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DISCUSSION PAPER

Risk Management in Electricity Markets: Hedging and Market Incompleteness [1]

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Abstract

The high volatility of electricity markets gives producers and retailers an incentive to hedge their exposure to electricity prices by buying and selling derivatives. This paper studies how welfare and investment incentives are affected when markets for derivatives are introduced, and to what extent this depends on market completeness. We develop an equilibrium model of the electricity market with risk-averse firms and a set of traded financial products, more specifically: forwards and an increasing number of options. Using this model, we first show that aggregate welfare in the market increases with the number of derivatives offered. If firms are concerned with large negative shocks to their profitability due to liquidity constraints, option markets are particularly attractive from a welfare point of view. Secondly, we demonstrate that increasing the number of derivatives improves investment decisions of small firms (especially when firms are risk-averse), because the additional financial markets signal to firms how they can reduce the overall sector risk. Also the information content of prices increases: the quality of investment decisions based on risk-free probabilities, inferred from market prices, improves as markets become more complete Finally, we show that government intervention may be needed, because private investors may not have the right incentives to create the optimal number of markets.

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1 Introduction

The electricity sector has been subject to major structural changes during the last decade. Liberalization policies all over the world required a separation of formerly vertically integrated monopolies into three parts: production, retail and network services. Network services are a natural monopoly and should remain regulated. Introducing competition is possible at the level of production and retail. In this paper we look at competition at the production level and the retail level, while assuming that prices in the retail market remain fixed in the short run.

The product electricity has special characteristics which greatly affects the way electricity markets are organized. Electrical energy *cannot be stored* economically, and therefore has to be produced the moment it is consumed. Intertemporal arbitrage is impossible, and the price for electricity is determined by the supply and demand conditions at each given hour. As *demand* for electrical energy is *very inelastic* and of *a stochastic nature* and generators face production capacity constraints, spot prices are very volatile.

Liberalized electricity markets are therefore typically organized around regional spot markets for energy, which determine hourly spot prices, complemented with markets for long term contracts, which help coordinate the actions of the players and hedge volume and price risks. The extent to which a firm can hedge its exposure, depends on the availability of markets, their liquidity (determined by such parameters as trading volume and bid-ask spread), and the presence of speculators who can absorb part of the risk. These factors change as markets evolve from pure OTC to sophisticated spot and futures markets, and to more complete markets in which there is a liquid trade of a broad set of derivatives[2].

The objective of this paper is to analyze the effect of *market completeness* on welfare and on investment incentives. Market completeness is measured as the number of electricity options available to producers and retailers, in addition to a forward contract.

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[2] Vertical integration of electricity production and retail is an alternative way of creating a "complete" set of hedging instruments between production and retail.

Indeed, as more options with different strike prices become available, firms obtain more instruments to trade risks and markets become more complete.[3]

This paper develops an equilibrium model of the electricity market, which includes the production process, the spot market trades and the trade of derivatives. The model is calibrated on the German electricity market. First, the results show that adding option markets is welfare-enhancing, but that most of the benefits are obtained with one to three options. In particular, if firms have strong aversion of negative shocks (shocks that would cause firm bankruptcy), then option contracts are needed to prevent retailers to go bankrupt, and welfare to be destroyed. Second, we analyze how investment decisions by small firms are affected when an increasing number of derivatives are traded. We show that investment decisions are improved when more derivatives are traded, especially when firms get more risk averse. The reason is that financial markets signal to new entrants how they can reduce the overall sector risk, and at the same time, they allow entrants to extract some of those sector-wide benefits. The amount of information that is contained in the equilibrium market prices, increases with the number of financial product being traded: it is shown that the quality of investment decisions that are based on risk-free probabilities inferred from market prices, improves with the number of contracts being traded.

Finally, we show that government intervention may be needed, because private investors do not have the right incentives to create the optimal number of markets. Indeed, adding additional derivatives may reduce broker revenue from previously existing contracts, despite being overall welfare-improving.

A side result of the model is that it shows how prices and price volatility are driven by the underlying market fundamentals, i.e. cost characteristics and demand uncertainty. The results of the model are complementary to the classical valuation models for financial derivatives. Classical models assume that the prices of the underlying asset follow some stochastic process and that a risk free portfolio can be built using delta hedging to price the derivative. Here we explain how price volatility is driven by the

[3] The paper assumes that demand shocks are the only cause of risk. Theoretically the market is then complete, if options at every strike price can be traded. However if there are also firm specific shocks, then additional derivatives should be added for the market to be complete.

market dynamics. The advantage of our approach is that pricing of all products will be consistent with the overall industry behavior and with the underlying physical reality.

The paper is organized as follows. First, section 2 provides an overview of relevant research on incomplete markets (including the applicability to electricity markets). Next, section 3 describes the electricity market model that is used to obtain the results of this paper, while section 4 describes the model data. Section 5 verifies the welfare effects of an increasing number of markets. Sections 6 and 7 analyzes the effect on investment incentives, based on welfare considerations (section 6) and on risk-free probabilities, i.e. the 'finance approach" (section 7), respectively. Section 8 describes practical conditions for financial innovations, and finally, section 9 summarizes our conclusions.

2 Literature review

The topic of this paper is closely related with the literature on incomplete markets and financial innovation, as well as with literature on hedging in electricity markets. In this section we first introduce the concept of incomplete markets. Next, we discuss the main results of the literature. Then, we highlight the relevance for electricity markets. Finally, we discuss related work on hedging in electricity markets. We base our discussion on market completeness mainly on (Staum, 2008) and (Duffie and Rahi, 1995).

2.1 Incomplete markets

Markets are incomplete when *perfect risk transfer* between the agents is *impossible*. There might be several reasons why this is the case. First, it might be that the marketed set of assets is insufficient to hedge the class of risk one wishes to hedge. This type of incompleteness deals with the spanning role of securities. See also (Allen and Gale, 1994).

Second, markets might be imperfect due to the existence of transaction costs and/or trading constraints. For instance, firms might not be able to take a short position in a traded security. These costs and/or constraints make it effectively impossibly to transfer risk perfectly.

In our paper we focus on the first type of market incompleteness: the missing markets problem.

In practice, markets are never complete, as not all risk factors are traded on a market. Hence, when might market incompleteness be relevant for hedging or pricing decisions? We mention two situations in which this might be the case. The first situation is when some of variables that one would like to hedge are derived from non-market prices, such as for weather derivatives. Another typical situation of market incompleteness occurs when the price of an asset does not follow a standard random walk process – where prices changes are 'infinitesimally small' – but contains 'large' price jumps. The problem with price jumps is that a hedging strategy where one dynamically adjusts a portfolio containing the risk-free asset and the asset itself is no longer possible, as the payout is non-linear in the size of the shock. In order to complete the market one would need to add a forward market and a set of option markets with different strike prices.

2.2 Research results on incompleteness

The main results of the literature on incomplete markets with respect to the welfare effects of additional markets and the pricing of these assets are the following.[4]

Welfare in an incomplete market is lower than in a complete market because not all risk is perfectly allocated in the market. This is a rather intuitive result: as in an incomplete market not all potential gains from trade are exhausted, total welfare can be improved by a sufficient number of additional markets until the market is complete. This simple intuition does, however, not carry over to situations where only one additional market is added to the economy, which does not complete the market. (Hart, 1975) shows that adding a financial product might make every one in the economy worse off. Extending this result, (Elul, 1995) and (Cass and Citanna, 1998) show that in an economy with

[4] In this paper we assume that a Walrasian equilibrium exists, even when markets are incomplete. In a general equilibrium setting with multiple goods, (where securities can contain different bundles of goods), this is not guaranteed. However, when we restrict ourselves to economies where financial claims only have a pay-off in terms of a single numeraire good, existence is satisfied. On existence of equilibria in a general equilibrium setting see (Duffie and Shafer, 1985) and (Duffie and Shafer, 1986).

many consumption goods one can always find an asset that makes everyone worse off, or an asset that makes everyone better off, or an asset that makes any combination of individuals better or worse off.[5] Based on (Diamond, 1967) however, one can conclude that this does not hold for single-good incomplete markets, since they are constrained suboptimal. In particular, this applies to the typical situation of financial markets, in which money is the single relevant good (this will also be the case in this paper: adding products always increases welfare). To create the conditions required by (Elul, 1995) and (Cass and Citanna, 1998) one would need to introduce multiple independent time periods, or assume that firms obtain a utility from a commodity *per se* (e.g., a "convenience yield"). Note that introducing all financial assets (completing the market) does not necessarily make everyone better off. Complete markets are Pareto efficient, but not necessarily Pareto dominant with all possible incomplete market allocations. (Willen, 2005) studies the impact of market innovation in more detail and shows that, when agents have exponential utility and with normally distributed risks, the effect of a financial innovation can be split up in a portfolio effect and a price effect. (Elul, 1999) studies the welfare effects of a financial innovation in a single-good market.

(Boyle and Wang, 2001) study the *pricing* of a new derivative in an incomplete market. They show that one should not use the standard arbitrage assumptions typically used in the financial (engineering) literature, as the prices of existing assets will change once a new asset is added to the economy.[6] Instead, they recommend to make explicit assumptions on the preferences of the agents in the economy and to use an equilibrium model to derive the prices of the different assets. (Staum, 2008) and (Carr et al., 2001) argue however that results of equilibrium models depend very much on the choice of the utility function, the initial endowment of the firms, and the parameters of the probability measure, and are therefore not useful for trading decisions.

[5] Similar results were earlier obtained by (Milne and Shefrin, 1987) in a specific model specification.

[6] They also show that the condition of arbitrage-free pricing does not determine a unique price for the newly created asset.

2.3 Incompleteness of electricity markets

Given the technical characteristics of the electricity system we described above, spot prices for electrical energy are not only very volatile, but electricity markets are typically also very incomplete. Hence, the risks are large and hard to hedge as not all financial contracts that span the market are traded.

As electricity cannot be stored, standard models for the pricing of forward contracts which rely on inter-temporal arbitrage and cost-of-carry models cannot be used. Instead, one has to rely on equilibrium models of the market and to make assumption about the relative risk aversion of the agents to derive predictions for the pricing of the goods. Bessembinder and Lemon (2002) develop such a model for the electricity market.[7]

Stochastic demand and supply shocks caused by weather changes or plant outages, lead to large jumps in equilibrium price and equilibrium demand levels. As a result of price and quantity jumps, firms are unable to hedge their full portfolio by only trading in forward markets and spot market. Instead, they should rely on trading option contracts with several strike prices, or vertical integration of generation and retail activities. Hence a full set of option contracts is needed to complete the market.

2.4 Hedging in electricity markets

This paper builds further upon existing studies on contracting and hedging in electricity markets, which we review now. In the review we limit ourselves – with one exception – to studies that rely on equilibrium models of the electricity market.[8]

(Bessembinder and Lemmon, 2002) develop a partial equilibrium model of the spot market and *one* forward market. They derive analytical solutions for forward and spot prices in a setting where firms are risk averse, production cost are convex, retail prices

[7] Even tough storage of electrical energy is very expensive; one can still rely on arbitrage principle for pricing electricity forwards when there is a liquid forward market for natural gas and a liquid market for generation capacity able to convert natural gas in electricity. The lack of the latter market makes this strategy impossible.

[8] The alternative to equilibrium models is the study of *one* firm's contracting and production decisions for a exogenously given stochastic spot price process and forward price.

are fixed and demand is stochastic. Their theoretical predictions on risk premia are verified empirically: the model correctly predicts when markets should be in backwardation or in contango.

Our paper extends the framework of (Bessembinder and Lemmon, 2002) and allows for multiple financial products to be traded – not just one forward contract[9]. It further analyzes the effects of speculators trading in a number of derivatives markets and studies alternative, more realistic formulations of risk aversion.

The usefulness of financial instruments other than forwards to hedge risks in electricity markets is discussed by (Oum et al., 2006). They show that a regulated retail firm can use a combination of forwards, call and put options to hedge its volumetric risk, and draw attention to the regulated firm's difficulty to hedge, when regulators might forbid trade in derivatives which look speculative such as weather derivatives and rebuff contracting positions that require the firm to pay a sum ex-ante. The optimal hedging strategy is found by optimizing the firm's utility, subjective to the financing constraint. The results are derived for the CARA and the mean-variance utility functions, with an endogenously given price and quantity distribution function. In our paper we develop an equilibrium model of the market and show that option contracts are important instrument *to transfer volumetric risks* from generators to retailers, even more so when firms might face liquidity constraints.

(Von Der Fehr, 2007) study vertical integration, forward contracting and hedging in an equilibrium electricity market model. They show that vertical integration might increase the equilibrium risk premia in the market and lower overall welfare, compared with forward contracting. The reason why this happens in their model is that it assumed that a vertically merged firm has a "smaller capacity" to take up risk than two separate entities. In our model we also represent vertical integration, but assume that a vertical integrated firm has the "same capacity" to take up risk than two separate companies. To our opinion, this is a more realistic assumption. We see the difference between vertical integration and contracting by means of a forward contract, as follows: Within the

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[9] With a forward contract this paper refers to a contract for future delivery of a fixed quantity of a good and a fixed price. We will not explicitly specify whether these contracts are traded over the counter (OTC), or whether they are traded as 'futures' on a centralized power exchange.

vertical integrated firm risk sharing between generation and retail is perfect, while risk sharing by trading forward contracts is imperfect, leaving part of the risk untraded.

Also (Aid et al., 2006) study vertical integration, forward contracting, hedging, and retail competition. They develop an equilibrium model where firms have a mean-variance utility function and show that both vertical integration and forward contracting allows for a better risk sharing between retailers and generators and leads to lower retail prices, increased market share for small generators, and a reduction of the profits of retailers. Compared with long term contracts, vertical integration leads to perfect risk sharing between generators and retailers. Additionally, forward markets might not develop under some parameters of the game in which case no risk is shared between upstream and downstream firms. The results of (Aid et al., 2006) on the comparison of vertical integration and forward contracting are driven by the change of the utility function (and the implied capacity of firms to take-up risks) and the quality of risk transfer between upstream and downstream firms (market completeness). In our paper we single out the effect of market completeness. We do not, however, study retail competition.

Our paper assumes perfect competitive market and neglect strategic issues associated with long-term contracting that have been reported in the literature.

(Allaz and Vila, 1993) study the role of forward contracts, not as a tool to hedge risks, but as an instrument used by oligopolist to strategically affect market outcomes. It is shown that in a Cournot setting, generation firms will sell forward contracts in order to commit to compete more aggressively in the spot market. Hence forward contracts make markets more competitive. (Willems, 2006) shows that a similar mechanism is at work with financial call options: the market equilibrium is even more competitive than with future contracts.

(Green, 2003) studies the combined hedging and strategic roles of forward contracts while at the same time examining different types of competition in the retail market. He shows that retail competition might lower the amount of forward contracts firms will sign. The current paper does not allow for retail competition, -- consumers cannot switch retail supplier -- and assumes, as in (Bessembinder and Lemmon, 2002), that retail prices are fixed.

Our paper does not model relation between market completeness and the investment decisions of the generation firms. This remains a topic for future study. (Green, 2007) models investment decisions and the technology choice in a long-term oligopolistic equilibrium model with risk averse firms in which firms can sign forward contracts.

3 Model description

We extend the competitive market equilibrium model of the forward and spot markets developed by (Bessembinder and Lemmon, 2002). The main difference with their model is that we allow for multiple financial products to be traded on the market. We start with a description of the spot market and continue with a description of the forward markets.

Demand for electricity D is inelastic and stochastic. The total production costs of the industry is the sum of a fixed cost F and a variable cost

$$C(Q) = F + \frac{a}{c} Q^c \tag{1}$$

where F, a and c are parameters that determine the shape of the cost function.

The spot market is perfectly competitive, and the wholesale price for electricity P is determined by market clearing.

$$P = C'(D) = a\, D^{c-1} \tag{2}$$

As demand is random, also the spot price is a random variable.

The generator's profit is equal to spot market revenue minus production costs:

$$\pi_g = P \cdot D - C(D) \tag{3}$$

Retailers buy the energy on the spot market and sell their energy at a regulated retail rate R to consumers. Their profit is equal to:

$$\pi_r = (R - P)D \tag{4}$$

Both retailers and generators' profit are affected by the stochastic nature of demand.

In the forward market, a derivative $i \in \{1,..,I\}$ is traded at a price F_i which promises a payment $T_i(P)$ which is conditional on the spot price P. This paper assumes that the only derivatives which are traded are call options. Hence:

$$T_i(P) = \max(P - S_i, 0) \tag{5}$$

with S_i the strike price of option i.

The total profit Π_j that a firm $j = r, g$ makes when it buys k_i^j derivatives in the forward market is equal to:

$$\Pi_j = \pi_j(P) + \sum_{i=1}^{I} k_i^j \cdot (T_i(P) - F_i) \tag{6}$$

The firm's profit is the sum of the profit it makes in the spot market, and the profit it makes on the derivatives it has bought. Both terms are stochastic as they depend on the realization of the demand level.

We assume that the retailers and generators are risk averse, and that their utility can be described by profit - variance utility with a risk aversion parameter A:

$$j = r, g \qquad U_j = \mathrm{E}(\Pi_j) - \frac{A}{2}\mathrm{Var}(\Pi_j) \tag{7}$$

The risk aversion parameter A measures the risk aversion of the generator and the retail sector as a whole, which we assume to be identical across sectors.[10]

In the contracting stage, firm j will maximize its utility U_j, by choosing the amount of derivatives $k_1^j, ..., k_i^j, ..., k_I^j$ it will buy. The equilibrium quantities contract positions are given by

$$\vec{k}^j = \Sigma^{-1}\frac{\mathrm{E}(\vec{T}) - \vec{F}}{A} - \Sigma^{-1}\mathrm{Cov}\{\pi_j, \vec{T}\} \tag{8}$$

with $\vec{k}^j = (k_1^j, ..., k_I^j)$, the vector of equilibrium quantities bought by player j, $\Sigma = \mathrm{Cov}\{\vec{T}, \vec{T}\}$ the I by I covariance matrix of the contracts $\vec{T} = (T_1, ..., T_I)$, $\vec{F} = (F_1, ..., F_I)$ the derivative price vector, and $\mathrm{Cov}\{\pi_j, \vec{T}\}$ the 1 by I covariance matrix of contracts and firm j's profit.

Equation (8) shows that the amount of contracts firm j buys is the sum of two terms. The first term is the pure speculative amount of contracts a firm would like to buy. If a

10 If there are N identical firms in the generation sector, then each of the firms own 1/Nth of the generation capacity, and has a risk aversion parameter equal to nA.

financial derivative has an expected positive return, then the firm will buy some of it, as long as it does not increase the variance of its portfolio too much. The second term is the pure hedging demand by the firm. A firm j will buy derivatives in order to hedge its profit risk. It will buy more of a certain derivative, if it is more correlated with the profit it wants to hedge, and if the impact on the variance of the portfolio is smaller.

In equilibrium the demand and supply of derivative products should be equal. Hence, if there are no speculators active in the market i we find:

$$k_i^r + k_i^g = 0 \tag{9}$$

and using equation (8) the equilibrium price of derivative i is given by

$$F_i = \mathrm{E}(T_i) - \frac{A}{2}\mathrm{Cov}\{\pi_g + \pi_r, T_i\} \tag{10}$$

Hence, the price of a derivative is equal to expected pay-off of the derivative minus a term which reflects that the derivative is used to hedge the risk of the individual firms. The last term depends on the risk aversion of all the firms and the covariance of industry profit with financial instrument i.

It is worth noting that the price of the derivative does not depend on the amount of products which are traded in the market.[11]

If there are risk neutral speculators active in derivatives market i, the risk premiums become zero, and we find that the price of the derivative should be equal to the expected value of the derivative:

$$F_i = \mathrm{E}(T_i) \tag{11}$$

4 Model data

The model is loosely calibrated on the German electricity market. Demand is assumed to be normally distributed with a mean of 60 GW and a standard deviation of 17 GW.

The parameters of the aggregate production cost function are $c = 4$, $a = 1.852\ 10^{-4}$ where price, quantities, and profits are expressed in (EUR/MWh), (GWh), and (1000

[11] In standard mean-variance settings risk pricing is not affected. Specifically, in quadratic or CARA-normal economies, the price of any risky security relative to the bond is unaffected by changes in the span. See (Oh, 1996).

EUR/h). Figure 1 shows the industry marginal cost function with these calibration parameters.

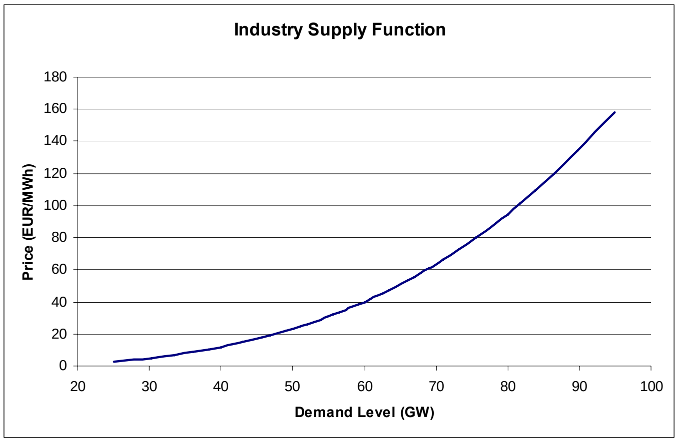

Figure 1: Industry marginal cost

Retailers and generators have the same risk aversion parameter $A = 0.0025$ which has the unit (h/ 1000 EUR).

Given the assumption on the supply and the demand side we can derive the wholesale price distribution. The distribution has a mean of 48 EUR/MWh and a standard deviation of 35 EUR/MWh. (Bessembinder and Lemmon, 2002) show that as the industry marginal cost function is convex, the price distribution is skewed, which can also be seen in Figure 2.

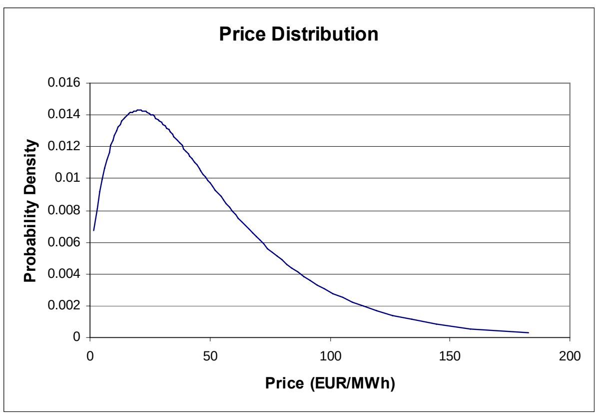

Figure 2: Wholesale price distribution

Further, we assume that the fixed cost parameter $F = 1200$, and that retailers sell their energy at a fixed price of 58 EUR/MWh.

5 Welfare effects

In this section, we use the model to calculate the optimal hedging strategy of the generator and the retailers, and analyze the welfare effects of adding additional derivatives to the market. In the first part of the simulations we assume that there are *no speculators* active on the market, and the supply and demand of financial contracts comes only from retailers and generators. We consider four scenarios with a different number of derivative markets present. In the first scenario, only a forward market exists. In the second through fourth scenario, the forward market is supplemented with one, three, and eleven additional option markets, respectively.[12]

Table 1 shows the simulation results for all scenarios. It shows for each of the twelve derivative contracts, the net amount traded by generators and retailers. The option contracts have strike prices ranging from 0 to 143 EUR / MWh, where the zero strike price corresponds to the forward contract.

Contract		Price	Net Contract Position			
Nr	Strike Price		Forward	Forward + 1 Option	Forward + 3 Options	All Contracts
1	*0*	-45.3	68.0	52.0	32.1	-5.3
2	*13*	-33.6				37.9
3	*26*	-25.2				16.2
4	*39*	-19.1			37.7	11.1
5	*52*	-14.5				8.3
6	*65*	-10.9				6.9
7	*78*	-7.9		45.4	15.9	5.9
8	*91*	-5.6				5.2
9	*104*	-3.8				4.3
10	*117*	-2.4			14.5	6.6
11	*130*	-1.3				-3.7
12	*143*	-0.7				13.1
		Welfare	**768.4**	**1224.1**	**1321.7**	**1337.0**

Table 1: Market equilibrium without speculation

The results show that if there are only forward contracts, generators will over hedge their position. They sell 68 GW forward, while in expected terms they will only sell 60

[12] The numerical model is written as a Mixed Complementarity Problem (MCP) in Gams. See Appendix A.

GW. The reason for this is that the generators and retailers want to hedge their quantity risk, and as price and quantity are positively correlated, they can do this by selling, respectively buying more contracts. The price of the forward contract is 45.3 EUR/MWh, which is below the expected price of 48 EUR/MWh in the market.

In scenarios 2 to 4, extra financial instruments are added to the market. Table 1 shows that once more instruments become available, generators will reduce the amounts of standard forward contract they sell and substitute these contracts with option contracts. The generator and the retailer reduce their supply and demand of forward contracts. Although both demand and supply functions shift, the price of the forward contract remains 45.3 EUR/MWh as shown in derivation (10).

The last row in Table 1 is the aggregate market welfare, measured in certainty equivalents (1000 EUR/h). Increasing the number of contracts traded clearly increases market efficiency. The introduction of one option contract, when none existed before, increases welfare with approximately 50 %. Adding extra markets for option contracts increases welfare, but to a lower extent. For instance, increasing the number of option markets from 3 to 11, increases welfare by 1.2 %. Hence risk sharing between generation and retail is close to optimal once a few option contracts are traded.

For the second part of the simulations, we assume that *speculators* can actively participate in the market, by taking positions in the electricity derivative markets and financially closing their position in the spot market. We assume they will trade away the risk premia in the market: the price of the derivatives becomes equal to the expected value of the derivative. As speculators provide extra liquidity to the market, the supply of derivatives by generators does no longer need to exactly balance the demand by retailers. The difference of generators' supply and retailers' demand is the position speculators take in the market. For the same four scenarios as before, Table 2 gives the net position of generators and retailers. In scenario 1, only forward contracts exist, and generators sell 69.1 GWh forward, retailers buy 67 GWh, and speculators buy 2.1 GWh. The results indicate that, the more derivative markets are introduced, the larger the gap between the supply and the demand for forward contracts, and the larger the role played by speculators. In scenario four, in which there are one forward market and eleven option markets, generators sell 34.2 GWh, retailers *sell* 44.8 GWh and speculators buy 79 GWh.

The introduction of speculators increases welfare, as the players can share their risk with players outside the market, the speculators. Hence, the addition of speculators does not change our previous conclusions. Speculators play an active role in the electricity market by taking up market risk and by decreasing the risk premia in the market. As the number of markets increases, the amount of risk that the speculators take away from market participants increases, but the positive welfare effect of additional markets levels off after a few products.

Contract		Price	Net Contract Position							
Nr	Strike Price		Forward		Forward + 1 Option		Forward + 3 Options		All Contracts	
1	*0*	-48.4	69.1	67.0	58.7	45.3	48.0	16.2	34.2	-44.8
2	*13*	-36.0							13.2	62.6
3	*26*	-25.9							8.8	23.6
4	*39*	-18.1					20.5	55.0	6.6	15.7
5	*52*	-12.4							5.5	11.2
6	*65*	-8.3							4.7	9.1
7	*78*	-5.5			29.3	61.5	12.2	19.7	4.2	7.6
8	*91*	-3.5							3.8	6.6
9	*104*	-2.1							3.2	5.4
10	*117*	-1.2					10.5	18.4	4.9	8.2
11	*130*	-0.7							-2.7	-4.7
12	*143*	-0.3							9.9	16.4
		Welfare		**771.6**		**1284.6**		**1402.8**		**1423.5**

Table 2: Market equilibrium with speculation

Finally, we repeated the last simulation with a different assumption for firms' utility functions: instead of the utility functions of equation (7), we used the well-known *CRRA utility function* (i.e., the utility function with *constant relative risk aversion*). As a result of the CRRA property, firms become very averse of potential shocks that would lead to very low or negative profits. In other words, the CRRA utility function models a world in which firms want to avoid the risk of liquidity problems or bankruptcy. Practically, we chose the coefficient of relative risk aversion to be 4 (i.e., in the middle of the typical 2-6 range).[13] The simulation results with speculation and CRRA utility

[13] Before applying the CRRA utility function, a shift of 1 million EUR/h was introduced, to model the effect that limited negative profits do not lead to immediate bankruptcy.

functions for producers and retailers are shown in Table 3. Generally speaking, the results are very similar to the results of Table 2, although the retailer seems to have a slightly increased preference for options over forwards (as compared to Table 2).

The most interesting observation is that with the CRRA utility function, it is not possible to find a sufficiently hedged solution in case only forwards are present. In other words, if no options are introduced, welfare remains "infinitely low" for CRRA utility functions. The reason is that forwards alone do not allow the retailer and the generator to limit their exposure in all "states-of-the-world". The intuition for this is the following: since a negative result in one potential state-of-the-world is strongly penalized by the CRRA function, the retailer and the generator would like to avoid – at all costs – any outcomes in which their profit is below a certain threshold, in order to avoid bankruptcy. The retailer faces a negative shock when demand is high (he faces a high wholesale price, and has to buy a large volume of power), and when demand is low (its sales volume is too low to cover fixed costs). The generator faces a negative shock when demand is very low (low volume and low volume of trade). As the retailer wants to avoid bankruptcy at all cost, its demand for forward contracts is undetermined for any quantity or price of forward contracts – at least within the price range that the generator can offer. Based on these results for a CRRA utility function, it is clear that the introduction of options is especially welfare-enhancing if there is a strong risk-aversion for negative shocks that could lead to bankruptcy.

Contract		Price	Net Contract Position						
Nr	Strike Price		Forward	Forward + 1 Option		Forward + 3 Options		All Contracts	
1	*0*	-48.4	No solution	58.8	38.7	48.1	11.3	33.5	-45.8
2	*13*	-36.0						13.4	63.6
3	*26*	-25.9						10.8	23.7
4	*39*	-18.1				20.5	59.6	0.0	15.7
5	*52*	-12.4						18.7	11.2
6	*65*	-8.3						-6.1	9.1
7	*78*	-5.5		29.3	63.8	12.1	20.2	3.7	7.9
8	*91*	-3.5						3.8	5.6
9	*104*	-2.1						9.9	9.4
10	*117*	-1.2				10.5	18.2	2.5	-9.6
11	*130*	-0.7						7.3	49.6
12	*143*	-0.3						-10.4	-47.0
		Welfare	**"-∞"**		**-1215.9**		**-349.3**		**-314.1**

Table 3: Market equilibrium with speculation, with CRRA utility functions

6 Investment decisions by small firms

Above we have shown that the welfare effect of adding contracts levels off after a relatively small number of contracts. However, implicitly we have assumed that the production firms have a diversified portfolio of generation plants. If some firms had only base load power plants and other firms had only peak load power plants, the social value of financial contracts could be higher.

In order to test the impact of the number of financial contracts on firms with different types of portfolio, we will calculate whether a small firm would invest in a single power plant with a marginal cost c, and fixed investment cost F. This small firm is assumed to be risk averse, with expectation-variance utility function (as in the first part of our simulations). We will assume no speculators in the market.

The firm will invest in this production plant when the investment increases its expected utility. The expected utility without investments is equal to

$$U^{NI} = \max_{k_1,\ldots,k_I} \left[\mathrm{E}\{\pi\} - \frac{a}{2} \mathrm{Var}\{\pi\} \right] \quad \text{with} \quad \pi = \sum_{i=1}^{I} k_i \cdot (T_i(P) - F_i) \tag{12}$$

while the expected utility with investments is equal to

$$U^{INV}(c,F) = \max_{k_1,\ldots,k_I} \left[\mathrm{E}\{\pi\} - \frac{a}{2} \mathrm{Var}\{\pi\} \right] \quad \text{with} \quad \pi = \sum_{i=1}^{I} k_i \cdot (T_i(P) - F_i) + (\max\{p-c,0\} - F) \tag{13}$$

The firm will invest as long as

$$U^{NI} \geq U^{INV}(c,F) \tag{14}$$

Equation (14) defines implicitly the maximal fixed cost for which the generator would willing to invest in new generation capacity with marginal cost c. Hence investment will occur as long as

$$F < F^{cr}(c) \tag{15}$$

Equation (15) describes the investment decision of the firm.

Given that the firms are risk averse, the investment decision is different from the standard NPV rule. A-risk neutral firm would invest as long as fixed costs are below the NPV of the investment:

$$F < NPV(c) = \mathrm{E}\{\max(P - c, 0)\} \tag{16}$$

Note that the investment decision of the firm depends on the number of contracts traded in the market. As more contracts are traded, the firm is able to better hedge the output of the production plant and reduce its risks. This makes it more interesting for the firm to build the power plant.

Figure 3 compares the optimal decision of the risk averse firm with the standard NPV approach. Note that the firm's risk aversion is chosen at a higher level than before, because we analyze a small firm and the expectation-variance utility function does not scale. The figure shows the ratio of the NPV value of the power plant and the “risk aversion adjusted” value of the plant. We call this ratio κ, the "risk adjustment factor":

$$\kappa = \frac{F^{cr}(c)}{NPV(c)} \tag{17}$$

If $\kappa < 1$, the firm will invest less in generation than predicted by the NPV approach, as the firm wants to be compensated for the risk it is taking in the market. If $\kappa > 1$ the firms are willing to invest more often than as predicted by the NPV value. The reason for this is that those firms lower the overall risk in the industry, for which they are compensated by the rest of the industry, as they receive a premium for their output.

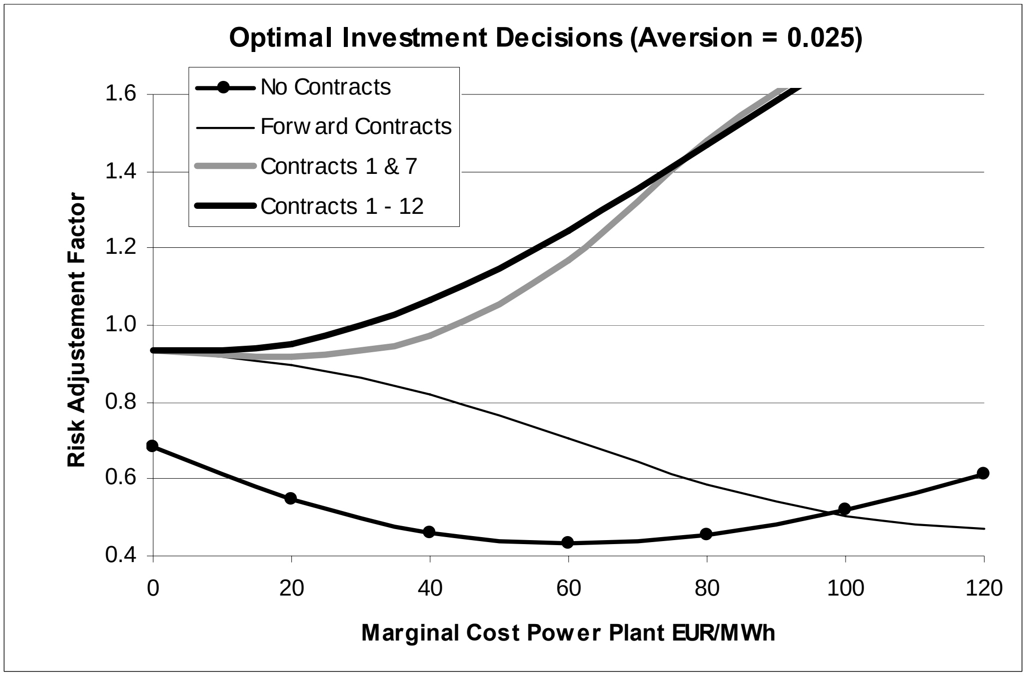

Figure 3: Optimal investment decisions.

Figure 3 shows that if no financial contracts are traded, firms will invest less in the generation assets than under the NPV rule. The reason for this is that investments are risky, and therefore they are only undertaken if they provide a premium profit. The effect of additional contracts on the investment decisions of peak-load power plants and base-load power plants is, however, very different.

Once the forward contracts is introduced, a firm with a *base load plant* ($c \approx 0$) would be able to completely hedge its position. However, hedging comes at a cost, which is reflected in the fact that $\kappa < 1$. Adding additional derivatives to the market does not change the investment decisions of the base load power plant, as the firm already has perfect information in evaluating the value of the power plant using the forward contracts. Speculation and investment decisions are decoupled.

For *peak load plants* (c large), the results are quite different. Once the forward markets are present (and no options), less is invested in peak load power plants. The reason for this is that it is often more profitable for the firm to speculate on the forward markets (without building a power plant), than to build a power plant and use financial contracts to hedge its portfolio. Hence, financial investments crowd-out the investments in physical assets; i.e. investment and speculation decisions are coupled. As more and more contracts are introduced, we see that investment in peak generation increases dramatically. The reasons are two-fold. On the one hand, there are better instruments to hedge the risk of the production output of the firm. As a result, the investment decision and the speculation decision become decoupled. On the other hand, the peak load power plants reduce the risk of the sector. For this they are refunded through a price premium for their product. As a result of this effect, investments in peak-load plants are higher than under a pure NPV reasoning without contracts, an interesting result. The figure also shows that for the technology with production costs approximately equal to 80, adding additional markets on top of market 7 does not change the results. Contract 7 has a strike price of approximately 80 EUR/MWh, hence the investment valuation of the firm is perfect.

Figure 4 gives the optimal investments decisions with more risk averse firms. It shows that as we increase the risk aversion of the firms, the effects are amplified. Comparing Figure 3 and Figure 4, (note the difference in scale) it can be observed that as the number of contracts increases, the effect of risk aversion decreases. In the limit when all financial contracts are traded, the optimal investment decisions no longer depend on the risk aversion of the firm. The reason for this is that the investment decision and the speculation decision are decoupled: the production plant can be valued using the prices of the financial products in the market (perfect hedging is almost possible), and the amount of speculation depends on the risk parameters of the firm.

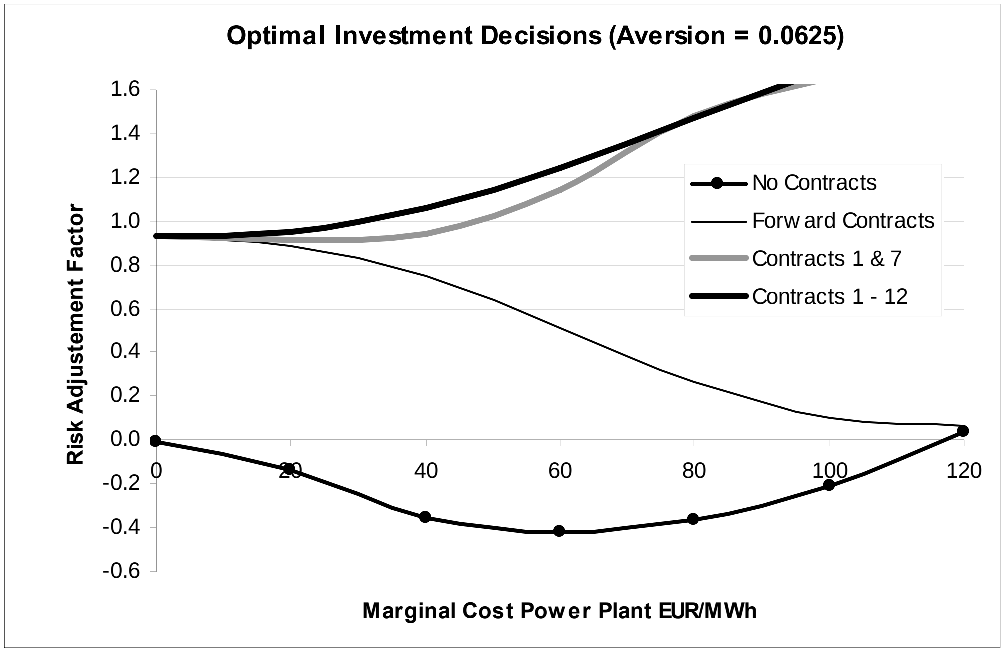

Figure 4: Optimal investment decisions with larger risk aversion parameter.

7 Information content of derivatives prices: risk-free probabilities

In the previous section, we studied the effect of financial contracts on the investment decisions of a firm as a function of its risk aversion parameter and the technology parameter c. In this section we look at the information content contained in the prices of financial products, and derive optimal investment decisions based upon a typical financial approach using risk-free probabilities. This approach uses the price data of the financial market to estimate the market value of an asset. The investment decision is made by comparing the market value of the asset with the investment costs of the asset. This approach assumes that the market is sufficiently complete to create a portfolio of contracts which recreates the pay-off of the physical asset. In this section we will test when the markets are sufficiently complete to use the risk-free probabilities approach by comparing the investment decisions in this section with the optimal decisions we found in the previous section.

The market equilibria in Table 1 can be represented by means of a risk-free probability distribution θ, different from the true distribution. Under the risk-free probability distribution, the contracts' prices are equal to their expected values:

$$\mathrm{E}_{\theta}(\vec{T}) = \vec{F} \quad (18)$$

and the generator and the retailer act as risk neutral agents who optimize expected profit:

$$j = r, g \qquad U_j = \mathrm{E}_{\theta}(\Pi_j) \quad (19)$$

Figure 5 gives the risk-free probabilities for different assumptions regarding the number of products being traded[14]. When all financial contracts are traded, the risk free probability distribution assumes that extreme events, especially low prices, are more likely to occur than they do in reality. When only forward contracts are traded, however, the risk free probabilities calculated on the basis of forward prices, give extreme events a too small probability. Adding just one extra financial market brings the distribution relatively close to the situation where all 12 financial products are traded, and greatly improves the information that firms receive.

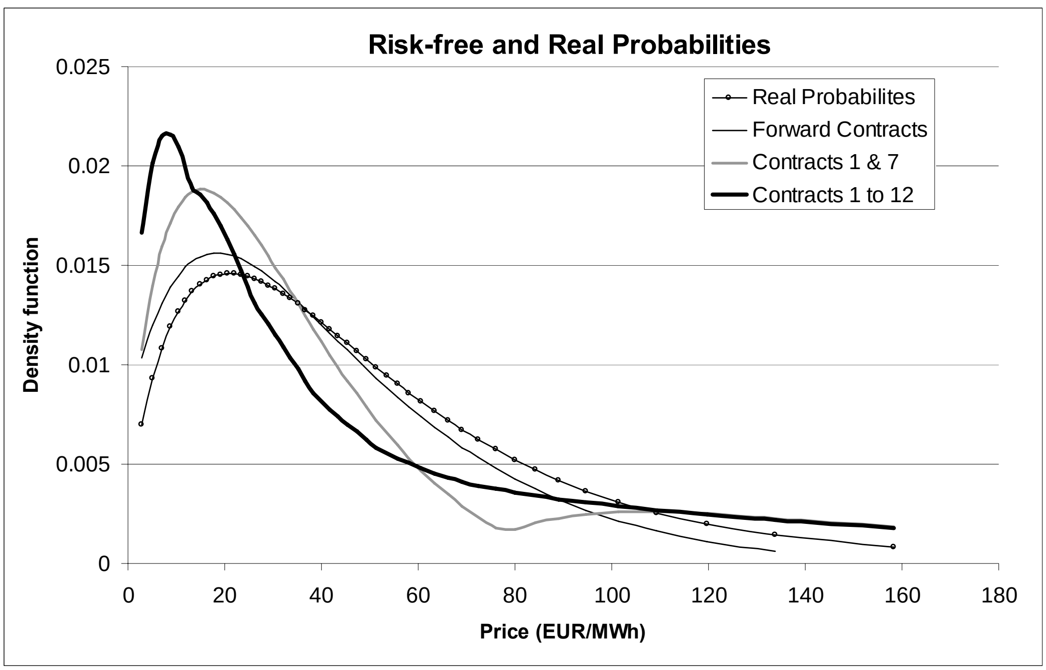

Figure 5: Risk free probabilities without speculators.

[14] As the set of forward and option markets is incomplete, the risk-free probability distribution is not unique. We rely on ex-ante information about the probability measure to determine it. This paper chooses to minimize the 'distance' between the true-probability of an event occurring and the-risk free probability of the same event.

Firms should invest in a power plant when the expected Net Present Value, calculated using the risk free probabilities, is larger than the fixed investment cost. Hence a firm will invest if

$$F < NPV_{\theta}(c) = \mathrm{E}_{\theta}\{\max(p - c, 0)\} \tag{20}$$

As in section 6 we will compare these critical values with $NPV(c)$, the net present value of a power plant with marginal cost c calculated using the true probabilities. Figure 6 gives – for different types of generation plants – the ratio of both numbers:

$$\kappa = \frac{NPV_{\theta}(c)}{NPV(c)} \tag{21}$$

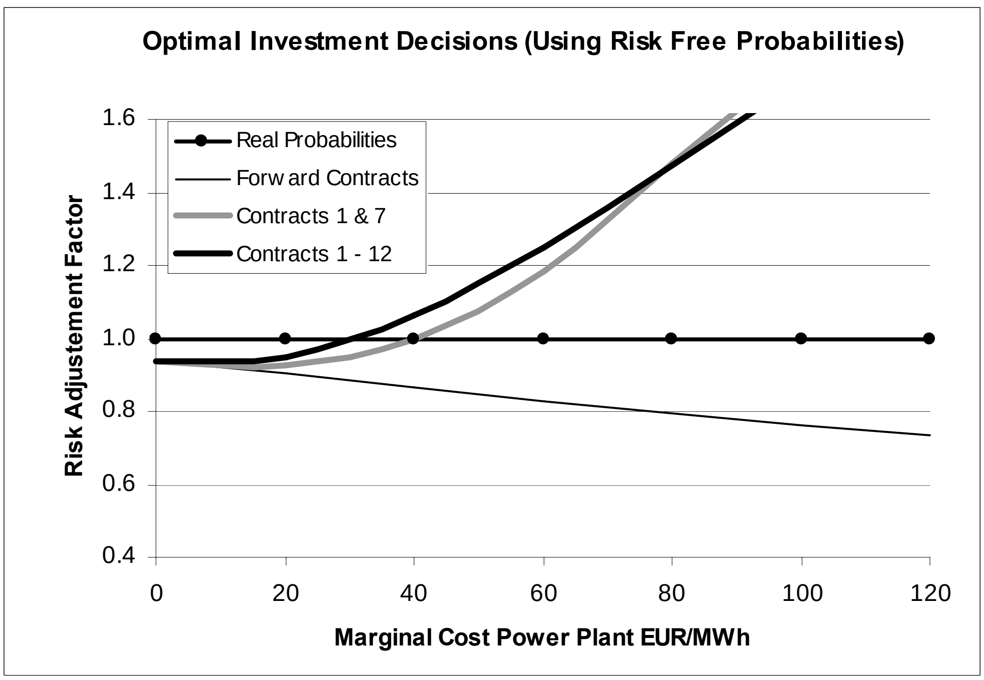

Figure 6: Investment decisions using risk free probabilities.

The results are very similar to those obtained in section 6. In order to see whether the information content is sufficient, we will compare the decisions of the risk-free probabilities model and the optimal decisions (as described in section 6). Figure 7 shows the difference between the two decision rules:

$$\kappa = \frac{NPV_{\theta}(c) - F^{cr}(c)}{NPV(c)} \tag{22}$$

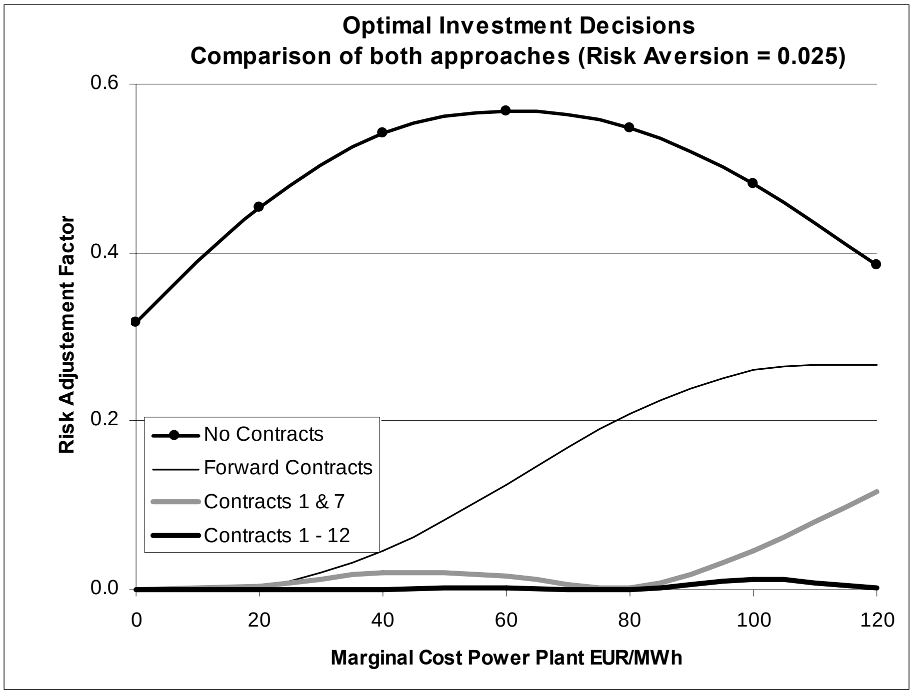

Figure 7: Difference of the risk free probabilities approach and the optimal investments

Figure 7 shows that if the marginal cost of the power plant is equal to the strike price of one of the options traded in the market, the two approaches produce identical results. In general, the error in using risk free probabilities is smaller when the contracts that are traded are better correlated with the power plant being built. Hence, the decisions to build base load power plants are always efficient when there is a forward contract. Similarly, if the market trades an option with a strike price very close to the marginal cost of a certain peak power plant, the investment decision for that power plant based on the risk-free probabilities is optimal.

On the other hand, the risk-free probabilities approach leads to overinvestment in power plants for which no close financial substitutes are traded in the market. For example, if only forward contracts are available, the investment decision for a peak power plant can be seriously distorted. For the risk aversion parameter equal to 0.025, the firms make an error of more than 20% when evaluating this decision. The addition of one option contract eliminates this error for almost all types of power plants. In general, as more contracts are being traded, the risk free probabilities approach will lead to decisions that are closer to the optimal decisions. If markets are complete, the two approaches yield identical results.

For larger risk aversion parameters, the effects are enlarged (see Figure 8). However, the figure shows again that adding one option contract eliminates most of the bias in the investment decisions, although the difference might remain economically significant (5-10% error).

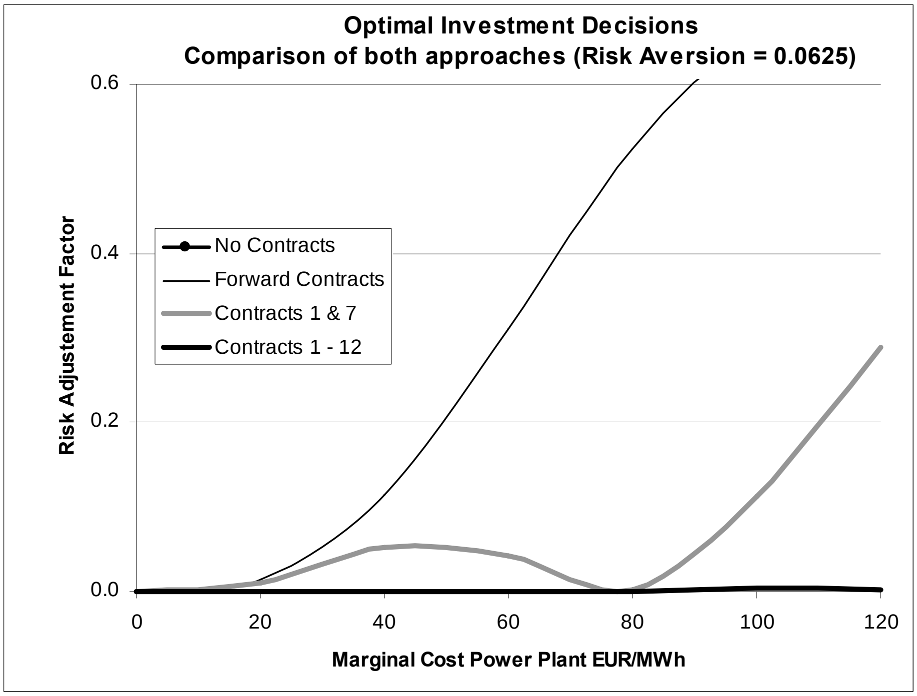

Figure 8: Difference of the risk free probabilities approach and the optimal investments

8 Conditions for financial innovations

The previous sections have shown that when electricity markets are made more complete, welfare increases, more information is transmitted, and firms are given incentives to build power plants which reduce overall sector risk. This in itself, however, does not guarantee that these markets will actually develop. There is a growing literature on financial innovations, i.e. the study of the creation of new securities. This body of the literature tries to understand the incentives of private actors to create new securities and compares their private incentives with the optimal type of financial contracts one should expect in the market. Strong predictions are not available, nevertheless some general rules could be derived (Duffie and Rahi, 1995).

Securities will only develop, when the underlying risk is sufficiently volatile, and the risk for the firms involved is substantial. Otherwise, the trade of the good will be too low, and the market will not develop. In the electricity market, price (and quantity) volatility is large and the effect on the firm's profit is very large. Hedging risk by vertical integration, or by trading derivatives becomes very important.

In order for a new financial market to be viable not only speculators should be present in the market, but also liquidity traders; i.e. participants which have a natural demand for hedging their positions. Markets in which only the only traders are speculators will not survive. In our model retailers and generators are the natural agents that have opposite risk exposure and who would like to trade their risk.

For markets to be functioning well, information asymmetries between the seller and the buyer of the market should not be too large. In case the seller has private information about the value of the security, the market will not develop. Hence, there might be an "information constrained number of optimal markets" (second best amount of markets). Risk which is caused by demand shocks (or weather factors) might be traded more easily on the market than risk which can be influenced by specific players. For instance plant, outages might not be insurable in the market.

If we assume that the (normalized) bid-ask spread is constant across markets, then the incentives for a broker to set-up a new market roughly depend on the volume of trade that he makes. The simulation results give us a qualitative impression of how market completeness affects the trading volumes for the different products. Adding extra markets increases the total volume which is traded. While it generally decreases trade in the products which were already traded before the market was, it might increase the participation of the speculators in some of the markets. Hence, the introduction of a security might create *spillovers* to other, already existing markets. In general, the direction of this effect on the trading volume is not clear-cut.

As an illustration of those spill-overs, Figure 9 discusses the incentives of a broker to introduce a new financial contract.[15] The results are simulated under the assumption that

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[15] The results are simulated under the assumption that all financial contracts in Table 1, except for contact number 6, are traded with a bid-ask spread of 0.50 EUR / MWh. The broker then decides to introduce contract number 6.

all financial contracts in Table 1 -- except for contact number 6-- are traded with a bid-ask spread of 0.50 EUR/MWh. A broker now decides to introduce contract number 6 and has to set its bid-ask spread. When the bid-ask spread is large, the product is not actively traded and revenue of the broker is zero on the contract. When the bid-ask spread decreases below 0.51 EUR/MWh, the product generates interest for generators and retailers as a hedging instrument. The broker maximizes its revenue on the contract when the bid-ask spread is equal to 0.38 EUR/MWh. Total welfare for generators and retailers would be maximized when the bid-ask spread is equal to zero. The introduction of the new contract has a negative spillover effect on the revenue that the other option contracts generate. Keeping the bid-ask spread of other contracts constant, total revenue over all contracts (existing and new) decreases after the introduction of the new contract.

Spillovers can be negative, as in the example, or positive when financial products are seen as complements for the hedging firms. For instance, a retailer might be willing to simultaneously buy a put and a call option to create a virtual forward contract if the forward contract would not be traded as such.

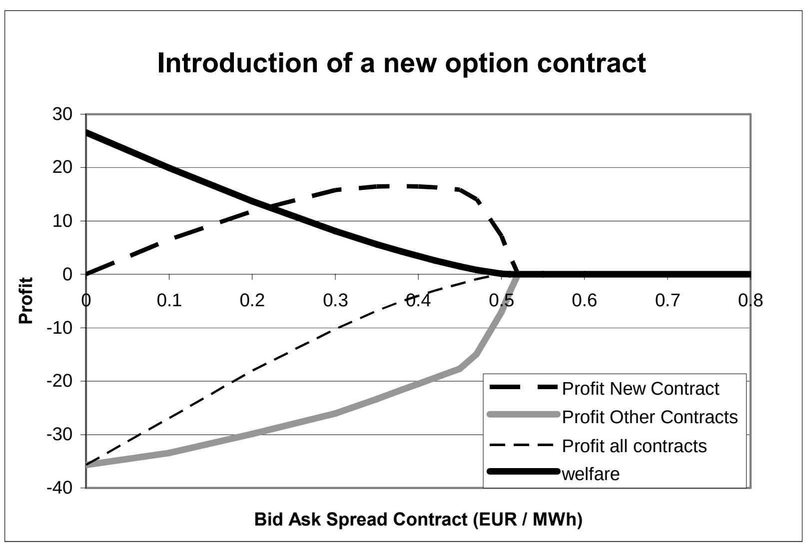

Figure 9: Effect of introducing a new financial contract on revenue and welfare

As a result of the spillovers, private investors may not have the right incentives to create a sufficiently complete set of markets, and government intervention may be needed to maximize welfare.

9 Conclusions

This paper derives an equilibrium model for spot and derivative markets in electricity and evaluates the effects of market completeness on welfare and on investment incentives.

With respect to *welfare*, the numerical results of the model (based on German market data) show that welfare is enhanced when options are offered in the market in addition to forward contracts. However, it turns out that most of the welfare benefits are achieved with 1 to 3 options. The need for options is especially relevant in case firms have a strong aversion of liquidity problems (bankruptcy risk): with a CRRA utility function, welfare is completely destroyed when no options are present. Allowing speculators to actively trade in the market, eliminates the risk premium, and increases aggregate welfare. The beneficial effect of speculators increases as more contracts are traded.

With respect to *investment incentives*, financial contracts are important for signaling to entrants how they could reduce the overall sector risk. When no financial contracts are traded, risk-averse firms would tend to underinvest compared to an approach based on expected NPV. When forward contracts are traded, investment in base-load power plants increases (investment and speculation are decoupled), but investment in peak load plants declines because it is more attractive to speculate with forward contracts instead (investment and speculation are coupled). When options are added to the market, investment in peak power plants increases again dramatically (investment and speculation become more and more decoupled), even beyond the level predicted by expected NPV, because firms investing in peak power plants are rewarded by the market for reducing overall sector risk. In general, investment decisions improve with increasing market completeness because of decoupling of investment and speculation – however, as soon as a firm can trade a derivative that exactly matches the risk profile of its investment, the investment decision becomes independent of market completeness. The above results are amplified when firms are more risk-averse.

When investment decisions are made based on a financial approach using *risk-free probabilities*, similar results are obtained. In fact, the results are identical for power plants for which a perfectly matching contract is traded. However, for power plants for which no close financial proxy is traded in the markets, the results can be severely distorted compared to the approach above. As more and more options are added to the markets, the results converge again, to become identical when markets are complete. This shows that the quality of the information contained in market prices improves as markets become more complete.

Finally, given the apparent benefits of increasing market completeness, we analyzed private investors' incentives to create liquid markets for additional derivatives. We found that adding a derivative may lead to (negative or positive) spill-over effects on already traded derivatives. As a result of negative spill-overs, private investors may not have the right incentives to create the optimal number of liquid markets, and government intervention may be needed to reach the welfare-optimum.

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Appendix A: Numerical model

The equilibrium model is solved as a Mixed Complementarity Problem (MCP) in Gams. By writing the problem as a MCP, we can simultaneously determine the equilibrium prices and production quantities. This formulation will also allow us to introduce trading frictions in financial markets, such as a bid-ask spread and extend the model to firms with capacity constraints.

Spot market

The following equations are considered for the spot market:

$$\begin{array}{ll} C_s = F + \frac{a}{c} Q_{gs}{}^{c} & C_s \\ MC_s = a\, Q_{gs}{}^{c-1} & MC_s \\ P_s = MC_s & Q_{gs} \\ Q_{gs} = Q_{rs} & P_s \\ Q_{rs} = D_s & Q_{rs} \\ \pi_{gs} = P_s \cdot Q_{gs} - C_s & \pi_{gs} \\ \pi_{rs} = (R - P_s) Q_{rs} & \pi_{rs} \end{array} \qquad (23)$$

with s the state of the world and the indices g and r indicating generator and retailers.

Call Option

The call option i pays T_{is} in state s.

$$T_{is} = \max(P_s - S_i, 0) \qquad T_{is} \qquad (24)$$

Equilibrium in the forward markets with trading costs

$$
\begin{array}{ll}
\Pi_{rs} = \pi_{rs} + \sum_{i=1}^{I} k_{rs} \cdot (T_{is} - F_{ir}) & \Pi_{rs} \\
\Pi_{gs} = \pi_{gs} + \sum_{i=1}^{I} k_{gs} \cdot (T_{is} - F_{ig}) & \Pi_{gs} \\
E\Pi_g = \sum_{s=1}^{S} \rho_s \Pi_{gs} & E\Pi_g \\
E\Pi_r = \sum_{s=1}^{S} \rho_s \Pi_{rs} & E\Pi_r \\
\sum_{s=1}^{S} \left(1 - \lambda\left(\Pi_{gs} - E\Pi_g\right)\right) \rho_s \, T_{is} = F_{ig} & k_{ig} \\
\sum_{s=1}^{S} \left(1 - \lambda\left(\Pi_{rs} - E\Pi_r\right)\right) \rho_s \, T_{is} = F_{ir} & k_{ir} \\
k_{ri} + k_{gi} = 0 & F_i \\
F_{ir} - F_i = Sp_{ir} & F_{ir} \\
F_{ig} - F_i = Sp_{ig} & F_{ig} \\
k_{ri} = 0 & \underline{Sp}_{ir} < Sp_{ir} < \overline{Sp}_{ir} \\
k_{gi} = 0 & \underline{Sp}_{ig} < Sp_{ig} < \overline{Sp}_{ig}
\end{array}
\qquad (25)
$$

Here Sp is the spread in the financial market.

Market outcome

$$
\begin{array}{ll}
V\Pi_r = \sum_{s=1}^{S} \rho_s \left(\Pi_{gs} - E\Pi_g\right)^2 & V\Pi_r \\
V\Pi_r = \sum_{s=1}^{S} \rho_s \left(\Pi_{gr} - E\Pi_r\right)^2 & V\Pi_r \\
U_g = E\Pi_g - \frac{\lambda}{2} V\Pi_g & U_g \\
U_r = E\Pi_r - \frac{\lambda}{2} V\Pi_r & U_r \\
W = U_g + U_r & W
\end{array}
\qquad (26)
$$

Risk free probabilities

The risk-free probabilities are chosen such that the expected value equal the price of the financial instrument and that the difference between the true probabilities is minimized:

$$
\begin{aligned}
&\sum_{s=1}^{S} \theta_s \; T_{is} = F_i && \mu_i \\
&\sum_{s=1}^{S} \theta_s \; = 1 && \omega \qquad (27) \\
&2(\theta_s - \rho_s) - \omega - \sum_{i=1}^{I} \mu_i T_{is} = 0 && \theta_s
\end{aligned}
$$