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Voting on Policies in Committees: A Welfare Analysis

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Abstract

This paper considers committees of n players that vote by (weighted) majority on policies that are binding for all members. The voting mechanism is implemented before the players learn their preferred policies. I derive a formula that measures ex-ante welfare and utility of such a committee as a function of the vote allocation. It will be shown that the simple one-player-one-vote rule is welfare maximizing if every player has the same weight in the social welfare function. For the case of different welfare weights numerical examples show that it might be optimal to include player with zero welfare weights in a committee.

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1 Introduction

Most decisions on policy issues are made by voting within committees, e.g. parliaments or councils of supranational or national institutions (as the EU council or central bank councils). The members of a committee represent usually some constituencies which consist of agents with heterogenous preferences. Hence the committee members will have diverging interests as well. Moreover, the preferences are normally only private information. The chosen policies, however, are in general binding for everyone. Under these constraints, the design of a committee involves two important problems. The first one is a problem of representation: How should the members of a committee be elected. The second one, which will be the focus of this paper, is the problem of mechanism design within a committee: Under which rules should decisions be made? This paper provides a welfare analysis of different voting mechanisms available for committees. Moreover, it derives conditions for the optimal design of a committee.

In this paper I introduce a framework that uses an abstract formulation of preferred policies which still delivers an explicit and simple welfare analysis of voting in committees. Preferred policies are modelled as random variables and the utility derived from a common policy is modelled assuming a quadratic loss function. Modelling preferred policies as random variables is a fruitful approach especially for an analysis focussing on the constitutional stage in which the committee is introduced. There a certain voting rule is implemented behind a veil of ignorance, i.e. before the preferences are known. Moreover, it is an appropriate technique to study committees that vote every period over one policy and where the interests of the different players are determined (or at least strongly influenced) by exogenous stochastic variables. An important example is the determination of monetary policy as in the Council of the future European Central Bank [see Brueckner (1997)]. The tool for comparing different committees in this framework is the expected value of an additive social welfare function (SWF). I compare the results with other simple non-voting mechanisms and with the outcome of a joint optimal decision. The latter is shown to be unfeasible in this model.

The main advantage of the approach adopted in this paper is that it directly measures expected utility of the players and social welfare as a function of the voting mechanism. Hence different mechanisms can easily be ranked according to their welfare effects. Since the assumptions needed for the results are fairly standard in economic analysis, this approach can be seen as an improvement on standard methods even if it is more limited in its scope. The most common traditional analytical tools for an analysis of voting in committees are power indices (PI)[1], spatial voting models[2] and, more general, the theory of voting as part of the social choice literature[3]. One main drawback of PI is that they measure the influence on decisions and not the utility derived from the decisions. Hence they are especially insufficient for a welfare analysis when there is voting over common policies. Spatial voting models analyze decisions when preferences of the players are given and are hence not suitable if one is interested in committees that work in a stochastic environment. Moreover, as in social choice theory in general the focus is more on the equilibrium decisions and less on their welfare effects, which is partly due to the fact that this literature works primarily only with ordinal preference relations.

The remainder of the paper is as follows. Section 2 presents the general model of the preferences. For this environment the joint optimal decision is characterized and it is illustrated why this decision is not feasible. Two simple decision mechanisms (dictator mechanism and a fixed policy mechanism) are presented as benchmark cases. In section 3 a specific voting mechanism is introduced. Due to the restriction on the domain of preferences in this paper, this mechanism leads to the standard median voter result for equilibrium policies. In section 4 I analyze how expected social welfare depends on the vote allocation in the committee. I develop a formula for the general case where the voting weights can be different across players. Moreover, I illustrate as well why PI are

[1]See Straffin (1995) for a survey.

[2]See e.g. Enelow and Hinich (1984)

[3]See especially Miller (1995) and Moulin (1995).

insufficient for a welfare analysis of this type. Section 5 characterizes the welfare maximizing vote allocation for different SWF. I will show that if every player has the same weight in the SWF, the vote allocation that maximizes expected welfare is one where every player has one vote. In the sequel I provide a complete analysis for small committees (up to 5 members) and an example for a large voting body. Hereby I illustrate how an optimal vote allocation can be derived as well in the case where the players have different weights in the SWF. By example I show that it might be welfare improving to include players in the committee who have no weight in the SWF. Section 6 concludes the paper.

2 The Model

The committee consists of n-players that decide jointly on a one-dimensional policy $x \in X \subset \Re$. This policy is binding for every player. The decision of forming the committee is taken as exogenously given. Hence it is implicitly assumed that the benefits of deciding jointly on a policy are (ex-ante) higher compared to the case of separate decisions.

The preferences of the members $i \in N$ over this policy are assumed to be independently uniformly distributed random variables. Formally, they are given by

$$\begin{aligned} x_i &\sim U[0,1] \quad \forall i \\ E(x_i\, x_j) &= E(x_i)\, E(x_j) \quad \forall j \neq i, \end{aligned} \tag{A1}$$

This assumption implies that all members are ex-ante identical with respect to their preferred policies. Moreover it says that the preferences are independent across players. The assumption of uniformity eases the analysis considerably. The restriction of the distribution on the interval $[0,1]$ is, however, without any loss of generality. In general the approach in this paper can be extended to other distribution functions, that might even differ between players and be correlated. The advantage of the chosen form is that it highlights the main welfare mechanisms of the voting procedure and gives clear cut, easily understandable solutions.

The time structure of the model is as follows. In period 0 the distribution of the preferences of all players are common knowledge. The decision mechanism is implemented in this period 0. In the following period every player learns his own preference x_i, which is assumed to be private information. The players can communicate with each other, but there is no mechanism that can enforce any kind of sidepayments. Then the committee decides according to the chosen mechanism on the policy x.

The ex-post utility of a player $i \in N$ is given by

$$U_i = -\left(x_i - x\right)^2 \tag{A2}$$

This is a standard quadratic loss function, the utility loss increases more than proportional in the distance between the own preferred policy and the jointly chosen policy.[4] In period 0, i.e. when the decision mechanism is implemented, expected utility of a player is given by

$$E\left(U_i\right) = E\left(-\left(x_i - x\right)^2\right) = -E\left(x_i^2 + x^2 - 2x_i x\right). \tag{1}$$

It is easy to show that under $(A2)$ the preferences x_i can be normalized on any interval without loss of generality as long as x is normalized in the same way.

In the following I will consider additive social welfare functions (SWF). In the general case the players might have different weights in the SWF. Ex-post welfare is then given as

$$W = \sum_{i \in N} \gamma_i U_i = \sum_{i \in N} -\gamma_i \left(x_i - x\right)^2, \quad \sum_{i \in N} \gamma_i = 1, \quad \gamma_1 \geq \gamma_2 \geq ... \geq \gamma_n \geq 0. \tag{A3}$$

Expected welfare in period 0 is

$$E\left(W\right) = \sum_{i \in N} \gamma_i E\left(-\left(x_{i,t} - x_t\right)^2\right) = -\sum_{i \in N} \gamma_i E\left(x_i^2 + x^2 - 2x_i x\right) \tag{2}$$

[4]Some results in this paper depend quite crucial on this assumption. I belief that a function with increasing marginal losses is more realistic than one with constant marginal losses.

The normalization of the welfare weights γ_i and the ordering of the players is made just for convenience. The most simple and standard case is the one of an equally weighted SWF, i.e. $\gamma_i = \frac{1}{n} \quad \forall i$. But for committees as in the European Union it is reasonable to allow for different weights in order to incorporate the different size of European countries. The SWF's considered in this paper have the feature that only the committee members and no constituencies choosing the members are taken into account. This can be justified by assuming that members of the committee pursue solely the homogenous interests of the people that choose them. Another justification might be that the welfare effects of the selection mechanisms are not part of the welfare effects of the committee decisions. This restriction is more natural if one considers the formation of supranational institutions as the EU than if one considers the welfare effects of decisions made in national parliaments.

As a benchmark case, consider first the solution for a social planner whose only constraint is that the decisions are binding for every player. Maximizing $(A3)$ with respect to x shows that the joint optimal decision x^* equals the weighted mean, where the weights are the those of the SWF. Formally,

$$x^* = \sum_{i \in N} \gamma_i x_i \tag{3}$$

Expected welfare in period 0 is given by the following lemma:

Lemma 1 *The expected welfare of the joint optimal decision is*

$$E(W) = -\frac{1}{12}\left(1 - \sum_i \gamma_i^2\right)$$

Proof. Consider a new normalization of the preferences such that the expected value of the preferred policy is equal to zero. Formally, $x_i' = x_i - \frac{1}{2} \quad \forall i$. Note that this renormalization does not change the values of the utility functions. We get

$$E\left(x_i'^2\right) = \int_{-\frac{1}{2}}^{\frac{1}{2}} x^2 dx = \frac{1}{12} \tag{4}$$

$$
\begin{aligned}
E\left(x_i' x_j'\right) &= E\left(x_i'\right) E\left(x_j'\right) = 0 \quad \forall j \neq i && (5) \\
E\left(x'^2\right) &= E\left(\left(\sum \gamma_i x_i'\right)^2\right) = E\left(\sum_i \gamma_i^2 x_i'^2 + \sum_i \sum_{j \neq i} \gamma_i \gamma_j x_i' x_j'\right) \\
&= \sum_i \gamma_i^2 E\left(x_i'^2\right) = \frac{1}{12} \sum_i \gamma_i^2 && (6) \\
E\left(x_i' x'\right) &= E\left(x_i' \sum_{i \in N} \gamma_i x_i'\right) = E\left(\gamma_i x_i'^2 + \sum_{j \neq i} \gamma_j x_i' x_j'\right) = \frac{\gamma_i}{12} && (7)
\end{aligned}
$$

From this it follows that

$$
\begin{aligned}
E(W) &= -\sum_{i \in N} \gamma_i E\left(x_i'^2 + x'^2 - 2 x_i' x'\right) \\
E(W) &= -\sum_{i \in N} \gamma_i \frac{1}{12}\left(1 + \sum_i \gamma_i^2 - 2\gamma_i\right) = -\frac{1}{12}\left(1 - \sum_i \gamma_i^2\right)
\end{aligned}
$$

■

However, it is important to note that this solution is not feasible in the model because we have private information about the preferences and no enforceable contracts for sidepayment mechanisms. Since preferences are single-peaked, we can apply the standard result that the only mechanisms that are strategy-proof (i.e. revealing the true preferences is a dominant strategy) and respect voter sovereignty (i.e. no alternative is a-priori excluded) are median voter schemes.[5] Since the main focus of this paper is the efficiency of decision mechanisms, it is worth to investigate briefly (non voting) mechanisms that violate voter sovereignty.

The first one is the dictator mechanism. With this mechanism one player chooses the policy regardless of the realized preferences of the other players. It is obvious, that from the standpoint of efficiency the best dictator mechanism is the one where the most important player (here player 1) decides, i.e. $x^d = x_1$. The following lemma describes the expected welfare of this mechanism.

[5]See e.g. Barbera et al. (1993) or Ching (1997).

Lemma 2 *The expected welfare of the optimal dictator mechanism is*

$$E(W) = -\frac{1}{6}(1-\gamma_1)$$

Proof. Consider the same renormalization of preferences as in the proof of lemma 1. Then

$$\begin{aligned} E(W) &= -\left(\gamma_1 \cdot 0 + \sum_{i=2}^{n} \gamma_i \left(E\left(x_i'^2\right) + E\left(x_1'^2\right)\right)\right) \qquad (8) \\ &= -\frac{1}{6}(1-\gamma_1) \qquad \blacksquare \end{aligned}$$

Note that in the case of an equally weighted SWF a random dictator mechanism is one optimal dictator mechanism.

Another simple mechanism is a fixed policy mechanism, i.e. $x^f = x$. Obviously, the most efficient fixed policy is $x = E(x_i) = \frac{1}{2}$. In this case the result is

Lemma 3 *Expected welfare of the optimal fixed policy is* $E(W) = -\frac{1}{12}$.

Proof.

$$\begin{aligned} E\left(x_i^2\right) &= \int_0^1 x^2 dx = \frac{1}{3} \qquad (9) \\ E(W) &= -\sum_{i \in N} \gamma_i E\left(x_i^2 + \frac{1}{4} - 2x_i \frac{1}{2}\right) \\ &= -\sum_{i \in N} \gamma_i \left(\frac{1}{3} + \frac{1}{4} - \frac{1}{2}\right) = -\frac{1}{12} \qquad \blacksquare \qquad (10) \end{aligned}$$

Since voting is the most common decision mechanism within committees and sidepayments are excluded by assumption, I do not consider mechanisms that might violate strategy-proofness or that rely on transfers among the players. Instead I concentrate in the following on the properties and welfare implications of different voting mechanisms.

3 The Voting Mechanism

The decision mechanism that I consider is voting among the member with a weighted majority rule. The set of members (players) in the committee is denoted by N. The voting game within a committee is described by $(d, \mathbf{w})$. The vector of voting weights (or simple the votes) $\mathbf{w} = (w_1, w_2, ...w_n)$ are chosen at the constitutional stage and remain fixed over time. The value of d gives the decision (majority) rule, i.e. the minimum number of votes required for a majority. A voting game is usually characterized by its coalitional (or characteristic) function, i.e. by a function $v : 2^N \to \Re$ that assigns to every coalition $S \subseteq N$ a value as its worth. For a voting game $(d, \mathbf{w})$ this function is given by

$$v(S) = \begin{cases} 1 \text{ if } w_S = \sum\limits_{i \in S} w_i \geq d \\ 0 \text{ if } w_S = \sum\limits_{i \in S} w_i < d \end{cases}. \tag{11}$$

The number of players in a coalition is denoted by $s = |S|$. A coalition is called a minimum winning coalition (MWC) if there exist at least one player whose exit would turn the coalition from a winning into a loosing coalition. Formally[6],

$$S \text{ is a } MWC \text{ iff } (v(S) = 1) \wedge (\exists i \mid v(S \backslash i) = 0). \tag{12}$$

In addition I make the following two assumptions

$$v(S) = 1 \Rightarrow v(N \backslash S) = 0 \tag{13}$$

$$\nexists \; i \mid v(\{i\}) = 1 \tag{14}$$

The first assumption is a natural restriction for committees since it excludes that two distinct coalitions could implement different policies at the same time. The second one serves only for distinguishing a dictator mechanism from a voting mechanism.[7] The easiest decision rule, that will

[6]For simplicity I write $S\backslash i$ instead of $S\backslash\{i\}$.

[7]In a simple voting game the condition

$$v(S) = 1 \Longrightarrow v(T) = 1 \quad \forall T \supseteq S \tag{15}$$

always holds. Occasionally any simple game fulfilling (13) and (15) is called a committee, see e.g. Peleg (1984).

play a major rule in the proceeding, is the simple majority rule without possibility for a tie. In this case the voting game is constant-sum in its coalitional function, i.e.

$$(v(S)=1) \Leftrightarrow (v(N\backslash S)=0)\,, \text{ or } \qquad (w_S \geq d) \Leftrightarrow \left(w_{N\backslash S} < d\right)\,. \quad (16)$$

The voting mechanism itself is the following multi-stage game. When the committee meets all players learned their preferences. Moreover there is a status-quo policy $x^q \in X$ which is the policy valid until the committee makes a final decision. There are infinitely many voting rounds, indexed by τ. Each voting round occurs an infinitesimally small cost c to every member[8], which can be thought of as disutility from being in the meeting. At the beginning of each meeting every player announces simultaneously a policy x_i^a that he wants to be implemented. In each voting round a randomly chosen member makes a proposal x_i^p. Then voting takes place. Every player votes either 'yes' or 'no'[9], formally

$$a_i = \begin{cases} 1 & \text{if 'yes'} \\ 0 & \text{if 'no'} \end{cases}$$

If a majority votes 'yes', i.e. $\sum a_i w_i \geq d$, this policy will be implemented. If $\sum a_i w_i < d$, a new round starts and another randomly chosen player (possibly the same) makes a proposal. This procedure continues until a proposal can be implemented.

I first consider that d is the simple majority rule, i.e. I assume (16) to hold. Under this condition it is straightforward to show that the game has a unique stationary perfect equilibrium. In equilibrium the preferred policy of the median voter is implemented without any delay. The unique stationary perfect equilibrium is characterized by the following proposition:

[8] Formally, $0 < c < \varepsilon$ for any positive number ε.

[9] Abstentions are regarded as 'no' votes.

Proposition 4 *Assume d is the simple majority rule. Then, in the unique stationary perfect equilibrium every player announces his preferred policy, every player proposes the preferred policy of the median voter (x_m) , this policy is implemented, and every player votes 'yes' if and only if a proposal gives him a utility at least as high as in the equilibrium. Formally*

$$\begin{aligned} x_i^a &= x_i, \quad x_i^p = x_m \quad \forall i, \quad x = x_m \\ a_i\left(x^p\right) &= \begin{cases} 1 & if \quad |x_i - x^p| \leq |x_i - x_m| \\ 0 & if \quad |x_i - x^p| > |x_i - x_m| \end{cases} \qquad \forall i. \end{aligned}$$

Proof. see appendix ■

It should be noted that due to the single-peakedness of the preferences the median voter theorem applies. Moreover, the implemented policy does not depend on the status-quo policy x^q. The introduction of the announcement stage simply avoids time-consuming pairwise voting. Evidently, there are possibly many variants of this voting mechanism that ensure that the preferred policy of the median voter is the chosen policy.

In principle, the voting procedure described above can be applied as well to committees using a supra-majority rule. But in this case the equilibrium depends on the status-quo policy. Moreover, it is possible that multiple stationary perfect equilibria arise. The appendix contains an example illustrating this point. The multiplicity results from the fact that with a supra-majority rule the set of policies that cannot be beaten (if players are rational) by an alternative is (generically) no longer single-valued. To avoid this complication, I concentrate in the following on voting games that fulfill condition (16).

4 Welfare and Vote Allocation

In this section I derive a formula that measures the welfare effects of a committee. As mentioned above, I restrict the analysis to the case that d is the simple majority rule. The welfare measure I use is the expected value of the SWF in period 0, i.e. when the voting mechanism is implemented. It is obvious that ex-post (i.e. after the preferences become known) the optimal vote allocation is one that makes the player as the median who is closest to the social optimum, i.e. in our model the weighted mean. But state-dependent vote allocations are excluded from the analysis for reasons of reality.

I start this analysis by regarding the expected utility of an individual player, computing expected welfare afterwards is relatively straightforward. We can rewrite (1) as

$$E\left(U_i\right) = -E\left(x_i^2\right) + E\left(2x_i x_m - x_m^2\right) \tag{17}$$

These values are functions of the distribution of the preferences and the vote allocation, since these two together determine the distribution of the (weighted) median.

Consider first any ordering of preferences

$$x_{(1)} < x_{(2)} < ... < x_m < x_{(m+1)} < ...x_{(n)},$$

where x_m is the position of the median voter. The density function of the median position for this given ordering is [cf e.g. Mood et al. (1974)]

$$f_{x_m}\left(x\right) = \frac{n!}{(m-1)!\,(n-m)!}\left[F\left(x\right)\right]^{m-1}\left[1-F\left(x\right)\right]^{n-m} f\left(x\right)$$

The density function $f\left(x\right)$ and the corresponding cumulative distribution function are given by $(A1)$. Hence we have

$$f_{x_m}\left(x\right) = \frac{n!}{(m-1)!\,(n-m)!}x^{m-1}\left(1-x\right)^{n-m} \tag{18}$$

From this we get the following three expression for the conditional expected value of $x_i x_m$:

$$\begin{aligned} E\left(x_i x_m \mid x_i = x_m\right) &= \int_0^1 x^2 f_{x_m}\left(x\right)dx \\ = \quad & \frac{n!}{(m-1)!\,(n-m)!}\int_0^1 x^2 x^{m-1}\left(1-x\right)^{n-m}dx \\ = \quad & \frac{n!}{(m-1)!\,(n-m)!}\frac{(m+1)!\,(n-m)!}{(n+2)!} \end{aligned} \tag{19}$$

$$E(x_i x_m \mid x_i < x_m) = \int_0^1 \int_0^x \frac{1}{x} yxdy \quad f_{x_m}(x)\,dx$$
$$= \frac{n!}{(m-1)!\,(n-m)!}\frac{1}{2}\int_0^1 x^2 x^{m-1}(1-x)^{n-m}\,dx \tag{20}$$
$$= \frac{n!}{(m-1)!\,(n-m)!}\frac{1}{2}\frac{(m+1)!\,(n-m)!}{(n+2)!}$$

$$E(x_i x_m \mid x_i > x_m) = \int_0^1 \int_x^1 \frac{1}{1-x} xydy \quad f_{x_m}(x)\,dx$$
$$= \frac{n!}{(m-1)!\,(n-m)!}\frac{1}{2}\int_0^1 \left(x+x^2\right) x^{m-1}(1-x)^{n-m}\,dx \tag{21}$$
$$= \frac{n!}{(m-1)!\,(n-m)!}\frac{1}{2}\left[\frac{(m+1)!\,(n-m)!}{(n+2)!}+\frac{m!\,(n-m)!}{(n+1)!}\right]$$

I first consider the case that every player has exactly one vote and that n is odd. In this case the position of the median is always the same, i.e. $m = \frac{n+1}{2}$. Thus equations (19), (20) and (21) simplify to

$$E(x_i x_m \mid x_i = x_m) = E\left(x_m^2\right) = \frac{\frac{n+1}{2}\frac{n+3}{2}}{(n+1)(n+2)} = \frac{n+3}{4(n+2)} \tag{22}$$
$$E(x_i x_m \mid x_i < x_m) = \frac{n+3}{8(n+2)} \tag{23}$$
$$E(x_i x_m \mid x_i > x_m) = \frac{1}{2}\left(\frac{n+3}{4(n+2)}+\frac{\frac{n+1}{2}}{n+1}\right) = \frac{n+3}{8(n+2)}+\frac{1}{4} \tag{24}$$

The expected welfare of the voting mechanism where every player has one vote is then given by the following proposition

Proposition 5 *Expected social welfare of a committee fulfilling A1,A2,A3 and* (16) *where every player has one vote and n is odd is*

$$E(W) = -\frac{1}{3}+\frac{1}{4}\frac{(n+1)^2}{(n+1)^2-1} \tag{25}$$

Proof. Consider first expected utility for an individual player (17). We have

$$E\left(x_i^2\right) = \int_0^1 x^2 dx = \frac{1}{3}. \tag{26}$$

For the remaining term we get

$$E\left(2x_i x_m - x_m^2\right) = \begin{cases} E\left(x_m^2\right) = \frac{n+3}{4(n+2)} \text{ if } x_i = x_m \\ 2\frac{1}{2}\frac{n+3}{4(n+2)} - E\left(x_m^2\right) = 0 \text{ if } x_i < x_m \\ 2\frac{1}{2}\left(\frac{n+3}{4(n+2)} + \frac{1}{2}\right) - E\left(x_m^2\right) = \frac{1}{2} \text{ if } x_i > x_m \end{cases} \tag{27}$$

Since every player has the same votes and all orderings are equally likely, symmetry implies

$$prob\left(x_i = x_m\right) = \frac{1}{n}, \quad prob\left(x_i < x_m\right) = prob\left(x_i > x_m\right) = \frac{n-1}{2n} \tag{28}$$

Hence we get

$$\begin{aligned} E\left(U_i\right) &= -\frac{1}{3} + \frac{1}{n}\frac{n+3}{4\left(n+2\right)} + \frac{n-1}{4n} = -\frac{1}{3} + \frac{n+3+\left(n-1\right)\left(n+2\right)}{4n\left(n+2\right)} \\ &= -\frac{1}{3} + \frac{1}{4}\frac{\left(n+1\right)^2}{\left(n+1\right)^2 - 1} \end{aligned} \tag{29}$$

$$E\left(W\right) = \sum \gamma_i E\left(U_i\right) = -\frac{1}{3} + \frac{1}{4}\frac{\left(n+1\right)^2}{\left(n+1\right)^2 - 1} \qquad \blacksquare$$

Comparing proposition 2 with the two benchmark mechanism, i.e. comparing (25) with (9) and (8) leads to the following two corollaries

Corollary 6 *The one-player-one-vote rule gives strictly higher welfare than the optimal fixed policy.*

Proof.

$$\begin{aligned} -\frac{1}{3} + \frac{1}{4}\frac{\left(n+1\right)^2}{\left(n+1\right)^2 - 1} &> -\frac{1}{12} \\ \Longleftrightarrow \frac{1}{4}\frac{\left(n+1\right)^2}{\left(n+1\right)^2 - 1} &> \frac{1}{4} \\ \Longleftrightarrow \frac{\left(n+1\right)^2}{\left(n+1\right)^2 - 1} &> 1 \quad \blacksquare \end{aligned}$$

Corollary 7 *The one-player-one-vote rule gives strictly higher welfare than the optimal dictator mechanism if* $\gamma_1 < \frac{3}{2}\frac{(n+1)^2}{(n+1)^2-1} - 1$.

Proof.

$$\begin{aligned}
-\frac{1}{3}+\frac{1}{4}\frac{(n+1)^2}{(n+1)^2-1} &> -\frac{1}{6}(1-\gamma_1) \\
\Longleftrightarrow 1-\gamma_1 &> 2-\frac{3}{2}\frac{(n+1)^2}{(n+1)^2-1} \\
\Longleftrightarrow \gamma_1 &< \frac{3}{2}\frac{(n+1)^2}{(n+1)^2-1}-1 \quad \blacksquare
\end{aligned}$$

Now I turn to the more general case where the players might have different votes. Since there are far more permutations ($n!$) than possible coalitions (2^n-1), it is convenient to determine expected utility not over permutations but over coalitions. To see this point, consider one ordering where player i is the median voter. Denote[10]

$$S\backslash i=\{j \mid x_j<x_i\}, \quad N\backslash S=\{k \mid x_k>x_i\}.$$

From equations (19) – (21) we know that for computing the conditional expected utility the ordering among the players 'left' of player i as well of those 'right' of player i do not matter. Thus there are $(s-1)!\,(n-s)!$ permutations that have an identical effect on the expected welfare. Moreover, recall that under $(A1)$ all of the $n!$ permutations are equally likely. Finally, we have from (11) that $x_i=x_m \Rightarrow v(S)-v(S\backslash i)=1$. With these preliminary results in mind, expected welfare of a committee is given by the following formula

Proposition 8 *In a committee fulfilling A1, A2, A3 and* (16)*, expected social welfare is given by*

$$\begin{aligned}
E(W) &= -\frac{1}{3}+\sum_{i\in N}\sum_{S\ni i}\left[v(S)-v(S\backslash i)\right] \qquad (30)\\
&\quad \left[\frac{(s+1)!\,(n-s)!}{(n+2)!}\gamma_i+\frac{s!\,(n-s)!}{(n+1)!}(1-\gamma_s)\right]
\end{aligned}$$

where $\gamma_s=\sum_{i\in S}\gamma_i$

[10]Note that the events $x_j=x_i$ and $x_k=x_i$ have zero probability.

Proof. 1. We have

$$E\left(x_i^2\right) = \int_0^1 x^2 dx = \frac{1}{3}. \tag{31}$$

2. For the expected value of the squared median decision we have

$$E\left(x_m^2\right) = \sum_{i \in N} \sum_{S \ni i} \left[v(S) - v(S \backslash i)\right] \frac{(s+1)!\,(n-s)!}{(n+2)!} \tag{32}$$

3. The influence of a median on the terms $2E\left(x_i x_m\right)$ is

$$2 \sum_{S \ni i} \left[v(S) - v(S \backslash i)\right] \left[\frac{(s+1)!\,(n-s)!}{(n+2)!} 1 + \frac{1}{2} \frac{(s+1)!\,(n-s)!}{(n+2)!} (s-1) \right.$$
$$\left. + \frac{1}{2} \left(\frac{(s+1)!\,(n-s)!}{(n+2)!} + \frac{s!\,(n-s)!}{(n+1)!} \right) (n-s) \right] \tag{33}$$

The first term within the last brackets gives the impact on the welfare of the median himself, the second term the impact on other players in S and the last term the impact on players outside S.

Combining these three expressions and multiplying them with the social welfare weights gives

$$\begin{aligned} E(W) &= -\frac{1}{3} + \sum_{i \in N} \sum_{S \ni i} \left[v(S) - v(S \backslash i)\right] \\ &\quad \left[\frac{(s+1)!\,(n-s)!}{(n+2)!} \left(-1 + 2\gamma_i + \gamma_s - \gamma_i + 1 - \gamma_s\right) \right. \qquad (34) \\ &\quad \left. + \frac{s!\,(n-s)!}{(n+1)!} \left(1 - \gamma_s\right) \right] \\ &= -\frac{1}{3} + \sum_{i \in N} \sum_{S \ni i} \left[v(S) - v(S \backslash i)\right] \qquad (35) \\ &\quad \left[\frac{(s+1)!\,(n-s)!}{(n+2)!} \gamma_i + \frac{s!\,(n-s)!}{(n+1)!} \left(1 - \gamma_s\right)\right] \end{aligned}$$

■

Using an indicator function for the membership of a player in a coalition, i.e.

$$I_{i,S} = \begin{cases} 1 & if \quad i \in S \\ 0 & if \quad i \notin S \end{cases}$$

and rearranging terms gives the following expression for the expected utility of an individual player:

$$\begin{aligned} E(U_i) &= -\frac{1}{3} + \sum_{S \ni i} [v(S) - v(S \backslash i)] \frac{(s+1)!\,(n-s)!}{(n+2)!} \\ &\quad + \sum_{j \neq i} \sum_{S \ni j} [v(S) - v(S \backslash j)] (1 - I_{i,S}) \frac{s!\,(n-s)!}{(n+1)!} \end{aligned} \tag{36}$$

Formula (36) illustrates clearly why power indices (PI) are insufficient to measure the welfare effects of these kind of models. In any stochastic game where all permutations of ordered preferences are equally likely, the Shapley-Shubik (1954) index of a voting game gives the probability that a player can enforce his preferred policy in the voting game.[11] Formally, the Shapley-Shubik value ϕ_i is given by

$$\phi_i = \sum_{S \ni i} [v(S) - v(S \backslash i)] \frac{(s-1)!\,(n-s)!}{n!} \tag{37}$$

The comparison between (36) and (37) shows, that the Shapley-Shubik value does not give the right effect of the decision of the median on himself and neglects the effects on the other players. The fact that other people receive nothing from the median decision, illustrates that the Shapley-Shubik index might be appropriate for voting over private goods, but not for voting over public goods like policies.[12]

5 Optimal Voting Games

In this section the conditions for a welfare optimal voting game are derived. Since the attention is restricted to the simple majority rule, we have to maximize $E(W)$ with respect to the votes. Formally, optimal

[11] See e.g. Owen (1995).

[12] For an early critic why standard PI are insufficient for decisions on public-goods see Barry (1980).

voting games are here defined as the solution of the following maximization problem

$$\begin{aligned}\max_{(d,\mathbf{w})} E(W) &= -\frac{1}{3}+\sum_{i\in N}\sum_{S\ni i}\left[v(S)-v(S\backslash i)\right] \\ &\quad \left[\frac{(s+1)!\,(n-s)!}{(n+2)!}\gamma_i+\frac{s!\,(n-s)!}{(n+1)!}(1-\gamma_s)\right] \\ s.t.\quad (v(S)=1) &\Leftrightarrow (v(N\backslash S)=0)\end{aligned} \tag{38}$$

I consider first the case in which each member of the committee has equal weight in the SWF. Starting with the case that n is odd, the following proposition shows that the very simple one-player-one-vote rule is welfare maximizing.

Proposition 9 *The vote allocation* $w_i=1\quad\forall i$ *and* $d=\frac{n+1}{2}$ *maximizes the expected welfare in a committee fulfilling A1,A2,A3 and* (16) *if* n *is odd and* $\gamma_i=\frac{1}{n}\quad\forall i$.

Proof. With $\gamma_i=\frac{1}{n}$ equation (30) becomes

$$\begin{aligned}E(W) &= -\frac{1}{3}+\sum_{i\in N}\sum_{S\ni i}\left[v(S)-v(S\backslash i)\right] \\ &\quad \left[\frac{(s+1)!\,(n-s)!}{(n+2)!}\frac{1}{n}+\frac{s!\,(n-s)!}{(n+1)!}\left(\frac{n-s}{n}\right)\right]\end{aligned}$$

Hence expected welfare depends only on the size of coalitions. Equivalently, expected welfare in any ordering of the players according to their preferences is determined by the position of the median. This implies that, if possible, all coalitions where $v(S)-v(S\backslash i)=1$, i.e. all MWC, have the same size in the optimal vote allocation. From the fact that the Shapley Shubik index sums up to 1 we get immediately that then

$$\sum_{i\in N}\sum_{S\ni i}\left[v(S)-v(S\backslash i)\right]=\frac{n!}{(s-1)!\,(n-s)!}$$

Hence the problem (38) reduces to

$$\max_s \frac{n!}{(s-1)!\,(n-s)!}\left[\frac{(s+1)!\,(n-s)!}{(n+2)!}\frac{1}{n}+\frac{s!\,(n-s)!}{(n+1)!}\left(\frac{n-s}{n}\right)\right]$$

or even simpler

$$\max_s s\,(s+1) + s\,(n-s)\,(n+2) \tag{39}$$

The FOC of this problem[13] is

$$s^* = \frac{n+1}{2}$$

The voting game $\left(\frac{n+1}{2}; \mathbf{1}\right)$ obviously guarantees that all MWC have exactly $\frac{n+1}{2}$ members. ■

The proof leads immediately to a complete characterization of all optimal vote allocations for the case in which n is odd.

Corollary 10 *All optimal voting games in a committee in which n is odd and $\gamma_i = \frac{1}{n}$ $\forall i$ and that fulfills A1, A2, A3 and* (16) *have the same coalitional function as $v\left[\frac{n+1}{2}; \mathbf{1}\right]$.*

The proof of proposition 4 leads as well to the characterization of all optimal voting games for the case that n is even. Since in this case the expression (39) is minimized at $s = \frac{n}{2}$ and $s = \frac{n}{2} + 1$, optimal voting games are characterized by the following corollary

Corollary 11 *In all optimal voting games in a committee fulfilling A1, A2, A3 and* (16) *with n even and $\gamma_i = \frac{1}{n}$ all MWC are of size $s = \frac{n}{2}$ or $s = \frac{n}{2} + 1$.*

The case of different weights in the SWF is analytically much more difficult to solve. The problem is that expected social welfare is neither continuous nor monotonic in the voting weights. For small n a complete characterization can be given, but for larger n the solution has to be found numerically. For simplicity I focus in the following on the case in which n is odd.

First insights can be found by checking the optimality of the benchmark cases. We know from corollary 1 that the fixed policy mechanism is never optimal since the one-player-one vote rule leads to higher welfare. Comparing (30) with (8) leads to the following corollary.

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[13] Since the function is concave in s, the second order condition is fulfilled as well.

Corollary 12 *A sufficient condition for a dictator mechanism to be suboptimal is* $\gamma_1 < \frac{n}{n+2}$.

Proof. See appendix. ■

Further interesting results can be found by a complete characterization of all possible different values of (30) for a small number of players. The appendix gives a complete treatment for the case that $n = 5$ and d is the simple majority rule. Consider e.g. the following social welfare weights: $\boldsymbol{\gamma}^e = (0.48, 0.26, 0.26, 0, 0)$, i.e. player 4 and 5 do not count for social welfare. In this case the vote allocation $\mathbf{w}^5 = (3, 2, 2, 1, 1)$ leads to a value of $E(W) = -0.065$ which is higher than for any possible vote allocation that assign no votes to players 4 and 5.[14] To see this point, assume that player 2 and 3 want a policy close to zero and player 1 wants a policy close to 1. If player 4 and 5 prefer a policy around $\frac{1}{2}$, the chosen policy with the vote allocation $\mathbf{w}^5$ is much closer to the joint optimal policy than the equilibrium policy in voting games where $w_4 = w_5 = 0$. Obviously, there are realizations where the inclusion of player 4 and 5 in the committee is actually welfare reducing. This happens e.g. if

$$x_4 = x_5 = 0, \quad x_2 = \frac{1}{4}, \quad x_1 = \frac{1}{2}, \quad x_2 = \frac{3}{4}.$$

If player 4 and 5 have no votes the equilibrium policy would be close to the joint optimum. The vote allocation $\mathbf{w}^5$, however, leads to the suboptimal policy $x = \frac{1}{4}$. But for the social welfare weights $\boldsymbol{\gamma}^e$ the welfare reducing effects of the vote allocation $\mathbf{w}^5$ are ex-ante smaller than the welfare improving effects. The conclusion from this example is that there exist committees where it is welfare improving to include players that have no weights in the SWF since they might help to moderate policies.

For large committees it becomes tedious to compute all possible values of $E(W)$. Thus the optimal solution can be better found by applying an appropriate search algorithm. As mentioned above, the problem is that $E(W)$ is neither continuous nor monotonic in the voting weights or

[14]The game $v(51; 48, 26, 26) = v(2; 1, 1, 1)$ gives $E(W) = -0.0667$ and the dictator mechanism gives $E(W) = -0.0866$.

the decision rule. In order to avoid that one finds only local maxima of $E(W)$, the use of multistage algorithms that use many starting points should deliver the solution of the welfare maximization problem. As an illustration, regard the 11-player committee with social welfare weights given as

$$\boldsymbol{\gamma} = (0.312, 0.213, 0.186, 0.11, 0.056, 0.037, 0.031, 0.025, 0.018, 0.011, 0.002)$$

This example characterizes the ECB-Council of the future European Monetary Union consisting of Germany, France, Italy, Spain, Netherlands, Belgium, Austria, Portugal, Finland, Ireland and Luxembourg. The weights in the SWF are the importance measures as they are laid down in the Maastricht treaty.[15] The optimal vote allocation that I could find is

$$\mathbf{w} = (414, 318, 291, 203, 127, 122, 115, 115, 99, 87, 68)$$

The relative votes in this allocation are

$$\begin{aligned}\frac{\mathbf{w}}{\sum w_i} &= (0.211, 0.162, 0.149, 0.104, 0.065, 0.062, 0.059, 0.059, 0.051, \\ &\quad 0.044, 0.035)\end{aligned}$$

The optimal vote allocation lies somehow in between the one-player-one-vote rule and the rule $\mathbf{w} = \boldsymbol{\gamma}$.[16] Hence one might conclude from this example that in the solution to the welfare maximization problem differences in the welfare weights should be only partially taken into account. Moreover, it is apparent that Luxembourg has a remarkable influence in the voting game even though its influence on welfare is quasi negligible. This indicates that there are potential benefits of including players with weights in the SWF of zero (or almost zero) not only in small but as well in larger committees.

[15] See Brueckner (1997) for a model of voting and bargaining over monetary policy in the ECB with public information of preferences.

[16] If one considers the Shapley-Shubik values of this game, we get

$$\phi = (0.235, 0.164, 0.154, 0.101, 0.056, 0.056, 0.051, 0.051, 0.048, 0.041, 0.041)$$

6 Conclusion

In this paper it was shown that the expected welfare of a committee where players with diverging interests decide on jointly binding policies can be expressed in a simple formula. It was shown that with an equally weighted social welfare function the simple one-player-one-vote rule is optimal. Hence the question of optimal mechanism design has an easy solution in this case. For unequal welfare weights the problem becomes more complicated but a numerical solution can always be provided. Moreover, it was shown that it is optimal in some cases to include players in a committee whose weight in the SWF is negligible or even zero. The analysis in this paper should not only lead to new insights for an understanding of existing committees but may help as well for the design of new committees.

As in many voting models, the assumption of single-peaked preferences is probably the most restrictive. Many committees, most notably parliaments and the EU Council, decide about many policies that might not correctly characterized by (multidimensional) single-peakness of preferences. By bundling many decisions, these committees could eventually moderate conflicts more effectively. But due to the problems arising from bargaining costs and private information, jointly optimal policies are still likely to be not feasible. Hence it remains an interesting theoretical and empirical question whether a system of many (small) independent committees deciding each on single issues or a system of one central committee leads to socially more preferable policies.

A Proofs

Proof of Proposition 1

This proof is basically a combination of the proof of the median voter theorem and the proof of stationary subgame perfect equilibrium in n-player bargaining games with random proposers (see e.g. Winter (1992) for the latter).

Proof. First I define the median voter. Denote the set of players with preferences 'left' resp. 'right' of player i with

$$L_i = \{j \mid x_j \leq x_i, \quad j \neq i\}, \quad R_i = \{j \mid x_j \geq x_i, \quad j \neq i\}$$

The median m is the player i who fulfills the condition

$$\left(\sum_{j \in L_i} w_j < d\right) \wedge \left(\sum_{j \in R_i} w_j < d\right) \tag{40}$$

Then, note that in any stationary equilibrium the final decision is made in the first voting round, since delay is costly. Next I show that the only policy that can be implemented in equilibrium is x_m.

$$\begin{aligned} \forall i \in \{L_m \cup m\} : x_m &\succeq \hat{x} \quad if \quad x_m < \hat{x} \\ (w_{L_m} + w_m > d) &\Rightarrow (x \leq x_m) \qquad (41) \\ \forall i \in \{R_m \cup m\} : x_m &\succeq \hat{x} \quad if \quad x_m > \hat{x} \\ (w_{R_m} + w_m > d) &\Rightarrow (x \geq x_m) \qquad (42) \\ ([41] \wedge [42]) &\Rightarrow x = x_m \qquad (43) \end{aligned}$$

Since x_m is the unique equilibrium policy, proposing this policy is a strictly dominating strategy for every player, i.e. $x_i^p = x_m \quad \forall i$.

For determining the voting strategies $a_i(x^p)$, I consider first the strategies in equilibrium, i.e. $a_i(x_m)$. It is easy to see that $a_i(x_m) = 1$ is a weakly dominating strategy for all players. If

$$\left(\sum_{j \neq i} a_j(x_m) w_j < d\right) \wedge \left(\sum_{j \neq i} a_j(x_m) w_j + w_i \geq d\right), \tag{44}$$

$a_i(x_m) = 1$ leads to a strictly higher payoff for player i. For all strategies of the other players that do not fulfill (44), both (pure) voting strategies result in the same pay-offs. Hence in any trembling-hand perfect equilibrium we must have $a_i(x_m) = 1 \quad \forall i$. To complete the characterization of the voting stage, it remains to determine the out of equilibrium voting strategies $a_i(x^p)$. Since every player accepts votes 'yes for $x^p = x_m$, subgame perfectness requires that he votes 'yes' for all proposals that give him at least the same utility. Conversely, he votes 'no' when x^p gives him a lower pay-off than the equilibrium policy.

Finally, it remains to proof that announcing the preferred policy is the unique perfect equilibrium in the announcement stage. It is straightforward to show that $x_i^a = x_i$ is a weakly dominating strategy. In case i is the median voter, $x_m^a = x_m$ is the unique best response given the equilibrium strategies in the following subgames. If i is not the median voter, there are two possible cases. Suppose (without loss of generality), that $x_i < x_m$. Any $x_i^a < x_m$ does not affect the following stages in the game. Any $x_i^a > x_m$ moves the (announced) median position to the right and leads to a strictly lower pay-off for player i. Since the announcements are made simultaneously, $x_i^a = x_i$ is the only trembling hand perfect equilibrium strategy in the first stage. ■

Example for multiplicity with supra-majority rule

Consider a committee with three players that decide by unanimity, i.e. $N = 3, \quad \mathbf{w} = (1,1,1), \quad d = 3$. For the status-quo policy assume $x_q = 0$. Take the realizations

$$x_1 = \frac{1}{3}, \quad x_2 = \frac{1}{2}, \quad x_3 = \frac{3}{4}.$$

Any policy $x \in \left[\frac{1}{3}, \frac{2}{3}\right]$ can be supported in a stationary perfect equilibrium. First note that $x > \frac{2}{3}$ cannot be an equilibrium since player 1 would be worse off than in the status-quo. Secondly, $x < \frac{1}{3}$ cannot be an equilibrium either since all players prefer a policy $x = \frac{1}{3}$. If player 1 uses in the second stage the stationary strategy

$$a_1(x^p) = \begin{cases} 1 & if \quad \left|\frac{1}{3} - x^p\right| \leq 0 \\ 0 & if \quad \left|\frac{1}{3} - x^p\right| > 0 \end{cases}$$

proposing and accepting the policy $x^p = \frac{1}{3}$ is the best response of player 2 and 3. Hence $x = \frac{1}{3}$ is an equilibrium policy. On the other hand, if player 3 uses in the second stage the stationary strategy

$$a_3(x^p) = \begin{cases} 1 & if \quad \left|\frac{3}{4} - x^p\right| \leq \frac{1}{12} \\ 0 & if \quad \left|\frac{3}{4} - x^p\right| > \frac{1}{12} \end{cases}$$

all players propose and accept the equilibrium policy $x = \frac{2}{3}$. With the same kind argument it can be shown that all policies $x \in \left[\frac{1}{3}, \frac{2}{3}\right]$ can be equilibrium policies.

Proof of Corollary 4

Proof. Take the difference between (8) and (30) for a voting game where player 1 forms a winning coalition with any other single player, formally $v(\{1,i\}) = 1 \quad \forall i \neq 1$. The difference can be written as

$$\begin{aligned} dv &= (1-\gamma_1)\left[\frac{3!(n-2)!}{(n+2)!} + \frac{(n-2)\,2!\,(n-2)!}{(n+1)!} + \frac{n!1!}{(n+2)!}\right] \\ &\quad + \gamma_1 \frac{(n-1)\,1!\,(n-1)!}{(n+1)!} - \gamma_1\left[\frac{2!(n-1)!}{(n+2)!} + \frac{(n+1)!0!}{(n+2)!}\right] \\ &\quad - (1-\gamma_1)\frac{1!(n-1)!}{(n+1)!} \end{aligned}$$

Simplification shows that

$$\begin{aligned} dv &> 0 \\ \Leftrightarrow 2n^2 - 2n &> \gamma_1\left(2n^2 + 2n - 4\right) \\ \Leftrightarrow \gamma_1 &< \frac{n}{n+2} \qquad \blacksquare \end{aligned}$$

Welfare in a 5-player committee

For the case that $n = 5$ and d is the simple majority without possibility of a tie, there are only 6 different voting games when the votes are (weakly) ordered according to the social welfare weights γ_i. They can be described by the following six vote vectors

$$\begin{aligned} \mathbf{w}^1 &= (3,3,3,0,0), \mathbf{w}^2 = (3,1,1,1,1), \mathbf{w}^3 = (2,2,1,1,1) \\ \mathbf{w}^4 &= (3,2,2,2,0), \mathbf{w}^5 = (3,2,2,1,1), \mathbf{w}^6 = (1,1,1,1,1). \end{aligned}$$

These vote allocations lead to the following values for period 0 welfare.

$$
\begin{aligned}
W_0^1 &= -\frac{1}{3} + \frac{1}{420}\left(112\gamma_1 + 112\gamma_2 + 112\gamma_3 + 84\gamma_4 + 84\gamma_5\right) \\
W_0^2 &= -\frac{1}{3} + \frac{1}{420}\left(132\gamma_1 + 90\gamma_2 + 90\gamma_3 + 90\gamma_4 + 90\gamma_5\right) \\
W_0^3 &= -\frac{1}{3} + \frac{1}{420}\left(114\gamma_1 + 114\gamma_2 + 100\gamma_3 + 100\gamma_4 + 100\gamma_5\right) \\
W_0^4 &= -\frac{1}{3} + \frac{1}{420}\left(126\gamma_1 + 98\gamma_2 + 98\gamma_3 + 98\gamma_4 + 84\gamma_5\right) \\
W_0^5 &= -\frac{1}{3} + \frac{1}{420}\left(120\gamma_1 + 106\gamma_2 + 106\gamma_3 + 92\gamma_4 + 92\gamma_5\right) \\
W_0^6 &= -\frac{1}{3} + \frac{1}{420}\left(108\gamma_1 + 108\gamma_2 + 108\gamma_3 + 108\gamma_4 + 108\gamma_5\right)
\end{aligned}
$$

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