When is it Optimal to Exhaust a Resource in a Finite Time?

Y. Hossein Farzin and Ken-Ichi Akao

NOTA DI LAVORO 23.2006

FEBRUARY 2006

NRM – Natural Resources Management

Y. Hossein Farzin, *Department of Agricultural and Resource Economics, University of California*
Ken-Ichi Akao, *School of Social Sciences, Waseda University*

When is it Optimal to Exhaust a Resource in a Finite Time?

Summary

Exhaustion of a natural resource stock may be a rational choice for an individual and/or a community, even if a sustainable use for the resource is feasible and the resource users are farsighted and well informed on the ecosystem. We identify conditions under which it is optimal not to sustain resource use. These conditions concern the discounting of future benefits, instability of social system or ecosystem, nonconvexity of natural growth function, socio-psychological value of employment, and strategic interaction among resource users. The identification of these conditions can help design policies to prevent unsustainable patterns of resource use.

Keywords: Renewable resource management, Sustainability, Finite-time exhaustion, Optimal path, Policy implications

JEL Classification: Q20

Address for correspondence:

Y. Hossein Farzin
Department of Agricultural and Resource Economics
University of California
Davis CA 95616
USA
E-mail: farzin@primal.ucdavis.edu

1. Introduction

Sustainability has long been a primary objective of renewable resource management. The notion of sustained yield goes back to at least the 18th century European forestry (Carlowitz 1713. Also see Vanclay 1996). After the publication of “Our Common Future” by the World Commission on Environment and Development (WCED 1987) and the United Nation Conference on Environment and Development in Rio de Janeiro, 1992, the concept of sustainability has been popularized and regarded as one of the basic social goals. At the same time, most scholars have recognized the vagueness of the concept, raising questions such as: Is it to keep physically intact a natural resource or environmental asset? If so, how should one think about the sustainability of a nonrenewable resource? How does it relate to intergenerational equity and intertemporal efficiency? Not surprisingly, economists have come to various definitions of sustainability and differing views about their merits. (See, for example, Pearce *et al.* 1990; Turner *et al.* 1994; Nordhaus, 1994; Solow, 1998; Heal 1999; Farzin, 2004.)

Whatever the definition of sustainability, it is obvious that finite-time resource extinction defies sustainability. In this paper, we show how a rational agent willingly exhausts a resource in a finite time, even though a sustainable resource use is feasible, or, at least, the resource could be used up over an infinitely long period. The assumption of rationality is important: it enables us to avoid an unsustainable path of resource use by removing the very conditions that render finite-time extinction rational. Therefore, the aim of the paper is to identify the conditions under which finite-time exhaustion of a renewable resource is optimal. These conditions concern (a) the discounting of future benefits, (b) uncertainty about the future of the resource stock, (c) nonconvexity of natural growth function, (d) socio-psychological aspect of work incentives, and (e) strategic interaction among resource users.

The paper is organized as follows: the next section introduces a simple model for resource management to show that heavy discounting makes finite-time extinction optimal. We show that a source of a high discount rate could be the uncertainty about the future ecological state of the resource stock or about its future ownership and management. In Section 3, we modify the model by allowing nonconvexity in the resource's natural growth function. If an inbreeding depression or an Alee effect exists, the growth function takes a shape that it is convex when the population size (resource stock) is small and concave when it is large. We will see that even with a low discount rate, if the initial stock of the resource is small, the optimal path is finite-time exhaustion. In Section 4, the model is extended to incorporate socio-psychological value of employment. We show that even with a low discount rate and an abundant resource stock, finite-time exhaustion becomes optimal. This is because in this case it is optimal for the resource user to harvest the resource at the maximum harvesting ability. This is an extreme case of extinction: the most rapid extinction. In Section 5, we consider a common property resource problem, assuming that multiple agents use the resource. Again, we show that the most rapid extinction is optimal for each *individual* resource user. At the same time, we show that, under the same condition, sustainable resource use is optimal too. However, one cannot be sure which optimal path is adopted. Section 6 concludes with some policy implications of these findings.

2. Discounting and Uncertainty

Let us start with a rudimentary model in resource economics, characterized by the following problem:

$$\max_{c(t)\geq 0} \int_0^{\infty} u(c(t))e^{-\rho t}dt \tag{1.a}$$

$$\text{subject to } \dot{x}(t) = f(x(t)) - c(t), \tag{1.b}$$

$$x(t), c(t) \geq 0,\ x(0) = x_0 \text{ given.} \tag{1.c}$$

Here x denotes the stock of a renewable resource. The natural growth of the resource is described by function $f:\mathbf{R}_+ \to \mathbf{R}$. Variable $c(t)$ denotes the amount of harvest at time t. Therefore, the evolution of the resource stock is described by equation (1.b), where $\dot{x}(t)$ denotes the time derivative of $x(t)$. The consumption of harvest yields utility to the resource user according to the utility function $u:\mathbf{R}_+ \to \mathbf{R}$. We assume that the natural growth function f is hump-shaped and strictly concave, and the utility function u is bounded from below, strictly increasing and strictly concave. See Figure 1. Formally, we make the following assumptions:

[Assumption 1] $f:\mathbf{R}_+ \to \mathbf{R}$ satisfies $f(0) = f(K) = 0,\ K > 0,\ f'' < 0$.

[Assumption 2] $u:\mathbf{R}_+ \to \mathbf{R}$ satisfies $u(0) = 0,\ u' > 0$ and $u'' < 0$.

<Figure 1>

The objective of the resource user is to maximize the sum of his discounted utilities from the present time to infinite future, as seen in the objective functional (1.a). The discount rate ρ is the user's time preference parameter. If ρ is zero, the user values the utilities equally between now and any time in future, whereas if it is positive, the utilities in future are valued less than the present utility. In particular, by the power of discounting, the present value of the well-being of a generation living in a far distant future is almost negligible. This implies that the choice of a discount rate raises an ethical problem for intergenerational equity. In fact, in the seminal paper which initiated dynamic analysis in economics, Frank Ramsey, who was a philosopher and mathematician as well as economist, wrote that [discounting] is ethically indefensible and arises merely from the weakness of the imagination (Ramsey 1928). It should be noted, however, that discounting could be rationalized from a *non*-ethical standpoint. It is known that there is no (lifetime) utility function which satisfies Pareto criterion and intergenerational equity in an infinite time horizon model (Basu and Mitra 2003. Also see Svensson 1980, Diamond, 1965; Koopmans, 1960). Pareto criterion is minimum essential requirement for rational decision making. For a discounting model ($\rho > 0$), the lifetime utility function is well defined and satisfies the Pareto criterion, but does not satisfy the intergenerational equity. For now, let us assume a positive discount rate $\rho > 0$. Later, we will justify it for a non-ethical reason: uncertainty.

In order to obtain finite-time extinction as an optimal path, we make an additional assumption:

[Assumption 3] $\lim_{x \to 0} f'(x) = r < \infty$ and $\lim_{c \to 0} u'(c) < \infty$.

While the finiteness of the intrinsic growth rate r is quite plausible, the assumption for the utility function is relatively restrictive, because it rules out the case where c is an essential good like water or oxygen. If your consumption of water decreases to zero, the value of one unit of water for you, $u'(c)$, will rise up to infinity. Therefore, you will never exhaust the source of water. This explains the necessity of this assumption for optimal finite-time resource exhaustion. However, in a setup with multiple resource users, as in Section 5, finite-time resource extinction could be optimal without this assumption.

To solve the rudimentary problem, it is a routine to define the (maximized) Hamiltonian $H^*: \mathbf{R}_+^2 \to \mathbf{R}$ as

$$H^*(x,\lambda) = \max[u(c) + \lambda(f(x) - c) \mid c \geq 0], \tag{2}$$

where λ is called the costate variable. By Pontryagin's maximum principle, there is a nonnegative function of time $\lambda(t)$, with which the optimal control $c^*(t)$ and the optimal state $x^*(t)$ satisfy (2), and

$$\dot{x} = \partial H^*(x,\lambda)/\partial\lambda, \tag{3.a}$$

$$\dot{\lambda} = \rho\lambda - \partial H^*(x,\lambda)/\partial x, \tag{3.b}$$

on the interior of the domain of H^*. The system of autonomous differential equations (3) is called (modified) Hamiltonian dynamics. This system is rewritten as

$$\dot{x} = f(x) - u'^{-1}(\lambda),$$
$$\dot{\lambda} = [\rho - f'(x)]\lambda,$$

where u'^{-1} is the inverse function of u'. The phase diagram of this system is depicted as Figure 2. A solution path is optimal if it converges to a steady state, by Arrow's sufficiency theorem.[1] From this, it follows that the interior steady state, which satisfies $f'(x^\rho) = \rho$, is an *optimal steady state* (OSS). This interior OSS exists if the intrinsic growth rate exceeds the discount rate:

$$\rho < r \ (= \lim_{x \downarrow 0} f'(x)). \tag{4}$$

<Figure 2>

---

1 See, for example, Dockner *et al*. 2000, Chapter 3.

Figure 2 Panel (a) illustrates this case, where every optimal path converges to the unique interior OSS. Thus, the optimal resource use is sustainable.

If the discount rate is too high to satisfy inequality (4), the interior OSS disappears. The phase diagram for this case is drawn in Figure 2 Panel (b). Still, an optimal path for each initial stock x_0 exists and it is unique.[2] Furthermore, the optimal path is monotone (Hartl 1987). Therefore, the path converging to $(x,\lambda)=(0,u'^{-1}(0))$ is the unique optimum. Along the optimal path, the resource stock decreases and eventually becomes extinct. The extinction occurs in a finite time, since $u'^{-1}(0)$ takes a finite value and the time derivatives of the Hamiltonian dynamics do not degenerate at the limit point: $\dot{x}\neq 0, \dot{\lambda}\neq 0$ at $(0,u'^{-1}(0))$. Therefore, we have the following proposition.

Proposition 1: *For the rudimentary problem (1) with Assumptions 1-3, if the discount rate exceeds the intrinsic growth rate, finite-time extinction is optimal for the resource user.*

As mentioned before, the choice of discount rate raises an ethical problem: that is, how should we value the well-being of future generations? Another problem is that if idiosyncratic preferences are the underlying cause of unsustainable resource use, then there is little one can do to prevent extinction without regulating the resource harvesting in some fashion. However, independently of any ethical argument, there is another reason for discounting; that is, uncertainty about the future of the resource stock or its ownership. Imagine that a sudden disaster completely destroys the resource, or the resource owner is suddenly deprived of his ownership by, say, confiscation of the resource stock by a corrupt or politically radical government.[3]

We assume that this sort of fatal event occurs with a positive probability. Formally, we suppose that the agent does not discount future utilities at all. Instead, the parameter ρ expresses

[2] For the proof of the existence, see Magill 1981. The uniqueness follows from the strictly concavity of the functions f and u.

[3] Note that for environmental conservation or other reasons, a politically radical government may suddenly impose a resource tax whose rate is sufficiently high and/or increases steadily at a constant proportional rate. It will have the equivalent effect as a stochastic resource confiscation/catastrophe, and as shown here, it will cause finite-time exhaustion to be optimal.

the arrival rate of the Poisson process for the occurrences of the fatal event. Once the fatal event occurs, the utility levels of the resource user thereafter are zero forever. Note that the probability with which the event occurs from $t = 0$ to $t = \tau$ is $1 - e^{-\rho\tau}$, and thus the probability density of occurrence is $\rho e^{-\rho t}$. The objective functional (1.a) is modified to be

$$E\left[\int_0^T u(c(t))dt\right],$$

where stochastic variable T is the time at which a fatal event occurs and E is the expectation operator. The following calculation is well known:[4]

$$\begin{aligned}
&E\left[\int_0^T u(c(t))dt\right] \\
&= \int_0^\infty \left[\int_0^T u(c(t))dt\right]\rho e^{-\rho T} dT = -\int_0^\infty \left[\int_0^T u(c(t))dt\right]\frac{de^{-\rho T}}{dT} dT \\
&= \left[e^{-\rho T}\int_0^T u(c(t))dt\right]_{T=0}^\infty + \int_0^\infty u(c(T))e^{-\rho T} dT \\
&= \int_0^\infty u(c(t))e^{-\rho t} dt.
\end{aligned}$$

The last equality holds since every feasible consumption path satisfies $\limsup_{t\to\infty} c(t) \le \max\left[f(x) \mid x \ge 0\right]$ and there exists $\bar{u} < \infty$ such that $\operatorname{ess\,sup}_{t\ge 0} u(c(t)) \le \bar{u}$ for each feasible $c(t)$. Note $\lim_{T\to\infty} e^{-\rho T}\int_0^T u(c(t))dt \le \lim_{T\to\infty} \bar{u}T / e^{\rho T} = \lim_{T\to\infty} \bar{u} / \rho e^{\rho T} = 0.$

After all, we are back to the rudimentary problem (1), although now ρ expresses the magnitude of the probability of the fatal event. We interpret this result as the following corollary:

Corollary of Proposition 1: *Finite-time extinction may be optimal, if the ecosystem and/or the socio-political system is so unstable that the probability of the arrival of the ecological catastrophe or socio-political upheaval is so high as to exceed the intrinsic growth rate,* $\rho > r$.

---

4 See, for example, Dasgupta and Heal (1979).

3. Nonconvexity of a Natural Growth Function

In this section, we focus on the natural growth function. A concave growth function implies that the natural growth rates increase as the size of the resource stock decreases. However, if the stock size is very small, the growth rate may be small, for example, due to an Alee effect or an inbreeding depression. Then, we may have a convex-concave shape of the growth function, as in figure 3.

<Figure 3>

Consider the rudimentary problem (1) with modification of Assumption 1 as follows:

[Assumption 4] $f(0)=0,\ \lim_{x\to 0} f'(x)<\rho,\ \exists x_I>0: f'(x_I)>\rho,\ f''(x)>(<)0 \text{ if } x<(>)x_I.$

<Figure 4>

Figure 4 illustrates the associated phase diagram. There are two stock levels at which the slope of the growth function coincides to discount rate ρ. The larger one x^ρ corresponds to the OSS for the original rudimentary model (1). The smaller one x_ρ is new. It is readily seen that x_ρ and the associated costate λ constitute another steady state of the Hamiltonian dynamics (3). However, it can be shown that x_ρ is not an OSS.[5] Also notice that it is ambiguous whether x^ρ is an OSS, due to the nonconvexity of the growth function. Even the existence of optimal paths is a subtle problem for a nonconvex problem. However, they exist if we impose a certain growth condition for the evolution of the resource stock (See Romer 1987). Furthermore, it can be shown that every optimal path is monotonic (See Long *et al.* 1997). Then, we have the following proposition:

Proposition 2: *Suppose the existence of optimal paths. Under Assumption 4, there exists a*

[5] It can be shown that given the initial point $(x_0, \lambda(0))$, the lifetime utility induced by the Hamiltonian dynamics coincides with $H^*(x_0, \lambda(0))/\rho$. By construction, H^* is a strictly convex function in λ. Therefore, the lifetime utility is *minimized* by staying the lower steady state x_ρ.

threshold $x_c \in (0,\infty]$ *such that if* $x_0 < (>) x_c$*, the optimal state trajectory monotonically converges to* 0 *(* x^ρ *). Furthermore, the threshold satisfies* $x_c < x^\rho$ *, if the following mild discounting condition holds:*

$$\rho < \max[f(x)/x \mid x \geq 0].$$

Proof: See the Appendix.

Figure 5 illustrates the optimal paths. Even with a very low discount rate, in the presence of non-concavity of natural growth function, finite-time extinction may be optimal if the resource stock has been already degraded (by, for example, overexploitation so far) below the critical threshold. This threshold x_c is called Skiba point or DNS point.[6]

<Figure 5>

4. Non-Pecuniary Value of Employment: A Socio-Psychological Aspect

It is natural to think that working is not only a means of earning income, but also a form of social involvement. Because of this, unemployment usually brings a person unhappiness more than loss of income does, which may include, for example, losses of dignity, confidence and identity. In other words, there is a non-pecuniary value of employment. Curiously enough, this fact has been ignored in traditional economics until recently. Farzin and Akao (2005) incorporate this socio-psychological aspect explicitly into a bio-economic model and find that the optimal resource use may be finite-time extinction. In this section, we introduce their results with a simpler model than theirs.

We modify the utility part of the rudimentary problem (1) as follows:

[Assumption 5] $u(c,E),\ E \in [0,\bar{E}],\ E =$ working time. $\partial u/\partial E > 0, \partial u^2/\partial E^2 < 0.$

The utility stems from two sources. One is consumption of harvests as before. The other source is

[6] "DNS" is the initials of three authors, Dechert, Nishimura, and Skiba. Skiba (1978) first introduced the convex-concave production function in the theory of optimal growth in economics. A rigorous analysis for a discrete time model is given by Dechert and Nishimura (1983). Gustav Feichtinger and his collaborators have recently studied the continuous time models and their applications. See, for example, Deissenberg *et al*. (2001) and Hartl *et al.* (2004), which contain the literature review in economic dynamics with nonconvexity, including environmental economics.

working. Different from a standard economic model, here working is not a source of disutility, but a source of utility. Although this assumption may seem curious, Farzin and Akao (2004) show that non-pecuniary value of work exceeds the value of leisure at very low income levels.

Assume that all the labor is used to extract the resource, which is the case where there is no alternative employment other than resource extraction. The relationship between labor input E and resource output c is described with the cost function $E(c)$. There is an upper bound for working time $\bar{E}$, which limits the maximum harvest level. Denote the maximum harvest level by $\bar{H}$, which satisfies $\bar{E} = E(\bar{H})$. We assume that with the maximum effort $\bar{E}$, the resource is certainly exhausted in a finite time:

[Assumption 6] The maximum harvest level with full employment exceeds the maximum sustained yield (MSY): $\bar{H} > f(x_{MSY})$, where x_{MSY} is the solution of $\max[f(x) \mid x \geq 0]$).

It is important to notice that even though working is a source of utility, the full employment $\bar{E}$ is *not necessarily* an optimal choice. This is because the full employment may degrade the resource too much to allow sustaining future consumptions. Recall that we have supposed that the resource user is rational enough and in particular farsighted.

Although it is mathematically invariant, let us add a flavor of macro economics to the rudimentary model (1). Consider a community, in which the local people are governed by a benevolent government. Everyone has identical preferences and the same harvesting technology, as described above. Let n be the population size. The problem of the benevolent government is:

$$\max_{c(t)\geq 0} \int_0^\infty u\left[c(t), E(c(t))\right] e^{-\rho t} dt \tag{7.a}$$

$$\text{subject to } \dot{x}(t) = f(x(t)) - nc(t), \tag{7.b}$$

$$0 \leq E(c) \leq \bar{H}, \quad x(0) = x_0 \text{ given.} \tag{7.c}$$

Pontryagin's maximum principle suggests that an optimal control c^* maximizes the Hamiltonian:

$$H(c, x, \lambda) = U(c) + \lambda[f(x) - nc], \text{ where } U(c) = u[c, E(c)].$$

Assume that the reduced form utility function U is strictly *convex*. The following example shows that such a convex utility function is obtained with standard assumptions in economics, if we allow working to be a source of utility.

[Example]

Let utility function have a form of $u(c,E)=c^{\alpha}E^{\eta}$, with $0<\alpha<1$ and $0<\eta<1-\alpha$ (so that u is concave and increasing, a standard assumption of economics). The harvesting technology is expressed by $E(c)=c^{\beta}$, with $\beta>1$, which is also a standard assumption of economics: a cost function is convex and increasing. If the elasticity of marginal utility of employment is sufficiently high to satisfy $\beta>(1-\alpha)/\eta$, then $U(c)=u[c,E(c)]=c^{\alpha+\beta\eta}$ is strictly convex ($d^2U/dc^2>0$).

If $d^2U/dc^2>0$, the maximum of the Hamiltonian is attained at a corner of c. That is, the optimal control $c*$ is either of the full harvesting $c*=\bar{H}$ or no harvesting $c*=0$. Also, notice that there is no interior optimal control and thus no interior OSS. Therefore, the optimal path of the resource stock converges either to the carrying capacity or to zero. It is, however, obvious that the path going to the carrying capacity is suboptimal, because there is no chance to harvest at all. Therefore, we have:

Proposition 3: *Under Assumptions 5 and 6, if* $d^2u[c,E(c)]/dc^2>0$, *full employment is always optimal. On an optimal path the resource stock decreases most rapidly and becomes extinct in a finite time.*[7]

In Farzin and Akao's framework, if the harvest level with full employment exceeds the MSY, full employment and sustainable resource management are incompatible objectives, and the former is chosen over the latter as the optimal policy. Population growth and technological progress in resource extraction may bring about such a situation. The optimal path has two novelties. First, the

[7] The formal proof, including the existence of an optima path, is found in Farzin and Akao (2005).

optimal resource extinction is an extreme one, the most rapid extinction.[8] Second, resource extinction is optimal irrespective of the state of the resource stock and the magnitude of the discount rate. Notice that in this section, we have referred neither to the discount rate nor to the initial level of the resource stock, which were crucial factors for finite-time extinction to be optimal in the previous sections.

5. Strategic Interaction

In this section, we consider a natural resource used by multiple users, who are not cooperative.[9] Such a resource may be categorized by its physical property into two types. The first type is a resource for which it is difficult to establish a definite property right. The global atmosphere, underground aquifers, and highly migratory fish stocks are few examples. The second type is a resource which, although its private or governmental holding is physically possible, is owned communally for institutional or historical reasons. An example is the high seas defined in the United Nations Convention on the Law of the Sea. Another example is a local communal forest in Japan, which is a relic of the Edo era (1603-1868), at which private ownership of a forest was prohibited.

We will show that for those resources, finite-time extinction may be optimal from the viewpoint of each resource user, despite the fact that it is by no means socially or cooperatively optimal. In other words, we will see the individual optimality of the so-called "tragedy of the commons." We also show that a sustainable resource use can be *individually* an optimal path, although it differs from a socially optimal path. Therefore, the tragedy of the commons is not an inevitable destiny. This could explain the fact that some communal resources have been managed in a sustainable way, at least apparently and so far.

A fundamental change from the previous models is that not a single agent but many agents use the resource. We assume that the number of resource users $n \geq 2$ is fixed. In terminology of

8 Without invoking non-pecuniary value of employment, we could obtain the most rapid extinction as an optimal path. It is necessary, however, to specify the utility and natural growth functions that satisfy the restrictive conditions derived by Spence and Starrett (1975). Heavy discounting is also needed.

9 We have used "not cooperative" as a synonym of "individually rational."

economics, this sort of resource is called a common property resource or a common pool resource.[10] The resource users are identical in their preferences and harvesting technology. As in the previous section, there exists the upper bound of the harvest ability $\bar{h}(=\bar{H}/n) > 0$. Modifying the rudimentary model (1), we study the following differentiable game model:

$$\max_{c(t)\geq 0} \int_0^\infty u(c(t))e^{-\rho t}dt \tag{8.a}$$

$$\text{subject to } \dot{x}(t) = f(x(t)) - (n-1)\sigma(x) - c(t), \tag{8.b}$$

$$c(t) \in [0, \bar{h}],\ x(0) = x_0 \text{ given.} \tag{8.c}$$

Let us introduce a few terms of differential game theory. A strategy is a way to harvest. While strategy is a quite broad concept, we restrict our analysis to stationary Markovian strategy. “Stationary” means that the strategy is time independent. “Markovian” means that the strategy is a function of the resource stock x. An example of stationary Markovian strategy $\sigma(x)$ is the most rapid extinction strategy:

$$\sigma(x) = \begin{cases} \bar{h} & \text{if} \quad x > 0 \\ 0 & \quad\quad x = 0 \end{cases}, \tag{9}$$

which harvests with the maximum effort as far as the resource exists. When the other players use a Markovian strategy $\sigma(x)$, the problem for each user is written as in (8) above. If the optimal solution is described by the same strategy $\sigma(x)$, it is said that $\sigma(x)$ constitutes a symmetric *Markov perfect Nash equilibrium* (shortly, MPNE). It is a Nash equilibrium, because once all users choose the strategy, then no one wants to change the strategy. “Perfect” implies that the equilibrium is subgame perfect. In other words, the strategy is strongly time consistent in the sense that even if some players deviate from the equilibrium strategy, the equilibrium strategy is still optimal as far as the players return to the equilibrium strategy. Finally, it is symmetric since all users take the same

---

[10] Another resource modeling with multiple users is that of an *open access resource problem*, for which the number of the users varies. Anonymous agents freely enter to extract the resource, until their temporal profits equilibrate to zero. For this setup, the mechanism causing finite-time extinction is quite simple. That is, it occurs if the aggregate harvest levels corresponding to zero profit always exceed the natural growth rates of the resource (Berck 1979). Also, notice that an open access resource problem can be regarded as the limit case of the problem (1) with $\rho \to \infty$ (Beddington *et al.* 1975). Therefore, we omit this problem in this paper.

strategy. For analytical simplicity, we will restrict our concern to the class of symmetric MPNEs. Notice that the problem of each resource user now becomes more complicated than the ones in the previous sections, because other users also harvest the resource and their harvest rates affect the user's action, which affects, in turn, other users' actions. This is the strategic interaction.

The cooperative or social optimization problem, compared with non-cooperative problem (8), is formulated as follows:

$$\max_{c(t)\geq 0} \int_0^\infty u(c(t))e^{-\rho t}dt$$

$$\text{subject to } \dot{x}(t) = f(x(t)) - nc(t),\ c(t) \in [0, \bar{h}],\ x(0) = x_0 \text{ given.}$$

Maintain Assumptions 1, 2, and

[Assumption 7] $\lim_{x \downarrow 0} f'(x) > \rho.$

It is easily verified that every optimal cooperative path of the resource stock monotonically converges to a unique social OSS, $x^\rho > 0$, such that $f'(x^\rho) = \rho$ (See Figure 1). Therefore, the cooperative solution is sustainable.

We want to show that finite-time exhaustion is a Nash equilibrium, i.e. individuals' rational choice. To do so, unlike the previous section, we do not need Assumption 3 (the finiteness of the marginal utility u' and the marginal productivity f' at the origin). Instead, we assume

[Assumption 8]

$$n\bar{h} > f(x_{MSY}),\ (n-1)\bar{h} > f(x^\rho),\ \beta(c) = \frac{-cu''(c)}{u'(c)} \leq \frac{n-1}{n},\ \lim_{c\to 0}\beta(c) > 0.$$

The first inequality is the same assumption as Assumption 6 in the previous section. That is, with the maximum harvesting effort, the resource is exhausted in a finite time. The second inequality ensures that if other users harvest the resource with the maximum effort, there is no way for an individual user to sustain the social OSS x^ρ. The third and forth inequalities restrict the curvature of utility function. These assumptions are technical, but standard in economics.

The following proposition on equilibrium resource use is given by Gerhard Sorger (1998).

Proposition 4 (Sorger, 1998): *(a) The most rapid extinction strategy (9) constitutes a MPNE if and only if the following inequality holds:*

$$u'(\bar{h}) \geq \frac{u(\bar{h})}{n\bar{h} - f(x^{\rho})} \exp\left[-\rho \int_0^{x^{\rho}} \frac{dy}{n\bar{h} - f(y)} \right]. \tag{10}$$

(b) There is a continuum of the other MPNE strategies if $f(x^{\rho}) > nf'(x^{\rho})x^{\rho}$. *Each strategy is sustainable in the sense that the associated equilibrium path of the resource stock converges to a positive stock level in* $(0, x^{\rho})$.

We refer to the strategy in Proposition 4(b) as Sorger's strategy. Interestingly, there is a pair of a Sorger's strategy and the initial resource stock with which the associated equilibrium path of the resource stock stays in an arbitrarily small neighborhood of the social OSS x^{ρ} for an arbitrarily long time. This implies that if a "good" Sorger's strategy is chosen and if the initial stock of the resource lies sufficiently near x^{ρ} , the lifetime utility of each resource user is approximately the same as would be with the social optimum. Eventually, the stock diminishes to less than x^{ρ} , though.

In contrast, the most rapid extinction is the worst strategy, from the viewpoint of sustainability. The inequality in Proposition 4(a) holds when the number of users n is large, the maximum harvest rate $\bar{h}$ is high, or the discount rate of each user ρ is high. In any of the cases, finite-time extinction is optimal from an individual's viewpoint.

A troublesome, but interesting, point is that we cannot predict which equilibrium is chosen, since all MPNEs are subgame perfect. Furthermore, the most rapid extinction may coexist with Sorger's strategies as equilibrium strategy. This is illustrated in Figure 6 with the following specification: the utility function is isoelastic, the natural growth function is described by a logistic equation, and ρ is fixed at a rather low level, since we have already seen how heavy discounting brings finite-time extinction. In Figure 6, there are two areas: one is the area on which Sorger's strategies constitute equilibria and the other is the area on which the most rapid extinction strategy becomes an equilibrium strategy. Observe that these two areas overlap.

<Figure 6>

The coexistence of the equilibrium strategies means that the resource use for a common property resource is ambiguous and unstable. It is possible on a theoretical ground, that for two communities with the same conditions, one uses its natural resource in a sustainable way, whereas the other exhausts its resource with the most rapid speed. Also, it is possible that a community which was using its resource in a sustainable way until yesterday suddenly starts a ruinous resource use path without any evident trigger.

6. Concluding Remarks

We conclude these observations with their policy implications.

First, we have seen how uncertainty raises discount rate and how a high discount rate brings finite-time resource extinction. To prevent such a situation, we need to mitigate the risk of the fatal events. For example, political stability matters. Furthermore, such a policy should be implemented early on, if the growth function of the resource exhibits nonconvexity and the resource is being degraded. This is because when the resource has been already degraded, finite-time extinction is more prone to be an optimal resource use policy even with a low discount rate.

Second, we have seen that non-pecuniary value of employment makes people give priority to full employment over sustainable resource use. Farzin and Akao (2005) show that the remedy is none but to create alternative employment sources to absorb labor force which is excessive from the viewpoint of sustainable resource use. They also suggest that earlier policy implementation is more prudent, since when the resource is more degraded, higher wage rates may be necessary to prevent resource exhaustion.

Third, we have seen a strategic interaction cause the most rapid extinction, which is an extreme case of a finite-time resource extinction. Although all common property resources do not have such a fate as predicted by Garrett Hardin (1968), all of them share the possibility. Breaking such an interaction is the primal policy to prevent the most rapid extinction. Akao (2001, 2004) shows that among standard economic policy measures, a tax on harvest does not work, whereas tradable permits or quota system works well, because it establishes property rights to resource use. Another prospective prescription is privatization. However, a caution is given by Dasgupta and Maler (1997). They have pointed out that, in the real world, the consequence of privatization of a common property resource may be further resource degradation. This is due to the existing inequality in a rural community. If the resource is not favorably distributed to the poor, they cannot help but to encroach on the resource.

Finally, resource-sector technological assistance and income assistance may not help to prevent finite-time extinction. In particular, if a technological assistance improves the harvesting efficiency, and hence the maximum harvesting ability, it may even accelerate resource extinction.

Appendix: Proof of Proposition 2

Suppose x_c does not exist. Then, every optimal path converges to x^ρ by its monotonicity. The monotonicity also implies that there is an interval $(0, \tilde{x}]$ such that the optimal control is zero harvesting (a corner solution) while the resource stock stays in the interval. The situation is depicted in Figure 7. Let $V(x)$ be the optimal value function, that is, the lifetime utility gained when the initial stock is x. For $x_0 \in (0, \tilde{x}]$,

$$V(x_0) = V(\tilde{x})e^{-\rho T(x_0)} = V(\tilde{x})\exp\left[-\rho\int_{x_0}^{\tilde{x}} \frac{dx}{f(x)}\right], \tag{5}$$

where $T(x_0)$ is the arrival time at $\tilde{x}$. Let $\lambda(t; x_0)$ be the optimal costate at time t when the initial stock is x_0. Since $V(x_0)$ is twice continuously differentiable, $\lambda(0; x_0) = V'(x_0)$ (See Arrow and Kurz, 1970). By (5), $V'(x_0) = \rho V(x_0)/f(x_0)$. Combine these and take the limit, $x_0 \to 0$. Using the l'Hospital's rule, we have

$$\lim_{x_0 \to 0} \lambda(0; x_0) = \lim_{x_0 \to 0} V'(x_0) = \lim_{x_0 \to 0} \frac{\rho V(x_0)}{f(x_0)} = \lim_{x_0 \to 0} \frac{\rho V'(x_0)}{f'(x_0)}. \tag{6}$$

<Figure 7>

Since $\lim_{x_0 \to 0} \lambda(0; x_0)$ exists and finite (see the phase diagram in Figure 5), the equality in (6) holds if and only if $\lim_{x_0 \to 0} f'(x_0) = \rho$, which contradicts Assumption 4. The second part of the proposition is proved as follows: Let $x_{MSY} = \arg\max[f(x)/x \mid x \geq 0]$. The mild discounting condition implies that $x_{MSY} < x_\rho$. Consider a new growth function $\tilde{f}(x)$,

which is defined by the convex hull of the graph $(x, f(x))$. Since $f(x) \leq \tilde{f}(x)$ all $x \geq 0$, an optimal control for the problem (1) with $\tilde{f}(x)$ is optimal for the problem with $f(x)$, if the control is feasible with $f(x)$. It is easily seen that these two problems share the optimal path in a neighborhood of x^ρ, which is the OSS when the growth function $\tilde{f}(x)$.

(Q.E.D.)

References

[1] Akao, K. (2004) Tax schemes in a class of differential games, mimeo.

[2] Akao, K.(2001) Some results for resource games, *Institute for Research in Contemporary Political and Economic Affairs* (*Waseda University*) WP **2009**.

[3] Arrow, K.J. and M. Kurz (1970) *Public Investment, the Rate of Return, and Optimal Fiscal Policy*, Johns Hopkins University Press.

[4] Beddington, J.R., C.M.K. Watts and W.D.C. Wright (1975) Optimal cropping of self-reproducible natural resources, *Econometrica* **43**, 789-802.

[5] Berck, P. (1979) Open access and extinction, *Econometrica* **47**, 877-882.

[6] Basu, K. and T. Mitra (2003) Aggregating infinite utility streams with intergenerational equity: the impossibility of being Paretian. *Econometrica* **71**, 1557-1563.

[7] Carlowitz, H.C. von (1713) Sylvicultura oeconomica, oder, Hausswirthliche Nachricht und naturmassige Anweisung. zur wilden Baumzucht.

[8] Dasgupta, P. and G. Heal (1974) The optimal depletion of exhaustible resources. *Review of Economic Studies*. Symposium on the Economics of Exhaustible Resources, 3-28.

[9] Dasgupta, P. and K. -G. Mäler (1997) The resource-basis of production and consumption: an economic analysis, in Dasgupta, P. and K. -G. Mäler (eds.) *The Environment and Emerging Development Issues*, vol.1, 1-32, Clarendon Press.

[10] Dechert, D. and K. Nishimura (1983) A complete characterization of optimal growth paths in an aggregate model with a nonconvex production function. *Journal of Economic Theory* **31**, 332-354.

[11] Deissenberg, C., G. Feichtinger, W. Semmler and F. Wirl (2001) History dependence and global dynamics in models with multiple equilibria, *Center for Empirical Macroeconomics Working Paper* **12**.

[12] Diamond, P. (1965) The evaluation of infinite utility streams. *Econometrica* **33**, 170-177.

[13] Dockner, E., S. Jørgensen, N. V. Long, and G. Sorger (2000) *Differential Games in Economics and Management Science*, Cambridge University Press.

[14] Farzin, Y.H. (2004) Is an exhaustible resource economy sustainable? *Review of Development Economics* **8**, 33-46.

[15] Farzin, Y.H. and K. Akao (2004) Non-pecuniary Value of Employment and Individual Labor Supply. *Fondazione Eni Enrico Mattei WP* **158.04**.

[16] Farzin, Y.H. and K. Akao (2005) Non-Pecuniary Value of Employment and Natural Resource Extinction. *SSRN eLibrary* http://ssrn.com/abstract=854585.

[17] Hardin, G. (1968) The tragedy of commons. Science **162**, 1243-1247.

[18] Hartl, R. F. (1987) A simple proof of the monotonicity of the state trajectories in autonomous control problems. *Journal of Economic Theory* **41**,211-215.

[19] Hartl, R.F., P.M. Kort, G. Feichtinger, and F. Wirl (2004) Multiple equilibria and threshold due to relative investment costs. *Journal of Optimization Theory and Applications* **123**, 49-82.

[20] Heal, G. (1998), *Valuing the Future: Economic Theory and Sustainability*, Columbia University Press, New York.

[21] Koopmans, T.C. (1960) Stationary ordinal utility and impatience. *Econometrica* **28**, 287-309.

[22] Long, N.V., K. Nishimura, and K. Shimomura (1997) Endogenous growth, trade and specialization under variable returns to scale: The case of a small open economy, in Jensen, B.S. and K.W. Wong (eds.) *Dynamics, Economic Growth, and International Trade*, 127-150, The University of Michigan Press.

[23] Magill, M.J.P. (1981) Infinite horizon programs. *Econometrica* **49**, 679-712.

[24] Nordhaus, W. D. (1994) Reflecting on the concept of sustainable economic growth, in Pasinetti, L. L. and R. M. Solow (eds.) *Economic Growth and the Structure of Long-Term Development*, 309-325, Macillan/St.Martin's Press.

[25] Pearce, D., E. B. Barbier and A. Markandya(1990) *Sustainable Development*: *Economics and Environment in the Third World*. Edward Elgar.

[26] Ramsey, F.P. (1928) A mathematical theory of saving. *Economic Journal* **38**, 543-559.

[27] Romer, P. (1986) Cake eating, chattering, and jumps: existence results for variational problems. *Econometrica* **54**, 897-908.

[28] Skiba, A.K. (1978) Optimal growth with a convex-concave production function. *Econometrica* **46**, 527-539.

[29] Solow, R.M. (1998) *An Almost Practical Step Toward Sustainability*, Resources for the Future.

[30] Sorger, G. (1998) Markov-perfect Nash equilibria in a class of resources games. *Economic Theory* **11**, 78-100.

[31] Spence, M. and D. Starrett (1975) Most rapid approach paths in accumulation problems. *International Economic Review* **16**, 388-403.

[32] Svensson, L.–G. (1980) Equity among generations. *Econometrica* **48**, 1251-1256.

[33] Turner, R.K., D. Pearce and I. Bateman.(1994) *Environmental Economics : An Elementary Introduction*, Harvester Wheatsheaf.

[34] Vanclay, J.K. (1996) Estimating Sustainable Timber Production from Tropical Forests. *Center for International Forestry Research Working Paper* **11**.

[35] World Commission on Environment and Development (1987) *Our common future*, Oxford University Press, 1987.

Figures

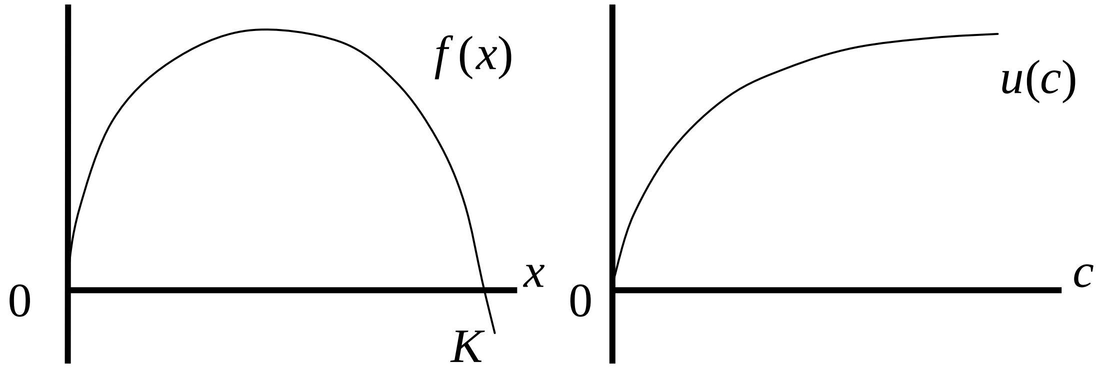

Figure 1 Natural growth function and utility function.

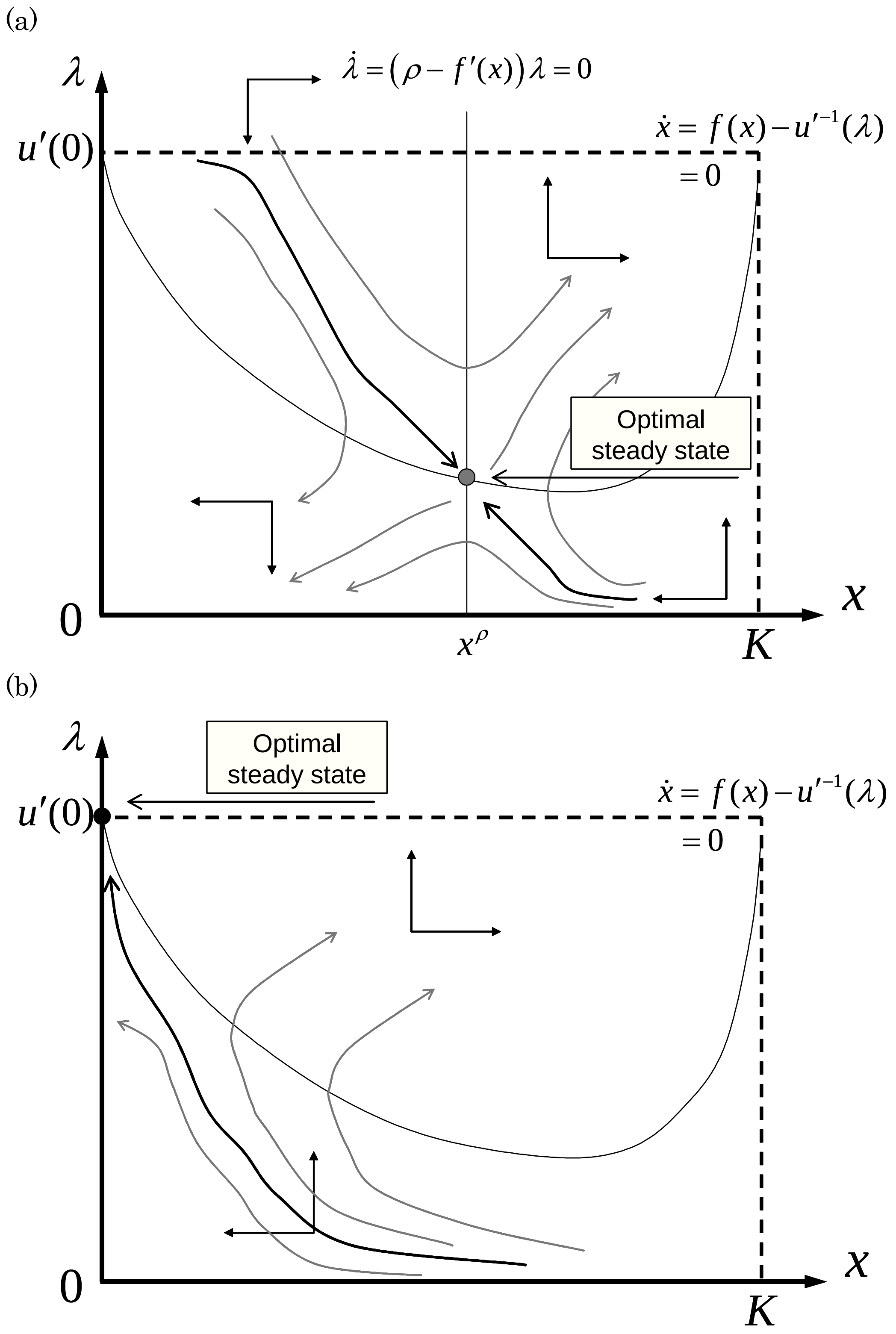

Figure 2 Phase diagrams for the rudimentary model

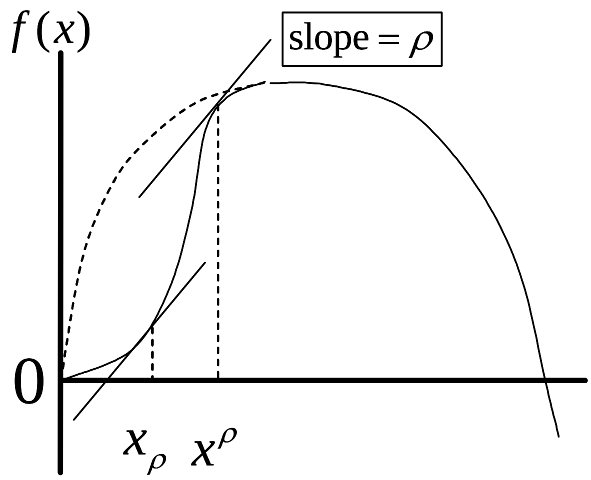

Figure 3 Convex-concave natural growth function.

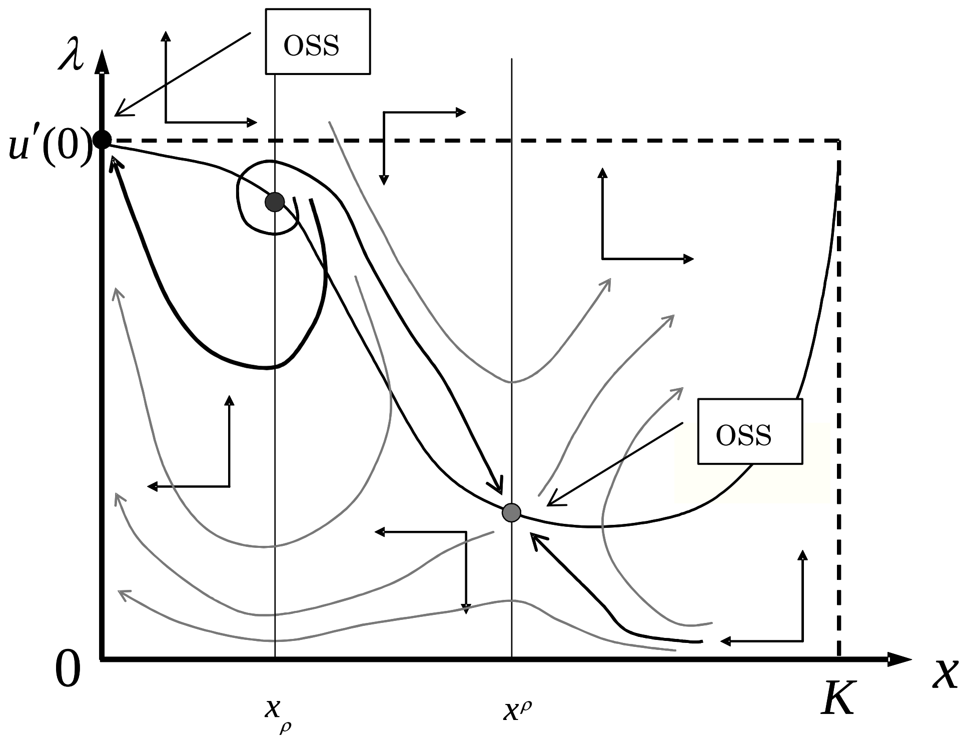

Figure 4 Phase diagram for the nonconvex model.

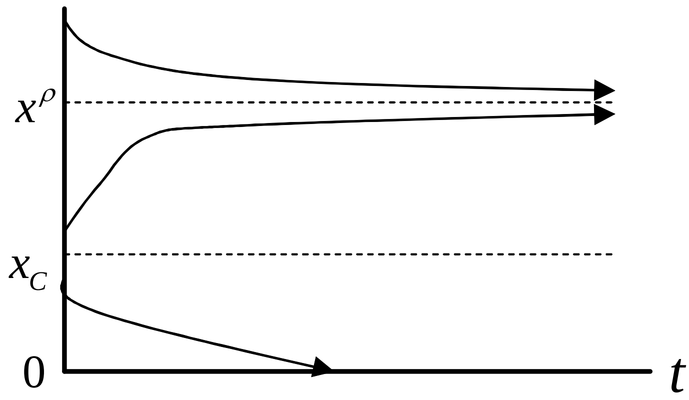

Figure 5 Optimal paths.

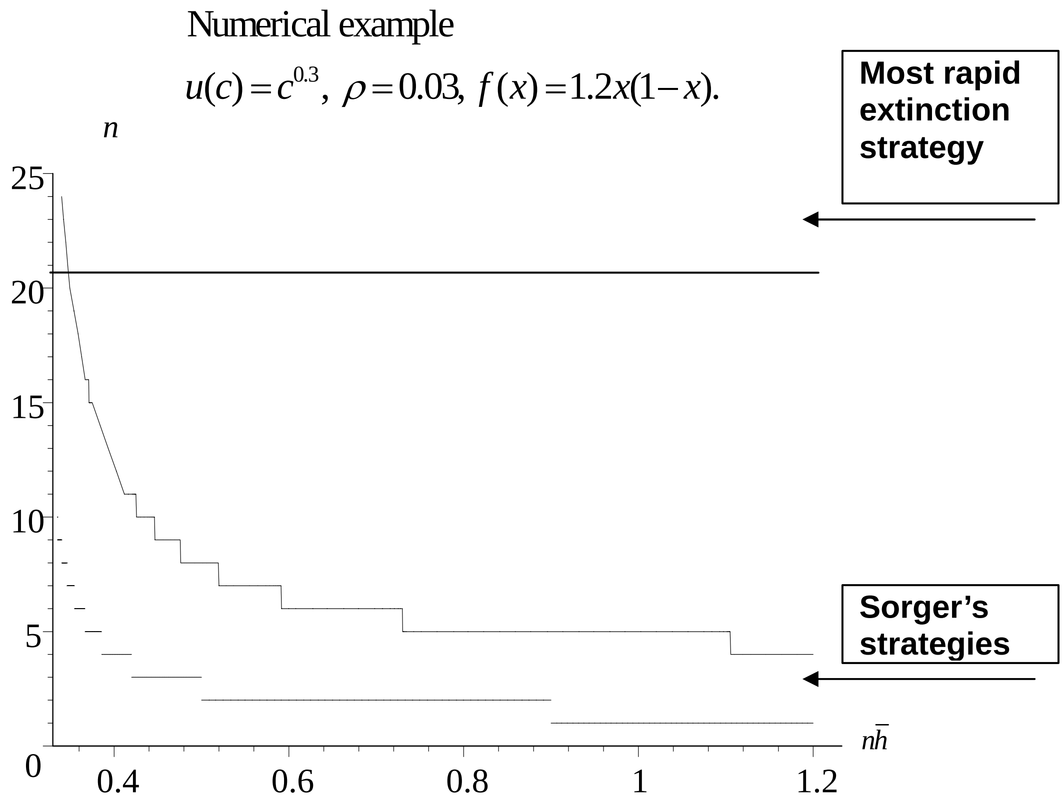

Figure 6 Coexistence of sustainable and unsustainable equilibria.

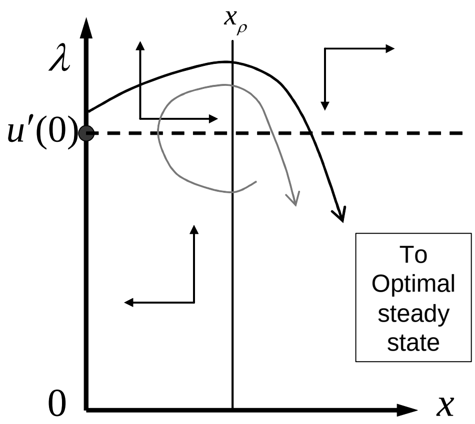

Figure 7　Phase diagram near the corner OSS.

NOTE DI LAVORO DELLA FONDAZIONE ENI ENRICO MATTEI

Fondazione Eni Enrico Mattei Working Paper Series

Our Note di Lavoro are available on the Internet at the following addresses:

http://www.feem.it/Feem/Pub/Publications/WPapers/default.html
http://www.ssrn.com/link/feem.html
http://www.repec.org
http://agecon.lib.umn.edu

NOTE DI LAVORO PUBLISHED IN 2006

SIEV	1.2006	*Anna ALBERINI*: Determinants and Effects on Property Values of Participation in Voluntary Cleanup Programs: The Case of Colorado
CCMP	2.2006	*Valentina BOSETTI, Carlo CARRARO and Marzio GALEOTTI*: Stabilisation Targets, Technical Change and the Macroeconomic Costs of Climate Change Control
CCMP	3.2006	*Roberto ROSON*: Introducing Imperfect Competition in CGE Models: Technical Aspects and Implications
KTHC	4.2006	*Sergio VERGALLI*: The Role of Community in Migration Dynamics
SIEV	5.2006	*Fabio GRAZI, Jeroen C.J.M. van den BERGH and Piet RIETVELD*: Modeling Spatial Sustainability: Spatial Welfare Economics versus Ecological Footprint
CCMP	6.2006	*Olivier DESCHENES and Michael GREENSTONE*: The Economic Impacts of Climate Change: Evidence from Agricultural Profits and Random Fluctuations in Weather
PRCG	7.2006	*Michele MORETTO and Paola VALBONESE*: Firm Regulation and Profit-Sharing: A Real Option Approach
SIEV	8.2006	*Anna ALBERINI and Aline CHIABAI*: Discount Rates in Risk v. Money and Money v. Money Tradeoffs
CTN	9.2006	*Jon X. EGUIA*: United We Vote
CTN	10.2006	*Shao CHIN SUNG and Dinko DIMITRO*: A Taxonomy of Myopic Stability Concepts for Hedonic Games
NRM	11.2006	*Fabio CERINA* (lxxviii): Tourism Specialization and Sustainability: A Long-Run Policy Analysis
NRM	12.2006	*Valentina BOSETTI, Mariaester CASSINELLI and Alessandro LANZA* (lxxviii): Benchmarking in Tourism Destination, Keeping in Mind the Sustainable Paradigm
CCMP	13.2006	*Jens HORBACH*: Determinants of Environmental Innovation – New Evidence from German Panel Data Sources
KTHC	14.2006	*Fabio SABATINI*: Social Capital, Public Spending and the Quality of Economic Development: The Case of Italy
KTHC	15.2006	*Fabio SABATINI*: The Empirics of Social Capital and Economic Development: A Critical Perspective
CSRM	16.2006	*Giuseppe DI VITA*: Corruption, Exogenous Changes in Incentives and Deterrence
CCMP	17.2006	*Rob B. DELLINK and Marjan W. HOFKES*: The Timing of National Greenhouse Gas Emission Reductions in the Presence of Other Environmental Policies
IEM	18.2006	*Philippe QUIRION*: Distributional Impacts of Energy-Efficiency Certificates Vs. Taxes and Standards
CTN	19.2006	*Somdeb LAHIRI*: A Weak Bargaining Set for Contract Choice Problems
CCMP	20.2006	*Massimiliano MAZZANTI and Roberto ZOBOLI*: Examining the Factors Influencing Environmental Innovations
SIEV	21.2006	*Y. Hossein FARZIN and Ken-ICHI AKAO*: Non-pecuniary Work Incentive and Labor Supply
CCMP	22.2006	*Marzio GALEOTTI, Matteo MANERA and Alessandro LANZA*: On the Robustness of Robustness Checks of the Environmental Kuznets Curve
NRM	23.2006	*Y. Hossein FARZIN and Ken-ICHI AKAO*: When is it Optimal to Exhaust a Resource in a Finite Time?

(lxxviii) This paper was presented at the Second International Conference on "Tourism and Sustainable Economic Development - Macro and Micro Economic Issues" jointly organised by CRENoS (Università di Cagliari and Sassari, Italy) and Fondazione Eni Enrico Mattei, Italy, and supported by the World Bank, Chia, Italy, 16-17 September 2005.

2006 SERIES	
CCMP	*Climate Change Modelling and Policy* (Editor: Marzio Galeotti)
SIEV	*Sustainability Indicators and Environmental Valuation* (Editor: Anna Alberini)
NRM	*Natural Resources Management* (Editor: Carlo Giupponi)
KTHC	*Knowledge, Technology, Human Capital* (Editor: Gianmarco Ottaviano)
IEM	*International Energy Markets* (Editor: Anil Markandya)
CSRM	*Corporate Social Responsibility and Sustainable Management* (Editor: Sabina Ratti)
PRCG	*Privatisation Regulation Corporate Governance* (Editor: Bernardo Bortolotti)
ETA	*Economic Theory and Applications* (Editor: Carlo Carraro)
CTN	*Coalition Theory Network*