Appraising Diversity with an Ordinal Notion of Similarity: An Axiomatic Approach

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NOTA DI LAVORO 45.2004

MARCH 2004

NRM – Natural Resources Management

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Summary

This paper provides an axiomatic characterization of two rules for comparing alternative sets of objects on the basis of the diversity that they offer. The framework considered assumes a finite universe of objects and an a *prior*i given ordinal quadernary relation that compares alternative *pairs* of objects on the basis of their ordinal dissimilarity. Very few properties of this quadernary relation are assumed (beside completeness, transitivity and a very natural form of symmetry). The two rules that we characterize are the maxi-max criterion and the lexi-max criterion. The maxi-max criterion considers that a set is more diverse than another if and only if the two objects that are the most dissimilar in the former are weakly as dissimilar as the two most dissimilar objects in the later. The lexi-max criterion is defined as usual as the lexicographic extension of the maxi-max criterion. Some connections with the broader issue of measuring freedom of choice are also provided.

Keywords: Diversity, Measurement, Axioms, Freedom of choice

JEL Classification: D63, D69, Q20

This paper has been presented at the 4th BioEcon Workshop on "Economic Analysis of Policies for Biodiversity Conservation", Venice, Italy, August 28-29, 2003, organised on behalf of the BIOECON Network by Fondazione Eni Enrico Mattei, Venice International University (VIU) and University College London (UCL).

We thank, without implying them for any remaining deficiencies, Jean-François Laslier, Alan Kirman and Yongshen Xu for their helpful comments.

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Appraising diversity with an ordinal notion of similarity: an Axiomatic approach

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1 Introduction

Would the killing of 50 000 thousand flies of a specific species have the same impact on the reduction of biological diversity than that of 200 white rhinoceros ? Is the diversity of opinions expressed in the written press larger in France than in the US ? Is the choice of models of cars offered by a particular retailer more diverse than that of another ? These are examples of questions whose answers require a precise notion of *diversity*.

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Biologists have probably been the first scientists interested in developing and implementing numerical indices that aim at measuring the biological diversity offered by alternative ecosystems. One of the most widely used of these indices is Shannon (1948) *weighted entropy* measure proposed in biology by Good (1953) (see e.g. Baczkowski, Joanes and Shamia (1997), Baczkowski, Joanes and Shamia (1998) and Magurran (1998) for other refinements and discussions of this class of indices). The generalized Good index evaluates the diversity of any ecosystem by counting, for each species, the frequency of living individuals within the species relative to the total number of living individuals and calculates a weighted entropy over these relative frequencies. Yet, and despite its wide use and computational convenience for applications, this index lacks sound justifications. Why after all should one use the specific entropy formula for appraising the impact of major changes on biodiversity ? Answering questions like this is important in these days where many countries who have ratified the UN 1992 convention on biological diversity have adopted economically costly environmental regulations in order to prevent a deterioration of biological diversity caused by human activities. It is all the most important as the generalized entropy measure suffers from the drawback of paying no attention whatsoever to either inter-species dissimilarities, or to the possibility for two individuals of the same species to be more dissimilar than two individuals coming from different species. For instance, according to the generalized entropy formula, a world in which all living individuals are equally split between two species of fly is just as diverse as one in which the living individuals are split equally between chimpanzees and hippocampi.

Recent efforts, actually due to economists (Weitzman (1992),Weitzman (1993), Weitzman (1998)), have been made to provide axiomatic foundations to the measurement of biodiversity. Weitzman approach is based on the primitive notion of a *cardinal* numerical measure of distance between living creatures. Such a numerical distance enables one to say things such as "the biological distance between a chimpanzee and a bee is twice as large as the biological distance between a rainbow trout and a kokanee salmon". Using such a numerical distance, Weitzman (1992) axiomatically characterizes a sophisticated iterative lexicographic method for appraising the diversity offered by a set of living individuals. Using a somewhat different setting, Bossert, Pattanaik and Xu (2002) also provide an axiomatic characterization of the Weitzman's method by taking as given a cardinal numerical measure of distances between the objects. Weitzman's procedure has been substantially generalized in a recent paper by Nehring and Puppe (2002) who propose to derive the basic numerical distance function assumed by Weitzman from an *a priori* grouping of the objects into collections of "attributes" (for instance being a mammal), each attributed being weighted by a (cardinally meaningful) numerical function.

Weitzman or Nehring and Puppe procedure, by taking due account of the (possibly) different distances that may exist between alternative pairs of living creatures, is clearly sensitive to inter-species dissimilarities. It also allows for the possibility of two individuals of a particular species (chimpanzee for instance) to be more diverse than two individuals coming from different species. On the other hand, it is not at all clear that the current state of knowledge in biology leads to such a precise cardinal measure of distance between living creatures as what is required by these approaches. Much biologists would probably agree that a chimpanzee and a bee are more dissimilar than a rainbow trout and a kokanee salmon. But would they agree to say that the dissimilarity between a chimpanzee and a bee is twice that between a rainbow trout and a kokanee salmon ? Does the discriminating power of current biology enable one to perform such precise cardinal statements ?

In the last 15 years or so, interest in diversity measurement has also arisen in *non-welfarist* normative economics, in connection with the issue of comparing alternative *opportunity sets* on the basis of their freedom of choice (see e.g. Arrow (1995), Bossert (1997), Bossert (2000), Bossert, Pattanaik and Xu (1994), Dutta and Sen (1996), Foster (1993), Gravel (1994), Gravel (1998), Gravel, Laslier and Trannoy (1998), Jones and Sugden (1982), Klemisch-Ahlert (1993), Kreps (1979), Nehring and Puppe (1999), Pattanaik and Xu. (1990), Pattanaik and Xu (1998), Pattanaik and Xu (2000b), Puppe (1995), Puppe (1996), Puppe (1998), Puppe and Xu (1996), Sen (1988), Sen (1991), Sugden (1985), Suppes (1987), Suppes (1996) and Van-Hees (1997) for representative pieces of this literature and Barberà, Bossert and Pattanaik (n.d.), Foster (2001) and Sugden (1998) for surveys). A major weakness of many rankings of opportunity sets examined in this literature is their insensitivity to the diversity of the options contained in opportunity sets. Even if the fact of being forced (by lack of available alternative) to drive a blue car to go to some destination can be considered freedom-wise equivalent to being forced to make the same trip by train, this does not imply that the possibility of getting to destination by driving either a blue or a red car offers the same freedom of choice as having the possibility of making the trip either by train or by a red car. Yet many rankings of opportunity sets examined in the literature fail to make the distinction.

While the present paper is primarily concerned with the issue of measuring diversity, it does provide some indication as to how the measurement of diversity may interfere with that of freedom of choice. Specifically, the object of this paper is to provide an axiomatic characterization of two rankings of sets on the sole basis of the diversity that they offer. As in Weitzman (1992),Weitzman (1993), Weitzman (1998) , Bossert et al. (2002) and, to some extent, Nehring and Puppe (2002), the two rankings characterized in this paper are based on an *a priori* notion of "proximity", or "dissimilarity", between the objects that is taken as given. However, the notion of similarity on which our axioms are based requires less information than what is necessary to define a cardinally meaningful numerical distance function such as that used in these contributions. Rather, the primitive notion of similarity on which we base our axiomatic construction is *ordinal*. That is, it requires the ability to perform statements like "the biological distance between a chimpanzee and a bee is larger than the biological distance between a rainbow trout and a kokanee salmon" but does *not* suppose the capacity of quantifying further these statements. In particular, it does not require the ability to make statements like "the biological distance between a chimpanzee and a bee is twice the distance between a rainbow trout and a kokanee salmon" that are required by a cardinal notion and which, in our view, exceed the current discriminating power of humans, even in disciplines as developed and sophisticated as biology.

To the best of our knowledge, Pattanaik and Xu (2000a) contribution, also discussed in Bossert et al. (2002), is the only one that examines a diversity-based ranking of sets of objects that refers explicitly to an *a priori* ordinal notion of similarity. However, the ordinal notion of similarity assumed in these papers is rather crude. For it only allows objects to be either pairwise dissimilar or pairwise similar. No intermediate categories of similarities are allowed. With this "zero-one" notion of similarity, Pattanaik and Xu (2000a) characterizes a ranking of sets based on the number of elements contained in the smallest (with respect to the number of elements) partition of the sets in subsets of similar objects. According to their ranking, set A offers at least as much diversity as set B if, and only if, the smallest partition of A into subsets of similar objects contains at least as many elements as the corresponding partition in B. While very interesting as a first step in the process of building a diversity ranking of sets based on an ordinal notion of similarity, this result suffers obviously from the paucity of the information conveyed by the "zero-one" notion of similarity used.

In this paper we characterize axiomatically two diversity rankings of sets based on an ordinal primitive notion of similarity that is not assumed to be "zero-one". Rather, the primitive notion of similarity used in this paper is an abstract quadernary relation (or a binary relation on the set of all pairs of objects) that is only restricted to be reflexive, transitive and complete as well as to satisfy a weak form of symmetry. Using this notion, the first ranking that we characterize is the *maxi-max* criterion that compares sets on the basis of the dissimilarity of their two most dissimilar objects. The other ranking characterized in this paper is the *lexi-max* criterion that is defined, as usual, as the lexicographic extension of the *maxi-max* one. It therefore compares any two sets by first comparing their two most dissimilar objects in terms of dissimilarity and, in case of a tie between these two most dissimilar objects, by looking at their two second most dissimilar objects and, if a tie is also obtained there, at their third most dissimilar pair and so on.

While these two rankings, and especially the second one, are of intrinsic interest for the problem at hand, we believe that the general methodology employed for obtaining a consistent method for assessing diversity on the basis of a primitive ordinal notion of similarity is more important than the rankings themselves. We further illustrate this by showing how our approach to diversity measurement can shed light on some aspects of the problem of ranking opportunity sets on the basis of their freedom of choice. For this sake, we adopt Pattanaik and Xu (1998) framework in which the freedom of choice offered by alternative opportunity sets is appraised by referring to a *set* of possible preference orderings that a reasonable person can have (see also Foster (1993), Nehring and Puppe (1999) or Puppe (1998) for other use and/or interpretation of this *multiple preferences* approach to freedom of choice). In this framework, the options of an opportunity set that are maximal with respect to some of the possible preference orderings is typically considered to be a sufficient information for appraising the freedom of choice offered by that opportunity set. Following this tradition, we provide in this paper an axiomatic characterization of a ranking of opportunity sets that compares their sets of maximal options with respect to some of the possible preference orderings by means of the maxi-max criterion alluded to above.

The rest of this paper is organized as follows. The next section presents the notation and the formal definitions of the axioms and the rankings characterized for the purpose of diversity measurement. Section 3 presents and briefly discusses the main characterization results that concern diversity measurement. Section 4 explores some of the connections between diversity and freedom of choice measurement and section 5 concludes.

2 Notations and definitions[1]

Let X be a finite set of options (living individuals, type of means of transportations, opinions expressed in newspapers, etc.) and $\mathfrak{P}(X)$ be the set of all non empty subsets of X with generic elements A, B,....

At the basis of our approach is a *quadernary* relation Q on X (alternatively, a binary relation on $X \times X$) (with asymmetric and symmetric factors Q_A and Q_S respectively) which reflects an *ordinal a priori* knowledge that one can have about the dissimilarity that exists between options. In this light, the statement $(w,\ z)\ Q\ (x,\ y)$ is interpreted as meaning "w is at least as dissimilar from z than x is from y". To motivate this interpretation, we assume throughout that, for every distinct objects x and $y \in X$, both $(x,\ y)$ Q (x, x) and (x, x) Q_S (y, y) hold (that is, two distinct objects are always

[1]The possibly non-standard notation of this paper is as follows. Given any set A, we denote by $|A|$ its cardinality. By a binary relation B on a set Ω, it is meant a subset of $\Omega \times \Omega$. Following common use we write $x\ B\ y$ instead of $(x, y) \in\ B$. For a binary relation B, its *asymmetric* factor B_A is defined by $x\ B_A\ y \Longleftrightarrow x\ B\ y \wedge \neg(y\ B\ x)$ and its *symmetric* factor B_S by $x\ B_S\ y \Longleftrightarrow (x\ B\ y) \wedge (y\ B\ x)$. A binary relation B on Ω is *reflexive* if $x\ B\ x$ for all $x \in \Omega$, is *complete* if $(x\ B\ y)$ or $(y\ B\ x)$ holds for every x and $y \in \Omega$, is *transitive* if $x\ B\ z$ follows from $x\ B\ y$ and $y\ B\ z$ for any x, y and z. A reflexive, complete and transitive binary relation is called an *ordering*.

weakly more dissimilar than any of the two objects in isolation, and pairs of identical objects are just equally similar (or dissimilar)). These two properties would clearly hold true if, as Weitzman (1992) or Bossert et al. (2002), we would accept to go as far as measuring the dissimilarity by a (cardinally measurable) distance function $d : X \times X \to \mathbb{R}$. We assume also that Q is *symmetric* in the sense that $(x, y)\ Q\ (y, x)$ holds for every objects x and y and, as a binary relation on $X \times X$, is *complete* and *transitive*. The reader can verify that these properties would also be satisfied by the ranking of pairs of objects induced by a conventional distance function d (in particular d is conventionally assumed to be symmetric). In order to simplify some of the proofs and formal statements, we further assume that Q is such that $(x,\ y)\ Q_A\ (x, x)$ for every two distinct x and y (two distinct options are always strictly more dissimilar than one of the two options and itself). Although there exists distance functions that violates this property, we believe it to be fairly natural in the current context. After all, if two objects x and y are considered to be distinct for the sake of the analysis performed, they should be considered to have some degree of "dissimilarity".

We let $\mathfrak{Q}$ denote the set of all ordinal diversity quadernary relations that satisfy all these properties. We record the obvious following fact (whose proof is omitted).

Fact 1 *If Q is a dissimilar quadernary relation in $\mathfrak{Q}$, then, for all distinct x and $y \in X$, and for all $z \in X$, $(x,y)\ Q_A\ (z,z)$*

Given a diversity quadernary relation Q and a set $P \subseteq X \times X$ of pairs of objects of X, we denote by $O^Q(P)$ the arrangement of the pairs in P in *decreasing order* of dissimilarity. That is $O^Q(P) = \{a_{(1)}, ..., a_{(|P|)}\}$ where, for every $i = 1, ..., |P| - 1$, $a_{(i)} \in P$, $a_{(i+1)} \in P$ and $a_{(i)}\ Q\ a_{(i+1)}$. As Q is symmetric, there is some arbitrariness in numbering the elements of $O^Q(P)$ as the order of appearance of any two symmetric pairs (x, y) and (y, x) is irrelevant.

Let now $\succeq$ (with asymmetric and symmetric factors $\succ$ and $\sim$ respectively) be a *transitive* binary relation on $\mathfrak{P}(X)$ that aims at reflecting the evaluation of the diversity offered by alternatives sets of objects in $\mathfrak{P}(X)$. We interpret the statement $A \succeq B$ as meaning "the set A offers at least as much diversity as the set B".

We wish to propose plausible properties (axioms) that $\succeq$ could satisfy in order to serve as a sensible method for appraising diversity, taking as given the ordinal notion of dissimilarity embodied in Q. In order to formulate these, we define as follows, for every set $A \in \mathfrak{P}(X)$, and any pair of objects x, $y \in X$, the sets $WD_A(x, y)$ and $D_A(x, y)$ of pairs of elements of A that are, respectively, weakly more diverse and strictly more diverse than x and y.

Definition 1 $WD_A(x, y) = \{(a, a') \in A \times A : \quad (a, a')\ Q\ (x, y)\}$

Definition 2 $D_A(x,y) = \{(a,a') \in A \times A: \ (a,a')\ Q_A\ (x,y)\}$

We now present the axioms used in the characterizations by what seems, in our view, to be their decreasing order of intuitive plausibility.

Axiom 1 $\forall\ w,x,\ y,\ z\ \in X,\ \ (w,\ z)\ Q\ (x,\ y) \Longleftrightarrow \{w,\ z\} \succeq \{x,\ y\}.$

Axiom 2 $\forall\ A,\ B \in P(X),$ *if* $A \supseteq B$*, then* $A \succeq B$*.*

Axiom 3 $\forall\ A,\ B, C$ *and* $D \in \mathfrak{P}(X)$ *such that* $B \cap C = B \cap D = C \cap D = \emptyset$, $(A \succeq B \cup C,\ A \succeq B \cup D$ *and* $A \succeq C \cup D) \Longrightarrow A \succeq B \cup C \cup D$ *and* $(A \succ (B \cup C)$, $A \succ (B \cup D)$ *and* $A \succ (C \cup D)) \Longrightarrow A \succ (B \cup C \cup D)$.

Axiom 4 $\forall\ A,\ B \in \mathfrak{P}(X)$ *such that* $|A| = |B|\ a_{(i)}\ Q\ b_{(i)}\ \forall\ i$ *with* $a(i) \in O^Q(A \times A)$ *and* $b(i) \in O^Q(B \times B) \Longrightarrow A \succeq B$.

Axiom 5 $\forall\ w,\ x,\ y,\ z \in X$*, if* $\{w,z\} \succsim \{x,y\}$ *and* $A,\ B\ \in \mathfrak{P}(X)$ *are such that* $\left|WD_{C \cup \{w,z\}}(w,z)\right| = \left|WD_{D \cup \{x,y\}}(x,y)\right|$ *and* $a_{(i)}\ Q\ b_{(i)}$ *for* $a_{(i)} \in O^Q(WD_{A \cup \{w,z\}}(w,z))$, $b(i) \in O^Q(WD_{B \cup \{x,y\}}(x,y))$ *and* $i = 1, ...,$ $\left|WD_{A \cup \{w,z\}}(w,z)\right|$*, then* $A \cup \{w,z\} \succeq B \cup \{x,y\}$.

Axiom 6 $\forall\ w,\ x,\ y,\ z \in X$*, if* $\{w,z\} \succ \{x,y\}$ *and* $A,\ B\ \in \mathfrak{P}(X)$ *are such that* $\left|D_{A\ \cup \{w,z\}}(w,z)\right| = \left|D_{B \cup \{x,y\}}(x,y)\right|$ *and* $a_{(i)}\ Q_A\ b_{(i)}$ *for* $a_{(i)} \in O^Q(D_{A \cup \{w,z\}}(w,z))$, $b(i) \in O^Q(D_{B \cup \{x,y\}}(x,y))$ *and* $i \in 1, ...,$ $\left|D_{A \cup \{w,z\}}(w,z)\right|$ *then* $A \cup \{w,z\} \succ B \cup \{x,y\}$.

Axiom 7 $\forall\ w,\ x,\ y,\ z \in X$*, if* $\{w,z\} \succ \{x,y\}$ *and* $A,\ B\ \in \mathfrak{P}(X)$ *are such that* $\left|WD_{C \cup \{w,z\}}(w,z)\right| = \left|WD_{D \cup \{x,y\}}(x,y)\right|$ *and* $a_{(i)}\ Q\ b_{(i)}$ *for* $a_{(i)} \in O^Q(WD_{A \cup \{w,z\}}(w,z))$, $b(i) \in O^Q(WD_{B \cup \{x,y\}}(x,y))$ *and* $i \in 1, ...,$ $\left|WD_{A \cup \{w,z\}}(w,z)\right|$ *then* $A \cup \{w,z\} \succ B \cup \{x,y\}$.

Axiom 1 just says that the ranking of sets made of two elements in terms of diversity must *coincide* with the ranking of the pairs in terms of dissimilarity as *per* the quadernary relation Q. It is difficult to imagine a diversity-ranking of sets based on an *a priori* notion of dissimilarity between options that would violate this axiom. Notice carefully that the formal statement of axiom 1 does *not* require the options w, x, y and z to be distinct.[2] Hence, when employed with a quadernary relation belonging to $\mathfrak{Q}$, axiom 1 implies the widely discussed (at least in the freedom of choice literature) Pattanaik and Xu. (1990)'s axiom of *indifference to no choice situation* saying that singleton sets should be considered equivalent. It also implies that any pair made of two distinct elements is strictly more diverse than any singleton.

[2] Hence, axiom 1 is formulated with a slight abuse of notation since, for $x = y$, the set $\{x,y\}$ is, in fact, the singleton $\{x\}$ (or $\{y\}$).

Axiom 2 is also well-known in the freedom of choice literature and is very natural in that context. It seems also plausible in the context of diversity measurement although perhaps not as much. At first sight, it seems indeed difficult to imagine a plausible conception of diversity that would consider that *adding* an object to a set could *reduce* its diversity. After all, if the added object is considered different, as an object, from those already contained in the set, how could its addition reduce the diversity of the world ? Yet the weighted enthropy indices used by biologists violate this axiom and, at second sight, one can see how a plausible "relativist" conception of diversity could, in some circumstances, contradict the partial ranking of sets provided by inclusion. Suppose a world in which the population of living individuals is equally split between two species 1 and 2. Consider now adding to this population a large number of living individuals of species 1 in such a way that the *ratio* of individuals from species 2 over those of species 1 becomes negligible. A "relativist" conception of biological diversity according to which diversity is maximized when all living individuals are equally split-up among the different categories could plausibly consider such a change as a reduction in diversity. To that extent, the rankings characterized in this paper are not relativist as they both satisfy axiom 2.

Axiom 3 is, perhaps, more disputable than the two preceding ones but is not unreasonable. Consider two sets A and B such that A is considered more diverse (weakly or strictly) than B. Consider then two processes of adding a bunch of options to B. One process consists in adding to B options collected into a set C while the other process consists in adding to B options gathered into some other set D (disjoint from C). Assume that the enlargement of diversity offered by B as a result of either of these two process is insufficient to reverse the relative ranking of the enlarged set with respect to A. In a situation like this, the axiom requires that, if A offers weakly or strictly more diversity than that provided by all options added to B by the two processes, then A should also be considered (weakly or strictly) more diverse than B enlarged by all objects in the two sets C and D. To give a somewhat more intuitive example, assume that A consists in all currently living creatures categorized as mammals, B contains all cartilaginous fishes, C contains all osseous fishes and D consists in all batracians. Axiom 3 would then require that if the diversity offered by mammals is larger than that offered by all fishes (cartilaginous and osseous), is larger than that offered by cartilaginous fishes and batracians and is also larger than that offered by all batracians and osseous fish, then the set of all mammals should also be considered more diverse than the set of all fishes and batracians.

Axiom 4 connects, for sets that contain the same number of options, the judgements made with respect to the dissimilarity of their options and those with respect to their relative standing in terms of diversity. As sets with the same number of options have the same number of pairs, they also have the same number of *positions* occupied by the pairs in terms of dissimilarity.

Suppose that a set A dominates a set B in the sense that, for any position occupied by a pair in the scale of dissimilarity, the pair that occupies that position in A is weakly more dissimilar than the corresponding pair in B. Axiom 4 would then require set A to offer weakly more diversity than B.

Axioms 5 to 7 are slight variant of the same principle and are probably the more difficult to accept (and to understand). They say that if a pair of objects is considered more diverse (strictly for axioms 6 and 7 or weakly for axiom 5) than another, then the ranking of the pair in terms of diversity should be robust to certain form of addition of options to both sets. The restriction imposed on the addition of options is that all the new pairs of options that they create that are (weakly for axioms 5 and 7 or strictly for axiom 6) more dissimilar, in each set, than the initial pair should be in same number in both sets and should be related, pair by pair, by a dominance relation with respect to dissimilarity. Here again, an example may help. Assume that the diversity ranking of sets considers the pair $\{$*bee*,*chimpanzee*$\}$ to be strictly more diverse than the pair $\{$*kokanee salmon*, *rainbow trout*$\}$. Suppose we add a fly to the first set and a *brown trout* to the second set and assume that the following (plausible) dissimilarity statements hold with respect to the living individuals:

1) a chimpanzee is weakly more similar to a bee than to a fly,

2) a bee is weakly more similar to a fly than to a chimpanzee,

3) a kokanee salmon is closer to a rainbow trout than to a brown trout,

4) brown and rainbow trouts are more similar than a rainbow trout and a kokanee salmon,

5) kokanee salmon and brown trout are more similar than chimpanzee and fly.

We note that, in this example, the addition of the fly to the first set and of the brown trout to the second set creates only, in each set, one pair of objects that are (strictly in this example) more dissimilar than the original pairs. These pairs are (*chimpanzee*, fly) for the first set and (*brown trout*, *kokanee salmon*) for the second. We note also that the first of these pairs is strictly more diverse than the second one. Axioms 6 and 7 would then require in this example the set $\{$*bee*, *chimpanzee*, $fly\}$to be strictly more diverse than the set $\{$*brown trout*, *kokanee salmon*, *rainbow trout*$\}$ while axiom 5 would only require $\{$*bee*, *chimpanzee*, $fly\}$to be weakly more diverse than $\{$*brown trout*, *kokanee salmon*, *rainbow trout*$\}$.

It should be noticed that, despite their similar looking aspect, axioms 5 to 7 are pairwise independent. Only axiom 7 plays a crucial role in the characterization of the *lexi-max* criterion. Axioms 5 and 6 are only brought in to illustrate, in an alternative characterization of the *maxi-max* criterion presented below, the formal differences that distinguish the *maxi-max* and the *lexi-max* criteria as methods for comparing sets on the basis of their diversity.

We now turn formally to the definitions of these rankings. The first of these rankings is the *maxi-max* criterion that ranks sets according to the relative dissimilarities of their most dissimilar pairs. This ranking $\succeq_{\max}$is defined as follows.

Definition 3 *For all A, $B \in \mathfrak{P}(X)$, $A \succeq_{\max} B \Longleftrightarrow a_{(1)}\ Q\ b_{(1)}$ for $a_{(1)} \in O^Q(A \times A)$ and $b_{(1)} \in O^Q(B \times B)$.*

To illustrate, suppose that X is the set of all means of transportation available to perform a certain trip between two cities defined specifically as $X = \{train,\ car, bike,\ foot\}$. Assume also that the ordinal notion of dissimilarities between these means of transportations is given by the quadernary relation Q defined by $(train, foot)\ Q_A\ (car, foot)\ Q_A\ (train, bike)\ Q_A\ (car, bike)$ $Q_A\ (train,\ car)\ Q_A\ (bike, foot)$. Then the maxi-max criterion would consider that the set $\{train,\ foot\}$ offers just as much diversity as the set $\{train,$ $car, bike,\ foot\}$, a judgement which may sounds at odd with one's intuition of what is diversity. Its biggest weakness is obviously to focus only on the two most dissimilar objects in the sets and to ignore completely the contribution to diversity made by the presence of less dissimilar objects.

The *lexi-max criterion* $\succsim_{lex}$ that will now be defined avoids to some extent this weakness. The formal statement of the definition exploits the obvious fact that, for every set A, $|A \times A| = |A|^2$ as well as the property stated in fact 1.

Definition 4 *For all A, $B \in \mathfrak{P}(X)$, $A \succ_{lex} B \Longleftrightarrow \exists\ k \in \{1, ..., |A|\,(|A| - 1)\}$ such that $a_{(k)}\ Q_A\ b_{(k)}$ and $a_{(i)}\ Q_S\ b_{(i)}$ for $i = 1, ...k - 1$ and $A \sim_{lex} B \Longleftrightarrow |A| = |B|$ and $a_{(i)}\ Q_S\ b_{(i)}$ for $i = 1, ..., |B|^2$ where, for every j, $a_{(j)} \in O^Q(A \times A)$ and $b_{(j)} \in O^Q(B \times B)$*

Albeit this ranking expresses some sensitivity with respect to the contributions of options that are not maximally dissimilar to diversity (for instance by considering that the set $\{train,\ car, bike,\ foot\}$ is strictly more diverse than the pair $\{train, foot\}$), this sensitivity is not as great as one would like. For it nonetheless gives a "veto power" to the most dissimilar two objects in the sets with respect to the appraisal of their diversity. In the example above, the set $\{train, foot\}$ would be considered strictly more diverse than the sets $\{train,\ car,\ bike\}$ and the sets $\{car, bike, foot\}$ as per the criterion $\succsim_{lex}$ even though this judgement may hurt the intuition of someone who nonetheless accepts the notion of dissimilarity Q assumed in this example.

It would therefore be nice to have a diversity ranking that enables more trade-off between the contributions of alternative pairs of options to diversity than what is allowed by the two diversity orderings characterized in this paper.

We now turn to the characterization.

3 Characterization results for diversity measurement

We first provide the characterization of $\succeq_{\max}$by means of axioms 1-3.

Theorem 1 *Let $\succeq$ be transitive binary relation defined on $\mathfrak{P}(X)$ and let Q be an ordinal notion of similarity belonging to $\mathfrak{Q}$. Then $\succeq$ satisfies Axioms 1 to 3 if and only if $\succeq \;=\; \succeq_{\max}$*

Proof. Necessity. It is immediate to see that the transitive binary relation $\succeq_{\max}$satisfies axioms 1 and 2. As for axiom 3, suppose that $A \succeq_{\max} B \cup C = E$, $A \succeq_{\max} B \cup D = F$ and $A \succeq_{\max} C \cup D = G$ and let $H = B \cup C \cup D$. Then $a_{(1)}\ Q\ e_{(1)}$, $a_{(1)}\ Q\ f_{(1)}$ and $a_{(1)}\ Q\ g_{(1)}$ for $a_{(1)}$, $e_{(1)}$, $f_{(1)}$ and $g_{(1)}$ denoting, respectively, the first element of the sets $O^Q(A \times A)$, $O^Q(E \times E)$, $O^Q(F \times F)$ and $O^Q(G \times G)$. We therefore have $a_{(1)}\ Q\ \max_Q(e_{(1)}, f_{(1)}, g_{(1)}) = \max_Q H \times H$ and, therefore, $A \succeq_{\max} H$.

Sufficiency. We first show that if $\succeq$ is transitive and satisfies axioms 1 to 3, then we have, for every A and $B \in \mathfrak{P}(X)$, $A \succeq B \Longrightarrow A \succeq_{\max} B$. Suppose $A \succeq B$ and let $a_{(1)} = (a_1, a_2)$ and $b_{(1)} = (b_1, b_2)$ denote the most dissimilar pairs of objects in A and B respectively. By axiom 2, we have $B \succeq \{b_1, b_2\}$ and by transitivity, $A \succeq \{b_1, b_2\}$. In the trivial case where $|B| = |A| = 1$, we can write that $A = \{x\}$ and $B = \{y\}$ for some options x and y so that $a_{(1)} = (x, x)$ and $b_{(1)} = (y, y)$. Since $(x, x)\ Q_S\ (y, y)$ for every x, $y \in X$, we therefore have $a_{(1)}\ Q\ b_{(1)}$ and $A \succeq_{\max} B$ in this case. We can rule out the case where $|A| = 1$ and $|B| \geq 2$ which would imply that $\{x\} \succeq \{b_1, b_2\}$ for some distinct b_1 and $b_2 \in B$, in contradiction with axiom 1 and fact 1. Assume now that $|A| = 2$ and, therefore, that $A = \{a_1, a_2\}$. Then $\{a_1, a_2\} \succeq \{b_1, b_2\}$ and by axiom 1, $a_{(1)}\ Q\ b_{(1)}$, which implies $A \succeq_{\max} B$. As the last case, assume that $|A| > 2$, write $A = \{a_1, ..., a_{|A|}\}$ and assume by contradiction that $a_{(1)}\ Q\ b_{(1)}$ is false. Since Q is complete, this amounts to assuming that $b_{(1)}\ Q_A\ a_{(1)}$ and, therefore, that $b_{(1)}\ Q_A\ (a_i, a_j)$ for all i, $j \in \{1, ..., |A|\}$. Pick any option a_1 in A. One has by axiom 1 that $\{b_1, b_2\} \succ \{a_1, a_i\}$, $\{b_1, b_2\} \succ \{a_1, a_j\}$ and $\{b_1, b_2\} \succ \{a_i, a_j\}$ for all i, $j \in \{1, ..., |A|\}$. By axiom 3, we must have $\{b_1, b_2\} \succ \{a_1, a_i, a_j\}$. Redoing the same procedure while replacing the option a_j by some option $a_h \in A$, one obtains that $\{b_1, b_2\} \succ \{a_1, a_i, a_h\}$. Using axiom 3 again and the fact that $\{b_1, b_2\} \succ \{a_j, a_h\}$, one is led to the conclusion that $\{b_1, b_2\} \succ \{a_1, a_h, a_i, a_j\}$. Redoing the last procedure if necessary while replacing a_h by $a_g \in A$, one can analogously obtain the statement $\{b_1, b_2\} \succ \{a_1, a_g, a_i, a_j\}$ and combining the last two statements and the fact that $\{b_1, b_2\} \succ \{a_g, a_h\}$, one obtains again by axiom 3 that $\{b_1, b_2\} \succ \{a_1, a_g, a_h, a_i, a_j\}$. This procedure can clearly be repeated with as many options in A as needed to finally obtain, using transitivity of $\succeq$ and axiom 2, the required contradictory conclusion that $B \succeq \{b_1, b_2\} \succ A$.

We now show that for every sets A and B in $\mathfrak{P}(X)$, $A \succeq_{\max} B$ implies $A \succeq B$ for every transitive binary relation $\succeq$ on $\mathfrak{P}(X)$ satisfying axioms 1 to 3. Suppose $A \succeq_{\max} B$. Then $a_{(1)}\ Q\ b_{(1)}$ where again $a_{(1)} = (a_1, a_2)$ and $b_{(1)} = (b_1, b_2)$ denote the most dissimilar pairs of objects in A and B respectively. Let $|B| = m$ and write $B = \{b_1, b_2, ..., b_m\}$. For the same reason as above, we can rule out from the start the case $m = 1$. If $m = 2$, then, by axiom 1, $\{a_1, a_2\} \succeq \{b_1, b_2\}$ and, by axiom 2, $A \succeq \{a_{(1)}\}$, so that, by transitivity, $A \succeq B$. For the other cases, we show the result by induction. For that purpose, we start with the case $m = 3$ and we write $B = \{b_1, b_2, b_3\}$. Because $a_{(1)}\ Q\ b_{(1)}$, we have $a_{(1)}\ Q\ (b_1, b_2)$, $a_{(1)}\ Q\ (b_1, b_3)$ and $a_{(1)}\ Q\ (b_2, b_3)$. Using axiom 1, we can write $\{a_1, a_2\} \succeq \{b_1\} \cup \{b_2\}$, $\{a_1, a_2\} \succeq \{b_1\} \cup \{b_3\}$ and $\{a_1, a_2\} \succeq \{b_2\} \cup \{b_3\}$. By axiom 3, it follows that $\{a_1, a_2\} \succeq \{b_1\} \cup \{b_2\} \cup \{b_3\} = B$ and, by axiom 2 and transitivity, that $A \succeq B$. The case $m = 3$ is then proved. Now suppose the result is true for any $m \in \{3, ..., |X| - 1\}$. That is, suppose that if A is a set in $\mathfrak{P}(X)$ and B is another set in $\mathfrak{P}(X)$ such that $|B| = m$, then $A \succeq_{\max} B \Longrightarrow A \succeq B$ and suppose $A \succeq_{\max} B'$ where $B' = B \cup \{b_{m+1}\}$ for some $b_{m+1} \in X \backslash B$. We wish to show that $A \succeq B'$ Let $b'_{(1)}$ denote the pair of two most dissimilar objects in B' and write $\overline{B} = \{b_1, b_2, ..., b_{m-1}\}$, $C = \{b_m\}$ and $D = \{b_{m+1}\}$. As $\succeq_{\max}$ is transitive and satisfies axiom 2, we have that $A \succeq_{\max} \overline{B} \cup C$ and, since $|\overline{B} \cup C| = m$, we have by the induction hypothesis that $A \succeq \overline{B} \cup C$. Because $a_{(1)}\ Q\ b'_{(1)}$, we have, by the transitivity of the quadernary relation Q, $a_{(1)}\ Q\ (b_m, b_{m+1})$ and, by virtue of axioms 1 and 2 and the transitivity of $\succeq$, $A \succeq C \cup D$. Finally, let $B'' = \overline{B} \cup D$. Then $b'_{(1)}\ Q\ b''_{(i)}$ and, by transitivity of Q, $a_{(1)}\ Q\ b''_{(i)}$ for all $b''_{(i)} \in O^Q(B'' \times B'')$. We therefore have $A \succeq_{\max} B''$. Yet $|B''| = m$ so that, by the induction hypothesis, we have $A \succeq B''$. By axiom 3, we have $A \succeq \overline{B} \cup C$, $A \succeq \overline{B} \cup D$ and $A \succeq C \cup D$, so that $A \succeq \overline{B} \cup C \cup D$, and this concludes the proof. ■

We first remark that, albeit this was not required, we obtain the completeness of the ranking as a by-product of the axioms 1 to 3. It is also worth noticing that this first characterization of $\succeq_{\max}$is obtained from the (reasonably) intuitive axioms 1 to 3 that only uses properties of sets. Only axiom 1 makes the connection between the ranking of pairs in terms of dissimilarity and the ranking of sets. Unfortunately the characterization of the more interesting $\succeq_{lex}$criterion is not as "nice" since it is obtained from the axioms 1,4 and 7, all of which are explicitly formulated in terms of the relationship between the diversity ranking of sets and the dissimilarity rankings of the pairs that these sets contain.

Before turning to this characterization, we show that axioms 1, 2 and 3 used to characterize $\succeq_{\max}$are independent.

Proposition 1 *For any $Q \in \mathfrak{Q}$, Axioms 1 to 3 are independent.*

Proof. Let $\succeq_*$be defined by: $A \succeq_* B \iff b_{(|B|(|B|-1))}\ Q\ a_{(|A|(|A|-1))}$ where $b_{(|B|(|B|-1))} \in O^Q(B\times B)$ and $a_{(|A|(|A|-1))} \in O^Q(A\times A)$.This transitive and complete binary relation on $\mathfrak{P}(X)$ considers that set A offers at least as much diversity as B if and only if the two most *similar distinct* objects in B are weakly more *dissimilar* than the two most similar distinct objects in A. It is certainly a peculiar criterion for comparing sets on the basis of their diversity. It is immediate to see that $\succeq_*$violates axiom 1. To see that it satisfies axiom 2, consider A and B in $\mathfrak{P}(X)$ such that $A \supseteq B$. As the two most similar objects in B are also in A, we must have the two most similar objects in A are weakly more similar than the two most similar objects in B. Hence the two most similar objects in B are weakly more dissimilar than the two most similar objects in A and, for this reason, one has $A \succeq_* B$. To see that $\succeq_*$satisfies axiom 3, assume that $A \succeq_* B\cup C$, $A \succeq_* B\cup D$ and $A \succeq_* C\cup D$. Write $E = B\cup C$, $F = B\cup D$ and $G = C\cup D$. One has by definition of $\succeq_*$that $e_{(|B\cup C|(|B\cup C|-1))}\ Q\ a_{(|A|(|A|-1))}$, $f_{(|B\cup D|(|B\cup D|-1))}\ Q\ a_{(|A|(|A|-1))}$ and $g_{(|C\cup D|(|C\cup D|-1))}\ Q\ a_{(|A|(|A|-1))}$ where $e_{(|B\cup C|(|B\cup C|-1))} \in O^Q(E\times E)$, $f_{(|B\cup D|(|B\cup D|-1))} \in O^Q(F\times F)$ and $g_{(|C\cup D|(|C\cup D|-1))} \in O^Q(G\times G)$. Let now $H = B\cup C\cup D$ and consider $h_{(|B\cup C\cup D|(|B\cup C\cup D|-1))} \in O^Q(H\times H)$. Clearly, since either

$$\begin{aligned} h_{(|B\cup C\cup D|(|B\cup C\cup D|-1))} &= e_{(|B\cup C|(|B\cup C|-1))}, \\ h_{(|B\cup C\cup D|(|B\cup C\cup D|-1))} &= f_{(|B\cup D|(|B\cup D|-1))},\ or \\ h_{(|B\cup C\cup D|(|B\cup C\cup D|-1))} &= g_{(|C\cup D|(|C\cup D|-1))} \end{aligned}$$

one has

$$h_{(|B\cup C\cup D|(|B\cup C\cup D|-1))}\ Q\ a_{(|A|(|A|-1))}$$

and, therefore, $A \succeq_* B\cup C\cup D$. Now let $\succeq_D$ be defined by $A \succeq_D B \iff a_{(|A|(|A|-1))}\ Q\ b_{(1)}$ where $a_{(|A|(|A|-1))} \in O^Q(A\times A)$ and $b_{(1)} \in O^Q(A\times A)$. This rule says that set A offers at least as much diversity as B if and only if the two most similar *distinct* objects in A are at least as dissimilar as the two most dissimilar distinct objects in B. It is immediate to see that $\succeq_D$ satisfies axioms 1 and is transitive. To see that it verifies axiom 3, assume that $A \succeq_D (B\cup C) = E$, $A \succeq_D (B\cup D) = F$ and $A \succeq_D C\cup D = G$. Then $a_{(|A|(|A|-1))}\ Q\ e_{(1)}$, $a_{(|A|(|A|-1))}\ Q\ \ f_{(1)}$ and $a_{(|A|(|A|-1))}\ Q\ \ g_{(1)}$ and, therefore, $a_{(|A|(|A|-1))}\ Q\ h_{(1)}$ where $H = B\cup C\cup D$ and $e_{(1)},\ \ f_{(1)}\ \ g_{(1)}$ and $h_{(1)}$ denote, respectively, the first elements of the sets $O^Q(E\times E)$, $O^Q(F\times F)$, $O^Q(G\times G)$ and $O^Q(H\times H)$. To see that $\succeq_D$ violates axiom 2, let $B = \{b_1,\ b_2\}$ and $A = \{b_1,\ b_2,\ b_3\}$ and assume that Q is such that $(b_1,\ b_2)\ Q_A\ (b_1,\ b_3)\ Q\ (b_2,\ b_3)$. Clearly, $b_{(1)} = (b_1,\ b_2)$, $b_{(6)} = (b_2, b_3)$ and, since $(b_2, b_3)\ Q\ (b_1,\ b_2)$ does not hold, $A \succeq_D B$ does not hold either. Finally, let $\succeq_{add}$ be defined by: $A \succeq_{add} B \iff \sum_{i=1}^{|A|^2} v(a_{(i)}) \geq \sum_{i=1}^{|B|^2} v(b_{(i)})$ for some function $v \in X \times X \to \mathbb{R}_+$ such that, for all (w,z), $(x,y) \in X \times X$, $v(w,z) \geq v(x,y) \Leftrightarrow (w,z)\ Q\ (x,y)$. The existence of such a (distance) real valued function does not pose any difficulty. The binary relation $\succeq_{add}$ is reflexive, transitive and complete and satisfies axioms 1 and 2. Yet, it may violates axiom 3 if, for instance, $X = \{w,x,y,z\}$ and v is such that $v(w,z) = 7$, $v(w,y) = 5$, $v(w,x) = 3 = v(x,y)$. In such a case, defining $A = \{w,z\}$, $B = \{w\}$, $C = \{y\}$ and $D = \{x\}$, one has $A \succeq_{add} B \cup C \Leftrightarrow 7 \geq 5$, $A \succeq_{add} B \cup D \Leftrightarrow 7 \geq 3$ and $A \succeq_{add} C \cup D \Leftrightarrow 7 \geq 3$. Yet $A \prec_{add} B \cup C \cup D$ as $v(w,z) = 7 < v(w,y) + v(w,x) + v(x,y) = 11$. ■

We now turn to the axiomatic characterization of $\succeq_{lex}$.

Theorem 2 *Let $\succeq$ be a transitive binary relation on $\mathfrak{P}(X)$ and let Q be an ordinal notion of similarity belonging to $\mathfrak{Q}$. Then $\succeq$ satisfies Axioms 1, 4 and 7 if and only if $\succeq\ =\ \succeq_{lex}$*

Proof. It is immediate to verify that $\succeq_{lex}$ is a reflexive, transitive and complete binary relation on $\mathfrak{P}(X)$ that satisfies axioms 1, 4 and 7. Assume now that $A \succ_{lex} B$. Then, by definition, there exists a $k \in \{1,...,(|A|(|A|-1)\}$ such that $a_{(k)}\ Q_A\ b_{(k)}$ and $a_{(i)}\ I\ b_{(i)}$ for $i = 1,...k-1$ where, for all $j = 1,...,k$ $a_{(j)} \in O^Q(A \times A)$ and $b_{(j)} \in O^Q(B \times B)$. Writing $a_{(j)} = (a_0^j, a_1^j)$ and $b_{(j)} = (b_0^j, b_1^j)$ for $a_i^j \in A$ and $b_i^j \in B$ for every $j \in \{1,...,k\}$, $i = 0,1$ we have by axiom 1 that $\{a_0^k, a_1^k\} \succ \{b_0^k, b_1^k\}$. Let $C = A \backslash \{a_0^k, a_1^k\}$ and $D = B \setminus \{b_0^k, b_1^k\}$. Sets C and D are clearly like sets A and B of axiom 7. In particular, one has $\left|WD_{\{a_0^k,a_1^k\}\cup C}(a_{(k)})\right| = \left|WD_{\{b_0^k,b_1^k\}\cup D}(b_{(k)})\right| = k$ and $a_{(i)}\ Q\ b_{(i)}\ for\ a_{(i)} \in WD_{\{a_0^k,a_1^k\}\cup C}(a_{(k)})$ and $b_{(i)} \in WD_{\{b_0^k,b_1^k\}\cup D}(b_{(k)})$. By axiom 7, we therefore have $A \succ B$. Suppose now $A \sim_{lex} B$. Then $a_{(i)}\ Q\ b_{(i)}\ \forall\ i$ and $b_{(i)}\ Q\ a_{(i)}\ \forall\ i$ and $|A| = |B|$ for $a_{(i)} \in O^Q(A \times A)$ and $b_{(i)} \in O^Q(B \times B)$ .By axiom 4, we then have $A \succeq B$ and $B \succeq A$ which entails $A \sim B$. Now, we have to show that if $\succeq$ is transitive and satisfies axioms 1, 4 and 7, then we have, for every A and $B \in P(X)$, $A \succeq B \Longrightarrow A \succeq_{lex} B$. Assume by contradiction that the implication is false. Then, since $\succeq_{lex}$is complete, this amounts to say that $B \succ_{lex} A$ holds which, by virtue of what we just established, implies that $B \succ A$, a contradiction. ■

Although a characterization of $\succeq_{lex}$ which would, like that of $\succeq_{\max}$provided by theorem 1, only use axioms that refer to *elements* of the sets rather than to *pairs of elements* would be more elegant, such an axiomatization is not easy to obtain. The *lexi-max* criterion is a ranking of sets for which the precise positions occupied by the pairs in the dissimilarity scale matters a great deal. Yet, it is difficult to axiomatically control these positions by using properties of elements in the set that could be retired or added in the spirit of axiom 3 for instance. The *maxi-max* criterion possesses the property that, when it compares sets with more than two elements, the ranking of the sets that it provides is *robust* to the deletion, in both sets, of specific elements one by one (clearly, taking away in two sets containing more than two elements an element that is not part of the most dissimilar pair does not affect the ranking of the two sets). The *lexi-max* criterion does *not* possess this robustness with respect to the deletion of its elements. Suppose for instance that $A = \{t, u, v\}$, $B = \{w, x, y, z\}$ with the dissimilarity ranking of the relevant pairs as follows:

$(t, u)\ Q_S\ (t, v)\ Q_S\ (w, x)\ Q_S\ (y, z)\ Q_A\ (u, v)\ Q_A\ (w, y)\ Q_S\ (x, z)\ Q_S\ (w, z)\ Q_S\ (x, y)$.

Here, $A \succ_{lex} B$ but there is no way to withdraw options from both A and B (or from A only) that would preserve the ranking. This difficulty of decomposing the properties of the ranking of sets $\succeq_{lex}$ into more basic properties involving only the sets and their elements - as in standard set theory - is, we believe, a significant one.

It is to a large extent our ability to escape from such a difficulty in the case of the $\succeq_{\max}$criterion, but not in the case of $\succeq_{lex}$, that explains the rather *different* axiomatic routes used to characterize, in theorem 1 and 2, the two *similar* looking rankings. It is however possible to provide an alternative - but admittedly uglier - characterization of $\succeq_{\max}$that emphasizes the similarity that exists between this ranking and $\succeq_{lex}$. This alternative characterization is provided in the next theorem.

Theorem 3 *Let $\succeq$ be a transitive binary relation defined on $\mathfrak{P}(X)$ and let Q be an ordinal notion of similarity belonging to $\mathfrak{Q}$. Then $\succeq$ satisfies Axioms 1, 5 and 6 if and only if $\succeq\ =\ \succeq_{\max}$.*

Proof. *We know that $\succeq_{\max}$ is transitive and satisfies axiom 1. It is immediate to see that it satisfies axioms 5 and 6 (notice that it does* not *satisfy axiom 7). Suppose that $A \succ_{\max} B$. Then $a_{(1)}\ Q_A\ b_{(1)}$ where $a_{(1)} = (a_1, a_2)$ and $b_{(1)} = (b_1, b_2)$ denote the most dissimilar pairs of objects in A and B respectively. By axiom 1, we have that $\{a_1, a_2\} \succ \{b_1, b_2\}$. Now, let $C = A \setminus \{a_1, a_2\}$ and $D = B \setminus \{b_1, b_2\}$. Trivially, C and D are just like the sets A and B in axiom 6 with, in particular $\left|D_{C\cup\{a_1,a_2\}}(a_1, a_2)\right| = \left|D_{D\cup\{b_1,b_2\}}(b_1, b_2)\right| = 0$. Hence we can apply axiom 6 trivially to obtain $A = C \cup \{a_1, a_2\} \succ D \cup \{b_1, b_2\} = B$. Suppose now $A \sim_{\max} B$. Then, using the same notation, we have $a_{(1)}\ Q_S\ b_{(1)}$ which entails by axiom 1 that $\{a_1, a_2\} \succeq \{b_1, b_2\}$ and also $\{b_1, b_2\} \succeq \{a_1, a_2\}$. Using the same (trivial) reasoning than for the preceding case but with axiom 5 instead, we obtain $A \succeq B$ as well as $B \succeq A$ and, therefore, $A \sim B$. We now wish to prove that if $A \succeq B$ for all transitive binary relations on $\mathfrak{P}(X)$ that satisfy axioms 1, 5 and 6, then $A \succeq_{\max} B$. Suppose not. Since $\succeq_{\max}$is complete, this amounts to assuming that $B \succ_{\max} A$. Yet by what has just been established above, this implies $B \succ A$ for all transitive binary relations on $\mathfrak{P}(X)$ that satisfy axioms 1, 5 and 6, a contradiction.* ■

We conclude this section by proving that axioms 1, 4 and 7 used in the characterization of $\succeq_{lex}$ are independent.

Proposition 2 *For any $Q \in \mathfrak{Q}$, axioms 1, 4 and 7 are independent.*

Proof. Consider first $\succeq_{add}$ as defined in the proof of proposition 1. It satisfies as we have seen axiom 1 and it is not hard to see that it also satisfies axiom 4. To see that it violates axiom 7, consider $X = \{w, x, y, z\}$ and assume that the function v which defines $\succeq_{add}$ is such that $v(w, z) = 7$, $v(w, y) = 5$, $v(w, x) = 3$, $2 = v(x, y) = v(x, z)$ and $v(y, z) = 0$. In such a case, we have that $\{w, y\} \succ \{w, x\}$ and the sets $A = \{z\} = B$ are just as in the antecedent of axiom 7 (with $WD_{\{w,y,z\}}(w, y) = \{(w, z), (w, y)\}$ and $WD_{\{w,x,z\}}(w, x) = \{(w, z), (w, x)\}$) (neglecting the duplication of symmetric pairs) . Yet $\{w, y, z\} \preceq \{w, x, z\}$ since $7+5+0 \leq 7+3+2$, which contradicts the requirement of axiom 7. Consider now the widely discussed Pattanaik and Xu. (1990) cardinality ordering $\succsim_{CARD}$defined by $A \succsim_{CARD} B \Leftrightarrow |A| \geq |B|$. This ordering obviously violates axiom 1 for any dissimilarity notion contained in $\mathfrak{Q}$. It satisfies however axiom 4 and, trivially, axiom 7 (whose antecedent never applies when there is universal indifference between all pairs of objects). Consider finally the incomplete transitive binary relation $\widehat{\succeq}_{lex}$ defined by $A \ \widehat{\succeq}_{lex} \ B \Longleftrightarrow A \succ_{lex} B$. As can be seen in the proof of theorem 2, this binary relation satisfies axioms 1 and 7. Yet it fails to satisfy axiom 4 (since $\widehat{\succeq}_{lex}$ considers as non comparable any two sets, such as those mentioned in axiom 4, that would be considered indifferent by $\succeq_{lex}$). ■

4 Diversity and freedom of choice

The diversity of options available for choice to a decision maker can arguably be seen as an essential element of the *freedom of choice* of this decision maker. Yet most rankings of opportunity sets examined in the freedom of choice literature mentioned in introduction have not exhibited a great sensitivity to diversity. In this section, we briefly explore the extent to which the methodology used in this paper can serve to bridge the gap between concerns for diversity and concerns for freedom.

There are roughly two approaches to the issue of defining and measuring freedom of choice in the literature. In the *first* approach, freedom of choice is conceived as an *intrinsic* criterion for comparing opportunity sets, roughly related to the "size" of the opportunity sets, and whose importance is, using the words of Sen (1988) (p. 290), "beyond that of providing the means of choosing the particular alternative that happens to be chosen". Hence, in this approach, the freedom of choice offered by a particular opportunity set is conceived as being completely independent from the preferences that the decision maker will use for choosing from that set. The widely discussed *cardinality rankings* of sets (characterized differently by Jones and Sugden (1982), Pattanaik and Xu. (1990) and Suppes (1987)) as well as their additive generalization (see e.g. Klemisch-Ahlert (1993) or Gravel et al. (1998)) belong clearly to this approach, as do the definition of freedom as enthropy in Suppes (1996) or the examination, made by VanHees (1997), of the distinction between negative and positive freedom. In the *second* approach, freedom of choice is defined with respect to a set of possible preferences that the decision maker could have when making its choice. In this approach, freedom of choice is important only in so far as it enables the decision maker to make better choice from the view point of some of the possible preference that he may use when making the choice. For this reason, when evaluating the freedom of choice offered by some opportunity set, this approach attaches a particular attention to the set of options which would be considered best in this set from the view point of some of the possible preference of the decision maker. We refer the reader to Arrow (1995), Barberà et al. (n.d.), Dutta and Sen (1996), Foster (1993), Jones and Sugden (1982), Nehring and Puppe (1999), Pattanaik and Xu (1998), Puppe (1998), Romero-Medina (2001) and Sugden (1998) for further justification of this *multi-preferences approach* to freedom of choice.

The methodology presented in this paper is directly relevant for this first approach if one accepts the view that the diversity of options contained in a particular opportunity set is a natural measure of the freedom of choice offered by that opportunity set. As there is, after all, some rationale for this view. After all someone who has only the choice between two slightly different cars for commuting from home to work can arguably be considered to have less freedom of choice - in terms of means of transportation - than an individual who can go to work either by one car or by a suburban train. Hence it is quite possible to interpret the *maxi-max* and the *lexi-max* criteria characterized in the preceding section as freedom of choice rankings rather than diversity ones. Of course the acceptability of the rankings, both from a diversity or a freedom of choice perspective, rides upon the acceptability of the underlying dissimilarity quadernary relation that is taken a given.

But diversity can also contribute to defining freedom of choice in the context of the multi-preference approach. To see how, adopt Pattanaik and Xu (1998) framework and let $\mathfrak{R} = \{R_1, ..., R_n\}$ be the set of all possible preference orderings over X that a "reasonable person" may have. For all $i = 1, ..., n$, let P_i and I_i denote, respectively, the asymmetric factor and the symmetric factor of R_i. In this setting, the binary relation $\succeq$ on $\mathfrak{P}(X)$ is explicitly interpreted in terms of freedom of choice rather than of diversity.

For all $A \in \mathfrak{P}(X)$, let $Max_{\mathfrak{R}}A = \{a \in A : \exists\ R_i \in \mathfrak{R}$ for which $a\ R_i\ a'$ $\forall\ a' \in A\}$ be the set of all options in A that are maximal for some of the possible preferences in $\mathfrak{R}$.

Taking as given the set $\mathfrak{R}$, Pattanaik and Xu (1998) characterizes the ranking of all sets in $\mathfrak{P}(X)$ defined by the comparison of the *cardinality* of their sets of elements which are maximal from the view point of at least one of the possible preferences in $\mathfrak{R}$. Formally, this ranking $\succeq^{\mathfrak{R}}_{card}$ is defined by $A \succeq^{\mathfrak{R}}_{card} B \iff |Max_{\mathfrak{R}}A| \geq |Max_{\mathfrak{R}}B|$. In this paper, taking as given *both* the set of possible preferences *and* the primitive notion of dissimilarity between options Q, we provide a characterization of the ranking $\succeq^{\mathfrak{R}}_{\max}$defined as follows.

Definition 5 $A \succeq^{\mathfrak{R}}_{\max} B \iff Max_{\mathfrak{R}} A \succeq_{\max} Max_{\mathfrak{R}} B$

Hence the ranking $\succeq^{\mathfrak{R}}_{\max}$ considers that opportunity set A offers at least as much freedom of choice as opportunity set B if and only if the set of elements in A that any reasonable person would choose is at least as diverse, in the sense of the ordering $\succeq_{\max}$of definition 3, than the set of options in B that any reasonable person would choose. This ranking provides therefore an alternative to the ranking $\succeq^{\mathfrak{R}}_{card}$of Pattanaik and Xu (1998) which attaches intrinsic importance to the diversity of the options that a reasonable person could choose.

The characterization that we provide of $\succeq^{\mathfrak{R}}_{\max}$uses the following axioms.

Axiom 8 $\forall\ A,\ B \in \mathfrak{P}(X),\ \forall\ x \in X,$ *if* $x \notin Max_{\mathfrak{R}} A \cup \{x\}$, *then* $[A \succeq B \iff A \cup \{x\} \succeq B]$ *and* $[B \succeq A \iff B \succeq A \cup \{x\}]$

Axiom 9 $\forall\ w, x, y, w, z \in X,$ *if and* $\{w, z\} = Max_{\mathfrak{R}}\{w, z\}$ *and* $\{x,\ y\} = Max_{\mathfrak{R}}\{x, y\}$ *then* $(w,\ s)\ Q\ (x,\ y) \iff \{w,\ z\} \succeq \{x,\ y\}$

Axiom 10 $\forall\ A,\ B \in Y$ *if* $B \subseteq Max_{\mathfrak{R}} A$, *then* $A \succeq B$

Axiom 8 has been introduced in Pattanaik and Xu (1998). It requires that if x is an option that no reasonable preference would consider strictly better than all options in a set A, then the ranking of A with respect to B should not be affected by the addition of x to A. Axioms 9 is a weakening of axioms 1. Like axiom 1, axiom 3 requires the ranking of sets that are made of two elements, each of which being a best choice over the other by some of the possible preferences, to coincide with the dissimilarity ranking of the pair made of these two elements as per the quardernary relation Q. On the other hand, and contrary to axiom 1, axiom 9 does *not* require the coincidence of dissimilarity comparisons and sets comparisons for pairs in which one element - say the fact of being beheaded at dawn - is considered worse than the other by all preferences in $\mathfrak{R} = R$. It should be noticed that, as for axiom 1, the formal statement of axiom 9 does not require the options to be distinct. Hence, since $Max_{\mathfrak{R}}\{x\} = \{x\}$ for all $x \in X$, axiom 9 implies also Pattanaik and Xu. (1990)'s axiom of *indifference to no-choice situations.* Axiom 10 is a weakening of axiom 2 which considers that a set A offers weakly more freedom of choice than any subset of the sets of options which, in A, could be considered best by some of the preferences in $\mathfrak{R}$.

These axioms, along with axiom 3, characterize the ordering $\succeq^{\mathfrak{R}}_{\max}$, as established in the following theorem.

Theorem 4 *Let $\succeq$ be a transitive binary relation on $\mathfrak{P}(X)$. Then $\succeq$ satisfies axioms 3, 8, 9 and 10 for a given dissimilarity quadernary relation $Q \in \mathfrak{Q}$ and a given set $\mathfrak{R}$ of possible preference orderings if and only if $\succeq\ =\ \succeq^{\mathfrak{R}}_{\max}$*

Proof. We leave to the reader the task of verifying that $\succeq^{\mathfrak{R}}_{\max}$ is transitive and satisfies axioms 3, 8, 9 and 10. We now establish that, for all transitive binary relations $\succeq$ on $\mathfrak{P}(X)$, $A \sim^{\mathfrak{R}}_{\max} B \Longrightarrow A \sim B$ and $A \succ^{\mathfrak{R}}_{\max} B \Longrightarrow A \succ B$. By a reasoning analogous to that conducted in the proof of Theorem 2, using the completeness of $\succeq^{\mathfrak{R}}_{\max}$, this suffices to prove the result. For every sets A and $B \in \mathfrak{P}(X)$, let us write $A = C \cup Max_{\mathfrak{R}}A$ and $B = D \cup Max_{\mathfrak{R}}B$ where $C = \{c_1, c_2, ...c_l\} = A\backslash\ Max_{\mathfrak{R}}A$ and $D = \{d_1, d_2, ..., d_m\} = B\backslash Max_{\mathfrak{R}}B$. We also write $Max_{\mathfrak{R}}A = \{a_1, a_2, ...a_g\}$ and $Max_{\mathfrak{R}}B = \{b_1, b_2, ..., b_h\}$. We recall that, as the elements of $\mathfrak{R}$ are orderings, neither $Max_{\mathfrak{R}}A$ nor $Max_{\mathfrak{R}}B$ is empty while either C or D can be empty. Assume $A \sim^{\mathfrak{R}}_{\max} B$ i. e. $Max_{\mathfrak{R}}A \sim_{\max} Max_{\mathfrak{R}}B$. Assume first that $|Max_{\mathfrak{R}}A| = |Max_{\mathfrak{R}}B| = 1$. By axiom 9, we must then have $Max_{\mathfrak{R}}A \sim Max_{\mathfrak{R}}B$. By axiom 8, $(Max_{\mathfrak{R}}A) \cup c_1 \sim Max_{\mathfrak{R}}A$. Using axiom 8 repeatedly with as many elements in C as necessary, we obtain $A \sim Max_{\mathfrak{R}}A$. Analogously, using axiom 8 with set B, we obtain $B \sim Max_{\mathfrak{R}}B$ and, by the transitivity of $\succeq$, $A \sim B$. For trivial reasons, we can not have $|Max_{\mathfrak{R}}A| > 1$ and $|Max_{\mathfrak{R}}B| = 1$ or $|Max_{\mathfrak{R}}A| = 1$ and $|Max_{\mathfrak{R}}B| > 1$ when $A \sim^{\mathfrak{R}}_{\max} B$ because of the fact that $(x, y)\ Q_A\ (z, z)$ for every x, y and z with x and y distinct. Consider therefore the last case, where $|Max_{\mathfrak{R}}A| > 1$ and $|Max_{\mathfrak{R}}B| > 1$. Since $Max_{\mathfrak{R}}A \sim_{\max} Max_{\mathfrak{R}}B$, we have $a_{(1)}\ Q_S\ b_{(1)}$ where $a_{(1)} = (a_1, a_2)$ and $b_{(1)} = (b_1, b_2)$ are the most dissimilar pair in $Max_{\mathfrak{R}}A$ and $Max_{\mathfrak{R}}B$ respectively. By axiom 9, $\{a_1, a_2\} \sim \{b_1, b_1\}$. For every $a_i \in Max_{\mathfrak{R}}A$, we have $\{a_1, a_2\} \succeq \{a_1, a_i\}$ and $\{a_1, a_2\} \succeq \{a_2, a_i\}$. Furthermore, $\{a_1, a_2\} \succeq \{a_1, a_2\}$ so that we can use axiom 3 to obtain $\{a_1, a_2\} \succeq \{a_1, a_2, a_i\}$. We can use the same procedure to add all the remaining elements from $Max_{\mathfrak{R}}A$, until we have $\{a_1, a_2\} \succeq Max_{\mathfrak{R}}A$. Now $\{a_1, a_2\} \subseteq Max_{\mathfrak{R}}A$ so that, by axiom 10, $Max_{\mathfrak{R}}A \succeq \{a_1, a_2\}$ and, therefore, $\{a_1, a_2\} \sim Max_{\mathfrak{R}}A$. Applying the same treatment to B gives us $\{b_1, b_2\} \sim Max_{\mathfrak{R}}B$. By transitivity, we have $Max_{\mathfrak{R}}A \sim Max_{\mathfrak{R}}B$. Repeated use of axiom 8 guarantees, as in the first case, that $A \sim Max_{\mathfrak{R}}A$ and $B \sim Max_{\mathfrak{R}}B$, which in turn, by transitivity, gives the result. Assume now that $A \succ^{\mathfrak{R}}_{\max} B$ and, therefore, that $Max_{\mathfrak{R}}A \succ_{\max} Max_{\mathfrak{R}}B$. Using the same notation as above for $a_{(1)} = (a_1, a_2)$ and $b_{(1)} = (b_1, b_2)$, this means that $a_{(1)}\ Q_A\ b_{(1)}$ which implies, by axiom 9, that $\{a_1, a_2\} \succ \{b_1, b_2\}$. As before, for every $a_i \in Max_{\mathfrak{R}}A$, we have $\{a_1, a_2\} \succeq \{a_1, a_i\}$, $\{a_1, a_2\} \succeq \{a_2, a_i\}$ and $\{a_1, a_2\} \succeq \{a_1, a_2\}$ so that, by axiom 3, we obtain $\{a_1, a_2\} \succeq \{a_1, a_2, a_i\}$. Repeating the argument with all elements of $Max_{\mathfrak{R}}A$, we obtain $\{a_1, a_2\} \succeq Max_{\mathfrak{R}}A$ and, using axiom 10, $\{a_1, a_2\} \sim Max_{\mathfrak{R}}A$. As the same treatment can be applied to B, we are led by transitivity to the statement $Max_{\mathfrak{R}}A \succ Max_{\mathfrak{R}}B$. Finally, and just in the same way as before, a repeated use of axiom 8 will give us $A \succ B$, as needed ■

While $\succeq^{\mathfrak{R}}_{\max}$provides a method for evaluating freedom of choice in the multi-preference approach that incorporate a concern for diversity, it is worth mentioning that, as its cousin $\succeq^{\mathfrak{R}}_{card}$, it does *not* satisfy the full-fledged weak monotonocity with respect to set inclusion as expressed in axiom 2. As this property appears to be a very minimal requirement to impose on a ranking of opportunity sets based on freedom of choice (which conception of freedom could say that making available for choice an option may reduce freedom of choice ?), we believe that the violation of this axiom by both $\succeq^{\mathfrak{R}}_{card}$and $\succeq^{\mathfrak{R}}_{\max}$limits somehow the usefulness of these rankings as appropriate measures of freedom of choice.

5 Conclusion

The purpose of this paper was to investigate the possibility of deriving axiomatic ranking of sets of objects on the basis of their diversity by using only an ordinal primitive information about the similarities of the objects. This approach is to be contrasted with the most recent sophisticated ones such as those proposed by Weitzman (1992),Weitzman (1993), Weitzman (1998), Nehring and Puppe (2002) or Bossert et al. (2002) which assume a cardinally measurable primitive notion of similarities. While this investigation has been proved successful, we are aware that the specific rankings which we have characterize in this paper are not perfect. As mentioned earlier, a basic flaw with these two rankings is that they do not allow smooth trade off between the contributions of alternative pairs of objects to diversity. Both rankings give a very large "veto power" to the two most dissimilar options in the sets to compare the relative diversity that they offer. It would be nice to obtain "smoother" rankings of sets than the two characterized in this paper. An interesting class of these rankings would be an additive one, a typical member of which would view the diversity of as set as the sum of values assigned to each of its pairs by a function that numerically represent, in the sense of Debreu (1954), the binary relation R defined on $X \times X$. An example of such a ranking is the ordering $\succeq_{add}$considered in the proof of propositions 1 and 2. Finding an axiomatic characterization of such a family of diversity rankings is a worthwhile objective for further research.

Another one is to explore further the connections between measurement of diversity and measurement of freedom of choice. While some comments and results have been presented in the last section, we believe that much more could be done. An interesting thing to do in that context would be to dig further behind the "black box" of the primitive quadernary relation of dissimilarity used to define diversity. This appears particularly important in the context of the multiple preference approach to freedom of choice. If diversity is conceived in the context of a decision theoretic model, it may well be relevant to connect the primitive notion of dissimilarity to the possible preference of the decision maker for the options that she will choose in the various opportunity sets. Why for example do a car and a bicycle look intuitively more different - or dissimilar - than two cars with slightly different characteristics ? It is, probably, because we think that most users of the modes of transportation are likely to experiment less difference in satisfaction in changing from one type of car to the other than in changing from one type of car to the bike. These differences in satisfaction could, it seems to us, be expressed in terms of a family of *utility functions* that a “reasonable” decision maker could use when choosing between modes of transportation, in just the same fashion as the notion of freedom of choice has been expressed in terms of a family of *possible preferences* for the decision maker. The resort to cardinally meaningful utility functions, rather than mere ordinal preference orderings, to explain a notion of dissimilarity represented by a quadernary relation seems unavoidable. For it seems very difficult *a priori* to produce rankings of *pairs of objects* in terms of dissimilarity from a mere knowledge of a set of rankings of the objects themselves. We think that the exploration of this area is very promising for future research.

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(lix) This paper was presented at the ENGIME Workshop on "Mapping Diversity", Leuven, May 16-17, 2002
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(lxii) This paper was presented at the ENGIME Workshop on "Communication across Cultures in Multicultural Cities", The Hague, November 7-8, 2002
(lxiii) This paper was presented at the ENGIME Workshop on "Social dynamics and conflicts in multicultural cities", Milan, March 20-21, 2003
(lxiv) This paper was presented at the International Conference on "Theoretical Topics in Ecological Economics", organised by the Abdus Salam International Centre for Theoretical Physics - ICTP, the Beijer International Institute of Ecological Economics, and Fondazione Eni Enrico Mattei – FEEM Trieste, February 10-21, 2003
(lxv) This paper was presented at the EuroConference on "Auctions and Market Design: Theory, Evidence and Applications" organised by Fondazione Eni Enrico Mattei and sponsored by the EU, Milan, September 25-27, 2003
(lxvi) This paper has been presented at the 4th BioEcon Workshop on "Economic Analysis of Policies for Biodiversity Conservation" organised on behalf of the BIOECON Network by Fondazione Eni Enrico Mattei, Venice International University (VIU) and University College London (UCL) , Venice, August 28-29, 2003
(lxvii) This paper has been presented at the international conference on "Tourism and Sustainable Economic Development – Macro and Micro Economic Issues" jointly organised by CRENoS (Università di Cagliari e Sassari, Italy) and Fondazione Eni Enrico Mattei, and supported by the World Bank, Sardinia, September 19-20, 2003
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