

Rationing in IPOs

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NOTA DI LAVORO 26.2003

MARCH 2003

PRIV – Privatisation, Regulation, Antitrust

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Summary

We provide a model of bookbuilding in IPOs, in which the issuer can choose to ration shares. We consider two allocation rules. Under share dispersion, before informed investors submit their bids, they know that, in the aggregate, winning bidders will receive only a fraction of their demand. We demonstrate that this mitigates the winner's curse, that is, the incentive of bidders to shade their bids. It leads to more aggressive bidding, to the extent that rationing can be revenue-enhancing. In a parametric example, we characterize bid and revenue functions, and the optimal degree of rationing. We show that, when investors' information is diffuse, maximal rationing is optimal. Conversely, when their information is concentrated, the seller should not ration shares. We determine the optimal degree of rationing in a class of credible mechanisms. Our model reconciles the documented anomaly that higher bidders in IPOs do not necessarily receive higher allocations.

JEL: D44, G2

This paper has been presented at the EuroConference on "Auctions and Market Design: Theory, Evidence and Applications" organised by Fondazione Eni Enrico Mattei and sponsored by the EU, Milan, September 26-28, 2002.

Some of the material in this paper was circulated under the title, "Rationing and Common Values." The authors are grateful to Peter Boatwright, Francesca Cornelli, Simon Gervais, Paul Klemperer, Adam Koch, Vlad Mares, Kjell Nyborg, Ann Sherman, Fallaw Sowell, Chester Spatt, Jeroen Swinkels, the editor, and an anonymous referee for helpful comments, and to participants at the Summer 2001 Econometric Society Meetings, the 2002 FEEM Auction Design Conference, and seminars at Carnegie Mellon University, Columbia University, University of Illinois, University of Maryland, University of Minnesota (Carlson), and University of Pennsylvania (Wharton). All errors remain the authors' responsibility.

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1 Introduction

Rationing in IPOs has been extensively documented.¹ Typically, at the offer price there is excess demand, and shares are rationed to investors. All investors, both informed and uninformed are rationed. Although explanations for the rationing of uninformed investors have been offered,² empirically, we observe that informed investors are also rationed. This is particularly puzzling, since informed investors submit price-contingent bids. If an investor submits a bid above the offer price, why don't underwriters increase the offer price and completely fill the bids of high bidders?

Empirically, they do not: informed investors who submit high bids in IPOs often do not receive the full quantity they bid for. For example, Cornelli and Goldreich (2001a) describe the bookbuilding process and examine the bids and allocations of one bookrunner in the UK. They find that informed bidders (who they describe as limit bidders) receive allocations only between 48.2% and 54.2% of their bids. Further, allocations that are received by limit bidders do not appear to depend on the level of their bid (contingent on it being higher than the offer price).

Allocations in IPO's have recently come to the forefront of policy debate.³ Is rationing direct evidence of mispricing? That is, could a seller simply choose a higher offer price and increase revenue? If the interests of the investment bank and entrepreneur differ, how much is the entrepreneur hurt by the investment banker's ability to allocate shares? Further, when does the selective allocation of shares affect seller revenue? Given the literature on optimal mechanism design in common value environments, which allocation mechanisms do we plausibly expect to see and of those, which are revenue maximal?

The rationing of informed investors is counterintuitive for two reasons. First, it contradicts the optimal IPO mechanism of Benveniste and Spindt (1989), who postulate that, amongst informed bidders, those with higher signals receive their full allocation before any shares are given to bidders with lower signals. Second, it seems to contradict profit-maximization on the part of the seller. An IPO often proceeds by a bookbuilding process, during which a demand curve is generated for the shares

¹See, for example, Cornelli and Goldreich (2001a), and the survey paper by Ritter (1998).

²One argument is that the investment bank needs to reward informed investors to convince them to yield their information (see Benveniste and Spindt, 1989, for example).

³For example, see Ritter and Welch (2002) for a survey of the academic questions. The SEC has also recently expressed in this issue ("Harvey Pitt wants people to look at IPO pricing and allocations," *Wall Street Journal*, Aug 23, 2002).

to be sold.⁴ Yet, faced with this demand curve, the seller frequently chooses a price below the market-clearing price. Why not choose a higher price, reduce the degree of rationing, and hence increase revenue?

In this paper, we construct a stylized version of the bookbuilding process for a common value asset. We consider different allocation rules the seller may use, including high bid priority, pro rata rationing, and aggregate rationing with discretionary allocation. We find that, since the book is private information (known only to the underwriter), pro rata rationing of regular investors (which includes market-clearing as a special case) is the only credible way of allocating shares. Within this class of credible mechanisms, it can be optimal to ration, rather than choose a market-clearing price. Thus, oversubscription and rationing are not *prima facie* evidence that the issue price was sub-optimal, and the issuer could have raised more revenue.

We quantify the effects of different allocation schemes in a parametric model. This allows us to determine the benefits and costs of credibility in commitments made by the seller in terms of their impact on issuer revenue. We find that high bid priority (which requires greater commitment on the part of the seller) can yield higher revenue than pro rata rationing (which is a credible mechanism). However, the magnitude of the gain in revenue from using high bid priority is small. Further, rationing allows for discretionary allocations among investors, which may have long-term benefits for the seller. This suggests that the current debate over discretionary allocations is misplaced to the extent that seller revenue is not hurt by such allocations.

Rationing shares at the offer price mitigates informed investors' fear of the winner's curse, and may thus increase seller revenue. Our mechanism includes the standard auction format as a special case. Indeed, our view corresponds to that expressed by Benveniste and Busaba (1990) and Sherman (2001), that auctions are merely a special case of book-building.

We show that a commitment to rationing can be revenue-maximizing (i.e., optimal for the seller) even in a single issue, absent any long-term effects. That is, we provide conditions under which the rules chosen by an investment banker do not hurt an entrepreneur, even if their interests diverge. For a specific class of signals, we solve numerically for the optimal degree of rationing. Thus, within our model, we are able to characterize the observed dispersion of bids and offer prices, and relate these to the allocations.

⁴A survey of international IPO practices is presented in Sherman (2001).

The effects of rationing on revenue are small; even when rationing is sub-optimal (compared to market clearing), it does not lead to a great loss of revenue. Hence, our paper does not offer an explanation for the large amounts of underpricing documented on the first day of trading.⁵ However, our model suggests that rationing is not the cause of this underpricing: instead, other explanations must be sought.

Since rationing can increase the proceeds of the IPO, “money left on the table” cannot be estimated by the difference between the offer price and the long-term value of the asset. The offer price is determined by the bids submitted by investors. We demonstrate that the observed demand curve depends on the mechanism offered by the seller: if investors anticipate a different allocation mechanism (in particular, a different degree of rationing), they will submit different bids. Thus, caution must be exercised in performing thought experiments of this nature on the demand curve.

Finally, we add to the auction literature in two ways: first, we characterize equilibria under commitment to ex ante rationing. We demonstrate that, for a wide class of allocation distributions, investors make the same bids that would be made given pro rata rationing, with no discretionary allocation, at the offer price. Further, the symmetric equilibrium for such allocations is the same as that in the multi-unit auction of Milgrom (1981). Second, we characterize the equilibrium in the third and high price common value auction.

The intuition that drives our results is straightforward: changing the degree of rationing potentially affects both elements of the winner’s curse, the expected consumption value of the asset conditional on winning, and the price which the winner expects to pay. If informed investors are not rationed, then they only get shares when they are among the highest bidders. If they win the asset, their ex ante estimate of the value of the asset is higher than that of any bidder who did not get the asset. They take this fact into account when they bid; i.e., they reduce their bids to avoid this “winner’s curse.” By contrast, consider pro rata rationing. Under this scheme, an investor can win the asset even when many other investors have higher signals. This causes every bidder to bid higher than they would in the absence of rationing. However, if a seller increases the degree of rationing, he selects a bidder with a lower estimate of the value of the shares. The optimal degree of rationing is determined by this tradeoff.

While we do not explicitly distinguish between issuers (i.e., firms going public)

⁵Ritter (1998) surveys the underpricing literature across different countries.

and underwriters, it is reasonable to suppose that their interests may diverge.⁶ Indeed, NASD has recently suggested explicit prohibitions of the preferential allocation of shares to investors, provided in consideration of future business. This practice, (called spinning) suggests that investment bankers do provide preferential allocations to some investors. While not all preferential allocations are necessarily bad, it is important to recognize that such discretion cannot exist unless there is oversubscription. However, we predict that the differences in revenues between oversubscription and market clearing are small. Thus, even if rationing is sub-optimal, issuers are not substantially damaged by these idiosyncratic allocations. This view is supported empirically by Ljungqvist and Wilhelm (2002), who conclude that discretionary allocation does not harm issuers.

In our model, a bookrunner has k units for sale and commits to allocate them across t bidders at the $(t + 1)^{st}$ highest bid. In particular, t may be greater than k , in which case the seller uses an allocation rule. For example, with pro rata rationing, all winners may be allocated an amount $\frac{k}{t}$. Under high bid priority, the highest k bidders are allocated the asset. Bidders in our model all receive signals about the value of the asset from the same distribution. They then submit sealed bids for the asset. Thus, we are considering informed bidders who submit limit bids (in particular, their bid consists of a price they are willing to pay).

Our model is, of course, a highly stylized description of an IPO process. In the U.S., an IPO is typically preceded by a “road show,” during which the underwriter makes presentations to groups of buyers in various cities, and often meets with important buyers one-on-one (see, for example, Ritter, 1998). During this road show, the lead investment banker also solicits information from the buyers on quantities they are interested in buying, and the associated prices at which they are willing to buy. In this book-building process, effectively “a demand curve is constructed” (Ritter, 1998). In our model, we interpret the construction of this book or demand curve as analogous to soliciting sealed bids from potential buyers. To the extent that a buyer is unaware of the price and quantity pair submitted by another buyer, this is equivalent to the simultaneous submission of sealed bids.

There are several features about the IPO process that we omit from our model.

⁶Biais, Boessarts and Rochet (2002) present a model in which institutional investors collude with bankers against issuers. The recent \$100 million settlement between Credit Suisse First Boston and the SEC supports this view.

First, we fix the demand for each buyer to be a single unit.⁷ Second, we consider a simultaneous game, in which bidders bid only once. With IPOs, the bookbuilding process is usually followed by the seller announcing a price at which shares will be sold. Bidders are then allowed to re-submit quantities they wish to buy at this price. In other words, they are given a chance to revise their bids, which we do not allow in our model.⁸

A seminal theoretical piece on rationing, the winner's curse, and IPO underpricing is Rock (1985). Though our model contains both rationing and a winner's curse, it is starkly different. In Rock's model, rationing is *ex post* (that is, occurs at the allocation stage in some states of the world), and is therefore an embodiment of the winner's curse. There are two states of the world, and rationing is evidenced only in the good state. In the bad state, there is no rationing: informed bidders reduce their demand, and uninformed bidders obtain the shares they requested. Hence, in this state, they are subject to the winner's curse.

In our model, all bidders are informed and the rationing is *ex ante* (that is, the seller chooses a rationing mechanism before any bids are submitted), and thus mitigates the winner's curse. Importantly, the seller commits to an equal degree of rationing in all states of the world. *Ex post* rationing hurts Rock's seller (it leads to lower revenue); *ex ante* rationing benefits the seller in our model. Thus, in Rock's model *ex post* rationing is the embodiment of the winner's curse and generates underpricing while in our model, *ex ante* rationing mitigates the winner's curse.

The distinction between *ex post* and *ex ante* rationing also distinguishes our work from Benveniste and Spindt (1989). Biais and Faugeron-Crouzet (2000) demonstrate that, given a proper choice of parameters, the French *mise en vente* mechanism replicates the optimal Benveniste–Spindt one. In the Benveniste–Spindt framework, the underwriter presells some of the issue to informed regular investors. As an incentive to reveal information, investors with good information are provided the quantity demanded by them, and are not rationed. We note that, in our model, we consider only informed bidders, and hence (unlike Benveniste and Spindt), we provide no predictions on the relative allocations between informed and uninformed bidders. As in Rock (1985), uninformed investors are rationed when a large number of informed

⁷In multi-unit settings, Back and Zender (1993), building on work by Wilson (1979), show that the uniform price auction has self-enforcing collusive equilibria, leading to lower revenue than discriminatory auctions.

⁸See footnote 13, however.

investors have good signals. In contrast, in our model, the degree of rationing is committed to in advance of any bids being received. Hence, this degree of rationing is constant across states of the world.

While both Benveniste and Spindt (1989) and the model in this paper consider financial assets to be common value objects, a critical difference is that our investors receive correlated signals. In Benveniste and Spindt, investors have independent signals. As we show in this paper, these two different sets of assumptions have very different implications for seller's revenue.

Much previous work on IPOs, empirical and theoretical, has focused on the underpricing phenomenon. Short-term underpricing has been extensively documented across various countries (Ritter, 1998, surveys these studies). We note in passing that in recent work, Brav and Gompers (1998) argue that long-run underperformance is “not an IPO effect. Similar size and book-to-market firms which have not issued equity perform as IPOs.”

However, the presence of underpricing is not surprising; indeed, it should be expected. Given a liquid secondary market, it is reasonable to consider shares in an IPO as common value assets. In most common value environments, the price a seller receives is lower than the full information value of the object, or every seller makes a loss.⁹ Buyers know that, if they are awarded the good, their information is more optimistic than that of other investors. Thus, everyone bids less aggressively to avoid being subject to the “winner's curse.” This makes underpricing endemic in common value markets.¹⁰ If rationing mitigates the winner's curse, then it can increase seller revenue, or, equivalently, decrease the degree of underpricing.

Some previous work in this area (including Krigman, Shaw, and Womack, 1999, and Loughran and Ritter, 1999) considers the first-day price gain in the secondary market to be money left on the table. Loughran and Ritter (1999) provide an explanation for this based on prospect theory, relying on the issuer having asymmetric utility over gains and losses, and a large increase in the reference level of wealth fol-

⁹Pesendorfer and Swinkels (1997) and Kremer (2000a), document conditions under which the price converges to the true value of the asset. Briefly, these conditions include both the number of bidders and the number of shares going to infinity.

¹⁰McAfee, Macmillan and Reny (1989) show that it is possible for sellers to extract full surplus (eliminate underpricing) if bidders' signals are correlated. However, their stylized mechanism requires that buyers agree ex ante to buy the asset at a price designated by the seller, even if on hearing the price, they might not want to trade. This mechanism therefore, bears little resemblance to real-world institutions.

lowing an IPO. Our model, on the other hand, suggests that it may be misleading to measure money left on the table by the first-day price gain in the secondary market. Changing the IPO mechanism will lead agents to revise their bids, and may lead to lower revenue than currently obtained. In other words, it is misleading to simply look at the eventual demand curve, and claim that there is money left on the table: the demand curve itself is constructed from investors playing best responses to a mechanism in place, and changes in that mechanism will inevitably lead to changes in the demand curve.

Our model draws upon previous work in auction theory, notably Milgrom (1981) and Pesendorfer and Swinkels (1997). Milgrom considers an auction for a common-value object, where the seller has t units for sale and each buyer demands one unit. The t highest bidders are each given one unit, at the $(t+1)^{st}$ highest bid. Pesendorfer and Swinkels (1997) show that the t -unit auction has a unique symmetric equilibrium (albeit under conditions that are stronger than those in our paper) and examine properties of the convergence of the price to the true value of the object. We show that the equilibria in this model are identical to those in our rationing mechanism.

To the extent that bidders bid one price in our model, the framework is different from Wilson (1979). In the latter, bidders submit demand functions. We assume constant marginal valuation for each fractional amount of one unit, or flat demand curves. Hence, our bidders submit a single price, which may be interpreted as the amount they are willing to pay for the entire quantity of the good that is for sale.¹¹

In a model with three agents, independent signals, and almost common values, Bulow and Klemperer (2001) demonstrate conditions under which rationing is the optimal mechanism for the seller. The precise condition they identify is that the hazard rate of signals must be decreasing. Under this condition, the optimal mechanism involves the seller posting a price, at which all three buyers are willing to buy the good. While this condition is violated in our example with a uniform distribution in Section 4, Bulow and Klemperer offer an intuition for our results in terms of a game they introduce called the “Maximum Game.” We comment on this in greater detail in Section 4.

The rest of this paper is organized as follows. Section 2 outlines our general

¹¹Other papers that build on Wilson, (1979) and consider models in which bidders submit demand (or supply) functions include Klemperer and Meyer (1989), Kyle (1989), Back and Zender (1993), and Kremer (2000b).

model. Section 3 examines some properties of bid and revenue functions in the general case. Section 4 discusses a special case of the signal distribution that proves more tractable and allows for explicit revenue comparisons, and is followed by some concluding remarks, in Section 5. All proofs are relegated to the Appendix.

2 Model

We model the bidding behavior of informed investors in an IPO. The seller wishes to sell k shares to $n > k$ investors, each of whom demands one share. Following Benveniste and Busaba (1996) and Sherman (2001), we model the bookbuilding mechanism as a generalization of a multi-unit common value auction. In practice, of course there are many distinctive features of bookbuilding, including information sharing and gathering.¹² However, once bids have been entered in the book, they are essentially firm.¹³ We thus view them as sealed bids.

In the bookbuilding mechanism, the seller announces a rationing rule, which has two components. First, the seller chooses a t , where $k \leq t < n$. He promises that the IPO offer price will be equal to the $(t+1)^{st}$ highest bid. If $t > k$, he further announces an allocation rule, to divide the shares across the t highest bidders. An investor then receives a private signal about the value of the asset, and submits a bid for one share. Finally, based on the book which only he observes, the seller announces an offer price and the k shares are allocated to investors. Investors' bids (i.e., the details of the book) are confidential throughout, and are not released even after the IPO.

Thus, the output of the bookbuilding mechanism is an offer price and a set of allocations to investors. When $t = k$, each of the highest k bidders is allocated one share. This effectively represents market-clearing. That is, there is no rationing, and the allocation received by the winning bidders is 100% of what they requested. In contrast, for $t > k$, there is rationing. Each of the t bidders have indicated a willingness to buy the item at strictly greater than the offer price. Hence, in the aggregate, potential winners must be rationed, to a degree $1 - \frac{k}{t}$.

We distinguish between two allocation rules conditional on an offer price. The

¹²Spatt and Srivastava (1991) discuss some of these features, including the communication between the underwriter and the investors.

¹³For example, Welch (1999) mentions that "In reality, an institutional investor who backs out after such informal requests (especially if it is close to the effective date) may not receive shares in future offerings; consequently, such indications of interest are practically firm."

first is to allocate one share to each of the k highest bidders, and the second allows the remaining $(t - k)$ bidders who also bid above the offer price to receive some shares.

Definition 1 Consider some integer $t \geq k$. In a (k, t) -bookbuilding mechanism, the offer price is set to the $(t + 1)^{st}$ highest bid. Each bidder who receives a positive allocation pays this price. Further,

(i) under *high bid priority*, each of the highest k bidders receives one share, and the remaining $(t - k)$ bidders with bids greater than or equal to the offer price receive no allocation.

(ii) under *share dispersion*, the k shares are allocated amongst the t highest bidders. No bidder receives more than her demand (i.e., one share), and, conditional on being amongst the t highest bidders, the allocation received by an agent is independent of her bid.

Many allocation rules are consistent with share dispersion, including:

(i) pro rata rationing: each investor in the set of potential winners receives $\frac{k}{t}$ shares,
(ii) random allocation: the seller randomly chooses k of the t potential winners; each of these k bidders receives their full demand (one share), and the others receive nothing.

(iii) discretionary allocation: the seller wishes to reward a set of long-term customers, investor 1 through i . These customers (preferred investors) receive their full quota (i.e., one share) if they are in the set of potential winners, and nothing otherwise. Other winners (regular investors) have their allocations reduced accordingly, either in pro rata or random fashion.

We show in Section 4, in a parametric model, that high bid priority may yield higher revenue for the seller than share dispersion. However, as shown below in Proposition 1, it requires greater commitment on the part of the seller than pro rata rationing, and is thus less credible. There is then potentially a trade-off between the two schemes.

The shares have a common value to all investors, V , which is drawn from a uniform distribution on $[v_\ell, v_h]$. V represents the long-term value of the asset.¹⁴ We emphasize that we do not expect V to be represented by the price at the end of the first day (or the first week) of trading in the secondary market.

¹⁴Notationally, we use upper case letters denote random variables, and lower case ones to denote particular realizations of random variables. Thus, v denotes a realization of random variable V .

Investors have private information about the long-term value of the asset. The information of investor i is represented by a signal, S_i . The signals of different investors are conditionally independent (given V), but all depend on V in the following manner. Each S_i , for $i = 1, \dots, n$, is independently drawn from the same atomless distribution $G(\cdot | V = v)$, with support $[v - \epsilon, v + \epsilon]$ for some $\epsilon > 0$. Here, ϵ represents dispersion of opinion about the value of the asset. We assume that $v_h - v_\ell > 2\epsilon$, so that informed investors' beliefs are more precise than the prior over V .

Hence, given V , the height of the conditional signal distribution at any signal, s , depends only on the position of the signal, relative to the lowest possible value ($v - \epsilon$). Therefore, a higher value of v leads to a shift in the support of the signal distribution, but the distribution has the same shape, given the support.

Formally,

Assumption 1 (i) $G(s | v)$ is an atomless distribution with support $[v - \epsilon, v + \epsilon]$, with density $g(s | v)$.

(ii) For any pairs (s, v) and (\tilde{s}, \tilde{v}) , if $s - v = \tilde{s} - \tilde{v}$, then $G(s | v) = G(\tilde{s} | \tilde{v})$.

(iii) (MLRP): $\frac{g(s|v)}{g(s|v')} \geq \frac{g(s'|v)}{g(s'|v')}$ for all $s > s'$, $v > v'$ such that s, s' are both in the support of $G(\cdot | v)$ and $G(\cdot | v')$ respectively.

Part (ii) of the assumption further implies that $g(s | v) = g(\tilde{s} | \tilde{v})$ when $s - v = \tilde{s} - \tilde{v}$. Part (iii) is a variant of the Monotone Likelihood Ratio Property.¹⁵

Note that, in our model, ϵ is independent of V . Intuitively, this means that the dispersion of investors' opinions over value does not depend on whether V is high or low. That is, ϵ does not depend on the long-term per share price of the asset. Empirically, ϵ can be inferred from the range of analyst forecasts over the value of the asset.¹⁶

We consider one-shot equilibria of the bookbuilding game. In practice, the set of bidders in an IPO varies from issue to issue. For example, in the 39 issues that Cornelli and Goldreich (2001a) analyze, only 16.8% bid in at least 10 issues. There are, therefore, several bidders who rarely participate in more than one deal. These players will perform bid as if in the one-shot game. Further, amongst the long-term

¹⁵In our model, MLRP cannot hold for all s, s', v, v' because a particular s can lie outside the support of $G(\cdot | v)$, and hence have a density of zero.

¹⁶Analyst forecasts have been used to proxy for the dispersion of beliefs over asset values in the accounting literature; see, for example, Ajinkya, Atiase, and Gift (1991), and the subsequent literature.

players, absent explicit collusion, there may not be a knowledge of which bidders are participating in which transaction. Since long-term bidders also pick and choose transactions, the usual punishments seen in repeated games are difficult to enforce. Hence, we consider equilibria of the one-shot game amongst investors, and ignore repeated game effects in bidding. Note, however, that the underwriter, being a long-term player, may be interested in maintaining a reputation.

2.1 Detectable Deviations by the Seller

Benveniste and Spindt (1989) provide an optimal mechanism for IPOs when investors' signals are independent. Since we have correlated signals, their mechanism does not apply. The optimal mechanism with our signal structure is exhibited by McAfee, MacMillan, and Reny (1989), who demonstrate that it is possible for the seller to extract the full surplus from buyers.¹⁷ Their mechanism includes a strong individual rationality constraint on the buyers. A seller must have the ability to potentially inflict large punishments on the buyers if they reject the offer.

Such mechanisms are not observed in practice, partly because the seller's ability to penalize the buyers is limited. We therefore consider mechanisms in which no buyer can be forced to accept the asset at a price higher than her bid. Further, we assume the seller can commit to choosing a price that is equal to one of the received bids (without necessarily committing to the $(t + 1)^{st}$ bid). Such a commitment, of course, is required in *any* auction process, be it private or common values.¹⁸

After the IPO, the seller announces the issue price and informs each investor of her allocation. For each investor, the contents of the book remain unknown, as do allocations awarded to other investors. Potentially, the seller could deviate from the announced pricing rule, and choose a higher bid to be the price.

Some allocation rules allow the seller to credibly demonstrate that he has not

¹⁷In particular, the seller asks a subset of buyers to reveal their signal, and then makes a take-it-or-leave-it offer to some subset of the remaining buyers. In equilibrium, the second set of buyers accepts this offer.

¹⁸For example, consider a first-price private values auction. Suppose bidders bid as if the seller had committed to selling at the first-price. A seller for whom this commitment was not binding could invert the bidding function to determine the value of the object to the highest bidder, and make a take-it-or-leave-it offer to the bidder at that value. Similarly, in the second-price auction, a seller could make a take-it-or-leave-it offer at a price equal to the winner's bid. Of course, such ex post violations of an implicit commitment change the bid functions in equilibrium. Our point is that every auction mechanism requires such commitment.

cheated in this manner. Suppose the seller deviates from his announced scheme of setting the offer price equal to the $(t + 1)^{st}$ bid. The only incentive compatible deviation is to choose a higher bid as the offer price (since choosing a lower bid leads to lower revenues).¹⁹ We say a deviation of this nature is detectable if there is an investor who can determine, from her allocation alone, that the issuer must have deviated from the announced pricing scheme. In particular, since the book is private throughout, deviations can only be detected by an investor based on her own bid and allocation, and the announced offer price.

Let $b_{(i)}$ denote the i^{th} highest bid, with $b_{(t+1)}$ representing the $(t + 1)^{st}$ highest bid.

Definition 2 Suppose the seller announces that he will price at $b_{(t+1)}$, but instead offers the IPO at $b_{(\tilde{t}+1)}$, where $\tilde{t} < t$. The pricing deviation is *detectable* if, for some investor i , the actual allocation is not feasible, given t , the announced IPO price, and her own bid.

Of the allocation schemes discussed, pro rata rationing has the appealing property that it represents a credible commitment on the part of the seller: deviations are detectable. Conversely, under high bid priority, we show that the seller could choose the $(k + 1)^{st}$ highest bid to set the price, instead of the $(t + 1)^{st}$ highest one, and not be detected.

Proposition 1 *Pricing deviations by the seller are:*

- (i) *always detectable under pro rata rationing with no discretionary allocation, and sometimes with discretionary allocation*
- (ii) *are not detectable under high bid priority or random allocation.*

Besides credibility, there are other advantages to pro rata rationing. Many exchanges have requirements on the distribution of shares across investors.²⁰ Thus, one of the goals of an IPO must be to generate a dispersed shareholder base. It has also been suggested that share dispersion per se increases the value of a firm.²¹ In addi-

¹⁹Since underwriters are typically paid a constant proportion of revenues from the transaction, we assume here the seller's goal is to maximize revenue.

²⁰For example, the NYSE requires at least 500 holders of round lots, while the NASD requires 400.

²¹Booth and Chua (1996) consider a model in which dispersed ownership increases secondary market liquidity and hence the value of the firm.

tion, Brennan and Franks (1996) argue that rationing is used because current owners want to reduce the block size of new shareholders.

Markets which exhibit pro rata rationing, include Singapore (Koh and Walter, 1989), Israel (Amihud, Hauser and Kirsh, 2002), and the UK (Levis, 1990). Ljungqvist and Wilhelm (2002) describe the allocation methods used in the UK, Germany, France and the US. In Germany, the June 7, 2000, guidelines promulgated by the Federal Ministry of Finance tries to rule out “subjective” criteria for allocating shares to retail investors. It recommends that issuers draw lots, allocate pro rata either within certain order sizes or across the whole offer, or allocate according to time priority or some other “objective” criteria. One of the mechanisms adopted in France, the *offre à prix ferme*, has pro rata allocations at a fixed price. Further, the current legal troubles of Salomon, Smith Barney over preferential allocation of shares in IPOs²² would appear to enhance the appeal of pro rata rationing, with no discretionary allocation, in the US.

2.2 Equilibrium in the Bookbuilding Mechanism

Next, we consider symmetric equilibria of the mechanism under the two classes of allocation rules: high bid priority and share dispersion. As observed in Proposition 1, the allocation rules under either high bid priority or share dispersion (except for pro rata rationing) allow the seller to deviate and charge a higher price than announced, without being detected by the investors. To evaluate the tradeoff between the revenue raised by an allocation rule under ideal conditions and the credibility of the seller’s commitment, we assume from now on that the seller can commit to charging a price equal to the $(t + 1)^{st}$ highest bid.

In many cases, the equilibrium in the absence of such commitment is clear. Consider high bid priority, for example. Once the bids have been received, in the absence of commitment the seller should charge a price equal to the $(k + 1)^{st}$ highest bid. Since the investors anticipate this, they bid exactly as they would if $t = k$.²³ In principle, the ability to commit to the $(t + 1)^{st}$ highest bid should yield greater revenue to the seller. However, as we show in a parametric model in Section 4, the gain in revenue is small.

²²See, for example, “Ex-Broker Says Salomon Gave IPOs to CEOs to Win Business,” *Wall Street Journal*, July 18, 2002.

²³The bidding function identified in Proposition 4 below therefore applies to this case, with $t = k$.

Given an allocation rule, a bidder observes her own signal s , and chooses a bid. We consider a symmetric Bayesian-Nash equilibrium, in which all bidders choose the same bid function, and a bidder with signal s bids $b(s)$. It is natural to consider equilibria in which $b(\cdot)$ is strictly increasing in s ; that is, bidders with higher signals submit higher bids.

For high bid priority, in the general case, we demonstrate that the bid function must satisfy a particular restriction. In Section 4, in a parametric model, we use this restriction to derive the bid function.

Let $Y_{j,n}$ be a random variable representing the j^{th} highest order statistic of bidders' signals, where n signals are drawn. Hence, $Y_{j,n} \geq Y_{j+1,n}$ for all $j = 1, \dots, n - 1$.

Proposition 2 *Suppose $b(\cdot)$ is a symmetric equilibrium of the (k, t) -bookbuilding mechanism with high bid priority, and $b(\cdot)$ is strictly increasing in s . Then, for each s ,*

$$E[b(Y_{t,n-1}) \mid Y_{k,n-1} = s] = E[V \mid S_1 = Y_{k,n-1} = s]. \quad (1)$$

Condition (1) highlights the two elements of the winner's curse. The right-hand-side is the expected consumption value to the marginal bidder who wins and is exactly tied with one other bidder. In that circumstance, the left-hand-side reflects how much this bidder expects to pay for the asset.

Next, we turn to share dispersion. We show that the equilibrium bids (and, by implication, seller revenue) do not depend on the particular allocation rule used, provided it satisfies the property that the allocations are independent of the actual bids of agents who bid more than the offer price. The intuition is that, conditional on being amongst the t highest bids, the allocation rule exposes each agent to a lottery. If the agent does not have one of the t highest bids, she gets zero allocation, with no access to the lottery. As long as the probability of receiving an allocation in the lottery is independent of the bids of any agent in the set of potential winners, the behavior of a risk-neutral agent will be unaffected.

Proposition 3 *Consider any two allocation rules that satisfy share dispersion. These rules result in the same set of equilibria.*

Proposition 3 implies that, under commitment, all allocation rules that satisfy share dispersion are equivalent (in terms of bids and revenues) to the pro rata rationing rule with no discretionary allocation, though each of these implies a very different set of final allocations. Thus, if a seller allocates shares in a way that benefits clients with whom he has a long term relationship, this is not to the detriment of the issuer. We note again, in this class of allocation rules, pro rata rationing of regular investors is special, in that the seller's commitment is credible.

Seller discretion over the eventual allocations is absent under high bid priority. In particular, in the latter mechanism, the underwriter does not have any means of differentially rewarding preferred investors. To the extent that rewarding preferred investors contributes any benefits in the long-term, this would be another reason to prefer share dispersion over high bid priority.²⁴

Now, suppose $t = k$. In this case, the allocations under high bid priority and share dispersion are identical: each of the t highest bidders gets her full demand (one unit). The bookbuilding mechanism in this case is, therefore, equivalent to a common value auction with t units in which each bidder demands one unit, and winning bidders all pay the $(t + 1)^{st}$ highest price. This auction was first analyzed by Milgrom (1981). We refer to this as the t -unit auction.

We show that the set of equilibria when the seller has k shares for sale, and rations across $t > k$ investors, is identical to the corresponding set when the seller actually sells t shares with no rationing. For a fixed t , in either of these mechanisms, a winning bidder has the same information: she knows she has one of the t highest signals. Hence, her bids in the two mechanisms are the same.

The symmetric equilibrium of the t -unit common-value auction was characterized by Milgrom (1981). Without loss of generality, consider the behavior of bidder 1. As Milgrom (1981) shows, bidder 1, with signal s , bids as if her signal is equal to the t^{th} highest (or pivotal) signal amongst the remaining $(n - 1)$ bidders. This is the price at which she is indifferent between winning and losing the asset. In a symmetric equilibrium, all bidders choose this strategy.

Proposition 4 *For a (k, t) -bookbuilding mechanism under share dispersion*

(i) the set of equilibria is equivalent to the set of equilibria in the t -unit auction.

²⁴See, for example, Sherman (2000) on benefits to the seller and Stoughton and Zechner (1998) on benefits to the issuer.

(ii) there is a symmetric equilibrium in which a bidder with signal s chooses a bidding function

$$b(s; t) = E[V \mid S = Y_{t,n-1} = s].$$

3 Bids and Revenue Under Share Dispersion

Under share dispersion, we have a closed form expression for the bid function, as given by part (ii) of Proposition 4. By contrast, under high bid priority, we have only an implicit bid function in Proposition 2. In this section, we examine the equilibrium bid function under share dispersion, and in particular demonstrate how it depends on the level of rationing. We further determine the revenue earned by the seller. We return to high bid priority in a parametric model, in Section 4.

3.1 Bids

Recall that, under share dispersion, a bidder bids the expected value of the asset, conditional on her signal being equal to the pivotal signal, or the t^{th} highest of the remaining $(n - 1)$ signals. In Proposition 5 below, we demonstrate that the bid function can be represented as the signal plus an adjustment term. Significantly, the adjustment term is constant over a range of signals, rendering the bid function linear in this range. To demonstrate this, we first characterize the inference problem (over V) of a bidder, given her own signal.

Let $\mu(v \mid s)$ be a bidder's posterior distribution over V , given her own signal s . Notice that the support of this posterior distribution depends on the signal.

Lemma 1 *Given a signal s , the support of μ , the agent's posterior belief over V , has range*

- (i) $s + \epsilon - v_\ell < 2\epsilon$ if $s \in [v_\ell - \epsilon, v_\ell + \epsilon]$
- (ii) 2ϵ if $s \in [v_\ell + \epsilon, v_h - \epsilon]$
- (iii) $v_h - s + \epsilon < 2\epsilon$ if $s \in (v_h - \epsilon, v_h + \epsilon]$.

Therefore, if signals are very low (i.e., within ϵ of v_ℓ) or very high (i.e., within

ϵ of v_h), the support of the posterior is truncated to a range less than 2ϵ , because agents know that the lowest and highest values of V are v_ℓ and v_h respectively. For all other signals, the posterior has support 2ϵ . We term such signals *interior signals*. An important implication of this is that, when signals are in the interior, the distribution of the posterior is the same relative to the support (in the same sense that $G(s | v)$ depends only on $(s - v)$).

We now proceed to show that the bid functions are linear in signal for interior signals, regardless of the actual distribution $G(s | v)$. In doing this, we use the fact that the inference problem for interior signals is independent of the actual value of the signal. Below, we define an inference problem over “pseudo-signals” that helps determine the bid function for interior signals.

First, consider the usual inference problem faced by a bidder in this environment: given her own signal s , and the rules of the mechanism, she wishes to determine the expected value of V . Let $\underline{v}(s) = \max(v_\ell, s - \epsilon)$, and $\bar{v}(s) = \min(v_h, s + \epsilon)$. Thus, $\underline{v}(s)$ and $\bar{v}(s)$ are the possible bounds of V , given a signal s . Since $\mu(v | s)$ represents the posterior distribution over V , given a signal s , the corresponding density is then defined as $d\mu(v | s) = \frac{g(s|v)}{\int_{\underline{v}(s)}^{\bar{v}(s)} g(s|v)dv}$.

Therefore, to infer the value of V , given her own signal, a bidder must determine the relative probability that each possible v could have generated that signal. From assumption 1, this probability depends only on $s - v$, and not on the individual values of s or v . Thus, agents’ beliefs over v can be re-expressed as beliefs over $(s - v)$. In particular, what is important is the distance between s and the lowest possible signal that can be drawn for a particular v ; that is, $v - \epsilon$.

Hence, we define a pseudo-signal, $x = s - (v - \epsilon) = s + \epsilon - v$. Notice that x has support $[0, 2\epsilon]$, and is increasing in s and decreasing in v . For a fixed value of v , instead of thinking about a sample of signals being independently drawn from $[v - \epsilon, v + \epsilon]$, we can equivalently interpret this as a sample of values of x drawn from $[0, 2\epsilon]$.

Notice that, when v is fixed, x will have the same distribution as s . Let $H(\cdot)$ denote the distribution of x , with $h(\cdot)$ the density. From assumption 1, the distributions of the order statistics of the pseudo-signals will match those of the actual signals.

The standard inference problem for an investor in this mechanism is: given her own signal, s , she does not know the true value v . This inference depends on her beliefs over other agents’ signals. With our transformation, we can equivalently think

of an agent observing her s , and not knowing her pseudo-signal x . In turn, her inference of x will depend on her beliefs over other agents' pseudo-signals.

Define

$$\begin{aligned}\underline{x}(s) &= s + \epsilon - \bar{v}(s) \\ \bar{x}(s) &= s + \epsilon - \underline{v}(s).\end{aligned}$$

Since $\bar{v}(s) > \underline{v}(s)$, it follows that $\underline{x}(s) < \bar{x}(s)$. These represent the lowest ($\underline{x}(s)$) and highest ($\bar{x}(s)$) possible values of a pseudo-signal x , given a signal s (recall that v is unknown to the investors).

Consider a particular pseudo-signal, X . Let x denote the realization of X , and let $Z_{t,n-1}$ denote the highest of the remaining $(n-1)$ pseudo-signals. Next, when $\underline{x}(s) < \bar{x}(s)$, define

$$\delta(s, n, t, \epsilon) = \frac{\int_{\underline{x}(s)}^{\bar{x}(s)} y H(y)^{n-t-1} (1-H(y))^{t-1} h(y)^2 dy}{\int_{\underline{x}(s)}^{\bar{x}(s)} H(y)^{n-t-1} (1-h(y))^{t-1} h(y)^2 dy}$$

If $\underline{x}(s) = \bar{x}(s)$, let $\delta = \underline{x}(s)$.

Conditional on winning in a (k, t) -bookbuilding mechanism, an agent does not know her own x , but does know that it is equal to $Z_{t,n-1}$, that is, the t^{th} highest of the remaining $(n-1)$ pseudo-signals. She evaluates her expectation of x , conditioning on this information. This expectation is denoted by δ .²⁵

Proposition 5 *Consider the (k, t) -bookbuilding mechanism with share dispersion. For all s , $b(s; t) = s + \epsilon - \delta(s, n, t, \epsilon)$.*

The term $\delta(s, n, t, \epsilon)$ is readily interpreted as an adjustment for the winner's curse. That is, it compensates for a bidder having received a signal that is equal to the t highest of the remaining signals. By definition, it depends on $H(\cdot)$ (and hence $G(\cdot)$) for all signal values.

When signals are in the interior, the support of x is $[0, 2\epsilon]$, and is independent of s . Thus, $\delta(\cdot)$ is independent of s and this implies that, for signals in this range, the bid function is linear, with a slope of 1.

²⁵Recall that the density of the t^{th} highest order statistic of a sample of n independent draws is given by $\nu(y) = \frac{n!}{(n-t)!(t-1)!} H(y)^{n-t-1} (1-H(y))^{t-1} h(y)$.

Corollary 5.1 *If $s \in [v_\ell + \epsilon, v_h - \epsilon]$, then, under share dispersion, the bid function is linear in s . In particular, $b(s; t) = s + \epsilon - \delta(n, t, \epsilon)$.*

In other words, regardless of the shape of $G(\cdot)$, the bid function over interior signals is linear, and takes the form of the signal plus a constant (which may be positive or negative). Further, a change in t leads to a parallel shift of the bid function. Since this holds regardless of the shape of $G(\cdot | v)$, this considerably simplifies the revenue function when all signals lie in this range.

Note that the winner's curse adjustment, $\delta(\cdot)$, may be greater or less than ϵ . As mentioned earlier, the signals $v_\ell - \epsilon$ and $v_h + \epsilon$ are fully revealing (in the former, the true value must be v_ℓ , with the latter, it must be v_h). For all other signals, $\delta(\cdot)$ is greater than ϵ for small values of t , and less than ϵ for large values of t . Hence, when t is low, bidders bid less than their signal, and, when t is high, they bid more than their signal.

For a fixed k , the bid function, $b(s; t)$ is strictly increasing in t .²⁶ Since t is directly related to the degree of rationing (which is $1 - \frac{k}{t}$), the bid function therefore increases with rationing. Intuitively, a higher value of t implies that a bidder may win the object even if he did not receive the highest signal. Thus, the "winner's curse" is mitigated: conditional on winning, a bidder in the (k, t) -bookbuilding mechanism knows only that her signal was among the t highest. Hence, the greater the degree of rationing in the mechanism, the more aggressive each bidder is.

The implications of this are that there is a benefit to the seller from rationing: bidders bid more aggressively. There is, of course, the obvious cost of choosing a lower ranked bid to set the offer price. We show later that, under some conditions, the seller can increase her revenue by rationing.

Proposition 6 *In the (k, t) -bookbuilding mechanism with share dispersion, bid functions are strictly increasing in the degree of rationing. That is, for all $s \in (v_\ell - \epsilon, v_h + \epsilon)$, and all $t = 1, \dots, n - 2$, $b(s; t + 1) > b(s; t)$.*

Therefore, even when the number of shares being sold is held fixed, the equilibrium bids contained in the book will vary with the seller's commitment to rationing. Thus, inferences drawn from the book must condition on the degree of rationing.

²⁶Except, of course, for the extreme signal values, $v_\ell - \epsilon$ and $v_h + \epsilon$. At these values, which occur with zero probability, the true value of the asset is known with certainty.

What can we infer from realized bids (i.e., the book)? We show that, for low t , bidders shade their bids, in the sense of bidding less than the conditional value of the asset, given their signal. However, for high t , they increment their bids, by bidding more than this conditional asset value. This effect, therefore, must be taken into account in inferring bidders' beliefs on the conditional value of the asset from an observed book.

Proposition 7 *Consider the (k, t) -bookbuilding mechanism with share dispersion. For all $s \in (v_\ell - \epsilon, v_h + \epsilon)$, there exists a $\hat{t}(s)$ such that, for $t \leq \hat{t}(s)$, $b(s; t) \leq E(V | s)$, and for $t > \hat{t}(s)$, $b(s; t) > E(V | s)$.*

In Section 4, in a parametric model, we characterize the dispersion of bids submitted to the book and the dispersion of bids that receive positive allocations, and tie these to the degree of rationing.

3.2 Revenue

We next examine the effect of rationing on revenue. As the degree of rationing increases, the seller is awarding the good at the bid of a bidder with a lower signal. Thus, the seller trades off the increase in the bid function against the fact that he is awarding it to a lower order statistic.

To illustrate this, we first characterize the revenue of the seller. Let $R(v; t)$ denote the seller's expected revenue in the (k, t) -bookbuilding mechanism, conditional on the event $V = v$. In the (k, t) -bookbuilding mechanism, the seller earns the expected $(t + 1)^{th}$ highest bid, with n signal draws. That is, $R(v; t) = E [b(Y_{t+1, n}; t) | V = v]$.

The seller in our model is uninformed, and earns an ex ante revenue of $\hat{R}(t) = \int_{v_\ell}^{v_h} R(v; t) dv$ in the (k, t) -bookbuilding mechanism. The seller's goal in our model is to choose an optimal t ; i.e., the t that maximizes $\hat{R}(t)$. Market-clearing corresponds to choosing $t = 1$, whereas higher values of t imply rationing.

A necessary condition for seller revenue, $\hat{R}(t)$, to increase in t (over any range of t) is that $R(v; t)$ be increasing in t for some t and some v . Hence, we focus initially on $R(v; t)$.

When signals are in the interior, the bid function is linear in s . When $V \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, all signals will be interior. Hence, it follows immediately that the revenue function is linear in v over this range.

Proposition 8 *Consider the (k, t) -bookbuilding mechanism with share dispersion. Suppose $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. Then, for all $t = 1, \dots, n - 1$, $R(v; t) = v + E[Z_{t+1, n}] - \delta(n, t, \epsilon)$.*

This form of the revenue function illustrates the tradeoff for the seller. For each value of V in the relevant range, increasing t leads to a lower winner's curse, and hence a lower last term in the expression for $R(v; t)$. As mentioned before, this term proxies for the extent of the winner's curse. However, it also leads to a reduced order statistic being used to set the price of the item, which is captured by the $E(Z_{t+1, n})$ term.

The linear revenue function allows us to determine when rationing increases revenue. Indeed, the seller compares the increase in revenue (in going from a (k, t) - to a (k, \tilde{t}) -bookbuilding mechanism) obtained as a result of the increase in the bid functions to the decrease in revenue as a result of choosing the $(\tilde{t} + 1)^{st}$ bid to set the price of the item, rather than the $(t + 1)^{st}$ one. The left-hand side of equation 2 below is the increase in revenue and the right-hand side is the decrease.

Corollary 8.1 *Suppose $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. Then, for any $t, \tilde{t} = 1, \dots, n - 1$, $R(v; \tilde{t}) > R(v; t)$ if and only if*

$$\delta(n, t, \epsilon) - \delta(n, \tilde{t}, \epsilon) > E[Z_{t+1, n}] - E[Z_{\tilde{t}+1, n}]. \quad (2)$$

Hence, when $V \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, the comparison of $R(v; t)$ and $R(v; \tilde{t})$ is independent of the actual value of V in this range. That is, if (2) holds for some t, \tilde{t} , then $R(v; \tilde{t}) > R(v; t)$ for all $V \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. This condition is easy to check for any distribution.

While (2) is a necessary condition for higher degrees of rationing to yield higher revenue, clearly there are values of ϵ (given v_ℓ and v_h) such that it is also a sufficient condition. In particular, if (2) holds and ϵ is small relative to $v_h - v_\ell$, then ex ante revenue must be increasing in the degree of rationing.

Suppose (2) holds for some distribution G and some $\tilde{t} > k$, so that $R(v; \tilde{t}) > R(v; t)$ for all $V \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. In the corner regions, $V \in [v_\ell, v_\ell + 2\epsilon)$, and $V \in (v_h - 2\epsilon, v_h]$, we may have $R(v; \tilde{t}) < R(v; t)$ over some range. However, the difference, $R(v; t) - R(v; \tilde{t})$ is bounded. Hence, it follows that, if ϵ is small enough

relative to $(v_h - v_\ell)$, then $\hat{R}(\tilde{t}) > \hat{R}(t)$. In other words, if the signal is relatively precise, relative to the prior over V , seller's ex ante revenue increases when the higher degree of rationing, \tilde{t} , is chosen. In the next section, we demonstrate that $R(v; t)$ is increasing in t for interior values of v when the signal distribution is diffuse enough.

Under share dispersion, therefore, we can think of the seller choosing an optimal t , to maximize revenue. Since pro rata rationing of regular investors is credible for any t , this amounts to choosing an optimal mechanism in this class of credible mechanisms.

4 Parametric Model

While theory illustrates the tradeoffs between commitment and revenue maximization, it is useful to get a sense of the magnitude of these effects. This is important for policy questions. For example, we show in a parametric model in this section that share dispersion involves at most a small revenue loss, relative to the optimal rationing mechanism. Since discretionary allocations are one form of share dispersion, this suggests that, in terms of the revenues raised by the issuing firms, the current debate over discretionary allocations is a red herring.

To determine the magnitudes of these effects, we present a parametric model of signal distributions that has the uniform as a special case. We explicitly solve for the equilibrium bid functions under high bid priority and share dispersion. This allows us to compare seller revenue under the two schemes. We further compare this revenue to the revenue with no rationing, that is, under market-clearing. Interestingly, the bid functions in the latter case are those that would obtain if buyers believed that sellers would cheat, i.e., report a higher bid (in particular, the $(k + 1)^{st}$ bid) as the equilibrium IPO price. Thus, the difference in revenue between market clearing and rationing, can also be interpreted as the cost of a lack of a credible commitment on behalf of the seller.

These comparative statics are holding a t , fixed. From the tradeoff explored in the previous section, it is clear that the specific choice of t depends on the signal distribution. Thus, we solve for the optimal t as a function of the signal distribution. We find that for diffuse signal distributions, that is if there is uncertainty about the underlying value of the asset (as there might be in the IPO of a new industry), then a high degree of rationing is optimal. However, for signal distributions which are less diffuse, i.e., for firms for which there is less uncertainty about the cash flows, rationing

becomes less attractive.

From the tradeoff explored in the previous section, it is clear that the specific choice of t depends on the signal distribution. To explore this further, and to provide some economic intuition on this tradeoff, we consider a class of signal distributions that has the uniform as a special case.

For this class of distributions, we determine the bid and revenue functions under share dispersion, and exhibit the bid function under high bid priority. We show that high bid priority yields higher revenues to the seller. Given the advantages of pro rata rationing mentioned earlier, we numerically solve for the optimal t , given a signal distribution. We then provide comparative statics on the book, and show how allocations change with the exogenous variables in our model. This allows us to compute endogenous bid ranges, which generates empirical predictions.

We assume that the signal distribution is uniform, with an extra parameter that allows for strict convexity of the distribution function.

Assumption 2 For $s \in [v - \epsilon, v + \epsilon]$, and some $c \geq 1$,

$$G(s | v) = \left(\frac{s - v + \epsilon}{2\epsilon} \right)^c, \quad \text{and} \quad g(s | v) = \frac{c}{2\epsilon} \left(\frac{s - v + \epsilon}{2\epsilon} \right)^{c-1}.$$

When $c = 1$, this distribution reduces to the uniform. It is strictly convex for $c > 1$. Increases in c thus represent first-order stochastic shifts in the underlying density function. For all $c \geq 1$, this distribution satisfies Assumption 1 (MLRP).

Figure 1 plots the density and distribution functions for different values of c .

When $c = 1$, signals are most diffuse; that is, all signals in the range are equally likely. As c becomes large, the distribution becomes more concentrated. Indeed, as $c \rightarrow \infty$, the signal distribution converges to a point mass at $v + \epsilon$.

Observe that, when c is large and t is low, there is potentially a large winner's curse. Consider, for example, the case of $c = 2$ and $t = 1$. If a bidder wins, since the distribution function is steep at high s , her signal is likely to be further away from other bidder's signals than if c were, say, 1. Hence, for low values of t , bidders should shade their bids (relative to their signals) more for higher values of c . When t is high, the reverse logic holds: there is a probability that some bidders have higher signals, and, when the distribution function is steep, other bidder's signals can be

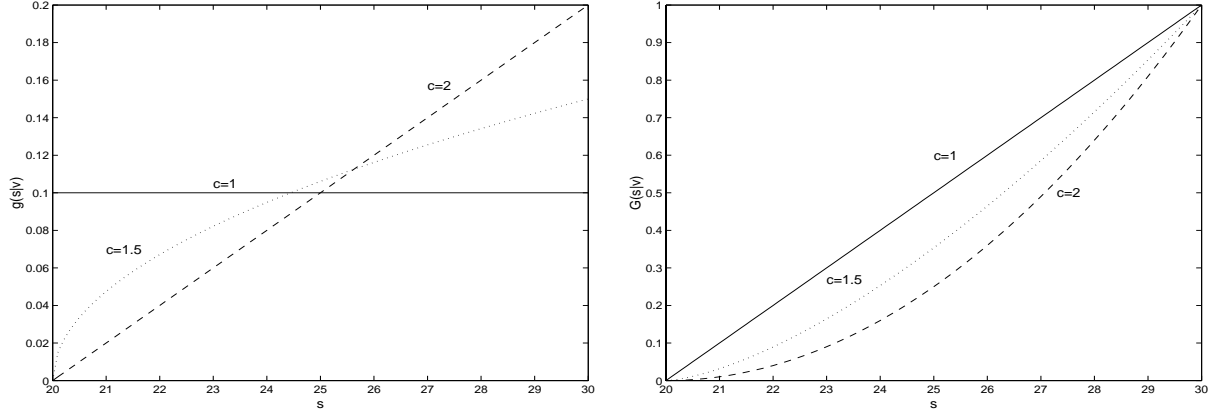


Figure 1: Density (left) and distribution (right) functions, for different values of c

significantly higher than that of the winning bidder. This intuition will be reflected in the equilibrium bid functions.

Recall that signals in the range $[v_\ell + \epsilon, v_h - \epsilon]$ are interior signals. We first derive the bid function under share dispersion and high bid priority for interior signals. In doing this, we assume that, under high bid priority, the seller can commit to allocating the good to the k highest bidders at the $(t + 1)^{st}$ price. We find that the bid function under high bid priority is (weakly) higher than the bid function under share dispersion. Since we analyze share dispersion extensively, we refer to the bid function in that regime as $b(\cdot)$, with $b^h(\cdot)$ denoting the bid function under high bid priority.

Define

$$\mu(n, k) = \prod_{j=0}^{k-1} \frac{n - j - \frac{1}{c}}{n - j}$$

$$\lambda(n, k, t) = \prod_{j=k+1}^t \frac{n - j}{n - j + \frac{1}{c}}$$

Proposition 9 *Suppose $s \in [v_\ell + \epsilon, v_h - \epsilon]$. Consider the (k, t) -bookbuilding mechanism.*

(i) *The equilibrium bid functions under share dispersion and high bid priority are*

(respectively)

$$\begin{aligned} b(s; t) &= s - \epsilon + 2\epsilon \mu(n, t) \\ b^h(s; t) &= s - \epsilon + 2\epsilon \lambda(n, k, t) \mu(n, k). \end{aligned}$$

(ii) $b^h(s; t) = b(s; t)$ when $c = 1$, and $b^h(s; t) > b(s; t)$ for $c > 1$.

In particular, therefore, the bid functions under high bid priority and share dispersion are identical (given a (k, t)) when signals are uniform; that is, when the signal distribution is diffuse. As the signal dispersion becomes more concentrated, the bid function for high bid priority becomes greater.

Now, consider the comparative effects of these two allocation schemes on the revenue earned by the seller. Suppose the seller announces a t . Under either scheme, the $(t + 1)^{st}$ bid is chosen as the price. Suppose $c > 1$. Since $b^h(s) > b(s)$ for any interior s , if an interior v is drawn, the revenue earned by the seller is correspondingly greater under high bid priority. If $c = 1$, then either allocation scheme results in the same revenue. In particular, there is no loss of revenue from allocating pro rata across the t highest bidders, instead of giving the asset to the k highest bidders.

Since we have a closed form expression for the bid function under share dispersion, for completeness, we provide this bid function for signals in the corners as well. Define

$$\phi(x, y) = \sum_{i=0}^{t-1} \frac{x^{c(n-i-y)} (1 - x^c)^i}{i! \prod_{j=i}^{t-1} (n - j - y)}.$$

Lemma 2 *Suppose $s \notin [v_l - \epsilon, v + \epsilon]$. Then, the equilibrium bid function under share dispersion is given by*

$b(s; t) = s + \epsilon - \delta(s, n, t, \epsilon)$, where

$$\frac{\delta(s, n, t, \epsilon)}{2\epsilon} = \begin{cases} \frac{\phi(\bar{x}(s)/2\epsilon, 0)}{\phi(\bar{x}(s)/2\epsilon, \frac{1}{c})} & \text{if } s < v_l + \epsilon \\ \frac{\phi(1, 0) - \phi(\underline{x}(s)/2\epsilon, 0)}{\phi(1, \frac{1}{c}) - \phi(\underline{x}(s)/2\epsilon, \frac{1}{c})} & \text{if } s > v_h - \epsilon \end{cases}$$

Cornelli and Goldreich (2001a; Table III, page 2349) document that, in their data,

(i) there are an average of 38.9 limit bids and 9.1 step bids²⁷ per issue, (ii) the average rationing across limit and step bids is approximately 50% (Table III), (iii) 21.5% of limit bids and 9.8% of step bids are allocated no shares, since the prices are below the offer price (page 2343) and (iv) the mean IPO offer price is \$23.6 (Table I).

For our numeric example, therefore, we assume $n = 50$. Consider the effects of pro rata rationing, with no discretionary allocation.²⁸ Since approximately 20% of all bidders get no shares, and the rationing is about 50%, we use $t = 40$ and $k = \frac{t}{2} = 20$. Thus, $t = 20$ represents the case of 100% allocation, and $t = 40$ the case of 50% allocation. We further use $v_\ell = 10$, $v_h = 40$, and $\epsilon = 5$.

Figure 2 plots the bid functions under share dispersion for the case of $c = 1.5$ when $t = 20$ and $t = 40$. The vertical difference between the two functions represents the difference in the winner's curse adjustment between the two levels of t .

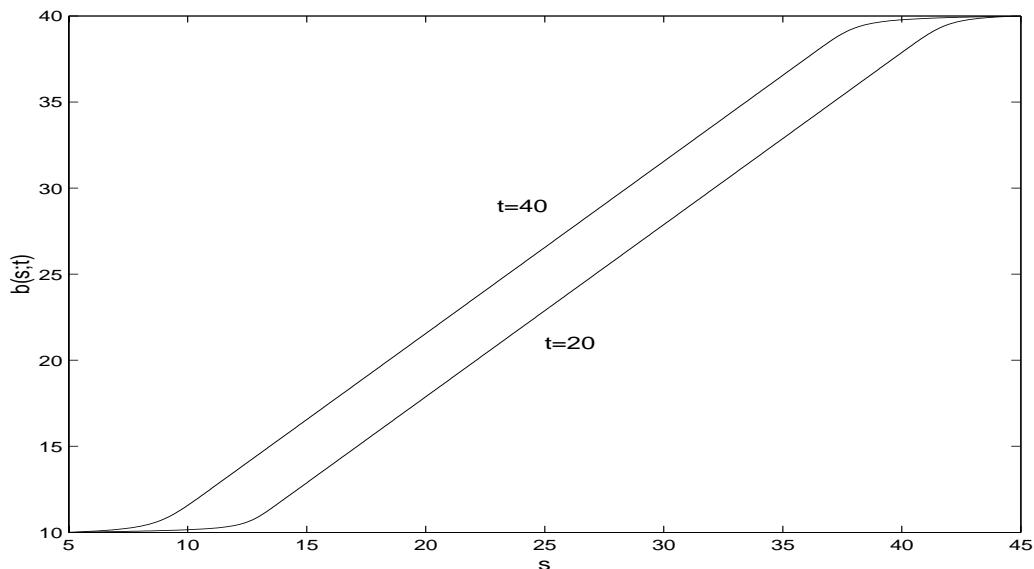


Figure 2: Bid functions for $c = 1.5$, $t = 20$, $t = 40$

While it is immediate that $b(s; t)$ is increasing in t , the extent of the increase depends on c . For $c = 1$, increasing t has a large effect on bids. As c gets large, changing t has a smaller effect on bid functions. The t that we expect to observe in IPO deals is the t that maximizes seller revenue.

We analyze the effects of changing the degree of rationing on the revenue of the seller, under share dispersion. We choose to examine share dispersion rather than

²⁷These are the two kinds of bids that are price-contingent in their study.

²⁸Recall that the bid function, for all investors, is the same with or without discretionary allocation.

high bid priority because, as discussed in Section 2, it requires less commitment on the part of the issuer. Further, we have a closed form expression for the bid function in the corners as well as the interior. The qualitative effects of rationing on revenue are similar under high bid priority or share dispersion, so the intuition gleaned under share dispersion carries over to high bid priority.

We start with the revenue of the seller, for a given value of v and t , $R(v; t)$. Analytically, we restrict attention to interior values of v . We first determine the revenue function in this range, and then derive a condition under which $R(v; t + 1) > R(v; t)$ (that is, rationing yields higher revenue than market-clearing). Subsequently, we examine both $R(v; t)$ and the ex ante revenue, $\hat{R}(t) = \int_{v_\ell}^{v_h} R(v; t) dv$ in the context of our numeric example.

Proposition 10 *Suppose v is in the interior; that is, $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. Then, under share dispersion,*

$$(i) R(v; t) = v - 2\epsilon \left\{ \prod_{j=0}^{t-1} \frac{n-j-\frac{1}{c}}{n-j} - \prod_{j=0}^t \frac{n-j}{n-j+\frac{1}{c}} \right\}$$

$$(ii) R(v; t + 1) > R(v; t) \text{ if and only if}$$

$$\left(\frac{n-t-1+\frac{1}{c}}{n-t} \right) \prod_{j=0}^{t-1} \frac{n-j-\frac{1}{c}}{n-j} > \prod_{j=0}^t \frac{n-j}{n-j+\frac{1}{c}}. \quad (3)$$

When $G(\cdot | v)$ is uniform, the bid and revenue functions simplify considerably. As shown by Harstad and Bordley (1996), the bid function for interior signals reduces to a function of the ratio $\frac{t}{n}$, and the revenue function for interior values of v is clearly increasing in t .

Corollary 10.1 *Suppose $c = 1$. Then under share dispersion,*

$$(i) \text{ for } s \in [v_\ell + \epsilon, v_h - \epsilon], b(s; t) = s + \epsilon \left(\frac{2t}{n} - 1 \right).$$

$$(ii) \text{ for } v \in [v_\ell + 2\epsilon, v_h - 2\epsilon], R(v; t) = v - \frac{2\epsilon(n-t)}{n(n+1)}.$$

Hence, when $G(\cdot | v)$ is uniform, $R(v; t + 1) > R(v; t)$ for all $t = 1, \dots, n - 2$. Therefore, in this case, the optimal t in the share dispersion class of mechanisms is $t = (n - 1)$. That is, the seller should choose maximal rationing when $c = 1$.

Note that the above comparison of revenues for different t assumes a given v in the interior of possible values; that is, $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. As argued before, if rationing

leads to higher revenue for all v in this region, and if ϵ is small relative to $(v_h - v_\ell)$, this implies that ex ante revenue $\hat{R}(t)$ is also higher for the seller with rationing.

Next, we consider the effects of rationing when V is in the corners; that is, $V < v_\ell + \epsilon$ or $V > v_h - \epsilon$. For v in this range, there is positive probability that the signals will lie outside the interior segment, so that the linear bid function no longer applies. In this case, an analytical examination of the change in revenue due to rationing is difficult. Instead, we numerically evaluate the effects of rationing.

Figure 3 below provides the difference in revenues, under share dispersion, in going from $t = 20$ to $t = 40$ for all values of v (including the corners) in our numeric example.

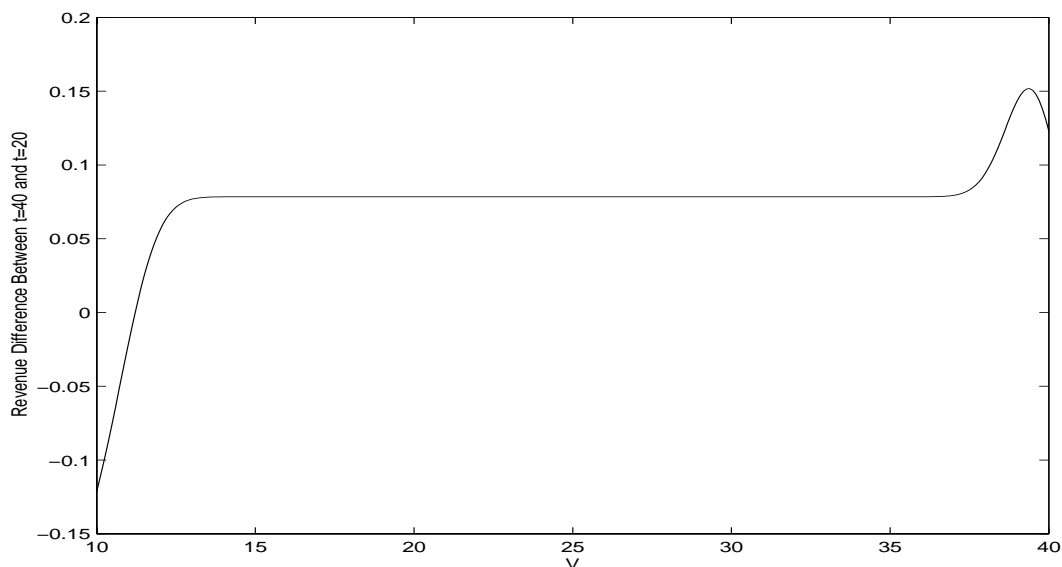


Figure 3: Revenue difference between $t = 40$ and $t = 20$, for $c = 1$.

Note that, as v approaches v_ℓ , rationing loses its luster, and leads to lower revenue. From the seller's point of view, the benefit to rationing is that, by mitigating the winners' curse, it induces bidders to bid more aggressively. This effect is at its strongest when a bidder's posterior over v is diffuse, given his own signal. In our context, since the prior over v is diffuse, a diffuse signal distribution translates to a diffuse posterior over v . In general, as v approaches v_ℓ or v_h , the posterior becomes more concentrated (at least, its support shrinks in size), potentially reducing the benefits of rationing.

Given a value of c , the seller chooses an optimal degree of rationing to maximize ex ante revenue $\hat{R}(t) = \int_{v_\ell}^{v_h} R(v; t) dv$. The optimal level of rationing depends on c . Specifically, as shown above, maximal rationing ($t = n - 1$) is optimal when $c = 1$.

As c increases, the optimal level of rationing decreases,²⁹ and when c is large, market-clearing ($t = k$) is optimal.

We first demonstrate analytically, for interior values of v , that market-clearing is optimal when c becomes large enough.

Proposition 11 *Under share dispersion, for $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, there exists a $\bar{c} > 1$ such that, for $c \geq \bar{c}$, market-clearing is preferable to rationing.*

Since analytic results on the corners are difficult to obtain, we use our numeric example to compute the optimal allocation proportion when the seller maximizes ex ante revenue, $\hat{R}(t) = \int_{v_\ell}^{v_h} R(v; t) dv$. Figure 4 demonstrates the results.

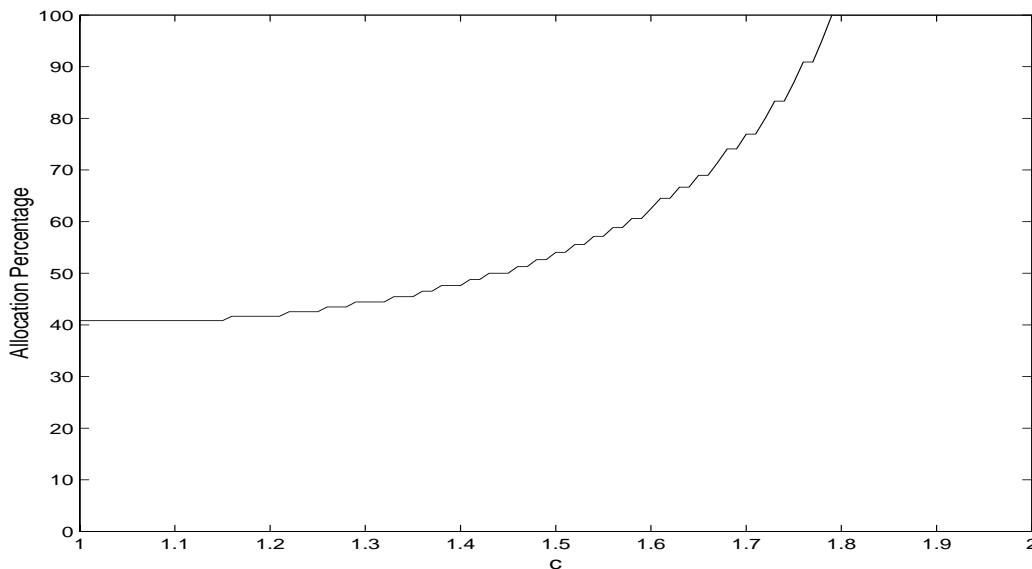


Figure 4: Optimal Allocation Percentage as c changes

This suggests that the oversubscription in IPOs of firms in new industries (where investor beliefs are diffuse) should be high.

Bulow and Klemperer (2001) offer an intuition for this result in terms of the “Maximum Game,” a game in which the common consumption value to all agents depends only on the maximum of all signals.³⁰ If the signal distribution is uniform, the signal range (the maximum minus the minimum signal) is a sufficient statistic for all other properties of the distribution, including the mean, which, in our parametric

²⁹Since t must be an integer, this decrease is in steps, rather than continuous.

³⁰We are grateful to Paul Klemperer for bringing this to our attention.

model, is the expected consumption value given all signals. Selling to the lowest signal bidder is not optimal, and eliminating this bidder recovers the intuition of the Maximum Game. In our model, as c approaches 1 from above, the model more closely resembles a Maximum Game, and hence maximal rationing is optimal.

Armed with an optimal t for each c , we can determine the endogenous distribution of bids in the book. In particular, we can compute the maximum and minimum bids (and hence the range), and the offer price (and thus the range of bids that received a positive allocation). This leads to some empirical predictions linking the degree of rationing with observed properties of bids.

We emphasize that these comparative statics predictions assume all else held constant; in particular, ϵ , n , k , and $[v_\ell, v_h]$. The optimal value of t is endogenously determined given these parameters and c .

First, note that the range of bids received (the maximum bid minus the minimum bid) declines with c .³¹ Hence, this range should be directly related to the degree of rationing; i.e., inversely related to the allocation percentage across IPO deals.

Second, the observed range of bids that receive a positive allocation (i.e., the maximum bid minus the offer price) also declines with c (since t declines with c). Hence, this range should also be inversely related to the degree of rationing (or alternatively, positively related to the aggregate allocation percentage).

Under what circumstances would we expect c to change in a predictable manner? Consider a sequence of IPOs of firms in the same industry. One would expect information on the first firm to be diffuse. Since information about other firms is likely to be correlated, as more firms go public, information should become more precise.³² Thus, we would expect c to increase with each subsequent IPO.

The same comparative static holds when comparing IPOs to seasoned equity offerings (SEOs). If a firm has been publicly traded for a length of time, information about it is more precise, and hence c should be larger. Indeed, Cornelli and Goldreich (2001b) find some support for this view. They find that, in their sample, the average elasticity of demand for SEOs is much larger than the average elasticity of demand for IPOs. This average elasticity measures the proportion of bids within 1% of the offer price, and hence proxies for the precision in the information of bidders.

³¹This is straightforwardly determined as the difference between the maximum and minimum order statistics.

³²Alti (2002) suggests that such an information structure should be a feature of observed IPOs.

Given a book with many price-contingent bids for a single IPO deal, the parameters c and ϵ can be structurally estimated. From this, signals can then be inferred from bids, and hence a more accurate measure of the true value of the asset (which depends on all signals) can be generated.

4.1 Underpricing

We conclude this section with a comment on underpricing. If we assume that the price of the asset in the secondary market is equal to v , then the difference $v - R(v; t)$ represents the level of underpricing for a given v .

In general, we expect this to be positive, so that rationing will not remove underpricing. Some underpricing is inevitable given that this is a common value item. Suppose, for example, $c = 1$. Suppose further that $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$. In the (k, t) -bookbuilding mechanism with share dispersion, we have $R(v; t) = v - 2\epsilon \left(\frac{n-t}{n(n+1)} \right)$. Hence, underpricing is on the order of $2\epsilon \left(\frac{n-t}{n(n+1)} \right)$. Therefore, underpricing decreases with rationing (as t increases), and is at its highest when $t = k$.

Suppose $c = 1$. Within our numeric example, choosing $t = n - 1$ instead of $t = k$ leads to an increase in ex ante revenue from 24.89 to 24.99, or 0.42%. This difference increases when k falls. For example, if $k = 10$, it is 0.57%. These numbers are valid under either share dispersion or high bid priority, since the bid function is equivalent under the two schemes when $c = 1$.

Conversely, suppose $c = 2$. As shown above in our numeric example, it is optimal to have no rationing under share dispersion. Suppose, instead, the seller chooses a pro rata allocation rule with a rationing level of 50% (i.e., the seller chooses $t = 40$). How much revenue does he lose? We compute the expected loss of revenue to be 0.12% of the optimal revenue. In other words, even if the goal of discretionary allocation is to reward long-term investors, the loss in revenue in the current transaction as a result of rationing is small.

The range of c over which high bid priority is optimal is different than the corresponding range for share dispersion. In particular, we expect this range to be larger (since the bids under high bid priority are greater than those under share dispersion, for a fixed t). A result similar to Proposition 11 holds. In the particular example above, if $c = 2$, if t is fixed at 40, high bid priority leads to a gain in revenue of 0.4% over share dispersion.

These numbers are all an order of magnitude smaller than observed levels of short-term underpricing.³³ Cornelli and Goldreich (2001a), for example, show that the average underpricing in their sample, relative to the first secondary market price, is 3.7% (page 2340); Ritter (1998) estimates it to be 15.8% over a sample of U.S. IPOs. Our model suggests that explanations for underpricing do not lie in the rationing that is also observed in IPOs. Indeed, this rationing is either revenue-enhancing, or, at worst, leads to a small loss of revenue for the seller.

In an empirical study, Kandel, Sarig and Wohl (1999) find that underpricing persists in IPO issuances in Israel, where the IPO mechanism is a uniform-price auction. This lends support to the theory that underpricing is inevitable given the presence of the winner's curse (unless a mechanism along the lines of the full surplus extraction one of McAfee, McMillan and Reny (1989) is used). While we do not have any estimates on whether the rationing seen in US IPOs leads to greater or lower underpricing compared to market-clearing, a simple resort to market-clearing is unlikely to remove underpricing.

5 Conclusion

In this paper, we provide a model of rationing in IPOs. We demonstrate that the bids of the agents depend on the rationing rule: higher rationing mitigates the winner's curse, and leads to higher bids. This effect is beneficial to the seller; the cost of the rationing is that the bidder who sets the price has a lower signal.

We consider the case of a uniform prior over true value, and show that rationing leads to higher revenue than market-clearing when the signal distribution is diffuse enough. In other words, when an agent's posterior over V , conditional on his signal, is diffuse, there is a large winner's curse. In such cases, mitigating the winner's curse has a relatively significant impact on bids, and outweighs the effect of choosing a lower order bid to set the price. Conversely, when agents' signals are precise, the latter effect dominates, and increasing the degree of rationing leads to lower revenue.

This effect is in contrast to the earlier literature on rationing and the winner's curse, such as Rock (1986) and Benveniste and Spindt (1989). In this earlier work,

³³This is in keeping with empirical tests of other models of the winner's curse. For example, Levis (1990) conducts a test of the Rock (1986) model and reports that actual levels of underpricing significantly exceed those estimated from the model.

rationing embodies the winner’s curse, and hurts seller revenue. In our model, even within a one-shot game, rationing can be revenue-enhancing.

We consider two allocation rules, high bid priority and share dispersion. Pro rata rationing, a special case of share dispersion, is credible on the part of the seller, in that deviations in pricing are detectable by investors. High bid priority and random allocation, require a stronger commitment. We find that, conditional on this commitment, high bid priority yields higher revenues than share dispersion. However, in numeric terms, the gains in revenue are small. The implication of this is that discretionary allocation, which necessarily involves rationing, has a minimal effect on seller revenue.

In the class of share dispersion allocation rules, our model allows allocations to vary widely across the set of winning bidders. Indeed, our results depend only on the aggregate rationing level chosen by the issuer, not on individual rationing across investors. This is also in contrast to Benveniste and Spindt (1989), whose model implies that investors with high bids should have their demands filled before investors with low bids receive any shares. The empirical literature suggests that departures from this latter paradigm are common (see, for example Cornelli and Goldreich, 2001a).

Rationing affects the allocation of the good in the primary market. In a pure common value case, which we consider, it should have no effect on the secondary market (in fact, there should be no trade in the secondary market, by the Milgrom and Stokey, 1982, no-trade theorem). However, if the value of the asset to an agent includes a private component, the allocation in the primary market will, in general, affect volume and value in the secondary market. Such a model is an avenue for further research.

6 Appendix: Proofs

Proof of Proposition 1

(i) First, consider pro rata rationing with no discretionary allocation. Under this scheme, if $b_i > b_{(t+1)}$, then the prescribed allocation is $a_i = \frac{k}{t}$. Suppose the seller deviates by choosing an offer price $b_{(\tilde{i}+1)} > b_{(t+1)}$. Then, some bidder i with $b_i \in (b_{(t+1)}, b_{(\tilde{i}+1)})$ receives no shares. Hence, the k shares are distributed over fewer investors, so some investor must receive more than $\frac{k}{t}$ shares, and can detect the devia-

tion.

Next, consider pro rata rationing with discretionary allocation. One outcome is that there are no preferred investors amongst the t highest bidders, in which case the above argument applies, and deviations are detectable. Hence, it suffices to demonstrate one example under which a deviation is not detectable. Suppose all the j highest bidders, where $j < k$ are preferred investors. The seller allocates j shares to the j preferred investors. The $(t - j)$ regular investors who have the remaining bids above $b_{(t+1)}$ should all receive $\frac{k-j}{t-j}$ shares. Consider the following deviation: the seller allocates $\frac{k}{t}$ shares to these bidders. The seller can then set a price equal to the $(j + (k - j)\frac{t}{k} + 1)^{th}$ highest bid, where $j + (k - j)\frac{t}{k} + 1 < j + (k - j)\frac{t-j}{k-j} + 1 = t + 1$. For a suitable choice of t, k, j , this is an integer, providing an example of a deviation that goes undetected.

(ii) Consider high bid priority. Suppose the investor deviates, and issues the IPO at $b_{(k+1)} > b_{(t+1)}$. Then, an investor receives 1 share if her bid is above the IPO price, and has no way to determine how many bids below her are also above the IPO price. Investors who bid below the IPO price receive zero shares, which is their prescribed allocation. Hence, this deviation is not detectable.

Finally, consider random allocation. As under high bid priority, the seller could deviate and charge a price equal to $b_{(k+1)} > b_{(t+1)}$. Again, investors who receive a share merely know that they bid above the offer price, and had a positive probability of receiving a share. Investors who bid below the offer price receive nothing, which is their prescribed allocation. ■

Proof of Proposition 2

Suppose $b(\cdot)$ is a symmetric equilibrium with a strictly increasing bid function. Then, under high bid priority, bidder 1 obtains the good if and only if $S_1 \geq Y_{k,n-1}$.³⁴ Suppose bidder 1 receives a signal $S_1 = s$, for which

$$E[b(Y_{t,n-1}) \mid S_1 = Y_{k,n-1} = s] > E[V \mid S_1 = Y_{k,n-1} = s].$$

If $Y_{k,n-1} = s$, bidder 1 wins the good with positive probability. In this case, the expected value of the good is given by the RHS, and the expected price by the LHS.

³⁴If $S_1 = Y_{k,n-1}$, bidder 1 obtains the good with positive probability; note that, since $G(s \mid v)$ is atomless, this is a zero probability event.

Since the expected price exceeds the expected value, bidder 1 can improve her payoff by reducing her bid to some $\tilde{b} < b(s)$. Hence, $b(\cdot)$ cannot be an equilibrium.

Suppose now that

$$E[b(Y_{t,n-1}) \mid S_1 = Y_{k,n-1} = s] < E[V \mid S_1 = Y_{k,n-1} = s].$$

Then, if $Y_{k,n-1} = s$, bidder 1 can improve her payoff by increasing her bid to some $\tilde{b} > b(s)$, since the expected price is less than the expected value of the asset. Hence, $b(\cdot)$ cannot be an equilibrium. ■

Proof of Proposition 3

Let ν_i define an allocation rule faced by agent i under share dispersion, conditional on i having one of the t highest bids (that is, being in the set of potential winners). ν_i can be a function of any variable observed by the seller (including, for example, the identity of agent i or some other agent j), except the bid of any agent.

Formally, ν_i is a probability distribution over $[0, 1]$, where 0 is the minimum number of shares agent i can receive, and 1 (her demand) the maximum. The allocation rule has the property that agent i faces the probability distribution ν_i if her bid is among the t highest bids, and receives 0 shares otherwise.

Consider agent i 's payoff when she is in the set of winners. Since all allocations pay the $(t + 1)^{st}$ highest bid, this payoff is represented as

$$\int_0^1 E[V - b_{t+1} \mid S_i = s, b_i \geq b_{t+1}] \text{Prob}(b_i \geq b_{t+1}) d\nu_i$$

The expectation of V is with respect to the agent's posterior distribution over V . Applying Fubini's theorem to interchange the two expectations, and noting that ν_i is independent of all bids, this expected payoff can be written as

$$\begin{aligned} & E[V - b_{t+1} \mid S_i = s, b_i \geq b_{t+1}] \text{Prob}(b_i \geq b_{t+1}) \int_0^1 d\nu_i \\ & = \bar{\nu}_i E[V - b_{t+1} \mid S_i = s, b_i \geq b_{t+1}] \text{Prob}(b_i \geq b_{t+1}), \end{aligned}$$

where $\bar{\nu}_i$ is the mean of ν_i .

Now, $\bar{\nu}_i$ is a constant unaffected by agent i 's signal, her bid, or the bids of any other agent. Hence, if all agents have the same bidding strategies across the two rationing rules, the optimal strategy of agent i too must be the same. Therefore, if

the bidding functions $\{b_i(\cdot)\}_{i=1}^n$ constitute an equilibrium under one rationing rule, they must also comprise an equilibrium under any other rationing rule that satisfies Definition 1. ■

Proof of Proposition 4

(i) Consider the decision faced by agent 1 in the (k, t) -bookbuilding mechanism with share dispersion. Let b_2, \dots, b_n denote the bids of the other $(n - 1)$ agents, in decreasing order. Then, agent 1 chooses a bid b_1 that maximizes

$$\bar{v}_1 E(V - b_{t+1} \mid S_1 = s, b_1 \geq b_{t+1}) \text{Prob}(b_1 \geq b_{t+1}),$$

where \bar{v}_1 is the rationing rule faced by her, conditional on being among the t highest bidders.

Next, consider a t -unit auction. In this auction, bidder 1 chooses a bid \tilde{b}_1 to maximize

$$E(V - b_{t+1} \mid S_1 = s, b_1 \geq b_{t+1}) \text{Prob}(b_1 \geq b_{t+1}).$$

That is, the payoffs differ only by a multiplicative constant, \bar{v}_i . This term is independent of any signals or bids. Clearly, the set of maximizers is the same in either case. Hence, the best response correspondences of the two mechanisms are identical, as are their equilibria.

(ii) Milgrom (1981) demonstrates that the t -unit auction has a symmetric equilibrium with $b(s; t)$ as defined. From (i) above, it follows that this is also an equilibrium of the (k, t) -bookbuilding mechanism under share dispersion. ■

Proof of Lemma 1

Immediate. ■

Proof of Proposition 5

Let $\underline{v}(s)$ and $\bar{v}(s)$ be the possible bounds of v , given a signal s . Since V is uniformly

distributed over $[v_\ell, v_h]$, under share dispersion we have

$$b(s; t) = \frac{\int_{\underline{v}(s)}^{\bar{v}(s)} v G(s | v)^{n-t-1} (1 - G(s | v))^{t-1} g(s | v)^2 dv}{\int_{s-\epsilon}^{s+\epsilon} G(s | v)^{n-t-1} (1 - G(s | v))^{t-1} g(s | v)^2 dv}.$$

We perform the following change of variables. Let the pseudo-signal, x , be defined as $x = s - (v - \epsilon)$. Then, $v = s + \epsilon - x$, and $dv = -dx$. Finally, let $\bar{x}(s) = s - (\underline{v}(s) - \epsilon)$, and $\underline{x}(s) = s - (\bar{v}(s) - \epsilon)$. Then, when $\underline{x}(s) < \bar{x}(s)$,

$$\begin{aligned} b(s; t) &= \frac{-\int_{\bar{x}(s)}^{\underline{x}(s)} (s + \epsilon - x) H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx}{-\int_{\bar{x}(s)}^{\underline{x}(s)} H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx} \\ &= \frac{\int_{\underline{x}(s)}^{\bar{x}(s)} (s + \epsilon - x) H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx}{\int_{\underline{x}(s)}^{\bar{x}(s)} H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx} \\ b(s; t) &= s + \epsilon - \frac{\int_{\underline{x}(s)}^{\bar{x}(s)} x H(x)^{n-t-1} (1 - H(x))^{t-1} g(x)^2 dx}{\int_{\underline{x}(s)}^{\bar{x}(s)} H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx} \\ &= s + \epsilon - \delta(s, n, t, \epsilon). \end{aligned}$$

The two cases when $\underline{x}(s) = \bar{x}(s)$ occur when $s = v_\ell - \epsilon$ or $v_h + \epsilon$, the lowest and highest possible signals. When $s = v_\ell - \epsilon$, $\delta = 0$, and when $s = v_h + \epsilon$, $\delta = 2\epsilon$. Hence, the bid function reduces to $b(s; t) = s + \epsilon - 2\delta$ in these cases as well. ■

Proof of Corollary 5.1

Suppose $s \in [v_\ell + \epsilon, v_h - \epsilon]$. Then, $\underline{x}(s) = 2\epsilon$ and $\bar{x}(s) = 0$. In particular, both of these are independent of s . Therefore, δ , as defined in equation (2), does not depend on s . Hence, the bid function reduces to $b(s; t) = s + \epsilon - \delta(n, t, \epsilon)$, which is linear in s . ■

Proof of Proposition 6

Consider the signal S_1 , and some $\tilde{t} > t$. By the definition of order statistics, the distribution of $Y_{\tilde{t}, n+1}$ is first-order stochastically dominated by the distribution of $Y_{t, n+1}$. Hence, the joint distribution of $Y_{\tilde{t}, n+1}$ and S_1 is first-order stochastically dominated by the joint distribution of $Y_{t, n+1}$ and S_1 .

Now, the equilibrium bid in the (k, t) -bookbuilding mechanism with share dispersion is $b(s; t) = E[V | Y_{t, n+1} = S_1 = s]$. It follows immediately that this is increasing

in t . ■

Proof of Proposition 7

Recall that $Y_{1,n-1}$ denotes the highest order statistic out of the signals of the $(n-1)$ bidders except bidder 1, and $Y_{n-1,n-1}$ denotes the lowest order statistic amongst these signals. Now,

$$E[V | S_1 = s] = \text{Prob}(Y_1 \leq s) E \{E[V | S_1 = s, Y_{1,n-1} \leq s] + \text{Prob}(Y_1 > s) E[V | S_1 = s | Y_{1,n-1} > s] \}$$

Since $E[V | S_1 = s, Y_{1,n-1} \leq s] < E[V | S_1 = s, Y_{1,n-1} > s]$, it follows that

$$E[V | S_1 = s] > E[V | S_1 = s, Y_{1,n-1} \leq s]$$

Similarly, it is straightforward to show that $E[V | S_1 = s] < E[V | S_1 = s, Y_{n-1,n-1} \geq s]$.

Hence, for $t = 1$, $b(s; t) < E[V | s]$, and for $t = n - 1$, $b(s; t) > E[V | s]$. Since $b(s; t)$ is increasing in t , it follows that there exists a $\hat{t}(s) \in \{1, \dots, n - 1\}$ such that $b(s; t) \leq E[V | s]$ for $t < \hat{t}(s)$, and $b(s; t) > E[v | s]$ for $t > \hat{t}(s)$. ■

Proof of Proposition 8

When $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, all observed signals will lie in $[v_\ell + \epsilon, v_h - \epsilon]$. Hence, $\underline{x}(s) = 2\epsilon$ and $\bar{x}(s) = 0$, so that each bidder bids $b(s; t) = s + \epsilon - \delta(n, t, \epsilon)$. We have

$$\begin{aligned} R(v; t) &= E_{Y_{t+1,n}|v} b(s; t) = E_{Y_{t+1,n}|v} (s + \epsilon - \delta(n, t, \epsilon)) \\ &= E_{Y_{t+1,n}|v} (s + \epsilon) - \delta(n, t, \epsilon). \end{aligned}$$

Noting that $x = s - (v - \epsilon)$, we have $s + \epsilon = v + x$. Therefore,

$$R(v; t) = E_{Z_{t+1,n}}(v + x) - \delta(n, t, \epsilon) = v + E(Z_{t+1,n}) - \delta(n, t, \epsilon). ■$$

Proof of Corollary 8.1

Immediate. ■

Proof of Proposition 9

The following two integration facts are used in this proof and the next two ones. Let α, β be real-valued, with $\alpha > -1$, and $\gamma > 0$ be an integer. Then, repeated integration by parts yields

$$\int x^\alpha (1 - x^\beta)^\gamma dx = \sum_{i=0}^{\gamma} \frac{\gamma! \beta^i x^{\alpha+1+i\beta} (1 - x^\beta)^{\gamma-i}}{(\gamma - i)! \prod_{j=0}^i (\alpha + 1 + j\beta)} + A \quad (4)$$

$$\int_0^1 x^\alpha (1 - x^\beta)^\gamma dx = \frac{\gamma! \beta^\gamma}{\prod_{j=0}^{\gamma} (\alpha + 1 + j\beta)}. \quad (5)$$

where A in equation (4) is a constant of integration.

(i) *Equilibrium under share dispersion:*

Now, consider the bid function under share dispersion, $b(\cdot)$. This bid function is exhibited in Proposition 5; what remains is to evaluate $\delta(s, n, t, \epsilon)$.

When $G(s | v) = \left(\frac{s-v+\epsilon}{2\epsilon}\right)^c$, the distribution of $x = s - (v - \epsilon)$ is $H(x) = \left(\frac{x}{2\epsilon}\right)^c$. Hence, $h(x) = \frac{c}{2\epsilon} \left(\frac{x}{2\epsilon}\right)^{c-1}$.

Now, when $\underline{x}(s) < \bar{x}(s)$,

$$\delta(s, n, t, \epsilon) = \frac{\int_{\underline{x}(s)}^{\bar{x}(s)} x H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx}{\int_{\underline{x}(s)}^{\bar{x}(s)} H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx} \quad (6)$$

Consider the denominator first. We have

$$\int_{\underline{x}(s)}^{\bar{x}(s)} H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx = \left(\frac{c}{2\epsilon}\right)^2 \int_{\underline{x}(s)}^{\bar{x}(s)} (x/2\epsilon)^{c(n-t+1-\frac{2}{c})} (1 - (x/2\epsilon)^c)^{t-1} dx.$$

Define $y = \frac{x}{2\epsilon}$, so that $dx = (2\epsilon)dy$. Then,

$$\int_{\underline{x}(s)}^{\bar{x}(s)} H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx = \frac{c^2}{2\epsilon} \int_{\underline{x}(s)/2\epsilon}^{\bar{x}(s)/2\epsilon} y^{c(n-t+1-\frac{2}{c})} (1 - y^c)^{t-1} dy \quad (7)$$

Note further that, when $v \in [v_\ell + \epsilon, v_h - \epsilon]$, we have $\frac{\underline{x}(s)}{2\epsilon} = 0$ and $\frac{\bar{x}(s)}{2\epsilon} = 1$. Hence, using (5),

$$\int_{\underline{x}(s)}^{\bar{x}(s)} H(x)^{n-t-1} (1 - H(x))^{t-1} h(x)^2 dx = \frac{c^2}{2\epsilon} \frac{(t-1)! c^{t-1}}{\prod_{j=0}^{t-1} c(n-t+j+1-\frac{1}{c})}$$

In a similar manner, the numerator reduces to

$$\begin{aligned} \int_{\underline{x}(s)}^{\bar{x}(s)} x H(x)^{n-t-1} (1-H(x))^{t-1} h(x)^2 dx &= c^2 \int_{\underline{x}(s)/2\epsilon}^{\bar{x}(s)/2\epsilon} y^{c(n-t+1-\frac{1}{c})} (1-y^c)^{t-1} dy \quad (8) \\ &= c^2 \frac{(t-1)! c^{t-1}}{\prod_{j=0}^{t-1} c(n-t+j+1)} \end{aligned}$$

Since both numerator and denominator are independent of s , we have

$$\delta(n, t, \epsilon) = 2\epsilon \prod_{j=0}^{t-1} \frac{n-t+j+1-\frac{1}{c}}{n-t+j+1} = 2\epsilon \mu(n, t)$$

Hence, the bid function under share dispersion is

$$b(s; t) = s - \epsilon + 2\epsilon \mu(n, t).$$

Equilibrium under high bid priority:

Step 1: Solving equation (1).

We solve equation (1) (in the statement of Proposition 2) to find b^h , and then show that this function represents an equilibrium (equation (1) provides only a necessary condition that b^h must satisfy).

Now, the RHS of (1) is given by $b(s; k)$. Consider the LHS. Conjecture a linear equilibrium, with $b^h(s; t) = s + \alpha$ (where α may be positive or negative). Then, $E[b(Y_{t,n-1}) | Y_{k,n-1} = s] = E(Y_{t,n-1} | Y_{k,n-1} = s) + \alpha$.

Recall that $Z_{t,n-1}$ represents the t^{th} highest pseudo-signal, from a sample of $(n-1)$ draws from the distribution H . Since $x = s - v + \epsilon$, we have $E(Y_{t,n-1} | Y_{k,n-1} = s, V = v) = v - \epsilon + E(Z_{t,n-1} | Z_{k,n-1} = x)$. We evaluate this expression first, and then return to the bid function.

The conditional density of $Z_{t,n-1}$, given $Z_{k,n-1}$, is

$$h_t(z | Z_{k,n-1} = x) = \frac{(n-k-1)!}{(n-t-1)! (t-k-1)!} \frac{H(z)^{n-t-1} [H(x) - H(z)]^{t-k-1} h(z)}{H(x)^{n-k-1}}$$

Let $\Gamma = \frac{(n-k-1)!}{(n-t-1)! (t-k-1)!}$. Then, $E[Z_{t,n-1} | Z_{k,n-1} = x] = \int_0^x z h_t(z | x, v) dz$

$$= \frac{\Gamma}{H(x)^{n-k-1}} \frac{c}{2\epsilon} \int_0^x z (z/2\epsilon)^{c(n-t-1)} ((x/2\epsilon)^c - (z/2\epsilon)^c)^{t-k-1} (z/2\epsilon)^{c-1} dz$$

$$= c \frac{\Gamma}{(x/2\epsilon)^{c(n-k-1)}} (x/2\epsilon)^{c(n-k-1)} \int_0^x (z/x)^{c(n-t)} (1 - (z/x)^c)^{t-k-1} dz.$$

Substitute $y = \frac{z}{x}$, so that $dz = xdy$. Then,

$$\begin{aligned} E[Z_{t,n-1} | Z_{k,n-1} = x] &= c \Gamma x \int_0^1 y^{c(n-t)} (1 - y^c)^{t-k-1} dy \\ &= c \Gamma x \frac{(t-k-1)! c^{t-k-1}}{\prod_{j=0}^{t-k-1} c(n-t + \frac{1}{c} + j)} = (s - v + \epsilon) \lambda(n, t, k) \end{aligned}$$

where the second equation follows from (5). Hence, $E[Y_{t,n-1} | Y_{k,n-1} = s, V = v] = v - \epsilon + \lambda(s - v + \epsilon) = (1 - \lambda)(v - \epsilon) + \lambda s$.

Since s and $(Y_{1,n-1}, \dots, Y_{n-1,n-1})$ are independent, $E[Y_{t,n-1} | Y_{k,n-1} = s, V = v] = E[Y_{t,n-1} | Y_{k,n-1} = S_1 = s, V = v]$.³⁵ Hence,

$$\begin{aligned} E[Y_{t,n-1} | Y_{k,n-1} = s] &= E\{E[Y_{t,n-1} | Y_{k,n-1} = S_1 = s] | V = v\} \\ &= (1 - \lambda)(E[V | Y_{k,n-1} = S_1 = s] - \epsilon) + \lambda s \\ &= (1 - \lambda)(s - \delta(s, n, k, \epsilon)) + \lambda s = s - (1 - \lambda)\delta(s, n, k, \epsilon). \end{aligned}$$

This yields the LHS of (1). Putting the two sides together,

$$s - (1 - \lambda)\delta(s, n, k, \epsilon) + \alpha = s + \epsilon - \delta(s, n, k, \epsilon), \quad \text{or,} \quad \alpha = \epsilon - \lambda\delta(s, n, k, \epsilon).$$

For $s \in [v_\ell + \epsilon, v_h - \epsilon]$, $\delta(s, n, k, \epsilon) = 2\epsilon\mu(n, k)$, as defined. Hence,

$$b^h(s; t) = s + \epsilon - 2\epsilon \lambda(n, k, t) \mu(n, t).$$

Step 2: No profitable deviation.

By construction, b^h satisfies the necessary condition for a symmetric equilibrium; it remains to show that there are no profitable deviations. Suppose all agents other than agent 1 are bidding according to $b^h(\cdot)$. Suppose agent 1 with signal s_1 bids $\hat{b} > b^h(s_1)$. Then, if $b(Y_{k,n-1}) > b^h(s_1)$, or if $b(Y_{k,n-1}) < \hat{b}$, there is no difference to the payoff of agent 1. However, if $b(Y_{k,n-1}) \in (b^h(s_1), \hat{b})$, then agent 1 wins with the bid \hat{b} , but not with the bid $b^h(s_1)$.

In this latter case, it must be that $Y_{k,n-1} > s$. Suppose $Y_{k,n-1} = y > s$. Let

³⁵In particular, the support of the posterior over V is unchanged if $S_1 = Y_{k,n-1} = s$.

$p(s, y) = E[V \mid Y_{k,n-1} = y, S_1 = s]$. From above, $E[Y_{t,n-1} \mid Y_{k,n-1} = y, S_1 = s] = (1 - \lambda)(p(s, y) - \epsilon) + \lambda y$. Hence, in this case, agent 1 expects to pay

$$\begin{aligned} E[b^h(Y_{t,n-1}) \mid S_1 = s, Y_{k,n-1} = y] &= b(E[Y_{t,n-1} \mid S_1 = s, Y_{k,n-1} = y]) \\ &= (1 - \lambda)(p(s, y) - \epsilon) + \lambda y + \epsilon - \nu \delta(n, k, \epsilon) \\ &= (1 - \lambda)(p(s, y) - \epsilon) + \lambda y + \epsilon - \delta(n, t, \epsilon) \end{aligned}$$

Further, if $y > s$, then $p(s, y) < p(y, y) = y + \epsilon - \delta(n, t, \epsilon)$. Hence, the consumption value of the asset, conditional on winning when $Y_{k,n-1} = y$ instead of s , increases by $p(s, y) - p(s, s) < y - s$. However, the expected bid increases by $(1 - \lambda)(p(s, y) - p(s, s)) + \lambda(y - s) > p(s, y) - p(s, s)$. Hence, in expectation bidder 1 loses money by bidding \hat{b} instead of $b^h(s_1)$.

A similar argument shows that bidder 1 loses money by bidding $\hat{b} < b^h(s_1)$.

(ii) Given the forms of b, b^h , $b^h(s) \geq b(s)$ at some $s \in [v_\ell + \epsilon, v_h - \epsilon]$ if and only if

$$\begin{aligned} \lambda(n, t, k)\mu(n, k) &\leq \mu(n, t) \\ \prod_{j=k+1}^t \frac{n-j}{n-j+\frac{1}{c}} &\leq \prod_{j=k}^{t-1} \frac{n-j-\frac{1}{c}}{n-j} \\ \prod_{j=k+1}^t \frac{n-j}{n-j+\frac{1}{c}} &\leq \prod_{j=k+1}^t \frac{n-j+1-\frac{1}{c}}{n-j+1}. \end{aligned}$$

Now, the j^{th} term on the LHS is $\frac{n-j}{n-j+\frac{1}{c}}$, and the j^{th} term on the RHS is $\frac{n-j+1-\frac{1}{c}}{n-j+1}$.

Further, $\frac{n-j}{n-j+\frac{1}{c}} < \frac{n-j+1-\frac{1}{c}}{n-j+1}$ if and only if

$$\begin{aligned} (n-j)(n-j+1) &< (n-j+\frac{1}{c})(n-j+1-\frac{1}{c}) \\ (n-j)(n-j+1) &< (n-j)(n-j+1) + \frac{1}{c} - \frac{1}{c^2}, \end{aligned}$$

which is true for all $c > 1$. Further, for $c = 1$, the LHS and RHS are equal.

Hence, for all $c > 1$, the LHS is (term-by-term, and hence overall) strictly less than the RHS. Therefore, for all $c > 1$, $b^h(s) > b(s)$ for interior signals, and for $c = 1$, the two functions coincide. \blacksquare

Proof of Lemma 2

Under share dispersion, the denominator and numerator of δ are given by equations (7) and (8) in the Proof of Proposition 9.

Consider the denominator first. Let $\lambda(z)$ denote the value of the integral $\int y^{c(n-t+1-\frac{2}{c})}(1-y^c)^{t-1}dy$ evaluated at z . Using equation (4), we have $\alpha = c(n-t+1-\frac{2}{c})$, so that $\alpha+1 = c(n-t+1-\frac{1}{c})$, $\beta = c$, and $\gamma = t-1$. Hence, ignoring the constant of integration A ,

$$\begin{aligned}\lambda(z) &= \sum_{i=0}^{t-1} \frac{(t-1)! c^i z^{c(n-t+1+i-\frac{1}{c})} (1-z^c)^{t-1-i}}{(t-1-i)! \prod_{j=0}^i c(n-t+1+j-\frac{1}{c})} \\ &= \frac{(t-1)!}{c} \sum_{i=0}^{t-1} \frac{z^{c(n-i-\frac{1}{c})} (1-z^c)^i}{i! \prod_{j=i}^{t-1} (n-j-\frac{1}{c})} = \frac{(t-1)!}{c} \phi(z, \frac{1}{c}).\end{aligned}$$

Hence,

$$\int_{\underline{x}(s)}^{\bar{x}(s)} H(x)^{n-t-1} (1-H(x))^{t-1} h(x)^2 dx = (t-1)! \left(\frac{c}{2\epsilon}\right) \left\{ \phi(\bar{x}(s), \frac{1}{c}) - \phi(\underline{x}(s), \frac{1}{c}) \right\}.$$

Next, consider the numerator. Let $\hat{\lambda}(z)$ denote the integral the integral $\int y^{c(n-t+1-\frac{1}{c})}(1-y^c)^{t-1}dy$, evaluated at z . Repeating the above steps, we have

$$\hat{\lambda}(z) = \frac{(t-1)!}{c} \phi(z, 0)$$

Hence,

$$\delta(s, n, k, \epsilon) = 2\epsilon \frac{\phi(\bar{x}(s)/2\epsilon, 0) - \phi(\underline{x}(s)/2\epsilon, 0)}{\phi(\bar{x}(s)/2\epsilon, \frac{1}{c}) - \phi(\underline{x}(s)/2\epsilon, \frac{1}{c})}.$$

Finally, note that (i) $\underline{x}(s) = 0$ for $x \leq v_h + \epsilon$, (ii) $\bar{x}(s) = 2\epsilon$ for $x \geq v_\ell - \epsilon$, and (iii) $\phi(0, y) = 0$ for all y . Now, the statement of the Lemma follows. \blacksquare

Proof of Proposition 10

(i) When $v \in [v_\ell + 2\epsilon, v_h - 2\epsilon]$, we have $s \in [v_\ell + \epsilon, v_h - \epsilon]$. From Proposition 9, for signals in this range, under share dispersion

$$\delta(n, t, \epsilon) = 2\epsilon \prod_{j=0}^{t-1} \left(\frac{n-j-\frac{1}{c}}{n-j} \right).$$

Next, consider $E[Z_{t,n}]$. For s in this range, we have

$$\begin{aligned} E[Z_{t+1,n}] &= \frac{n!}{(n-t-1)! t!} \int_0^{2\epsilon} x H(x)^{n-t-1} (1-H(x))^t h(x) dx \\ &= \frac{n! c}{(n-t-1)! t!} \int_0^{2\epsilon} \frac{x}{2\epsilon} (x/2\epsilon)^{c(n-t-1)} (1-(x/2\epsilon)^c)^t (x/2\epsilon)^{c-1} dx \end{aligned}$$

Substitute $y = \frac{x}{2\epsilon}$, and $dx = 2\epsilon dy$, to get

$$\begin{aligned} E[Z_{t+1,n}] &= \frac{n! 2\epsilon c}{(n-t-1)! t!} \int_0^1 y^{c(n-t)} (1-y^c)^t dy \\ &= \frac{n! 2\epsilon c}{(n-t-1)! t!} \left(\frac{t! c^t}{c^{t+1} \prod_{j=0}^t (n-t+j+\frac{1}{c})} \right) = 2\epsilon \prod_{j=0}^t \frac{n-j}{(n-j+\frac{1}{c})}, \end{aligned}$$

where the second equality applies equation (5), and the last step follows from $\prod_{j=0}^t (n-t+j+\frac{1}{c}) = \prod_{j=0}^t (n-j+\frac{1}{c})$.

Substitute the expressions for $\delta(s, n, t, \epsilon)$ and $E Z_{t,n}$ into the revenue function in Proposition 8, and the statement of part (i) follows.

(ii) Consider condition (2) from part (ii) of Proposition 8, when $m = t + 1$. The left-hand side is

$$\begin{aligned} \delta(n, t, \epsilon) - \delta(n, t+1, \epsilon) &= 2\epsilon \prod_{j=0}^{t-1} \left(\frac{n-j-\frac{1}{c}}{n-j} \right) - 2\epsilon \prod_{j=0}^t \left(\frac{n-j-\frac{1}{c}}{n-j} \right) \\ &= 2\epsilon \prod_{j=0}^{t-1} \left(\frac{n-j-\frac{1}{c}}{n-j} \right) \left(1 - \frac{n-t-\frac{1}{c}}{n-t} \right) \\ &= 2\epsilon \frac{1}{c(n-t)} \prod_{j=0}^{t-1} \left(\frac{n-j-\frac{1}{c}}{n-j} \right) \end{aligned}$$

On the right-hand side, we have

$$\begin{aligned} E[Z_{t+1,n}] - E[Z_{t+2,n}] &= 2\epsilon \prod_{j=0}^t \frac{n-j}{(n-j+\frac{1}{c})} - 2\epsilon \prod_{j=0}^{t+1} \frac{n-j}{(n-j+\frac{1}{c})} \\ &= 2\epsilon \left(1 - \frac{n-t-1}{n-t-1+\frac{1}{c}} \right) \prod_{j=0}^t \frac{n-j}{n-j+\frac{1}{c}} \\ &= 2\epsilon \frac{1}{c(n-t-1+\frac{1}{c})} \prod_{j=0}^t \frac{n-j}{n-j+\frac{1}{c}}. \end{aligned}$$

Hence, $R(v; t + 1) > R(v; t)$ if and only if

$$\left(\frac{n - t - 1 + \frac{1}{c}}{n - t} \right) \prod_{j=0}^{t-1} \frac{n - j - \frac{1}{c}}{n - j} > \prod_{j=0}^t \frac{n - j}{n - j + \frac{1}{c}}. \quad (9)$$

■

Proof of Corollary 10.1

(i) Substitute $c = 1$ in the expression for the bid function under share dispersion in Proposition 9 when $s \in [v_\ell + \epsilon, v_h - \epsilon]$. This yields $\delta(n, t, \epsilon) = 2\epsilon \prod_{j=0}^{t-1} \left(\frac{n-j-1}{n-j} \right) = 2\epsilon \left(\frac{n-t}{n} \right) = 2\epsilon \left(1 - \frac{t}{n} \right)$. Hence, $b(s; t) = s + \epsilon - 2\epsilon \left(1 - \frac{2t}{n} \right) = s + \epsilon \left(\frac{2t}{n} - 1 \right)$.

(ii) Substitute $c = 1$ in the expression for $R(v; t)$ in Proposition 10, part (i). ■

Proof of Proposition 11

Consider condition (3) for revenue comparison under share dispersion, expressed in Proposition 10 (ii). Reversing that inequality, we have $R(v; t) > R(v; t + 1)$ if and only if

$$\left(\frac{n - t - 1 + \frac{1}{c}}{n - t} \right) \prod_{j=0}^{t-1} \frac{n - j - \frac{1}{c}}{n - j} < \prod_{j=0}^t \frac{n - j}{n - j + \frac{1}{c}}.$$

For market-clearing to be optimal, this inequality must hold for all $t \geq k$. Consider $c \rightarrow \infty$. In the limit, the condition reduces to $\frac{n-t-1}{n-t} < 1$, which clearly holds for all $t \geq k$. Since both sides of the inequality are continuous in c , it follows that there exists a \bar{c} such that the condition holds for all $t \geq k$ and all $c \geq \bar{c}$. Finally, since maximal rationing is optimal at $c = 1$, it must be that $\bar{c} > 1$. ■

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- (li) This paper was presented at the Fourth Toulouse Conference on Environment and Resource Economics on “Property Rights, Institutions and Management of Environmental and Natural Resources”, organised by Fondazione Eni Enrico Mattei, IDEI and INRA and sponsored by MATE, Toulouse, May 3-4, 2001
- (lii) This paper was presented at the International Conference on “Economic Valuation of Environmental Goods”, organised by Fondazione Eni Enrico Mattei in cooperation with CORILA, Venice, May 11, 2001
- (liii) This paper was circulated at the International Conference on “Climate Policy – Do We Need a New Approach?”, jointly organised by Fondazione Eni Enrico Mattei, Stanford University and Venice International University, Isola di San Servolo, Venice, September 6-8, 2001
- (liv) This paper was presented at the Seventh Meeting of the Coalition Theory Network organised by the Fondazione Eni Enrico Mattei and the CORE, Université Catholique de Louvain, Venice, Italy, January 11-12, 2002
- (lv) This paper was presented at the First Workshop of the Concerted Action on Tradable Emission Permits (CATEP) organised by the Fondazione Eni Enrico Mattei, Venice, Italy, December 3-4, 2001
- (lvi) This paper was presented at the ESF EURESCO Conference on Environmental Policy in a Global Economy “The International Dimension of Environmental Policy”, organised with the collaboration of the Fondazione Eni Enrico Mattei, Acquafredda di Maratea, October 6-11, 2001
- (lvii) This paper was presented at the First Workshop of “CFEWE – Carbon Flows between Eastern and Western Europe”, organised by the Fondazione Eni Enrico Mattei and Zentrum für Europäische Integrationsforschung (ZEI), Milan, July 5-6, 2001
- (lviii) This paper was presented at the Workshop on “Game Practice and the Environment”, jointly organised by Università del Piemonte Orientale and Fondazione Eni Enrico Mattei, Alessandria, April 12-13, 2002
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