Merger Performance under Uncertain Efficiency Gains

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Summary

In view of the uncertainty over the ability of merging firms to achieve efficiency gains, we model the post-merger situation as a Cournot oligopoly wherein the outsiders face uncertainty about the merged entity's final cost. At the Bayesian equilibrium, a bilateral merger is profitable provided that non-merged firms sufficiently believe that the merger will generate large enough efficiency gains, *even if ex post none actually materialize*. The effects of the merger on market performance are shown to follow similar threshold rules. The findings are broadly consistent with stylized facts, and provide a rationalization for an efficiency consideration in merger policy.

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1   Introduction

Mergers and acquisitions constitute a major feature of the economic landscape of most industrialized countries. Historically, mergers have displayed a clear tendency to occur in waves, stretching from the end of the nineteenth century to the present. To provide an idea of the numbers and resources involved, over the period 1981-1998, there were nearly 70,000 merger announcements worldwide, with each deal worth at least 1 million U.S. dollars, of which nearly 45,000 were actually implemented. The average deal was valued at 220 million U.S. dollars (base year 1995.) Of these, 42% were horizontal mergers, defined as those involving two companies with sales in the same 4-digit industry, 54% were conglomerate mergers, and 4% were vertical mergers (Gugler, Mueller, Yurtoglu and Zulehner, 2003; henceforth GMYZ.)

By their very nature, mergers pose an extremely complex conceptual challenge, wherein structure and conduct are inextricably intertwined. Mergers have been an important source of increase in market concentration, particularly outside the U.S. (Schmalensee, 1989.) Not surprisingly, the topic has received considerable scholarly attention from industrial, business and financial economists over the last two decades. An extensive empirical and theoretical literature has explored the motives and consequences of mergers on business activity. While both approaches have yielded some useful insights, allowing industrial economists to reach a consensus on various aspects of merger performance, major points of controversy remain. In particular, important discrepancies exist between key theoretical findings and stylized facts based on empirical and event studies.

The theoretical literature on horizontal mergers relies largely on the standard Cournot model. A central postulate is that the pre-merger and the post-merger situations are represented as Cournot equilibrium points involving different market structures, with the merged entity being treated as a single player in the post-merger game (Salant, Switzer and Reynolds, 1983, henceforth SSR, and Farrell and Shapiro, 1990). The first of these studies showed that in the context of an n-firm symmetric Cournot oligopoly with linear demand and costs, for a merger to be profitable, it should engender a post-merger market share of at least 80%. Allowing the merging firms to exploit production synergies, thereby lowering their post-merger costs, leads to a somewhat wider scope for profitable mergers (Perry and Porter, 1985, Farrell and Shapiro, 1990, and McAfee and Williams, 1992). A similar result prevails under sufficiently concave demand, Fauli-Oller (1996). By contrast, postulating Bertrand competition with differentiated products in an otherwise analogous setting, Deneckere and Davidson (1985) established that every merger would be profitable.

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While some degree of controversy, mostly of a quantitative sort, persists, the empirical literature has delineated some important stylized facts, despite the diversity of data sets, countries, time periods, methodologies, and comparison standards adopted. Rather than reviewing the entire extensive literature on the topic[1], we concentrate on the more important conclusions that have direct bearing on the model proposed in the present paper. On the key issue of profitability, in the largest cross-national study to date (based on the data set described above), GMYZ reports that nearly 60% of all horizontal mergers were profitable, with this proportion being somewhat higher in services than in manufacturing. As for sales (or revenues), it is essentially the other way around, with nearly 60% of merged firms experiencing a drop in sales. A similar negative effect is also reported for the post-merger market shares of the firms involved in the merger (e.g. Mueller, 1985). On the other hand, two other broad-based studies concluded that the profitability of acquired firms declined after the merger (Meeks, 1977) for U.K. firms and Ravenscraft and Scherer, 1987 for U.S. firms.[2] The overall conclusion one can draw from this somewhat mixed picture is that while horizontal mergers have some limited negative effects on sales, they do not appear to have, on average, a clear-cut impact on profitability.[3]

Many other consequences of horizontal mergers were investigated in the empirical literature. For the effect on industry price, there is a consensus that mergers typically lead to a price increase, although there are surprisingly few studies to this effect: See e.g. Kim and Singal (1993) and Borenstein (1990) for airline mergers. Regarding the effects on share prices, near-unanimity has emerged around the fact that initially the target firm's shareholders earn a substantial premium of about 30% on the merger while those of the acquirer tend to have more variable fortunes, with an average on the low side (see e.g. Mueller, 1985). Furthermore, the acquirer's shares tend to experience a subsequent fall in value, a few years after the merger. For firms outside a merger, the evidence does not seem conclusive for mergers in recent times, but Banerjee and Eckard (1998) report significant losses of about 10% for the merger wave at the turn of the 19th century.

As to the issue of whether mergers generate efficiency gains, of paramount importance for all aspects of the economics of mergers, the evidence is not direct as such gains are difficult to estimate, but rather deductive, and the findings are controversial.

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[1] A somewhat detailed, if perhaps nondiscriminating, survey and discussion is given in Tichy (2002).

[2] There are many other studies on this central point, and the results are quite mixed. In particular, a multitude of specific studies involving O.E.C.D. countries have produced divergent results (Mueller, 1980).

[3] The comparison standard adopted by GMYZ and many other empirical studies is to compare the performance of the merged firms against a control group composed of (an average of) non-merged peers in the same industry, over a period of the first five years or so following the merger.

While many studies, including Ravenscraft and Scherer (1987), report little support for a positive relationship, GMYZ concludes that 29.1% of all mergers engendered efficiency gains, as suggested by the finding that these mergers experienced an increase in both profits and sales.[4] Naturally, it is very difficult to disentangle the efficiency gain and the market power effects due to a merger. On the other hand, there appears to be a consensus reached on the basis of case studies and casual observation that while some mergers were successful in securing substantial efficiency gains[5], there is great variability on this issue.

In view of the lack of congruence between theoretical and empirical findings[6], the primary challenge of theoretical work on mergers is to come up with alternative models of merger behavior that would close this gap, while at the same time preserving the equilibrium nature of both the pre and post-merger situations, a consensual feature of the theoretical literature (at least since Farrell and Shapiro, 1990.) This paper constitutes an attempt in this direction within the framework of static analysis. The key novel ingredient is that the non-merging firms face uncertainty as to the efficiency gains that the merged firm could achieve. More precisely, they believe that with some fixed probability, the merged firm will end up with a lower unit cost than before and with the complementary probability, it will retain its original unit cost. The lower cost may correspond e.g. to the claim made by the merging firms to the antitrust agency, possibly appropriately discounted by the rival firms, or to a past average achieved by comparable mergers in related industries. Pre-merger competition is modelled as a standard Cournot oligopoly with identical firms while short-run post-merger competition involves a Bayesian Cournot equilibrium, with the merged firm alone being informed about its true cost. Demand and costs are assumed linear, both for simplicity and for ease of comparison with much of the literature.

On the face of it, this simple formulation seems natural and appropriate, in view of the aforementioned stylized facts and other common casual observations following merger announcements. Indeed, for the merger to obtain the approval of the antitrust authorities in most countries, the candidate firms have to convincingly document scope for significant efficiency gains, via the exploitation of organizational and production synergies. In most cases, the approval of the merger presumes that the antitrust authority has been swayed by the firms' claims of lurking efficiency gains. Likewise, the initial

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[4]See also Tichy (2001), who reports that a quarter of all mergers lead to efficiency gains.

[5]For instance, a case study in Scherer et. al. (1975) documents an increase of 40% in output per worker upon post-merger reorganization. Many other success stories may be found in Fisher and Lande (1983).

[6]Observe that with the above stylized fact on profitability, the conclusions reached under the Cournot and the Bertrand approaches to mergers are equally far off the mark, albeit in different directions.

positive reaction of the financial markets provides some support for the presumption that the merger is likely to lead to strong efficiency gains. In this respect, the magnitude of the upward shift in share prices suggests that an increase in market power alone is unlikely to yield the concomitant increase in expected profit. Another point is that the firms in the industry frequently react with apprehension to a merger announcement by two of their rivals. These typical facts lend clear credence to the postulate that all concerned parties generally hold beliefs about the prospect of efficiency gains that are captured by a Bayesian formulation.

The main result states that if the non-merged firms believe with a sufficiently high probability that the merged firm will experience a high enough efficiency gain, then the merger will be profitable, even if one takes the worst-case scenario for the merged firm, wherein it ends up not experiencing any efficiency gain at all. Given that the answer to this central question rests on two threshold values, the belief and the efficiency gain, or more concisely on the product of the two, a natural question to ask is whether these would typically be reasonable. In a number of plausible cases, we show that these threshold values are indeed quite plausible, even in the worst case scenario. Similar threshold rules are shown to govern the effects of a merger on the merged firm's and outsiders' outputs as well as on industry price, using worst case, best case and ex ante (or expected terms) benchmarks.

In all theoretical models with complete information, whether based on Cournot or on Bertrand competition, mergers always exert a positive externality on non-merged firms. In a Bayesian formulation, the nature of this externality also follows a threshold rule depending on the same pair of parameter values, so that it may well be negative. Similar remarks may be made about market shares and sales. The set of possible outcomes following a merger is substantially expanded, with one or both the merged firm and the outsiders, or neither of them, being possible beneficiaries.

In analyzing the welfare effects of mergers, we proceed both along the worst-case and the ex ante expected scenarios. For both consumer surplus and social welfare, the former benchmark yields a negative effect of mergers while the latter leads to a threshold rule depending again on the belief and the efficiency gain. Interestingly, the implications for merger policy are that the classical dilemma of antitrust authorities as to which standard to base policy on –consumer surplus or social welfare– is of no practical consequence, since the two standards yield the same prescriptions. The threshold rule associated with the ex ante expected benchmark confirms the central role played by expected efficiency gains in gauging the welfare effects of mergers. The present set-up also demonstrates that the merged firms have a strong incentive to overstate the extent of their potential efficiency gains ex ante, not only to secure approval of the merger by

antitrust authorities, but also to twist the terms of Bayesian Cournot competition in their favor, in a short-run perspective.

All in all, our results form a major departure from the complete information equilibrium analysis of the literature starting with SSR. Particularly noteworthy is the fact that the novel features of the present paper hold even in the worst-case ex post outcome of no efficiency gains. In such a case, the *only difference* between the post-merger markets in this paper and in SSR is the *informational market power* of the merged firm. An important consequence of this difference is that, unlike most previous theoretical results, our conclusions are quite consistent with many empirical findings and stylized facts on the effects of mergers on profitability, sales and market shares, both for the merged firm and for the outsiders to the merger. This will be reviewed in detail later on. *Informational market power* thus emerges as a very natural candidate for the fundamental asymmetry that mergers seem to trigger in favor of the merged firm, which previous models have not attempted to capture.

Furthermore, we can add a plausible dynamic extrapolation of our model to capture the resolution of uncertainly over the merged firm's cost. Our results are consistent with GMYZ's finding that over their five-year data window, from one year to the next, realized profits increased for profitable mergers but decreased for unprofitable mergers. A similar mechanism may be invoked to account for the initial substantial rise in share values that typically accompanies merger announcements, which often ends up spiraling downwards after one to three years.

2   The Model

In the pre-merger situation, consider an industry composed of $n + 1$ identical firms choosing quantity levels of a homogenous product in a market with inverse demand $P = a - bQ$, with $a > 0$ and $b > 0$. Each firm has constant marginal cost c, with $a > c > 0$. Each firm's Cournot equilibrium output and profits are then:

$$q = \frac{a - c}{b\left(2 + n\right)} \quad \text{and} \quad \pi = \frac{\left(a - c\right)^2}{b\left(2 + n\right)^2}$$

We consider a two-firm merger only. In the post-merger situation, we postulate that the non-merged firms are uncertain over the resulting unit cost of the merged firm. In particular, they believe that with probability p the merged firm will end up with marginal cost $c_l < c$, thus having experienced efficiency gains equal to $\Delta c \doteq c - c_l$, while with probability $(1 - p)$ nothing will change and its cost will remain c. Here, c_l may for instance be an average value attained by comparable mergers, or the value reflected in post-merger simulations accepted by the merger authorities, or the actual

value claimed by the merging firms. The value of p reflects the subjective perception rivals have formed about the merged firm's ability to achieve the posited efficiency gain, given the information available to them about the case.

Let q_m^h and q_m^l be the merged firm's quantities according as it ends up as a low-cost type (i.e. with unit cost c_l) or as a high-cost type (i.e. with unit cost c), and Eq_m be its expected quantity. Each *non-merged firm's* or outsider's quantity is denoted by q_o. The Bayesian Nash equilibrium quantities are as follows (recall that *after* the merger, the industry has n firms, and $\Delta c = c - c_l > 0$ is the efficiency gain):

$$
\begin{aligned}
q_o &= \frac{a - c - p\Delta c}{b\,(n+1)}, \qquad q_m^l = \frac{2(a-c) + \Delta c(1 + n + p(n-1))}{2b\,(n+1)} \\
q_m^h &= \frac{2(a-c) + p\Delta c(n-1)}{2b\,(n+1)} \quad \text{and} \quad Eq_m = \frac{a - c + np\Delta c}{b\,(n+1)}.
\end{aligned}
$$

All these quantities are strictly positive if one assumes[7] $p < \frac{a-c}{\Delta c}$.

The corresponding market prices are

$$
P^l = \frac{2\,(a + nc_l) + \Delta c(n-1)(p+1)}{2\,(n+1)}
$$

$$
P^h = \frac{2\,(a + nc) + p\Delta c\,(n-1)}{2\,(n+1)}, \qquad EP = \frac{a + cn - p\Delta c}{n+1}
$$

The expected equilibrium profit of each outsider firm and of *the merged* firm, conditional on its cost type, c or c_l, are respectively

$$
\pi_o = \frac{(a - c - p\Delta c)^2}{b\,(n+1)^2}, \qquad \pi_m^h = \frac{(2\,(a-c) + p\Delta c\,(n-1))^2}{4b\,(n+1)^2}
$$

$$
\text{and} \quad \pi_m^l = \frac{(2\,(a-c) + \Delta c\,(1 + n + p\,(n-1)))^2}{4b\,(n+1)^2}
$$

Note that it is always the case that $\pi_o \leq \pi_m^h \leq \pi_m^l$, with equality throughout if and only if $p = 0$. In other words, the informational asymmetry created by the merger always works in favor of the merged firm, which now outperforms its rivals even in the worst case situation wherein their costs are all equal. Whether this informational rent is sufficient to compensate for the fact that the merged firm must now divide its profit between its two pre-merger partners is investigated in our main result below.

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[7]Observe that certainty-equivalence holds, due to the linear specification, in that the merged firm expected output is the p-weighted average of its outputs in the corresponding full information oligopolies. This suggests that our conclusions extend to more general formulations, instead of our highly stylized version, of the key informational feature of this model.

3 Effects on Market Performance

This section provides a detailed account of the consequences of the merger on profits and outputs for both the merged firm and the outsider firms, as well as on industry price. In dealing with these effects, several options are possible. One is obviously to use expected profits and outputs at the Bayesian Cournot equilibrium. This profit measure is arguably the relevant indicator that determines the movement and magnitude of the merged firm's share price. Instead, we focus on the worst case scenario, wherein the merged firm fails in achieving any ex-post efficiency gains at all, so that its post-merger realized profit is given by π_m^h, which is clearly its lowest possible realized profit. Our results can then only be reinforced in the event that efficiency gains have been realized ex-post.

In the worst case scenario, the merger is profitable if $\pi_m^h > 2\pi$, the solution of which leads us one of our main results.

Proposition 1 *If the non-merged firms believe sufficiently, i.e. with*

$$p > p^{h*} = \frac{2(a-c)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{\Delta c\,(n+2)\,(n-1)}$$

that the merged firm will experience large enough efficiency gains

$$\Delta c > \frac{2(a-c)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{(n+2)\,(n-1)}$$

then the merger is profitable, even in the worst case scenario.

These gains can occur only if the original cost is high enough, i.e.

$$c > \frac{2a\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{2\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)+(n+2)\,(n-1)}.$$

To provide some illustrative idea of the plausibility of the two threshold values given in the above Proposition, the following graphs depict the regions of (p, c_l) space for which a bilateral merger is profitable when the merged firm experiences no actual efficiency gains ex-post, for $a = 10$, $n = 5, 10$, or 15, and $c = 3$ or 7. In each case, the merger is profitable below the given curve and unprofitable above it.
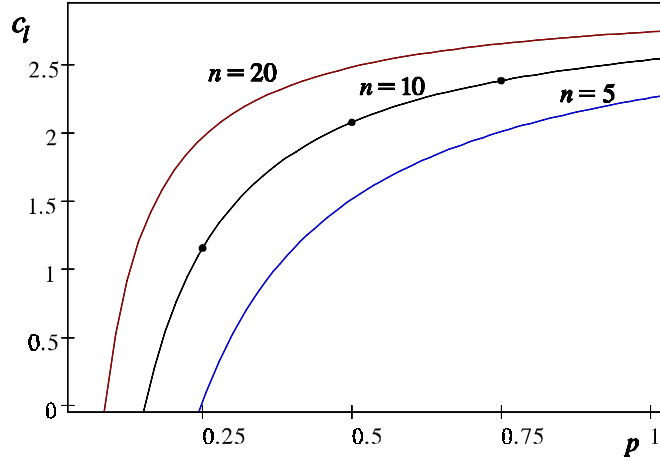
Profitability thresholds when $c = 3$
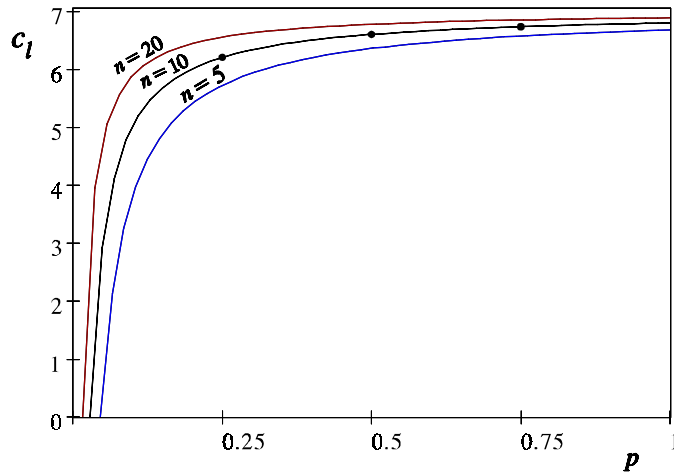
Profitability thresholds when $c = 7$

The marked coordinates on the graphs when $n = 10$ are provided in the table below for both cost cases:

$a = 10$ and $n = 10$	$p = 0.25$	$p = 0.5$	$p = 0.75$
$c = 7$	6.2097	6.6049	6.7366
$c = 3$	1.1560	2.0780	2.3853

For each of the curves, the merger is profitable, even for the worst case scenario of no efficiency gains, whenever the associated parameters lie under the curve. For the given parameter values, it thus appears that the scope for two-firm mergers to be profitable in our setting is quite broad. This finding is quite consistent with the empirical facts on profitability of mergers, whether one goes by the more optimistic picture presented by GMYZ or by the more pessimistic numbers of Ravenscraft and Scherer (1987) and others.

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In expected rather than worst-case terms, mergers are even more likely to be profitable for the merging firms. It is easy to see that

$$\pi_m = (1-p)\pi_m^h + p\pi_m^l \geq \pi_m^h$$

and hence the thresholds for merger profitability, p^* (the exact formula is rather long and is provided in the appendix) are less demanding providing thus, wider scope for mergers. The following example offers a direct comparison between the profitability cost-threshold (for c_l) ex-post in the worst case and ex-ante.

$a = 10 \; n = 10 \; c = 7$	$p = 0.25$	$p = 0.5$	$p = 0.75$
$\pi_m^h > 2\pi$	6.2097	6.6049	6.7366
$\pi_m > 2\pi$	6.6754	6.8280	6.8828

Lastly, we can perform the same kind of comparison between the profits of the merged firm in the best case scenario, i.e., in the event the merged firm experiences efficiency gains, π_m^l and the pre-merger profits. Naturally, since $\pi_m^l \leq \pi_m \leq \pi_m^h$, the probability threshold, p^{l*}, is even lower than p^*. Where p^{l*} is given by

$$p^{l*} = \frac{2(a-c)\left[n(\sqrt{2}-1) - (2-\sqrt{2})\right] - (n+1)(n+2)\Delta c}{(n-1)(n+2)\Delta c}$$

We now investigate the effects of the merger on outputs and outsiders' profits.

Proposition 2 *The merged firm expands output in the worst case scenario (i.e. $q_m^h \geq 2q$) if and only if*

$$p \geq p_m^h = \frac{2n(a-c)}{\Delta c(n-1)(n+2)}.$$

and in the best case scenario (i.e. $q_m^l \geq 2q$) if and only if

$$p \geq p_m^l = \frac{2n(a-c) - (n+1)(n+2)\Delta c}{(n+1)(n+2)\Delta c}.$$

As for outsiders, the same threshold on the value of p governs the direction of change of their output as well as their profit, at the Bayesian Cournot equilibrium.

Proposition 3 *The merger increases an outsider firm's expected profit and output (i.e. $\pi_o \geq \pi$ and $q_o \geq q$) if and only if*

$$p \leq p_o = \frac{a-c}{\Delta c(n+2)}.$$

The merged firm expands output in expected terms (i.e., $q_m \geq 2q$) if and only if $p \geq p_o$.

The following graph summarizes all the possibilities in expected terms. It also includes the threshold belief, p^*, above which the merged firm will be profitable in expected terms. In the appendix it is shown that $p^* < p_o$.

If $\dfrac{a-c}{c-c_l} < n+2$

$\Delta\pi_m < 0$ $\qquad\qquad\qquad$ $\Delta\pi_m > 0$

$\Delta\pi_o > 0$ $\qquad\qquad\qquad$ $\Delta\pi_o < 0$

$\Delta q_o > 0 \quad \Delta EQ < 0$ $\qquad$ $\Delta q_o < 0 \quad \Delta EQ > 0$
$\Delta q_m < 0 \quad \Delta EP > 0$ $\qquad$ $\Delta q_m > 0 \quad \Delta EP < 0$

$0 \qquad\qquad p^* \qquad\qquad p_o \qquad\qquad\qquad\qquad$ Belief $p \qquad 1$

If $\dfrac{a-c}{c-c_l} \geq n+2$ $\quad$ then $p_o \geq 1$

$\Delta\pi_m < 0$ $\qquad\qquad\qquad$ $\Delta\pi_m > 0$

$\Delta q_o > 0 \quad \Delta EQ < 0$
$\Delta\pi_o > 0 \quad \Delta q_m < 0 \quad \Delta EP > 0$

$0 \qquad\qquad\qquad\qquad p^* \qquad\qquad\qquad$ Belief $p \qquad 1$
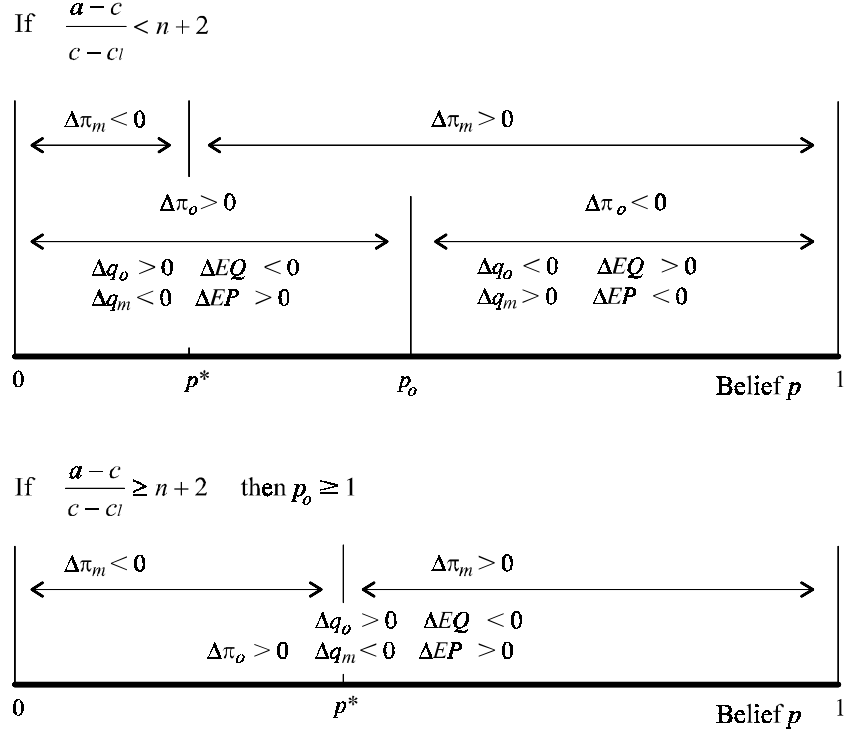
Figure 1. Market performace in expected terms

A more detailed exposition of all the possible ex-post realizations (high and low costs) is given in the appendix. Depending on the belief and efficiency gain levels, a much richer picture emerges, relative to the SSR model. All possible combinations on the contraction or expansion of output by the merged firm and the outsiders emerge. Due to the informational asymmetry, the usual business stealing argument does not quite hold in the worse case scenario, where both the merged firm and the outsiders actually contract output when $p_o \leq p \leq p_m^h$. Likewise, the worst-case profit of the merged firm and the expected profit of the outsiders can both increase (for $p^* \leq p \leq p_o$).

All in all, the emerging picture squares well with the stylized facts, even from a (suggestive) quantitative standpoint. The theory at hand predicts a narrower scope for output expansion ($p \geq p_m^h$) than for profitability of the merger ($p \geq p^{h*}$). The corresponding empirical averages reported by GMYZ are about 60% and 40%, respectively! Thus a merger may be profitable in our setting in case the merged firm contracts output, even if this means a loss in market share (which happens in a majority of cases, Mueller, 1985.)

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These results suggest that the underlying informational asymmetry endows the merged firm with a new form of market power, of an informational nature, relative to the outsider firms. Whether this power is sufficient to overcome the usual mechanism that makes mergers more favorable to outsiders than to insiders depends on the levels of the potential efficiency gains and the associated belief.

As to the effect on industry price, it is unambiguously upwards only when no efficiency gain was realized ex post.

Proposition 4 *(a) In the worst case scenario, industry price is higher after the merger (i.e. $P < P^h$.)*

(b) The expected price is higher after the merger (i.e. $P < EP$) if and only if $p < p_o$

(c) In the best case scenario, industry price is higher after the merger (i.e. $P < P^l$) if and only if

$$p > p^l = \frac{(n+1)(n+2)\Delta c - 2(a-c)}{\Delta c(n+1)(n-1)}.$$

This finding is consistent with the empirical results reported in the few studies that have dealt directly with the price effects of mergers. Overall, these studies typically report modest to significant price increases, ranging from around 10% (Kim and Singal, 1993) to none (Borenstein, 1990 for the airline merger between TWA and Ozark in 1986.)

We now provide an intuitive explanation and interpretation of our results. Recall that in the full information model of SSR, each merged firm wishes to reduce output in the post-merger situation as it now takes into account the business-stealing externality it inflicts on its merging partner (this is also the only way to hope for a price increase, which is a necessary condition for profits to increase for the merged firm). Non-merging firms optimally react to this contraction by expanding output, due to the same externality. The resulting price increase is then not sufficient to imply higher profits for the merged firm.[8]

By contrast, in the present Bayesian setting, the merged firm exploits its informational market power that lies in the inability of the outsiders to adapt their outputs to its true unit cost. Depending on the belief held by the outsiders, this new market power may well lead to the merged firm producing more than before the merger, despite the fact that the aforementioned externality effect is still present here. While a tendency for the outsiders' output to move in the opposite direction is still there, there is a range

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[8]By contrast, in the Bertrand case with substitute products, the merging partners raise their prices, but now the other firms react by raising prices as well, in view of the strategic complementarity property of price competition. So prices rise even more, and the merger is profitable for both insiders and outsiders.

of values of p (between p_0 and p_m^h) for which all firms decrease their output after the merger, even in the worst case scenario. Similarly, there is a range of p (between p^{h*} and p_o) such that all firms' profits increase.

4   Welfare Analysis

Producer surplus, PS, consumer surplus, CS, and total (social) welfare, TW, before the merger are given below:

$$PS = \frac{(n+1)(a-c)^2}{b(2+n)^2}, \quad CS = \frac{(c-a)^2(n+1)^2}{2(n+2)^2 b} \text{ and } TW = \frac{(c-a)^2(n+1)(n+3)}{2(n+2)^2 b}.$$

Ex-post producer surplus, conditional on the realized cost being high (no efficiency gains), takes the following forms:

$$PS^h = \frac{(2(a-c)+p\Delta c(n-1))^2}{4b(n+1)^2} + (n-1)\left(\frac{a-c-p\Delta c}{b(n+1)}\right)\left(\frac{2(a-c)+p\Delta c(n-1)}{2(n+1)}\right)$$

In dealing with the welfare effects of merger, one should keep in mind that none of the three possible evaluation benchmarks – worst-case scenario ($\Delta c = 0$), best-case scenario ($\Delta c = c - c_l > 0$), and in expected value terms – can be dispensed with a priori. For instance, while the merged firm obviously prefers the state $\Delta c > 0$ over the state $\Delta c = 0$ and the outsider firms have the reverse preference (so that industry profits are ambiguous), it is also not clear a priori which way consumers would lean. Hence, the same ambiguity carries over to social welfare.

Expected producer surplus, EPS, is provided below:

$$
\begin{aligned}
EPS \;=\; & p\frac{(2(a-c)+\Delta c(1+n+p(n-1)))^2}{4b(n+1)^2} \\
& +(1-p)\frac{(2(a-c)+p\Delta c(n-1))^2}{4b(n+1)^2} + (n-1)\frac{(a-c-p\Delta c)^2}{b(n+1)^2}
\end{aligned}
$$

The expected change in producer surplus is:

$$
\begin{aligned}
\Delta EPS \;=\; & EPS - PS \\
\;=\; & p\frac{(2(a-c)+\Delta c(1+n+p(n-1)))^2}{4b(n+1)^2} + (1-p)\frac{(2(a-c)+p\Delta c(n-1))^2}{4b(n+1)^2} \\
& +(n-1)\frac{(a-c-p\Delta c)^2}{b(n+1)^2} - (n+1)\frac{(a-c)^2}{b(2+n)^2}.
\end{aligned}
$$

Proposition 5 *In the worst-case scenario as well as in expected terms, the merger increases producer surplus, i.e., $\Delta PS^h = PS^h - PS > 0$ and $\Delta EPS > 0$.*

As in the standard model of SSR, a merger is beneficial to the industry as a whole since it increases aggregate profits. How these gains are divided between the insiders and the outsiders to the merger in our Bayesian setting depends of course on the usual pair of key parameters $(p, \Delta c)$. The result is nevertheless noteworthy for its robustness with respect to the level of informational market power held by the merged firm, in that it holds for all values of p.

In the U.S.A., the antitrust authorities generally take consumer welfare as the key indicator in merger cases. So a separate analysis of consumer surplus is highly desirable. Ex-post consumer surplus, conditional on the actual realized cost, takes the following forms:

$$CS^h = \frac{1}{2b} \left(\frac{2n\,(a-c) - p\Delta c\,(n-1)}{2\,(n+1)} \right)^2$$

$$CS^l = \left(\frac{2n\,(a-c_l) - \Delta c(n-1)(p+1)}{8b\,(n+1)^2} \right) (2n(a-c) - \Delta c(n-1)(p+1))$$

Recall that in case of no efficiency gain ex post, price always increases, and so ex post consumer welfare always decreases in that case.

Proposition 6 *In the worst-case scenario, the merger decreases consumer surplus, i.e.,* $\Delta CS^h = CS^h - CS < 0$.

Expected consumer surplus is clearly the weighted average of the two conditional consumer surpluses:

$$
\begin{aligned}
ECS &= pCS^l + (1-p)CS^h \\
&= p \left(\frac{2n\,(a-c_l) - \Delta c(n-1)(p+1)}{8b\,(n+1)^2} \right) (2n(a-c) + \Delta c(1+n-p(n-1))) \\
&\quad + (1-p)\frac{1}{2b} \left(\frac{2n\,(a-c) - p\Delta c\,(n-1)}{2\,(n+1)} \right)^2
\end{aligned}
$$

We now show that expected consumer welfare can go either way. The following two examples (for different initial costs) show how the change in ECS can be positive or negative, depending on the parameters of the model.

Example 7 *Consider the parameter values* $a = 10$, $b = 1$, $n = 10$ *and* $c = 3$.
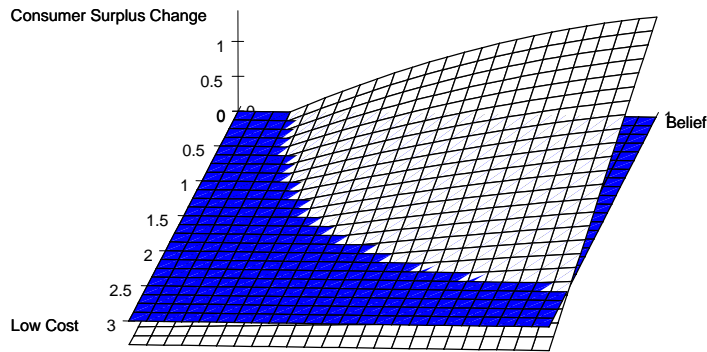
The dark shaded surface is the 0 plane whereas the light shaded area denotes the change in consumer surplus. Hence, when the light shaded surface appears above the dark one it indicates an increase in consumer surplus and identifies the pairs of belief and low cost that can yield such a positive change. Observe that the higher the belief and the lower the cost the more likely it is for consumer surplus to increase. The graph below compares the expected profitability threshold (dotted line) with the threshold for an increase in expected consumer surplus (solid line). It is clear that if expected consumer surplus increases so do the merger's expected profits, but not the other way around. In fact, only if the merger is expected to be sufficiently profitable it will become beneficial to consumers as well. Hence, antitrust authorities should examine the impact on consumer surplus even if the merger is expected to be profitable, especially if the expected increase in profitability is marginal.
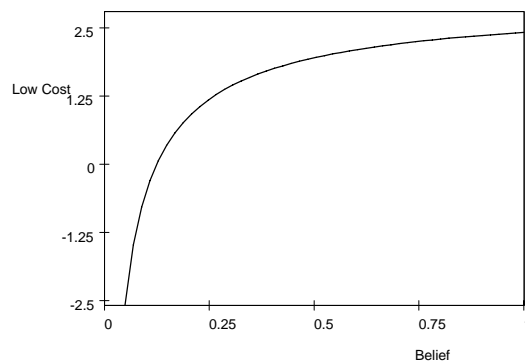
Example 8 *Similarly to the previous example consider the parameter values $a = 10$, $b = 1$, $n = 10$, and $c = 7$.*

Consumer Surplus Change
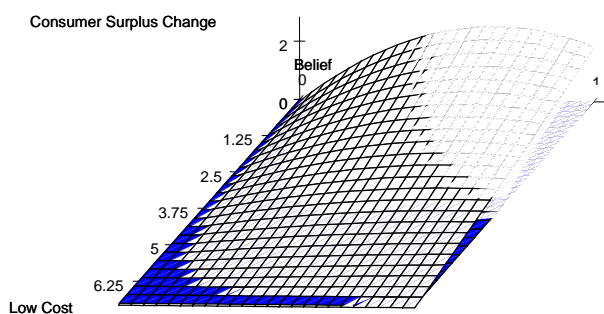
We examine the expected change in consumer surplus. Again the light shaded surface denotes the change in consumer surplus [The part of the surface that is eliminated refers to the area where $p > \frac{a-c}{c-c_l}$ and the solution is, hence, not interior]. The comparison between the expected profitability threshold and the expected change in consumer surplus is depicted in the graph below. The lower curve (indicated by circles) denotes the condition for interior solution, that is, for all (p, c_l) located in lower right corner of the diagram our model does not admit an interior solution.
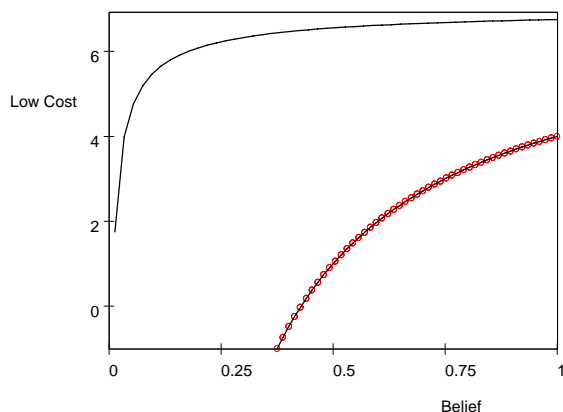
The results of the two examples suggest that when the starting cost is higher it is more likely that the merger will be beneficial to consumers. Naturally, higher starting costs provide more opportunities for efficiency gains that are advantageous to consumers as well.

It is not surprising that since expected producer surplus always increases and expected consumer surplus is ambiguous, expected social welfare can also go either way, as illustrated below.

$$ETW = p^2 (\Delta c)^2 (5n + 7) (n-1) + p\Delta c \left[8 (a - c) (n + 2) + 3\Delta c (n + 1)^2\right] + 4n (a - c)^2 (n + 2)$$

The threshold belief for the change in expected total welfare to be positive is rather

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long and is provided in the appendix. However, as the following examples illustrate, a merger is not likely to be harmful to the society as a whole.

Example 9 *Consider the parameter values $a = 10$, $b = 1$, $n = 10$, and $c = 3$.*
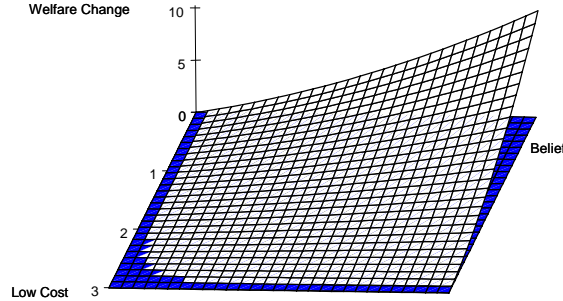
It is clear that unless the belief is extremely low or the efficiency gains insignificant the merger will be advantageous to the society, i.e., expected total welfare will increase.

Example 10 *Consider the parameter values $a = 10$, $b = 1$, $n = 10$ and $c = 7$.*
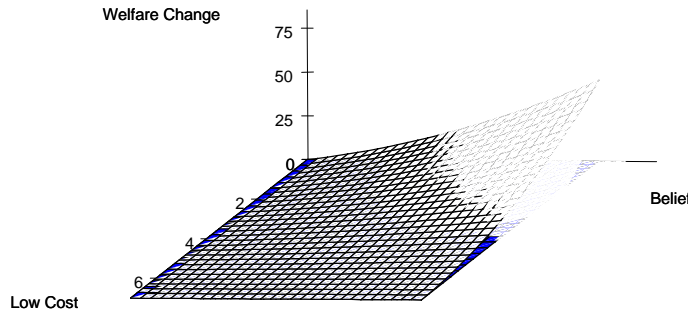
The observation made in the previous example concerning the impact of a merger to society are reinforced. In fact, similarly to consumer surplus, the opportunities for a socially beneficial merger are increased when the starting cost is higher.

More importantly, if the merger is expected to be profitable, then expected total welfare will also increase as is formally stated in the following proposition.

Proposition 11 *(a) In the best case scenario as well as in expected terms, the merger increases social welfare whenever the merger is expected to be profitable, i.e.,*

$$\Delta\pi_m > 0 \implies \Delta TW^l > 0 \text{ and } \Delta ETW = ETW - TW > 0.$$

(b) In the worst case scenario, the merger always lowers social welfare, i.e., $\Delta TW^h < 0$.

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The implications of this result are significant in that they suggest that if two firms' wish to merge were fully based on an expected profits calculation, then the antitrust authorities would better adopt a laissez-faire policy. However, if efficiency gains do not materialize ex post, then the merger will always be detrimental to society. Thus if the antitrust institution adopted an absolutely conservative standard requiring that social welfare increase in the worst case scenario, then no merger would ever be permitted. To the extent that it is widely believed that in some cases, mergers are, at least partly, motivated by managerial hubris or empire-building, the implications of such a result should be viewed with due care. The result also raises in a clear-cut manner the issue of risk-bearing in merger policy.

The two graphs below illustrate this result by plotting the threshold beliefs for the same parameters used in the previous examples. The threshold for social welfare (solid line) is milder in the sense that it is always lower than the threshold for expected profits (dotted line). As in all previous graphs for the case of $c_l = 7$, in the lower right corner of the graph (depicted by the circled line) our model does not admit an interior solution.
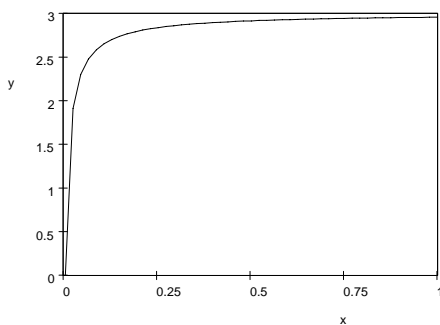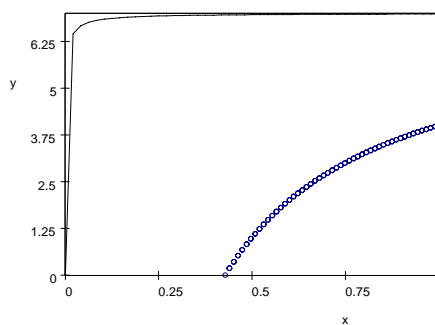
$$c_l = 3 \qquad\qquad\qquad\qquad c_l = 7$$

5 Further Results and Observations

5.1 The "Dynamics" of Profitability

A finer empirical finding of GMYZ is that, over their five-year data window, from one year to the next, realized profits increased for profitable mergers but decreased for unprofitable mergers. This seemingly strange finding turns out to be quite consistent with our results if one adds a plausible dynamic extrapolation capturing the resolution of uncertainty in our model. Assuming that profitable mergers tend to be those that indeed generate efficiency gains, the outsiders progressively learn about these gains, realize with more and more certainty that they face a lower-cost merged rival and react accordingly. Likewise, if unprofitable mergers are identified with those that failed to generate much efficiency gain, rivals will progressively find out over time that they

face a high-cost (or no efficiency gain) merged rival, and the latter's profits will move accordingly lower. In both cases, the process eventually settles at the full information Cournot equilibrium that reflects the true efficiency gains actually achieved by the merger.

To better grasp the above argument, consider the following example where the merger decision is based on expected merger profits.

Example 12 *Suppose $a = 10, b = 1$, $n = 10$, $c = 7$, $p = 0.5$, and $c_l = 6.6200 \in$ $(6.6049, 6.8280)$. That is, c_l is in between the 2 critical values of c_l depicted in the Table above. As a result, $\pi_m^h = 0.122\,82 < 2\pi = 0.125$ and $\pi_m = 0.207\,46 > 2\pi = 0.125$. If the merger decision is made based on the ex ante expected profit, then, ex post, with probability p the cost is c_l and the profits are $\pi_m^l = 0.292\,09 > 2\pi = 0.125$ while with probability $(1-p)$ the cost is c and profits are $\pi_m^h < 2\pi$. Our static setting is equivalent to a dynamic setting with myopic firms. Given this extreme setting, in subsequent competition, the Cournot game is either one with the merged firm having cost c^l or one with the merged firm having cost c. In the former case, the merged firm's profits are $0.382\,15 > \pi_m^l = 0.292\,09$ and in the latter case, its profits are $0.074\,38 < \pi_m^h = 0.122\,82$.*

5.2 Sales and Profitability

In their typology of merger outcomes, GMYZ propose four possible cases depending on whether profits and sales each increased or decreased. They report that the case where profits decline and sales rise accounts for 15% of all mergers, and suggest that this case is hard to reconcile with any theoretical predictions. They conclude that such mergers are likely to be driven by empire building or growth concerns as opposed to profit maximization. In the following illustration, we can confirm their typology by proving that the aforementioned case cannot take place within our model, and by identifying parameter regions that account for the other three cases.

Proposition 13 *A bilateral merger cannot lead to declining profits combined with rising sales.*

The argument for the worst case scenario is as follows: As profits decline, we have $p < p^{h*}$, and $\pi_m^h < 2\pi$ or $P^h q_m^h - cq_m^h < 2Pq - 2cq$ or $P^h q_m^h - 2Pq < c(q_m^h - 2q)$. As sales rise, $P^h q_m^h - 2Pq > 0$, so from the previous inequality $q_m^h > 2q$, which from Proposition 2 yields $p > p_m^h$. Overall then, we have $p_m^h < p < p^{h*}$, a contradiction since we know (shown in the appendix) that $p_m^h > p^*$. Therefore, if profits go down, sales are decreasing as well.

When the merger is profitable, we are unable to find analytically the belief or cost thresholds for increasing sales. The following example where $a = 10$, $n = 10$ and $c = 3$ illustrates the three possible cases:
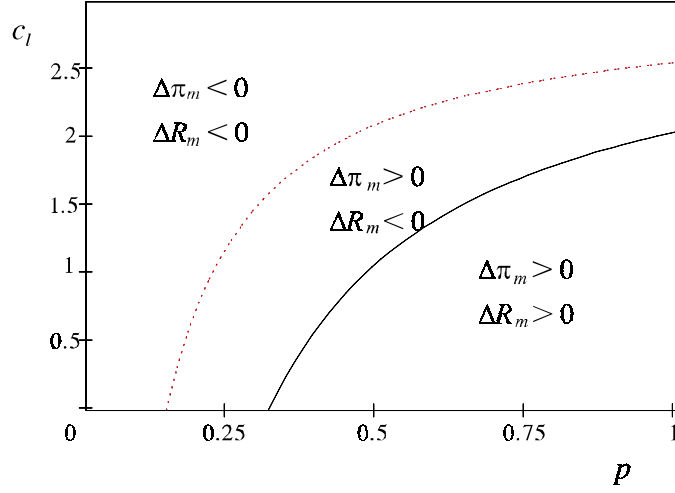
5.3 The Profitability of Multilateral Mergers

As attention was restricted at the outset to bilateral mergers, it is worthwhile to examine mergers involving more firms. We provide a plausible illustration showing that (i) larger mergers are not necessarily better than bilateral mergers and (ii) larger mergers may not even be profitable when bilateral mergers are.

With s merging partners, each of the merged partners' profits are

$$\frac{\pi_m^h}{s} = \frac{(2a - 2c + p(c - c_l)(n - s + 1))^2}{4sb(n - s + 3)^2}$$

Consider $n = 10$, $a = 10$, $c = 3$, $c_l = 2$, $p = 0.6$, $b = 1$. The following is a plot of π_m^h/s as a function of s.
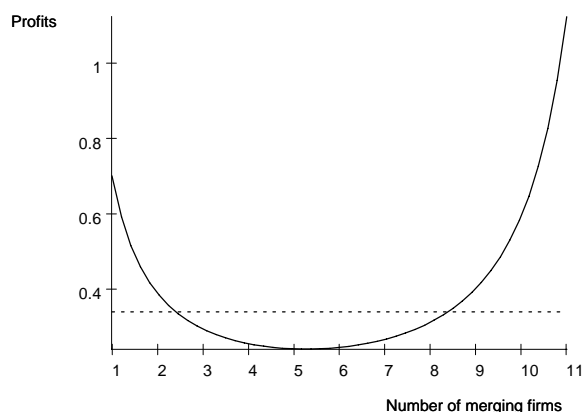
The only profitable mergers are those involving 2, 10 or all 11 firms. By contrast, recall that for this example, only mergers with 9, 10 or 11 firms are profitable in the complete information Cournot model of SSR.

6   Conclusion

This paper argues that many of the circumstances surrounding mergers call for a theoretical model wherein the firms outside the merger face a new type of rival, characterized by unknown unit costs, reflecting their natural initial uncertainty about the ability of the merged firm to achieve any (of the claimed) efficiency gains. This pervasive uncertainty also affects the approval decision of antitrust authorities, and triggers the favorable response by financial markets. Within the obvious confines of a static model, the proposed Bayesian Cournot equilibrium leads to an equilibrium outcome that is broadly consistent with much of the empirical evidence on the industry effects of mergers, including on profits, price and market shares for the merged firm as well as for outsiders, at least in the short run. All in all, the model at hand reflects a simple and natural modification to the standard Cournot approach, based on an informational asymmetry between the merged firm and outsiders, which confers additional market power to the merged firm, bringing about a surprising level of congruence with stylized facts, hitherto elusive.

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8   Appendix

8.1   Nash Equilibrium

Each firm produces $q = \frac{a-c}{b(2+n)}$ and the total quantity is $Q = \frac{(n+1)(a-c)}{b(n+2)}$. The equilibrium price is $P = \frac{a+c(n+1)}{n+2}$ and each firm's profits are $\pi = \frac{(a-c)^2}{b(2+n)^2}$.

8.2   Bayesian Nash Equilibrium

Each (outsider) firm's expected payoffs are:

$$
\begin{aligned}
E\pi_i &= p\pi_i^h + (1-p)\pi_i^l \\
&= p\left((a - bQ_{-i}^h - bq_i)q_i - cq_i\right) + (1-p)\left((a - bQ_{-i}^l - bq_i)q_i - cq_i\right) \\
&= aq_i - bEQ_{-i}q_i - bq_i^2 - cq_i.
\end{aligned}
$$

1st order condition yields the best response function:

$$
q_i = \frac{a - bEQ_{-i} - c}{2b}.
$$

But now everybody knows the cost of $n-1$ but not the cost of the nth firm and the best response function of the *each outsider* becomes:

$$
q_o = \frac{a - c - bEq_m}{bn}.
$$

The *merged* firm's best response functions are:

$$q_m^h = \frac{a - c - b(n-1)q_o}{2b}$$

and

$$q_m^l = \frac{a - c_l - b(n-1)q_o}{2b}.$$

Therefore the *outsiders'* quantity is:

$$q_o = \frac{a - c - p\,(c - c_l)}{b\,(n+1)}.$$

Merged firms quantity:

$$q_m^h = \frac{2\,(a-c) + p(c-c_l)(n-1)}{2b\,(1+n)}$$

$$q_m^l = \frac{2(a-c) + (c-c_l)(1+n+p(n-1))}{2b\,(1+n)}$$

$$q_m = \frac{a - c + np(c-c_l)}{b\,(n+1)}.$$

Total quantity:

$$Q^l = \frac{2n\,(a-c) + (c-c_l)\,(n+1-p\,(n-1))}{2b\,(n+1)}$$

$$Q^h = \frac{2n\,(a-c) - p\,(c-c_l)\,(n-1)}{2b\,(n+1)}$$

$$EQ = \frac{n\,(a-c) + p\,(c-c_l)}{b(n+1)}.$$

Price:

$$P^h = \frac{2\,(a+cn) + p\,(c-c_l)\,(n-1)}{2\,(n+1)}$$

$$P^l = \frac{2\,(a+cn) - (c-c_l)\,(1+n-p\,(n-1))}{2\,(n+1)}$$

$$EP = \frac{a + nc - p\,(c-c_l)}{n+1}.$$

Conditional expected profits of each outsider firm:

$$\pi_o^h = \pi_o + \frac{p(c-c_l)\,[(a-c) - p(c-c_l)]}{2b(n+1)}$$

$$\pi_o^l = \pi_o + \frac{(c-c_l)(1-p)\,[p(c-c_l) - (a-c)]}{2b(n+1)}$$

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Expected Profits of a *each outsider* firm:

$$\pi_0 = \frac{(a - c - p(c - c_l))^2}{b(n+1)^2}.$$

Conditional expected profits of the merged firm:

$$\pi_m^l = \frac{[2(a-c) + (c - c_l)(1 + n + p(n-1))]^2}{4b(n+1)^2}$$

$$\pi_m^h = \frac{(2(a-c) + p(c - c_l)(n-1))^2}{4b(n+1)^2}.$$

Expected profits of the merged firm:

$$\pi_m = p\pi_m^l + (1-p)\pi_m^h$$

$$\pi_m = \frac{\left(\begin{array}{c} p^2(c_l - c)^2(3n+1)(n-1) \\ +p(c - c_l)\left(8n(a-c) + (c - c_l)(n+1)^2\right) \\ +4(c-a)^2 \end{array} \right)}{4b(n+1)^2}.$$

Change in expected profits:

$$\Delta\pi_m = \pi_m - 2\pi$$

$$\frac{\left(\begin{array}{c} p^2(3n+1)(n-1)(n+2)^2(c_l - c)^2 \\ +p(n+2)^2(c - c_l)\left(8n(a-c) + (c - c_l)(n+1)^2\right) \\ -4(c-a)^2(n^2 - 2) \end{array} \right)}{4b(n+1)^2(2+n)^2}$$

Threshold belief above which expected profits are higher than pre-merger profits:

$$p^* = \frac{\left(\begin{array}{c} -(n+2)\left[8n(a-c) + (c - c_l)(n+1)^2\right] \\ +\sqrt{\begin{array}{c}(n+2)^2\left[8n(a-c) + (c - c_l)(n+1)^2\right] \\ +16(3n+1)(n-1)(a-c)^2(n^2 - 2)\end{array}} \end{array} \right)}{2(3n+1)(n-1)(n+2)(c - c_l)}$$

8.3 Welfare analysis details

Expected total welfare:

$$
\begin{aligned}
ETW \;=\;& p\frac{\left(2\left(a-c\right)+\left(c-c_l\right)\left(1+n+p\left(n-1\right)\right)\right)^2}{4b\left(n+1\right)^2}\\
&+\left(1-p\right)\frac{\left(2\left(a-c\right)+p\left(c-c_l\right)\left(n-1\right)\right)^2}{4b\left(n+1\right)^2}+\left(n-1\right)\frac{\left(a-c-p\left(c-c_l\right)\right)^2}{b\left(n+1\right)^2}\\
&+p\frac{1}{2}\left(a-\frac{2\left(a+nc_l\right)+\left(c-c_l\right)\left(n-1\right)\left(p+1\right)}{2\left(n+1\right)}\right)\\
&\frac{1}{2b\left(n+1\right)}\left(2n(a-c)+\left(c-c_l\right)\left(1+n-p(n-1)\right)\right)\\
&+\left(1-p\right)\frac{1}{2}\left(a-\frac{2\left(a+nc\right)+p\left(c-c_l\right)\left(n-1\right)}{2\left(n+1\right)}\right)\\
&\frac{1}{2b\left(n+1\right)}\left(2n(a-c)-p(n-1)\left(c-c_l\right)\right)
\end{aligned}
$$

which reduces to:

$$
\begin{aligned}
ETW \;=\;& p^2\left(c_l-c\right)^2\left(5n+7\right)\left(n-1\right)\\
&+p\left(c-c_l\right)\left[8\left(a-c\right)\left(n+2\right)+3\left(c-c_l\right)\left(n^2+2n+1\right)\right]\\
&+4n\left(c-a\right)^2\left(n+2\right)
\end{aligned}
$$

Change in expected total welfare:

$$
\Delta ETW=\frac{\left(\begin{array}{c}p^2\left(5n+7\right)\left(n-1\right)\left(n+2\right)^2\left(c_l-c\right)^2\\+p\left(n+2\right)^2\left(c-c_l\right)\left(8\left(a-c\right)\left(2+n\right)+3\left(c-c_l\right)\left(n+1\right)^2\right)\\-4\left(c-a\right)^2\left(2n+3\right)\end{array}\right)}{8\left(n+1\right)^2b\left(n+2\right)^2}
$$

Threshold belief above which expected total welfare is higher than pre-merger total welfare

$$
p'=\frac{\left(\begin{array}{c}-\left(n+2\right)\left[8(a-c)(n+2)+3(c-c_l)\left(n+1\right)^2\right]+\\\sqrt{\begin{array}{c}\left(n+2\right)^2\left[8(a-c)(n+2)+3(c-c_l)\left(n+1\right)^2\right]\\+16(n-1)(5n+7)\left(a-c\right)^2\left(2n+3\right)\end{array}}\end{array}\right)}{2(5n+7)(n-1)(n+2)(c-c_l)}
$$

8.4 Proofs

Proof of Proposition 1. The merged firm is profitable if

$$
\pi_m^h>2\pi
$$

which reduces to

$$p > \frac{2(a-c)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{(c-c_l)(n+2)(n-1)} = p^{h*}$$

It is easy to see that $p^{h*} > 0$ if $n \geq 2$. Moreover, $p^{h*} < 1$ if the cost gains are sufficient:

$$\frac{2(a-c)\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{(n+2)(n-1)} < c - c_l.$$

For the lower bound on cost gains to be feasible $(c - c_l < c)$ it must be that the pre-merger cost is at least:

$$\frac{2a\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)}{2\left(\left(\sqrt{2}-2\right)+\left(\sqrt{2}-1\right)n\right)+(n+2)(n-1)} < c.$$

Note that the latter lower bound on c is always below a. ■

Proof of Proposition 2. In the worst case scenario the merged firm expands if and only if

$$q_m^h \geq 2q$$

which yields

$$p \geq p_m^h = \frac{2n(a-c)}{(c-c_l)(n-1)(2+n)}.$$

Whereas, in the best case scenario the merged firm expands if and only if

$$q_m^l \geq 2q$$

which yields

$$p \geq p_m^l = \frac{2n(a-c)-(c-c_l)(n+1)(n+2)}{(c-c_l)(2+n)(n-1)}.$$

■

Proof of Proposition 3. Outsider firms benefit from the merger and expand their output if and only if

$$\pi_o \geq \pi \text{ and } q_o \geq q$$

Both inequalities reduce to

$$p \leq p_o = \frac{a-c}{(c-c_l)(n+2)}.$$

Similarly, the merged firm expands its output if and only if

$$q_m \geq 2q$$

which reduces to

$$p \geq p_o.$$

∎

Figure 1 explanation. Note that if $\frac{a-c}{c-c_l} \geq n+2$ then $p_o \geq 1$ hence, $\pi_o \geq \pi$ and $q_o \geq q$ for all $p \in [0,1]$. If the merger is profitable in the worst case scenario, then $p > p^{h*}$ and it easy to show that $p_o > p^{h*}$ for all $(a-c) > 0$ and for all $(c-c_l) > 0$ since the inequality reduces to $n > \frac{2(\sqrt{2}-2)}{3-2\sqrt{2}}$ which is always true. If the merger is profitable in the worst case scenario then it is profitable in expected terms as well, i.e., $p^* < p^{h*}$, hence, $p^* < p_o$. It follows immediately from the expressions presented in the previous section that $EQ > Q$ if and only if $p > p_o$. ∎

Proof of Proposition 4. (a) The inequality $P^h > P$ reduces to $2(a-c) + p(c-c_l)(n-1)(n+2) > 0$ which is always true. (b) The inequality $EP > P$ reduces to $p < \frac{a-c}{(c-c_l)(n+2)} = p_o$. (c) The inequality $P^l > P$ reduces to

$$p > p^l = \frac{(n+1)(n+2)(c-c_l) - 2(a-c)}{(c-c_l)(n+1)(n-1)}.$$

∎
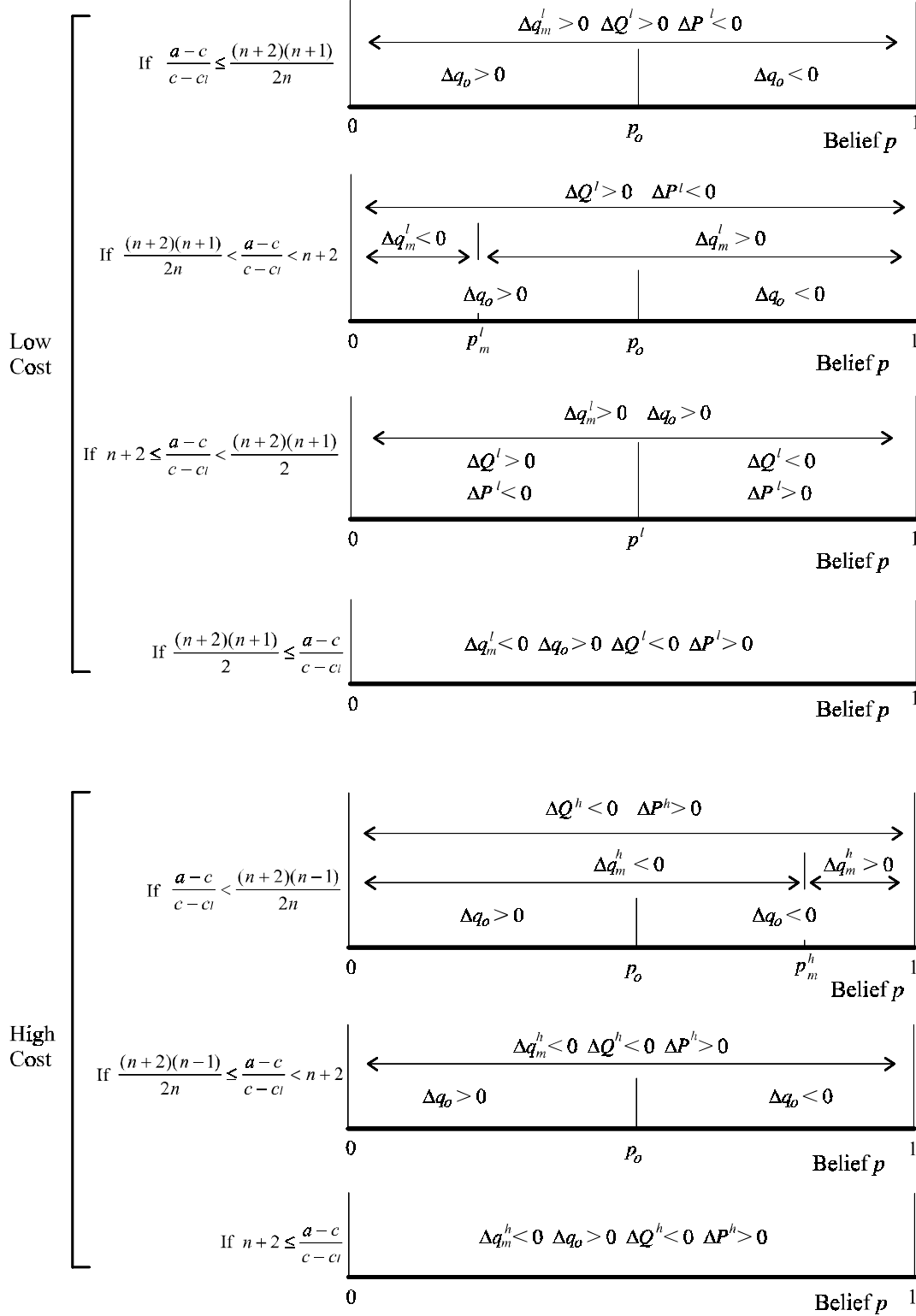
Figure 2. Market performance in low and high cost realizations

Figure 2 explanation. For information regarding p_o look in explanation of Figure 1.

Low Cost:

Further note that $p_m^l \geq 0$ if and only if $\frac{a-c}{c-c_l} \geq \frac{(n+2)(n+1)}{2n}$ and $p_m^l \leq 1$ if and only if

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$\frac{a-c}{c-c_l} \leq n+2$. It is easy to see that $\frac{(n+2)(n+1)}{2n} < n+2$. Observe that $p_m^l < p_o$ if and only if $\frac{a-c}{c-c_l} < n+2$ but otherwise both p_o and p_m^l are greater than 1 hence their relationship is insignificant. Similarly, notice that $p^l \leq 1$ if and only if $\frac{a-c}{c-c_l} \geq n+2$ and $p^l \geq 0$ if and only if $\frac{a-c}{c-c_l} \leq \frac{(n+2)(n+1)}{2}$. Again it is easy to see that $n+2 \leq \frac{(n+2)(n+1)}{2}$ is always true. Finally, the inequality $Q^l > Q$ reduces to $p < p^l$. Observe that if p^l is effective (i.e., $p^l < 1$) p_o and p_m^l are ineffective (i.e., greater than 1).

High Cost:

Note that it is always the case that $p_m^h \geq 0$ and $p_m^h \leq 1$ if and only if $\frac{a-c}{c-c_l} \leq \frac{(n+2)(n-1)}{2n}$. Similarly, it is easy to show that $p_m^h > p_o$ for all $(a-c) > 0$ and for all $(c-c_l) > 0$ since the inequality reduces to $n > -1$ which is always true. Finally the inequality $Q^h > Q$ reduces to $2(a-c) + p(c-c_l)(n-1)(n+2) > 0$ which is always true. ∎

Proof of Proposition 5. The change in Producer Surplus in the worst case scenario is given below:

$$\Delta PS^h = \frac{(2n(a-c) - p(n-1)(c-c_l))(2(a-c) + p(n-1)(c-c_l))}{4b(n+1)^2} - (n+1)\frac{(a-c)^2}{b(2+n)^2}.$$

Recall that we require that $p < \frac{a-c}{c-c_l}$ for an interior solution. Now observe that ΔPS^h is increasing in p since

$$\frac{d\Delta PS^h}{dp} = \frac{2(n-1)^2(c-c_l)^2\left(\frac{a-c}{c-c_l} - p\right)}{4b(n+1)^2} > 0.$$

Hence, it suffices to show that $\Delta PS^h|_{p=0} > 0$. Indeed

$$\Delta PS^h|_{p=0} = \frac{(a-c)^2}{b}\left(\frac{n}{(n+1)^2} - \frac{n+1}{(2+n)^2}\right) > 0.$$

To show that $\Delta EPS > 0$ we first take its derivative with respect to p and see that is it positive:

$$\frac{d\Delta EPS}{dp} = \frac{\left(\begin{array}{c} 2p(3n+5)(n-1)(n+2)^2(c_l-c)^2 \\ + (n+2)^2(c-c_l)\left(8(a-c) + (1+n)^2(c-c_l)\right) \end{array}\right)}{4b(n+1)^2(2+n)^2} > 0$$

Note that $\Delta EPS > 0$ if $\Delta EPS|_{p=0} > 0$. Then observe that

$$\Delta EPS|_{p=0} = \frac{(a-c)^2(n^2+n-1)}{b(n+1)^2(n+2)^2} > 0.$$

∎

Proof of Proposition 6. The change in consumer surplus in the worst case scenario is given below:

$$\Delta CS^h = \frac{1}{2} \frac{(2n(a-c) - p\,(n-1)\,(c-c_l))^2}{4b\,(n+1)^2} - \frac{1}{2} \frac{(a-c)^2\,(n+1)^2}{(n+2)^2\,b} < 0$$

Observe that ΔCS^h is a decreasing function of p because $2n(a-c) - p\,(n-1)\,(c-c_l) > 0$. It is easy to see that at $p = 0$ where ΔCS^h takes its highest value it is already negative. ∎

Proof of Proposition 11. Observe that for $\Delta \pi_m > 0$ it suffices to show that

$$p^2\,(3n+1)\,(n-1)\,(n+2)^2\,(c_l - c)^2 \;+$$
$$p\,(n+2)^2\,(c-c_l)\,\big(8n\,(a-c) + (c-c_l)\,(n+1)^2\big) \;-$$
$$4\,(c-a)^2\,\big(n^2 - 2\big) > 0$$

. Let the coefficient of p^2 be denoted by

$$\alpha_m = (3n+1)\,(n-1)\,(n+2)^2\,(c_l - c)^2\,,$$

the coefficient of p be denoted by

$$\beta_m = (n+2)^2\,(c-c_l)\,\big(8n\,(a-c) + (c-c_l)\,(n+1)^2\big)$$

and the constant be denoted by

$$\gamma_m = -4\,(c-a)^2\,\big(n^2 - 2\big)$$

. Note that $\alpha_m, \beta_m > 0$, hence $\Delta \pi_m$ is increasing in p.

Now observe that for $\Delta ETW > 0$ it suffices to show that

$$p^2\,(5n+7)\,(n-1)\,(n+2)^2\,(c_l - c)^2 \;+$$
$$p\,(n+2)^2\,(c-c_l)\,\big(8\,(a-c)\,(2+n) + 3\,(c-c_l)\,(n+1)^2\big) \;-$$
$$4\,(c-a)^2\,(2n+3) > 0.$$

Similarly, let the coefficient of p^2 be denoted by

$$\alpha_w = (5n+7)\,(n-1)\,(n+2)^2\,(c_l - c)^2\,,$$

the coefficient of p be denoted by

$$\beta_w = (n+2)^2\,(c-c_l)\,\big(8\,(a-c)\,(2+n) + 3\,(c-c_l)\,(n+1)^2\big)$$

and the constant term by $\gamma_w = -4\,(c-a)^2\,(2n+3)$. Again, note that $\alpha_w, \beta_w > 0$, hence ΔETW is increasing in p.

Next note that $\alpha_w > \alpha_m$, $\beta_w > \beta_m$ and $\gamma_w > \gamma_m$. When $\Delta\pi_m(p^*) = 0$ we have $p^{*2}\alpha_m + p^{*2}\beta_m = -\gamma_m$. since $\alpha_w > \alpha_m$ and $\beta_w > \beta_m$ we have $p^{*2}\alpha_w + p^{*2}\beta_w > p^{*2}\alpha_m + p^{*2}\beta_m$ hence $p^{*2}\alpha_w + p^{*2}\beta_w > -\gamma_m$ but $\gamma_m < \gamma_w \Leftrightarrow -\gamma_m > -\gamma_w$. Thus, $p^{*2}\alpha_w + p^{*2}\beta_w > -\gamma_w$ which implies that $p^{*2}\alpha_w + p^{*2}\beta_w + \gamma_w > 0 \Leftrightarrow \Delta ETW(p^*) > 0$.

To conclude, when $p = p^*$ we have $\Delta\pi_m(p^*) = 0$ and $\Delta ETW(p^*) > 0$ and for all $p > p^*$ we have $\Delta\pi_m(p^*) > 0$ and $\Delta ETW(p^*) > 0$ since both $\Delta\pi_m(p)$ and $\Delta ETW(p)$ are increasing functions of p as argued earlier.

The change in Total Welfare in the worst case scenario is given by the formula below:

$$
\begin{aligned}
\Delta TW^h \;=\; & \frac{(2n\,(a-c) - p(n-1)\,(c-c_l))\,(2(n+2)(a-c) + p(n-1)(c-c_l))}{8b\,(n+1)^2} \\
& -(n+1)\frac{(a-c)^2}{b\,(2+n)^2} - \frac{1}{2}\frac{(a-c)^2\,(n+1)^2}{(n+2)^2\,b}.
\end{aligned}
$$

It is easy to see that ΔTW^h is a decreasing function of p, thus, it suffices to show that $\Delta TW^h < 0$ at $p = 0$ where ΔTW^h takes its highest value. Indeed

$$
\Delta TW^h|_{p=0} = \frac{(a-c)^2}{b}\left(\frac{-2n-3}{2\,(n+1)^2\,(2+n)^2}\right) < 0.
$$

∎

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