is contradicted. ■

Theorem 3.2 is completely analogous to Theorem 3.1: (i) if all members of the class $\mathscr{M}$ imply a strict preference over a pair of social alternatives, then the most inequality averse social preferences in the general class $\mathscr{W}$ unanimously agree weakly with that preference, and, conversely, (ii) if all most inequality averse members of $\mathscr{W}$ weakly prefer one social alternative over another, then the members of $\mathscr{M}$ unanimously strictly agree with this preference.

Connection between Chapters 3 and 4: Chapter 3 provided a justification on the basis of the Arrow-Pratt concept of viewing the class of weakly maximin social welfare orderings as extremely inequality averse. The discussion of the weakly maximin class in Section 3.2 revealed that the class is very broad: for choices over alternatives in which the worst off is equally well off, the property of being weakly maximin does not impose anything at all. In Chapter 4, this observation is used to criticize the Arrow-Pratt concept. It is argued that the weakly maximin class includes members that we would not usually refer to as particularly egalitarian, and that, by consequence, the Arrow-Pratt concept is not demanding enough as a concept of inequality aversion—see the discussion following Proposition 4.4 in Section 4.5. Accordingly, Chapter 4 studies two such more demanding concepts of inequality aversion. In Section 4.5, it is revealed that the weaker of the two, the RD-concept, also characterizes the weakly maximin class as extremely inequality averse. By contrast, according to the stronger of the two alternative concepts, the L-concept, the class of monotonic extremely inequality averse social welfare orderings is empty.

Chapter 4

Comparing Degrees of Inequality Aversion

4.1 Introduction

How should we compare different social preference relations over income distributions with respect to the degree of inequality aversion, i.e., the degree of dislike towards inequality, they express? We propose a procedure for comparing degrees of inequality aversion that can be loosely formulated as follows:

Procedure ($\star$): A social welfare ordering (SWO) R is *at least as inequality averse as* an SWO R' if and only if, for all income distributions x and y such that x is *less unequal than* y according to a pre-specified inequality quasi-ordering, (i) R strictly prefers x to y (xPy) whenever R' strictly prefers x to y ($xP'y$), and (ii) R weakly prefers x to y (xIy or xPy) whenever R' is indifferent between x and y ($xI'y$).

Note that in order to make this procedure operational, an inequality quasi-ordering must first be chosen. This feature of Procedure ($\star$) makes explicit the fact that, underlying any concept for comparing degrees of inequality aversion, there necessarily has to be a criterion for making comparisons according to inequality—obviously, to be able to check whether an SWO expresses more or less *dislike towards inequality* than another SWO, it must be clear what is meant by inequality in the first place. Once an inequality quasi-ordering is chosen, Procedure ($\star$) turns into a fully operational concept of inequality aversion which entails a straightforward check for dominance: an SWO is referred to as at least as inequality averse as another if it implies, in all relevant choice situations (i.e., those pairs of income distributions that are strictly ranked using the chosen inequality quasi-ordering), an at least as inequality averse choice as the other (as defined in (i) and (ii) of Procedure ($\star$)). Procedure ($\star$) can furthermore be shown to be consistent with the common approach of measuring the degree of inequality aversion by the amount of mean income an SWO is prepared to forego in exchange for a given decrease in inequality (see Section 4.3).

Interestingly, the traditional Arrow-Pratt concept for comparing degrees of inequality aversion[1] is a special case of Procedure ($\star$). Roughly speaking, the Arrow-Pratt concept of inequality aversion is obtained in the case where the chosen inequality quasi-ordering is the extremely simplistic one which allows only (strict) inequality comparisons between, on the one hand, unequal income distributions and, on the other hand, perfectly equal ones (see Section 4.4). In this chapter, we take the point of view that while Procedure ($\star$) is the appropriate way to approach the problem of comparing degrees of inequality aversion, the Arrow-Pratt version of the procedure is unattractive because it is based on an unduly restrictive inequality quasi-ordering. Taking into consideration its central place in the literature on inequality measurement, the Lorenz inequality quasi-ordering seems a much more suitable candidate for this role. This critique of the Arrow-Pratt concept echoes that of Ross (1981) in the context of decision under risk. Ross argues that for a comparison of risk aversion between two expected utility maximizers, it is not sufficient to compare the premia they are maximally prepared to pay for an insurance against all risks, as the Arrow-Pratt concept prescribes, but it is also necessary to consider premia for insurances that decrease risk to a lower, but still risky, level. Our proposal to consider the concept of inequality aversion based on Procedure ($\star$) using the Lorenz inequality quasi-ordering is similar to that proposed by Ross since his criterion of decreasing risk is close to the Lorenz criterion.

Throughout the chapter, we will often be concerned with comparing results yielded by, on the one hand, the version of Procedure ($\star$) that is equivalent to the Arrow-Pratt concept and, on the other hand, the favoured version of Procedure ($\star$) which uses the Lorenz inequality quasi-ordering. It is interesting, however, to consider also a third concept that is intermediate between the Arrow-Pratt concept and the Lorenz-based concept. This third concept is based on the relative differentials quasi-ordering, an inequality criterion that is stronger than the minimalist inequality criterion underlying the Arrow-Pratt concept and weaker than the Lorenz quasi-ordering (see Moyes, 1994). Henceforth, we refer to the inequality aversion concept obtained from Procedure ($\star$) using the Lorenz quasi-ordering as the '*L*-concept,' and to that obtained from the procedure using the relative differentials quasi-ordering as the '*RD*-concept.'

We first compare the three concepts of inequality aversion for the class of continuous and monotonic SWOs, the broadest class of SWOs to which the conventional Arrow-Pratt concept is commonly applied. We show that the *RD*-concept

[1] The Arrow-Pratt approach is discussed thoroughly in Lambert (2001).

yields the same results as the Arrow-Pratt concept if SWOs are in addition separable, but not necessarily otherwise. Unfortunately, such consistency turns out not to hold between the *L*-concept and the Arrow-Pratt concept, not even with respect to the important class of constant elasticity of substitution (CES) SWOs (a subclass of the class of continuous, monotonic and separable SWOs). Usually, a CES SWO with a higher value of the single parameter, ε, is considered more inequality averse than one with a lower value of ε. This role of ε as a measure for the degree of inequality aversion is justified in the framework of the Arrow-Pratt concept of inequality aversion. However, as straightforward examples show, this role of ε is *not* justified if the *L*-concept is adopted: given two income distributions such that one is less unequal than the other according to the Lorenz inequality quasi-ordering, it is quite possible that a CES SWO with ε strictly prefers the less unequal income distribution, while a CES SWO with $\varepsilon' > \varepsilon$ strictly prefers the more unequal one. Moreover, using a result by Ross (1981) we show that such examples can be found for any two CES SWOs. In other words, if the *L*-concept is adopted, then no two CES SWOs can be compared with respect to degree of inequality aversion.

Second, we study the idea of 'extreme inequality aversion' for the three different concepts of inequality aversion. We call an SWO extremely inequality averse in a class of SWOs *S* if and only if it is at least as inequality averse as all SWOs in *S* (and, moreover, is itself a member of *S*). In the literature, leximin is often seen as a typical example of an SWO that combines extreme inequality aversion with monotonicity. We show that, in the class of monotonic SWOs, both the Arrow-Pratt concept and the *RD*-concept identify the entire class of weakly maximin SWOs as extremely inequality averse—an SWO is said to be *weakly maximin* if and only if it implies a strict preference for a given income distribution over another whenever the worst off is strictly better off in the given income distribution. The class includes leximin and, by consequence, the Arrow-Pratt concept and the *RD*-concept can be said to support the conventional view (see also Tungodden and Vallentyne, 2005). However, if the *L*-concept is adopted, this view has to be abandoned: we show that in this case the set of extremely inequality averse monotonic SWOs is empty. Finally, we demonstrate that the incompatibility between extreme inequality aversion and monotonicity is robust with respect to certain reasonable changes in the definition of the idea of extreme inequality aversion.

The chapter is structured as follows. Section 4.2 deals with preliminaries. In Section 4.3 we formally introduce and discuss the three concepts for comparing degrees of inequality aversion that constitute the topic of the chapter. The questions of how the three concepts compare with respect to the class of continuous and monotonic SWOs, and with respect to the idea of extreme inequality aversion, are dealt with in Sections 4.4 and 4.5, respectively. Some concluding remarks are given in Section 4.6. All proofs are contained in Appendix 4.A.

4.2 Preliminaries

An *income distribution* is a vector $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n_{++}$ where $n \geq 3$ is the (fixed) number of individuals in society and x_i is the income of individual i. The set of individuals is N and the set of income distributions is X. We assume that, for any income distribution $x \in X$, individuals are indexed such that $x_1 \leq x_2 \leq \cdots \leq x_n$. In accordance with this assumption, we suppose that all considered concepts for welfare and inequality comparisons satisfy *anonymity*—that is, any given income distribution is treated equivalently as each of its permutations. The arithmetic mean of any income distribution $x \in X$ is written as $\mu(x)$. We use the symbol 1_n to denote an n-dimensional vector of which all components are equal to 1. For any pair of income distributions, $x, y \in X$, we write $x > y$ if $x_i \geq y_i$ for all $i \in N$ with at least one strict inequality, and we write $x \gg y$ if $x_i > y_i$ for all $i \in N$.

Social preferences are represented by a *social welfare ordering* (SWO) R ('is at least as good as') on X.[2] The asymmetric and symmetric parts of R are denoted by P ('is better than') and I ('is equally good as'), respectively. A *social welfare function* is a function $W : X \to \mathbb{R}$ which represents some SWO.

We shall require certain axioms in our analysis. Roughly speaking, continuity ensures that small changes in an income distribution cause only small changes in its social welfare ranking against other income distributions.

Continuity. For all $x \in X$, $\{y \,|\, y \in X, yRx\}$ and $\{y \,|\, y \in X, xRy\}$ are closed in X.

Monotonicity says that it is an improvement if some individuals get better off without any individuals getting worse off.

Monotonicity. For all $x, y \in X$, if $x > y$, then xPy.

Separability requires that the social welfare ranking of any pair of income distributions is not influenced by the incomes that are the same in both income distributions.

Separability. For all $\hat{N} \subset N$ and for all $x, y, x', y' \in X$, if $x_i = y_i$ and $x'_i = y'_i$ for all $i \in \hat{N}$, and $x_i = x'_i$ and $y_i = y'_i$ for all $i \in N \setminus \hat{N}$, then $xRy \Leftrightarrow x'Ry'$.

Any SWO that satisfies continuity, monotonicity and separability can be represented by a social welfare function of the following form:

$$W(x) = \sum_{i=1}^{n} u(x_i) \quad \text{for all } x \in X, \tag{4.1}$$

where $u : \mathbb{R}_{++} \to \mathbb{R}$ is a continuous and strictly increasing function, referred to as a *utility function* (see Bossert and Weymark, 2004, Theorem 20). We shall pay

[2] An ordering is a reflexive, transitive and complete binary relation.

special attention in our analysis to the constant elasticity of substitution (CES) class of SWOs, an important subclass of the class of continuous, monotonic and separable SWOs. An SWO R_ε is a member of the CES class if and only if there exists a nonnegative scalar ε such that R_ε can be represented by (4.1) with utility function $u(t) = \frac{1}{1-\varepsilon}t^{1-\varepsilon}$ for all $t \in \mathbb{R}_{++}$.

Since comparisons of income distributions *with respect to inequality* are conceptually prior to comparisons of SWOs *with respect to degree of inequality aversion*, we require the concept of an *inequality quasi-ordering* (IQO) $\preceq$ ('is at most as unequal as') on X.[3] The asymmetric and symmetric parts of $\preceq$ are denoted by $\prec$ ('is less unequal than') and $\sim$ ('is equally unequal as'), respectively. An *inequality measure* is a function $J : X \to \mathbb{R}$ which represents some complete IQO. The strongest IQO to receive broad acceptance amongst economists is the Lorenz IQO. The *Lorenz IQO*, written as $\preceq_L$, is defined as follows: for all $x, y \in X$,

$$x \preceq_L y \Leftrightarrow \sum_{i=1}^{k} \frac{x_i}{\mu(x)} \geq \sum_{i=1}^{k} \frac{y_i}{\mu(y)} \quad \text{for all } k = 1, 2, \ldots, n.$$

An IQO $\preceq$ will be referred to as *Lorenz consistent* if it agrees with all comparisons made by the Lorenz IQO, i.e., if $\prec_L \subset \prec$ and $\sim_L \subset \sim$. We shall refer to an SWO as Lorenz consistent if it follows the asymmetric part of the Lorenz IQO for comparisons between income distributions with the same mean incomes.[4]

Lorenz Consistency. For all $x, y \in X$, if $\mu(x) = \mu(y)$ and $x \prec_L y$, then xPy.

In the literature, social welfare functions are often assumed to depend on mean income and inequality only, i.e., it is assumed that there exists an inequality measure J and a function $f : (\mathbb{R}_{++} \times \mathbb{R}) \to \mathbb{R}$, increasing in the first argument and decreasing in the second, such that $W(x) = f(\mu(x), J(x))$ for all $x \in X$. In this framework, Lorenz consistency is a weak requirement for SWOs—it is sufficient that the underlying inequality measure is Lorenz consistent.[5] We note that all CES SWOs are Lorenz consistent and can be written as a function of mean income and inequality. Specifically, it can be shown that any CES SWO R_ε can be represented by a social welfare function of the form $W(x) = \mu(x)[1 - J_\varepsilon(x)]$ for all $x \in X$, where J_ε is a Lorenz consistent inequality measure.[6] We emphasize that

[3] A quasi-ordering is a reflexive and transitive binary relation.

[4] So we use the same term for two different concepts of Lorenz consistency. However, confusion is avoided because it will always be clear from the context whether the Lorenz consistency concept for IQOs or that for SWOs is meant.

[5] Note that, for any continuous, monotonic and separable SWO R, Lorenz consistency is satisfied if the following weaker criterion is satisfied: $\mu(x)1_n Px$ for all $x \in X$ such that x is not perfectly equal. See Chateauneuf and Moyes (2004, Proposition 4.1).

[6] See Atkinson (1970).

our results—with the sole exception of Proposition 4.8—do not assume that social welfare is a function of mean income and inequality only. To the contrary, we do not make any assumptions at all about the determinants of social welfare (see the discussion at the end of Section 4.3). Because the mean income-inequality representation of inequality is popular, we will however occasionally interpret results in that light.

4.3 Three Concepts of Inequality Aversion

In this section, we define three concepts for comparing degrees of inequality aversion based on Procedure ($\star$). We give a formal outline of this procedure. First, a set is determined that contains exactly all pairs of income distributions such that one income distribution is strictly more unequal than the other according to some 'reference' IQO (clearly, this set is simply the asymmetric part of the reference IQO on X). These are exactly all pairs for which each SWO either implies an inequality averse choice (the less unequal income distribution is chosen), a neutral choice (indifference), or an inequality prone choice (the more unequal one is chosen)—three choices which can of course be unambiguously ranked from most inequality averse to least inequality averse. Second, two SWOs are compared with respect to the choices implied for each of the pairs of income distributions in the asymmetric part of the reference IQO: one SWO is referred to as at least as inequality averse as the other if and only if it implies an at least as inequality averse choice for all pairs belonging to the reference set. The procedure can be defined formally as follows, with $\preceq_A$ taking the role of the reference IQO.

Definition 4.1. Let $\preceq_A$ be some IQO. Let R and R' be any two SWOs. Then, R is *at least as A-inequality averse as* R' if and only if, for all $x, y \in X$ such that $x \prec_A y$, we have (i) if $xP'y$, then xPy, and, (ii) if $xI'y$, then xRy.

As is conventional, we say that R is *more A-inequality averse than* R' if R is at least as A-inequality averse as R' while R' is not at least as A-inequality averse as R, and we say that R is *equally A-inequality averse as* R' if R is at least as A-inequality averse as R' and R' is at least as A-inequality averse as R.

In principle, any IQO can be chosen to determine the reference set $\prec_A$ in the outlined procedure. However, since different people may have different reasonable views with respect to inequality comparisons, it seems preferable to consider the common part of all these views. Now, this is exactly the role that is often attributed to the Lorenz criterion in the literature. We argue, therefore, that it is most appropriate to use as the set of pairs of income distributions for which two SWOs are compared, the set $\prec_L$. We refer to the concept of inequality aversion based on Definition 4.1 with $\preceq_A$ equal to $\preceq_L$ as the L-concept.

The L-concept is closely related to the concept of 'strong risk aversion' studied by Ross (1981). Ross' concept is obtained if the L-concept is restricted to SWOs of the expected utility form, i.e., SWOs satisfying continuity, monotonicity and separability, and if the absolute version of the Lorenz IQO is used instead of the regular (relative) version.[7]

Given the broad acceptance of the Lorenz IQO, we consider the L-concept to be the ideal concept for comparing degrees of inequality aversion, but to allow for a stronger link with the existing literature on the topic, we shall consider also two alternative concepts based on Definition 4.1 that will appear to be closer to the conventional Arrow-Pratt framework (as will be shown in Section 4.4). For these concepts, two IQOs are used that are (weaker) alternatives for the Lorenz IQO—that is, in both cases, in comparing two SWOs a set is considered which is a proper subset of $\prec_L$. The first alternative IQO we consider is the *minimalist* IQO, written as $\preceq_M$: for all $x, y \in X$,

$$x \preceq_M y \Leftrightarrow x = e1_n \text{ for some scalar } e.$$

The minimalist IQO only allows inequality comparisons between pairs of income distributions of which at least one is perfectly equal. The second alternative is the *relative differentials* IQO, written as $\preceq_{RD}$: for all $x, y \in X$,

$$x \preceq_{RD} y \Leftrightarrow \frac{x_i}{y_i} \geq \frac{x_{i+1}}{y_{i+1}} \quad \text{for all } i = 1, 2, \ldots, (n-1).$$

The relative differentials IQO, which was introduced into the literature on income distribution by Moyes (1994), says that any progressive redistribution decreases inequality. Setting $\preceq_A$ in Definition 4.1 equal to $\preceq_M$ or $\preceq_{RD}$, we obtain the M-concept and the RD-concept, respectively.

The M-concept is sometimes considered in the literature on risk aversion, but in a restricted version that makes the concept applicable only to SWOs of the expected utility form. It is an established result in this context that, for SWOs of the expected utility form, the M-concept and the Arrow-Pratt concept are equivalent.[8] A more general result will be shown to hold in Section 4.4.

Since the three concepts of inequality aversion rely on comparisons of choices over pairs of income distributions that are members of some set which represents a view on inequality, $\prec_M$, $\prec_{RD}$ and $\prec_L$, respectively, and given the fact that $\prec_M \subset \prec_{RD} \subset \prec_L$, the following remark is straightforwardly established.

[7]Definition 4.1 has, moreover, a different phrasing than the concept of Ross (1981). Statement (ii) of Proposition 4.1 below and condition (4.3) in the proof of Lemma 4.2 below, are closer to the formulation used by Ross.

[8]See, e.g., Mas-Colell et al. (1995, Proposition 6.C.2)—the restricted version of the M-concept is close to their statement (v).

Remark 4.1. Let R and R' be any two SWOs. Then, of the following three statements, (i) implies (ii), but (ii) does not imply (i), and (ii) implies (iii), but (iii) does not imply (ii):

(i) R is at least as L-inequality averse as R';
(ii) R is at least as RD-inequality averse as R';
(iii) R is at least as M-inequality averse as R'.

The relationships described in Remark 4.1 also hold for the relation 'is equally inequality averse as,' but not for the relation 'is more inequality averse than.'

Remark 4.1 shows that the RD-concept is more demanding than the M-concept and, in turn, the L-concept is more demanding than the RD-concept. A consequence is that if, for instance, the M-concept and the L-concept yield a different conclusion, then this disagreement will typically be of the type where the M-concept ranks two SWOs whereas the L-concept does not. The converse case, as well as cases in which the M-concept and the L-concept rank two SWOs in opposite ways, are excluded by Remark 4.1. In this respect, it is important to note that if two SWOs, say R and R', are incomparable according to one of the three concepts of inequality aversion, this does not simply mean that there is not sufficient evidence to refer to one SWO as at least as inequality averse as the other, but, more strongly, it means that the evidence is pointing in different directions: for some pair(s) of income distributions, R is locally more inequality averse than R', while, for (an)other pair(s) R', is locally more inequality averse than R.

Comparisons of inequality aversion are often interpreted as comparisons of the willingness of SWOs to sacrifice mean income in return for a given decrease in inequality. Since this view of inequality aversion as essentially describing a trade-off between mean income and equality is popular, we wish to demonstrate that the L-concept, the M-concept and the RD-concept are consistent with it—i.e., that these three concepts can be rephrased in terms of the mean income-equality trade-off. The following proposition shows that according to each of the three concepts, for any continuous and monotonic SWOs R and R', R is at least as inequality averse as R' if and only if, starting from any income distribution, R accepts a move to a given lower level of inequality at a loss of at least as much income as R' does.

Proposition 4.1. *Let $\preceq_A$ be equal to either $\preceq_L$, $\preceq_M$ or $\preceq_{RD}$. Let R and R' be any two continuous and monotonic SWOs. Then, the following two statements are equivalent:*

(i) *R is at least as A-inequality averse as R';*
(ii) *for all $x, x', y \in X$ such that $x \sim_A x'$, $x \prec_A y$, xIy and $x'I'y$, we have $\mu(x) \leq \mu(x')$.*

To conclude the section, we mention two reasons for preferring the simple formulation used in Definition 4.1—i.e., the formulation in terms only of preferences over pairs of income distributions—to the more traditional formulation in terms of the mean income-equality trade-off. First, the formulation in Definition 4.1 has the advantage that it allows application of the inequality aversion concepts to *all* SWOs—also for instance to non-continuous SWOs, which will be useful in the discussion of extreme inequality aversion in Section 4.5. Second, a deeper concern is that an explicit reference to a mean income-equality trade-off may in certain cases misrepresent what comparisons of inequality aversion are really about. In general, there is no reason why equality should be traded off *only* with mean income. SWOs may express interest for other concerns, such as poverty alleviation for instance—then, the trade-off with mean income is just one of several trade-offs that are relevant for the idea of inequality aversion. As the neutral formulation used in Definition 4.1 does not refer to any particular trade-off, it seems to better capture the general essence of the idea of inequality aversion.

4.4 The Three Concepts and the Arrow-Pratt Approach

The objective of this section is to compare the conventional Arrow-Pratt concept with the three concepts of inequality aversion that were presented in the previous section.

We first define the Arrow-Pratt concept. The analysis of Pratt (1964) concerning risk aversion has provided several equivalent concepts that can be applied to the problem of comparing degrees of inequality aversion (see also Lambert, 2001, pp. 94-97). Some of these concepts can only be used to compare SWOs that can be written in the expected utility form, i.e., SWOs that satisfy continuity, monotonicity and separability. This class is important and we shall pay attention to it in this section. However, because we wish to initially consider the entire class of continuous and monotonic SWOs, we focus on the strongest of Pratt's concepts that is applicable also to non-separable SWOs, viz., the criterion based on the *equally distributed equivalent income.* The equally distributed equivalent income, $\xi(R;x)$, for any income distribution x and any SWO R, is the income that, when equally distributed, yields the same level of welfare according to R as the income distribution x.[9] Formally, for any SWO R and any $x \in X$, we have $\xi(R;x) = e$ if and only if $e1_nIx$. The Arrow-Pratt concept of inequality aversion is defined as follows.

[9]See Atkinson (1970) and Kolm (1969).

Definition 4.2. Let R and R' be any two continuous and monotonic SWOs. Then, R is *at least as Arrow-Pratt inequality averse as* R' if and only if, for all $x \in X$, $\xi(R;x) \leq \xi(R';x)$.[10]

The 'more inequality averse than' and 'equally inequality averse as' relations corresponding to the Arrow-Pratt concept are defined in the same way as for the inequality aversion concept of Definition 4.1.

Although we are most interested in the L-concept for the reason specified in Section 4.3, it is convenient for expositional purposes to start with the comparison of the Arrow-Pratt concept with the M-concept and RD-concept. These criteria will turn out to be closer to the Arrow-Pratt concept than the L-concept is. Consider first the following lemma.

Lemma 4.1. *Let R and R' be any two continuous, monotonic and separable SWOs. Consider any $x, y \in X$ such that*

$$\text{there is an integer } k \text{ such that } x_i \geq y_i \text{ for all } i < k, \text{ and } x_i \leq y_i \text{ for all } i \geq k. \quad (4.2)$$

Then, of the following two statements, (i) *implies* (ii)*:*

(i) *R is at least as Arrow-Pratt inequality averse as R';*
(ii) *if $xP'y$, then xPy, and, if $xI'y$, then xRy.*

The following proposition summarizes the relationships between the Arrow-Pratt concept, the M-concept and the RD-concept.

Proposition 4.2. *Let R and R' be any two continuous and monotonic SWOs. Consider the following three statements:*

(i) *R is at least as Arrow-Pratt inequality averse as R';*
(ii) *R is at least as M-inequality averse as R';*
(iii) *R is at least as RD-inequality averse as R'.*

Then, we have:

(a) *statements* (i) *and* (ii) *are equivalent;*
(b) *statement* (iii) *implies statement* (i)*, but statement* (i) *does not imply statement* (iii)*;*
(c) *if, in addition, R and R' are separable, then statements* (i), (ii) *and* (iii) *are equivalent.*

[10] Note that the Arrow-Pratt concept compares, for all income distributions, how much sacrifice of mean income SWOs maximally allow in order to move from a given income distribution to a perfectly equal one—for an SWO R and an income distribution x, this sacrifice equals $[\mu(x) - \xi(R;x)]$.

We mentioned in the previous section that the M-concept and the Arrow-Pratt concept are equivalent for continuous, monotonic and separable SWOs. As statement (a) of Proposition 4.2 shows, this equivalence also holds if separability is not demanded. This result is not very surprising, given the fact that the definitions of both the M-concept and the Arrow-Pratt concept refer to preferences over pairs of income distributions of which one is perfectly equal. Statement (b) shows that, if we take the step from the minimalist IQO to the relative differentials IQO as the underlying inequality criterion for the concept of inequality aversion, then we move away from convention. The inconsistency of the RD-concept and the Arrow-Pratt concept consists of there being SWOs such that the Arrow-Pratt concept ranks them while the RD-concept does not. Finally, statement (c) shows that the RD-concept and the Arrow-Pratt concept agree on how to rank any pair of SWOs of the expected utility form.

With respect to the M-concept and the RD-concept we may conclude that the former, and to a lesser extent the latter, support the claims made traditionally in the literature on the basis of the Arrow-Pratt concept. An important question we now turn to is whether the favoured L-concept is consistent with these claims. We already know, by Remark 4.1 and Proposition 4.2, that the L-concept and the Arrow-Pratt concept cannot be equivalent for the entire class of monotonic and continuous SWOs, so the question becomes whether this equivalence holds for the expected utility class of SWOs (as for the RD-concept), or at least for the popular CES subclass. This appears *not* to be the case. According to the Arrow-Pratt concept, an SWO in the CES class is more inequality averse as the value of its corresponding ε is greater.[11] For this reason, ε is traditionally interpreted as being a parameter of inequality aversion. Now, there are several pairs of CES SWOs R_ε and $R_{\varepsilon'}$ such that $\varepsilon > \varepsilon'$, and several pairs of income distributions $x, y \in X$ such that $x \prec_L y$, for which we have $yP_\varepsilon x$ while $xP_{\varepsilon'}y$. This is illustrated in the following example.

Example 4.1. The example is for the case $n = 3$. Take the income distributions $x = (19, 57, 76)$ and $y = (20, 20, 130)$. We have $x \prec_L y$. However, for all CES SWOs with ε such that $0.403 < \varepsilon < 14.513$, we have $xP_\varepsilon y$, while for all CES SWOs with $\varepsilon > 14.514$, we have $yP_\varepsilon x$.[12]

Note that the example exploits the fact that the parameter ε plays a double role in the CES class: it is a parameter of inequality aversion in the Arrow-Pratt sense, but it is also a parameter that measures the sensitivity of the SWO to inequality in the bottom of the income distribution relative to inequality in the top

[11] In fact, ε is the value of the relative Arrow-Pratt measure of risk/inequality aversion.

[12] Note that we have $\mu(x) < \mu(y)$ in the example. This is no coincidence since if we would have $\mu(x) \geq \mu(y)$ and $x \prec_L y$, then all CES SWOs would strictly prefer x over y, as can be easily established using the fact that all these SWOs satisfy Lorenz consistency.

(see Cowell, 1985). Indeed, while $y = (20, 20, 130)$ is overall more unequal than $x = (19, 57, 76)$ according to the Lorenz IQO, which is all that is required for our purposes, it is true that y has less inequality in the bottom of the income distribution (i.e., between individuals 1 and 2) than x. Accordingly, the CES SWOs with higher bottom sensitivity, those with higher values of ε, prefer y to x.

Using a result by Ross (1981) it is possible to draw even stronger conclusions with respect to the CES class. Ross' critique of the Arrow-Pratt framework can be interpreted as a confrontation of the L-concept and the M-concept in the framework of expected utility theory. The following lemma is based on one of his results.

Lemma 4.2. *Let R_u and R_v be any two continuous, monotonic and separable SWOs such that the respective corresponding utility functions u and v, are twice differentiable. Then, the following two statements are equivalent:*

(i) *R_u is at least as L-inequality averse as R_v;*

(ii) *there exist a decreasing and concave function $f : \mathbb{R}_{++} \to \mathbb{R}$ and a scalar $\lambda > 0$ such that, for all $t \in \mathbb{R}_{++}$, $u(t) = \lambda v(t) + f(t)$.*

It can be shown now that in the entire class of CES SWOs there are no two SWOs that can be compared using the L-concept of inequality aversion.

Proposition 4.3. *Let R_ε and $R_{\varepsilon'}$ be any CES SWOs such that $\varepsilon \neq \varepsilon'$. Then, R_ε and $R_{\varepsilon'}$ are incomparable according to the L-concept, i.e., R_ε is not at least as L-inequality averse as $R_{\varepsilon'}$, and $R_{\varepsilon'}$ is not at least as L-inequality averse as R_ε.*

The CES class of SWOs is often considered to be very useful in practice because, according to the conventional Arrow-Pratt approach, it encompasses a continuum of positions with respect to inequality aversion from the completely non-egalitarian mean income rule ($\varepsilon = 0$) to leximin ($\varepsilon \to \infty$) which is often viewed as extremely inequality averse (see Section 4.5, but also Chapter 3). The class owes its popularity furthermore to the fact that it has attractive properties from the theoretical perspective: all CES SWOs satisfy the basic axioms continuity, monotonicity and separability, and allow a natural decomposition into mean income and a Lorenz consistent inequality measure as explained in Section 4.2. However, the deep inconsistency between, on the one hand, the conventional interpretation of the parameter ε and, on the other hand, the L-concept may be seen as somewhat damaging for the CES class to operate as a canonical class of SWOs. The problem is aggravated by the fact that all members of the CES class ascribe importance to the Lorenz IQO—and thus the L-concept—because they are all Lorenz consistent. Is it possible to find another class of SWOs which both has attractive properties and encompasses a continuum of degrees of inequality aversion according to the

L-concept? Although we shall not attempt to answer this question here, we wish to note that a sacrifice will have to be made irrespective of the direction in which an answer is sought. For instance, the analysis of Ross (1981) can be used to construct a class of SWOs to play a role similar to that of the CES class, in which case continuity, monotonicity and separability will still be satisfied. However, it may possibly be seen as a drawback that in that case the natural link between welfare and an underlying criterion of (Lorenz consistent) inequality will be lost. Alternatively, such a natural link can be taken as a starting point to construct an alternative to the CES class, but at the cost of separability.[13]

4.5 Extreme Inequality Aversion

In this section, we characterize the classes of SWOs that reconcile monotonicity with an extreme form of inequality aversion for each of the three concepts proposed in Section 4.3.[14]

Conventionally, maximin and leximin, both of which give absolute priority to the worst off, are seen as typical examples of extremely inequality averse SWOs. Maximin implies indifference in all cases in which the worst off is equally well off, i.e., an SWO R is *maximin* if and only if, for all $x,y \in X$, $xRy \Leftrightarrow x_1 \geq y_1$. Leximin, on the other hand, gives priority to the second worst off in the cases where the worst off is equally well off in both alternatives, and so on, i.e., an SWO R is *leximin* if and only if, for all $x,y \in X$, we have

$$xRy \Leftrightarrow x = y, \text{ or, there is an integer } k \text{ such that } x_i = y_i \text{ for all } i < k \text{ and } x_k > y_k.$$

Maximin and leximin are both members of the class of weakly maximin SWOs, which is the class of SWOs that all have in common the asymmetric part of maximin, i.e., an SWO R is *weakly maximin* if and only if, for all $x,y \in X$, if $x_1 > y_1$, then xPy. It can be shown that maximin is the only continuous member of the class of weakly maximin SWOs and that leximin is the only separable member of the class.[15] It will be of interest to see what role leximin plays in our analysis, since this is the only popular SWO that is commonly viewed as combining extreme inequality aversion with monotonicity—maximin, by contrast, does not satisfy monotonicity.

[13] See Champernowne and Cowell (1998, pp. 107-108) on a similar point.

[14] How the ideals of extreme inequality aversion and monotonicity can be combined is an important question in egalitarian social ethics. See Tungodden (2003, pp. 10-23) for an overview of the economic and philosophical literature concerning this topic.

[15] See Chapter 5.

The starting point of our analysis is the following definition of the idea of extreme inequality aversion.

Definition 4.3. Let S be a class of SWOs. An SWO R is *extremely inequality averse* in the class S if and only if R is a member of S and R is at least as inequality averse as any member of S.

This definition assures that an extremely inequality averse SWO in S never implies a choice over a pair of income distributions that is less inequality averse than that implied by any other member of S. Note also that all extremely inequality averse SWOs are equally inequality averse.

In what follows, we identify the members of the class of monotonic SWOs that are extremely inequality averse according to the M-concept, the RD-concept and the L-concept. Since we do not require continuity, the standard Arrow-Pratt concept cannot be applied in this context—however, it is natural to interpret the M-concept as being the evident extension of the Arrow-Pratt concept capable of such comparisons. Again, it is convenient to begin the analysis by considering the M-concept and the RD-concept.

Proposition 4.4. *Let R be any monotonic SWO. Then, the following five statements are equivalent:*

(i) *R is extremely M-inequality averse in the class of monotonic SWOs;*
(ii) *R is extremely RD-inequality averse in the class of monotonic SWOs;*
(iii) *for all* $x, y \in X$ *such that not* $x < y$*, we have, if* $x \prec_M y$*, then* xPy*;*
(iv) *for all* $x, y \in X$ *such that not* $x < y$*, we have, if* $x \prec_{RD} y$*, then* xPy*;*
(v) *R is weakly maximin.*

The equivalence of (i) and (v) in Proposition 4.4 says that, the case of extreme M-inequality aversion in the class of monotonic SWOs is covered by the monotonic weakly maximin SWOs. To a certain extent, this result supports the conventional view that leximin constitutes the case of extreme inequality aversion. The reason is that the literature focuses virtually exclusively on separable SWOs when studying extreme inequality aversion, combined with the fact that leximin is the only separable weakly maximin SWO.[16] The finding that (i) and (v) are equivalent is important for two reasons. Firstly, given Remark 4.1, it follows from this

[16]The interpretation of leximin as being extremely inequality averse can be defended on the basis of the Arrow-Pratt concept. As we saw in Chapter 3, Hammond (1975) has demonstrated that leximin can be interpreted as the limit case, $\varepsilon \to \infty$, of the CES class of SWOs, a point which Lambert (2001, Theorem 4.4) has generalized with respect to the entire class of continuous, monotonic and separable SWOs. In Chapter 3, we have shown that using an approach analogous to that of Hammond (1975) and Lambert (2001) the weakly maximin class can be identified as extremely inequality averse on the basis of the Arrow-Pratt concept if separability is dropped. Proposition 4.4 confirms the latter result.

result that the classes of extremely inequality averse SWOs that are implied by the *RD*-concept and the *L*-concept must be subsets of the class of monotonic weakly maximin SWOs. Secondly, it presents another way of seeing why the *M*-concept is unattractive. As an illustration of this point, consider the following SWO R: for all $x,y \in X$, we have,

$$\text{if } x_1 > y_1 \text{, then } xPy \text{, and}$$
$$\text{if } x_1 = y_1 \text{, then } [xRy \Leftrightarrow \mu(x) \geq \mu(y)].$$

Clearly, this SWO is both monotonic and weakly maximin. Now note that whenever two income distributions have the same lowest incomes, this SWO ranks them according to the completely non-egalitarian mean income rule.[17] Probably, many would hesitate to refer to such an SWO as extremely inequality averse, thus implicitly accepting that the *M*-concept is too undemanding as a criterion for comparing degrees of inequality aversion. However, as the equivalence of (ii) and (v) shows, moving on to the *RD*-concept does not solve anything: the class of monotonic weakly maximin SWOs is still identified as the extremely inequality averse subclass of the class of monotonic SWOs. Before we consider which monotonic weakly maximin SWOs survive the test of Definition 4.3 when we move to the *L*-concept, we consider the other statements of Proposition 4.4.

The conditions expressed in statements (iii) and (iv) of Proposition 4.4 constitute a natural way of giving meaning to extreme inequality aversion for SWOs that satisfy monotonicity—the conditions say that one should prefer, for any pair of income distributions, the one which is less unequal (according to the minimalist IQO and the relative differentials IQO in statements (iii) and (iv), respectively) unless the income distribution is worse for some and better for none. In a recent study on the possibility of combining extreme inequality aversion and monotonicity, Tungodden and Vallentyne (2005) have taken natural conditions as those expressed in statements (iii) and (iv) as a starting point (so, relying only implicitly on the concepts defined in our Definitions 4.1 and 4.3). They have considered a condition similar to that of statement (iii) and also show that statements (iii) and (v) are equivalent. Later, we draw a more interesting parallel between the present work and theirs.

Now, we come to the important question of which SWOs are extremely inequality averse according to the *L*-concept. Note first that while the *M*-concept and the *RD*-concept identify all weakly maximin SWOs as extremely inequality averse, according to the *L*-concept no member of this class is extremely inequality averse.

[17] Note that the comparison of such income distributions is probably even quite common in practice—think of a change in the tax system that leaves the existing minimally guaranteed income unaffected.

Proposition 4.5. *Let R be any continuous and monotonic SWO satisfying Lorenz consistency. Then, there are several pairs* $x, y \in X$ *for which* $x \prec_L y$*, such that we have xPy, while all weakly maximin SWOs strictly prefer y to x.*

Proposition 4.5 demonstrates that, given the *L*-concept, the weakly maximin SWOs do not only fail the test of extreme inequality aversion described in Definition 4.3, they do so in a particularly bad way. The proposition says that, for instance, it is possible to find pairs of income distributions such that a CES SWO with ε arbitrarily close to, but greater than, zero, and hence arbitrarily close to the completely non-egalitarian mean income rule, is locally more inequality averse than all weakly maximin SWOs for these pairs. As an illustration, note that, for the income distributions x and y in Example 4.1, all weakly maximin SWOs strictly prefer y to x. Because any extremely *L*-inequality averse SWO in the class of monotonic SWOs must be weakly maximin by Remark 4.1 and Proposition 4.4, the following result follows immediately from Proposition 4.5.

Proposition 4.6. *There is no SWO that is extremely L-inequality averse in the class of monotonic SWOs.*

So, we conclude that if we accept the *L*-concept, then extreme inequality aversion is incompatible with monotonicity. In their work, Tungodden and Vallentyne (2005) reach a similar conclusion. However, they implicitly use a criterion that lies in between the *M*-concept and the *RD*-concept, and find an incompatibility.[18] This is possible because they use a slightly (but significantly) different framework than the one used here: their result is driven by the fact that they reject anonymity as a property of SWOs, but accept it for IQOs. The present study shows that without this assumption, there is no incompatibility between their version of extreme inequality aversion and monotonicity (this is implied by the equivalence of (ii) and (v) in Proposition 4.4), but that the incompatibility crops up again when the *L*-concept is accepted (Proposition 4.6).[19]

What should egalitarians who agree with the *L*-concept and want both monotonicity and extreme inequality aversion choose as an SWO? It might at first glance seem natural to regard leximin or other monotonic weakly maximin SWOs as being ‘close enough’—these SWOs satisfy a necessary condition for being extremely inequality averse (they are extremely inequality averse if one looks only at the pairs in $\prec_M$ or $\prec_{RD}$), and a sufficient condition cannot be satisfied (being extremely inequality averse for those in $\prec_L$ is impossible), hence why not content

[18] More precisely, they use a condition similar to that stated in statements (iii) and (iv) of Proposition 4.4, but with, instead of the minimalist or relative differentials IQO, an IQO that is a proper subrelation of the relative differentials IQO and a proper superrelation of the minimalist IQO.

[19] In Tungodden (2000) it is also shown that, without rejecting anonymity, their extreme inequality aversion condition and monotonicity are compatible.

ourselves with these? Proposition 4.5 illustrates already how unattractive it is to settle for a conclusion based on the less demanding criteria *M*-concept and *RD*-concept if the *L*-concept is the one which is deemed ideal. There is also a deeper reason for extreme egalitarians not to (necessarily) focus on the class of weakly maximin SWOs. It is perfectly acceptable to consider the pairs ordered by the minimalist IQO (i.e., the set $\prec_M$) as not being more important than some alternative set of pairs ordered by the Lorenz IQO (i.e., a subset of $\prec_L$ which differs from $\prec_M$). If one accepts the Lorenz IQO, these former pairs of income distributions are not special in any way. If such an alternative set of pairs is used in a criterion for comparing degrees of inequality aversion, in accordance with the explanation at the beginning of Section 4.3, then the set of extremely inequality averse monotonic SWOs need not be empty, nor contain any weakly maximin SWOs. For instance, if the income distributions from Example 4.1 are members of this alternative set, then none of the weakly maximin SWOs pass the test of extreme inequality aversion of Definition 4.3, while (depending on the other elements of the set) other SWOs may pass the test.

To conclude the section, we consider two alternative ways of giving meaning to the view that inequality reduction should always be preferred unless no one gains by it. However, as we shall see, neither alternative produces a convincing way out of the incompatibility.

The first alternative is to consider the SWOs for which no monotonic SWO is more inequality averse according to the *L*-concept, instead of the ones that are at least as inequality averse as all the other monotonic ones according to the *L*-concept (as in Definition 4.3). Consider the following definition of this alternative concept of 'maximal inequality aversion.'

Definition 4.4. Let *S* be a class of SWOs. An SWO *R* is *maximally inequality averse* in the class *S* if and only if *R* is a member of *S* and no member of *S* is more inequality averse than *R*.[20]

The subset of maximally *L*-inequality averse SWOs in the set of monotonic SWOs is not empty: as the following proposition shows, at least leximin is a member.

Proposition 4.7. *Leximin is maximally L-inequality averse in the set of monotonic SWOs.*

[20] Note that the ideas of extreme inequality aversion and maximal inequality aversion do not, in general, coincide if the concept of inequality aversion does not provide a complete ranking (as is the case for all inequality aversion concepts discussed in this chapter). The distinction between extreme inequality aversion and maximal inequality aversion is analogues to the distinction made by Sen (1997) between optimization and maximization, respectively, in individual choice theory.

However, the concept of maximal inequality aversion seems too undemanding, because it is not excluded that there are SWOs, which are themselves unlikely candidates for being considered extremely inequality averse, that are more inequality averse for at least some pairs of income distributions—in the case of leximin, Proposition 4.5 should suffice to make this point.

A second alternative is to start from the view that SWOs are functions of an underlying inequality measure or IQO, a view not uncommon in the literature as we saw in Section 4.2. In that perspective, the following approach to combining monotonicity and an absolute preference for inequality reduction seems reasonable: choose an SWO that, for all pairs of income distributions to which monotonicity applies, follows monotonicity, and, for all pairs to which monotonicity does not apply, prefers the income distribution which minimizes inequality according to some IQO (or some inequality measure). Note that this approach does not require a complete IQO since the IQO need not order pairs of income distributions to which monotonicity applies—it does have to order all pairs to which monotonicity does not apply, however. The question is whether it is possible to find an SWO and a corresponding IQO that satisfy the required condition. First we need to consider some minimal criteria that a sensible IQO ought to satisfy. The first is that it should have the minimalist IQO as a subrelation. The second is that it satisfies some invariance criterion. An invariance criterion defines the transformation which when applied to all incomes leaves inequality invariant. For instance, the invariance criterion underlying the Lorenz IQO and the relative differentials IQO is scale invariance, which says: for all $x \in X$ and all scalars $\lambda > 0$, $x \sim \lambda x$. However, we will demand only that a much weaker invariance criterion is satisfied. Minimal invariance says that for any given income distribution there must exist an income distribution in which everyone is better off and which is at least as unequal as the given income distribution.

Minimal Invariance. For all $x \in X$, there is a $x' \in X$ such that $x' \gg x$ and $x \preceq x'$.

The following proposition shows that no SWO and IQO with the described properties exist.

Proposition 4.8. *Let R be any monotonic SWO and let $\preceq$ be any IQO that satisfies minimal invariance and for which $\prec \supset \prec_M$. Then, the following condition is not satisfied: for all $x, y \in X$ such that not $x < y$, $x \preceq y \Leftrightarrow xRy$.*

The proposition implies that whatever the concept of inequality used (requiring only that it satisfies minimal conditions—far weaker than Lorenz consistency for instance), say, leximin at least for some pairs of income distributions will not choose the least unequal one according to this concept of inequality, even though this income distribution is not worse by monotonicity. Moreover, this is not only true for leximin, but for all monotonic SWOs.

4.6 Concluding Remarks

In this chapter, we studied a straightforward dominance procedure for comparing SWOs with respect to degree of inequality aversion. We considered three versions of the procedure based on three inequality criteria: the L-concept which we argued to be the ideal version, the M-concept which is roughly equivalent to the traditional Arrow-Pratt approach, and the RD-concept which is intermediate in strength between the other two concepts.

It was shown that the L-concept is in general incompatible with the M-concept. In the case of the CES class of SWOs, the difference between the conclusions produced by the two concepts was especially pronounced: whereas the M-concept ranks all members of this class, the L-concept ranks none. As we have said already, it would be interesting to think about theoretically agreeable alternatives to the CES class of which the members can be ranked using the L-concept and which covers a wide spectrum of positions with respect to inequality aversion. Probably the most attractive solution is to give up separability and to consider classes of SWOs such as those given by $W(x) = \mu(x)[1 - J(x)]^{\alpha}$ for all $x \in X$, where J is a Lorenz consistent inequality measure and α is a parameter that measures inequality aversion in accordance with the L-concept. It may be interesting to see whether classes of SWOs in the spirit of this example can be constructed in a theoretically and philosophically sound way starting directly from the idea of the natural decomposition of welfare in mean income and inequality.

We showed, furthermore, that if we accept the L-concept, then monotonicity and extreme inequality aversion are incompatible. Hence, egalitarians committed to monotonicity have to content themselves with being less than extremely inequality averse: it is always possible to find pairs of income distributions for which a less inequality averse choice than possible must be made. Those who are attracted to both the ideals of monotonicity and extreme inequality aversion have to determine which of the two to weaken. We have discussed that if extreme inequality aversion is weakened, nothing forces one to opt for a weakly maximin SWO such as leximin. It is perfectly possible to choose a different set over which one wants to make inequality averse choices than the set that forces one to give full priority to the worst off. The other possibility, not yet discussed, is to weaken monotonicity. For instance, a possibility is to demand only *ray-monotonicity*: for all $x \in X$ and all $\lambda > 1$, $\lambda x P x$. It can easily be shown that there exist extremely inequality averse SWOs according to the L-concept in the class of ray-monotonic SWOs.[21] Interestingly, not only does the weakening to ray-monotonicity make it possible to have extremely inequality averse SWOs, but none of them is weakly

[21]Consider the example of an SWO R: for all $x, y \in X$, we have, if $x \prec y$, then xPy, and if $x \sim y$, then $[xRy \Leftrightarrow \mu(x) \geq \mu(y)]$, where $\preceq$ is a Lorenz consistent and complete IQO.

maximin (and this is true even if we use the *M*-concept instead of the *L*-concept). In other words, whichever of the two ideals egalitarians choose to weaken in order to deal with the incompatibility, they should not feel required to restrict their consideration to leximin or other weakly maximin SWOs.

Appendix 4.A: Proofs

Proof of Proposition 4.1. We provide a proof for the case where $\preceq_A$ is equal to $\preceq_L$. The proofs for the cases where $\preceq_A$ is equal to $\preceq_M$ or $\preceq_{RD}$ are very similar and are, therefore, omitted.

First, we show that statement (i) implies statement (ii). Assume that (i) is true. Take any $x, x', y \in X$ such that $x \sim_L x'$, $x \prec_L y$, xIy and $x'I'y$. We have to show that $\mu(x) \le \mu(x')$. Note that, for all $z, w \in X$, $z \sim_L w$ if and only if there exists a scalar $\lambda > 0$ such that $\lambda z = w$. Hence, there exists a $\lambda > 0$ such that $\lambda x' = x$. Now, we have $x'Ry$ by (i). Since also xIy, it follows that $\lambda \le 1$ by monotonicity. By consequence, we obtain $\mu(x) \le \mu(x')$.

Second, we show that statement (ii) implies statement (i). Assume that (ii) is true. Take any $x, y \in X$ such that $x \prec_L y$. By continuity and monotonicity, there exist $\lambda, \lambda' > 0$ such that λxIy and $\lambda' xI'y$. To check statement (ii), suppose, first, that $xP'y$. We have to show that in this case xPy. Note that $xP'y$ implies $\lambda' < 1$ by monotonicity. Since, furthermore, $\lambda \le \lambda'$ by (ii), we have $\lambda < 1$. Hence, monotonicity implies xPy. By a similar reasoning, $xI'y$ is seen to imply xRy. ■

Proof of Lemma 4.1. Let R_u and R_v be any two continuous, monotonic and separable SWOs with u and v as utility functions in (4.1), respectively. Suppose that statement (i) is true for R_u and R_v, i.e., $\xi(R_u; x) \le \xi(R_v; x)$ for all $x \in X$. We denote the sets of individuals who gain and individuals who lose in going from any $y \in X$ to any $x \in X$ by $G(y,x) = \{i \,|\, x_i > y_i\}$ and $L(y,x) = \{i \,|\, x_i < y_i\}$, respectively. Throughout the proof we consider the pair $x, y \in X$, which is any pair for which condition (4.2) of Lemma 4.1 is satisfied. What has to be shown is that statement (ii) is true for this pair, i.e., if xI_vy, then xR_uy, and, if xP_vy, then xP_uy. Therefore, it is assumed that we have either xI_vy or xP_vy. Since statement (ii) is trivially true for the pair x, y in the cases where $G(y,x) = \varnothing$ or $L(y,x) = \varnothing$, we only consider cases in which both sets are nonempty.

We require a tool to bring any income distribution $z \in X$ closer to x by replacing at least one of the components of z by a component of x, and this in such a way that for the resulting income distribution z' we have zI_vz'. Consider a function T_{ij} which transforms any $z \in X$ into z' by replacing two and only two incomes, z_i and z_j. The function T_{ij} has z in its domain if and only if $z_i < x_i \le x_j < z_j$, in other words, if and only if $i \in G(z,x)$ and $j \in L(z,x)$, and, furthermore, $i < j$. The values of z'_i and z'_j are determined as follows: (a) if $v(x_i) + v(x_j) = v(z_i) + v(z_j)$, then $(z'_i, z'_j) = (x_i, x_j)$, (b) if $v(x_i) + v(x_j) > v(z_i) + v(z_j)$, then $(z'_i, z'_j) = (s, x_j)$, where s is such that $z_i < s < x_i$ and $v(z_i) + v(z_j) = v(s) + v(x_j)$, (c) if $v(x_i) + v(x_j) < v(z_i) + v(z_j)$, then $(z'_i, z'_j) = (x_i, t)$, where t is such that $x_j < t < z_j$ and $v(z_i) + v(z_j) = v(x_i) + v(t)$. The s and t considered in cases (b) and (c), respectively, always exist by continuity and monotonicity. Note that indeed zI_vz'.

Now, using T_{ij}, we transform y step by step into x. First, transform y into y' by applying T_{ij} for some $i \in G(y,x)$ and some $j \in L(y,x)$. If $G(y',x)$ and/or $L(y',x)$ is empty, stop. Otherwise, perform the transformation on y' by again applying T_{ij} for some $i \in G(y',x)$ and some $j \in L(y',x)$ such that $i < j$. Repeat this until the set of those who gain and/or the set of those who lose is empty after the transformation, then stop. Note that the transformation can always be performed if the sets are both nonempty due to the fact that $x, y \in X$ satisfy condition (4.2) of Lemma 4.1, since the condition implies that, for all $i \in G(y,x)$ and all $j \in L(y,x)$, we have $i < j$, and this continues to hold after every step. So, the income distribution that results from the final step, say y'', has the property that $G(y'',x)$ and/or $L(y'',x)$ is empty and $y'' I_v y$. Furthermore, it is impossible that $G(y'',x)$ is empty while $L(y'',x)$ is not, since otherwise $x < y''$ and so $y'' P_v x$ by monotonicity, so that the fact that $y'' I_v y$ would imply that $y P_v x$, contrary to what we assumed. So, given that $L(y'',x)$ must be empty, we have either $y'' < x$ or $y'' = x$ and, by consequence, x is weakly preferred to y'' by any monotonic SWO.

We shall now show that $y'' I_v y$ implies $y'' R_u y$. It is known from Pratt (1964) that $\xi(R_u;x) \leq \xi(R_v;x)$ for all $x \in X$ implies that $u = f \circ v$ where the function $f : \mathbb{R} \to \mathbb{R}$ is strictly increasing and concave. Now for any $z \in X$, $z I_v z'$ is equivalent to $v(z_i') + v(z_j') = v(z_i) + v(z_j)$ or $v(z_i') - v(z_i) = v(z_j) - v(z_j')$. Since, furthermore, $v(z_j) > v(z_j') > v(z_i') > v(z_i)$, we have $u(z_i') - u(z_i) \geq u(z_j) - u(z_j')$ by strict increasingness and concavity of f. So, we have $z' R_u z$. Hence, by transitivity, it follows that $y'' R_u y$

We can now conclude the following. In the case where $x I_v y$, we have indeed $x R_u y$, since $y'' R_u y$ and since x is weakly preferred to y'' by any monotonic SWO. In the case where $x P_v y$, we have indeed $x P_u y$ since $x P_v y$ and $y'' I_v y$ imply $x P_v y''$ and hence $x > y''$, so that, by $x P_u y''$ and $y'' R_u y$, we obtain $x P_u y$. ■

Proof of Proposition 4.2. (a) Statement (i) of Proposition 4.2 is equivalent to statement (ii) of Proposition 4.1 for $\preceq_A = \preceq_M$. Hence, the result follows from Proposition 4.1.

(b) That (iii) implies (i) follows from statement (a) and Remark 4.1. We give an example to show that (i) does not imply (iii). Consider an SWO R such that, for all $x, y \in X$,

$$xRy \Leftrightarrow F(x) \geq F(y),$$

where

$$F(x) = \begin{cases} \frac{2}{3}x_1 + \frac{1}{3}x_2 + \sum_{i=3}^{n} x_i & \text{if } x_1 \geq \frac{2}{5}x_2; \\ \frac{3}{13}(4x_1 + x_2) + \sum_{i=3}^{n} x_i & \text{if } x_1 \leq \frac{2}{5}x_2. \end{cases}$$

Consider also an SWO R' such that, for all $x, y \in X$,

$$xR'y \Leftrightarrow G(x) \geq G(y),$$

where

$$G(x) = \begin{cases} \frac{3}{5}x_1 + \frac{2}{5}x_2 + \sum_{i=3}^{n} x_i & \text{if } x_1 \geq \frac{1}{2}x_2; \\ \frac{7}{30}(4x_1 + x_2) + \sum_{i=3}^{n} x_i & \text{if } x_1 \leq \frac{1}{2}x_2. \end{cases}$$

Both SWOs are clearly continuous and monotonic.

First, we show that R is at least as Arrow-Pratt inequality averse as R'. We consider, in turn, the three possible cases. (I) Case where $x \in A = \{x \in X \mid x_1 \geq \frac{1}{2}x_2\}$: Note that $\xi(R;x) = \frac{1}{n-1}(\frac{2}{3}x_1 + \frac{1}{3}x_2 + \sum_{i=3}^{n} x_i)$ and $\xi(R';x) = \frac{1}{n-1}(\frac{3}{5}x_1 + \frac{2}{5}x_2 + \sum_{i=3}^{n} x_i)$ for all $x \in A$. By consequence, $\xi(R;x) \leq \xi(R';x)$ for all $x \in A$. (II) Case where $x \in B = \{x \in X \mid \frac{2}{5}x_2 \leq x_1 \leq \frac{1}{2}x_2\}$: Note that $\xi(R;x) = \frac{1}{n-1}(\frac{2}{3}x_1 + \frac{1}{3}x_2 + \sum_{i=3}^{n} x_i)$ for all $x \in B$. To calculate $\xi(R';x)$ for any given $x \in B$, we first find an $y \in X$ such that $xI'y$ and $(y_1, y_2) = (\frac{1}{2}t, t)$ and then use that $\xi(R';x) = \xi(R';y)$. Now, y is such that $4x_1 + x_2 = 4\frac{1}{2}t + t$, so that $t = \frac{4x_1+x_2}{3}$, and $y_i = x_i$ for all $i = 3, 4, \ldots, n$. Since $y \in A$, we can calculate $\xi(R';y)$ as in the previous case, so that $\xi(R';x) = \xi(R';y) = \frac{1}{n-1}(\frac{28}{30}x_1 + \frac{7}{30}x_2 + \sum_{i=3}^{n} x_i)$. For all $x = (x_1, x_2) \in X$, $\xi(R;x) < \xi(R';x)$ if and only if the condition is met that $x_1 > \frac{3}{8}x_2$, a condition that holds for all $x \in B$. (III) Case where $x \in C = \{x \in X \mid x_1 \leq \frac{2}{5}x_2\}$: To calculate $\xi(R;x)$ and $\xi(R';x)$ for any given $x \in C$, we use the same method as in the previous case. So, first we find $y, y' \in X$ such that xIy, $xI'y'$, $(y_1, y_2) = (\frac{2}{5}t, t)$ and $y_i = x_i$ for all $i = 3, 4, \ldots, n$, $(y'_1, y'_2) = (\frac{2}{5}t', t')$ and $y'_i = x_i$ for all $i = 3, 4, \ldots, n$, and then calculate $\xi(R;y)$ and $\xi(R';y')$, which are equal to $\xi(R;x)$ and $\xi(R';x)$, respectively. Note, however, that $t = t'$, so that, since $y, y' \in B$, we have that $\xi(R;y) < \xi(R';y')$, and hence $\xi(R;x) < \xi(R';x)$ for all $x \in C$. We conclude from (I), (II) and (III) that R is at least as Arrow-Pratt inequality averse as R'.

We now show that R is not at least as *RD*-inequality averse as R'. Consider x and y such that $(x_1, x_n) = (120, 785)$, $(y_1, y_n) = (100, 800)$ and $x_i = y_i = 240$ for all $i = 2, 3, \ldots, (n-1)$. Clearly, $x \prec_{RD} y$, but yPx while $xP'y$.

(c) That (iii) implies (i) follows from (b). That (i) implies (iii) follows from Lemma 4.1 and the fact that, for any pair $x, y \in X$, if $x \prec_{RD} y$, then condition (4.2) of Lemma 4.1 is met. ■

Proof of Lemma 4.2. Ross (1981) shows (ii) to be equivalent to the condition: for all $x, y \in X$ such that $x \prec_L y$, if $x - \pi_u 1_n I_u y$ and $x - \pi_v 1_n I_v y$, then $\pi_u \geq \pi_v$. We consider the following condition:

$$\text{for all } x, y \in X \text{ such that } x \prec_L y, \text{ if } \gamma_u x I_u y \text{ and } \gamma_v x I_v y, \text{ then } \gamma_u \leq \gamma_v. \tag{4.3}$$

If this latter condition is fitted into the proof of Ross instead of the former, it is easily seen that they play the same role and are equivalent.

What remains to be shown is that the condition in (4.3) is equivalent to (i).

First we show that (4.3) is equivalent to

$$\text{for all } x, x', y \in X \text{ such that } x \sim_L x',\ x \prec_L y,\ xI_u y \text{ and } x'I_v y, \text{ we have } \mu(x) \leq \mu(x'). \tag{4.4}$$

It is immediate that (4.4) implies (4.3). That (4.3) implies (4.4) follows from the fact that if there exist $x, x', y \in X$ such that $x \sim_L x'$, $x \prec_L y$, $xI_u y$ and $x'I_v y$, then there exists a $z \in X$ and scalars γ_u, γ_v such that $x = \gamma_u z$ and $x' = \gamma_v z$. Now, since (4.4) is equivalent to (i) by Proposition 4.1, the required result follows. ■

Proof of Proposition 4.3. Seeking a contradiction, suppose that, without loss of generality, $\varepsilon > \varepsilon'$ and that R_ε is at least as L-inequality averse as $R_{\varepsilon'}$. Then, by Lemma 4.2, there exist a decreasing and concave function $f : \mathbb{R}_{++} \to \mathbb{R}$ and a scalar $\lambda > 0$ such that, for all $t \in \mathbb{R}_{++}$,

$$\frac{t^{1-\varepsilon}}{1-\varepsilon} = \lambda \frac{t^{1-\varepsilon'}}{1-\varepsilon'} + f(t).$$

Decreasingness and concavity of f imply

$$\frac{df(t)}{dt} = t^{-\varepsilon} - \lambda t^{-\varepsilon'} \leq 0 \quad \text{for all } t \in \mathbb{R}_{++}, \tag{4.5}$$

and

$$\frac{df^2(t)}{dt^2} = -\varepsilon t^{-(1+\varepsilon)} + \lambda \varepsilon' t^{-(1+\varepsilon')} \leq 0 \quad \text{for all } t \in \mathbb{R}_{++}. \tag{4.6}$$

From (4.5) and (4.6) it follows that

$$\lambda \geq t^{-(\varepsilon-\varepsilon')} \quad \text{for all } t \in \mathbb{R}_{++}, \tag{4.7}$$

and

$$\lambda \leq \frac{\varepsilon}{\varepsilon'} t^{-(\varepsilon-\varepsilon')} \quad \text{for all } t \in \mathbb{R}_{++}, \tag{4.8}$$

respectively. Since the functions $t \mapsto t^{-(\varepsilon-\varepsilon')}$ and $t \mapsto \frac{\varepsilon}{\varepsilon'} t^{-(\varepsilon-\varepsilon')}$ map $\mathbb{R}_{++}$ *onto* $\mathbb{R}_{++}$, there exist $s, t \in \mathbb{R}_{++}$ such that $s^{-(\varepsilon-\varepsilon')} > \frac{\varepsilon}{\varepsilon'} t^{-(\varepsilon-\varepsilon')}$. By consequence, λ cannot satisfy both (4.7) and (4.8) and we have a contradiction. ■

Proof of Proposition 4.4. Equivalence of (iii) and (v): That (v) implies (iii) is immediate. We prove using contraposition that (iii) implies (v). Suppose R is a monotonic SWO for which (v) is not true, that is, R is not weakly maximin. Then, there is a pair $x, y \in X$, where $x_1 > y_1$, such that yRx. Since $xRx_1 1_n$ by reflexivity and monotonicity, we have by transitivity that $yRx_1 1_n$ while not $x_1 1_n < y$ and $x_1 1_n \prec_M y$. Hence, (iii) is not true for R.

Equivalence of (i) and (iii): That (iii) implies (i) is immediate. We prove using contraposition that (i) implies (iii). Suppose that R is a monotonic SWO for which (iii) is not true, that is, there is a pair $x,y \in X$ such that not $x < y$ and $x \prec_M y$, while yRx. Now, take any monotonic SWO R' such that $xP'y$. Clearly, R is not at least as inequality averse as R' according to the M-concept. Hence, (i) is not true for R.

Equivalence of (iv) and (v): We first show that (v) implies (iv). Let $x,y \in X$ be any pair such that not $x < y$ and $x \prec_{RD} y$. Suppose, first, that $\mu(x) > \mu(y)$. Then, there must be an $i \in N$ such that $\frac{x_i}{y_i} > 1$. Since also $x \prec_{RD} y$, we have $\frac{x_1}{y_1} > 1$. Hence, xPy for any weakly maximin SWO R. Suppose alternatively that $\mu(x) \leq \mu(y)$. Then, because $x \prec_{RD} y$, $x_1 \leq y_1$ would imply that $\frac{x_i}{y_i} \leq 1$ for all $i \in N$ and $\frac{x_i}{y_i} < 1$ for at least one $i \in N$ and, hence, that $x < y$ which contradicts our premise. By consequence, $x_1 > y_1$ and xPy for any weakly maximin SWO R. Since (iv) implies (iii), because $\prec_M \subset \prec_{RD}$, and (iii) implies (v), as shown above, it follows furthermore that (iv) implies (v).

Equivalence of (ii) and (iv): The proof is very similar to that of the equivalence of (i) and (iii) and is therefore omitted. ■

Proof of Proposition 4.5. Take any $x,y \in X$ such that $x_1 \leq x_2 < x_3$ and $y = \left(\lambda x_1, \lambda \frac{\sum_{i=2}^n x_i}{n-1}, \lambda \frac{\sum_{i=2}^n x_i}{n-1}, \ldots, \lambda \frac{\sum_{i=2}^n x_i}{n-1}\right)$ where λ is a positive scalar. For any allowed value of λ, we have $y \prec_L x$. Whenever $\lambda = 1$, then yPx by Lorenz consistency. By continuity and monotonicity, there is an infinite number of λs such that $0 < \lambda < 1$ and yPx. Now, for any such λ we have that x is strictly preferred to y by all weakly maximin SWOs. ■

Proof of Proposition 4.7. Suppose R is a monotonic SWO that is more inequality averse than leximin. Then, there is some pair $x,y \in X$ such that $x \prec_L y$, xRy and leximin strictly prefers y to x. By the latter condition, we have that, either (a) $x_1 < y_1$, or (b) there is a $k > 1$ such that, for all $i = 1,2,\ldots,(k-1)$ we have $x_i = y_i$ while $x_k < y_k$. Now, consider a $z \in X$ such that, in case (a), $x_1 < z_1 < y_1$ and $z = z_1 1_n$ and, in case (b), $z_i = x_i = y_i$ for all $i = 1,2,\ldots,(k-1)$, $x_k < z_k < \min\{\frac{\sum_{i=k}^n x_i}{n-k+1}, y_k\}$ and $z_i = z_k$ for all $i = (k+1),(k+2),\ldots,n$. Then, by monotonicity, yPz, and hence by transitivity xPz. Now, $z \prec_L x$ and leximin strictly prefers z to x. By consequence, R is not more inequality averse than leximin and we have a contradiction. ■

Proof of Proposition 4.8. Proposition 4.4 implies that a monotonic SWO R can only satisfy the condition stated in the proposition if it is weakly maximin. Next, take an income distribution $x \in X$ where $x_1 < x_2 < x_3$. By minimal invariance there must be some $x' \in X$ such that $x' \gg x$ and $x \preceq x'$. Now consider an y such that $x_1 < y_1 < x'_1$, $y_2 < x_2 < x'_2$, and $x_3 < x'_3 < y_3$. Clearly, for any weakly maximin SWO R, we have yPx and $x'Py$. Now, suppose $\preceq$ is an inequality quasi-ordering that satisfies the condition specified in the proposition. Then, $x \preceq x'$, $y \prec x$ since not $y > x$, and $x' \prec y$ since not $x' > y$. The IQO is intransitive, which is a contradiction. ■

Connection between Chapters 4 and 5: In Chapter 4 (as well as in Chapter 3), concepts of inequality aversion were used to determine the class of extremely inequality averse social welfare orderings. An alternative egalitarian approach is to focus on a particular basic inequality decreasing transformation and demand that the social welfare ordering always considers the transformation as welfare increasing. In Chapter 5, we study Hammond equity, which is a famous example of an axiom in this spirit. We argue that Hammond equity is too demanding as a general egalitarian principle because it sometimes forces social welfare orderings to make choices over pairs of income distributions that cannot be ranked using the Lorenz inequality criterion. This critique is in a sense the counterpart of the critique of the Arrow-Pratt criterion in Chapter 4: there we argued that the Arrow-Pratt concept is not demanding enough because in comparing social welfare orderings it does not use all pairs of income distributions that can be ranked using the Lorenz inequality criterion. We introduce in Chapter 5 a modified version of Hammond equity which does not share the mentioned shortcoming of Hammond equity. This new axiom is then used to characterize once again the weakly maximin class (Proposition 5.1), and, moreover, to characterize maximin (Theorem 5.1), which is a prominent member of this class.

Chapter 5

A Characterization of Maximin

This chapter is based on joint work with Erwin Ooghe.

5.1 Introduction

Maximin and leximin are two well known social rankings which give priority to the worst off. Leximin receives considerable attention in the axiomatic social choice literature—see, e.g., Sen (1986), and more recently Bossert and Weymark (2004) and d'Aspremont and Gevers (2002), for excellent overviews. Among the many available characterizations, we mention Hammond's (1976) seminal one, in which an equity axiom—Hammond equity—is reconciled with the standard axioms anonymity and strong Pareto.

Contrary to leximin, maximin traditionally receives very little attention—e.g., none of the above overviews provides a characterization of maximin. In the context of Hammond's characterization of leximin, maximin is often just quoted as an example of a different rule that also satisfies Hammond equity. In addition, maximin satisfies anonymity and—contrary to leximin—continuity, but only weak Pareto instead of the more demanding strong Pareto. As far as we know, no attempts have been made to characterize maximin along this line. Roemer (1996, p. 35) characterizes maximin on the basis of a strong information invariance requirement (allowing only ordinal measurability and full comparability of utilities). In a recent contribution, Segal and Sobel (2002) provide a joint characterization of maximin, maximax and the sum of utilities rule using a partial separability axiom.[1]

Our main contribution is to provide the characterization of maximin alluded

[1] Lauwers (1997) characterizes the related infimum rule in the context of social choice with infinite populations. For choice under uncertainty, a characterization of maximin has been provided by Barberá and Jackson (1988).

to above, i.e., on the basis of anonymity, continuity, weak Pareto and Hammond equity. In addition, we (i) show that this characterization result remains valid under a weaker and possibly more interesting set of axioms, and (ii) present an alternative characterization of leximin along this way.

5.2 Preliminaries

The problem of social choice concerns the determination of a ranking of all possible alternatives open to society. A society consists of $n \in \mathbb{N}_0$ individuals, gathered in a set $N = \{1, \ldots, i, \ldots, n\}$. In line with the so called welfarist doctrine, the different alternatives for society are represented by individual utility vectors, denoted by $u = (u_1, \ldots, u_n) \in \mathbb{R}^n$. Let $(u_{(1)}, u_{(2)}, \ldots, u_{(n)})$ be a permutation of the utility vector u such that $u_{(1)} \leq u_{(2)} \leq \cdots \leq u_{(n)}$. The idea of a social ranking is captured by a binary relation R ('is at least as good as') on $\mathbb{R}^n$. The asymmetric and symmetric parts of R are denoted P ('is better than') and I ('is equally good as'), respectively. We assume R to be a quasi-ordering, i.e., a reflexive and transitive (but not necessarily complete) relation.

We can now define maximin. A quasi-ordering R on $\mathbb{R}^n$ is *maximin* if and only if, for all $u, v \in \mathbb{R}^n$,

$$uRv \text{ if and only if } u_{(1)} \geq v_{(1)}.$$

Maximin is obviously a complete quasi-ordering: it fully corresponds with the ranking of the worst off. We note that maximin is a member of the class of weakly maximin quasi-orderings. A quasi-ordering R on $\mathbb{R}^n$ is *weakly maximin* if and only if, for all $u, v \in \mathbb{R}^n$,

$$\text{if } u_{(1)} > v_{(1)}, \text{ then } uPv.$$

To characterize maximin, we need to define some axioms. The first four axioms are well known. Anonymity requires that the identities of the individuals do not matter.

Anonymity. For all $u \in \mathbb{R}^n$, uIu', with u' any permutation of u.

Continuity ensures, loosely speaking, that small changes in a utility vector can only cause small changes in its social ranking with respect to other utility vectors.

Continuity. For all $u \in \mathbb{R}^n$, if a sequence of vectors $(v^k)_{k \in \mathbb{N}_0}$ converges to v and we have uRv^k (respectively, v^kRu) for all $k \in \mathbb{N}_0$, then uRv (respectively, vRu).[2]

[2]Contrary to the version of the continuity axiom used in Chapters 3 and 4, the version in this chapter is suitable also for incomplete relations. For complete relations, the two versions are equivalent.

Weak Pareto demands that an increase in the utilities of all individuals is considered as an improvement.

Weak Pareto. For all $u, v \in \mathbb{R}^n$, if $u_i > v_i$ for all $i \in N$, then uPv.

According to Hammond equity, social welfare should (weakly) increase whenever the utilities of two individuals become more equal while keeping the other utilities constant.

Hammond Equity. For all $u, v \in \mathbb{R}^n$, if $v_i < u_i < u_j < v_j$ for some $i, j \in N$, and $u_k = v_k$ for all $k \neq i, j$, then uRv.

We wish to introduce an alternative equity axiom. It is convenient to first point out what some may consider to be a shortcoming of Hammond equity. Hammond equity is not an egalitarian principle in the following sense: if Hammond equity implies uRv for some utility vectors u and v, then it is not necessarily the case that u is less unequal than v according to the Lorenz criterion, the strongest inequality concept to enjoy wide acceptance. For example, for the utility vectors $u = (10, 21, 22, 40)$ and $v = (10, 20, 30, 40)$, Hammond equity implies uRv, but u does not Lorenz dominate v. Furthermore, since u does not Lorenz dominate v, there exist Lorenz consistent inequality measures which indicate a *higher* level of inequality for u than for v. In other words, Hammond equity would force an egalitarian who defines inequality using such an inequality measure to prefer the, in her opinion, more unequal alternative u over v. Therefore, it may be interesting to consider a new axiom, modified Hammond equity, which, in contrast to Hammond equity, qualifies as an egalitarian principle in the above sense. Modified Hammond equity says that if there is exactly one worst off individual and all those better off have equal utility levels, then society must weakly approve of any change that increases the utility level of the worst off (such that this individual remains the worst off) and decreases the utility levels of all the best off (such that they all remain the best off).

Modified Hammond Equity. For all $u, v \in \mathbb{R}^n$, if $v_{(1)} < u_{(1)} < u_{(2)} = u_{(3)} = \cdots = u_{(n)} < v_{(2)} = v_{(3)} = \cdots = v_{(n)}$, then uRv.

It can straightforwardly be established that if modified Hammond equity implies uRv for any utility vectors u and v, then u strictly Lorenz dominates v. Also, it is important to note that if transitivity is given, Hammond equity implies modified Hammond equity. By consequence, in the present context of quasi-orderings modified Hammond equity can be treated as a weakening of Hammond equity.

5.3 Result and Discussion

Our main result says that maximin is characterized by anonymity, continuity, weak Pareto and Hammond equity, and that an alternative characterization is obtained if Hammond equity is replaced by modified Hammond equity.

Theorem 5.1. *Let R be a quasi-ordering on $\mathbb{R}^n$. Then, the following three statements are equivalent:*

(i) *R is maximin;*
(ii) *R satisfies anonymity, continuity, weak Pareto and Hammond equity;*
(iii) *R satisfies anonymity, continuity, weak Pareto and modified Hammond equity.*

Before we prove Theorem 5.1, we consider two intermediate results that are interesting in their own right, and are therefore presented as propositions. The first proposition says that, if anonymity is given, then weak Pareto and modified Hammond equity characterize the class of weakly maximin quasi-orderings. Anonymity has to be given because not all weakly maximin quasi-orderings satisfy this axiom.

Proposition 5.1. *Let R be a quasi-ordering on $\mathbb{R}^n$ that satisfies anonymity. Then, R satisfies weak Pareto and modified Hammond equity if and only if R is weakly maximin.*

Proof. It is easy to verify that any weakly maximin quasi-ordering satisfies weak Pareto and modified Hammond equity. Therefore, we concern ourselves only with the reverse implication. Suppose R is any quasi-ordering on $\mathbb{R}^n$ which satisfies anonymity, weak Pareto and modified Hammond equity. Consider, moreover, any two vectors $u, v \in \mathbb{R}^n$.

We have to show that if $u_{(1)} > v_{(1)}$, then uPv. Note that it is possible to construct vectors $w, z \in \mathbb{R}^n$ such that

$$v_{(1)} < w_{(1)} < z_{(1)} < z_{(2)} = \cdots = z_{(n)} < u_{(1)} \leq \max\{u_{(n)}, v_{(n)}\} < w_{(2)} = \cdots = w_{(n)}.$$

Using anonymity, weak Pareto and transitivity, we have uPz and wPv. Using modified Hammond equity, we also have zRw. Hence, we obtain uPv by transitivity. ■

We note that Tungodden (2000) presents a weaker version of Proposition 5.1. He shows that anonymity, strong Pareto and a stronger axiom than modified Hammond equity (but also weaker than Hammond equity) imply that the quasi-ordering must be weakly maximin.

Our second proposition identifies maximin as the only weakly maximin quasi-ordering that satisfies continuity.

Proposition 5.2. *Let R be a quasi-ordering on $\mathbb{R}^n$. Then, R is weakly maximin and satisfies continuity if and only if R is maximin.*

Proof. It is easily verified that maximin satisfies continuity and is weakly maximin. Therefore, we consider only the reverse implication. Suppose R is any quasi-ordering on $\mathbb{R}^n$ which is weakly maximin and satisfies continuity. Consider, moreover, any two vectors $u, v \in \mathbb{R}^n$.

Since maximin is weakly maximin, what remains to be shown is that if $u_{(1)} = v_{(1)}$, then uIv. Note that it is possible to construct two sequences of vectors $(x^k)_{k\in\mathbb{N}_0}$ and $(y^k)_{k\in\mathbb{N}_0}$ with

$$x^k = \big(u_1 + \frac{1}{k}, u_2 + \frac{1}{k}, \ldots, u_n + \frac{1}{k}\big) \text{ and } y^k = \big(u_1 - \frac{1}{k}, u_2 - \frac{1}{k}, \ldots, u_n - \frac{1}{k}\big).$$

By construction, both sequences converge to u. Furthermore, $x^k_{(1)} = u_{(1)} + \frac{1}{k} > u_{(1)} = v_{(1)}$ and $y^k_{(1)} = u_{(1)} - \frac{1}{k} < u_{(1)} = v_{(1)}$ for all $k \in \mathbb{N}_0$. Thus, using the fact that R is weakly maximin, we also have $x^k R v$ and vRy^k for all $k \in \mathbb{N}_0$. Using continuity, we get uRv and vRu, and thus uIv. ■

We now prove Theorem 5.1 using Propositions 5.1 and 5.2.

Proof of Theorem 5.1. It is immediate both that (i) implies (ii) and that (ii) implies (iii). What remains to be shown is that (iii) implies (i). Suppose R is any quasi-ordering on $\mathbb{R}^n$ which satisfies anonymity, continuity, weak Pareto and modified Hammond equity.

From Proposition 5.1 it follows that if R satisfies anonymity, weak Pareto and modified Hammond equity, then it is weakly maximin. Proposition 5.2 implies that if R satisfies in addition continuity, then it is maximin. ■

To conclude, we note that a characterization of leximin is obtained using the axioms mentioned in statement (ii) of Theorem 5.1 but with continuity replaced by separability—the latter axiom demands that the social ranking is independent of the utilities of indifferent individuals.[3] So, leximin is characterized by anonymity, weak Pareto, modified Hammond equity and separability. We omit the formal statement and proof of this result because a similar result has already been presented by Tungodden (2000)—to prove the result, he builds on his alternative version of Proposition 5.1 that was mentioned above. This characterization of leximin testifies of the fact that Hammond equity is not a purely egalitarian concept but, rather, carries with it also instances of the idea of separability.

[3]For formal definitions of leximin and separability, see Chapters 3 and 4.

Part III

Inequality and the Claims Problem

Chapter 6

Lorenz Rankings of Nine Division Rules for Claims Problems

6.1 Introduction

Claims problems are distribution problems concerned with the allocation of an amount of money (referred to as the *estate*) among a group of individuals who typically have different entitlements (referred to as *claims*) and where the estate falls short of the sum of the claims. Several distribution problems can be framed as claims problems. In the taxation problem, the estate equals the sum of the pre-tax incomes minus the desired tax revenue and the claims are the pre-tax incomes. In the bankruptcy problem, the estate is the liquidation value of the firm and the claims are the entitlements of the creditors. The literature on claims problems focuses on the axiomatic study of *rules* that associate with each claims problem a division between the individuals of the estate (referred to as an *awards vector*).[1]

In this chapter, we analyze the Lorenz dominance relationships that exist between several well known division rules for claims problems. A glance at the literature on the measurement of income inequality shows that the Lorenz dominance relation is the most prominent criterion for making inequality comparisons between distributions.[2] Our analysis will therefore enable us to compare rules with respect to progressivity—a rule is more progressive than another rule if, for any claims problem, the awards vector it selects is less unequal than that of the other rule.[3]

[1]For an extensive overview of the literature, see Thomson (2003).

[2]See Cowell (2000) and Lambert (2001) for overviews of this literature.

[3]See also Moreno-Ternero and Villar (2006), who compare the members of the so-called TAL family of rules—a one-parameter family which encompasses the constrained equal awards rule, the constrained equal losses rule and the Talmud rule—with respect to progressivity using the concept of Lorenz dominance.

Concretely, we analyze the Lorenz dominance relationships that exist between the following nine well known division rules: the proportional rule, the constrained equal awards rule, the constrained equal losses rule, the Talmud rule, Piniles' rule, the constrained egalitarian rule, the adjusted proportional rule, the random arrival rule and the minimal overlap rule. It is shown that several of these rules can be characterized as most or least progressive among a class of rules of which the members satisfy certain axioms. Thus, we provide a generalization of a characterization in the same spirit of the constrained equal awards rule by Schummer and Thomson (1997). The other characterizations along this line concern the constrained equal losses rule, the Talmud rule, the constrained egalitarian rule,[4] and Piniles' rule. We also provide several results involving the other rules in order to uncover the complete set of Lorenz dominance relationships that hold between the nine rules.

The chapter is structured as follows. Section 6.2 introduces notation, defines rules and axioms and defines and discusses the Lorenz dominance relation. In Section 6.3, we provide the characterization results. In Section 6.4, we provide several additional results in order to be able to give a detailed description of the Lorenz relationships holding between the division rules. Section 6.5 concludes. All proofs are relegated to Appendix 6.A.

6.2 Preliminaries

The set of individuals is $N = \{1, 2, \dots, n\}$ with n an integer greater than 1. Each individual $i \in N$ has an associated *claim* $c_i \in \mathbb{R}_+$. We let $C = \sum_{i \in N} c_i$. A *claims problem* is an ordered pair (c, E) where $c = (c_i)_{i \in N} \in \mathbb{R}^n_+$ is the *claims vector*, $E \in \mathbb{R}_+$ is the *estate* which has to be divided among the members of N, and where $C \geq E$. The symbol $\mathscr{C}$ denotes the set of all claims problems. A *rule* is a function R which associates with each $(c, E) \in \mathscr{C}$ an element of $\mathbb{R}^n_+$ such that $\sum_{i \in N} R_i(c, E) = E$ and $0 \leq R(c, E) \leq c$.[5] We refer to $R(c, E)$ as the *awards vector* of rule R for (c, E). For any vector $x \in \mathbb{R}^n_+$, we let $(x_{(1)}, x_{(2)}, \cdots, x_{(n)})$ be a rearrangement of x such that $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$.

In this chapter, we focus on nine division rules for claims problems.[6] The rule that is without a doubt the most often used in practice is the proportional rule, which makes awards proportional to claims.

Proportional rule (*P*). For all $(c, E) \in \mathscr{C}$, we have $P(c, E) = \lambda c$ where λ solves $\sum_{i \in N} \lambda c_i = E$.

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[4] A result also considered by Chun, Schummer and Thomson (2001).

[5] For any $x, y \in \mathbb{R}^n_+$, the vector inequality $x \leq y$ signifies that $x_i \leq y_i$ for all $i = 1, \dots, n$.

[6] For a thorough discussion of the rules and axioms defined in this section, see Thomson (2003).

The next two rules both implement the idea of equality, but in very different ways. In the case of the constrained equal awards rule, all individuals receive equal awards as long as their claim is not exceeded.

Constrained equal awards rule (*CEA*). For all $(c,E) \in \mathscr{C}$ and all $i \in N$, we have $CEA_i(c,E) = \min\{c_i, \lambda\}$ where λ solves $\sum_{i \in N} \min\{c_i, \lambda\} = E$.

The constrained equal losses rule equalizes losses—the *loss* of an individual is equal to her claim minus her award—instead of awards, under the condition that no individual receive less than zero.

Constrained equal losses rule (*CEL*). For all $(c,E) \in \mathscr{C}$ and all $i \in N$, we have $CEL_i(c,E) = \max\{0, c_i - \lambda\}$ where λ solves $\sum_{i \in N} \max\{0, c_i - \lambda\} = E$.

The Talmud rule specifies two different regimes depending on whether the estate is smaller or greater than the sum of the half-claims. In the former case, the formula of the constrained equal awards rule is used in the definition of the Talmud rule and, in the latter case, that of the constrained equal losses rule is used, in both cases applied to the vector of half-claims instead of to the claims vector itself.

Talmud rule (*T*). For all $(c,E) \in \mathscr{C}$, we have

(i) if $E \leq \frac{1}{2}C$, then $T(c,E) = CEA(\frac{1}{2}c, E)$; and
(ii) if $E \geq \frac{1}{2}C$, then $T(c,E) = \frac{1}{2}c + CEL(\frac{1}{2}c, E - \frac{1}{2}C)$.

The four rules defined above constitute the classical rules for solving claims problems.[7] All rules subsequently defined are, as we shall see, in some way related to the Talmud rule.

The next two rules are equal to the Talmud rule whenever the estate is smaller than the sum of the half-claims, but different whenever it is greater. Piniles' rule utilizes the formula of the constrained equal awards rule instead of that of the constrained equal losses rule whenever the estate is greater than the sum of the half-claims.

Piniles' rule (*Pin*). For all $(c,E) \in \mathscr{C}$, we have

(i) if $E \leq \frac{1}{2}C$, then $Pin(c,E) = CEA(\frac{1}{2}c, E)$; and
(ii) if $E \geq \frac{1}{2}C$, then $Pin(c,E) = \frac{1}{2}c + CEA(\frac{1}{2}c, E - \frac{1}{2}C)$.

The constrained egalitarian rule uses a different egalitarian procedure in the case where the estate exceeds the sum of the half-claims.

[7] For a comparative study of these four rules, see Herrero and Villar (2001).

Constrained egalitarian rule (*CE*). For all $(c,E) \in \mathscr{C}$, we have

(i) if $E \leq \frac{1}{2}C$, then $CE(c,E) = CEA(\frac{1}{2}c,E)$; and

(ii) if $E \geq \frac{1}{2}C$, then, for all $i \in N$, $CE_i(c,E) = \max\{\frac{c_i}{2}, \min\{c_i,\lambda\}\}$ where λ solves $\sum_{i\in N} \max\{\frac{c_i}{2}, \min\{c_i,\lambda\}\} = E$.

The last three rules we define in this section are equal to the Talmud rule for claims problems with only two individuals, but may deviate otherwise.[8] To define the adjusted proportional rule, we require the concept of a minimal right. The minimal right of an individual is the part of the estate that is left for her if the others are all compensated fully. Formally, for all $(c,E) \in \mathscr{C}$ and all $i \in N$, $m_i(c,E) = \max\{E - \sum_{j\in N\setminus\{i\}} c_j, 0\}$ is the minimal right of individual i. We let $m(c,E) = (m_i(c,E))_{i\in N}$. The adjusted proportional rule gives every individual her minimal right and applies the proportional rule for allocating the remainder.

Adjusted proportional rule (*A*). For all $(c,E) \in \mathscr{C}$, we have

$$\begin{aligned} A(c,E) &= m(c,E) \\ &\quad + P\Big(\big(\min\{c_i - m_i(c,E), E - \sum_{j=1}^{n} m_j(c,E)\}\big)_{i\in N}; E - \sum_{j=1}^{n} m_j(c,E)\Big). \end{aligned}$$

For defining the random arrival rule, suppose the individuals arrive one at a time and are completely compensated until the estate runs out. The division selected by the random arrival rule is the average over all orders of arrival of the awards vectors obtained in this way. We let Π^N denote the class of all bijections that map N onto itself.

Random arrival rule (*RA*). For all $(c,E) \in \mathscr{C}$ and all $i \in N$, we have

$$RA_i(c,E) = \frac{1}{n!} \sum_{\pi\in\Pi^N} \min\{c_i, \max\{E - \sum_{j\in N, \pi(j)<\pi(i)} c_j, 0\}\}.$$

To explain and formally define the minimal overlap rule, we assume that the members of N are indexed such that $c_1 \leq c_2 \leq \cdots \leq c_n$. This can be done without loss of generality since the rule—as all other rules defined in this section—is symmetric. To understand the minimal overlap rule, regard individuals as claiming specific parts of the interval $[0,E]$. Each part is equally distributed among all individuals claiming it. For instance, the interval $[0,c_1]$ is claimed by everyone, and so everyone gets $\frac{c_1}{n}$. The interval $(c_1,c_2]$ is claimed by everyone except individual 1, and so each member of $N \setminus \{1\}$ receives in addition $\frac{c_2-c_1}{n-1}$. This process

[8] See Thomson (2003). The two-individual version of the Talmud rule and these three rules is conventionally referred to as the *contested garment rule* or as *concede-and-divide*.

continues until the entire interval $[0,E]$ is covered. If there are individuals who have claims higher than the estate, then their claims are simply truncated by the estate. If, on the other hand, there are no individuals who have claims higher than the estate, then a scalar t is sought such that all individuals with claims higher than t exclusively claim a specific part of the interval $(t,E]$.

Minimal overlap rule (*MO*). For all $(c,E) \in \mathscr{C}$, we have

(i) if $c_i \geq E$ for some $i \in N$, then, for all $i \in \{j \in N \mid c_j < E\}$,

$$MO_i(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1} + \cdots + \frac{c_i - c_{i-1}}{n-(i-1)},$$

and, for all $i \in \{j \in N \mid c_j \geq E\}$,

$$MO_i(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1} + \cdots + \frac{c_{k-1} - c_{k-2}}{n-(k-2)} + \frac{E - c_{k-1}}{n-(k-1)},$$

where $k = \min\{j \in N \mid c_j \geq E\}$; and

(ii) if $c_i < E$ for all $i \in N$, then, for all $i \in \{j \in N \mid c_j < t\}$,

$$MO_i(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1} + \cdots + \frac{c_i - c_{i-1}}{n-(i-1)},$$

and, for all $i \in \{j \in N \mid c_j \geq t\}$,

$$MO_i(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1} + \cdots + \frac{c_{k-1} - c_{k-2}}{n-(k-2)} + \frac{t - c_{k-1}}{n-(k-1)} + c_i - t,$$

where t solves $\sum_{i \in \{j \in N \mid c_j \geq t\}} (c_i - t) = E - t$ and $k = \min\{j \in N \mid c_j \geq t\}$.

In our analysis we make use also of several axioms. Order preservation requires that if an individual has a higher claim than another, then she should get a higher award and should carry a greater absolute loss.

Order preservation. For all $(c,E) \in \mathscr{C}$ and all $i, j \in N$, if $c_i \geq c_j$, then $R_i(c,E) \geq R_j(c,E)$ and $c_i - R_i(c,E) \geq c_j - R_j(c,E)$.

Resource monotonicity demands that if the estate increases, then all individuals should receive at least as much as they did initially.

Resource monotonicity. For all $(c,E) \in \mathscr{C}$ and for all $E' \in \mathbb{R}_+$, if $C \geq E' > E$, then $R(c,E') \geq R(c,E)$.

Super-modularity requires that if the estate increases, then individuals with higher claims should receive a greater part of the increment than those with lower claims.

Table 6.1. Satisfaction of Axioms

Axiom	*P*	*CEA*	*CEL*	*T*	*Pin*	*CE*	*A*	*RA*	*MO*
Order preservation	yes	yes	yes	yes	yes	yes	yes	yes	yes
Resource monotonicity	yes	yes	yes	yes	yes	yes	yes	yes	yes
Super-modularity	yes	yes	yes	yes	yes	NO	yes	yes	yes
Midpoint property	yes	NO	NO	yes	yes	yes	yes	yes	NO

Super-modularity. For all $(c,E) \in \mathscr{C}$, all $E' \in \mathbb{R}_+$ and all $i,j \in N$, if $C \geq E' > E$ and $c_i \geq c_j$, then $R_i(c,E') - R_i(c,E) \geq R_j(c,E') - R_j(c,E)$.

The midpoint property, finally, says that if the estate is equal to the sum of the half-claims, then every individual should get its half-claim.

Midpoint property. For all $(c,E) \in \mathscr{C}$ such that $E = \frac{1}{2}C$, we have $R(c,E) = \frac{1}{2}c$.

Table 6.1 indicates which axioms are satisfied by each of the nine rules defined in this section. The only results that are not well established in the literature, are those saying that the midpoint property is satisfied by the adjusted proportional rule and the random arrival rule. However, these results easily follow from the fact that the adjusted proportional rule and the random arrival rule are self-dual—self-duality requires that, for all $(c,E) \in \mathscr{C}$, we have $R(c,E) = c - R(c,C-E)$.[9] If $2E = C$, then we have $2R(c,E) = c$ for all self-dual rules, and hence all these rules satisfy the midpoint property.

The objective of this chapter is to provide a description of the Lorenz dominance relationships that hold between the nine rules defined above. In the literature on income inequality measurement, it is broadly accepted that if a vector x Lorenz dominates a vector y, then x is less unequally distributed than y—if the means and dimensions of x and y are equal, then x can be obtained from y by a finite number of richer to poorer transfers (and permutations).[10]

For a formal definition of Lorenz dominance, take any $x,y \in \mathbb{R}^n_+$ such that $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$. We say that x *Lorenz dominates* y if and only if

$$\sum_{i=1}^{k} x_{(i)} \geq \sum_{i=1}^{k} y_{(i)} \quad \text{for all } k = 1,2,\ldots,n-1.$$

We say that x *strictly* Lorenz dominates y if and only if, moreover, at least one of these inequalities holds strictly. Obviously, for any $x,y \in \mathbb{R}^n_+$ such that $\sum_{i=1}^n x_i =$

[9] This is well known for the random arrival rule. For the adjusted proportional rule, self-duality can be easily checked using the rule's relation to the proportional rule and Theorem 9 in Thomson (2003).

[10] See, for instance, Cowell (2000) or Lambert (2001).

$\sum_{i=1}^{n} y_i$, if x Lorenz dominates y and $(x_{(1)}, x_{(2)}, \cdots, x_{(n)}) \neq (y_{(1)}, y_{(2)}, \cdots, y_{(n)})$, then x strictly Lorenz dominates y.

In what follows, we frequently call a rule R *more progressive than* a rule R' if and only if $R(c,E)$ Lorenz dominates $R'(c,E)$ for all $(c,E) \in \mathscr{C}$. We call R *strictly* more progressive than R' if and only if, moreover, $R(c,E)$ strictly Lorenz dominates $R'(c,E)$ for some $(c,E) \in \mathscr{C}$. Similar statements will be used whenever these Lorenz relationships hold only for all claims problems in a sub-domain $\hat{\mathscr{C}} \subset \mathscr{C}$.

To conclude this section, we consider a remark that describes two useful properties of the Lorenz dominance relation. The remark says that if a rule is more progressive than another, then, for each claims problem, its awards vector has (i) a greater minimal component and a smaller maximal component, and (ii) a lower variance than the awards vector of the other rule. We do not provide a proof of the remark as it is easily established.

Remark 6.1. Let R and R' be two rules such that $R(c,E)$ Lorenz dominates $R'(c,E)$ for all $(c,E) \in \hat{\mathscr{C}} \subseteq \mathscr{C}$. For all $(c,E) \in \hat{\mathscr{C}}$, we have

(i) $R_1(c,E) \geq R'_1(c,E)$ and $R_n(c,E) \leq R'_n(c,E)$; and
(ii) the variance is at least as low for $R(c,E)$ as for $R'(c,E)$.[11]

Of course, statement (i) in Remark 6.1 ensures that the range—defined as the absolute difference between the minimal and maximal components of the vector in question—is also always lower for the awards vectors of the more progressive rule. Remark 6.1 will enable us to link two of our characterization results in the next section to results of Schummer and Thomson (1997) and Chun, Schummer and Thomson (2001).

6.3 Characterizations

The constrained equal awards rule, the constrained equal losses rule, the constrained egalitarian rule, Piniles' rule and the Talmud rule can be completely characterized as the rules that are most or least progressive among all rules satisfying particular axioms.

Not surprisingly, the constrained equal awards rule and the constrained equal losses rule form the two extreme positions with respect to progressivity. The former is most progressive among all rules.

Proposition 6.1. *Let R be any rule. For all $(c,E) \in \mathscr{C}$, $CEA(c,E)$ Lorenz dominates $R(c,E)$.*

[11] Statement (ii) holds for all inequality measures that are symmetric functions and satisfy the transfer principle. See Chapter 2 for several examples besides the variance.

Proposition 6.1 is related to two characterizations of the constrained equal awards rule by Schummer and Thomson (1997, Propositions 3 and 4). Their two results state that, for all claims problems, respectively the range and the variance of the awards vector of the constrained equal awards rule are smaller than for all other rules. Using Remark 6.1, it is easily seen that Proposition 6.1 generalizes these results.

The constrained equal losses rule is the least progressive rule among all rules satisfying order preservation. Remember that all rules that we have defined in the previous section satisfy this axiom.

Proposition 6.2. *Let R be any rule that satisfies order preservation. For all* $(c,E) \in \mathscr{C}$, $R(c,E)$ *Lorenz dominates* $CEL(c,E)$.

The following characterization of the constrained egalitarian rule is equivalent to a result proven by Chun, Schummer and Thomson (2001, Theorem 3). It says that the constrained egalitarian rule is most progressive among all rules satisfying resource monotonicity and the midpoint property. We state the result for the sake of completeness and provide a significantly different proof.

Proposition 6.3. *Let R be any rule that satisfies resource monotonicity and the midpoint property. For all* $(c,E) \in \mathscr{C}$, $CE(c,E)$ *Lorenz dominates* $R(c,E)$.

Chun, Schummer and Thomson (2001, Theorems 1 and 2) also provide characterization results for the constrained egalitarian rule in terms of the range and the variance, analogous to those of Schummer and Thomson (1997) for the constrained equal awards rule mentioned above. Again, using Remark 6.1, it is easily seen that Proposition 6.3 implies these alternative results.

From Proposition 6.3, it follows that the constrained egalitarian rule is more progressive than Piniles' rule. As mentioned in the previous section, the former rule does not satisfy super-modularity whereas the latter does. The following result shows that, if super-modularity is demanded in addition to resource monotonicity and the midpoint property, then Piniles' rule is the most progressive rule available.

Proposition 6.4. *Let R be any rule that satisfies the midpoint property.*

(i) *If R satisfies in addition resource monotonicity, then, for all* $(c,E) \in \mathscr{C}$ *such that* $E \leq \frac{1}{2}C$, $Pin(c,E)$ *Lorenz dominates* $R(c,E)$*; and*

(ii) *if R satisfies in addition super-modularity, then, for all* $(c,E) \in \mathscr{C}$ *such that* $E \geq \frac{1}{2}C$, $Pin(c,E)$ *Lorenz dominates* $R(c,E)$.

Note that statement (i) in Proposition 6.4 immediately follows from Proposition 6.3.

Since the Talmud rule is equivalent to the constrained egalitarian rule and Piniles' rule in the case where the estate is smaller than the sum of the half-claims, it follows from Proposition 6.3 and 6.4 that the Talmud rule can be interpreted as being rather egalitarian in that case. However, as the following result shows, the Talmud rule goes to the other extreme whenever the estate exceeds the sum of the half-claims. Among all rules satisfying order preservation, resource monotonicity and the midpoint property, the Talmud rule is the most progressive rule in the case where the estate is smaller than the sum of the half-claims, but the least progressive rule in the other case.

Proposition 6.5. *Let R be any rule that satisfies resource monotonicity and the midpoint property.*

(i) *For all $(c,E) \in \mathscr{C}$ such that $E \leq \frac{1}{2}C$, $T(c,E)$ Lorenz dominates $R(c,E)$; and*

(ii) *if R satisfies in addition order preservation, then, for all $(c,E) \in \mathscr{C}$ such that $E \geq \frac{1}{2}C$, $R(c,E)$ Lorenz dominates $T(c,E)$.*

Again, statement (i) in Proposition 6.5 is an immediate corollary of Proposition 6.3.

Proposition 6.5 is the last of our characterization results. In the next section, we consider several results that help to complete the picture with respect to the Lorenz dominance relationships holding between the nine defined rules.

6.4 Rankings

Although the characterization results of the previous section allow us to rank quite a few rules on the basis of Lorenz dominance, they do not allow us to rank all of them. In this section, we first provide the additional results that are required to complete the ranking, and then summarize all results in a number of diagrams.

First consider two lemmas.

Lemma 6.1. *For all $(c,E) \in \mathscr{C}$, if $c_1 + c_2 + \cdots + c_{n-1} \leq E \leq c_n$, then $A_i(c,E) = \frac{1}{2}c_i$ for all $i \in N \setminus \{n\}$.*

Lemma 6.2. *For all $(c,E) \in \mathscr{C}$,*

(i) *if $c_i \geq E$ for some $i \in N$, then $MO_i(c,E) \leq \frac{c_i}{2}$ for all $i \in \{j \in N \mid c_j < E\}$; and*

(ii) *if $c_i < E$ for all $i \in N$, then $MO_i(c,E) \leq \frac{c_i}{2}$ for all $i \in \{j \in N \mid c_j < t\}$, where t solves $\sum_{i \in \{j \in N \mid c_j \geq t\}} (c_i - t) = E - t$.*

The following proposition shows that the adjusted proportional rule is more progressive than the proportional rule if the estate is smaller than the sum of the half-claims, and less progressive otherwise.

Proposition 6.6. *For all* $(c,E) \in \mathscr{C}$,

(i) *if* $E \leq \frac{1}{2}C$, *then* $A(c,E)$ *Lorenz dominates* $P(c,E)$*; and*

(ii) *if* $E \geq \frac{1}{2}C$, *then* $P(c,E)$ *Lorenz dominates* $A(c,E)$.

The final three propositions deal with rankings of the minimal overlap rule against other rules. The following result casts the minimal overlap rule as a rather non-egalitarian rule whenever the estate is greater than the sum of the half-claims. In that case, the minimal overlap rule is less progressive than each member of the class of rules satisfying order preservation, resource monotonicity and the midpoint property. Note that the minimal overlap rule does not itself satisfy the midpoint property.

Proposition 6.7. *Let* R *be any rule that satisfies order preservation, resource monotonicity and the midpoint property. For all* $(c,E) \in \mathscr{C}$ *such that* $E \geq \frac{1}{2}C$, $R(c,E)$ *Lorenz dominates* $MO(c,E)$.

It follows immediately from Proposition 6.7 that the adjusted proportional rule and the random arrival rule are more progressive than the minimal overlap rule whenever the estate is greater than the sum of the half-claims. The following result says that, for the adjusted proportional rule, this is also true whenever the estate is smaller than the sum of the half-claims.

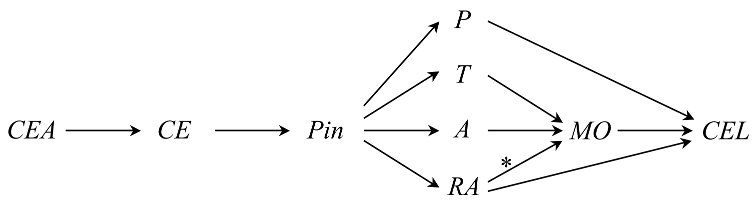
Proposition 6.8. *For all* $(c,E) \in \mathscr{C}$ *such that* $E \leq \frac{1}{2}C$, $A(c,E)$ *Lorenz dominates* $MO(c,E)$.

Whenever the number of individuals is four or less, a similar result holds for the random arrival rule.

Proposition 6.9. *Let* $n \leq 4$. *For all* $(c,E) \in \mathscr{C}$ *such that* $E \leq \frac{1}{2}C$, $RA(c,E)$ *Lorenz dominates* $MO(c,E)$.

It is an open question whether it is possible to generalize this result to all sizes of N.

To conclude, we present several diagrams which make use of Propositions 6.1 to 6.9. These diagrams summarize in detail the Lorenz relationships that hold between the nine division rules considered in this chapter. As Figure 6.1 shows, the Lorenz ranking is incomplete: there are several rules R and R' such that neither is R more progressive than R', nor is R' more progressive than R. Appendix 6.A



An arrow from R to R' indicates that, for all $(c,E) \in \mathscr{C}$, $R(c,E)$ Lorenz dominates $R'(c,E)$. The absence of an arrow indicates either that there is no Lorenz relationship, or that a Lorenz relationship can be established using transitivity. The arrow with $*$ indicates a relationship that has only been established for $n \leq 4$.

Figure 6.1. Lorenz Rankings of Nine Division Rules

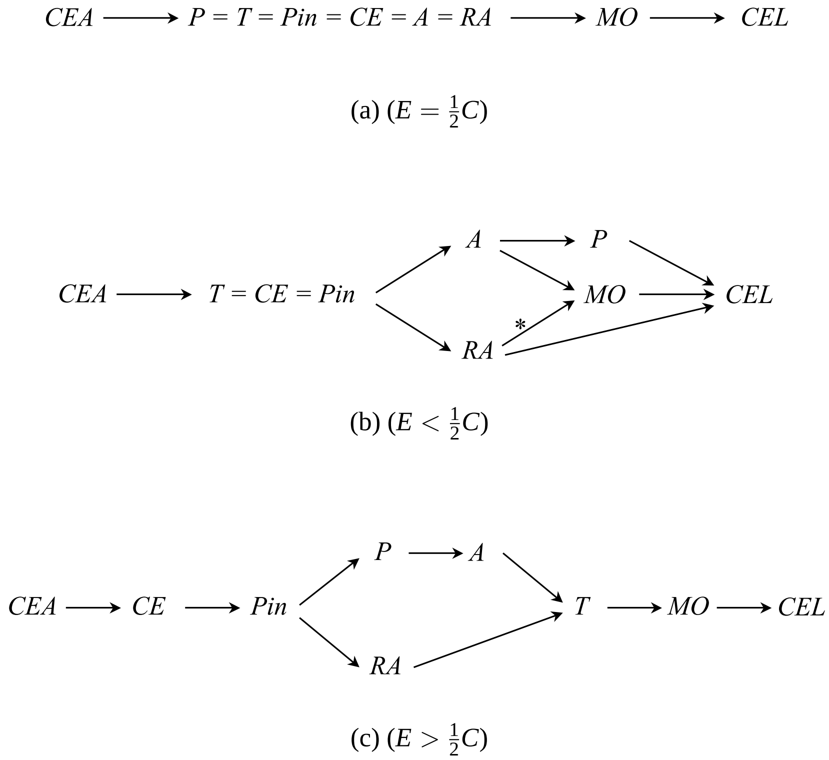
presents examples for the incomparabilities that cannot be established using the propositions.

The Lorenz ranking is much less incomplete if attention is restricted to claims problems where (a) the estate equals the sum of the half-claims, (b) the estate is strictly smaller than the sum of the half-claims, or (c) the estate is strictly greater than the half-claims. Figure 6.2 deals with these three sub-domains of $\mathscr{C}$. In case (a), several rules select the same awards vector due to the midpoint property. Moreover, note that in this case all rules can be ranked on the basis of the Lorenz dominance relation. With respect to case (b), note that there are only three cases of incomparability. In case (c), there are only two cases of incomparability, both involving the random arrival rule.

Finally, we note that, by Remark 6.1, the ranking results presented in Figures 6.1 and 6.2 can straightforwardly be used to compare rules with respect to the sizes of the minimal and maximal components of their awards vectors, or with respect to the variance of their awards vectors. For instance, it can be directly seen from Figure 6.1 that, for each claims problem, say, the minimal component of the awards vector of Piniles' rule is greater than that of the Talmud rule, which in turn is greater than that of the minimal overlap rule.

6.5 Conclusion

In this chapter, we have studied the Lorenz dominance relationships that hold between nine well known division rules for claims problems. It was shown that the constrained equal awards rule, the constrained equal losses rule, the constrained egalitarian rule, Piniles' rule and the Talmud rule can be characterized as most or least progressive among a class of rules satisfying certain axioms. We also



An arrow from R to R' indicates that, for all $(c,E) \in \mathscr{C}$ with either (a) $E = \frac{1}{2}C$, (b) $E < \frac{1}{2}C$, or (c) $E > \frac{1}{2}C$, $R(c,E)$ Lorenz dominates $R'(c,E)$. An equality sign between R and R' indicates equality of the awards vectors of R and R' for the given restricted domain. The absence of an arrow indicates either that there is no Lorenz relationship, or that a Lorenz relationship can be established using transitivity. The arrow with $*$ indicates a relationship that has only been established for $n \leq 4$.

Figure 6.2. Lorenz Rankings of Nine Division Rules over Restricted Domains

provided results on Lorenz dominance rankings involving the other rules: the proportional rule, the adjusted proportional rule, the random arrival rule and the minimal overlap rule. The analysis revealed that, in describing the Lorenz dominance relationships holding between the nine rules, it is interesting to consider separately the sets of claims problems for which the estate is smaller than, equal to, and greater than, the sum of the half-claims, respectively. Over each of these three sub-domains, we obtained an almost complete Lorenz dominance ranking of the considered rules.

Appendix 6.A: Proofs

In all proofs except those of Propositions 6.1 to 6.5, we assume for convenience that individuals are indexed such that $c_1 \leq c_2 \leq \cdots \leq c_n$ for all considered $(c,E) \in \mathscr{C}$. We can assume this without loss of generality because the proofs in question only deal with symmetric rules.

Proof of Proposition 6.1. Seeking a contradiction, let R be a rule and let $(c,E) \in \mathscr{C}$ be such that $CEA(c,E)$ does not Lorenz dominate $R(c,E)$. From the definition of the constrained equal awards rule, we have that there exist $j \geq 0$ and $\mu > 0$ such that

$$CEA_{(i)}(c,E) = \begin{cases} c_{(i)} & \text{for all } i = 1,\ldots,j; \\ \mu < c_{(i)} & \text{for all } i = j+1,\ldots,n. \end{cases}$$

Since $CEA(c,E)$ does not Lorenz dominate $R(c,E)$, it must be the case that there exists a $k \geq 1$ such that

$$\begin{aligned} &R_{(1)}(c,E) + R_{(2)}(c,E) + \cdots + R_{(k)}(c,E) \\ &\qquad > CEA_{(1)}(c,E) + CEA_{(2)}(c,E) + \cdots + CEA_{(k)}(c,E). \end{aligned}$$

By consequence, for some $\ell \leq k$, we have $R_{(\ell)}(c,E) > CEA_{(\ell)}(c,E)$. Obviously, $CEA_{(\ell)}(c,E) = \mu$ because otherwise $R_{(\ell)}(c,E) > c_{(\ell)}$. Now, there must also be an $m > k$ such that $R_{(m)}(c,E) < CEA_{(m)}(c,E)$ since we require $\sum_{i \in N} R_i(c,E) = \sum_{i \in N} CEA_i(c,E)$. However, because $CEA_{(m)}(c,E) = \mu$, $R_{(\ell)}(c,E) > R_{(m)}(c,E)$ while $\ell < m$, and we have a contradiction. ■

Proof of Proposition 6.2. Seeking a contradiction, let R be a rule that satisfies order preservation and let $(c,E) \in \mathscr{C}$ be such that $R(c,E)$ does not Lorenz dominate $CEL(c,E)$. From the definition of the constrained equal losses rule, we know that there exist $j \geq 0$ and $\mu > 0$ such that

$$CEL_{(i)}(c,E) = \begin{cases} 0 & \text{for all } i = 1,\ldots,j; \\ c_{(i)} - \mu > 0 & \text{for all } i = j+1,\ldots,n. \end{cases}$$

Because $R(c,E)$ does not Lorenz dominate $CEL(c,E)$, there must exist a $k \geq 1$ such that

$$\begin{aligned} &R_{(1)}(c,E) + R_{(2)}(c,E) + \cdots + R_{(k)}(c,E) \\ &\qquad < CEL_{(1)}(c,E) + CEL_{(2)}(c,E) + \cdots + CEL_{(k)}(c,E). \end{aligned}$$

So, $R_{(\ell)}(c,E) < CEL_{(\ell)}(c,E)$ for some $\ell \leq k$. Note that it must be the case that $CEL_{(\ell)}(c,E) = c_{(\ell)} - \mu$ because otherwise $R_{(\ell)}(c,E) < 0$. Now, there is also an $m > k$ such that $R_{(m)}(c,E) > CEL_{(m)}(c,E)$ since $\sum_{i \in N} R_i(c,E) = \sum_{i \in N} CEL_i(c,E)$.

We also have $CEL_{(m)}(c,E) = c_{(m)} - \mu$. Since $c_{(\ell)} - R_{(\ell)}(c,E) > \mu > c_{(m)} - R_{(m)}(c,E)$ while $c_{(m)} \geq c_{(\ell)}$, R violates order preservation and we have a contradiction. ■

Proof of Proposition 6.3. Seeking a contradiction, let R be a rule that satisfies resource monotonicity and the midpoint property and let $(c,E) \in \mathscr{C}$ be such that $CE(c,E)$ does not Lorenz dominate $R(c,E)$.

Suppose first that $E \geq \frac{1}{2}C$. From the definition of the constrained egalitarian rule, we have that there exist $j \geq 0$, $k \geq j$ and $\mu > 0$ such that

$$CE_{(i)}(c,E) = \begin{cases} c_{(i)} & \text{for all } i = 1,\ldots,j; \\ \mu < c_{(i)} & \text{for all } i = j+1,\ldots,k; \\ \frac{c_{(i)}}{2} > \mu & \text{for all } i = k+1,\ldots,n. \end{cases}$$

Since $CE(c,E)$ does not Lorenz dominate $R(c,E)$, it must be the case that there exists an $\ell \geq 1$ such that

$$\begin{aligned} &R_{(1)}(c,E) + R_{(2)}(c,E) + \cdots + R_{(\ell)}(c,E) \\ &\qquad > CE_{(1)}(c,E) + CE_{(2)}(c,E) + \cdots + CE_{(\ell)}(c,E). \end{aligned}$$

By consequence, there exists an $m \leq \ell$ such that $R_{(m)}(c,E) > CE_{(m)}(c,E)$. It must be the case that $CE_{(m)}(c,E)$ is equal either to μ or to $\frac{c_{(m)}}{2}$ because otherwise $R_{(m)}(c,E) > c_{(m)}$. Now, there must also exist a $p > \ell$ such that $R_{(p)}(c,E) < CE_{(p)}(c,E)$ since $\sum_{i \in N} R_i(c,E) = \sum_{i \in N} CE_i(c,E)$. If $CE_{(p)}(c,E) = \mu$, then also $CE_{(m)}(c,E) = \mu$, so that $R_{(m)}(c,E) > R_{(p)}(c,E)$ while $m < p$ and we have a contradiction. If $CE_{(p)}(c,E) = \frac{c_{(p)}}{2}$, then $R_{(p)}(c,E) < \frac{c_{(p)}}{2}$, which is excluded by the joint use of resource monotonicity and the midpoint property, and again we have a contradiction.

The proof for the case where $E \leq \frac{1}{2}C$ uses lines of reasoning very similar to those used in the proof for the case where $E \geq \frac{1}{2}C$ and those used in the proof of Proposition 6.1, and is therefore omitted. ■

Proof of Proposition 6.4. We first consider part (ii). Seeking a contradiction, let R be a rule that satisfies the midpoint property and super-modularity and let $(c,E) \in \mathscr{C}$ be such that $E \geq \frac{1}{2}C$ and such that $Pin(c,E)$ does not Lorenz dominate $R(c,E)$.

From the definition of the Piniles' rule, we know that there exist $j \geq 0$, and $\mu > 0$ such that

$$Pin_{(i)}(c,E) = \begin{cases} c_{(i)} & \text{for all } i = 1,\ldots,j; \\ \frac{c_{(i)}}{2} + \mu < c_{(i)} & \text{for all } i = j+1,\ldots,n; \end{cases}$$

Since $Pin(c,E)$ does not Lorenz dominate $R(c,E)$, it must be the case that there exists a $k \geq 1$ such that

$$R_{(1)}(c,E)+R_{(2)}(c,E)+\cdots+R_{(k)}(c,E) > Pin_{(1)}(c,E)+Pin_{(2)}(c,E)+\cdots+Pin_{(k)}(c,E).$$

So, there is an $\ell \leq k$ such that $R_{(\ell)}(c,E) > Pin_{(\ell)}(c,E)$. Note that $Pin_{(\ell)}(c,E) = \frac{c_{(\ell)}}{2}+\mu$ since otherwise $R_{(\ell)}(c,E) > c_{(\ell)}$. There must also be an $m > k$ such that $R_{(m)}(c,E) < Pin_{(m)}(c,E)$ because $\sum_{i \in N} R_i(c,E) = \sum_{i \in N} Pin_i(c,E)$. Of course, we also have $Pin_{(m)}(c,E) = \frac{c_{(m)}}{2}+\mu$. The midpoint property implies that $R(c,\frac{1}{2}C) = \frac{1}{2}c$. Hence, we have $R_{(\ell)}(c,E)-R_{(\ell)}(c,\frac{1}{2}C) > \mu$ and $R_{(m)}(c,E)-R_{(m)}(c,\frac{1}{2}C) < \mu$. Since $c_{(m)} \geq c_{(\ell)}$, we have a violation of super-modularity and thus a contradiction.

Part (i) deals with $(c,E) \in \mathscr{C}$ such that $E \leq \frac{1}{2}C$. Since, for all such $(c,E) \in \mathscr{C}$, $Pin_{(\ell)}(c,E) = CE_{(\ell)}(c,E)$, we refer to the proof of Proposition 6.3. ■

Proof of Proposition 6.5. The proof uses ideas which are very similar to the ones used in the proofs of Propositions 6.1 to 6.4 and is therefore omitted. ■

Proof of Lemma 6.1. Take any $(c,E) \in \mathscr{C}$ such that $c_1+c_2+\cdots+c_{n-1} \leq E \leq c_n$. We consider only the case where there is some $i \in N \setminus \{n\}$ such that $c_i > 0$ since the result is trivially true for other cases.

Note that, since $c_i + c_n \leq C$ for all $i \in N \setminus \{n\}$, we have that $c_i \leq E-(E-C+c_n)$ for all $i \in N \setminus \{n\}$. The latter is equivalent to

$$c_i - m_i(c,E) \leq E - \sum_{j=1}^{n} m_j(c,E) \quad \text{for all } i \in N \setminus \{n\}, \tag{6.1}$$

since $c_n \geq E$ implies that $m_i(c,E) = 0$ for all $i \in N \setminus \{n\}$ and since $m_n(c,E) = E-C+c_n > 0$. Moreover, we have

$$c_n - m_n(c,E) \geq E - \sum_{j=1}^{n} m_j(c,E), \tag{6.2}$$

since $c_n \geq E$. Given (6.1) and (6.2), the definition of the adjusted proportional rule yields, for all $i \in N \setminus \{n\}$,

$$\begin{aligned} A_i(c,E) &= P_i\big((c_1,c_2,\ldots,c_{n-1},E-m_n(c,E));E-m_n(c,E)\big) \\ &= \frac{E-(E-C+c_n)}{\sum_{i \in N\setminus\{n\}} c_i + E-(E-C+c_n)} c_i \\ &= \frac{C-c_n}{2(C-c_n)} c_i \\ &= \frac{c_i}{2}. \end{aligned}$$

■

Proof of Lemma 6.2. The proofs of parts (i) and (ii) are very similar. Therefore, we only prove part (i).

Take any $(c,E) \in \mathscr{C}$ for which there is at least one $i \in N$ for which $c_i \geq E$. For all $i \in \{j \in N \mid c_j < E\}$,

$$MO_i(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1} + \frac{c_3 - c_2}{n-2} + \cdots + \frac{c_{i-1} - c_{i-2}}{n-(i-2)} + \frac{c_i - c_{i-1}}{n-(i-1)}$$
$$= \frac{c_i - v}{n-(i-1)},$$

where

$$v = \left[1 - \tfrac{n-(i-1)}{n-(i-2)}\right] c_{i-1} - \left[\tfrac{n-(i-1)}{n-(i-3)} - \tfrac{n-(i-1)}{n-(i-2)}\right] c_{i-2} - \cdots - \left[\tfrac{n-(i-1)}{n-1} - \tfrac{n-(i-1)}{n}\right] c_1.$$

We have $v \geq 0$. Note, moreover, that $i \leq n-1$ for all $i \in \{j \in N \mid c_j < E\}$. Hence, we have that, for all $i \in \{j \in N \mid c_j < E\}$,

$$\frac{c_i - v}{c_i} \leq \frac{n-(i-1)}{2}.$$

From this it follows that, for all $i \in \{j \in N \mid c_j < E\}$,

$$MO_i(c,E) = \frac{c_i - v}{n-(i-1)} \leq \frac{c_i}{2}.$$

■

Proof of Proposition 6.6. We first prove part (i). Take any $(c,E) \in \mathscr{C}$ such that $E \leq \frac{1}{2}C$.

We first consider the case where $m_n(c,E) = 0$. In that case, $m_i(c,E) = 0$ for all $i \in N$. We then have $A(c,E) = P((\min\{c_i, E\})_{i \in N}, E)$, which Lorenz dominates $P(c,E)$.

Second, we consider the case where $m_n(c,E) > 0$. Since $m_n(c,E) > 0$, we have $\sum_{i \in N \setminus \{n\}} c_i < E$. Because $C \geq 2E$, we have furthermore $c_n > E$. So, by Lemma 6.1, we have $A_i(c,E) = \frac{c_i}{2}$ for all $i \in N \setminus \{n\}$. Since if $E \leq \frac{1}{2}C$, then $P_i(c,E) \leq \frac{c_i}{2}$ for all $i \in N \setminus \{n\}$, the result follows.

Now we consider part (ii). Take any $(c,E) \in \mathscr{C}$ such that $E \geq \frac{1}{2}C$.

Suppose first that $m_n(c,E) = 0$. In this case we have $\sum_{i \in N \setminus \{n\}} c_i \geq E$, and so $c_n \leq E$ because also $2E \geq C$. Since $m_i(c,E) = 0$ and $c_i \leq E$ for all $i \in N$, we have that $A(c,E) = P(c,E)$.

Suppose now that $m_n(c,E) > 0$. Define the set $S = \{i \in N \mid m_i(c,E) > 0\}$ and let $r = \min S$. Since, for all $i \in S$, $m_i(c,E) = E - C + c_i$,

$$c_i - m_i(c,E) = C - E \quad \text{for all } i \in S. \tag{6.3}$$

Moreover, we show that if $c_n \leq E$, then

$$C - E \leq E - \sum_{j=1}^{n} m_j(c,E). \tag{6.4}$$

In the case where $c_n > E$, Lemma 6.1 can be used (since also $m_n(c,E) > 0$) to see that $P(c,E)$ Lorenz dominates $A(c,E)$. Therefore, we only consider the case $c_n \leq E$ in what follows. Note that (6.4) holds if and only if

$$C - E \leq E - (E - C + c_r) - (E - C + c_{r+1}) - \cdots - (E - C + c_n),$$

which can be rewritten as

$$0 \leq ((n-r+1)-1)C - ((n-r+1)-2)E - c_r - c_{r+1} - \cdots - c_n,$$

which, in turn, can be rewritten as

$$0 \leq ((n-r+1)-2)(C-E) + C - c_r - c_{r+1} - \cdots - c_n.$$

The latter expression indeed holds since $C \geq E \geq c_n$. Note also that

$$c_i \leq E - \sum_{j=1}^{n} m_j(c,E) \quad \text{for all } i \in N \setminus S, \tag{6.5}$$

since $m_i(c,E) = 0$ implies $E - C + c_i \leq 0$ and by (6.4).

So, using (6.3), (6.4) and (6.5), the definition of the adjusted proportional rule yields

$$\begin{aligned} A(c,E) &= (0,0,\ldots,0,m_r(c,E),m_{r+1}(c,E),\ldots,m_n(c,E)) \\ &\quad + P\Big((c_1,c_2,\ldots,c_{r-1},C-E,C-E,\ldots,C-E);E - \sum_{j=1}^{n} m_j(c,E)\Big). \end{aligned}$$

Hence, we have

$$A_i(c,E) = \begin{cases} \frac{E-\sum_{j=1}^{n} m_j(c,E)}{C-\sum_{j=1}^{n} m_j(c,E)} c_i & \text{for all } i \in N \setminus S; \\ (E - C + c_i) + \frac{E-\sum_{j=1}^{n} m_j(c,E)}{C-\sum_{j=1}^{n} m_j(c,E)}(C-E) & \text{for all } i \in S. \end{cases}$$

So, it follows that

$$\begin{aligned} &A_i(c,E) < \frac{E}{C} c_i = P_i(c,E) \quad \text{for all } i \in N \setminus S; \\ &A_i(c,E) = c_i - \mu \quad \text{for all } i \in S, \end{aligned}$$

for some $\mu \geq 0$. The rest of the proof is similar to the proof of Proposition 6.2 and is therefore omitted. ■

Proof of Proposition 6.7. Given Proposition 6.5 (ii), it is sufficient to show that $T(c,E)$ Lorenz dominates $MO(c,E)$ for all $(c,E) \in \mathscr{C}$ such that $E \geq \frac{1}{2}C$. Take any $(c,E) \in \mathscr{C}$ such that $E \geq \frac{1}{2}C$.

Note that, there exist $j \geq 0$ and $\mu \geq 0$ such that

$$T_i(c,E) = \begin{cases} \frac{c_i}{2} & \text{for all } i = 1,\ldots,j; \\ c_i - \mu \geq \frac{c_i}{2} & \text{for all } i = j+1,\ldots,n. \end{cases}$$

Suppose first that there exists an $i \in N$ such that $c_i \geq E$. Because $2E \geq C$, we have $c_{n-1} \leq E$. If $c_{n-1} = E$, then $c_n = E$ and $c_i = 0$ for all $i \in N \setminus \{n-1,n\}$ since $2E \geq C$. By consequence, $MO(c,E) = T(c,E) = \frac{1}{2}c$ in this case. If $c_{n-1} < E$, then $MO_i(c,E) \leq \frac{c_i}{2}$ for all $i \in N \setminus \{n\}$ by Lemma 6.2. It follows that $T(c,E)$ Lorenz dominates $MO(c,E)$ in this case.

Suppose now that there exists no $i \in N$ such that $c_i \geq E$. Seeking a contradiction, let $(c,E) \in \mathscr{C}$ be such that $T(c,E)$ does not Lorenz dominate $MO(c,E)$. Using Lemma 2, there exist $k \geq 0$ and $\nu > 0$ such that

$$MO_i(c,E) \leq \frac{c_i}{2} \quad \text{for all } i = 1,\ldots,k;$$
$$MO_i(c,E) = c_i - \nu \quad \text{for all } i = k+1,\ldots,n,$$

Since $T(c,E)$ does not Lorenz dominate $MO(c,E)$, it must be the case that there exists an $\ell \geq 1$ such that

$$MO_1(c,E) + MO_2(c,E) + \cdots + MO_\ell(c,E) > T_1(c,E) + T_2(c,E) + \cdots + T_\ell(c,E).$$

By consequence, there must exist an $m \leq \ell$ such that $MO_m(c,E) > T_m(c,E)$. We obviously have that $MO_m(c,E) = c_m - \nu$. There must also exist a $p > \ell$ such that $MO_p(c,E) < T_p(c,E)$ since $\sum_{i \in N} MO_i(c,E) = \sum_{i \in N} T_i(c,E)$. We also have $MO_p(c,E) = c_p - \nu$. Now, since $c_m - T_m(c,E) > \nu > c_p - T_p(c,E)$ while $c_m \leq c_p$, the Talmud rule violates order preservation and we have a contradiction. ■

Proof of Proposition 6.8. Take any $(c,E) \in \mathscr{C}$ such that $E \leq \frac{1}{2}C$.

Suppose first that $c_1 + c_2 + \cdots + c_{n-1} \leq E \leq c_n$. We then have $A_i(c,E) = \frac{c_i}{2}$ for all $i \in N \setminus \{n\}$ by Lemma 6.1, and $MO_i(c,E) \leq \frac{c_i}{2}$ for all $i \in N \setminus \{n\}$ by Lemma 6.2. By consequence, $A(c,E)$ Lorenz dominates $MO(c,E)$ in this case.

Suppose now that either (a) $c_n \geq E$, $c_1 + c_2 + \cdots + c_n > E$, and $c_i < E$ for all $i \in N \setminus \{n\}$, (b) $c_i \geq E$ for at least two $i \in N$, or (c) $c_i < E$ for all $i \in N$. Let $\hat{c} = (\min\{c_i, E\})_{i \in N}$ and $\hat{C} = \sum_{i=1}^{n} \hat{c}_i$. We have $MO(c,E) = MO(\hat{c},E)$. In case (a), we have $m_n = 0$, so that $A(c,E) = P(\hat{c},E)$ (see the proof of Proposition 6.6 (i)). Since in both case (b) and case (c) we have $E \leq \frac{1}{2}\hat{C}$, Proposition 6.6 (i) implies that $A(c,E)$ Lorenz dominates $P(\hat{c},E)$. Hence, it suffices to show that in cases (a), (b) and (c), $P(\hat{c},E)$ Lorenz dominates $MO(\hat{c},E)$.

We first consider cases (a) and (b). For all $i \in N$,

$$MO_i(\hat{c},E) = \frac{\hat{c}_1}{n} + \frac{\hat{c}_2 - \hat{c}_1}{n-1} + \cdots + \frac{\hat{c}_i - \hat{c}_{i-1}}{n-(i-1)}, \tag{6.6}$$

and

$$P_i(\hat{c},E) = \frac{E}{\hat{C}}\hat{c}_1 + \frac{E}{\hat{C}}(\hat{c}_2 - \hat{c}_1) + \cdots + \frac{E}{\hat{C}}(\hat{c}_i - \hat{c}_{i-1}). \tag{6.7}$$

Since $E \geq \frac{\hat{C}}{n}$, we have that $P_1(\hat{c},E) \geq MO_1(\hat{c},E)$. Now suppose that, for some $i \in N$, we have $MO_i(\hat{c},E) > P_i(\hat{c},E)$. Using (6.6) and (6.7) it then follows that, for some $j \leq i$,

$$\frac{\hat{c}_j - \hat{c}_{j-1}}{n-(j-1)} > \frac{E}{\hat{C}}(\hat{c}_j - \hat{c}_{j-1}),$$

or

$$\frac{\hat{C}}{n-(j-1)} > E.$$

But, the latter implies that, for all $k > j$,

$$\frac{\hat{c}_k - \hat{c}_{k-1}}{n-(k-1)} > \frac{E}{\hat{C}}(\hat{c}_k - \hat{c}_{k-1}).$$

Hence, using (6.6) and (6.7), it follows that if $MO_i(\hat{c},E) > P_i(\hat{c},E)$ for some $i \in N$, then $MO_j(\hat{c},E) > P_j(\hat{c},E)$ for all $j > i$. Consequently, it is clear that $P(\hat{c},E)$ Lorenz dominates $MO(\hat{c},E)$.

The proof for case (b) is similar to that for case (a) and is therefore omitted. ■

Proof of Proposition 6.9. Take any $(c,E) \in \mathscr{C}$ such that $E \leq \frac{1}{2}C$.

Suppose there is an $i \in N$ such that $c_i \geq E$. We first show that, for all $i = 1,2$ such that $c_i < E$, $RA_i(c,E) \geq MO_i(c,E)$. For $i = 1$, this is immediate since $RA_1(c,E) \geq \frac{c_1}{n} = MO_1(c,E)$ whenever $c_1 < E$. If $c_2 < E$, then $RA_2(c,E)$ is at least as great as

$$\begin{aligned} &\frac{c_2}{n} + \frac{(n-2)!}{n!}\min\{c_2, E - c_1\} \\ &\qquad = \frac{c_1}{n} + \frac{c_2 - c_1}{n} + \frac{c_2 - c_1}{n(n-1)} + \frac{\min\{c_1, E - c_2\}}{n(n-1)}. \end{aligned} \tag{6.8}$$

Furthermore, if $c_2 < E$, then

$$MO_2(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1}. \tag{6.9}$$

We have that (6.8) is at least as great as (6.9), and, hence, $RA_2(c,E) \geq MO_2(c,E)$ if $c_2 < E$.

Suppose now there exists no $i \in N$ such that $c_i \geq E$. Let t be such that it solves $\sum_{i\in\{j\in N|c_j\geq t\}}(c_i - t) = E - t$. Using the same reasoning as above, we get that, for all $i = 1,2$ such that $c_i < t$, $RA_i(c,E) \geq MO_i(c,E)$.

In the case where $n = 2$, $RA(c,E) = MO(c,E)$ for all $(c,E) \in \mathscr{C}$. In what follows, we only consider the case where $n = 4$ since the proof for the case where $n = 3$ is very similar.

If there is some $i \in N$ such that $c_i \geq E$, then there are four possible cases: (a) $c_1 \geq E$, (b) $c_1 < E$ and $c_2 \geq E$, (c) $c_i < E$ for all $i = 1,2$ and $c_3 \geq E$, (d) $c_i < E$ for all $i = 1,2,3$ and $c_4 \geq E$. In case (a), we clearly have $RA(c,E) = MO(c,E)$. In case (b), we have $RA_1(c,E) \geq MO_1(c,E)$ and, moreover, $RA_2(c,E) = RA_3(c,E) = RA_4(c,E)$ and $MO_2(c,E) = MO_3(c,E) = MO_4(c,E)$. By consequence, $RA(c,E)$ Lorenz dominates $MO(c,E)$. In case (c), we have $RA_i(c,E) \geq MO_i(c,E)$ for all $i = 1,2$ and, furthermore, $RA_3(c,E) = RA_4(c,E)$ and $MO_3(c,E) = MO_4(c,E)$. Again, it is clear that $RA(c,E)$ Lorenz dominates $MO(c,E)$. In case (d), we have again $RA_i(c,E) \geq MO_i(c,E)$ for all $i = 1,2$. Seeking a contradiction, suppose that $RA(c,E)$ does not Lorenz dominate $MO(c,E)$. We then have

$$MO_1(c,E) + MO_2(c,E) + MO_3(c,E) > RA_1(c,E) + RA_2(c,E) + RA_3(c,E).$$

By consequence, it must be the case that $MO_3(c,E) > RA_3(c,E)$. It also follows that $MO_4(c,E) < RA_4(c,E)$ since $\sum_{i\in N} RA_i(c,E) = \sum_{i\in N} MO_i(c,E)$. Hence, we have $MO_4(c,E) - MO_3(c,E) < RA_4(c,E) - RA_3(c,E)$. However, we have $MO_4(c,E) - MO_3(c,E) = E - c_3$, while $RA_4(c,E) - RA_3(c,E) \leq E - c_3$ by order preservation, and we have a contradiction.

If there is no $i \in N$ such that $c_i \geq E$, then there are three possible cases: (e) $c_1 \geq t$, (f) $c_1 < t$ and $c_2 \geq t$, (g) $c_i < t$ for all $i = 1,2$ and $c_3 \geq t$. In case (e), $MO(c,E) = CEL(c,E)$, and the desired result follows from Proposition 6.2. In case (f), $RA_1(c,E) \geq MO_1(c,E)$ and there is a $\mu > 0$ such that $MO_i(c,E) = c_i - \mu$ for $i = 2,3,4$. Seeking a contradiction, suppose that $RA(c,E)$ does not Lorenz dominate $MO(c,E)$. Then we have $MO_j(c,E) > RA_j(c,E)$ for some $j = 2,3$. Note that $MO_j(c,E) = c_j - \mu$. We also have $MO_k(c,E) < RA_k(c,E)$ for some $k > j$ because $\sum_{i\in N} RA_i(c,E) = \sum_{i\in N} MO_i(c,E)$. Also $MO_k(c,E) = c_k - \mu$. So we have $c_j - RA_j(c,E) > \mu > c_k - RA_k(c,E)$, while $c_j \leq c_k$. We have a violation of order preservation and hence a contradiction. The proof for case (g) is similar as that for case (f) and therefore omitted. ■

Finally, we consider several examples to illustrate the Lorenz incomparabilities between rules given in Figures 6.1 and 6.2. Examples for higher numbers of individuals are obtained by simply adding individuals with claims equal to 0.

– The adjusted proportional rule and the random arrival rule are incomparable if $E < \frac{1}{2}C$: Consider the claims problems $(c,E) = (10,10,10,20;20)$ and $(c',E') = (10,20,30;20)$. Now, $RA(c,E) \approx (4.2,4.2,4.2,7.5)$ strictly Lorenz dominates $A(c,E) = (4,4,4,8)$, while $A(c',E') = (4,8,8)$ strictly Lorenz dominates $RA(c',E') \approx (3.3,8.3,8.3)$.
– The adjusted proportional rule and the random arrival rule are incomparable if $E > \frac{1}{2}C$: Consider the claims problems $(c,E) = (10,10,10,20;30)$ and $(c',E') = (10,20,30;40)$. Now, $A(c,E) = (6,6,6,12)$ strictly Lorenz dominates $RA(c,E) \approx (5.8,5.8,5.8,12.5)$, while $RA(c',E') \approx (6.7,11.7,21.7)$ strictly Lorenz dominates $A(c',E') = (6,12,22)$.
– The proportional rule and the random arrival rule are incomparable whenever $E < \frac{1}{2}C$: Consider $(c,E) = (20,30;10)$ and $(c',E') = (10,30,30;30)$. We have that $RA(c,E) = (5,5)$ strictly Lorenz dominates $P(c,E) = (4,6)$, while $P(c',E') \approx (4.3,12.9,12.9)$ strictly Lorenz dominates $RA(c',E') \approx (3.3,13.3,13.3)$.
– The proportional rule and the random arrival rule are incomparable whenever $E > \frac{1}{2}C$: Consider the claims problems $(c,E) = (20,30;40)$ and $(c',E') = (10,30,30;40)$. We have that $P(c,E) = (16,24)$ strictly Lorenz dominates $RA(c,E) = (15,25)$, whereas $RA(c',E') \approx (6.7,16.7,16.7)$ strictly Lorenz dominates $P(c',E') \approx (5.7,17.1,17.1)$.
– The proportional rule and the minimal overlap rule are incomparable if $E < \frac{1}{2}C$: Consider the claims problems $(c,E) = (20,30;10)$ and $(c',E') = (10,10,10,40;20)$. We have that $MO(c,E) = (5,5)$ strictly Lorenz dominates $P(c,E) = (4,6)$, while $P(c',E') \approx (2.9,2.9,2.9,11.4)$ strictly Lorenz dominates $MO(c',E') = (2.5,2.5,2.5,12.5)$.

Connection between Chapters 6 and 7: Chapter 6 studied Lorenz dominance comparisons between awards vectors of different rules for the same claims problem. A possibly interesting path for further research is to broaden progressivity comparisons to intra-rule comparisons, i.e., Lorenz dominance comparisons of awards vectors of the same rule for different claims problems. It could be examined, for example, how the degree of progressivity of a rule varies as the estate changes while the claims vector remains the same, or as the inequality level of the claims vector changes while the estate remains the same. It is not obvious, however, how to define the concept of progressivity for intra-rule comparisons. For instance, while it seems natural to view the constrained equal awards rule as having a constant degree of progressivity over all claims problems, the Lorenz dominance relation would say that it has a varying degree of progressivity since the rule does not always select perfectly equal awards vectors. It seems inappropriate, therefore, to use Lorenz dominance directly for this purpose. In Chapter 7, which presents the results of a questionnaire study on the claims problem, variations of progressivity under changes of the characteristics of the claims problem are defined using the proportional rule, the constrained equal awards rule and the constrained equal losses rule as benchmark cases—see especially the discussion in Subsection 7.4.3. The results of the questionnaire study reveal that a substantial number of respondents vary their position with respect to progressivity in simple ways under changes of the claims problem.

Chapter 7

Equality Preference in the Claims Problem: A Questionnaire Study

This chapter is based on joint work with Erik Schokkaert.

7.1 Introduction

How ought an amount of money to be distributed among a group of individuals if these individuals have differing claims with respect to the money and the amount available falls short of the sum of these claims? The study of this type of distribution problem—referred to as the 'claims problem'—is relevant in several economic contexts such as taxation, bankruptcy and inheritance. The literature on claims problems focuses to a large extent on the axiomatic examination of rules, which associate with every claims problem a division of the amount of money among the individuals.[1]

Many of the axioms used in the literature on claims problems have ethical content, which implies that the rules they characterize are open to interpersonal disagreement. The question thus arises to which degree the various proposed rules are found attractive by empirical subjects in solving concrete claims problems.[2] In the theoretical literature, any claims problem is completely given by two characteristics, viz., the vector of claims and the amount to be distributed. However, if rules are to be applied in practice, then the particular economic context is likely to be important as well, as it influences the ethical status of the characteristics of the claims problem. Therefore, from an empirical perspective, the following two question appear to be of importance. (i) *Within-context consistency:* for a

[1] For overviews of this literature see Moulin (2002) and Thomson (2003).

[2] There is an increasing interest in economics in the empirical study of the acceptance of theories of distributive justice. See Konow (2003) and Schokkaert (1999) for recent overviews.

given economic context, to which degree do people use the same rule for different claims problems, i.e., claims problems with different claims vectors and/or available money amounts? (ii) *Between-context uniformity:* for a given claims problem, to which degree do people propose the same division for different economic contexts?

In the existing questionnaire studies on claims problems—Schokkaert and Overlaet (1989), Béhue (2003) and Gächter and Riedl (2006)—the focus is largely on the question of between-context uniformity, while within-context consistency receives only modest attention. These studies consider only a small number of claims problems and, moreover, only provide an analysis on the basis of aggregate data. By ignoring the complete choice patterns of respondents over several claims problems, the studies neglect the individual level analysis which is particularly important to asses within-context consistency.

In this chapter, we deal with both the question of between-context uniformity and that of within-context consistency. The former question is tackled by consideration of a wide variety of claims problems which are all presented to each respondent and by giving special attention to individual level data. The question of between-context uniformity is dealt with by using two versions of the questionnaire with the same claims problems—in the Firm version, three firm owners have to distribute a loss, and in the Pensions version, a shortage in funds has to be distributed over three pensioners. The consideration of a greater variety of claims problems in our questionnaire also benefits the analysis of between-context uniformity in terms of robustness. In the discussion of the results of the questionnaire, we focus especially on the question of how respondents vary tolerance for inequality under variations of the characteristics of the claims problem and of the economic context.

The chapter is structured as follows. In Section 7.2 we present several rules and define the concept necessary for comparing divisions with respect to how much tolerance for inequality they imply. Section 7.3 discusses the setup of our questionnaire. In Section 7.4, the results of the questionnaire are presented and discussed. Section 7.5 concludes.

7.2 Theory

A claims problem involves the allocation of an amount $E \in \mathbb{R}_+$, referred to as the *estate*, among the members of a set $N = \{1, 2, \ldots, n\}$ who have claims adding up to more than E. The *claims vector* is a vector $(c_1, c_2 \ldots, c_n) \in \mathbb{R}^n_+$ where, for all $i \in N$, c_i denotes individual i's *claim*. We let $C = \sum_{i \in N} c_i$. Formally, a *claims problem* is a pair $(c, E) \in \mathbb{R}^n_+ \times \mathbb{R}_+$ where $C \geq E$. The symbol $\mathscr{C}$ denotes the set of all claims problems. The literature on claims problems focuses on rules which select for each claims problem a division of the estate between the individuals. Formally, a *rule* is a function R which associates with each $(c,E) \in \mathscr{C}$ an element of $\mathbb{R}^n_+$, referred to as an *awards vector*, such that $\sum_{i\in N} R_i(c,E) = E$ and $0 \leq R_i(c,E) \leq c_i$ for all $i \in N$.

One of the objectives in the discussion of our questionnaire results will be to compare various well known rules for solving claims problems with respect to how well they explain the choices of the respondents.[3] The three most prominent rules that we consider are the proportional rule, the constrained equal awards rule and the constrained equal losses rule. The proportional rule makes awards proportional to claims.

Proportional rule (*P*). For all $(c,E) \in \mathscr{C}$, we have $P(c,E) = \lambda c$ where λ solves $\sum_{i\in N} \lambda c_i = E$.

The constrained equal awards rule proposes equal awards for all individuals, conditionally on not exceeding anyone's claim.

Constrained equal awards rule (*CEA*). For all $(c,E) \in \mathscr{C}$ and all $i \in N$, we have $CEA_i(c,E) = \min\{c_i, \lambda\}$ where λ solves $\sum_{i\in N} \min\{c_i, \lambda\} = E$.

The constrained equal losses rule divides the estate in such a way that all individuals endure an equal loss (defined as the difference between claim and award), under the restriction that no individual receive less than zero.

Constrained equal losses rule (*CEL*). For all $(c,E) \in \mathscr{C}$ and all $i \in N$, we have $CEL_i(c,E) = \max\{0, c_i - \lambda\}$ where λ solves $\sum_{i\in N} \max\{0, c_i - \lambda\} = E$.

We shall consider also five other rules. The Talmud rule defines two regimes depending on whether the estate is smaller or greater than the sum of the half-claims. If the estate is smaller, then the formula of the constrained equal awards rule is used, applied to the vector of half-claims instead of to the claims vector itself. If the estate is greater, then the constrained equal losses rule formula is used, again applied to the half-claims.

Talmud rule (*T*). For all $(c,E) \in \mathscr{C}$, we have

(i) if $E \leq \frac{1}{2}C$, then $T(c,E) = CEA(\frac{1}{2}c, E)$; and
(ii) if $E \geq \frac{1}{2}C$, then $T(c,E) = \frac{1}{2}c + CEL(\frac{1}{2}c, E - \frac{1}{2}C)$.

The next two rules are equal to the Talmud rule in the case where the estate is smaller than the sum of the half-claims, but different in the other case. Piniles' rule utilizes the constrained equal awards rule formula instead of that of the constrained equal losses rule whenever the estate is greater than the sum of the half-claims.

[3] For a thorough discussion of all the rules considered in this section, see Thomson (2003).

Piniles' rule (*Pin*). For all $(c,E) \in \mathscr{C}$, we have

(i) if $E \leq \frac{1}{2}C$, then $Pin(c,E) = CEA(\frac{1}{2}c,E)$; and
(ii) if $E \geq \frac{1}{2}C$, then $Pin(c,E) = \frac{1}{2}c + CEA(\frac{1}{2}c, E - \frac{1}{2}C)$.

Like Pinile's rule, the constrained egalitarian rule uses an egalitarian procedure if the estate exceeds the sum of the half-claims.

Constrained egalitarian rule (*CE*). For all $(c,E) \in \mathscr{C}$, we have

(i) if $E \leq \frac{1}{2}C$, then $CE(c,E) = CEA(\frac{1}{2}c,E)$; and
(ii) if $E \geq \frac{1}{2}C$, then, for all $i \in N$, $CE_i(c,E) = \max\{\frac{c_i}{2}, \min\{c_i,\lambda\}\}$ where λ solves $\sum_{i\in N} \max\{\frac{c_i}{2}, \min\{c_i,\lambda\}\} = E$.

To define the random arrival rule, assume the individuals arrive one at a time and receive full compensations until the estate runs out. The random arrival rule proposes as a division the average over all orders of arrival of the awards vectors obtained in this way. Let Π^N denote the class of all bijections that map N onto itself.

Random arrival rule (*RA*). For all $(c,E) \in \mathscr{C}$ and all $i \in N$, we have

$$RA_i(c,E) = \frac{1}{n!} \sum_{\pi\in\Pi^N} \min\{c_i, \max\{E - \sum_{j\in N, \pi(j)<\pi(i)} c_j, 0\}\}.$$

For defining the minimal overlap rule, we assume, without loss of generality, that the members of N are indexed such that $c_1 \leq c_2 \leq \cdots \leq c_n$. To understand the minimal overlap rule, individuals have to be seen as claiming specific parts of the interval $[0,E]$. Each part of the interval is distributed equally among all individuals claiming it. For instance, the interval $[0,c_1]$ is claimed by everyone, and so everyone gets $\frac{c_1}{n}$. The interval $(c_1,c_2]$ is claimed by everyone except individual 1, and so each member of $N \setminus \{1\}$ receives in addition $\frac{c_2-c_1}{n-1}$. This process goes on until the entire interval $[0,E]$ is covered. In the case where there are individuals who have claims higher than the estate, the claims of these individuals are truncated by the estate. In the case where there are no individuals who have claims higher than the estate, a scalar t is sought such that all individuals with claims higher than t exclusively claim a specific part of the interval $(t,E]$.

Minimal overlap rule (*MO*). For all $(c,E) \in \mathscr{C}$, we have

(i) if $c_i \geq E$ for some $i \in N$, then, for all $i \in \{j \in N \mid c_j < E\}$,

$$MO_i(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1} + \cdots + \frac{c_i - c_{i-1}}{n-(i-1)},$$

and, for all $i \in \{j \in N \mid c_j \geq E\}$,

$$MO_i(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1} + \cdots + \frac{c_{k-1} - c_{k-2}}{n-(k-2)} + \frac{E - c_{k-1}}{n-(k-1)},$$

where $k = \min\{j \in N \mid c_j \geq E\}$; and

(ii) if $c_i < E$ for all $i \in N$, then, for all $i \in \{j \in N \mid c_j < t\}$,

$$MO_i(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1} + \cdots + \frac{c_i - c_{i-1}}{n-(i-1)},$$

and, for all $i \in \{j \in N \mid c_j \geq t\}$,

$$MO_i(c,E) = \frac{c_1}{n} + \frac{c_2 - c_1}{n-1} + \cdots + \frac{c_{k-1} - c_{k-2}}{n-(k-2)} + \frac{t - c_{k-1}}{n-(k-1)} + c_i - t,$$

where t solves $\sum_{i \in \{j \in N \mid c_j \geq t\}} (c_i - t) = E - t$ and $k = \min\{j \in N \mid c_j \geq t\}$.

The various rules imply different attitudes to inequality, an aspect we shall pay special attention to in discussing the results of the questionnaire. In order to make inequality comparisons, we define the concept standardly used in the literature on inequality measurement, viz., the Lorenz dominance relation.[4] Take any $x, y \in \mathbb{R}^n_+$ and let $(x_{(1)}, x_{(2)}, \cdots, x_{(n)})$ and $(y_{(1)}, y_{(2)}, \cdots, y_{(n)})$ denote permutations of x and y, respectively, such that $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$ and $y_{(1)} \leq y_{(2)} \leq \cdots \leq y_{(n)}$. We say that x *Lorenz dominates* y if and only if

$$\sum_{i=1}^{k} \frac{x_{(i)}}{\frac{1}{n}\sum_{i=1}^{n} x_i} \geq \sum_{i=1}^{k} \frac{y_{(i)}}{\frac{1}{n}\sum_{i=1}^{n} y_i} \quad \text{for all } k = 1, \ldots, n.$$

We say that x *strictly* Lorenz dominates y if and only if, moreover, at least one of these inequalities holds strictly. Of two awards vectors proposed for the same claims problem, one is referred to as less unequal than the other if it Lorenz dominates the other. The rule (or respondent) that proposed the less unequal awards vector, will be referred to as *more progressive* for the given claims problem than the one that proposed the more unequal awards vector. The general Lorenz dominance relationships that hold between the rules are described in Chapter 6.

The three prominent rules constitute benchmark cases with respect to progressivity: they imply a uniform attitude in terms of progressivity irrespective of the specific characteristics of the claims problem at hand. The proportional rule is neutral with respect to progressivity in the sense that it chooses from the available awards vectors the (only) one which is Lorenz equivalent to the claims vector. The

[4]See Cowell (2000) and Lambert (2001) for recent overviews of this literature.

constrained equal awards rule and the constrained equal losses rule are extreme cases. The former always selects the least unequal awards vector available, while the latter always selects the most unequal one. In contrast to these three prominent rules, the behaviour of the other five with respect to progressivity is not so clear-cut—as the solutions of the various rules for the claims problems used in our questionnaire show (see Subsection 7.4.1), it is not the case that these rules behave in the same way with regard to progressivity for all claims problems.

7.3 The Setup of the Questionnaire

In the questionnaire, respondents were asked to state their preferred awards vector for nine different claims problems which are presented in Table 7.1. Each of the nine claims problems is a combination of one of three possible claims vectors and one of three possible estates. The three possible claims vectors have the same sum of claims, but differ in terms of inequality (in the Lorenz sense). We mention the claims vectors in order of increasing inequality: $(1500, 2000, 2500)$, $(1000, 2000, 3000)$ and $(500, 2000, 3500)$. The three possible estates, 4500, 3000 and 1500, are greater than, equal to, and smaller than the sum of the half-claims, respectively. By using all nine combinations of these claims vectors and estates in our questionnaire, we obtain data for a wide variety of claims problems. The questionnaire design allows us to analyze the effect on responses of a change in estate for a given claims vector (questions 1, 2 and 3; questions 4, 5 and 6; questions 7, 8, and 9), as well as the effect of a change in claims inequality for a given estate (questions 1, 4 and 7; questions 2, 5 and 8; questions 3, 6, and 9). For each of the nine questions, respondents were presented with a list of alternative awards vectors to choose from—these lists of alternatives will be given in the next section where we discuss the results of the questionnaire. To be able to examine *within-context consistency*, i.e., the degree to which respondents use the same rule for different claims problems, the awards vectors selected by each of the rules defined in the previous section are included in the lists of alternatives for every question.

In order to allow us to tackle the question of *between-context uniformity*, i.e., the question of how the economic context in which the claims problems are presented affects responses, we consider two versions of the questionnaire with the same nine claims problems, but with different background stories. The questions in the 'Firm' version are formulated as follows (here we consider question 1):

> Persons A, B and C own a firm together. A, B and C contribute to the activities of the firm in different degrees, and for this reason they have agreed that their salaries differ. They receive monthly €1,500,

Table 7.1. The Questions, All Amounts in Euros

Question 1	c = (1500, 2000, 2500)	E = 4500
Question 2	c = (1500, 2000, 2500)	E = 3000
Question 3	c = (1500, 2000, 2500)	E = 1500
Question 4	c = (1000, 2000, 3000)	E = 4500
Question 5	c = (1000, 2000, 3000)	E = 3000
Question 6	c = (1000, 2000, 3000)	E = 1500
Question 7	c = (500, 2000, 3500)	E = 4500
Question 8	c = (500, 2000, 3500)	E = 3000
Question 9	c = (500, 2000, 3500)	E = 1500

> €2,000 and €2,500, respectively. Each of the three persons have still other sources of income. Due to an unexpected deterioration of the economic circumstances, the part of the revenue of the firm that can be used for salaries in a certain month amounts to only €4,500, not enough to compensate the three firm directors. What is in your view the most just distribution of the sum of €4,500 between persons A, B and C?

In the ‘Pensions’ version, the questions were formulated as follows (here we again consider question 1):

> Persons A, B and C go on retirement. On the basis of the contributions they have paid during their active career, they are entitled to a monthly pension of €1,500, €2,000 and €2,500, respectively. Due to the demographic ageing, these pension amounts can no longer be paid. The government only has €4,500 monthly to spend on the pensions of A, B and C. What is in your view the most just distribution of the sum of €4,500 between persons A, B and C?

The two versions of the questionnaire differ, explicitly or implicitly, in several respects. First, the status of the differences between the claims of the three individuals is different. In the Firm version, these differences are agreed upon by the three firm owners, while in the Pensions version they are explained by contributions in the past of the three pensioners and hence by wage differences during the active career. Therefore, in the Firm version respondents are likely to interpret the differences between claims to be caused more by desert and less by talent than in the Pensions version. Second, the two versions of the questionnaire differ with respect to the relation between the claims or awards and the ultimate outcomes relevant to the three individuals. In the Firm version it is specified that the individuals have also other sources of income. In the Pensions version on the other hand, it is likely that respondents view the pension amounts as very important, perhaps even the only, sources of income of the three individuals. Finally, the scope of the decision is different in the two versions of the questionnaire. Whereas in the Firm version awards pertain only to one monthly pay, in the Pensions version payments are implied to be determined by the decision for much longer.

Questions were presented in series of three: with the long introductions given above for the first of the three questions and shorter introductions for the second and third. After each series of three questions, respondents were encouraged to provide written comments on their choices. The questionnaire was anonymous. In order to test for order effects, we used several variants of the questionnaire with different orders of the questions and different orders of the alternatives. There were no significant differences between these alternative variants, and we therefore pooled all the data.

The questionnaire was conducted among the first year undergraduate economics students and business students of the Catholic University of Leuven in May 2005. At that time, the students had not been exposed to the theory of claims problems in their study programs. In the course of one week, the questionnaires were filled in by the students at the start of several exercise sessions on 'Banking and Finance.' In each session, roughly half of the students participated in the Firm version of the questionnaire, and the other half in the Pensions version. The sample sizes are 123 respondents and 118 respondents for the Firm version and the Pensions version, respectively.

7.4 Results

7.4.1 A First Look

Tables 7.2 to 7.10 report the percentages of the respondents who chose each of the alternative awards vectors in questions 1 to 9, respectively. The awards vectors presented to the respondents appear in sequence of increasing inequality (in the sense of the Lorenz criterion). For convenience of exposition, we have distinguished three sets of alternatives in the tables. First, the *neutral* awards vector is the one consistent with the proportional rule—it is referred to as neutral because it is Lorenz equivalent to the claims vector. Second, the awards vectors that Lorenz dominate the neutral solution are referred to as *egalitarian*. Third, the *anti-egalitarian* awards vectors are the ones that are Lorenz dominated by the neutral solution. The tables also report which rules are consistent with each of the alternative awards vectors.

We first focus on the question of between-context uniformity, i.e., on the differences between the results obtained with the two versions of the questionnaire. *Overall, the evidence suggests that responses are less egalitarian in the Firm ver-*

Table 7.2. Question 1, $c = (1500, 2000, 2500)$ and $E = 4500$

	Rule	Awards vector		Firm	Pensions
Egalitarian:	CEA, CE	(1500, 1500, 1500)		4	4
	Pin	(1250, 1500, 1750)		12	21
			Total	*16*	*25*
Neutral:	P	(1125, 1500, 1875)		55	46
Anti-egalitarian:		(1050, 1500, 1950)		2	7
	CEL, T, RA, MO	(1000, 1500, 2000)		27	22
			Total	*29*	*29*

Table 7.3. Question 2, $c = (1500, 2000, 2500)$ and $E = 3000$

	Rule	Awards vector		Firm	Pensions
Egalitarian:	CEA	(1000, 1000, 1000)		2	5
		(850, 1000, 1150)		14	22
			Total	*16*	*27*
Neutral:	P, T, CE, Pin, RA	(750, 1000, 1250)		58	47
Anti-egalitarian:		(650, 1000, 1350)		7	12
	CEL, MO	(500, 1000, 1500)		20	14
			Total	*27*	*26*

Table 7.4. Question 3, $c = (1500, 2000, 2500)$ and $E = 1500$

	Rule	Awards vector		Firm	Pensions
Egalitarian:	CEA, T, CE, Pin, RA, MO	(500, 500, 500)		9	11
		(450, 500, 550)		9	24
			Total	*18*	*36*
Neutral:	P	(375, 500, 625)		59	44
Anti-egalitarian:		(250, 500, 750)		20	16
	CEL	(0, 500, 1000)		2	3
			Total	*22*	*19*

Table 7.5. Question 4, $c = (1000, 2000, 3000)$ and $E = 4500$

	Rule	Awards vector		Firm	Pensions
Egalitarian:	CEA, CE	(1000, 1750, 1750)		1	0
	Pin	(1000, 1500, 2000)		11	17
			Total	*12*	*17*
Neutral:	P	(750, 1500, 2250)		73	64
Anti-egalitarian:	RA	(666, 1416, 2416)		4	5
	CEL, T, MO	(500, 1500, 2500)		11	14
			Total	*15*	*19*

Table 7.6. Question 5, $c = (1000, 2000, 3000)$ and $E = 3000$

	Rule	Awards vector		Firm	Pensions
Egalitarian:	CEA	(1000, 1000, 1000)		2	3
		(700, 1000, 1300)		14	24
			Total	*16*	*27*
Neutral:					
	P, T, CE, Pin, RA	(500, 1000, 1500)		73	61
Anti-egalitarian:	MO	(333, 833, 1833)		9	9
	CEL	(0, 1000, 2000)		2	3
			Total	*11*	*12*

Table 7.7. Question 6, $c = (1000, 2000, 3000)$ and $E = 1500$

	Rule	Awards vector		Firm	Pensions
Egalitarian:	CEA, T, CE, Pin	(500, 500, 500)		11	14
	MO, RA	(333, 583, 583)		4	17
			Total	*15*	*31*
Neutral:					
	P	(250, 500, 750)		70	58
Anti-egalitarian:		(150, 500, 850)		12	8
	CEL	(0, 250, 1250)		3	2
			Total	*15*	*10*

Table 7.8. Question 7, $c = (500, 2000, 3500)$ and $E = 4500$

	Rule	Awards vector		Firm	Pensions
Egalitarian:	CEA, CE	(500, 2000, 2000)		3	5
	Pin	(500, 1625, 2375)		10	18
		(450, 1600, 2450)		2	12
		(400, 1500, 2600)		12	13
			Total	*27*	*48*
Neutral:	P	(375, 1500, 2625)		48	34
Anti-egalitarian:	RA	(333, 1333, 2833)		3	7
	T	(250, 1375, 2875)		12	6
	MO	(166, 1416, 2916)		7	2
	CEL	(0, 1500, 3000)		4	2
			Total	*26*	*17*

Table 7.9. Question 8, $c = (500, 2000, 3500)$ and $E = 3000$

	Rule	Awards vector		Firm	Pensions
Egalitarian:	CEA	(500, 1250, 1250)		6	17
		(350, 1100, 1550)		13	30
			Total	*19*	*47*
Neutral:	P, T, CE, Pin, RA	(250, 1000, 1750)		67	44
Anti-egalitarian:	MO	(166, 916, 1916)		9	4
	CEL	(0, 750, 2250)		3	2
			Total	*12*	*6*

Table 7.10. Question 9, $c = (500, 2000, 3500)$ and $E = 1500$

	Rule	Awards vector		Firm	Pensions
Egalitarian:	CEA	(500, 500, 500)		4	17
	T, CE, Pin	(250, 625, 625)		6	16
	MO, RA	(166, 666, 666)		5	3
			Total	*15*	*36*
Neutral:	P	(125, 500, 875)		62	46
Anti-egalitarian:		(100, 450, 950)		9	6
		(50, 450, 1000)		8	3
	CEL	(0, 0, 1500)		4	3
			Total	*21*	*12*

Table 7.11. Chi-Square Tests for Homogeneity of Firm and Pensions Versions

Question	Test Statistic	p-Value
1	2.778	0.249
2	3.980	0.137
3	8.404	0.015
4	1.924	0.382
5	3.932	0.140
6	7.685	0.021
7	10.135	0.006
8	18.640	0.000
9	13.394	0.001
All Questions	70.873	0.000

sion than in the Pensions version. In all nine questions, the percentage of the respondents that chose egalitarian awards vectors is lower in the Firm version than in the Pensions version, and in six out of nine questions the percentage that chose anti-egalitarian awards vectors is higher in the Firm version than in the Pensions version. The chi-square test statistics given in the first nine lines of Table 7.11 test for each question separately the null hypothesis that the population proportions for the categories egalitarian, neutral and anti-egalitarian, respectively, are equal for the two questionnaire versions. The test statistic given in the last row does the same for the complete set of questions, i.e., for 27 (3×9) categories. Table 7.11 confirms that responses are significantly different for the two different versions. The evidence that respondents chose less egalitarian alternatives in the Firm version than in the Pensions version is particularly strong in the case of questions 3, 6, 7, 8 and 9. Note that in each of these questions the majority of the alternative award vectors feature incomes lower than €500, an amount close to the minimally guaranteed income in Belgium. The fact that respondents in the Pensions version are especially egalitarian in these cases, suggests that they may have some concern for a minimum level of income being respected—in fact, this concern was expressed explicitly by several respondents in the comments box of the questionnaire. Put differently, for especially low awards, considerations with respect to needs appear to override considerations with respect to claims. It is interesting to note that the evidence suggests that the reason why respondents choose more egalitarian alternatives in the Pensions version is not because they favour equality in itself but rather because they want to make sure individuals get a sufficient amount of income.

The aggregate data given in Tables 7.2 to 7.10 is not completely suitable for examining within-context consistency, i.e., for evaluating rules with respect to how well they describe the choices of the respondents. It is inherent in the definition of a rule that it proposes an awards vector for *every* claims problem. Hence, to evaluate them we need to look at the entire response pattern of respondents, not just at aggregates. However, one first impression on the basis of the aggregate data is worth mentioning. *The awards vectors consistent with the proportional rule perform very well in explaining responses: they are convincingly most popular in every question.* Although this is true for both versions of the questionnaire, the awards vectors of the proportional rule are clearly more popular in the Firm version than in the Pensions version. Note that, both for the Firm version and for the Pensions version, the proportional rule is especially popular in questions 4, 5 and 6 where the claims are $(1000, 2000, 3000)$. It is hard to find an economic explanation for this observation—perhaps it is simply due to the fact that, for the given claims vector, the awards vectors of the proportional rule are particularly easy to calculate.

In the next subsection, we will provide a more robust analysis based on individual level data to compare the empirical performance of the various rules. Before moving on, however, we discuss two basic intuitions of the respondents that are revealed in the aggregate data for both versions of the questionnaire: concerns for *strict order preservation* and *nonzero awards*. Both of these concerns were also stated explicitly in comments by several respondents.

Respondents in both questionnaire versions seem to want the order in claims to be preserved strictly in awards. Alternatives in which individuals with different claims get the same award only appear in the sets of egalitarian awards vectors of the questions. For all questions in which alternatives are available in the set of egalitarian awards vectors that respect strict order preservation (this is the case in all questions except 6 and 9), the awards vector that violates strict order preservation is almost always least popular among the egalitarian awards vectors for both questionnaire versions (the only exception being question 7 in the Firm version). Since the constrained equal awards rule never respects strict order preservation, its awards vectors perform rather badly in describing respondents' choices.

There seems to be a reluctance among respondents in both questionnaire versions to give an individual a zero award. Awards vectors in which an individual gets a zero award only appear as the least egalitarian alternative in the set of anti-egalitarian awards vectors of the questions. In almost all questions in which such an alternative is present (all questions except 1, 2 and 4), it is least popular among the anti-egalitarian awards vectors for both versions of the questionnaire (the only exception being question 7 in the Firm version). Since the constrained equal losses rule always selects awards vectors with zero awards if they are consistent with the definition of a rule, it does badly in the cases in which such awards vectors are among the alternatives. At the same time, it should be mentioned that in the questions in which the constrained equal losses rule gives everyone strictly more than zero (questions 1, 2 and 4), its awards vectors are most popular in the set of anti-egalitarian solutions.

Table 7.12. Distances to Award Vectors of Rules

	Rule	Firm		Pensions	
		Average	Ranked 1st	Average	Ranked 1st
Egalitarian:	CEA	7149	2	6344	4
	CE	4859	1	4685	0
	Pin	3209	6	3062	23
Neutral or unclear:	P	1391	75	1919	58
	T	3055	2	3285	2
	RA	2727	6	2989	5
	MO	3999	6	4384	6
Anti-egalitarian:	CEL	6974	2	7824	3

7.4.2 A Comparison of Rules

Strictly speaking, a respondent is consistent with a rule only if she chooses the awards vectors implied by the rule in all questions. Due to the relatively high number of questions in the questionnaire, this test is, however, rather stringent. *Nevertheless, 38% of the respondents were consistent with the proportional rule in all nine questions in the Firm version and 20% in the Pensions version.* Given this good performance, it is not surprising that it was often mentioned by respondents in their comments that they were applying a proportional procedure throughout the questionnaire. The only other rule that respondents have chosen consistently with in all nine questions is the constrained equal losses rule: however, this is the case only for 2% of the respondents in each of the two versions of the questionnaire.

A less stringent way for comparing the performance of the various rules in describing respondents' choices can be obtained by making use of the following concept of 'distance.' Let (c^ℓ, E^ℓ) be the claims problem used in question $\ell = 1, \ldots, 9$ and denote the awards vector chosen by respondent k in question ℓ by $A^k(c^\ell, E^\ell)$. We define the 'distance' between the set of awards vectors chosen by k and the set of awards vectors implied by rule R as $\sum_{\ell=1}^{9} \sum_{i=1}^{3} |A_i^k(c^\ell, E^\ell) - R_i(c^\ell, E^\ell)|$, i.e., as the total money amount that respondent k deviates from what is prescribed by rule R. The calculated distances can be used to compare the empirical performance of the different rules: the lower the distance, the better the performance of the rule in describing the choices of the given respondent.

Table 7.12 presents the average over all respondents of the distance of the response pattern implied by each of the rules to the response pattern chosen by each respondent. Also reported, for each rule, are the percentages of the respondents for whom the rule is ranked first, i.e., for whom the distance to the given rule is lower than that to each other rule. The categories *egalitarian*, *anti-egalitarian* and *neutral* are defined similarly as in Tables 7.2 to 7.10. A rule is categorized as *unclear* if it does not belong to any of the three other categories for all questions. *For* *the Firm version, the proportional rule clearly performs best, while all other rules pale in comparison. For the Pensions version, the proportional rule also comes out first, but less overwhelmingly so: Piniles' rule apparently picks up on the more egalitarian responses for this version and comes out as a convincing second.* It is remarkable that Piniles' rule outperforms the prominent constrained equal awards and constrained equal losses rules. Table 7.12 also confirms the conclusion with respect to the differences between the two questionnaire versions that was stated in the previous subsection: egalitarian rules do better in the Pensions version.

7.4.3 Variations in Degree of Egalitarianism

In the previous subsection, we studied the question of within-context consistency, i.e., whether responses use the same rule for each claims problem. Here, we consider a similar question but in terms of progressivity. We examine whether respondents take, for each claims problem, the same position with respect to progressivity, or whether they vary their position in a straightforward manner depending on the characteristics of the claims problem at hand. Specifically, we analyze how responses vary under two basic variations of the claims problem: (a) a decrease of the estate while the claims remain the same, and (b) an increase in the inequality of the claims vector while the estate remains the same.

In Table 7.13, the response patterns over the combinations of questions relevant for question (a) are summarized (in percentages). The category *same* covers the response patterns consistent with the proportional rule, the constrained equal awards rule or the constrained equal losses rule, i.e., the patterns in which the degree of progressivity remains unchanged. The other four categories—*decrease*, *increase*, *decrease-increase* and *increase-decrease*—describe simple variations in progressivity, and are also defined using the three prominent rules as benchmark cases. To give an example: a response pattern over questions 1, 2 and 3 which is consistent with the constrained equal awards rule in question 1, consistent with the proportional rule in question 2 and consistent with the constrained equal losses rule in question 3, would be categorized under *decrease*.[5] A complete description

[5]The reason why the Lorenz dominance relation was not used to define the categories, is that this relation is not appropriate for making progressivity comparisons between awards vectors proposed for *different* claims problems. To illustrate this point, suppose a respondent chooses the awards vectors consistent with the constrained equal awards rule in both questions 4 and 5. The Lorenz dominance relation would in that case say that the respondent is less progressive in question 4 than in question 5, while it seems more natural to conclude instead that the respondent takes the same position with respect to progressivity in the two question, viz., the extremely egalitarian one. The problem with the Lorenz dominance relation is that it does not take into account the restrictions standardly imposed in the literature on claims problems—in the case of the illustration, it fails to recognize that an individual should never receive an award greater than her claim.

Table 7.13. Evolution of Progressivity as Estate Decreases

Quest.	Context		Same		Decrease	Increase	Decrease-	Increase-
		P	CEA	CEL			Increase	Decrease
1, 2, 3	Firm	45	2	2	3 (0.998)	22 (0.000)	9 (0.923)	4 (1.000)
	Pensions	29	2	2	9 (0.872)	26 (0.000)	6 (1.000)	3 (1.000)
4, 5, 6	Firm	55	0	2	8 (0.471)	16 (0.000)	2 (1.000)	7 (0.962)
	Pensions	39	0	2	6 (0.978)	21 (0.000)	2 (1.000)	5 (1.000)
7, 8, 9	Firm	43	1	2	11 (0.106)	10 (0.179)	2 (1.000)	10 (0.952)
	Pensions	25	3	2	7 (0.881)	15 (0.025)	4 (1.000)	11 (0.990)

of the response patterns belonging to each of the categories is given in Appendix 7.A.

The category *same* performs best empirically, a result that can be ascribed to to the popularity of the proportional rule response patterns. It is interesting to examine how the various other categories perform in describing the choices of those respondents not consistent with the *same* category. Therefore, Table 7.13 provides *p*-values for the null hypothesis that the population proportions for each of the given categories is equal to what it would be if choices of respondents not consistent with the *same* category were completely random.[6] Obviously, only if the evidence allows this hypothesis to be rejected, the category is useful in explaining non-*same* consistent responses. *We find that, for both versions of the questionnaire, the category* increase*, describing an increase in progressivity as the estate decreases, performs well empirically, whereas all other categories fail.* The popularity of the *increase* category is consistent with the observation made in Subsection 7.4.1 that respondents seem to attribute importance to minimal income needs. What is interesting is that this pattern appears to be present not only for the Pensions version of the questionnaire, but also for the Firm version, albeit in a somewhat weaker form.

Table 7.14 presents similar results as Table 7.13 but for question (b). Again, the *same* consistent patterns perform very well. The question again arises how the other categories perform in describing the choices of the respondents who are not consistent with this category. *The table shows that, for both the Firm version and the Pensions version, the* increase *category, describing an increase in progressivity as claims inequality increases, performs very well empirically, whereas all other categories fail.* It is not immediately clear how this pattern can be given economic meaning. However, it is interesting to note that the same pattern was also found by Gächter and Riedl (2006) in a study on the basis of aggregate results for two questions.

[6]The *p*-values are for the one sided exact test based on the binomial distribution.

Table 7.14. Evolution of Progressivity as Claims Inequality Increases

Quest.	Context	P	Same CEA	CEL	Decrease	Increase	Decrease-Increase	Increase-Decrease
1, 4, 7	Firm	42	0	2	11 (0.242)	22 (0.000)	8 (0.988)	7 (0.998)
	Pensions	21	0	2	8 (0.908)	34 (0.000)	13 (0.983)	3 (1.000)
2, 5, 8	Firm	49	2	2	3 (0.996)	17 (0.001)	2 (1.000)	7 (0.966)
	Pensions	30	1	2	3 (1.000)	26 (0.000)	9 (0.994)	2 (1.000)
3, 6, 9	Firm	48	4	2	11 (0.106)	8 (0.414)	2 (1.000)	7 (0.957)
	Pensions	34	8	2	5 (0.972)	18 (0.002)	5 (1.000)	3 (1.000)

7.5 Conclusion

We discussed in this chapter the results of a questionnaire concerning claims problems that was held among Belgian students. Two versions of the questionnaire were considered—the Firm version and the Pensions version—in which the same claims problems were presented in different economic contexts. The questionnaire setup allowed us to consider (i) the question of within-context consistency, i.e., the degree to which respondents apply the same rule for different claims problems in the same economic context, and (ii) the question of between-context uniformity, i.e., the degree to which respondents propose the same awards vector for the same claims problem in different economic contexts.

With regard to the question of within-context consistency, we found that the proportional rule performed very well in describing the choices of the respondents in both versions of the questionnaire, this in stark contrast to the other two rules that play a prominent role in the literature, viz., the constrained equal awards rule and the constrained equal losses rule. These latter two rules fail to capture basic intuitions of the respondents. The constrained equal awards rule in many cases gives equal awards to individuals with different claims, whereas respondents seemed to prefer to respect these differences in the choice of the awards vector. Respondents also appeared to be reluctant to give a zero award to an individual with a nonzero claim, an intuition typically violated by the constrained equal losses rule.

As for the question of between-context uniformity: responses were clearly more egalitarian in the Pensions version of the questionnaire than in the Firm version, a phenomenon we suggested could be due to the fact that the given context induced respondents to give more weight to respect for basic needs in the choice of awards vectors. As a consequence, in a comparison of the empirical performance of all rules, the egalitarian Piniles' rule came in as a relatively convincing second to the proportional rule. In the Firm version, on the other hand, no rule distinguished itself as a serious competitor for the proportional rule.

We also considered the question of within-context consistency from the inequality perspective. The questionnaire design allowed us to examine variations in the tolerance of inequality of the respondents under simple changes of the characteristics of the claims problem. We found that, for those respondents who were not consistent with a uniform attitude towards inequality in all claims problems, response patterns describing an increase in progressivity as the estate decreases other things equal and as inequality of the claims vector increases other things equal performed well empirically. It is interesting to note as a suggestion for new lines of research that although, as we have shown, simple ideas stated in terms of inequality are useful in organizing empirical intuitions concerning claims problems, such ideas have remained largely unexamined in the theoretical literature.

Appendix 7.A: Description of Categories

Over two questions, the response patterns belonging to the categories *same*, *decrease* and *increase* can be defined using the proportional rule, the constrained equal awards rule and the constrained equal losses rule as benchmarks.

1. *Same* progressivity in question x and question y:
   (a) Consistent with the constrained equal awards rule in question x and in question y.
   (b) Consistent with the proportional rule in question x and in question y.
   (c) Consistent with the constrained equal losses rule in question x and in question y.
2. *Decrease* of progressivity from question x to question y:
   (a) Consistent with the constrained equal awards rule in question x and less progressive than the constrained equal awards rule in question y.
   (b) More progressive than the proportional rule in question x and at most as progressive as the proportional rule in question y.
   (c) Consistent with the proportional rule in question x and less progressive than the proportional rule in question y.
   (d) Less progressive than the proportional rule but not consistent with the constrained equal losses rule in question x and consistent with the constrained equal losses rule in question y.
3. *Increase* of progressivity from question x to question y:
   (a) Consistent with the constrained equal losses rule in question x and more progressive than the constrained equal losses rule in question y.
   (b) Less progressive than the proportional rule in question x and at least as progressive as the proportional rule in question y.
   (c) Consistent with the proportional rule in question x and more progressive than the proportional rule in question y.
   (d) More progressive than the proportional rule but not consistent with the constrained equal awards rule in question x and consistent with the constrained equal awards rule in question y.

Over three questions, the five categories *same*, *decrease*, *increase*, *decrease-increase* and *increase-decrease* are defined as follows.

1. *Same* progressivity over questions x, y, z: *Same* in x and y and *same* in y and z.
2. *Decrease* of progressivity over questions x, y, z:
   (a) *Decrease* from x to y and *decrease* from y to z.
   (b) *Decrease* from x to y and *same* from y to z.
   (c) *Same* from x to y and *decrease* from y to z.
3. *Increase* of progressivity over questions x, y, z:
   (a) *Increase* from x to y and *increase* from y to z.
   (b) *Increase* from x to y and *same* from y to z.
   (c) *Same* from x to y and *increase* from y to z.
4. *Decrease-increase* of progressivity over questions x, y, z: *Decrease* from x to y and *increase* from y to z.
5. *Increase-decrease* of progressivity over questions x, y, z: *Increase* from x to y and *decrease* from y to z.

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