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"Pareto Improvement and Agenda Control of
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Pareto Improvement and Agenda Control of Sequential Financial Innovations

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Abstract

In an exchange economy under uncertainty with two periods, one physical good, and finitely many states of the world, we show that for every (complete or incomplete) market span there exists a sequence of securities such that if they are introduced into markets one by one, the prices of any security is not affected by the subsequent introduction of newer securities and they together generate the given market span. Since these securities generate no pecuniary externalities, this result implies that every stage of such sequential financial innovations is Pareto-improving. Its implications on financial innovations via voting are also explored.

JEL Classification Codes: C72, C73, D51, D52, D61, G11.

Keywords: General equilibrium theory of incomplete security markets, financial innovation, Pareto improvement, agenda control, Nash equilibrium, subgame perfect equilibrium.

1 Introduction

General equilibrium theory of incomplete security markets has flourished during the last couple of decades, and much of the contributions are surveyed in Magill and Shafer (1991) and Magill and Quizii (1996). While the non-existence and inefficiency of equilibria in incomplete markets have been thoroughly investigated, a relatively unexplored topic is financial innovation. Most of the contributions in the field take a set of tradeable securities as exogenously given, without explaining how they have come to be traded. Exceptions are Duffie and Jackson (1989), Allen

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and Gale (1991), the papers in the special issue of *Journal of Economic Theory* (1995), Bisin (1998), Elul (1999), Schmedders (2001), Bettzüge and Hens (2001), Acharya and Bisin (2005), Rahi and Zigrand (2009), and others.

When trying to model financial innovations, several questions immediately pop up in the modeler's mind. First, who undertake financial innovations? Investment banks or futures exchanges? Second, what are the innovators' objectives? To generate cash flows from selling new contingent claims or earn commissions by acting as an intermediary? Third, what kind of costs do they incur? Costs to design new contingent claims or to maintain the marketplace for newly innovated securities? Finally, what is the welfare consequence of financial innovations? Are they beneficial to traders because they provide more hedging opportunities or are they responsible for the fragility of financial markets? There are, of course, no unique answers to these questions, and it is probably for this reason that there seems to be no "standard" or "benchmark" model of financial innovation in the literature.

In this paper, we offer a step towards a "benchmark" model of financial innovation. In the model we present, there is a single, monopolistic agent who undertakes financial innovation. Although the assumption of a monopolistic innovator is admittedly restrictive, his objective and cost functions are quite arbitrary. As for the welfare consequences of financial innovation, we ask whether the introduction of a new security makes every trader better off, that is, whether it leads to a Pareto-improving equilibrium allocation, taking the pecuniary externalities generated by financial innovations into account.

We start our analysis by establishing the possibility of *sequentially Pareto-improving* financial innovations. Specifically, recall that in a two-period model consisting of S states in the second period, where S is finite, each security is characterized by its second-period dividend vector $a \in \boldsymbol{R}^S$. When J securities $a_1, a_2, \dots, a_J$ are traded, the set of all possible income transfers across states coincides with the linear subspace spanned by $\{a_1, a_2, \dots, a_J\}$, called the *market span*. Then, our first main result states that for every market span $G \subseteq \boldsymbol{R}^S$ with $\dim G = J$, there is a sequence $(a_1, a_2, \dots, a_J)$ of J securities such that G coincides with the linear subspace spanned by $\{a_1, a_2, \dots, a_J\}$ and, for every $j \leq J$, the equilibrium prices in markets where $j-1$ securities $a_1, a_2, \dots a_{j-1}$ are traded remain to be equilibrium prices even when j securities $a_1, a_2, \dots, a_j$ are traded. In short, it is possible to generate any market span by introducing securities one by one, while preventing each security from affecting the prices for any preceding ones. To prove this result, we only need continuity, monotonicity, and strict quasi-concavity of traders' utility functions. Since the prices for the preceding securities are not affected by the introduction of any new security, any income transfers between time and states that are affordable before the introduction remain to be affordable even after the introduction. Hence, by a revealed preference argument, all traders' utility levels attained at equilibrium increase as more and more securities get traded in markets. It is in this sense that the securities are sequentially Pareto-improving.

We then move on to constructing a game of financial innovation played between a single innovator and traders. In this game, the innovator proposes to introduce a new security into markets and each trader simultaneously votes for or against the introduction of the proposed security. The proposed security will be introduced if and only if all traders unanimously vote for its introduction. Although this proposal-vote phase will be repeated infinitely many times, the market span generated by the approved securities can be reached after some finitely many phases, because there are only finitely many states of the world. The approved securities are then traded as in the standard models in general equilibrium theory of incomplete security markets, taking those securities as if they were exogenously given. The payoffs to the traders are the utility levels that they enjoy at equilibrium. The payoff to the innovator is arbitrary, as long as, just like the traders' payoff functions, it is uniquely determined by the resultant market span, not by which securities generate it or in which order they are proposed and approved of.

For this voting game of sequential financial innovations, we characterize the Nash and subgame perfect equilibria. Using this characterization and the existence of sequentially Pareto-improving securities, we show that the proposer has the advantage of *agenda control*, in the sense that he can introduce the securities that generate the market span of his highest payoff (among all the market spans). Recall that for a security to be introduced into markets, it needs to be unanimously approved of by the traders. What is surprising on this result is, therefore, that although the unanimity requirement seems to significantly limit the possibility of financial innovations, the innovator can still attain the most desirable outcome, regardless of his and the traders' payoff functions.

Let us now review the literature on the welfare consequence of introduction of new securities in markets. Introduction of new securities enhance risk-hedging opportunities, but it may cause unfavorable changes in the prices for the existing securities for some traders. If this negative pecuniary externality dominates the (direct) benefit of enhanced risk-hedging opportunities, such traders get worse off and, thus, introduction of new securities does not lead to Pareto-improving equilibrium allocations. Indeed, in models with sequential trades, Hart (1975) and Newbery and Stiglitz (1984) gave examples where all traders get worse off. Cass and Citanna (1998) and Elul (1995) went one step further by establishing the following result: If there are sequential trades and the numbers of the existing securities and of the (types of) traders are sufficiently smaller than that of the states of the world, then, whatever the existing securities are, it is generically possible to make all traders simultaneously better off or worse off by introducing an appropriately chosen new security.

The case without sequential trades, which we investigate in this paper, is more fortunate. Since there is no spot market, the constrained efficiency implies that it is impossible that all traders are made worse off by introducing a new security. It is, however, still non-trivial to show that all traders can be made better off. Using a similar assumption on the numbers of existing securities, states, and traders and a similar proof method to those in Cass and Citanna (1998) and Elul (1995), Elul (1999) showed that it is generically possible to make all consumers better off by introducing an appropriately chosen new security, whatever the existing securities are. Since our first main result (Theorem 1) implies that markets can be made complete by introducing some S securities in an appropriate order, it does not require that the number of the existing securities be sufficiently smaller than the number of states to guarantee the existence of a Pareto-improving security. It does not impose any upper bound on the number of traders, either. Our results, however, do not guarantee the existence of a Pareto-improving security when the existing securities are arbitrarily specified. Our and their results should, therefore, be understood as complementary to one another.

In concluding the introduction, we should note that the intellectual debt to Andreu Mas-Colell is evident throughout this paper. Indeed, the subsequent analysis depends, directly and indirectly, on Mas-Colell (1986), Mas-Colell (1987), Hirsh, Magill, Mas-Colell (1990), Geanakoplos and Mas-Colell (1990), and Gottardi and Mas-Colell (2000). Without his contribution to general equilibrium theory of incomplete security markets, this paper would not have been possible.

This paper is organized as follows. Section 2 presents the basic model of security markets. Section 3 establishes the existence of Pareto-improving financial innovations. Section 4 shows that generically, along every sequence of securities with invariant equilibrium prices, each security makes at least one trader strictly better off. Section 5 is devoted to the analysis of the voting game of sequential financial innovations. Section 6 summaries our results and suggests directions of future research. The appendix justifies our proof method, which might appear to be overly mathematical.

2 Model

There are two consumption periods. There is no uncertainty in the first period and there are S possible states of the world in the second period, indexed $s \in \{1, 2, \dots, S\}$. The first period is indexed $s = 0$. There is only one physical good in every period and state.

Each *consumer* (or *trader*), indexed $i \in \{1, 2, \dots, I\}$, is characterized by his utility function $u_i : \boldsymbol{R}_+ \times \boldsymbol{R}^S_+ \to \boldsymbol{R}$ and endowment vector $(e_{0i}, e_i) \in \boldsymbol{R} \times \boldsymbol{R}^S$. We assume throughout this paper that every consumer's consumption set is the nonnegative orthant, u_i is continuous, strongly monotone, and strictly quasi-concave,[1] and $(e_{0i}, e_i) \in \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$.

A *security* is characterized by its second-period dividend vector $a \in \boldsymbol{R}^S$. It is traded in the first period with other securities as well as with the first-period consumption.

Suppose that J securities $(a_1, a_2, \dots, a_J)$ are available for trade. Using the first-period consumption as the numeraire, we denote the security prices in the vector form $q = (q_1, q_2, \dots, q_J) \in \boldsymbol{R}^J$. This is a *security price vector.* The maximization problem of consumer i is

$$\begin{aligned} \max_{(w_i, y_i) \in \boldsymbol{R} \times \boldsymbol{R}^J} \quad & u_i \left(e_{0i} + w_i, e_i + \sum_{j=1}^{J} y_{jh} a_j \right), \\ \text{subject to} \quad & w_i + q \cdot y_i \leq 0. \end{aligned} \tag{1}$$

[1]These assumptions exclude some important utility functions, such as those exhibiting constant relative risk aversion greater than or equal to one. As we will see later, all we need is to guarantee that the net demands are single-valued and continuous when the wealth is strictly positive. This property could be derived even when the strong monotonicity and strict quasi-concavity restricted on the strictly positive orthant, given the strict positivity of initial endowments.

A security price vector q and an allocation of first-period consumptions and securities, $((w_1^*, y_1^*), \dots, (w_I^*, y_I^*))$, constitute a *security market equilibrium* of $(a_1, a_2, \dots, a_J)$ if, for every i, (w_i^*, y_i^*) is a solution to the above maximization problem and $\sum_{i=1}^I (w_i^*, y_i^*) = (0, 0)$. Since u_i is strictly quasi concave, the allocation of consumptions and securities is uniquely determined by the equilibrium security price vector. Also, by Walras law, it suffices to only require $\sum_{i=1}^I y_i^* = 0$ in the market-clearing condition.

For each subset A of $\boldsymbol{R}^S$, denote by span A the smallest linear subspace of $\boldsymbol{R}^S$ that includes A. For a finite or infinite sequence $(a_1, a_2, \dots, a_J)$ of $\boldsymbol{R}^S$ (where J may be infinite), we let span $(a_1, a_2, \dots, a_J)$ to mean span $\{a_1, a_2, \dots, a_J\}$. By the strong monotonicity of utility functions and strict positive of initial endowments, for every equilibrium security price vector $q \in \boldsymbol{R}^J$, there exists a $p \in \boldsymbol{R}^S_{++}$ such that $p \cdot z = q \cdot y$ whenever $z \in \boldsymbol{R}^S$, $y \in \boldsymbol{R}^J$, and $z = \sum_{j=1}^J y_j a_j$. A security structure and an equilibrium security price vector can thus be identified with a linear subspace of $\boldsymbol{R}^S$ and a vector in $\boldsymbol{R}^S_{++}$, which we refer to as a *market span* and a *state price vector*. Using this correspondence between the portfolios y and the second-period consumption z, and writing $G = \text{span}\{a_1, \dots, a_J\}$, we can rewrite the maximization problem (1) as follows:

$$
\begin{aligned}
\max_{(w_i, z_i) \in \boldsymbol{R} \times \boldsymbol{R}^S} \quad & u_i(e_{0i} + w_i, e_i + z_i), \\
\text{subject to} \quad & w_i + p \cdot z_i \leq 0, \\
& z_i \in G.
\end{aligned} \tag{2}
$$

More generally, without initially formulating the securities as vectors of $\boldsymbol{R}^S$, a state price vector p and an allocation of two-period consumptions $((w_1^*, z_1^*), \dots, (w_I^*, z_I^*))$ constitute an *equilibrium* of the market span G if, for every i, (w_i^*, z_i^*) is a solution to the above maximization problem (2) and $\sum_{i=1}^I (w_i^*, z_i^*) = (0, 0)$ as an *equilibrium state price vector* for the market span G. As before, the consumption allocation is uniquely determined by the equilibrium state price vector, and it suffices to only require $\sum_{i=1}^I z_i^* = 0$ in the market-clearing condition.

3 Pareto Improvement and Invariant Equilibrium Prices

In this section, we present the first main results of this paper. We start by defining a sequentially (weakly) Pareto-improving financial innovations as follows.

Definition 1 Let $J \in \{1, 2, \dots, S\}$. A linearly independent sequence $(a_1, a_2, \dots, a_J)$ of J securities has the *weak Pareto improvement property* (WPIP for short) if for every $j \in \{1, 2, \dots, J\}$, there exists a security market equilibrium allocation $\left(\left(w_1^j, y_1^j\right), \dots, \left(w_I^j, y_I^j\right)\right)$ of $(a_1, a_2, \dots, a_j)$ such that

$$
u_i \left(e_{0i} + w_i^{j-1}, e_i + \sum_{k=1}^{j-1} y_{ki}^{j-1} a_k \right) \leq u_i \left(e_{0i} + w_i^j, e_i + \sum_{k=1}^{j} y_{ki}^j a_k \right)
$$

for every $i \in \{1, 2, \dots, I\}$, where $\left(w_i^0, y_i^0\right) = (0, 0)$ by convention.

For any two nonnegative integers L and M with $L \leq M$, denote by $\mathscr{G}^{L,M}$ the Grassmann manifold that consists of all L-dimensional vector subspaces of $\boldsymbol{R}^M$. It is compact. For $J \in$

$\{0, 1, \ldots, S\}$, each element of $\mathscr{G}^{J,S}$ can be considered as the market span generated by a linearly independent set of J securities. Write $\mathscr{G}$ for $\bigcup_{J=0}^{S} \mathscr{G}^{J,S}$. This is the set of all market spans. It is endowed with the topology with respect to which a subset of it is open if and only if the subset can be represented as a union of opens subsets of the $\mathscr{G}^{J,S}$. It is compact with respect to this topology; and the union of compact subsets of $\mathscr{G}^{J,S}$ is also compact. For two nonnegative integers J and K satisfying $J \leq K \leq S$, we define $\mathscr{F}^{J,K}$ as the set of all $(G^J, G^{J+1}, \ldots, G^{K-1}, G^K) \in \mathscr{G}^{J,S} \times \mathscr{G}^{J+1,S} \times \cdots \times \mathscr{G}^{K-1,S} \times \mathscr{G}^{K,S}$ such that $G^J \subset G^{J+1} \cdots \subset G^{K-1} \subset G^K$. It is endowed with the product topology of the topologies of Grassmann manifolds. The following definition is equivalent to Definition 1 but given in terms of market span and state prices.

Definition 2 Let $J \in \{1, 2, \ldots, S\}$. A $(G^1, G^2, \ldots, G^J) \in \mathscr{F}^{1,J}$ has the *weak Pareto improvement property* (WPIP for short) if for every $j \in \{1, 2, \ldots, J\}$, there exists an equilibrium allocation $\left(\left(w_1^j, z_1^j\right), \ldots, \left(w_I^j, z_I^j\right)\right)$ of G^j such that

$$u_i\left(e_{0i} + w_i^{j-1}, e_i + z_i^{j-1}\right) \leq u_i\left(e_{0i} + w_i^j, e_i + z_i^j\right)$$

for every $i \in \{1, 2, \ldots, I\}$, where $\left(w_i^0, z_i^0\right) = (0, 0)$ by convention.

The first main result of this paper claims that for every market span, there is a WPIP sequence of financial innovations.

Theorem 1 *1. For every $J \in \{1, 2, \ldots, S\}$ and every $G \in \mathscr{G}^{J,S}$, there exists a WPIP sequence $(G^1, G^2, \ldots, G^J) \in \mathscr{F}^{1,J}$ such that $G^J = G$.*

2. For every $J \in \{1, 2, \ldots, S\}$ and every $G \in \mathscr{G}^{J,S}$, there exists a WPIP sequence $(a_1, a_2, \ldots, a_J)$ of J securities such that $\operatorname{span}\{a_1, a_2, \ldots, a_J\} = G$.

The special case of interest is the case of complete markets, with $G = \boldsymbol{R}^S$. This theorem, then, states that it is possible to arrive at complete markets by introducing S securities one by one so that at every stage of introducing a new security, no consumer gets worse off.

We prove this theorem in a series of preliminary results. To start, we need the following definition.

Definition 3 Let $J \in \{1, 2, \ldots, S\}$. A linearly independent sequence $(a_1, a_2, \ldots, a_J)$ of J securities has the *invariant equilibrium price property* (IEPP for short) if there exists a security price vector $(q_1, q_2, \ldots, q_J) \in \boldsymbol{R}^J$ such that for every $j \in \{1, 2, \ldots, J\}$, $(q_1, q_2, \ldots, q_j) \in \boldsymbol{R}^j$ is an equilibrium security price vector for $(a_1, a_2, \ldots, a_j)$.

Since the first-period consumption is the numeraire, the IEPP implies "value conservation" of Hakansson (1982). The following definition is equivalent to Definition 3 but is given in terms of market spans and a state price vector.

Definition 4 Let $J \in \{1, 2, \ldots, S\}$. A $(G^1, G^2, \ldots, G^J) \in \mathscr{F}^{1,J}$ has the *invariant equilibrium price property* (IEPP for short) if there exists a state price vector $p \in \boldsymbol{R}_{++}^S$ such that for every $j \in \{1, 2, \ldots, J\}$, p is an equilibrium state price vector for G^j.

Any sequence having the IEPP is referred to simply as an *IEPP sequence*. Along any IEPP sequence, the budget sets of the utility maximization problems (1) and (2) increase monotonically. An easy revealed preference argument, similar to the one in Hakansson (1982), then shows that every IEPP sequence is a WPIP sequence. To establish Theorem 1, therefore, it is sufficient to prove the following proposition, which claims that every market span G can be arrived at by some IEPP sequence starting from the zero security case. Technically, it is the most important result in this paper.

Proposition 1 1. *For every $J \in \{1, 2, \dots, S\}$ and every $G \in \mathscr{G}^{J,S}$, there exists an IEPP sequence $(G^1, G^2, \dots, G^J) \in \mathscr{F}^{1,J}$ such that $G^J = G$.*

2. *For every $J \in \{1, 2, \dots, S\}$ and every $G \in \mathscr{G}^{J,S}$, there exists an IEPP sequence $(a_1, a_2, \dots, a_J)$ of J securities such that* $\operatorname{span}\{a_1, a_2, \dots, a_J\} = G$.

Before proving this proposition, we should mention that although every IEPP sequence is a WPIP sequence, a WPIP sequence need not be an IEPP sequence. However, as is shown in the appendix, in economies in which there are many consumers with varieties of utility functions, any WPIP sequence must necessarily be an IEPP. We can, therefore, say that invoking the existence of an IEPP sequence does not excessively complicate our proof of the existence of an WPIP sequence.

To prove Proposition 1, for each $i \in \{1, \dots, I\}$, $p \in \boldsymbol{R}^S_{++}$, and $G \in \mathscr{G}$, denote by $z_i(p, G) \in \boldsymbol{R}^S$ the second period consumption of the solution to the maximization problem (2). Define $z(p, G) = \sum_{i=1}^{I} z_i(p, G)$. Then p is an equilibrium state price vector of the market span G if and only if $z(p, G) = 0$. The following lemma is a consequence of Theorem 2 of Hirsch, Magill, and Mas-Colell (1990) or Theorem A$'$ of Husseini, Lasry, and Magill (1990) but we give a proof here for completeness.

Lemma 1 *For every $J \in \{1, 2, \dots, S\}$, $G \in \mathscr{G}^{J,S}$, and $p \in \boldsymbol{R}^S_{++}$, there exists a $G^* \in \mathscr{G}^{J-1,S}$ such that $G^* \subset G$ and p is an equilibrium state price vector of G^*.*

Proof of Lemma 1 First, consider the vector bundle

$$\varXi^J = \{(V, v) \in \mathscr{G}^{J-1,J} \times \boldsymbol{R}^J \mid v \in V\}.$$

This is the vector bundle with base space $\mathscr{G}^{J-1,J}$ and fiber V at each $V \in \mathscr{G}^{J-1,J}$. The dimensions of the base space and each fiber are both equal to $J - 1$. As on pages 100–101 of Hirsch, Magill, and Mas-Colell (1990), we can show that the mod 2 Euler number of $\varXi^J$ is equal to one. Hence every continuous section of $\varXi^J$ intersects the zero section at least once.

Second, let $\pi : \boldsymbol{R}^J \to G$ be a linear isometry and define a section σ of $\varXi^J$ by

$$\sigma(F) = (F, \pi^{-1}(z(p, \pi(F)))$$

for every $F \in \mathscr{G}^{J-1,J}$. This is in fact a section of $\varXi^J$ because $z_i(p, \pi(F)) \in \pi(F)$ for every i. Its continuity follows from that of z_i. Hence there exists a $F^* \in \mathscr{G}^{J-1,J}$ such that $\pi^{-1}(z(p, \pi(F^*)) =$

0.

Finally, define $G^* = \pi(F^*)$, then $G^* \in \mathscr{G}^{J-1,S}$, $G^* \subset G$, and $z(p, G^*) = 0$ because π is a linear isometry. Hence p is an equilibrium state price vector for G^*. ///

We can now prove part 1 of Proposition 1 by applying Lemma 1 to $G = G^J$ and then repeatedly applying $G^{J-1}, G^{J-2}, \ldots, G^1$. Part 2 of the proposition follows from part 1. The proof of Theorem 1 is thus completed.

In closing this section, we give some remarks on the results.

Remark 1 (The First Period Consumption) We have so far assumed that consumption takes place in both the first and second periods. All the results, however, apply when consumption takes place only in the second period and the first period is the mere trading period. For example, in the absence of the first period consumption, part 1 of Proposition 1 is modified as follows.

Proposition 2 *For every $G \in \mathscr{G}$ and $G^1 \in \mathscr{G}^{1,S}$ with $\{0\} \neq G^1 \cap \boldsymbol{R}^S_+ \subseteq G^1 \subseteq G$, there exists an IEPP sequence $(G^1, G^2, \ldots, G^J) \in \mathscr{F}^{1,J}$ such that $G^J = G$.*

One can think of G^1 in Proposition 2 as being spanned by some nonzero, nonnegative security, such as the riskless bond and any Arrow security. While the proposition allows for an arbitrary $G^1 \in \mathscr{G}^{1,S}$ as long as $G^1 \cap \boldsymbol{R}^S_+ \neq \{0\}$ and $G^1 \subseteq G^J$, it is important to notice that this arbitrariness does not give rise to any essential multiplicity of the IEPP sequences. To see this, let $(G^1, G^2, \ldots, G^J)$ be an IEPP sequence in the proposition, with $G^J = G$, under a invariant state price vector $p \in \boldsymbol{R}^S_{++}$. For each $j \in \{1, 2, \ldots, J\}$, define $B^{j-1} = \{z \in G^j \mid p \cdot z = 0\}$, then $B^{j-1} \in \mathscr{G}^{j-1,S}$ because $G^j \cap \boldsymbol{R}^S_+ \neq \{0\}$. Let $F^1 \in \mathscr{G}^{1,S}$ be such that $F^1 \cap \boldsymbol{R}^S_+ \neq \{0\}$ and $F^1 \subset G^J$. For each $j \in \{2, \ldots, J\}$, define $F^j = F^1 + B^{j-1}$. Then $(F^1, F^2, \ldots, F^J)$ is another IEPP sequence with $F^J = G^J$ under p. The underlying fact of this result is that consumers care only about the B^{j-1}, not about how they are spanned by securities with positive prices.

Remark 2 (Equivalent Condition of WPIP) Part 2 of Theorem 1 states that for every $J \in \{1, 2, \ldots, S\}$ and every $G \in \mathscr{G}^{J,S}$, there exists a WPIP sequence $(a_1, a_2, \ldots, a_J)$ of J securities such that $G = \operatorname{span}\{a_1, a_2, \ldots, a_J\}$. Consider the following property similar to WPIP: for every $j \leq J$, there exists an equilibrium allocation $\left(\left(w_1^j, y_1^j\right), \ldots, \left(w_I^j, y_I^j\right)\right)$ of $(a_1, a_2, \ldots, a_j)$ such that

$$u_i\left(e_{0i} + w_i^j, e_i + \sum_{k=1}^{j} y_{ki}^j a_k\right) \leq u_i\left(e_{0i} + w_i^J, e_i + \sum_{k=1}^{J} y_{ki}^J a_k\right)$$

for every $i \geq 1$. This property, which we refer to as LBP, standing for the "the Last is the Best" Property, means that the equilibrium allocation of the last market span $G = \operatorname{span}\{a_1, a_2, \ldots, a_J\}$ is at least as desirable as the equilibrium allocation of any other market span in the sequence for every consumer, although the utility levels need not monotonically increase along the sequence $(a_1, a_2, \ldots, a_J)$. LBP is implied by WPIP and we will see that in the proof of the agenda control in the voting game of sequential financial innovations (Corollary 3), it is sufficient to guarantee that for every $J \in \{1, 2, \ldots, S\}$ and every $G \in \mathscr{G}^{J,S}$, there exists a LBP sequence $(a_1, a_2, \ldots, a_J)$ such that $G = \operatorname{span}\{a_1, a_2, \ldots, a_J\}$. We now show that part 2 of Theorem 1, in fact, follows from this property, and thus that WPIP and LBP are equivalent for our analysis.

Indeed, let $G \in \mathscr{G}^{J,S}$. Then there exists a LBP sequence $\left(a_1^J, a_2^J, \ldots, a_J^J\right)$ such that $G = \operatorname{span}\left\{a_1^J, a_2^J, \ldots, a_J^J\right\}$. Then $\operatorname{span}\left\{a_1^J, a_2^J, \ldots, a_{J-1}^J\right\} \in \mathscr{G}^{J-1,S}$ and, by assumption, there exists a LBP sequence $\left(a_1^{J-1}, a_2^{J-1}, \ldots, a_{J-1}^{J-1}\right)$ of $J-1$ securities such that $\operatorname{span}\left\{a_1^{J-1}, a_2^{J-1}, \ldots, a_{J-1}^{J-1}\right\} = \operatorname{span}\left\{a_1^J, a_2^J, \ldots, a_{J-1}^J\right\}$. Again by assumption, there exists a LBP sequence $\left(a_1^{J-2}, a_2^{J-2}, \ldots, a_{J-2}^{J-2}\right)$ such that $\operatorname{span}\left\{a_1^{J-2}, a_2^{J-2}, \ldots, a_{J-2}^{J-2}\right\} = \operatorname{span}\left\{a_1^{J-1}, a_2^{J-1}, \ldots, a_{J-2}^{J-1}\right\}$. We can continue this process to obtain a sequence $\left(a_1^1, a_2^2, \ldots, a_J^J\right)$ such that $G = \operatorname{span}\left\{a_1^1, a_2^2, \ldots, a_J^J\right\}$. By construction, it is a WPIP sequence.

Remark 3 (Sequence Starting from an Arbitrary Market Span) While we have shown that an arbitrary market span can be arrived at along some WPIP sequence starting from the no security case, we have not discussed whether it would be possible to construct an IEPP or WPIP sequence starting from an arbitrary market span. Given that there are certain types of assets, such as stocks and bonds, of which the prime function is not to facilitate risk-sharing (as would be the case for futures contracts) but to raise funds, the normative implications would be much richer if our results could be extended to the case where they are already traded securities. Such an extension would also fill in the gap between our approach and the approach of Cass and Citanna (1998) and Elul (1995, 1999), who took some collection of traded securities as given and asked whether it is possible to introduce one security to bring about a Pareto-improving allocation.

It is unfortunate but should not be surprising that such an extension is impossible, in view of recent developments to solve the equity premium puzzle and the risk-free rate puzzle. For example, Hara and Kajii (2006) found restrictions on utility functions, which are of constant absolute risk aversion type but allow for recursivity and multiple-priors, under which the risk-free bond price goes down (and hence the risk-free interest rate goes up) every time a new security is introduced. It is then easy to see, as proved in the appendix, that a consumer for whom the newly introduced security does not provide any extra risk-hedging opportunities and the changes in the interest rate or equity premium is unfavorable would get worse off. We can therefore conclude that it is impossible to guarantee the existence of an IEPP or WPIP sequence staring from an arbitrary market span.

4 Genericity of Pareto Improvement

In this section we show that, in almost every economy, along every IEPP sequence, every time a new security is introduced, some consumer becomes strictly better off. The genericity argument of this section owes much to Geanakoplos and Mas-Colell (1990). The following definition of Pareto improvement property is given for a sequence of market spans, but, of course, an analogous definition can also be given to a sequence of securities.

Definition 5 Let $J \in \{1, 2, \ldots, S\}$. A $(G^1, G^2, \ldots, G^J) \in \mathscr{F}^{1,J}$ has the *Pareto improvement property* (PIP for short) if for every $j \in \{1, 2, \ldots, J\}$, there exists an equilibrium allocation $\left((w_1^j, z_1^j), \ldots, (w_I^j, z_I^j)\right)$ of G^j such that

$$u_i\left(e_{0i} + w_i^{j-1}, e_i + z_i^{j-1}\right) \leq u_i\left(e_{0i} + w_i^j, e_i + z_i^j\right)$$

for every $i \in \{1, 2, \ldots, I\}$ with strict inequality for some $i \in \{1, 2, \ldots, I\}$, where $(w_i^0, z_i^0) = (0, 0)$ by convention.

PIP sequences may not exist. For example, if the initial endowment allocation is fully Pareto efficient, then, for every market span, the only equilibrium is the no trade equilibrium, and hence no consumer becomes strictly better off as the market span is enhanced. One should thus aim at establishing that every IEPP sequence is generically a PIP sequence.

Note that Definition 5 does not require every consumer to become strictly better off at every stage of span enhancement. There is indeed an example of utility functions such that, regardless of initial endowments, there always exists an IEPP sequence along which some consumer does not become strictly better off at some stage of span enhancement; and hence it is impossible to guarantee, even on a generic set of initial endowments, that every consumer becomes strictly better off at every stage of span enhancements along every IEPP sequence.

The CAPM is one such example: Oh (1996) showed, in our terminology, that every element of $\mathscr{F}^{1,S}$ is an IEPP sequence in the CAPM. To construct an IEPP sequence along which some consumer does not become strictly better off at every stage, fix any particular consumer i and denote by $z_i^S \in \boldsymbol{R}^S$ his excess demand for the second-period consumption at the complete market equilibrium. If $z_i^S = 0$, then he does not trade at all on any market span and thus does not become strictly better off by any span enhancement. If $z_i^S \neq 0$, then define $G^1 \in \mathscr{G}^{1,S}$ to be the line spanned by z_i^S and then define $(G^2, \ldots, G^S)$ so that $(G^1, G^2, \ldots, G^S) \in \mathscr{F}^{1,S}$, then his equilibrium consumption in the the second period remains to be z_i^1 all along the sequence. Hence he does not become strictly better off when G^1 is further enhanced.

In the rest of this section, the consumers' utility functions $u_1, u_2, \ldots, u_I$ remain fixed and an economy is identified with a vector $((e_{01}, e_1), \ldots, (e_{0I}, e_I)) \in (\boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++})^I$ of initial endowments. We make the following additional assumptions on the utility functions: $\{(x_{0i}, x_i) \in \boldsymbol{R}_+ \times \boldsymbol{R}^S_+ \mid u_i(x_{0i}, x_i) \geq u_i(e_{0i}, e_i)\} \subset \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$ for every $(e_{0i}, e_i) \in \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$; and u_i is sufficiently many times differentiable, $\nabla u_i(x_{0i}, x_i) \in \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$, and $\nabla^2 u_i(x_{0i}, x_i) \in \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$ is negative definite on the hyperplane with normal $\nabla u_i(x_{0i}, x_i)$, for every $(x_{0i}, x_i) \in \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$.[2]

For each $i \in \{1, 2, \ldots, I\}$, $p \in \boldsymbol{R}^S_{++}$, $G \in \mathscr{G}$, and $(e_{0i}, e_i) \in \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$, let $z_i(p, G, (e_{0i}, e_i)) \in \boldsymbol{R}^S$ be the second-period consumption (excess demand) of the solution to the maximization problem (2). Then z_i is sufficiently many times differentiable (when restricted on each $\boldsymbol{R}^S_{++} \times \mathscr{G}^{J,S} \times (\boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++})$) so that the subsequent transversality argument can be justified.

[2]The condition that $\{(x_{0i}, x_i) \in \boldsymbol{R}_+ \times \boldsymbol{R}^S_+ \mid u_i(x_{0i}, x_i) \geq u_i(e_{0i}, e_i)\} \subset \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$ for every $(e_{0i}, e_i) \in \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$ is stronger than we need; and it excludes utility functions exhibiting constant relative risk aversion less than one. All we need here is to guarantee that the demands are always in $\boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$.

The following theorem states that if there are sufficiently numerous consumers, then, generically in the space of initial endowments, every IEPP sequence is a PIP sequence.

Theorem 2 *If* $I > \max_{J\in\{1,2,\dots,S\}} (J(S-J)+J)$, *then there exists an open subset* E *of* $(\boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++})^I$, *with the complement of measure zero, such that for every* $((e_{01}, e_1), \dots, (e_{0I}, e_I)) \in E$, *every IEPP sequence in the economy* $((e_{01}, e_1), \dots, (e_{0I}, e_I))$ *is a PIP sequence.*

To prove this theorem, define

$$\Theta^J = \{(G, v) \in \mathscr{G}^{J,S} \times \boldsymbol{R}^S \mid v \in G \text{ and } \|v\| = 1\},$$

then Θ^J is a manifold of dimension $J(S-J)+(J-1)$. Define $F^J : \Theta^J \times \boldsymbol{R}^S_{++} \times (\boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++})^I \to \boldsymbol{R}^{I-1} \times \boldsymbol{R}^S$ by

$$\begin{aligned} &F^J((G, v), p, ((e_{01}, e_1), \dots, (e_{0I}, e_I))) \\ &= \left(v \cdot z_1(p, G, (e_{01}, e_1)), \dots, v \cdot z_{I-1}(p, G, (e_{0,I-1}, e_{I-1})), \sum_{i=1}^{I} z_i(p, \boldsymbol{R}^S, (e_{0i}, e_i)) \right). \end{aligned} \tag{3}$$

The following lemma constitutes the basis of our application of the transversality theorem. The equality $F^J((G, v), p, ((e_{01}, e_1), \dots, (e_{0I}, e_I))) = (0, \dots, 0, 0)$ means that when the initial endowment allocation is $((e_{01}, e_1), \dots, (e_{0I}, e_I))$, p is an equilibrium state price vector of the market span G and all consumers' second-period net demands are contained in a hyperplane in G with normal vector v.

Lemma 2 *The zero vector* $(0, \dots, 0, 0) \in \boldsymbol{R}^{I-1} \times \boldsymbol{R}^S$ *is a regular value of* F^J.

Proof of Lemma 2 It is sufficient to check that one can control each of the $(I-1)+1$ component functions on the right-hand side of (3) by perturbing $((e_{01}, e_1), \dots, (e_{0I}, e_I))$ without affecting the others. But the necessary line of argument, such as the proof of Lemma 3 of Geanakoplos and Mas-Colell (1990), is by now standard. Namely, one can control $\sum_{i=1}^I z_i(p, \boldsymbol{R}^S, (e_{0i}, e_i))$ by perturbing (e_{0I}, e_I) on the hyperplane with normal p; and one can control $v \cdot z_i(p, G, (e_{0i}, e_i))$ by perturbing (e_{0i}, e_i) in the direction of $(p \cdot v, -v)$ and perturbing (e_{0I}, e_I) in the direction of $(-p \cdot v, v)$ to keep $\sum_{i=1}^I z_i(p, \boldsymbol{R}^S, (e_{0i}, e_i))$ constant. ///

Proof of Theorem 2 By Lemma 2 and the transversality theorem, for every $J \in \{1, 2, \dots, S\}$, there exists a subset E^J of $(\boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++})^I$, with the complement of zero measure, such that for every $((e_{01}, e_1), \dots, (e_{0I}, e_I)) \in E^J$, the partial mapping $F^J(\cdot, \cdot, ((e_{01}, e_1), \dots, (e_{0I}, e_I)))$: $\Theta^J \times \boldsymbol{R}^S_{++} \to \boldsymbol{R}^{I-1} \times \boldsymbol{R}^S$ still has $(0, \dots, 0, 0)$ as a regular value. By our assumption on I,

$$\dim(\Theta^J \times \boldsymbol{R}^S_{++}) = J(S-J) + (J-1) + S < (I-1) + S = \dim(\boldsymbol{R}^{I-1} \times \boldsymbol{R}^S).$$

Hence, in fact, for every $((e_{01}, e_1), \dots, (e_{0I}, e_I)) \in E^J$ there is no $((G, v), p) \in \Theta^J \times \boldsymbol{R}^S_{++}$ such that $F^J((G, v), p, ((e_{01}, e_1), \dots, (e_{0I}, e_I))) = (0, \dots, 0, 0)$. By the completeness of Lebesgue measure, we can take E^J as the set of all $((e_{01}, e_1), \dots, (e_{0I}, e_I)) \in (\boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++})^I$ for which there is no $((G, v), p) \in \Theta^J \times \boldsymbol{R}^S_{++}$ such that $F^J((G, v), p, ((e_{01}, e_1), \dots, (e_{0I}, e_I))) = (0, \dots, 0, 0)$. Then E^J is open. The construction of F^J implies that, for every $((e_{01}, e_1), \dots, (e_{0I}, e_I)) \in E^J$, every $G \in \mathscr{G}^{J,S}$, and every complete market equilibrium state price vector $p \in \boldsymbol{R}^S_{++}$, the first $I-1$ consumers' excess demands for the second period consumption, $z_1(p, G, (e_{01}, e_1)), \dots, z_{I-1}(p, G, (e_{0,I-1}, e_{I-1}))$, span G.

Now define $E = \bigcap_{J=1}^S E^J$. Let $((e_{01}, e_1), \dots, (e_{0I}, e_I)) \in E$ and $(G^1, G^2, \dots, G^J) \in \mathscr{F}^{1,J}$ be an IEPP sequence. Then E is open with the complement of zero measure, and for every $j \leq J$, there exists an $i \in \{1, \dots, I-1\}$ such that $z_i(p, G^j, (e_{0i}, e_i)) \notin G^{j-1}$. By the strict quasi concavity of u_i and the revealed preference argument, this implies that consumer i becomes strictly better off as the market span is enhanced from G^{j-1} to G^j. Hence E has the desired property. ///

5 Voting Game of Sequential Financial Innovations

In this section, we present a dynamic game in which there is a player, called the proposer, who proposes introducing a new security, and each consumer can vote for or against introducing the proposed security. Then, we characterize the Nash and subgame-perfect equilibrium in terms of a condition similar to LBP defined in Remark 2 and show, using Theorem 1, that the proposer has the advantage of *agenda control*, in the sense that he can attain the highest utility among those he can possibly enjoy from introducing securities, as long as he can propose to introduce securities in any order he likes.

In our formulation of sequential financial innovations, there are $1+I$ players, of which player $i \in \{1, 2, \dots, I\}$ is a consumer as in the security market model of Section 2, and player 0 is an extra player, who proposes to introduce securities of any type in any order he wishes. A proposed security is introduced if it is unanimously approved of by the consumers, and the securities thus introduced will be traded in markets in the way described in Section 2.[3]

Under the assumptions set out at in Section 2, for every collection of securities, there exists an equilibrium security price vector; and for every market span, there exists an equilibrium state price vector. Throughout this section, we assume that these equilibrium price vectors are essentially unique.

Assumption 1 For every $G \in \mathscr{G}$, if $p \in \boldsymbol{R}^S_{++}$ and $p' \in \boldsymbol{R}^S_{++}$ are equilibrium state price vectors of the market span G, then $p \cdot z = p' \cdot z$ for every $z \in G$.

We can then define $v_i(G)$ as consumer i's equilibrium utility level, that is, the maximum of the

[3] In fact, it is not necessary to assume that all the consumers in the security market model of Section 2 vote. The subsequent argument is valid even if the voting right is granted only to a subset of consumers in the model of Section 2.

following utility maximization problem under any equilibrium state price vector p of G:

$$\begin{aligned} \max_{(w_i,z_i)} \quad & u_i(e_{0i}+w_i, e_i+z_i), \\ \text{subject to} \quad & w_i + p\cdot z_i \leq 0, \\ & z_i \in G. \end{aligned} \tag{4}$$

As for the utility function of the proposer, we assume that it depends on the market span generated by the introduced securities, but not on the securities themselves that generate the market span or the order in which they are introduced. That is, the utility function of the proposer is given by a function $v_0 : \mathscr{G} \to \boldsymbol{R}$. For example, if the proposer is a benevolent social planner, then he may have the utilitarian social welfare function $v_0(G) = \sum_{i\geq 1} v_i(G)$. Also, just like Allen and Gale (1991), we could assume that the proposer is an entrepreneur who issues several claims against his assets to maximize the revenue, in terms of the first-period consumption, from the sales of these claims, subtracted by the costs of issuing these claims. Specifically, suppose that the proposer has some underlying assets (initial endowments) $e_{10} \in \boldsymbol{R}^S$ and can issue any claims at some cost, which is a function $\tau : \mathscr{G} \to \boldsymbol{R}$ of the market spans. Then define his utility function v_0 by $v_0(G) = p\cdot e_{10} - \tau(G)$ whenever $e_{10} \in G$ and p is an equilibrium state price vector of G.[4]

The game we shall consider has discrete time periods, or *stages*. At each odd stage, the proposer proposes a security, and the voters make no move. At each even stage, the voters simultaneously vote for or against the introduction of the security proposed at the previous stage, while the proposer makes no move. Then the securities of which the introduction have been unanimously approved of are traded by the voters in the way described in Section 2.

We now formulate the game of sequential financial innovations as a game in extensive form with almost perfect information following Harris, Reny, and Robson (1995). The set of players is $\{0,1,\dots,I\}$ and the set of stages is $\{0,1,2,\dots\}$. Player 0 is the proposer and the other I players are (some or all of the) consumers in our model of an exchange economy. For each $t \in \{0,1,2,\dots\}$ and $i \in \{0,1,\dots,I\}$, denote by B_i^t the set of actions that player i may take at stage t. We write $B^t = B_0^t \times B_1^t \times \cdots \times B_I^t$. Then B^t is the set of all profiles of actions taken by the players at stage t. We also write $C^t = B^0 \times B^1 \times \cdots \times B^t$. Then C^t is the set of all histories up to and including stage t. Write C for $B^0 \times B^1 \times \cdots$. Then C is the set of all entire histories. For each player $i \in \{0,1,\dots,I\}$, $\pi_i : C \to \boldsymbol{R}$ is the payoff function for player i.

A (pure) *strategy* for player $i \in \{0,1,\dots,I\}$ is a sequence $(\sigma_i^1, \sigma_i^2, \dots)$ such that $\sigma_i^t : C^{t-1} \to B_i^t$ for every $t \in \{0,1,2,\dots\}$. The strategy $(\sigma_i^1, \sigma_i^2, \dots)$ is denoted by σ_i. The set of all strategies for player i is denoted by Σ_i. A profile $(\sigma_0, \sigma_1, \dots, \sigma_I)$ of strategies for all players is denoted by σ and the set of all such strategy profiles, $\Sigma_0 \times \Sigma_1 \times \cdots \times \Sigma_I$, is denoted by Σ. Each $\sigma \in \Sigma$ induces a $(b^0, b^1, \dots) \in C$ inductively by $b_i^t = \sigma_i^t(b^0, b^1, \dots, b^{t-1})$ for every $t \geq 1$ and $i \in \{0,1,\dots,I\}$; and we denote $(b^0, b^1, \dots) \in C$ by $\eta(\sigma)$. A *Nash equilibrium* (in pure strategies) of this game is a

[4] By Assumption 1, the value of $p\cdot e_{10}$ does not depend on the choice of an equilibrium price vector p. For each span G with $e_{10} \notin G$, the value $v_0(G)$ should be taken so low that it is always optimal for the proposer to choose a span containing e_{10}.

strategy profile $\sigma = (\sigma_0, \sigma_1, \ldots, \sigma_I)$ such that $\pi_i(\eta(\sigma)) \geq \pi_i(\eta(\widehat{\sigma}_i, \sigma_{-i}))$ for every $i \in \{0, 1, \ldots, I\}$ and every $\widehat{\sigma}_i \in \Sigma_i$, where $\sigma_{-i} = (\sigma_0, \sigma_1, \ldots, \sigma_{i-1}, \sigma_{i+1}, \ldots, \sigma_I)$. A *subgame* is identified by a $t \geq 1$ and a $c^{t-1} \in C^{t-1}$, from which it starts. To distinguish subgames from the original game, which starts from b_0, we sometimes refer to the latter as the *entire game*. A Nash equilibrium of a subgame can be defined analogously to one for the entire game. A *subgame perfect equilibrium* (in pure strategies) of the entire game is a Nash equilibrium (in pure strategies) of the entire game, whose restriction to every subgame is also a Nash equilibrium (in pure strategies) of the subgame.

The description so far is an abstract formulation of a game in extensive form with almost perfect information. We now move on to describe the specifics of our voting game.

For every $i \geq 0$, let B_i^0 be any singleton; for every odd $t \geq 1$, let $B_0^t = \boldsymbol{R}^S$; for every odd $t \geq 1$ and every $i \geq 1$, let B_i^t be any singleton; for every even $t \geq 2$, let B_0^t be any singleton; and for every even $t \geq 2$ and every $i \geq 1$, let $B_i^t = \{1, -1\}$. The Euclidean space $\boldsymbol{R}^S$ is interpreted as the set of the dividend vectors of all securities. The zero vector 0 of $\boldsymbol{R}^S$ is referred to as the *zero security*. Since it does not enhance any market span, by proposing it, the proposer essentially shows his intention not to introduce any security at that stage. The set $\{1, -1\}$ is interpreted as representing two possible choices for voting, 1 being the voting for the introduction of a proposed security, while -1 being voting against it. Since B_0^i is a singleton for every even t, to specify a strategy of player 0, it suffices to determine σ_0^t for each odd t. Similarly, to specify a strategy of player $i \geq 1$, it suffices to determine σ_i^t for each even $t \geq 2$.

For each $c = (b^0, b^1, \ldots) \in C$, let $\mathscr{J}$ be the set of all positive integers j such that $b_h^{2j} = 1$ for every $h \geq 1$. Let $J = |\mathscr{J}|$ (the number of the elements of $\mathscr{J}$, which may be infinite) and let κ be the strictly increasing sequence from $\{0, 1, 2, \ldots, J\}$ to $\{0\} \cup \mathscr{J}$ (and hence $\kappa(0) = 0$). Then let $\varXi(c)$ be the sequence $\left(0, b_0^{2\kappa(1)-1}, b_0^{2\kappa(2)-1}, \ldots, b_0^{2\kappa(J)-1}\right)$ (which may be of infinite length) in $\boldsymbol{R}^S$. This is the sequence of all approved securities listed up in the order they are proposed. Then, for each $i \geq 0$, define player i's payoff function $\pi_i : \Sigma \to \boldsymbol{R}$ by letting $\pi_i(\sigma) = v_i(\operatorname{span} \varXi(\eta(\pi)))$. This means that every player's payoff is determined by the market span generated by the securities that are unanimously approved of, and is independent of which securities generate the span or in which order they are proposed.[5] Note that since $\boldsymbol{R}^S$ is of finite dimension, span $\varXi(\eta(\pi))$ is well defined even when $|\mathscr{J}|$ is infinite, and, thus, there always exists a finite J^* such that span $\varXi(\eta(\pi)) = \operatorname{span} \left\{0, b_0^{\kappa(1)-1}, b_0^{\kappa(2)-1}, \ldots, b_0^{\kappa(J^*)-1}\right\}$.

The following theorem is the second main result of this paper. It shows that the proposer has the advantage of agenda control, in the sense that if he proposes "right" securities in the "right" order, then he can attain his most preferred market span.

Theorem 3 *Let $G^* \in \mathscr{G}$ satisfy $v_0(G^*) \geq v_0(G)$ for every $G \in \mathscr{G}$. Then there exists a Nash equilibrium σ of the voting game of sequential financial innovations such that* span $\varXi(\eta(\sigma)) = G^*$. *Moreover, if $I \geq 2$, then there exists a subgame perfect equilibrium σ such that* span $\varXi(\eta(\sigma)) = G^*$.

[5]But the players can use strategies that depend on, say, the order in which the securities are proposed. Indeed, we will construct such strategies in the proof of Proposition 3.

To prove this theorem, we first need the following definition.

Definition 6 A finite or infinite sequence $(a_0, a_1, \ldots, a_J)$ of securities (where J may be infinite) has the *extended last-is-best property* (ELBP for short) if $v_i(\operatorname{span}\{a_0, a_1, \ldots, a_j\}) \leq v_i(\operatorname{span}\{a_1, a_2, \ldots, a_J\})$ for every $j \in \{0, 1, \ldots, J\}$ and every $i \in \{0, 1, \ldots, I\}$.

ELBP is similar to but different from LBP, defined in Remark 2, in that ELBP respects the proposer's payoff while LBP does not, and ELBP may involve redundant securities while LBP must not. In fact, we often take $a_0 = 0$ in the above definition, as in the case of $(a_0, a_1, \ldots, a_J) = \Xi(c)$ for some $c \in C$. The following proposition shows that ELBP is sufficient for Nash and subgame perfect equilibria.[6]

Proposition 3 *1. For every ELBP sequence $(a_0, a_1, a_2, \ldots, a_J)$, where $a_0 = 0$ and J may be infinite, there exists a Nash equilibrium σ of the voting game of sequential financial innovations such that $\Xi(\eta(\sigma)) = (a_0, a_1, a_2, \ldots, a_J)$.*

2. If $I \geq 2$,[7] then for every ELBP sequence $(a_0, a_1, a_2, \ldots, a_J)$, where $a_0 = 0$ and J may be infinite, there exists a subgame perfect equilibrium σ of the voting game of sequential financial innovations such that $\Xi(\eta(\sigma)) = (a_0, a_1, a_2, \ldots, a_J)$.

Proof of Proposition 3 For any ELBP sequence $(a_0, a_1, a_2, \ldots, a_J)$, define $c = (b^0, b^1, \ldots) \in C$ by letting[8]

$$b_0^{2j-1} = \begin{cases} a_j & \text{if } j \leq J, \\ 0 & \text{otherwise,} \end{cases}$$

and, for each $i \geq 1$,

$$b_0^{2j} = \begin{cases} 1 & \text{if } j \leq J, \\ -1 & \text{otherwise,} \end{cases}$$

Write $c^{t-1} = (b^0, b^1, \ldots, b^t) \in C^{t-1}$ for each $t \geq 1$. Define a strategy profile $\sigma = (\sigma_0, \sigma_1, \ldots, \sigma_I)$ so that the players take the agreed actions b_i^t whenever the agreed history c^{t-1} has been realized, and, otherwise, the proposer proposes the zero security (that is, chooses $0 \in \boldsymbol{R}^S$) and the the voters vote against the introduction of any security (that is, choose $-1 \in \{1, -1\}$). We prove that if $I \geq 2$, then σ is a subgame perfect equilibrium of the voting game of sequential financial innovations such that $\Xi(\eta(\sigma)) = (0, a_1, a_2, \ldots, a_J)$. To do so, for any $t \geq 1$ and $\widetilde{c}^{t-1} = (\widetilde{b}^0, \widetilde{b}^1, \ldots, \widetilde{b}^{t-1}) \in C^{t-1}$, we need to show that the restrictions of σ is a Nash equilibrium of the subgame following the history $\widetilde{c}^{t-1}$. Denote the history induced by σ in this subgame by $(\widetilde{b}^t, \widetilde{b}^{t+1}, \ldots)$ and write $\widetilde{c} = (\widetilde{c}^{t-1}, \widetilde{b}^t, \widetilde{b}^{t+1}, \ldots)$. Take any player $i \geq 0$ and take any deviation strategy $\widehat{\sigma}_i$ of player i. Denote the history induced by $(\widehat{\sigma}_i, \sigma_{-i})$ in this subgame by $(\widehat{b}^t, \widehat{b}^{t-1}, \ldots)$,

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[6] The proof of the theorem owes much to a comment by Tadashi Sekiguchi.

[7] As noted in Footnote 3, the voters may constitute only a subset of consumers in the security market model of Section 2. What this assumption excludes is, thus, the case of a single voter, not just a single-consumer economy.

[8] We do not need to specify b_0^{2j} or b_i^{2j-1} with $i \geq 1$ because B_0^{2j} and B_i^{2j-1} are singletons. For the same reason, we do not need to specify σ_0^{2j} or σ_i^{2j-1} with $i \geq 1$.

and write $\widehat{c} = (\widetilde{c}^{t-1}, \widehat{b}^t, \widehat{b}^{t+1}, \dots)$. To prove that σ is a Nash equilibrium of the subgame following the history $\widetilde{c}^{t-1}$, we need to show that

$$v_i(\text{span } \Xi(\widehat{c})) \leq v_i(\text{span } \Xi(\widetilde{c})). \tag{5}$$

Suppose first that $\widetilde{c}^{t-1} \neq c^{t-1}$. Then, by the definition of σ_h, $\widetilde{b}_h^{2j} = -1$ for every j with $2j \geq t$ and every $h \geq 1$. Thus

$$\text{span } \Xi(\widetilde{c}) = \text{span } \left(\{0\} \cup \left\{\widetilde{b}_0^{2j-1} \mid 2j \leq t-1 \text{ and } \widetilde{b}_h^{2j} = 1 \text{ for every } h \geq 1\right\}\right).$$

Since $\widehat{c}^{t-1} = \widetilde{c}^{t-1}$, $\widehat{c}^{t-1} \neq c^{t-1}$. Since $I \geq 2$, there is an $h \geq 1$ such that $h \neq i$. For such an h, by the definition of σ_h, $\widehat{b}_h^{2j} = -1$ for every j with $2j \geq t$. Thus

$$\text{span } \Xi(\widehat{c}) = \text{span } \left(\{0\} \cup \left\{\widehat{b}_0^{2j-1} \mid 2j \leq t-1 \text{ and } \widehat{b}_h^{2j} = 1 \text{ for every } h \geq 1\right\}\right).$$

Again, since $\widehat{c}^{t-1} = \widetilde{c}^{t-1}$, $\widehat{b}_0^{2j-1} = \widetilde{b}_0^{2j-1}$ and $\widehat{b}_h^{2j} = \widetilde{b}_h^{2j}$ for every $h \geq 1$ and every j with $2j \leq t-1$. Thus span $\Xi(\widehat{c}) =$ span $\Xi(\widetilde{c})$ and (5) follows.

Suppose next that $c^{t-1} = \widetilde{c}^{t-1}$. By the construction of c and the definition of σ, $c = \widetilde{c}$ and $\Xi(c) = \Xi(\widetilde{c}) = (a_0, a_1, \dots, a_J)$. Without loss of generality, assume that $\widehat{c} \neq c$. Let J^* be the maximum j such that $\widehat{c}^{2j} = c^{2j}$. Then J^* is finite. If $\widehat{b}_0^{2J^*+1} \neq b_0^{2J^*+1}$, then $i = 0$ (that is, $\widehat{\sigma}_0 \neq \sigma_0$ and $\widehat{\sigma}_h = \sigma_h$ for every $h \geq 1$). By the definition of σ_h, $\widehat{b}_h^{2J^*+2} = -1$ for every $h \geq 1$. On the other hand, if $\widehat{b}_0^{2J^*+1} = b_0^{2J^*+1}$, then $i \geq 1$ and hence $\widehat{b}_i^{2J^*+2} \neq b_i^{2J^*+2}$. If $J^* + 1 \leq J$, this means that $\widehat{b}_h^{2J^*+2} = -1$, and if $J^* + 1 > J$, this means that $\widehat{b}_0^{2J^*+1} = 0$. In either case, by the definition of σ_0, $\widehat{b}_0^{2j+1} = 0$ for every $j \geq J^* + 2$. Therefore, span $\Xi(\widehat{c}) =$ span $\left\{a_0, a_1, \dots, a_{\min\{J^*,J\}}\right\}$. Since $(a_0, a_1, \dots, a_J)$ is an ELBP sequence, $v_i\left(\text{span}\left\{a_0, a_1, \dots, a_{\min\{J^*,J\}}\right\}\right) \leq v_i\left(\text{span}\left\{a_0, a_1, \dots, a_J\right\}\right)$. Thus (5) holds. This completes the proof that if $I \geq 2$, then σ is a subgame perfect equilibrium.

To prove that σ is a Nash equilibrium even when $I = 1$, it suffices to show that no player $i \geq 0$ can increase his payoff by changing his strategy from σ_i to $\widehat{\sigma}_i$ on the entire game. The entire game corresponds to the case of $t = 1$ in the above proof and, trivially, $c^0 = \widetilde{c}^0$. For the case where $c^{t-1} = \widetilde{c}^{t-1}$, the proof did not use the assumption that $I \geq 2$. Therefore, it shows that σ is a Nash equilibrium even when $I = 1$. ///

We can now prove Theorem 3 by using Proposition 3.

Proof of Theorem 3 If $G^* = \{0\}$, note that a single zero security constitutes an ELBP sequence. Then this corollary follows from Theorem 3. Suppose that $G^* \neq \{0\}$. By part 2 of Corollary 1, there exists a WPIP sequence $(a_1, a_2, \dots, a_J)$ such that $J = \dim G^* \geq 1$ and $G^* = \text{span}\left(\{a_1, a_2, \dots, a_J\}\right)$. Then $(a_0, a_1, a_2, \dots, a_J)$, with $a_0 = 0$, is an ELBP sequence, because $G^* = \text{span}\left(\{a_1, a_2, \dots, a_J\}\right)$ maximizes v_0 on the entire $\mathscr{G}$. This corollary, again, follows from Theorem 3. ///

Proposition 3 showed that any ELBP sequence can emerge at some Nash or subgame perfect equilibrium. In concluding this section, we prove its converse: the sequence of all approved securities at any Nash equilibrium is an ELBP sequence. Note that these two propositions together provide a full characterization of the sequence of all approved securities at Nash and subgame perfect equilibria: A sequence of securities is an ELBP sequence if and only if it is the sequence of all approved securities at some Nash equilibrium, and also, in the case of $I \geq 2$, if and only if it is the sequence of all approved securities at some subgame perfect equilibrium.

Proposition 4 *If σ is a Nash equilibrium of the voting game of sequential financial innovations, then $\Xi(\eta(\sigma))$ is an ELBP sequence.*

Proof of Proposition 4 Write $c = (b^0, b^1, \dots) = \eta(\sigma)$. Let $\mathscr{J}$, J, τ, and $\Xi(c)$ be as defined in the previous paragraph. Then it suffices to show that

$$v_i\left(\text{span}\left\{0, b_0^{2\kappa(1)-1}, b_0^{2\kappa(2)-1}, \dots, b_0^{2\kappa(j)-1}\right\}\right) \leq v_i(\text{span}\ \Xi(c)) \tag{6}$$

for every $i \geq 0$ and $j < J$.

Let $i \geq 1$ and $j < J$. Let $\widehat{\sigma}_i$ be a strategy that is identical to σ_i except on the subgame following the history $(b^0, b^1, b^2, \dots, b^{2\kappa(j)+1}) \in C^{2\kappa(j)+1}$, where he always rejects the introduction of any security. Let $\widehat{c} = \eta(\widehat{\sigma}_i, \sigma_{-i})$. Then $\Xi(\widehat{c}) = \left(0, b_0^{2\kappa(1)-1}, b_0^{2\kappa(2)-1}, \dots, b_0^{2\kappa(j)-1}\right)$. Since σ is a Nash equilibrium, $\pi_i(\sigma) \geq \pi_i(\widehat{\sigma})$. This is equivalent to (6).

As for the proposer, let $\widehat{\sigma}_0$ be a strategy that is identical to σ_0 except on the subgame following the history $\left(b^0, b^1, b^2, \dots, b^{2\kappa(j)}\right) \in C^{2\kappa(j)}$, where he always proposes the zero security. Let $\widehat{c} = \eta\left(\widehat{\sigma}_0, \sigma_{-0}\right)$. Then $\Xi(\widehat{c}) = \left(0, b_0^{2\kappa(1)-1}, b_0^{2\kappa(2)-1}, \dots, b_0^{2\kappa(j)-1}\right)$. Since σ is a Nash equilibrium, $\pi_i(\sigma) \geq \pi_i(\widehat{\sigma})$. This is equivalent to (6). ///

6 Conclusion

We have considered whether it is possible to introduce new securities into markets to make all consumers better off. We have shown that for any specification of utility functions and initial endowments, and for any given market span, complete or not, there is a sequence of securities that, if introduced one by one, eventually generate the given market span and make no consumer worse off at any stage. We have also presented a dynamic game in which there is a player, called the proposer, who proposes introducing a new security, and each consumer can vote for or against introducing the proposed security. We have then proved that the proposer has the advantage of agenda control.

There are some unsolved issues in our investigation. First, although we have established the existence of a Pareto-improving sequence of securities, we have not given any result on what such a sequence is like, except that in Remark 3, we pointed out that some securities must necessarily fail to constitute any Pareto-improving sequence. To increase the applicability of our first main theorem (Theorem 1), it is important to know more on the characterization of such sequences. Second, for the voting game of financial innovations, we should give an existence theorem of Markov perfect equilibria, where the market spans are the state variables. This is because, as mentioned in Remark 3, Cass and Citanna (1998) and Elul (1995, 1999) took some collections of traded securities as exogenously given, without specifying, say, in which order they have been introduced into markets; and, as such, the type of equilibria in our model that is comparable with the equilibria of their models is where the players choose actions in a way that is not history-dependent. These are, precisely, Markov perfect equilibria.

A Note on the Proof Method

We established the existence of WPIP sequences by proving, first, that there exists an IEPP sequence and, second, that every IEPP sequence is a WPIP sequence. Although our proof method might seem to be unnecessarily powerful, we argue, in this appendix, that it is likely to be the right one for our purpose.

Let's start with the second step first. To be sure, an IEPP sequence is a WPIP sequence, but the converse is in general not true. It may be conceived that there are other WPIP sequences that are easier to identify. This is true if there are relatively few consumers and securities, as assumed in Cass and Citanna (1998) and Elul (1995, 1999), but we argue, along the lines of Mas-Colell (1987), that this would be false if there are many consumers with varieties of characteristics.

Imagine that there is a continuum of consumers. Specifically, the set of (the names of) consumers is given by a probability space $(\boldsymbol{R}^S, \mathscr{B}(\boldsymbol{R}^S), \mu)$, where $\mathscr{B}(\boldsymbol{R}^S)$ is the Borel σ-field of $\boldsymbol{R}^S$ and μ is a Borel measure on $\boldsymbol{R}^S$ with full support (that is, the support of μ coincides with the entire $\boldsymbol{R}^S$). For example, we can take μ to be a S-dimensional normal distribution. Let $v : \boldsymbol{R}_+ \to \boldsymbol{R}$ be a function that is continuous on $\boldsymbol{R}_+$ and twice continuously differentiable on $\boldsymbol{R}_{++}$, and satisfies $v'(w) > 0 > v''(w)$ for every $w \in \boldsymbol{R}_{++}$ and Inada condition, that is, $v'(w) \to 0$ as $w \to \infty$ and $v'(w) \to \infty$ as $w \to 0$.[9] For each consumer $\iota = (\iota_1, \iota_2, \dots, \iota_S) \in \boldsymbol{R}^S$, define $u_\iota : \boldsymbol{R}_+ \times \boldsymbol{R}_+^S \to \boldsymbol{R}$ by letting

$$u_\iota(\widehat{x}_0, \widehat{x}) = v(\widehat{x}_0) + \sum_{s=1}^{S} (\exp \iota_s) v(\widehat{x}_s)$$

for every $(\widehat{x}_0, \widehat{x}) \in \boldsymbol{R}_+ \times \boldsymbol{R}_+^S$. This means that all consumers have expected utility functions exhibiting the same risk attitudes, but their probabilistic beliefs over the S states and time discount rates between the two periods differ in the way described by the probability measure μ. Let $(\widehat{e}_0, \widehat{e}) \in \boldsymbol{R}_{++} \times \boldsymbol{R}_{++}^S$. We take this to be the initial endowment vector for each consumer. Thus all the consumers have the same initial endowment vector.

We show that in this example economy, every WPIP sequence is an IEPP sequence. To do so, it suffices to prove that for all $G \in \mathscr{G}$, $G' \in \mathscr{G}$, $p \in \boldsymbol{R}_{++}^S$, and $p' \in \boldsymbol{R}_{++}^S$, if $G \subset G'$, p is an equilibrium state price vector of G, p' is an equilibrium state price vector of G', and there exists a $\widehat{z} \in G$ such that $p \cdot \widehat{z} \neq p' \cdot \widehat{z}$, then there is a group of consumers with positive measure who

[9]Since v is defined on $\boldsymbol{R}_+$, which contains 0, this assumption excludes the utility functions exhibiting constant relative risk aversion greater than or equal to one. However, the following argument is in fact valid even when v is defined only on $\boldsymbol{R}_{++}$, which excludes 0, as long as it satisfies $v'(w) > 0 > v''(w)$ for every $w \in \boldsymbol{R}_{++}$ and Inada condition.

are strictly worse off at the equilibrium of G' than at the equilibrium of G. In other words, let (w_ι, z_ι) be the solution to the utility maximization problem (2) when the market span is G and the state price vector is p, and let (w'_ι, z'_ι) be the solution to the utility maximization problem (2) when the market span is G' and the state price vector is p'. We wish to prove that

$$u_\iota\left(\widehat{e}_0 + w_\iota, \widehat{e} + z_\iota\right) > u_\iota\left(\widehat{e}_0 + w'_\iota, \widehat{e} + z'_\iota\right) \tag{7}$$

for a set of ι's with positive measure. Indeed, we can assume without loss of generality that $p \cdot \widehat{z} < p' \cdot \widehat{z}$ and $(\widehat{e}_0 - p' \cdot \widehat{z}, \widehat{e} + \widehat{z}) \in \boldsymbol{R}_{++} \times \boldsymbol{R}^S_{++}$. Define $\iota^* = (\iota^*_1, \iota^*_2, \ldots, \iota^*_S) \in \boldsymbol{R}^S$ by

$$\iota^*_s = \ln v'\left(\widehat{e}_0 - p' \cdot \widehat{z}\right) - \ln v'\left(\widehat{e}_s + \widehat{z}_s\right) + \ln p'_s$$

for every $s = 1, \ldots, S$. Then

$$p'_s = \frac{(\exp \iota^*_s) v'\left(\widehat{e}_s + \widehat{z}_s\right)}{v'\left(\widehat{e}_0 - p' \cdot \widehat{z}\right)}$$

for every $s = 1, \ldots, S$. Thus $(-p' \cdot \widehat{z}, \widehat{z})$ is the solution to the utility maximization problem (2) of consumer ι^* when the market span is $\boldsymbol{R}^S$ (complete markets) and the state price vector is p'. Since $\widehat{z} \in G$ and $G \subset G'$, $\widehat{z} \in G'$ and hence $(w'_{\iota^*}, z'_{\iota^*}) = (-p' \cdot \widehat{z}, \widehat{z})$. Moreover, since $p \cdot \widehat{z} < p' \cdot \widehat{z}$,

$$u_{\iota^*}\left(\widehat{e}_0 - p \cdot \widehat{z}, \widehat{e} + \widehat{z}\right) > u_{\iota^*}\left(\widehat{e}_0 + w'_{\iota^*}, \widehat{e} + z'_{\iota^*}\right).$$

Since $(-p \cdot \widehat{z}, \widehat{z})$ satisfies the constraint of (2) when the market span is G and the state price vector is p,

$$u_{\iota^*}\left(\widehat{e}_0 + w_{\iota^*}, \widehat{e} + z_{\iota^*}\right) \geq u_{\iota^*}\left(\widehat{e}_0 - p \cdot \widehat{z}, \widehat{e} + \widehat{z}\right).$$

Thus

$$u_{\iota^*}\left(\widehat{e}_0 + w_{\iota^*}, \widehat{e} + z_{\iota^*}\right) > u_{\iota^*}\left(\widehat{e}_0 + w'_{\iota^*}, \widehat{e} + z'_{\iota^*}\right).$$

Since the values of both sides depend continuously on ι, (7) holds for every ι sufficiently close to ι^*. Since μ has full support, (7) holds for a set of ι's with positive measure. We can therefore conclude that the IEPP and WPIP are equivalent properties in economies with varieties of characteristics.

Let's discuss the first step. We established the existence of an IEPP sequence by constructing a vector bundle whose mod 2 Euler number is one and then using the fact that every continuous section of such a vector bundle intersects the zero section at least once (Lemma 1). One might wonder if there is a more elementary proof. To argue that there seems none, let $p \in \boldsymbol{R}^S_{++}$ and $G \in \mathscr{G}^{J,S}$. To prove the existence of an IEPP sequence, we want to find a $G^* \in \mathscr{G}^{J-1,S}$ such that $G^* \subset G$ and p is an equilibrium state price vector of G^*. Such a market span G^* exists if *and only if* the section $C \mapsto (C, \pi^{-1}(z(p, \pi(C))))$ in the proof of Lemma 1 intersects with the zero section at least once. Hence the question of whether our mathematical machinery is unnecessarily powerful boils down to that of whether the section $C \mapsto \pi^{-1}(z(p, \pi(C)))$ has any other property than continuity. If there is no such property, then the existence of an IEPP sequence is mathematically equivalent to the zero intersection property on a mod 2 vector bundle, and, therefore, there is no fundamentally different approach other than the use of mod 2 Euler number. This latter question was posed by Mas-Colell (1986) in the context of the pseudo equilibrium existence problem in incomplete security markets. Recent answers to this question, such as those in Gottardi and Mas-Colell (2000) and Chiappori and Ekeland (1999), point to the infinitesimal and local arbitrariness of the section beyond continuity.[10] It would therefore be safe to conclude that our mathematical machinery was the right one to prove the existence of an IEPP sequence.

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