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JUST ONE OF US: CONSUMERS PLAYING OLIGOPOLY IN MIXED MARKETS

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JUST ONE OF US: CONSUMERS PLAYING OLIGOPOLY IN MIXED MARKETS

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Abstract. Consumer cooperatives constitute a highly successful example of democratic forms of enterprises operating in developed countries. They are usually organized as medium and large-scale firms competing with profit-maximizing firms in retail industries. This paper models such situation as a mixed oligopoly in which consumer cooperatives maximize the utility of consumer-members and distribute them a share of the profit equal to the ratio of their individual expenditure to the firm total sales. We show that when consumers possess quasilinear preferences over a bundle of symmetrically differentiated goods and firms operate with a linear technology, the presence of consumer cooperatives affects all industries output and social welfare positively. The effect of cooperatives on welfare proves more significant when goods are either complements or highly differentiated and when competition is *à la* Cournot rather than *à la* Bertrand.

Keywords: Consumer Cooperatives, Profit-maximizing Firms, Mixed Oligopoly.

JEL codes: L21, L22, L31

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1. Introduction

Since 1844, Rochdale pioneers' idea of cooperation has spread around the world and today more than 700 million cooperators are active through 100 countries (ICA 2006). Among the various cooperative forms of enterprises, consumer cooperatives (henceforth Coops) are typically firms that operate in retail industries pursuing the institutional objective to act on behalf of their consumer-members.[1] Nowadays these organizations represent one of the most successful examples of democratic and participative forms of enterprises, able to compete against well established and large-size profit-maximizing companies. Formed through a discontinuous process of sequential waves (see Finch, Trombley & Rabas 1998, for a brief account of the US case) Coops are well established in several countries without in general possessing a dominant position in retail industries, with a few exceptions such as Switzerland, Finland and Japan. Given their operational large scale Coops usually operate oligopolistically in developed countries. The *Cooperative Group* in the UK is one of the world's best known consumer cooperative providing a variety of retail and financial services. Japan is also known to possess a very relevant consumer cooperative movement with over 23.5 million members and with retail cooperatives alone scoring a turnover of about 374 billion yens in 2006 (Japanese Consumers' Cooperative Union, 2006). Italy's largest group of consumer cooperatives represents today a serious competitor to private companies operating in retail industry. Among the top 30 Italian retail companies, 9 are consumer cooperatives, with a recorded turnover of about 12.9 billion euro in 2009, corresponding to around 18% of total market share (E-coop 2010). The European Association of Consumer Cooperatives estimates approximately 3,200 consumer cooperatives active in Europe, (overall turnover of 70 billion euro), employing 300,000 workers and serving about 25 million consumer-members (Euro Coop 2008).

So far, the economic literature on consumer cooperatives (e.g. Enke 1945, Anderson, Porter & Maurice 1979 and 1980, Ireland and Law 1980, Sexton 1983, Sexton and Sexton 1987, Farrell 1985, and more recently, Hart and Moore 1996 and 1998, Kelsey and Milne 2010, Mikami 2003 and 2010) has mainly focussed on the behaviour of these firms under either perfect competition, monopoly or monopolistic competition. However, retail industries are characterized by large-scale companies in developed economies, therefore in most cases modern Coops strategically compete against traditional profit-maximizing firms (henceforth PMFs), thus giving rise to a specific instance of *mixed oligopoly.*[2]

To the best of our knowledge, there are no contributions specifically dealing with mixed oligopoly between Coops and PMFs with the exception of Kelsey and Milne (2008) and Goering (2008). Kelsey and Milne (2008) study the effects of the presence of consumer-shareholders on the firm decision-making under both monopoly and oligopoly. They show the presence of consumers among the firm's stakeholders may be a strategic advantage and

[1]Consumer-members are usually entitled to elect their representatives who participate in assemblies and hire the (professional or non professional) managers running the firm. In large Co-ops the assembly elects a board of directors that, on its behalf, controls managers.

[2]The term *mixed oligopoly* is usually adopted to describe a market in which one or more publicly-owned firms compete against PMFs oligopolistically. Publicly-owned firms are assumed to maximize social welfare, i.e the sum of consumers and producers' surplus.(see, for a survey, Delbono and De Fraja 1990). Alternatively, one can conceive a publicly-owned firm as financed directly by all consumers through income tax. As a result, the marginal-cost pricing only obtains in the special case in which the income of the median voter equals the average income (Corneo 1997).

ultimately increase firm's profit. In their model consumers have non zero mass, and therefore act strategically. In contrast to their model, in our setup firms are either pure Coops or pure PMFs competing in a differentiated oligopoly. Coops maximize the utility of a representative consumer, which is assumed atomistic, and therefore solely interested in his/her consumer surplus. We will further analyze the difference between modelling atomistic and non atomistic consumers later on in the paper.

The other paper, by Goering (2008), presents a homogeneous good duopoly between a PMF and a non-profit firm assumed to maximize a parametrized combination of profit and consumers' surplus. In the paper, such objective function is assumed exogenously.[3]

A wide number of related papers deal with labour-managed firms *à la* Ward (1958) and Vanek (1970) assumed to compete with PMFs in homogenous or differentiated good duopolies (see Law and Stewart 1983, Okuguchi 1986, Cremer and Cremér 1992 and Lambertini 2001). A few recent contributions model the behaviour of agricultural cooperatives under either imperfectly competitive (Rodhes 1983, Fulton 1989, Sexton 1990) or mixed duopoly with both homogeneous (Tennbakk 1992, Albaek and Schultz 1998) and vertically differentiated goods (Fulton and Giannakas 2001, Pennerstorfer and Weiss 2007). In general, both labour-managed firms and farmer-cooperatives represented in these models are not assumed to act on behalf of consumers. In the typical labour-managed firm of the literature, worker-members are assumed to maximize per-worker value added, which implies that labour-managed firms set their output more restrictively than standard profit-maximizing firms. On the other hand, agriculture cooperatives are generally modelled as firms using the inputs received from their farmer-members to deliver final goods to consumers. This objective function implies that agriculture cooperatives possess an incentive to overproduce, since farmers do not internalize their production externality on the final market price. However, if agriculture cooperatives buy inputs on behalf of their members, strong similarities with consumer cooperatives arise, as in this case they compete with profit-maximizing firms in selling inputs to farmers, who here act as consumers. Empirically the presence of agriculture cooperatives increases sales and reduces prices on input markets, breaking existing monopsonies (Hansmann, 1996). Therefore, in this respect, some of the results of our paper may also be applied to agriculture cooperatives selling inputs to farmers.

Our paper models a Coop as a firm maximizing the utility of a representative (atomistic) consumer that buys its good and receives a share of the firm's net profit proportional to the ratio of her individual expenditure to the firm's total sales.[4] As a result, every Coop is shown to set in equilibrium a price equal to its average production cost, thus affecting the equilibrium behaviour of rival PMFs. All firms are assumed to possess a constant-return-of-scale technology, and therefore in equilibrium every Coop sets a price equal to its constant marginal cost. The marginal cost pricing rule emerges endogenously in our model. This pricing rule makes our results comparable to those obtained in mixed oligopoly models with state-owned and PMFs (Cremèr, Marchand and Thisse 1989, De Fraja and Delbono 1989). The constant average cost assumption enables overcoming many of the issues related to

[3]Loffler, Kopel and Marini (2010) explore the effects arising when consumers delegate a manager to maximize a weighted sum of their aggregate utilities and total sales.

[4]In Coops this share takes the form of a patronage rebate applied to consumer-members' purchases.

Coops membership stability.[5] At the end of the paper we briefly consider the effects that may occur assuming increasing marginal costs.

The main purpose of our paper is to present a detailed taxonomy of the results obtained in an oligopoly in which an arbitrary number of PMFs and Coops compete strategically either in quantities or in prices and goods are differentiated. We show that, under consumers' quasi-linear preferences, the presence of Coops in the market positively affects both the total output and total welfare of the given industries (and market prices negatively). Under Cournot oligopoly and homogeneous goods it can be shown that the presence of Coops pushes all PMFs out of the market (or, alternatively, obliges them to behave as perfectly competitive firms) thus maximizing social welfare. When goods are differentiated, Coops effect on welfare proves more significant when goods are either complements or highly differentiated and when competition is *à la* Cournot (in quantities) rather than *à la* Bertrand (in prices). Based on these results, we should expect consumer cooperatives to be more often present in markets possessing such features.

This paper is organized as follows. Section 2 introduces the model. Sections 3 and 4 present the main results of mixed oligopoly with quantity and price competition and Section 5 offers our concluding remarks.

2. The Model

2.1. **Consumer Preferences.** The demand side of the market is represented by a *continuum* of atomistic consumers $i \in I$, whose mass is normalized to one, i.e. $I = [0,1]$. Every consumer is assumed to possess preferences defined on $(n+1)$ commodities, n symmetrically differentiated goods[6] x_k $(k = 1, ..., n)$ and a *numeraire* y, expressed by the following utility function $U_i : \mathbb{R}_+^{n+1} \rightarrow \mathbb{R}_+$

$$U_i\left(x_1^i, x_2^i, ., x_{k,}^i .., x_n^i, y^i\right) = u_i\left(x_1^i, x_2^i, ., x_{k,}^i .., x_n^i\right) + y^i \tag{2.1}$$

where $x_{k,}^i$ and y^i denote the individual consumption of these goods. Let $u_i(.)$ be smooth, increasing and strictly concave in all $x_{k,}^i$[7]

If the available income of each i-th consumer (denoted $\overline{y}^i$) is sufficiently high, every individual inverse demand can be obtained from the first-order conditions of the problem maximization (2.1) subject to budget constraint

$$\sum_{k=1}^{n} p_k\left(x_1, ., x_n\right) x_k^i + y^i \leq \overline{y}^i, \tag{2.2}$$

as

$$p_k = \frac{\partial u_i\left(x_1^i, x_2^i, ., x_n^i\right)}{\partial x_k^i}, \text{ for } x_k^i > 0 \text{ and } k = 1, 2, ...n. \tag{2.3}$$

[5]See Anderson, Maurice & Porter (1979), Sandler & Tschirhart (1981), Sexton (1983) and Sexton & Sexton (1990). In our paper all consumers buy Coops' goods and are therefore entitled to become members. This assumption is in line with the typical "open door" principle holding in cooperatives. Moreover, given the constant-return-of-scale technology, Coop efficiency cannot be affected by favouring the entry or the exit of members.

[6]A good here may also be interpreted as a bundle of goods sold by every firm in the market.

[7]The Hessian of U_i is negative semidefinite.for all $(x_1^i, x_2^i, ., x_n^i) .\in \mathbb{R}_+^n$.

In (2.2) the price of good x_k depends on the profile of quantities $(x_1, ., x_n)$ (the market is a oligopoly) and not on every individual purchase x_k^i of the good.

2.2. **Industry.** The retail industry consists of n firms supplying n differentiated goods (or bundles of goods), whose m are supplied by consumer cooperatives and $(n-m)$ by traditional profit-maximizing firms. Let $M \subset N$ denote the set of all Coops and $N \backslash M$ the set of all PMFs. As usual, PMFs are assumed to maximize their profit

$$\pi_k (x_1, ...x_n) = p_k (x_1, x_2, ...x_n) x_k - c_k(x_k). \tag{2.4}$$

In general we will assume linear variable costs and zero fixed costs for all firms. As anticipated, Coops act on behalf of atomistic consumers, and every consumer is assumed to receive a share of the Coop's net profit proportional to the amount of goods purchased over the Coop's total sales. This can be expressed by the following objective-function for a Coop $j \in M$,[8]

$$\begin{cases} \max\limits_{x_j^i} u_i \left(x_1^i, x_2^i, ., x_{k,}^i .., x_n^i\right) + y^i & \text{s.t} \\ \sum\limits_{k=1}^{n} p_k (x_1, ., x_n) x_k^i + y^i \leq \overline{y}^i + \sum\limits_{j \in M} \dfrac{x_j^i}{x_j} \left[p_j (x_1, ., x_n) x_j - c_j (x_j)\right]. \end{cases} \tag{2.5}$$

The problem (2.5) reduces to

$$\max_{x_j^i} \left\{ u_i \left(x_1^i, x_{2,}^i, .., x_n^i\right) + \overline{y}^i - \sum_{j \in M} \frac{c_j (x_j)}{x_j} x_j^i - \sum_{k \in N \backslash M} p_k (x_1, ., x_n) x_k^i \right\} \tag{2.6}$$

and, the FOC for interior maximum of (2.6) for every $j \in M$ can be written as

$$\frac{\partial u_i (x_1^i, .., x_n^i)}{\partial x_j^i} = \frac{c_j(x_j)}{x_j} \text{ for } x_j > 0. \tag{2.7}$$

as long as the price charged by a j-th Coop is sufficiently high to generate non negative profits, namely, for $p_j (x_1, .x_n) \geq \frac{c_j(x_j)}{x_j}$. Expression (2.7) indicates that a Coop acting on behalf of atomistic consumers sets its quantity to equate every consumer's willingness to pay for good j at its average cost, with the purpose to distribute the maximum consumer surplus to consumer-members (which here are all consumers).

Once (2.7) is respected for every single consumer, the Coop aggregates it for all consumers $i \in I$, obtaining

$$\frac{\partial u_i (x_1, .., x_n)}{\partial x_j} = \frac{c_j(x_j)}{x_j} \text{ for } x_j > 0. \tag{2.8}$$

Since all firms possess a constant-return-of-scale technology, every Coop makes total consumers' willingness to pay for good j equal to marginal cost.[9]

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[8]Note that when prices instead of quantities are firms' choice variables, PMFs and Coops' payoffs can be expressed as a function of a price vector $(p_1, p_2, ..p_n)$.

[9]Coop behaviour would be different if assumed to act on behalf of all consumers together. In this case, consumers could coordinate their actions to affect the prices of all goods in the market.

3. Oligopoly with Quantity Competition

In order to study the implications of the simultaneous presence of both PMFs and Coops in an oligopolistic market, let the following utility function represent the preferences of a i-th consumer in the economy:[10]

$$U_i\left(x_1, x_2, .., x_n, y\right) = \alpha \sum_{k=1}^{n} x_k^i - (1/2)\left[\sum_{k=1}^{n}\left(x_k^i\right)^2 + \beta \sum_{k=1}^{n} x_k^i \sum_{r\neq k} x_r^i\right] + y^i \tag{3.1}$$

where $\alpha > 0$ and $\beta \in [1/(1-n), 1]$ represents the degree of product differentiation. For $\beta = 0$, goods are independent and for $\beta = 1$ goods are perfect substitutes. Moreover, for $\beta < 0$ goods become complements.

Let also all firms $k = 1, 2, ...n$ possess identical strategy sets $X_k = [0, \infty)$ and identical technology, expressed by a linear cost function, $c_k\left(x_k\right) = cx_k$ with $0 < c < \alpha$.

By (3.1) and (2.3), the following individual linear inverse demand for every good $k = 1, 2, .., n$ is obtained

$$\alpha - x_k^i - \beta \sum_{h\neq k} x_h^i = p_k \quad \text{for } x_k^i > 0. \tag{3.2}$$

Inverse market demand for one good can simply be obtained by integrating (3.2) over all consumers $i \in I$. Moreover, the FOC of problem (2.6) yields the following FOC for every Coop producing the j-th good

$$\alpha - x_j^i - \beta \sum_{h\neq j} x_h^i = c. \tag{3.3}$$

Expression (3.3) is the FOC of a Coop acting on behalf of one atomistic consumer buying its product. A Coop will decide its own market quantity aggregating (3.3) for all consumers.

3.1. The Benchmark Case: Oligopoly with all PMFs.

We can start illustrating the case in which all firms are PMFs and the choice variables are quantities. Whereby firms are PMFs they simply maximize their profits with respect to the quantity of the k-th good,

$$\pi_k\left(x_1, x_2, ..., x_n\right) = (\alpha - x_k - \beta \sum_{r\neq k} x_r)x_k - cx_k. \tag{3.4}$$

Solving this simple maximization problem yields the following best-replies for each k-th PMF,

$$x_k\left(x_{-k}\right) = \frac{1}{2}\left(\alpha - \beta x_{-k} - c\right)$$

where $x_{-k} = (x_1, x_2, .., x_{k-1}, x_{k+1}, .., x_n)$, and therefore pure-PMF Nash equilibrium quantities $(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)$ are easily obtained as

$$\overline{x}_k = \frac{(\alpha - c)}{2 + \beta\left(n-1\right)} \tag{3.5}$$

[10] See Shubik ans Levitan (1971), Vives (1984) and Dixit (1983) for further details on this utility specification.

for $k = 1, 2, .., n$ and prices are given by

$$p_k(\overline{x}_1, ., \overline{x}_n) = \frac{\alpha + c + \beta c(n-1)}{\beta(n-1)+2}.$$

It is made evident by (3.5) that for $\beta = 1$ the usual Cournot solution with homogenous goods ($\overline{x}_k = (\alpha - c)/(n+1)$) occurs, while for $\beta = 0$ goods are independent and all PMFs act monopolistically ($\overline{x}_k = (\alpha - c)/2$).

3.2. **Mixed Cournot Oligopoly.** Let us now assume that a group of m firms in the market ($m \leq n$) turn into Coops accepting all consumers as their members. The market thus turns into a mixed oligopoly in which m Coops compete against $(n-m)$ traditional PMFs.

By aggregating (3.3) for all consumers and differentiating (3.4), the following best-replies are obtained, respectively,

$$x_j(\sum_{h\in N\backslash M} x_h, \sum_{r\in M\backslash\{j\}} x_r) = \alpha - \beta \sum_{h\in N\backslash M} x_h - \beta \sum_{r\in M\backslash\{j\}} x_r - c, \tag{3.6}$$

$\forall j \in M$,

$$x_h(\sum_{j\in M} x_j, \sum_{g\in(N\backslash M)\backslash\{h\}} x_g) = \frac{\alpha - \beta \sum_{j\in M} x_j - \beta \sum_{g\in(N\backslash M)\backslash\{h\}} x_g - c}{2}, \tag{3.7}$$

$\forall h \in N\backslash M$.

Exploiting the symmetry of the m Coop and of the $(n-m)$ PMFs, the following mixed oligopoly Nash equilibrium quantities are obtained for every Coop

$$x_j^* = \frac{(2-\beta)(\alpha - c)}{2 + (n+m-3)\beta - (n-1)\beta^2} \quad \forall j \in M, \tag{3.8}$$

and every PMF

$$x_h^* = \frac{(1-\beta)(\alpha - c)}{2 + (n+m-3)\beta - (n-1)\beta^2} \quad \forall h \in N\backslash M, \tag{3.9}$$

with corresponding equilibrium prices

$$p_j(x_1^*, x_2^*, .., x_n^*) = c$$

for every Coop and

$$p_h(x_1^*, x_2^*, .., x_n^*) = \frac{\alpha + c + \beta(c(2m+n-2) - \alpha(m+1)) + \beta^2(m\alpha - c(n+m-1))}{2 + (n+m-3)\beta - (n-1)\beta^2}$$

for every PMF, respectively.

It can be proved that, in general, if goods are perfect substitutes ($\beta = 1$) the model yields the extreme prediction that the presence of even just one Coop in the market pushes PMFs out of the market.[11] This could, alternatively, be interpreted as if the presence of Coops obliges all PMFs to adopt a perfectly competitive behaviour in order to stay in the market. Either way, as the equilibrium price coincides with all firms' average and marginal costs, every consumer's willingness to pay for the homogeneous good is just equal to every firm's marginal production cost thus implying welfare maximization (since $u' = c$).

These results are condensed in the next proposition. As an additional observation, please note that the total market output under mixed oligopoly $X^* = \sum_{k=1,..,n} x_k^*$ is equal to

$$X^* = mx_j^* + (n-m)x_h^* = \frac{(\alpha - c)\,(n(1-\beta)+m)}{2+(n+m-3)\,\beta-(n-1)\,\beta^2}. \tag{3.10}$$

For $m = 0$ the above expression coincides with pure n-PMF oligopoly

$$X^*(m=0) = \frac{n\,(\alpha - c)}{2+\beta\,(n-1)} \tag{3.11}$$

and for $m = n$ the expression turns into pure n-Coop total quantity, with

$$X^*(m=n) = \frac{n\,(\alpha - c)}{1+\beta\,(n-1)}. \tag{3.12}$$

From (3.11) and (3.12) pure Coop oligopoly clearly yields higher output than pure PMF oligopoly. Moreover, expression (3.10) makes clear that under mixed oligopoly the total output increases monotonically with the number of active Coops in the market.

Proposition 1. *Under a mixed oligopoly in quantities and homogeneous goods ($\beta = 1$), the presence of even one single Coop in the market implies that all PMFs become inactive, the industry output is greater than that obtained with all PMFs and the economy social welfare is maximized.*

Proof. See the Appendix. □

Moreover, some simple results can be obtained for the range $\beta \in [0,1]$.

Proposition 2. *Under a mixed oligopoly in quantities, for $\beta \in [0,1]$ Coop output is always greater than PMF output, namely, $x_j^* > x_h^*$ for all $j \in M$ and $h \in N \backslash M$. Moreover, for $\beta \in [0,1]$, $x_j^* > \overline{x}_k \geq x_h^*$.*

Proof. See the Appendix. □

3.3. **Welfare Analysis: PMFs vs. Mixed Oligopoly.** The analysis of social welfare under mixed oligopoly with differentiated goods requires careful calculation of the interacting effects of Coops and PMFs simultaneous presence on consumer surplus and profits in all markets. By the property of quasi-linear preferences, consumers' welfare can be measured with no approximation by using consumers' surplus which, in turn, corresponds to the value of consumers' utilities.

[11] Alternatively, one could assume that Coops are less efficent than PMFs or that PMFs enjoy some sort of cost advantage. In this case both types of firms can co-exist also when goods are perfectly homogeneous. (see for instance Cremer, Marchand and Thisse, 1998).

Under a pure PMF oligopoly, for all k-th goods produced, total welfare (TW_k) can be computed as the sum of consumers' surplus plus firms' profits,

$$\begin{aligned} TW_k^{PMF} &= \int_0^{x_k^*} p_k(\tau)d\tau - p_k\left(\overline{x}_1, \overline{x}_{2,}, .., \overline{x}_n\right) x_k^* + p_k\left(\overline{x}_1, \overline{x}_{2,}, .., \overline{x}_n\right) \overline{x}_k - c\overline{x}_k \\ &= U\left(\overline{x}_1, \overline{x}_{2,}, .., \overline{x}_n\right) - c\overline{x}_k + \overline{y}. \end{aligned}$$

Summing up the welfare generated in all n markets and using (3.1), the utility functions aggregated for all consumers, we obtain

$$TW^{PMF} = (\alpha - c)\sum_{k=1}^{n} \overline{x}_k - (1/2)\left[\sum_{i=1}^{n} \left(\overline{x}_k\right)^2 + \beta \sum_{k=1}^{n} \overline{x}_k \sum_{r\neq k} \overline{x}_r\right] + \overline{y},$$

which, by the symmetry of all firms, can be written as

$$TW^{PMF} = (\alpha - c)\, n \cdot \overline{x}_k - (1/2)\left[n\left(\overline{x}_k\right)^2 + \beta n(n-1)\overline{x}_k^2\right] + \overline{y}. \tag{3.13}$$

In a mixed oligopoly, total welfare generated in all market managed by a j-th Coop is given by the area under the demand in correspondence to the quantity for which $p_j(x_1^*, x_2^*, .., x_n^*) = c_j$,

$$TW_j^{COOP} = \int_0^{x_j^*} p_j(\tau)d\tau - cx_j^*, \tag{3.14}$$

which using (3.1), (3.14) can be simply expressed as

$$TW_j^{COOP} = \sum_{j\in M} (\alpha - c)\cdot x_j^* - (1/2)\left[\sum_{j\in M} \left(x_j^*\right)^2 + \beta \sum_{j\in M} x_j^* \sum_{r\neq j} x_r^*\right].$$

Finally, total welfare under mixed oligopoly can be expressed as

$$\begin{aligned} &\sum_{h\in N\setminus M} TW_h^{PMF} + \sum_{j\in M} TW_j^{COOP} = \\ &= \sum_{h\in N\setminus M} (\alpha - c)\, x_h^* - (1/2)\left[\sum_{h\in N\setminus M} \left(x_h^*\right)^2 + \beta \sum_{h\in N\setminus M} x_h^* \sum_{r\neq h} x_r^*\right] + \\ &+ \sum_{j\in M} (\alpha - c)\, x_j^* - (1/2)\left[\sum_{j\in M} \left(x_j^*\right)^2 + \beta \sum_{j\in M} x_j^* \sum_{r\neq j} x_r^*\right] + y^*. \end{aligned}$$

Now, plugging (3.5), (3.8) and (3.9) into the above expressions, we obtain the following values for total welfare (see Appendix)

$$TW^{PMF} = \frac{1}{2}\frac{n\left(\alpha - c\right)^2\left(3 + \beta\left(n-1\right)\right)}{\left(2 + \beta\left(n-1\right)\right)^2} + \overline{y}, \tag{3.15}$$

under pure PMF oligopoly

$$TW^{COOP} = \frac{1}{2}\frac{n\left(\alpha - c\right)^2}{1 + \beta\left(n-1\right)} + \widetilde{y} \tag{3.16}$$

pure Coop oligopoly

$$TW^{MO} = \underbrace{\frac{1}{2}\frac{(n-m)(\alpha-c)^2(1-\beta)\left(3+\beta(n+m-4)-\beta^2(n-1)\right)}{\left(2+\beta(n+m-3)-\beta^2(n-1)\right)^2}}_{\sum_{h\in N\setminus M} TW_h} + \underbrace{\frac{1}{2}\frac{m(\alpha-c)^2(2-\beta)}{2+\beta(n+m-3)-\beta^2(n-1)}}_{\sum_{j\in M} TW_j} + y^*. \tag{3.17}$$

and mixed oligopoly with m Coops and $(n-m)$ PMFs, respectively.

Expression (3.17) illustrates that social welfare in a mixed oligopoly accounts for the sum of welfare yielded in $(n-m)$ markets in which PMFs produce plus welfare yielded in m markets in which Coops are, in turn, active.

3.3.1. *Welfare under Duopoly.* For illustrative purposes we can focus on the case of mixed duopoly compared to a pure PMF and to a pure Coop duopoly, respectively. The presence of Coops can be relatively more beneficial in some circumstances than in others and, in particular, for specific levels of product differentiation. Using (3.15), (3.16) and (3.17) we obtain total welfare as

$$TW^{PMF} = \frac{(\alpha-c)^2(3+\beta)}{(2+\beta)^2}$$

under pure PMF duopoly,

$$TW^{COOP} = \frac{(\alpha-c)^2}{(1+\beta)}$$

under pure Coop duopoly, and

$$TW^{MO} = \frac{1}{2}\frac{(\alpha-c)^2\left(7+\beta\left(2\beta^2-2\beta-6\right)\right)}{\left(2-\beta^2\right)^2}$$

under mixed duopoly, respectively.

Figure 1 shows that in terms of total welfare a pure Coop duopoly (continuous line) outperforms both a pure PMF duopoly and a mixed duopoly for any degree of goods differentiation which is obvious, considering that a pure Coop basically acts as a welfare maximizer.

[FIGURE 1 - APPROXIMATELY HERE]

As already proven in proposition 1, under mixed duopoly (dotted line) for $\beta = 1$ (homogeneous goods), only the Coop remains in the market and welfare is, therefore, maximized. Moreover, it can be noticed that the relative efficiency of mixed duopoly versus pure PMF duopoly (circled line) is higher when goods are either complement ($\beta < 0$) or highly differentiated. When goods become more and more homogeneous, the welfare loss determined in a pure PMF versus a mixed duopoly or a pure Coop duopoly decreases progressively, yet it never vanishes. Similarly, mixed oligopoly better and better approximates maximum social welfare for goods becoming increasingly substitute.

3.3.2. *Welfare Comparison with More than Two Firms.* The results obtained above still hold with more than two firms that compete *à la* Cournot. It can be proven that the entry of new Coops in the market is always beneficial to social welfare.

Proposition 3. *Social welfare under mixed oligopoly increases with the number of m Coops regardless of the number of n firms active on the market.*

Proof. See the Appendix. □

The positive effect of Coops on welfare still holds true when the total number of firms in the market increases. Figure 2 illustrates that the entry of new firms, boosting competition, always exerts a favourable impact on market welfare. Consequently, if the new entrants are Coops, such impact is even stronger. Consumers should therefore exert pressure on respective Coops to set up new selling units, thus increasing competition and welfare.

[FIGURE 2 - APPROXIMATELY HERE]

However, a simple comparison shows that when goods are substitutes ($\beta > 0$), the welfare raised by a pure Coop oligopoly becomes less and less advantageous compared to a pure PMF oligopoly when both n and β increase. When competition is high (which happens for high n and β) the different forms of market do not perform so differently, and thus welfare is not so far. See next proposition.

Proposition 4. *When the total number of firms in the market increases (higher n) and goods become increasingly substitute (higher β), the difference between total welfare in pure Coop oligopoly and welfare in pure PMF oligopoly progressively decreases.*

Proof. See the Appendix. □

[FIGURE 3 - APPROXIMATELY HERE]

Figure 3 shows that when the number of firms increases and goods become increasingly substitutes, Coops become the relative welfare advantage yielded by Coops progressively shrinks. Therefore, if Coops aspire to match consumers' needs, we should see this type of firms more frequently in highly monopolistic markets in which goods are either highly differentiated or complements.

In the next section, we consider the case of price competition.

4. Price Competition

It can be interesting to compare the case of quantity competition to the case of price competition so as to verify whether differences arise. An obvious difference is that, when goods are perfectly homogeneous, Bertrand competition yields the extreme prediction that firms set prices equal to marginal cost, regardless of the objective functions of firms competing in the market.

4.1. **Oligopoly with all PMFs.** When all firms are PMFs, we first obtain the direct demand for each k-th good as price function,

$$x_k\left(p_1,p_2,..,p_n\right)=\frac{\alpha\left(1-\beta\right)-p_k-(n-2)\beta p_k+\beta\sum_{h\neq k}p_h}{\left(1-\beta\right)\left((n-1)\beta+1\right)}$$

for $k=1,2,..,n$ and $\beta\neq 1$.[12]

As a result, all PMFs' profit function can be written as

$$\pi_k\left(p_1,.,p_n\right)=\left(p_k-c\right)\left(\frac{\alpha(1-\beta)-p_k-(n-2)\beta p_k+\beta\sum_{h\neq k}p_h}{(1-\beta)((n-1)\beta+1)}\right). \tag{4.1}$$

Differentiating (4.1) with respect to p_k yields the best-reply of every k-th PMF as

$$p_k(p_{-k})=\frac{1}{2}\frac{\alpha\left(1-\beta\right)+c\left(n-2\right)\beta+c+\beta p_{-k}}{\beta\left(n-2\right)+1}$$

where $p_{-k}=(p_1,p_2,..,p_{k-1},p_{k+1},..,p_n)$.

By symmetry, the Nash equilibrium price of every k-th PMF can be obtained as

$$\begin{cases}\overline{p}_k=\dfrac{\left(\alpha\left(1-\beta\right)+\beta c\left(n-2\right)+c\right)}{\beta\left(n-3\right)+2}\ \text{for}\ \beta\neq 1\\[2ex] \qquad\qquad\overline{p}_k=c\ \ \text{for}\ \beta=1,\end{cases} \tag{4.2}$$

with associated quantities:

$$x_k\left(\overline{p}_1,\overline{p}_2,..,\overline{p}_n\right)=\frac{\left(\alpha-c\right)\left(1+\beta\left(n-2\right)\right)}{\left(1+\beta\left(n-1\right)\right)\left(2+\beta\left(n-3\right)\right)} \tag{4.3}$$

and profits

$$\pi_k\left(\overline{p}_1,.,\overline{p}_n\right)=\left(\overline{p}_k-c\right)\cdot x_k\left(\overline{p}_1,.,\overline{p}_n\right)=\frac{\left(\alpha-c\right)^2\left(1-\beta\right)\left(1+\beta\left(n-2\right)\right)}{\left(2+\beta\left(n-3\right)\right)^2\left(1+\beta(n-1)\right)}.$$

4.2. **Mixed Oligopoly with Price Competition.** Again we assume that $m\leq n$ firms start behaving as Coops. By (3.1) and (3.4), we obtain the following direct demands for a PMF $h\in N\backslash M$, given the price charged by other firms,

$$x_h\left(p_1,.,p_n\right)=\frac{\alpha\left(1-\beta\right)-p_h-\beta(n-2)p_h+\beta\sum\limits_{r\in(N\backslash M)\backslash h}p_r+m\beta c}{\left(1-\beta\right)\left(1+\beta(n-1)\right)} \tag{4.4}$$

and the price charged by a Coop $j\in M$

$$x_j\left(p_1,.,p_n\right)=\frac{\alpha\left(1-\beta\right)-c-(n-m-1)\beta c+\beta\sum\limits_{h\in N\backslash M}p_h}{\left(1-\beta\right)\left(1+\beta(n-1)\right)} \tag{4.5}$$

for $\beta\neq 1$.

By (4.4) we can write the profit-function of a PMF as a function of prices,

[12] Since demands are not defined for $\beta=1$, output level under homogeneous goods are simply defined as firms' direct demands for prices equal to marginal costs.

$$\pi_h\left(p_1,.,p_n\right)=\left(p_h-c\right)x_j\left(p_1,.,p_n\right)$$

and, after straightforward calculations, the following mixed oligopoly equilibrium prices are obtained

$$\left\{\begin{array}{l} p_h^*=\dfrac{\alpha\left(1-\beta\right)+c\left(1+\beta\left(n+m-2\right)\right)}{2+\beta\left(n+m-3\right)} \text{ for } \beta\neq 1 \\ \\ p_h^*=c \ \text{ for } \beta=1 \text{ and } p_j^*=c \end{array}\right. \tag{4.6}$$

with associated quantities

$$x_h\left(p_1^*,p_2^*,..,p_n^*\right)=\frac{\left(\alpha-c\right)\left(1+\beta\left(n-2\right)\right)}{\left(1+\beta\left(n-1\right)\right)\left(2+\beta\left(n+m-3\right)\right)} \tag{4.7}$$

for every PMFs and

$$x_j\left(p_1^*,p_2^*,..,p_n^*\right)=\frac{\left(\alpha-c\right)\left(2+\beta\left(2n-3\right)\right)}{\left(1+\beta\left(n-1\right)\right)\left(2+\beta\left(n+m-3\right)\right)}, \tag{4.8}$$

for every Coop, respectively.

Finally, every PMF's equilibrium profit is given by

$$\pi_h\left(p_1^*,p_2^*,..,p_n^*\right)=\frac{\left(\alpha-c\right)^2\left(1-\beta\right)\left(\beta\left(n-2\right)+1\right)}{\left(2+\beta\left(n+m-3\right)\right)^2\left(1+n\beta-\beta\right)}.$$

Proposition 5. *Under price competition and $\beta\in[0,1]$, mixed oligopoly prices are, for all firms, either lower than or equal to pure PMF oligopoly prices, namely, $\overline{p}_k\geq p_h^*\geq p_j^*$ for every $j\in M$, $h\in N\backslash M$ and $k=1,2,..n$. Moreover, $x_j\left(p^*\right)\geq x_k\left(\overline{p}\right)\geq x_h\left(p^*\right)$.*

Proof. See the Appendix □

4.3. **Welfare Comparison under Price Competition.** For the sake of brevity, we concentrate in the Appendix all calculations of total welfare under price competition. We here report the results of such calculations, which are not too dissimilar from those obtained in the case of quantity competition. Total welfare under mixed oligopoly with an arbitrary number of PMFs and Coops competing in prices is obtained as

$$TW^{MO}=\underbrace{\frac{1}{2}\frac{(n-m)(\alpha-c)^2(3+\beta(n+m-4))(1+\beta(n-2))}{(2+\beta(n+m-3))^2(1+\beta(n-1))}}_{\sum_{h\in N\backslash M}TW_h}+\underbrace{\frac{1}{2}\frac{m(\alpha-c)^2(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))}}_{\sum_{j\in M}TW_j} \tag{4.9}$$

In the expression above we have again decomposed the total welfare in two distinct parts. Setting $m=0$ in (4.9) we can obtain pure PMF oligopoly total welfare as

$$TW^{PMF}=\frac{1}{2}\frac{n\left(\alpha-c\right)^2\left(1+\beta\left(n-2\right)\right)\left(3+\beta\left(n-4\right)\right)}{\left(1+\beta\left(n-1\right)\right)\left(2+\beta\left(n-3\right)\right)^2},$$

while, by setting $n=m$, we obtain pure Coop total welfare as

$$TW^{COOP}=\frac{1}{2}\frac{\left(\alpha-c\right)^2n}{\left(1+\beta\left(n-1\right)\right)^2}.$$

It can be noticed that pure Coop oligopoly always yields the optimum welfare, regardless of whether competition is either *à la* Cournot or *à la* Bertrand. Again, for illustrative purposes, we use the duopoly case to highlight the main differences in welfare under price and quantity competition.

4.3.1. *Welfare in Bertrand Duopoly.* The expressions for the total welfare under Bertrand duopoly are the following:

$$TW^{PMF} = \frac{(\alpha - c)^2 (3 - 2\beta)}{(1+\beta)(2-\beta)^2},$$
$$TW^{MO} = \frac{1}{8}\frac{(\alpha - c)^2 (7 + \beta)}{(1+\beta)},$$
$$TW^{COOP} = \frac{(\alpha - c)^2}{(1+\beta)}.$$

By plotting the above expressions does not yield particular differences versus the Cournot competition case, except in that all types of markets (pure PMF-duopoly included), yield the marginal cost pricing and then maximum welfare for $\beta = 1$. Under Bertrand competition and homogeneous goods we observe perfect "isomorphism" in all firms' behaviours.

[FIGURE 4 - APPROXIMATELY HERE]

An important difference between Bertrand and Cournot competition emerges in terms of welfare loss for a pure PMF oligopoly versus a pure Coop oligopoly. As shown in Figure 4, the loss is definitively larger under quantity than under price competition and the difference is particularly high when goods are reasonably homogeneous. This is the case in which the presence of at least one Coop in the market is definitively more beneficial under Cournot than under Bertrand competition. Additional welfare comparisons between Cournot and Bertrand oligopolies are provided in the Appendix.

[FIGURE 5 - APPROXIMATELY HERE]

5. Concluding Remarks

Although consumer cooperatives are, in general, well established in several countries, their behaviour is still largely unknown and requires additional research, notably to identify the effects of the strategic interaction between consumer cooperatives and traditional profit-maximizing firms in oligopolistic markets. This paper has attempted to take a first step in this direction, showing the main effects arising in a mixed oligopoly with profit-maximizing firms and consumer cooperatives competing either *à la* Cournot or *à la* Bertrand in markets with heterogeneous goods. We have shown that the presence of Coops is particularly beneficial for industries output and social welfare in mainly two cases. The first under Cournot competition and homogeneous goods when Coops behave so expansively to expel PMFs from the market, or, if interpreted differently, to oblige them to behave as perfectly competitive firms, setting a price equal to the marginal cost and making zero profit as a result. In the second case when market competition is relatively weak, namely when goods are either complements or highly differentiated and the presence of Coops appears particularly valuable, by increasing output and welfare considerably. In this paper we have also shown that Coops affect total welfare comparatively more under Cournot than under Bertrand competition. Therefore, according to our model, consumer cooperatives are likely to behave not too dissimilarly to traditional profit-maximizing firms in all retail markets in which goods are highly (but not completely) homogeneous and competition occurs mostly in prices. As a reaction to these market forces, Coops may attempt to propose their customers genuinely differentiated goods and, consequently, enhance consumers' welfare.

Some of the paper results call for further analysis. First of all, we have assumed throughout the paper a constant return of scale technology for firms. Some of the recent literature on mixed oligopoly assumes decreasing returns of scale, thus implying increasing marginal costs. In this case a Coop, with its typical output expanding behaviour, could prove endogenously less efficient than a PMF, thus imposing negative externality on the society. This effect would be overturned if a Coop could be managed jointly by all consumers or by someone acting on their behalf. In this case consumers would no longer be atomistic and could play welfare-enhancing strategies, setting prices equal to marginal costs plus some distortive effects arising from manipulating to their advantage the pricing of rival PMFs. Developing a model of consumer cooperatives governed by coalitions of consumers acting strategically may constitute a topic of great interest.

6. Appendix

Proof of Proposition 1. For the first result, note that if $\beta = 1$, conditions (3.3) and (3.4) imply the following best-replies (expressed as function of the other type of firms' output only)

$$x_j\left(\sum_{h\in N\backslash M} x_h\right) = \frac{\alpha - (n-m)\,x_h - c}{m}, \forall j \in M, \tag{6.1}$$

$$x_h\left(\sum_{j\in M} x_j\right) = \frac{\alpha - m x_j - c}{(n-m+1)}, \forall h \in N\backslash M. \tag{6.2}$$

Hence:

$$\begin{cases} x_j^* = \dfrac{\alpha - c}{m} \\ x_h^* = 0. \end{cases} \tag{6.3}$$

The economy total output is thus given by

$$\sum_{j\in M} x_j^* + \sum_{h\in N\backslash M} x_h^* = m\frac{\alpha - c}{m} + 0 = (\alpha - c) > \sum_k \overline{x}_k = n\,(\alpha - c)\,/\,(n+1)$$

The economy social welfare is defined as the sum of consumers' surplus and firms' profits, here equal to zero. Using (3.1) and (6.3) it becomes:

$$TW = \int_0^1 U_i\left(x_1^i(t), .., x_n^i(t), y^i(t)\right) dt - \sum_{k=1}^n p_k x_k^* + \sum_{k=1}^n p_k x_k^* - c\sum_{k=1}^n x_k^* =$$

$$= (\alpha - c)\sum_{j\in M} x_j^* - (1/2)\left(\sum_{j\in M}\left(x_j^*\right)^2 + \sum_{r\neq j} x_r^* \sum_{j\in M} x_j^*\right) + y^* =$$

$$= (\alpha - c)\, m\left(\frac{\alpha - c}{m}\right) - \frac{1}{2}m\frac{(\alpha - c)^2}{m^2} - \frac{1}{2}m(m-1)\frac{(\alpha - c)^2}{m^2} + y^* =$$

$$= \frac{1}{2}(\alpha - c)^2 + y^*.$$

which is also the maximum welfare obtainable in the market for $\beta = 1$. ■

Proof of Proposition 2. The first result can be easily checked by direct inspection of expressions (3.9) and (3.8). The second result can be proved by noting that, for all $j \in M$ and $k \in N$,

$$x_j^* - \overline{x}_k = \frac{(\beta(n-m-1)+2)(\alpha - c)}{\left(\beta(n+m-3) - \beta^2(n-1) + 2\right)(\beta(n-1)+2)} \tag{6.4}$$

and expression (6.4) is always strictly positive for $\beta \in [0,1]$ and $n \geq 2$.

Finally, for all $h \in N\backslash M$

$$\overline{x}_k - x_h^* = \frac{(\alpha - c)}{2 + \beta(n-1)} - \frac{(\alpha - c)(1-\beta)}{2 + \beta(n+m) - \beta^2(n-1) - 3\beta}$$

is equal to zero for $\beta = 0$, since $\overline{x}_k(\beta = 0) = x_h^*(\beta = 0) = (\alpha - c)/2$, while for $\beta = 1$,

$\overline{x}_k(\beta = 1) = (\alpha - c)/(n+1) > x_h^*(\beta = 1) = 0$. Finally, straightforward manipulations show that for $\beta \neq 0$

$$\overline{x}_k - x_h^* = \frac{(\alpha - c)\, m\beta}{\left(\beta(n+m-3) + \beta^2(1-n) + 2\right)(n(\beta - 1) + 2)} > 0$$

for

$$\left(\beta(n+m-3) + \beta^2(1-n) + 2\right) > 0$$

which is always satisfied for $\beta \in (0,1)$. ■

.**Proof of Proposition 3.** By ispection of (3.17), it can be observed that the welfare raised by a Coop is higher than the welfare raised by a PMF

whenever

$$\frac{(2-\beta)\left(2 + \beta(n+m-4) - \beta^2(n-1)\right)}{\left(2 + \beta(n+m-3) - \beta^2(n-1)\right)^2}$$

$$> \frac{(1-\beta)\left(3 + \beta(n+m-4) - \beta^2(n-1)\right)}{\left(2 + \beta(n+m-3) - \beta^2(n-1)\right)^2}$$

which implies

$$\begin{aligned}&(2-\beta)\left(2+\beta(n+m-3)-\beta^2(n-1)\right)>\\&>(1-\beta)\left(3+\beta(n+m-4)-\beta^2(n-1)\right),\end{aligned}$$

and then

$$\begin{aligned}&(1-\beta)^2+(2-\beta)\left(\beta(n+m-3)-\beta^2(n-1)\right)>\\&>(1-\beta)\left(\beta(n+m-3)-\beta^2(n-1)\right)\end{aligned}$$

which always holds for $m \leq n$ and $\beta \in [1/(1-n), 1]$. ■

Proof of Proposition 4. Straightforward manipulations show that

$$\begin{aligned}TW^{COOPs}-TW^{PMFs}&=\frac{1}{2}\frac{n(\alpha-c)^2}{1+\beta(n-1)}-\frac{1}{2}\frac{n(\alpha-c)^2(3+\beta(n-1))}{(2+\beta(n-1))^2}=\\&=\frac{1}{2}\frac{n(\alpha-c)^2}{(1+\beta(n-1))(2+\beta(n-1))^2}\end{aligned}$$

which is monotonically decreasing both in β, for $n > 1$, and in n, for $1 \geq \beta > \underline{\beta}(n)$, where $\underline{\beta}(n)$ is a level of β not too far from zero (the higher the number of firms in the market, the closer $\underline{\beta}$ to zero). ■

Proof of Proposition 5. By expressions (4.2), (4.6) and by Bertrand equilibrium property, when goods are homogeneous ($\beta = 1$) no difference between mixed and pure oligopoly equilibrium prices occurs, since $\overline{p}_k = p_j^* = p_h^* = c$. When goods are independent ($\beta = 0$) all PMFs behave as monopolists under both pure and mixed oligopoly, with $p_h^* = \overline{p}_k = \dfrac{a+c}{2}$ whereas, also in this case, Coops behave as a perfectly competitive firm, setting $p_j^* = c$. Moreover, for $\beta \in (0, 1)$

$$(\overline{p}_k - p_h^*) = \frac{(\alpha-c)(1-\beta)m\beta}{(2+\beta(n+m-3))(2+\beta(n-3))},$$

which is zero for $m = 0$ and monotonically increasing in the number of Coops, since

$$\frac{d(\overline{p}_k - p_h^*)}{dm} = \frac{(1-\beta)(\alpha-c)\beta}{(2+\beta(n+m-3))^2} > 0$$

for $n \geq 1$. As to the second group of results, note that, for $\beta = 0$

$$x_k(\overline{p}) = x_h(p^*) = \frac{1}{2}(\alpha - c)$$

and, for every j-th Coops,

$$x_j(p^*, \beta = 0) = (\alpha - c), \tag{6.5}$$

and therefore

$$x_j(p^*, \beta = 0) > x_h(p^*, \beta = 0) = x_k(\overline{p}, \beta = 0).$$

Moreover, for $\beta = 1$ in all types of oligopoly the same quantities are chosen with

$$x_k(\overline{p}, \beta = 1) = x_h(p^*, \beta = 1) = x_j(p^*, \beta = 1) = \frac{(\alpha - c)}{n}.$$

When $\beta \in (0,1)$, a simple inspection of (4.3) and (4.7) shows that, for $m \geq 1$,

$$x_k(\overline{p}) > x_h(p^*).$$

Finally, for $\beta \in (0,1)$ see that

$$x_j(p^*) - x_k(\overline{p}) = \frac{\left(\beta(3n-m-5) + \beta^2(2m-4n+3-mn+n^2)+2\right)(\alpha-c)}{(2+\beta(n+m-3))(1+\beta(n-1))(2+\beta(n-3))}$$

whose both numerator and denominator are strictly positive within the defined range of parameters. ■

Welfare under Cournot Competition

Under Cournot competition, the welfare raised in a mixed oligopoly can be expressed as

$$\sum_{h\in N\setminus M} TW_h^{PMF} + \sum_{j\in M} TW_j^{COOP} =$$

$$= \sum_{h\in N\setminus M} (\alpha-c)x_h^* - (1/2)\left[\sum_{h\in N\setminus M}(x_h^*)^2 + \beta\sum_{h\in N\setminus M} x_h^* \sum_{r\neq h} x_r^*\right] +$$

$$+ \sum_{j\in M}(\alpha-c)x_j^* - (1/2)\left[\sum_{j\in M}\left(x_j^*\right)^2 + \beta\sum_{j\in M} x_j^*\sum_{r\neq j} x_r^*\right] + y^*,$$

which, by symmetry of all j-th Coop and all h-th PMF, can be written as

$$TW^{MO} = (n-m)\left[(\alpha-c)x_h^* - (1/2)\left(x_h^{*2} + \beta m x_j^* x_h^* + \beta(n-m-1)x_h^{*2}\right)\right] +$$
$$+ m\left[(\alpha-c)x_j^* - (1/2)\left(x_j^{*2} + \beta(n-m)x_j^* x_h^* + \beta(m-1)x_j^{*2}\right)\right] + y^*.$$

Plugging (3.8) and (3.9) into the above expression, mixed oligopoly welfare is obtained as

$$TW^{MO} = \tfrac{1}{2}\frac{(n-m)(\alpha-c)^2(1-\beta)\left(3+\beta(n+m-4)-\beta^2(n-1)\right)}{\left(2+\beta(n+m-3)-\beta^2(n-1)\right)^2} + \tfrac{1}{2}\frac{m(\alpha-c)^2(2-\beta)}{2+\beta(n+m-3)-\beta^2(n-1)} + y^*. \tag{6.6}$$

For $m=0$ (6.6) becomes

$$TW^{PMF} = \frac{1}{2}\frac{n(\alpha-c)^2(3+\beta(n-1))}{(2+\beta(n-1))^2} + \overline{y},$$

which is pure PMF oligopoly welfare. For $m=n$, (6.6) turns into

$$TW^{COOP} = \frac{1}{2}\frac{n(\alpha-c)^2}{1+\beta(n-1)} + \widetilde{y},$$

i.e., pure Coop oligopoly welfare.

Welfare under Bertrand Competition

By plugging (4.3) into

$$TW^{PMF} = (\alpha-c)n\cdot x_k(\overline{p}) - \tfrac{1}{2}\left[n\cdot x_k^2(\overline{p}) + \beta n(n-1)x_k^2(\overline{p})\right], \tag{6.7}$$

we obtain pure PMF oligopoly welfare under price competition as

$$TW^{PMF} = \frac{1}{2}\frac{n(\alpha-c)^2(1+\beta(n-2))(3+\beta(n-4))}{(1+\beta(n-1))(2+\beta(n-3))^2}.$$

Under a pure Coop Bertrand oligopoly with n firms we obtain

$$TW^{COOP} = n\left(\alpha - c\right) x_k\left(\widetilde{p}\right) - \tfrac{1}{2}\left[n\left(x_k\left(\widetilde{p}\right)\right)^2 + \beta n(n-1) x_k^2\left(\widetilde{p}\right)\right] \tag{6.8}$$

where $x_k\left(\widetilde{p}\right)$ denotes Coop quantity under a pure Coop Bertand equilibrium ($\widetilde{p} = \widetilde{p}_1, \widetilde{p}_2, ...\widetilde{p}_n$).

By plugging (4.8) for $m = n$ into (6.8), we obtain

$$TW^{COOP} = \frac{1}{2}\frac{n\left(\alpha - c\right)^2}{\left(1 + \beta\left(n-1\right)\right)}.$$

Moreover, by using (4.7) and (4.8) and knowing that

$$\begin{aligned} TW^{MO} &= (n-m)\left[\left(\alpha - c\right) x_h^* - \tfrac{1}{2}\left(x_h^{*2} + \beta m x_j^* x_h^* + \beta(n-m-1) x_h^{*2}\right)\right] + \\ &+ m\left[\left(\alpha - c\right) x_j^* - \tfrac{1}{2}\left(x_j^{*2} + \beta\left(n-m\right) x_j^* x_h^* + \beta(m-1) x_j^{*2}\right)\right]. \end{aligned}$$

yields

$$TW^{MO} = \tfrac{1}{2}\tfrac{(n-m)(a-c)^2(3+\beta(n+m-4))(1+\beta(n-2))}{(2+\beta(n+m-3))^2(1+\beta(n-1))} + \tfrac{1}{2}\tfrac{m(\alpha-c)^2(2+\beta(2n-3))}{(1+\beta(n-1))(2+\beta(n+m-3))}. \tag{6.9}$$

A welfare comparison between Bertrand (6.9) and Cournot welfare (6.6) for $(a - c) = 1$, yields the following expression:

$$TW_p^{MO} - TW_q^{MO} = \tfrac{1}{2}\tfrac{\left(6\beta - 2m\beta - 2n\beta - \beta^2 + n\beta^2 - 4\right)(n-m)(n-1)(1+\beta(m-1))(\beta-1)\beta^2}{(2+\beta(n+m-3))^2\left(m\beta - 3\beta + n\beta + \beta^2 - n\beta^2 + 2\right)^2(1+\beta(n-1))},$$

which, under duopoly ($n = 2$) becomes

$$TW_p^{PMF} - TW_q^{PMF} = \tfrac{\left(4-\beta^2-2\beta\right)\beta^2}{(\beta+2)^2(\beta-2)^2(\beta+1)},$$

when $m = 0$ (pure PMF duopoly) and

$$TW_p^{MO} - TW_q^{MO} = \tfrac{1}{8}\tfrac{(\beta+2)(\beta-2)(\beta-1)\beta^2}{\left(\beta^2-2\right)^2(\beta+1)}$$

when $m = 1$ (mixed duopoly). Firstly it is worth noticing that both expressions (**??**) and (**??**) are not monotonic in β. Moreover, welfare differences between price and quantity competition are generally larger under pure PMF duopoly than under mixed duopoly. In both cases such a difference is high when goods are complements. When goods are substitutes, in a pure PMF duopoly the welfare difference between Bertand and Cournot increases with β, and only when β is close to one does it starts to decrease. Conversely, in a mixed duopoly such a difference first increases and then decreases to eventually disappear for $\beta = 1$. These qualitative results also hold for $n > 2$.

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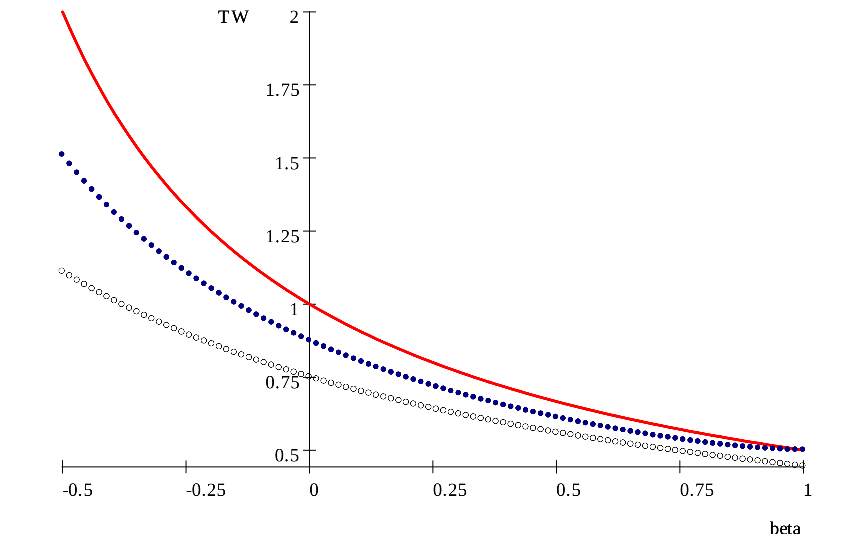

Figure 1 - Cournot competition: pure PMF (circled line), pure Co-op (continuous line) and mixed duopoly total welfare (dotted line), for $(\alpha - c) = 1$ and $\beta = [-0.5, 1]$.

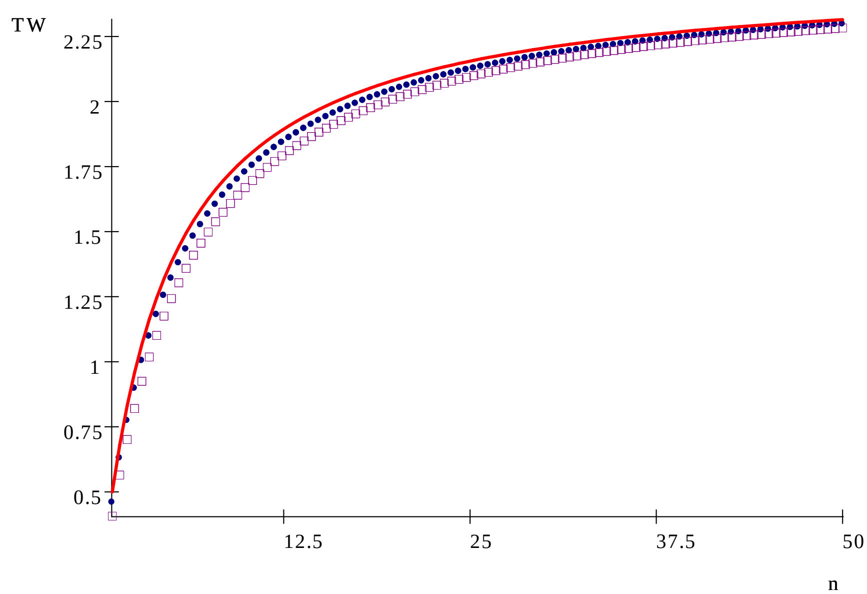

Fig. 2 - Cournot Oligopoly: Mixed Oligopoly welfare for $m = \frac{2}{3}n$ (dotted line), $m = \frac{n}{4}$ (squared line) and $m = n$ (optimum) (continuous line) for $\beta = 0.2$, $(a - c) = 1$, $n = 1, 2, .., 50$.

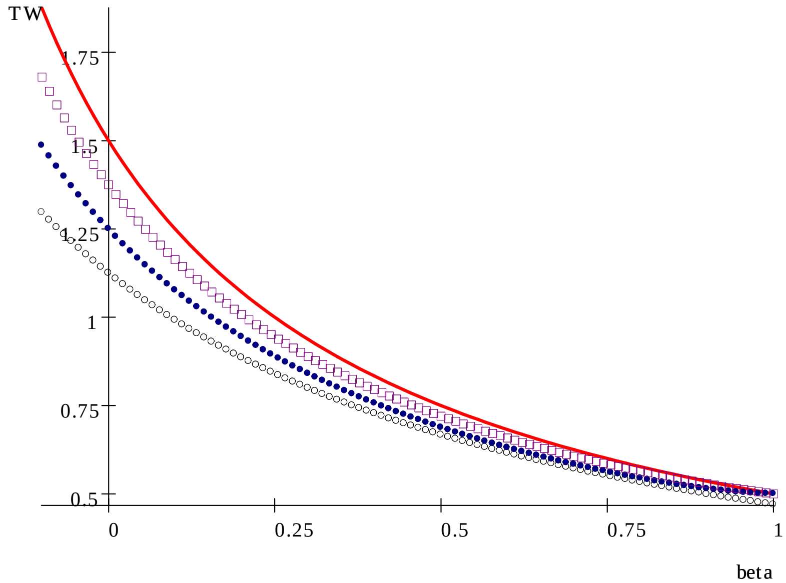

Figure 3 - Cournot competition: pure PMF (circled line), pure Co-op (continuous line) and mixed triopoly total welfare with $m = 1$ (dotted line), $m = 2$ (squared line), for $(\alpha - c) = 1$ and $\beta = [-0.1, 1]$.

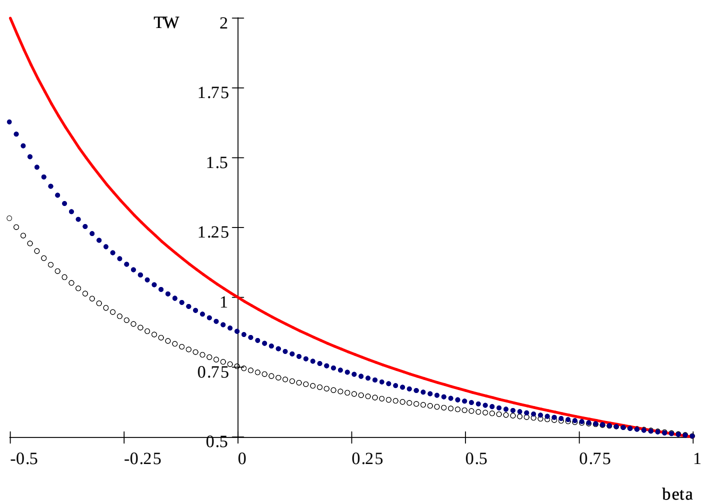

Figure 4- Bertrand competition: pure PMF (circled line), pure Co-op (continuous line) and mixed duopoly total welfare (dotted line), for $(\alpha - c) = 1$ and $\beta = [-0.5, 1]$.

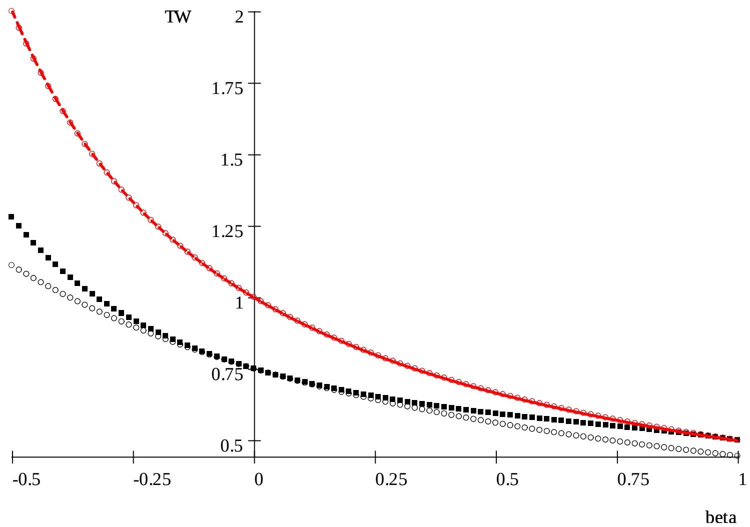

Fig.5- Total welfare in a pure PMF duopoly under Cournot (circled line) and Bertrand competition (dotted line) compared to a pure Co-op market (continuous line) for $(a-c)=1$ and $\beta \in [-0.5, 1]$.