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Keywords: Predictions, Minimax regret, Beliefs, Matching pennies, Experiments

JEL Classification: C72, C91

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Abstract

We present a simple procedure that selects the strategies most likely to be played by inexperienced agents who interact in one shot 2x2 matching pennies games. As a first step we axiomatically describe players' beliefs. We find the minimax regret criterion to be the simplest functional form that satisfies all the axioms. Then we hypothesize players act as if they were best responding to the belief their opponent plays according to minimax regret. When compared with existing experimental evidences about one shot matching pennies games, the procedure correctly indicates the choices of around 80% of the players. Applications to other classes of games are also explored.

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1 Introduction

Consider the situation faced by individuals who are involved in a one shot 2x2 game, possibly as subjects of a controlled experiment. The players do not have any knowledge of game theory and they never played before the specific game they are facing. In addition, given that the interaction is not repeated, they cannot expect to learn and improve on their performance over time.

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How should these players decide which strategy to play? Irrespective of the specific game under consideration, the rational approach for dealing with such a decision under uncertainty (in a spirit similar to Savage, 1954) is the following:

1 - each player forms a belief about what his opponent will play.

2 - each player chooses the strategy which best responds to this belief.

Interpreting it as a heuristics, this procedure cannot be expected to describe the behavior of all the agents. Single individuals may in fact use different decisional processes or they may incur in computational errors in choosing their best response. Still, in spite of this noise, the fact that in simple strategic situations the majority of individuals behave in a manner which is coherent with their beliefs finds confirmation in some recent papers[1]. Nyarko and Schotter (2002) study a 60 times repeated 2x2 game and find that around 75% of the players do indeed best respond to their stated beliefs. For the case of 3x3 games, Rey Biel (2004) considers 10 one shot games and finds a similar rate of compliance while 55% is the percentage found by Costa-Gomes and Weizsäcker (2005) using data about 14 (more complex) one shot games. Given these results, it is therefore a conservative guess to expect that, in one shot 2x2 games, at least half of the individuals play consistently with their beliefs. We want to capture the behavior of this majority of players.

The first part of this paper is focused on the process of beliefs formation. The goal is to find a single function (with the payoffs of the game as its argument) that may describe players' beliefs. In looking for this function we adopt an axiomatic approach: first, we list desirable properties that, according to us, should characterize a belief function. Then, we check existing concepts and criteria commonly used in game theory and decision theory to see which of them, if any, fulfills all the requirements.

We find the minimax regret criterion (originally proposed by Savage, 1951) to be the unique candidate that obeys all the axioms. Therefore we propose minimax regret as a proxy for players' beliefs[2] and we claim that the majority of players play "as if" they were best responding to these approximated beliefs.

This conjecture is tested in the second part of the paper. The predictions stemming from the suggested procedure (best respond to beliefs equal to the minimax regret distribution of the opponent) are compared with experimental evidences about different versions of 2x2 one shot matching pennies games. To forecast players' choices in this class of games is particularly problematic, also because the indication given by the Nash equilibrium is often

[1] These papers elicit players' beliefs using a proper quadratic scoring rule, such that for the players "telling the truth" is optimal.

[2] This proxy is "operationally" accurate in the sense that the conjectured minimax beliefs and the real subjective beliefs appear to lead, in the majority of cases, to the same best response.

misleading (see for instance Ochs, 1995 and Goeree and Holt, 2001). Our procedure proves to be an effective way to identify the strategies which are more likely to be played. In fact it correctly predicts the actual choices of around 80% of the players.

In a later section the same procedure is also applied to other kinds of 2x2 games and its relationship with the Nash prediction is explored. An interesting result is that the procedure selects a single outcome even in games that have multiple Nash equilibria and so it contributes to the debate on equilibrium selection (see Straub, 1995 and Haruvy and Stahl, 2004).

This paper thus aims to get some insights into the way people actually behave in simple strategic interactions and it is motivated by experimental results that traditional theory fails to explain. Therefore, despite the initial theoretic axiomatic approach, the paper places itself in the behavioral game theory literature. Behavioral game theory enriches pure game theory by adding elements which are typical of the human nature: limited rationality, heuristic decisions, psychological regularities, feelings and emotions. In the last few years it proved to be successful in narrowing the gap between theory and experimental data[3]. Camerer (2003) provides a very rich overview of the aims, the methods, the empirical evidence and the findings of this fast growing area of research.

A number of studies that focus on how people play one shot simultaneous games and investigate the issue of beliefs formation are more related to this paper. Stahl and Wilson (1995) and Costa-Gomes et al. (2000) test the existence and relative importance of various archetypes of players that differ in the prior they have about the degree of sophistication of their opponents. Results indicate that the majority of the individuals behave as if they were performing one or two steps of strategic thinking. A similar result is also found by Camerer et al. (2004) with experiments about market entry games, Nash demand games and stag hunt games.

There are also various papers that are more specifically focused on the experimental study of matching pennies games. Games of this family have in fact been extensively used to test the validity of the Nash prediction and to study the issues of individuals' learning and adaptive dynamics. These questions stimulated much research with important contributions by Mookherjee and Sopher (1994), Ochs (1995), McKelvey and Palfrey (1995), Erev and Roth (1998), McKelvey et al. (2000), Tang (2001) and Goeree et al. (2003). The natural design of these experiments consisted in letting subjects repeatedly play the same version of a matching pennies game. A different question is to study how agents behave in front of a single interaction: in fact in this case players cannot learn over time and their behavior

[3]For instance, and as already briefly mentioned, the concept of Nash equilibrium is sometimes too "radical" and it may lead to conclusions which are often rejected by experimental evidences.

is not affected by inter temporal considerations. Less work has been carried out to study individuals' play in one shot games[4] with a unique Nash equilibrium in mixed strategies, the reason possibly being the fact that agents' behavior is too erratic to reach some general conclusions. Our paper is focused on one shot games mainly for three reasons: first, we claim our theory is able to capture the behavior of inexperienced players; second, as just mentioned, one shot individuals' play has been less investigated; third, we think that enough real life situations are more likely to be similar to one off events rather than to repeated interactions.

The paper is structured as follows: Section 2 lists the axioms we feel characterize a beliefs function. Section 3 shows that minimax regret satisfies all the axioms being at the same time very simple in its functional form. In Section 4 other candidate functions are shown to under perform the minimax regret; in particular it is shown that various proposals connected with the concept of mixed strategy Nash equilibrium cannot be expected to adequately mimic players' beliefs. Section 5 formalizes the procedure that we claim is able to capture the behavior of the majority of individuals. Section 6 uses existing experimental evidences about matching pennies games to test the validity of our predictions. In Section 7 the procedure is applied, as a robustness check, to other classes of 2x2 games. Section 8 concludes.

2 An axiomatic approach to belief formation

The aim of this section is to provide an axiomatic description of players' beliefs. These beliefs will be later used as the starting point for a procedure that selects the strategies most likely to be played in one shot 2x2 games. We tackle the issue of beliefs formation in a very simplified framework. Still the axioms and the results that follow can be easily restated for the more general case.

Consider the following 2x2 game, where player $i = \{A, B\}$ can choose between strategies H_i and T_i. We assume that $x \geq 0$ and $y > 0$.

1)

		H_B	T_B
$g_H(x,y) \rightarrow$	H_A	$x, \cdot$	$0, \cdot$
$g_T(x,y) \rightarrow$	T_A	$0, \cdot$	$y, \cdot$

We focus on the beliefs of player B about what player A will play. This is the reason why

[4] A notable exception is Goeree and Holt (2004) that presents a model of iterated noisy introspection for one shot interactions which is then tested over a large number of games.

the payoff matrix is incomplete and only the payoffs of player A appear[5]. Whilst keeping in mind the example of an inexperienced boundedly rational player, B's beliefs are considered as being just a function of player A's payoffs. To further simplify the analysis and the exposition we assume two of the payoffs of player A (the ones for the outcomes in which players make different choices) to be equal and normalized to 0.

We indicate with:

- $g_H(x, y)$ the belief of player B about player A playing strategy H_A.
- $g_T(x, y)$ the belief of player B about player A playing strategy T_A.

According to us, a belief function must obey the following axioms:

[A1] Consistency with probability distribution: $g_H(x, y) \geq 0$, $g_T(x, y) \geq 0$ and $g_H(x, y) + g_T(x, y) = 1$, $\forall x, \forall y$.

Axiom 1 states the most basic properties a belief function must obey, namely that it has to identify a meaningful and complete probability distribution. Note that, because of the relationship $g_H(x, y) + g_T(x, y) = 1$, a single probability is enough to define the entire distribution. Therefore A1 allows us to focus just on $g_H(x, y)$. In addition the specific functional form of g_H implies that player B realizes A does respond to changes in his own payoffs. The own payoff effect is a robust feature of games played in experiments (for clear evidences of this effect in matching pennies games see, among others, Ochs, 1995 and Goeree and Holt, 2001).

[A2] Symmetry: $g_H(x, x) = \frac{1}{2}$, $\forall x$.

[A3] Dominance: $g_H(0, y) = 0$, $g_H(x, 0) = 1$, $\forall x, y > 0$.

Axioms 2 and 3 restrict the behavior of the beliefs function for some peculiar values of the payoffs x and y. Axiom 2, which is partly derived from A1, states that the function has to assign a uniform prior to player B whenever A's strategies look the same. Even if A may still have idiosyncratic preferences over his two pure strategies, these cannot be anticipated by B. Axiom 3 implies that players are able to recognize a weakly dominated strategy and that they assign a null probability to the event of the opponent playing such a strategy. The same holds a fortiori for strictly dominated strategies. A3 is thus in line with basic

[5] Depending on B's payoffs, the partial structure of Game 1 is compatible with games such as pure coordination, battle of the sexes and matching pennies, with the last class being our main interest.

rationality assumptions. In 2x2 one shot games the majority of players seems to be able to recognize and eliminate dominated strategies (Roth, 1995).

[A4] Continuity: $g_H(x, y)$ is a continuous function of both x and y.

[A5] Monotonicity: $g_H(x_1, y) > g_H(x_2, y)$ if $x_1 > x_2$, $g_H(x, y_1) < g_H(x, y_2)$ if $y_1 > y_2$.

[A6] Homogeneity of degree zero: $g_H(kx, ky) = g_H(x, y)$, $\forall k > 0$.

These three axioms list some more general properties that must characterize the function g_H. Continuity (A4) is required since there are no evident reasons for B's beliefs to jump in a discrete way given small changes in the arguments of the function. The monotonicity axiom (A5) defines the sign of the already mentioned own payoff effect. It states that players believe their opponent are attracted by strategies that "look better". This implies that if the payoffs associated with strategy H_A increase so does the probability that player B assigns to the event of A playing that strategy. The axiom therefore requires $g_H(x, y)$ to be strictly increasing (respectively decreasing) in x (resp. y). This requirement is in line with a large experimental evidences (among others Ochs, 1995; Goeree and Holt, 2001; Goeree et al., 2003). Players' beliefs have thus to respond to any change in the payoff structure with the exception of the case in which all payoffs are multiplied by a positive constant k. In fact such a transformation would not modify the relative attractiveness of the strategies. The homogeneity of degree zero axiom (A6) formalizes this requirement.

Two more axioms conclude the normative description of the belief function.

[A7] Insensitivity to column switch: $g_H(x, y)$ remains unchanged if the payoffs of the two columns are inverted.

[A8] Sensitivity to row switch: $g_H(x, y) = g_T(x, y)$ if the payoffs of the two rows are inverted.

These last two axioms are a bit more unusual but they still refer to very basic properties of a beliefs function. Axiom 7 states that the beliefs of player B have to remain the same if the payoffs of the columns of the game are inverted. In fact this would not affect the relative preferences of player A over his two pure strategies. To clarify this point with an example, the axiom implies $g_H(x, y) = g'_H(x, y)$ where $g_H(x, y)$ refers to the original Game 1 and $g'_H(x, y)$ refers to Game $1'$ below. For similar reasons Axiom 8 implies that B's beliefs distribution has to be the mirror image of the original one if the payoffs of the two rows in

Game 1 are inverted: $g_H(x,y) = 1 - g''_H(x,y) = g''_T(x,y)$ where $g''_T(x,y)$ refers to Game $1''$. In other words players' beliefs consistently react to the payoff structure of the game.

1′) $g'_H(x,y) \rightarrow$

	H_B	T_B
H_A	$0, \cdot$	$x, \cdot$
T_A	$y, \cdot$	$0, \cdot$

1″) $g''_T(x,y) \rightarrow$

	H_B	T_B
H_A	$0, \cdot$	$y, \cdot$
T_A	$x, \cdot$	$0, \cdot$

2.1 Some useful properties of the beliefs function

If a function satisfies axioms A1-A8 then it also fulfills some other more specific requirements. An important relation is easily obtained. Start from the equality stated in Axiom 1: $g_H(x,y) + g_T(x,y) = 1$. Because of homogeneity of degree zero (A6) every argument of g_H and of g_T can be divided by $y > 0$. Call $z = \frac{x}{y}$ with $z \geq 0$ to get $g_H(z,1) + g_T(z,1) = 1$. Invoking again Axiom 6 divide the arguments of g_T by z: $g_H(z,1)+g_T(1,\frac{1}{z}) = 1$. By axioms 7 and 8 the term $g_T(1,\frac{1}{z})$ is equivalent to $g_H(\frac{1}{z},1)$ so that $g_H(z,1) + g_H(\frac{1}{z},1) = 1$ must hold. Rename $g_H(\cdot,1)$ with $f(\cdot)$ and rearrange to get Relation 1:

$$f(z) = 1 - f\left(\frac{1}{z}\right) \tag{1}$$

Relation 1 implies that player B assigns the same probability to the event of player A playing strategy H_A in Game 2 and to the event of A playing strategy T_A in Game 3 shown below.

2) $f(z) \rightarrow$

	H_B	T_B
H_A	$z, \cdot$	$0, \cdot$
T_A	$0, \cdot$	$1, \cdot$

3) $1 - f(\frac{1}{z}) \rightarrow$

	H_B	T_B
H_A	$\frac{1}{z}, \cdot$	$0, \cdot$
T_A	$0, \cdot$	$1, \cdot$

Note that in Game 2 and 3 only the variable z appears. Moreover, because of Relation 1, to know $f(z)$ means to know $f\left(\frac{1}{z}\right)$. Given the one to one relation between $\forall z \in [0,1]$ and the reciprocal $\frac{1}{z} \in [1,\infty)$, the analysis of a function that obeys Relation 1 can be restricted to the partial domain $z \in [0,1]$.

It is therefore much more practical to study the beliefs function f that refers to Game 2 rather than the original function g_H in Game 1. Lemma 1 states the conditions under which the two functions are equivalent. In particular it provides sufficient conditions for a generic function g_H to be transformed in a simpler function f as well as conditions a generic function f has to satisfy for being used to approximate players' beliefs originally captured by g_H.

Lemma 1 *Consider g_H and let $g_H(z,1) = f(z)$. Then:*

a) $\forall g_H$ s.t. g_H satisfies axioms A1-A8 $\Longrightarrow \exists f$ s.t. $f(z) = 1 - f\left(\frac{1}{z}\right)$, $f(0) = 0$, $f(1) = \frac{1}{2}$ and f is strictly increasing on $[0,1]$.

b) $\forall f : [0,\infty] \to [0,1)$ s.t. $f(z) = 1 - f\left(\frac{1}{z}\right)$, $f(0) = 0$, $f(1) = \frac{1}{2}$, f is strictly increasing on $[0,1]$ and f is homogeneous of degree 0 $\Longrightarrow \exists g_H$ s.t. g_H satisfies axioms A1-A8.

Proof.

a) The proof replicates the steps used to derive Relation 1. Transformations applied to g_H in order to get f are innocuous, therefore the axioms are still valid and they just need to be restated. $f(0) = 0$ indicates f obeys dominance, $f(1) = \frac{1}{2}$ refers to symmetry and the requirement of f being strictly increasing on $[0,1]$ is equivalent to the monotonicity axiom about g_H.

b) Given that by definition $f(z) = g_H(z,1)$, then if f is homogeneous of degree 0 the following relation is also true: $f(z) = g_H(kz,k), \forall k > 0$. Therefore the original belief function g_H can be reconstructed starting from the relation $f(z) = 1 - f\left(\frac{1}{z}\right)$ and following backwards the proof used to get Relation 1. ■

From now on we restrict our attention to generic functions f that obey the requirements of Lemma 1 and the main object of study will be $f(z)$: the beliefs of player B about player A playing strategy H_A in the reduced Game 2.

Some other properties of $f(z)$ are implied by the axioms and by Relation 1. In particular the function f is continuous both at $z = 1$ ($\lim_{z\to 1^-} f(z) = \lim_{z\to 1^-} f\left(\frac{1}{z}\right) = \frac{1}{2}$ by Lemma 1) and at $z = 0$ ($\lim_{z\to 0} f(z) = 0$ and $f(0) = 0$ by Lemma 1) and $\lim_{z\to 0} f(\frac{1}{z}) = 1$ ($\lim_{z\to 0} f(z) = 0$ and A1). Moreover, as mentioned before, there is a one to one relationship between any z and the correspondent $\frac{1}{z}$ and so between $f(z)$ and $f\left(\frac{1}{z}\right)$.

$$f(z) : [0,1] \to \left[0, \frac{1}{2}\right] \Leftrightarrow f\left(\frac{1}{z}\right) : [1,\infty) \to \left[\frac{1}{2}, 1\right) \tag{2}$$

The following propositions are particularly important in the task of describing as precisely as possible the beliefs function because they investigate the possible curvature of f. Proposition 2 does not require f to be differentiable, but if differentiability is assumed then sharper results can be proven (Proposition 3).

Proposition 2 *On the entire domain, $f(z)$ is neither linear nor strictly convex.*

Proof. For any $z \in (0,1)$ take the linear combination between z and $\frac{1}{z}$ such that $\hat{\alpha}z + (1-\hat{\alpha})\frac{1}{z} = 1$ so that $\hat{\alpha} = \frac{1}{z+1}$. Then, by Axiom 2, we know that $f(\hat{\alpha}z + (1-\hat{\alpha})\frac{1}{z}) = f(1) = \frac{1}{2}$. Compare it with $\hat{\alpha}f(z) + (1-\hat{\alpha})f\left(\frac{1}{z}\right)$ which, using Relation 1, can be expressed as $\frac{1}{z+1}f(z) + \frac{z}{z+1}\left[1 - f(z)\right]$ and thus as $f(z)\left(\frac{1-z}{1+z}\right) + \frac{z}{z+1}$. We show that the relation defining concavity holds: $f(\hat{\alpha}z + (1-\hat{\alpha})\frac{1}{z}f(z) > \hat{\alpha}f(z) + (1-\hat{\alpha})$ which in this specific case means $\frac{1}{2} > f(z)\left(\frac{1-z}{1+z}\right) + \frac{z}{z+1}$. This last condition simplifies to $f(z) < \frac{1}{2}$ which is satisfied given that $z \in (0,1)$ and the monotonicity axiom. The function $f(z)$ is concave at least over part of its domain and thus it cannot be linear or strictly convex. ■

Proposition 3 *If $f(z)$ is differentiable then it is strictly concave at $z = 1$.*

Proof. Differentiate twice with respect to z the relation $f(z) = 1 - f\left(\frac{1}{z}\right)$ to get the first (3) and second (4) derivatives.

$$f'(z) = \frac{1}{z^2}f'\left(\frac{1}{z}\right) \tag{3}$$

$$f''(z) = -\frac{2}{z^3}f'\left(\frac{1}{z}\right) - \frac{1}{z^4}f''\left(\frac{1}{z}\right) \tag{4}$$

Evaluating (4) at $z = 1$ we get $f''(1) = -f'(1)$. Given that, because of monotonicity, $f'(1) > 0$ it follows that $f''(1) < 0$ and the function is strictly concave at $z = 1$. ■

2.2 Bounds on the function $f(z)$

The axioms and the derived properties imply a rather specific behavior of the beliefs function. A graphical description of the bounds that restrict f appears in Figure 1. In what follows we let $z \in [0,1]$ and therefore $\frac{1}{z} \in [1,\infty)$.

Axioms 2 and 3, when restated according to Lemma 1, provide the starting point: given that $f(0) = 0$ and $f(1) = \frac{1}{2}$ the function has to pass through points $a = (0,0)$ and $b = \left(1, \frac{1}{2}\right)$. However, this is not enough to identify the function given that $f(z)$ is not linear (Proposition 2). Still the fact that $f(1) = \frac{1}{2}$ together with the monotonicity axiom implies that $f(z) \in \left[0, \frac{1}{2}\right]$ and $f\left(\frac{1}{z}\right) \in \left[\frac{1}{2}, 1\right)$.

For what concerns the curvature of the function we know that, assuming the function to be differentiable (as we do), $f(z)$ has to be strictly concave at $z = 1$ (Proposition 3). Note that $z = 1$, given its mirroring properties captured by Relation 1, is the only "peculiar" point of the domain, i.e. the point in which a change in the sign of the second derivative may have been expected (think for instance of a logistic function). Moreover f has also to be concave at least in some part of the domain for $\frac{1}{z} \to \infty$ because it is strictly increasing but bounded above by 1. Because of these two facts and in order to find a function which is as simple as possible, we require f to be strictly concave over all the domain. In fact a function that changes concavity would have a more complex analytical form with respect to a "well behaved" strictly concave function[6].

The concavity requirement provides a lower bound for the function in the interval $[0,1]$. In fact $\forall z \in [0,1]$, $f(z) \geq \frac{1}{2}z$ has to hold where $\frac{1}{2}z$ is the equation of the line that connects points $a = (0,0)$ and $b = \left(1, \frac{1}{2}\right)$. Since $f(z) = 1 - f(\frac{1}{z})$ has to be always valid, this lower bound becomes an upper bound in the interval $[1,\infty)$ where $f(\frac{1}{z}) \leq 1 - \frac{1}{2}z$ has to hold.

Finally note that $\forall \frac{1}{z} \in [1,\infty)$, and given that $f(\frac{1}{z}) < 1$, the condition $f(\frac{1}{z}) < \frac{1}{z}$ holds and the same condition holds $\forall z \in \left(\frac{1}{2}, 1\right]$ as well. We require this condition to be valid also $\forall z \in \left(0, \frac{1}{2}\right]$ and thus $f(z) < z$, for any $z \neq 0$. This last assumption implies two things: (1) $f(z)$ increases less than proportionally with respect to z and (2) $f(z)$ approaches the upper limit 1 not too slow. In fact, given that $f(z) \leq z$, the lower bound $f(\frac{1}{z}) \geq 1 - z$ in the interval $\frac{1}{z} \in [1,\infty)$ is directly derived from Relation 1.

Figure 1 provides a graphical representation of the results of this section. The four thin lines define the two corridors in which the function f has to develop with the additional constraints that f has to pass through points $(0,0)$ and $\left(1, \frac{1}{2}\right)$ and be strictly increasing. These restrictions do not identify a unique function. Indeed any function that stays within the bounds could be used to approximate players' beliefs as stated by the following proposition.

Proposition 4 $\forall f$ *s.t.* $f(z) = 1 - f(\frac{1}{z})$, f *is strictly increasing on* $[0,1]$ *and* $\frac{1}{2}z \leq f(z) \leq \min\left\{z, \frac{1}{2}\right\}, \forall z \in [0,1] \Longrightarrow \exists g_H$ *s.t.* $f = g_H$.

Proof. If f satisfies Relation 1, it is strictly increasing on $[0,1]$ and it stays within the bounds then it means that f satisfies Lemma 1 and thus there exists a g_H s.t. $f = g_H$. ■

Among all these possible functions we now turn our attention to the one which appears in bold in Figure 1: this function is the minimax regret.

[6] Given that the beliefs retrieved through f will be used to predict the outcome of 2x2 games, a simple functional form is a valuable quality. Indeed our aim is to identify the simplest function among those that satisfy all the axioms and the derived properties.

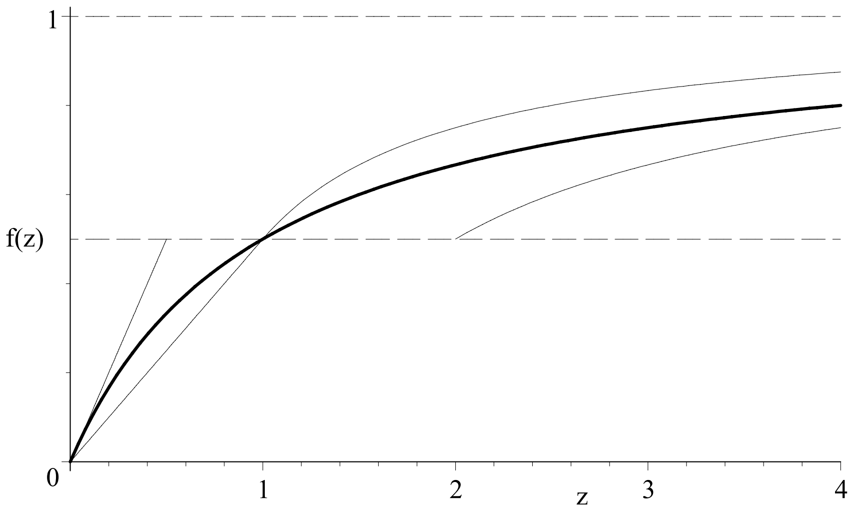

Figure 1: lower and upper bounds for f(z) vs. the minimax regret proposal.

3 The proposed beliefs function: the minimax regret

According to the conjecture of the paper the beliefs of player i about what player i' will play can be approximated by the minimax regret of player i'. Minimax regret, originally proposed by Savage (1951), is a concept which found its main applications as a selection criterion in decision theory (starting with Milnor, 1954). More recently minimax regret has also been used in modeling the behavior of subjects with limited rationality (for instance Bergermann and Schlag, 2005, for the case of boundedly rational monopolists) as well as a way to deal with missing data in econometrics (Mansky, 2005) and it also appears in the artificial intelligence literature (Brafman and Tennenholtz, 2000). The minimax regret criterion prescribes a player who has to make a decision under uncertainty to choose the action that minimizes his expected regret. The regret is defined as the difference between the best payoff player A could have got if he knew what his opponent (a player or Nature) had played and the payoff the player actually got. In fact the first step to compute the minimax regret consists in building the regret matrix which captures these differences. In the specific case of Game 2, which remains the game under study, and given that $z > 0$, the regret matrix is given by R_2:

2) $f(z) \rightarrow$		H_B	T_B		$R_2)$				
	H_A	$z, \cdot$	$0, \cdot$			H_A	$0, \cdot$	$1, \cdot$	(p_{mmr})
	T_A	$0, \cdot$	$1, \cdot$			T_A	$z, \cdot$	$0, \cdot$	$(1-p_{mmr})$

Strategy H_A attains minimax regret in pure strategies for any $z > 1$ while strategy T_A attains minimax regret for $z \in [0, 1)$. Taking this specification as a belief function would clearly be unsatisfactory since the pure version of the minimax regret fails both the continuity and the monotonicity axioms. The use of mixed strategies solves this problem. Before moving to the computation of the mixed minimax regret (*mmr*) note that, by construction, in any 2x2 game, the regret matrix contains at least two zeros, a feature that makes the computation of the *mmr* very easy.

To find the *mmr* means to find the probability distribution (defined by $\tilde{p}_{mmr}$) that equalizes the expected regret of the two strategies so that player A is indifferent in playing H_A or T_A. This optimal $\tilde{p}_{mmr}$ solves $p_{mmr}(1) = (1-p_{mmr})\, z$ so that $\tilde{p}_{mmr} = \frac{z}{z+1}$. According to the conjecture of this paper $f(z) = \tilde{p}_{mmr}$ should (heuristically) hold and thus:

$$f(z)_{mmr} = \frac{z}{z+1}$$

Once again, referring to Game 2 above, this means that player B approximately believes player A to play strategy H_A with probability $\frac{z}{z+1}$ and strategy T_A with the complementary probability of $\frac{1}{z+1}$. This candidate function obeys all the axioms and the derived properties and assumptions as shown by the following proposition (the subscript *mmr* is dropped wherever it is superfluous).

Proposition 5 *the mixed minimax regret function $f(z)_{mmr}$:*

i) satisfies axioms A1-A8.

ii) satisfies Relation 1.

iii) is differentiable and strictly concave.

Proof. i) Trivial for the first few axioms. In particular: $f(z) \in [0, 1]$ (A1), $f(1) = \frac{1}{2}$ (A2), $f(0) = 0$ (A3), f is continuous (A4) as well as strictly increasing (A5) in z and it is homogeneous of degree zero in its payoffs (A6). Concerning the last two axioms note that, with respect to Game 2, f is indeed insensitive to column switch (A7) as shown by Game $2'$ and sensitive to row switch (A8, Game $2''$).

	H_B	T_B
H_A	$0,\cdot$	$z,\cdot$
T_A	$1,\cdot$	$0,\cdot$

2′) $f(z)=\frac{z}{z+1}$

	H_B	T_B
H_A	$0,\cdot$	$1,\cdot$
T_A	$z,\cdot$	$0,\cdot$

2″) $f(z)=\frac{1}{z+1}$

ii) Compute $f\left(\frac{1}{z}\right)=\frac{\frac{1}{z}}{\frac{1}{z}+1}=\frac{1}{z+1}$ and verify that $f(z)+f\left(\frac{1}{z}\right)=\frac{z}{z+1}+\frac{1}{z+1}=1$.

iii) The second derivative is given by $f^{''}(z)=\frac{-2}{(z+1)^3}$ which is defined and strictly negative over all the domain. ■

Moreover the proposed function also "works" with strictly dominated strategies in the sense that $f(z)_{mmr}$ assigns to a player a null belief about the event of his opponent playing a strictly dominated strategy.

As explained in the previous section we cannot claim the minimax regret to be the unique function that satisfies all the axioms and the additional requirements. It is however an advantage that an already existing concept (though normally used for different purposes) may be used to approximate players' beliefs. In this way in fact there is no need to invoke new definitions or *ad hoc* formulas.

Moreover the following proposition underlines an appreciable feature of $f(z)_{mmr}$, namely that it is the unique one among all linear functions that obeys all the axioms and derived properties. As before, the results are proven in the simplified context of Game 2 but, because of Lemma 1, they also hold with more general payoff structures.

Proposition 6 *If* $f(z)=\frac{az+b}{cz+d},\forall z$ *and* f *satisfies Lemma 1* $\Longrightarrow f(z)=\frac{z}{z+1}=f(z)_{mmr},\forall z.$

Proof. For f to satisfy Lemma 1 we must have $f(0)=\frac{b}{d}=0$ (which implies $b=0$) and $f(1)=\frac{a}{c+d}=\frac{1}{2}$ (which implies $2a=c+d$). Now impose the condition $f(z)+f(\frac{1}{z})=\frac{az}{cz+d}+\frac{a}{c+dz}=1$ which is equivalent to $(z^2+1)(ad-cd)+z(2ac-c^2-d^2)=0,\forall z$. Consider for instance the cases of $z=\frac{1}{2}$ and $z=\frac{1}{4}$. If $z=\frac{1}{2}$ then (a) $\frac{5}{4}(ad-cd)+\frac{1}{2}(2ac-c^2-d^2)=0$. If $z=\frac{1}{4}$ then (b) $\frac{17}{16}(ad-cd)+\frac{1}{4}(2ac-c^2-d^2)=0$. Subtract (b) from (a) to get : (c) $\frac{1}{4}(2ac-c^2-d^2)=-\frac{3}{16}(ad-cd)$. Substitute (c) in (b) : $\frac{17}{16}(ad-cd)-\frac{3}{16}(ad-cd)=\frac{14}{16}(ad-cd)=0$ i.e. $ad-cd=0$. This last condition is verified if: (1) $d=0$, but given that $2a=c+d$ then we would have $f(z)=\frac{1}{2},\forall z$ which fails Lemma 1 because f would not be strictly increasing. (2) $a=c$, but given that $2a=c+d$ we must have $a=c=d$. This simplifies to our formulation: $f(z)=\frac{az+b}{cz+d}=\frac{az}{cz+d}=\frac{az}{az+a}=\frac{z}{z+1}=f(z)_{mmr}$. ■

4 Other candidate concepts

In the previous section the mixed version of the minimax regret has been shown to obey all the axioms and derived properties required for approximating players' beliefs in a 2x2 game, being at the same time very simple in its functional form.

In this section we check the compliance to the axioms of other existing concepts commonly used in game theory and decision theory. Two categories are recognizable among these candidates: the first one collects proposals that are connected with the concept of mixed strategy Nash equilibrium (subsections 4.1 and 4.2), the second one considers criteria which mainly find application in decision theory, namely the maxmin (4.3) and the Laplace (4.4) criteria. Subsection 4.5 considers the hypothesis that beliefs may be captured by a logit specification. In line with what has been done for the minimax regret we keep on referring to Game 2, considering how these candidate functions perform in approximating the beliefs of player B on what A will play. The table that appears in section 4.6 summarizes the results.

4.1 Mixed strategy Nash equilibrium of player A

According to this hypothesis player B believes player A randomizes over H_A and T_A following the probability distribution the mixed strategy Nash equilibrium (*msne*) attaches to player A. At first glance this may seem a good candidate given that mixed Nash equilibria, out of many different interpretations[7], have also been considered as mimicking players' beliefs. However this proposal does not even pass the requirements of the first axiom. First there is an issue of existence: in a 2x2 game a well defined *msne* exists only when there are no strictly dominant strategies. Second, and more important, the *msne* of player A depends by construction only on player B's payoffs given that the mix adopted by A has to make B indifferent among his strategies. In other words, this criterion does not capture any own payoff effect: no matter how the payoffs of player A could change, A's distribution in the mixed equilibrium (and thus B's beliefs) remains the same as far as B's payoffs remain fixed. The *msne* of player A thus fails Axiom 1 and as a consequence, it also fails all the remaining ones[8].

[7] Cfr. section 3.2 in the book "A course on game theory" by Osborne and Rubinstein (1994, MIT press).

[8] Moreover the predictive power of such a beliefs formulation would be very low. In fact any strategy in the support of the mixed equilibrium of B would be a best response.

4.2 Mixed strategy Nash equilibrium of player B

This alternative would imply that the probability distribution that the *msne* attaches to player B could be considered as B's beliefs about what A would play. The *msne* of player B still suffers from the problem of nonexistence in the presence of dominant strategies but it is indeed a function of the payoffs of player A. In order to assess the performance of this proposal, we apply it to Game 2, which is reproduced below.

2)

		(q)	$(1-q)$	
		H_B	T_B	
$f(z) \rightarrow$	H_A	$z, \cdot$	$0, \cdot$	(p_{mm})
	T_A	$0, \cdot$	$1, \cdot$	$(1-p_{mm})$

2″)

		(q)	$(1-q)$
		H_B	T_B
$f(z) \rightarrow$	H_A	$0, \cdot$	$1, \cdot$
	T_A	$z, \cdot$	$0, \cdot$

The probability distribution of the *msne* is defined by the q^* that solves $q^*z+(1-q^*)\,0 = q^*0+(1-q^*)\,1$, i.e. $q^* = \frac{1}{1+z}$. Then the beliefs of player B about A playing H_A should be captured by:

$$f(z)_{msne_B} = \frac{1}{1+z}$$

This specification does not obey the monotonicity axiom since $f(z)_{msne_B}$ is decreasing in z, the opposite behavior with respect to the one prescribed by Axiom 5. To see how misleading this interpretation could be, consider as an example the case in which $z = 9$. In such a situation the *msne* of player B proposal would imply that player B believes A will play H_A with probability $f(z) = 0.1$, clearly a counter intuitive indication. Indeed letting players best respond to these beliefs would lead to predictions which are often totally in contrast with experimental results.

To solve this problem one may be tempted to approximate the beliefs of player B with the complement to 1 of $f(z)_{msne_B}$. The functional form for the beliefs function would then be: $f(z) = 1 - q^*$, where again q^* defines the probability distribution of the *msne* of player B. If applied to Game 2, this proposal leads to the following functional form:

$$f(z)_{1-msne_B} = \frac{z}{1+z}$$

which indeed satisfies the monotonicity axiom. Actually this function identifies the same beliefs indicated by the minimax regret[9] and so, apparently, it should obey all the axioms.

[9] Gallice (2005) shows that in any 2x2 game where a well defined *msne* exists its probability distibution is either the same or the mirror image of the minimax regret distribution of the other player.

However it fails axioms 7 and 8. Consider for instance Game $2''$ above, a game in which the payoffs of the two rows have been inverted with respect to Game 2. The probability distribution of the mixed equilibrium of B is again defined by $q^* = \frac{1}{1+z}$. It follows that player B's beliefs on A playing H_A would then be the same as before: $f(z)_{1-msne_B} = \frac{z}{z+1}$, a fact that violates Axiom 8 for $\forall z \neq 1$. A similar demonstration shows that this proposal also fails Axiom 7.

4.3 Maxmin of player A

Are the strategies selected by the maxmin criterion a credible candidate for approximating players' beliefs? The pure version of the maxmin criterion predicts a player to choose the strategy which guarantees him the highest minimum payoff. As in the case of the minimax regret, such a "pure" formulation does not obey the continuity and monotonicity axioms. Allowing for mixed strategies the maxmin criterion assumes the player to mix over his strategies in such a way to maximize the expected minimum. Referring to Game 2 this would imply $f(z)_{mm} = p_{mm}$ where p_{mm} is such that $p_{mm}z = (1 - p_{mm})$ and thus

$$f(z)_{mm} = \frac{1}{1+z}$$

In the context of Game 2 this alternative leads to the same functional form which characterized the $msne_B$ proposal. Therefore this specification does not satisfy Axiom 5 (monotonicity). More in general, in 2x2 games where all the payoffs are different from 0, the probability distributions implied by the maxmin criterion and by the $msne$ are usually different. However they both continue to fail the monotonicity requirement.

4.4 Laplace

According to this possibility player B believes player A chooses the strategy to play following the Laplace criterion. This criterion assumes a player to best respond to uniform priors. The strategy to be chosen is then the one which has the highest sum of payoffs. In the case of Game 2, this means:

$$f(z)_{La} = \begin{cases} 1 & \text{if} \quad z > 1 \\ 0 & \text{if} \quad z < 1 \\ 0.5 & \text{if} \quad z = 1 \end{cases}$$

Clearly this criterion fails both the continuity and the strict monotonicity axioms.

4.5 Logit rule

If the beliefs of player B about player A playing H_A in Game 2 were approximated using a logit rule, they would take the following analytical form:

$$f(z)_{lo} = \frac{e^z}{e^z + e^1}$$

It is easy to see that this formulation fails Axiom 3 (dominance) given that $f(0)_{lo} \neq 0$ as well as Axiom 6 (homogeneity of degree 0) given that $\frac{e^{kz}}{e^{kz}+e^k} \neq \frac{e^z}{e^z+e^1}$.

4.6 A summary

Table 1 summarizes the compliance to the axioms of the candidate functions which have been considered till now. The axioms are identified as: consistency with probability distribution (1), symmetry (2), dominance (3), continuity (4), monotonicity (5), homogeneity of degree zero (6), insensitivity to column switch (7) and sensitivity to row switch (8).

Criteria\Axioms		1	2	3	4	5	6	7	8
Minimax regret of pl. A		y	y	y	y	y	y	y	y
Msne of pl. A		n	n	n	n	n	n	n	n
Msne of pl. B		y[10]	y	n	y	n	y	n	n
1-Msne of pl. B		y[11]	y	y	y	y	y	n	n
Maxmin		y	y	n	y	n	y	y	y
Laplace		y	y	y	n	n	y	y	y
Logit		y	y	n	y	y	n	y	y

Table 1: compliance to the axioms of the candidate functions.

5 A procedure to forecast outcomes

The proposal that the minimax regret may approximate players' beliefs has been until now discussed in the simplified framework of Game 2. Still we claimed from the beginning that results were also valid in more general cases. Here we apply our conjecture to a game that encompasses the cases of a matching pennies game and of a game with a dominant

[10] The axiom is satisfied if there are no dominant strategies.
[11] As before.

strategy. The example is meant to show how simple is the process to approximate beliefs, understressing once more the inadequacy of proposals connected with the concept of mixed strategy Nash equilibria. It also serves as a preliminary stage to introduce a simple procedure for forecasting individuals' strategies which are more likely to be observed when games are played "for real". The predictions of this procedure will then be compared with existing experimental evidences.

Consider the following game where $k \in [-\infty, \infty)$.

4)

	H_B	T_B
H_A	$k, -1$	$-1, 1$
T_A	$-1, 1$	$1, -1$

$\underset{k\in(-1,\infty)}{R_1} =$

	H_B	T_B
H_A	$0, 2$	$2, 0$
T_A	$k+1, 0$	$0, 2$

$\underset{k\in[-\infty,-1)}{R_2} =$

	H_B	T_B
H_A	$-1-k, 2$	$2, 0$
T_A	$0, 0$	$0, 2$

For $k \in (-1, \infty)$ Game 4 is a matching pennies game. With $k = 1$ the game is in its standard version, with $k \neq 1$ the game is asymmetric. In both cases the regret matrix is given by R_1. The minimax regret mixed strategy for player A is given by: $(\tilde{p}_A H_A + (1 - \tilde{p}_A) T_A)$ where $\tilde{p}_A = \frac{1+k}{3+k}$ is the probability that the *mmr* allocates to strategy H_A and thus our candidate to approximate B's beliefs about A playing that strategy. The function for $\tilde{p}_A$ appears as the bold concave curve in Figure 2 which focuses on the conjectured beliefs of player B about what A will play[12].

For $k \in (-\infty, -1]$ the game has a different structure since strategy H_A is dominated by T_A. The dominance is weak for $k = -1$ and strict otherwise. The regret matrix is given by R_2 and the minimax regret attaches probability 0 to A playing H_A. In Figure 2 this appears as the bold line that lies on the x-axis for $k \leq -1$.

The other two functions (thin lines) that appear in Figure 2 depict, respectively, the probability that the mixed strategy Nash equilibrium assigns to player A playing strategy H_A $\left(\frac{1}{2}\right)$ and to player B playing strategy H_B $\left(\frac{2}{3+k}\right)$. The figure thus highlights the problems which were mentioned in the previous section: the *msne* of player A does not respond to a change in A's payoff (in this case k) while the *msne* of player B does respond to a change in k but not in the desired direction. Note also that the functions for the minimax regret and

[12] For $\forall k \in (-\infty, \infty)$ the minimax regret of player B is $\left(\frac{1}{2}H_B + \frac{1}{2}T_B\right)$. According to our interpretation this implies that player A believes player B is equally likely to play any of his strategies no matter the specific value of k.

for the *msne* of the two players intersect just once. The intersection happens for the unique k (in this case $k = 1$, symmetric game) for which all the three functions reach a value of $\frac{1}{2}$.

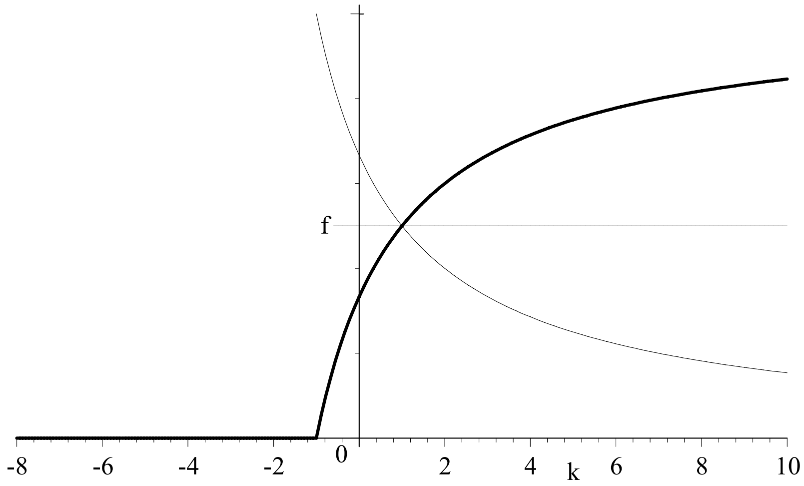

Figure 2: beliefs approximation through minimax regret in a specific game.

In front of such a game, a way to test our theory would be to set a certain k, elicit players' beliefs and check if these subjective beliefs lie close to the ones implied by the minimax regret. We do not pursue this testing strategy for two reasons. First, we claim that the minimax regret beliefs are a good approximation of the real ones from an "operational" point of view, in the sense that they both lead to the same best response[13]. In fact there is no need of extreme precision as far as the conjectured beliefs prove to be useful in forecasting the behavior of the majority of individuals. Second, the technique of beliefs elicitation is still a bit controversial in the literature. The risk is to get biased answers since agents, in declaring their beliefs, are pushed to think more strategically than they would normally do. Croson (2000) finds, for instance, significant differences in the experimental results of public good and prisoner's dilemma games played with and without belief elicitation. On the other side Nyarko and Schotter (2002) and Rey Biel (2004) do not observe different behavior in the context of normal form games.

Therefore, instead of testing the precision of the theory, we test its usefulness. Refer again to Game 4. For any possible k the minimax regret function indicates a unique beliefs distribution defined over the two strategies of the opponent. The expected payoff of the

[13]This happens whenever the subjective beliefs and the conjectured ones of player i lie on the same side of the unit interval with respect to the mixed equilibrium of player i', which sets the indifference point.

two pure strategies conditional on this distribution can then be easily computed. The strategy characterized by the highest expected payoff is the one which best responds to the conjectured beliefs and thus the one we would expect individuals to play. This is the simple structure of the procedure we now present. The testing strategy is equally simple: to check if the hypothetical behavior which stems by the conjectured beliefs is consistent with the one observed in experiments. The focus remains on matching pennies but the procedure, which we now formally define, can (and will) also be applied to other classes of games.

We consider a 2×2 matching pennies game played between players A and B. Let $S_i = \{H_i, T_i\}$ be the strategy space of player $i = \{A, B\}$ and $u_i(s_i, s_{i'})$ the payoffs of the game. The unique minimax regret distribution is given by:

$$\{(\tilde{p}_A H_A + (1 - \tilde{p}_A) T_A), (\tilde{p}_B H_B + (1 - \tilde{p}_B) T_B)\}$$

where $\tilde{p}_i$ defines the probability with which player i should play strategy H_i in order to minimize his expected regret. With a slightly different notation with respect to the previous sections where only B's beliefs were considered, define now as $f_i = [\theta, 1 - \theta]$ the beliefs player i holds on player i' playing strategies $H_{i'}$ with probability θ and strategy $T_{i'}$ with probability $1 - \theta$. $\beta_i(f_i)$ is the best reply function of player i. It uses i's beliefs as an input and provides as an output the strategy i must choose in order to maximize his expected payoff.

The procedure

1. Compute the minimax regret distribution for the two players and retrieve $\tilde{p}_A$ and $\tilde{p}_B$.
2. Assign the following beliefs to the two players:
   - $f_A = [\tilde{p}_B, (1 - \tilde{p}_B)]$
   - $f_B = [\tilde{p}_A, (1 - \tilde{p}_A)]$
3. Let the two players choose the strategy to play according to $\beta_i(f_i)$:
   - $\beta_i(f_i) = \begin{cases} \{H_i\} \text{ iff } u_i(H_i|f_i) > u_i(T_i|f_i) \\ \{T_i\} \text{ iff } u_i(H_i|f_i) < u_i(T_i|f_i) \\ \{0.5H_i + 0.5T_i\} \text{ iff } u_i(H_i|f_i) = u_i(T_i|f_i) \end{cases}$

The strategies selected by $\beta_i(f_i)$ are the ones which have the largest probability to be played in a one shot game or, equivalently, the ones which we would expect to be chosen with the highest frequency if the game is played in a large enough population. Whenever $u_i(H_i|f_i) \neq u_i(T_i|f_i), \forall i$, every player has a single best response and the intersection of the two selected strategies indicates a single outcome of the game as the most likely one. If $u_i(H_i|f_i) = u_i(T_i|f_i)$ for a unique $i = \{A, B\}$ then two are the outcomes selected by the procedure. Finally if $u_i(H_i|f_i) = u_i(T_i|f_i), \forall i$, it means that both players are indifferent on what to play and therefore all the four outcomes are equally likely.

The procedure thus provides a forecast in three simple steps: it is enough to compute the minimax regret, use its probability distributions to approximate players' beliefs and choose for each player the strategies (one or two) that best responds to these beliefs. Again we do not claim this procedure to be consciously used by players. What we claim is that, on average, the procedure is operationally valid i.e. the majority of individuals play the game "as if" they were applying it.

6 Experimental evidences for matching pennies games

We apply the proposed procedure to matching pennies games for which experimental results are available from other studies[14]. Given that the procedure aims to capture the behavior of inexperienced players the ideal data to test our conjecture come from experiments in which subjects played just once a single game (data are reported in Table 2, Section 6.1). Still, as a matter of comparison, data about the first round of repeated games are also considered provided that players were randomly matched in each round so that inter temporal effects are minimized (data appear in Table 3, Section 6.2). Also in this second case the procedure is able to predict the strategies which are overplayed even though the robustness of the results is lower. Despite of the games being different, this last result suggests that individuals' behavior is different (though still similar) in front of one shot interactions and first round of repeated games.

6.1 One shot games

The first three games in Table 2 ($GH1$, $GH2$ and $GH3$) and the correspondent experimental results are taken from Goeree and Holt (2001). Each game was played only once by a different pool of 50 subjects. In the original paper the authors use these games to evaluate the predictive power of the mixed strategy Nash equilibrium. The last three games appear in Goeree and Holt (2004) who took them from Guyer and Rapoport (1972). In the original

[14] With respect to the original papers strategies will be renamed in order to be consistent with previous sections.

experiment 214 subjects were asked to play in a random order 244 games belonging to different typologies. Note two things about these last three games: first, the payoff structure is more complex and second, despite of the fact that games were one shot, the huge number of strategic situations that the players had to face makes the experiment less reliable for our purposes.

The last four columns of Table 2 are the important ones: in the fourth to last column we report $\beta_i(f_i) = \{\cdot\}$, the prediction of the procedure. The third to last column presents the experimental results in the form $a/b\ S_i$, where a is the number of players that chose strategy $S_i = \{H_i, T_i\}$ and $b = 0.5N$ is the total number of row or column players.

The second to last column shows the hit rate which measures the performance of the prediction in forecasting actual behavior. The hit rate is a simple summary statistics which counts the number of hits, i.e. the proportion of player that chose the forecasted strategy. It is described in Verbeek (2004) and used for instance in Gneezy and Guth (2003). The hit rate ranges between 0% (all misses) and 100% (all hits) with 50% being the expected rate of randomly guessing between the two strategies and thus the benchmark for evaluating the value added of the procedure. So, when the procedure indicates a single strategy the hit rate simply captures the percentage of players who actually played it. In games in which the procedure indicates that subjects should uniformly randomize and b is odd (like in $GH1$), the hit rate reaches 100% if the players split as equally as possible. In game $GH1$ for instance the hit rate would have been 100% both if 12 or 13 out of the 25 row or column players chose H_A[15].

Finally in the last column we test for the significance in the difference between the proportions of actual plays observed in the experiments and the benchmark uniform distribution of choices. Our claim is in fact to be able to ex-ante individuate the strategies which are over played by agents. We use the Fisher's exact probability test which calculates the probability of the difference in the distribution between the observed data and the alternative uniform data. When our procedure selects a single strategy we would expect the null hypothesis (observed data being generated by a uniform distribution) to be rejected while when the procedure indicates that players should uniformly mix then we would expect the null hypothesis not to be rejected. The last column reports the (one sided) p-values in percentage: values below the critical value of 5% indicate that the observed proportions are unlikely to come from a uniform distribution, values above 5% are such that the null hypothesis cannot be rejected. For clarity purposes, p-values that are in lines with our procedure are preceded by an asterisk.

[15] A more precise formulation of the hit rate, which has to be used in more complex cases, but that encompasses the ones just described, is presented in the next subsection.

Game N		H_B	T_B	Notes	Procedure selects	Exper. results	Hit rate	Fisher p-values
$GH1$	H_A	$80, 40$	$40, 80$	1 shot	$\frac{1}{2}H_A+\frac{1}{2}T_A$	$12/25\ H_A$	100%	*22%
50	T_A	$40, 80$	$80, 40$		$\frac{1}{2}H_B+\frac{1}{2}T_B$	$12/25\ H_B$	100%	*22%
$GH2$	H_A	$320, 40$	$40, 80$	//	H_A	$24/25\ H_A$	96%	*0,04%
50	T_A	$40, 80$	$80, 40$		T_B	$21/25\ T_B$	84%	*1,6%
$GH3$	H_A	$44, 40$	$40, 80$	//	T_A	$23/25\ T_A$	92%	*0,2%
50	T_A	$40, 80$	$80, 40$		H_B	$20/25\ H_B$	80%	*3,6%
$GR4$	H_A	$24, 5$	$5, -10$	1 shot	H_A	$91/107\ H_A$	85%	*0%
214	T_A	$26, 9$	$-10, 26$	244 g.	H_B	$85/107\ H_B$	79%	*0%
$GR5$	H_A	$15, 5$	$5, -10$	//	H_A	$82/107\ H_A$	77%	*0%
214	T_A	$26, 9$	$-10, 26$		H_B	$81/107\ H_B$	76%	*0%
$GR6$	H_A	$9, 5$	$5, -10$	//	$\frac{1}{2}H_A+\frac{1}{2}T_A$	$74/107\ H_A$	62%	0,4%
214	T_A	$26, 9$	$-10, 26$		T_B	$32/107\ T_B$	30%	0,2%

Table 2: the hit rate of the procedure in one shot matching pennies games.

To have a better feeling of how the procedure works in practice consider a couple of examples. Game $GH1$ is a standard or symmetric matching pennies game. The minimax regret is obviously $\frac{1}{2}H_i + \frac{1}{2}T_i, \forall i = \{A, B\}$ and thus the procedure assigns uniform beliefs to both players. Indeed, in front of such a game, no player has any reason to expect his opponent to be biased in playing a specific strategy. Both strategies therefore lead to the same expected payoff and the procedure predicts all outcomes to be equally likely. Actual frequencies confirm that the distributions of choices of the two populations of players are as uniform as possible.

Things are different when the game is asymmetric like for instance in game $GH2$ where the payoff for players A in the outcome (H_A, H_B) has been modified. In these cases the minimax regret distribution remains the same for players B $\left(\frac{1}{2}H_B + \frac{1}{2}T_B\right)$ but it changes for players A $\left(\frac{7}{8}H_A + \frac{1}{8}T_A\right)$. It follows that, according to our conjecture, a generic player

A still has uniform beliefs about B while B's beliefs change. The procedure then selects strategy H_A as the most likely choice for players A. This strategy has an expected value of $\frac{1}{2}(320) + \frac{1}{2}(40) = 180$ which is larger than the expected value of $T_A : \frac{1}{2}(40) + \frac{1}{2}(80) = 60$. Strategy H_A was in effect chosen by 24 out of the 25 A players. The mechanism is the same for players B: according to the conjecture they strongly believe (probability of $\frac{7}{8}$) that their opponents will play H_A. The procedure thus selects T_B as B's most likely strategy given that $\frac{7}{8}(80) + \frac{1}{8}(40) = 75 > 45 = \frac{7}{8}(40) + \frac{1}{8}(80)$. Strategy T_B was indeed chosen by 84% of B players.

The prediction of the procedure is confirmed also in Game $GH3$ where strategies T_A and H_B are the selected ones and the hit rate is again considerably high. The hit rate remains above 75% and the p-values are in line with our conjecture also in games $GR4$ and $GR5$ while results are less good in the case of Game $GR6$ in which the procedure failed to predict that players over played strategies H_A and H_B. Again we stress that the last three games use data collected more than thirty years ago (1972), that they have a more complex structure involving also a substantially negative payoff and that the design of the experiment does not exactly fit our ideal framework of a single non repeated interaction.

Nevertheless the overall hit rate is above 50% in 11 out of the 12 predictions, being above 70% in 10 out of 12 cases. Considering just games where the procedure indicates a single outcome ($GH2, GH3, GR4, GR5$), the procedure correctly predicts the choices of 81% of the players. Note that in these cases the outcome selected is clearly not an equilibrium since a generic player A would always like to deviate. It may then seem that somehow players A act with a lower degree of rationality in comparison with players B[16]. However the behavior of the majority of both classes of players is consistent with the archetype of individuals that play as if they were best responding to the conjectured minimax beliefs.

6.1.1 A comparison with Nash equilibrium and maxmin prediction

We briefly compare the performance of the prediction of our procedure with the ones provided by the mixed Nash equilibrium and by the maxmin criterion. For brevity we just consider the first three simpler games (Goeree and Holt, 2001).

For what concerns the Nash equilibrium, the authors present the results for Game $GH1$ as supportive of the *msne* prediction, while they show the results of games $GH2$ and $GH3$ as evidences of its failure: in fact, given that player B's payoffs do not change, the *msne* predicts player A to keep on uniformly mixing in all the three games, a forecast which is clearly denied by the data. Therefore the authors write that "*The Nash analysis seems to*

[16] This obviously cannot be the case given the large number of subjects and the random allocation of players to roles.

work only by coincidence, when the payoff structure is symmetric and deviation risks are balanced"[17].

Analyzing the same results through the lens of our conjecture, it seems indeed that the fact that the Nash analysis works in game $GH1$ may be the result of a coincidence. But this coincidence has an explanation. In symmetric matching pennies games the probability distributions implied by the *msne* and by the minimax regret always coincide. In fact the situation of Game $GH1$ is analogous to the game depicted in Figure 2 with $k = 1$, the unique point for which the functions for the minimax regret and for the *msne* intersect.

Still individuals' behavior is by far better captured by our behavioral model rather than by the mixed strategy Nash equilibrium prediction. In fact in games $GH2$ and $GH3$ the Nash mixed equilibrium is extremely good in capturing players B's proportions but, because of the already mentioned "no own payoff effect", it totally fails in predicting that players A will over play strategy H_A. To have a feeling for this difference, Table 3 computes the hit rate of the Nash prediction for the first three games[18]. The table also computes the hit rate of the maxmin prediction: as in the case of the *msne*, maxmin works fine in the symmetric game $GH1$ but its prediction is completely misleading in games $GH2$ and $GH3$. Asterisks next to the hit rate indicate that the associated p-values do not reject the hypothesis of the prediction being able to selct the strategies which are overplayed.

Game N		H_B	T_B	Exper. results	Nash eq. prediction	Hit rate[19]	Maxmin prediction	Hit rate
$GH1$	H_A	$80, 40$	$40, 80$	$12/25\ H_A$	$\frac{1}{2}H_A+\frac{1}{2}T_A$	*100%	$\frac{1}{2}H_A+\frac{1}{2}T_A$	*100%
50	T_A	$40, 80$	$80, 40$	$12/25\ H_B$	$\frac{1}{2}H_B+\frac{1}{2}T_B$	*100%	$\frac{1}{2}H_B+\frac{1}{2}T_B$	*100%
$GH2$	H_A	$320, 40$	$40, 80$	$24/25\ H_A$	$\frac{1}{2}H_A+\frac{1}{2}T_A$	8%	$\frac{1}{8}H_A+\frac{7}{8}T_A$	5%
50	T_A	$40, 80$	$80, 40$	$21/25\ T_B$	$\frac{1}{8}H_B+\frac{7}{8}T_B$	*86%	$\frac{1}{2}H_B+\frac{1}{2}T_B$	32%
$GH3$	H_A	$44, 40$	$40, 80$	$23/25\ T_A$	$\frac{1}{2}H_A+\frac{1}{2}T_A$	16%	$\frac{10}{11}H_A+\frac{1}{11}T_A$	9%
50	T_A	$40, 80$	$80, 40$	$20/25\ H_B$	$\frac{10}{11}H_B+\frac{1}{11}T_B$	*88%	$\frac{1}{2}H_B+\frac{1}{2}T_B$	40%

Table 3: the hit rate of the Nash and maxmin prediction in the GH matching pennies games.

[17] Goeree, J. & Holt, C. (2001), "Ten Little Treasures of Game Theory and Ten Intuitive Contradictions", *American Economic Review*, Vol. 91, pp. 1419.

[18] Results are similar also for the three GR games where the Nash equilibrium indicates that players should randomize according to $\frac{17}{32}H_i+\frac{15}{32}T_i$, i.e. an almost uniform distribution.

[19] Given that *msne* and maxmin often indicate non uniform mixed strategies, the hit rate has been com-

6.2 First round of games with random matching

A similar behavior as the one found in the case of one shot games characterizes also individuals' play in the first round of repeated games. We restrict our attention to games in which players were randomly matched each round to minimize inter temporal strategic effects. Table 3 reports data about matching pennies games played in this way. The first game has been studied by Nyarko and Schotter (2002). It has been played for 60 rounds and over four treatments to investigate the issue of beliefs learning. Game NSa reports the data of treatment 4 (random matching and no belief elicitation) which, in the original paper, served as a control treatment. Game NSb is equal to the previous one but the number of players is larger because data from treatment 4 and treatment 1 (random matching and belief elicitation) are pooled together. The last four games (MPW) have been studied by McKelvey, Palfrey and Weber (2000)[20]. In the original paper these games were played 50 times and data were used to test a version of the quantal response equilibrium that allows for heterogeneity among subjects.

Table 4 has the same structure of Table 2 and it summarizes the results. Again asterisks in the last column indicate that the p-values are in line with the predictions of the procedure.

Game NS is a constant sum game. Minimax regret distribution are given by $\left(\frac{3}{5}H_A + \frac{2}{5}T_A\right)$ and $\left(\frac{2}{5}H_B + \frac{3}{5}T_B\right)$. The expected value of strategy H_A is then $\frac{2}{5}(6) + \frac{3}{5}(3) = \frac{21}{5}$ which is equal to the expected value of T_A: $\frac{2}{5}(3) + \frac{3}{5}(5) = \frac{21}{5}$ so that players A are indifferent on what to play. The procedure instead indicates that players B will select strategy T_B and it thus forecasts outcomes (H_A, T_B) and (T_A, T_B) as the most likely ones. The hit rate is particularly high when only the data about the treatment without beliefs elicitation are considered but it still remains above 75% when the data coming from the belief elicitation treatment are also considered. The p-values always indicate that the data for players A are not unlikely to come from a uniform distribution of play while they confirm that players B overplay strategy T_B.

The last four games all have a similar structure with the payoff for players A in the outcome $\{H_A, H_B\}$ being the largest one. The hit rate never goes below 50% even though its value is lower than before. In some cases, also because of small samples, the p-values do not allow us to make precise statements about the robustness of these results. Finally note

puted according to the formula: $H = \left[1 - \frac{|a-p|}{\max\{b-p,p\}}\right] \cdot 100$. As before, a is the number of player that played strategy $S_i = \{H_i, T_i\}$, $b = 0.5N$ is the number of total players in that role and p is the number of players that should have played strategy S_i according to the ($msne$ or maxmin) prediction. Loosely speaking the hit rate assigns a penalization (numerator) which increases in the distance between the predicted and the actual outcomes. This penalization is then scaled (denumerator) such that a hit rate of 0% is assigned to the case in which the distance is maximal. This formulation encompasses the simpler cases of the previous section so that the hit rates of tables 2 and 3 are directly comparable.

[20] I thank Roberto Weber for giving me access to the original data set.

that the procedure seems to work better in anticipating the behavior of B players (with an overall hit rate of 77% in all the MPW games, p-value of 0%) than the one of A players (hit rate of 55%, p-value of 8%).

Game n				notes	Procedure selects	Exper. results	Hit rate	Fisher p-values
		H_B	T_B					
NSa	H_A	$6,2$	$3,5$	1^{st} round	$\frac{1}{2}H_A+\frac{1}{2}T_A$	8/15 H_A	100%	*28%
30	T_A	$3,5$	$5,3$	random m.	T_B	13/15 T_B	87%	*3%
NSb	H_A	$6,2$	$3,5$	1^{st} round	$\frac{1}{2}H_A+\frac{1}{2}T_A$	11/29 H_A	76%	*13,5%
58	T_A	$3,5$	$5,3$	r.m.+bel.el.	T_B	23/29 T_B	79%	*1,4%
$MPWa$	H_A	$9,0$	$0,1$	1^{st} round	H_A	20/36 H_A	55%	16,7%
72	T_A	$0,1$	$1,0$	random m.	T_B	29/36 H_B	80%	*0,05%
$MPWb$	H_A	$9,0$	$0,4$	//	H_A	14/24 H_A	58%	19,3%
48	T_A	$0,4$	$1,0$		T_B	21/24 H_B	88%	*0,5%
$MPWc$	H_A	$36,0$	$0,4$	//	H_A	13/24 H_A	54%	21,8%
48	T_A	$0,4$	$4,0$		T_B	17/24 H_B	71%	8%
$MPWd$	H_A	$4,0$	$0,1$	//	H_A	6/12 H_A	50%	31,6%
24	T_A	$0,1$	$1,0$		T_B	8/12 T_B	75%	23,3%

Table 4: the procedure in first round of repeated matching pennies games with random matching.

7 The procedure in other games

The same procedure which until now has been applied only to matching pennies can also be used with other games. In fact the procedure provides predictions for all 2x2 games. In some of them it is probably not needed given that it is trivial to forecast the strategies the players adopt. Still it is good to know that the procedure selects a meaningful outcome, i.e. an outcome which has some theorethical foundations and which is confirmed by experimental evidences. In this respect the content of this section can be seen as a robustness check of our conjecture.

In any 2x2 game the steps to select the strategies more likely to be chosen by inexperienced players remain the same: compute the minimax regret, use its probability distribution to approximate players' beliefs and choose the pure strategies that best respond to these beliefs. Simple examples of a game with a single dominant strategy (SD), prisoner's dilemma (PD), pure coordination games (PC), stag-hunt games (SH) and symmetric (BS) and asymmetric (aBS) battle of the sexes games are shown in Table 5. The claim about the effectiveness of the prediction still refers to one shot interactions.

Game		H_B	T_B	Minimax regret	Procedure selects	Related outcome	Notes
SD	H_A	$3,1$	$1,0$	$1H_A+0T_A$	$\{H_A\}$	$\{H_A, H_B\}$	Unique
	T_A	$1,0$	$0,2$	$\frac{1}{3}H_B+\frac{2}{3}T_B$	$\{H_B\}$		NE
PD	H_A	$3,3$	$0,5$	$0H_A+1T_A$	$\{T_A\}$	$\{T_A, T_B\}$	Unique
		$5,0$	$1,1$	$0H_B+1T_B$	$\{T_B\}$		NE
PC	H_A	$2,2$	$0,0$	$\frac{1}{3}H_A+\frac{2}{3}T_A$	$\{T_A\}$	$\{T_A, T_B\}$	Pareto dominant
	T_A	$0,0$	$4,4$	$\frac{1}{3}H_B+\frac{2}{3}T_B$	$\{T_B\}$		NE
SH	H_A	$2,2$	$3,0$	$\frac{2}{3}H_A+\frac{1}{3}T_A$	$\{H_A\}$	$\{H_A, H_B\}$	Risk dominant
	T_A	$0,3$	$4,4$	$\frac{2}{3}H_B+\frac{1}{3}T_B$	$\{H_B\}$		NE
BS	H_A	$3,1$	$0,0$	$\frac{3}{4}H_A+\frac{1}{4}T_A$	$\{H_A, T_A\}$	$\{\cdot, \cdot\}$	All outcomes
	T_A	$0,0$	$1,3$	$\frac{1}{4}H_B+\frac{3}{4}T_B$	$\{H_B, T_B\}$		equally likely
aBS	H_A	$5,1$	$0,0$	$\frac{5}{6}H_A+\frac{1}{6}T_A$	$\{H_A\}$	$\{H_A, H_B\}$	Payoff dominant
	T_A	$0,0$	$1,3$	$\frac{1}{4}H_B+\frac{3}{4}T_B$	$\{H_B\}$		NE

Table 5: the procedure applied to other classes of 2x2 games.

In games that have at least a Nash equilibrium (NE) in pure strategies, if the procedure selects a single outcome, then this outcome is always a NE of the game (SD, PD, PC, SH, aBS). However it may be the case that the procedure does not select any outcome (or better it selects them all), even if pure Nash equilibria exist. This is what happens in the case of symmetric battle of the sexes (BS): in fact the expected payoffs of the two strategies conditional on the conjectured beliefs are equal. Indeed, because of the tension between the preferences of the two players, data coming from the laboratory confirm quite a dispersed distribution of choices. The situation is different in the asymmetric version of the game (aBS) where, in accordance with experimental evidences, the procedure selects the payoff dominant equilibrium. The reason is that a generic player B correctly believes that his opponent has stronger incentives in playing his preferred strategy.

The conjectured beliefs of the players sometimes happen to be incorrect in the sense that they are not in line with the strategies selected by the procedure (SD, BS, aBS). For instance in the SD game the row player expects his opponent to be biased toward playing strategy T_B but indeed player B plays strategy H_B. We do not perceive this to be a problem. In fact we axiomatized the beliefs of inexperienced, unsophisticated and boundedly rational players and therefore the possibility that in some cases the procedure allocates to players "incorrect" beliefs was embedded in our model since the beginning. What matters is that the players play according to their beliefs and that the prediction of the procedure is in line with existing evidences. In the case of the SD game for instance, player A chooses his strictly dominant strategy (which is a best response to any possible belief) and player B best responds choosing H_B

It is also interesting to note how the procedure performs in the case of coordination games. In accordance with theory, intuition and experimental results the Pareto dominant NE is the outcome selected in pure coordination games (PC). More controversial is the indication in stag hunt games (SH) where players face a trade off between an unsafe, but potentially more rewarding, strategy and a safer one. Such games have therefore two Nash equilibria in pure strategies: a Pareto dominant one (more rewarding) and a risk dominant one (less risky). The latter is the one indicated by the procedure. For this class of games the experimental evidence is mixed (see for instance Harsanyi and Selten, 1988; Straub, 1995; Haruvy and Stahl, 2004) but there is a prevailing consensus indicating indeed the risk dominant equilibrium. With this respect and because of its capacity to select a single outcome, the suggested procedure can also be considered as a tool for equilibrium selection in games with multiple equilibria.

8 Conclusion

2x2 one shot games remain a fundamental tool for modeling strategic interactions. These games capture the simplest relations (the number of players and strategies is minimal) but still they can be used to describe an uncountable number of situations. In fact many real life interactions take place among two subjects and many decisions are binary in nature.

No wonder therefore that the study of 2x2 games has always attracted a lot of attention. Theory provides elegant tools to individuate the equilibria of these games; these equilibria have often been given not only a normative interpretation (about how fully rational players should play) but also a positive one (about how, less rational, real players are indeed expected to play). The empirical relevance of these predictions may be weak, particularly for games in which players' interests are always in contrast (matching pennies). As a consequence to predict players' behavior in one off interactions remains a problematic issue.

This paper introduced a simple procedure to be used for forecasting the outcome of 2x2 one shot games. Using an axiomatic approach, we justified the use of minimax regret to approximate the beliefs of inexperienced individuals. Then we let players behave as if they were responding to these conjectured beliefs.

A nice feature of the procedure is that, in selecting the strategies more likely to be played, it considers all the payoffs of the game. In fact the beliefs of generic player i are mimicked by the minimax regret probability distribution of the opponent i' and thus they depend on the payoffs of the latter. But then, in computing best responses, also the payoffs of player i are taken into account. Traditional concepts like the Nash equilibrium and maxmin strategies do not display this "all payoff" effect being functions of only half of the payoffs of the game.

Indeed, when compared with existing experimental evidences about one shot matching pennies games, our suggested procedure proved to be an effective tool in anticipating the moves of the vast majority of the players. Far from having fully solved the problem (for instance the performance of the procedure seems to be much lower in games with more than two strategies), we think that this paper may contribute to the study of individuals' behavior in one shot games.

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