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The evolution of the Sino-American Co-dependency: modelling a regime switch in a growth setting
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THE EVOLUTION OF THE SINO-AMERICAN CO-DEPENDENCY:
MODELING A REGIME SWITCH IN A GROWTH SETTING

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ABSTRACT

This work presents a two-country two-stage growth model capturing the special relationship that
has emerged in recent years between the US and China (the so-called BWII regime described by
Dooley et al., 2003). The Chinese authorities maintain a competitive (i.e., undervalued) exchange
rate in order to sustain the high-productive exporting sectors, foster growth and absorb the large
amount of rural workers into the industrial sector. Thus, China runs current account surpluses
against the US and accumulates US assets in the form of foreign reserves. The US policy-makers
are supposedly more concerned with keeping high the consumption possibilities of the population
and exploit the Chinese willingness to finance the US external deficits. We consider three scenarios
for the future state of the Sino-American co-dependency. All the scenarios share phase 1,
resembling what has actually occurred in recent years, but differ in accordance with what fiscal
policy the Chinese authorities adopt, and whether and when China fully liberalizes its capital
account and floats the currency (thus starting phase 2). Scenario A is quite optimistic because the
Chinese fiscal policy is effective in partially substituting the mercantilist policy undertaken in phase
1 as a fundamental source of demand for tradables and as an engine of growth. Scenario B
emphasizes the risks for China of abandoning too early the peg of the exchange rate. Finally,
Scenario C shows that a Chinese continuation of the current export-led growth strategy can be
economically feasible and lead to the mobilization of the Chinese manpower into the advanced
sectors of the economy.

Keywords: Bretton Woods II, growth, global imbalances, regime switch

JEL codes: E42, F33, F41, F43

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1. Introduction

According to the “Sino-American co-dependency” view (put forward by Dooley et al., 2003, 2004a,b, 2007, 2009), China aims at maintaining a competitive (i.e. undervalued) exchange rate so as to sustain its high-productive exporting sectors and to foster growth in a measure sufficient to absorb the large amount of rural workers into the industrial sector.\(^1\) China, in addition, employs capital controls to avoid the speculative inflows generated by the expected appreciation of its currency and maintains a high degree of domestic financial repression in order to control the allocation among competing uses of the capital blocked in the country. As a result of this policy, the Chinese authorities accumulate large stocks of US financial assets. The US, in its turns, exploits the Chinese willingness to finance its persistent current account deficit by maintaining high consumption levels, thus boosting the American demand for Chinese products while maintaining low interest rates and subdued inflation.\(^2\)

Dooley and co-authors argued that this arrangement represented a successful development strategy for China and other Asian economies and, accordingly, maintained that this strategy could have continued as long as growth would have remained the principal goal of the policymakers of these emerging markets. After the crisis of 2008, however, it is has become apparent that US current account deficits of the size displayed in the years immediately preceding the crisis are unsustainable, and that the inevitable down-sizing of these deficits will permanently reduce the room for China’s export-led growth strategy. This recognition has accelerated the debate on the changes in policy that are necessary in the medium and longer run to “rebalance” the growth pattern of China. In recent years, indeed, several studies have advocated policy shifts that may help rebalance Chinese growth away from heavy dependence on external demand, investment and industry, and toward domestic demand, consumption and services (Aziz, 2006; Blanchard and Giavazzi, 2006; Kuijs and Wang, 2006; Lardy, 2006; Prasad, 2009; Prasad and Rajan, 2006; Zheng et al., 2009). This literature has been criticized on the ground that the key to China’s buoyant growth is the rapid transition into producing tradables (mainly manufactures), and that, therefore, a

\(^1\) Surplus labour in the agricultural sector in China is estimated to be between 100 and 200 million people (see Lipschitz et al., 2009).
\(^2\) The US plays the role of a financial intermediary (Caballero et al., 2008, and Mendoza et al., 2007), reflected also in the favorable return differential on the US international gross assets and liabilities (Gourinchas and Rey, 2007a,b).
correction in China’s policy mix which slows down this process of structural change will also result in a reduction in the economy’s longer-term growth rate (Rodrik, 2009a, 2009b): “This would be a bad bargain for China, and it is an important reason why the Chinese authorities have resisted a significant appreciation of their currency (which, in the absence of other compensating policies, would have the effect of reducing the profitability of investment in tradables)” (Rodrik, 2009b). It is implicit in Rodrik’s assessment the conviction that the abandonment of rapid growth as the main objective of the Chinese economic policy would be premature, at a stage in which China’s per capita GDP is still a small fraction of that of the most advanced countries. One could speculate that China’s leadership is likely to share this conviction, especially considering that economic growth is not only a necessary condition for raising people’s consumption levels, but it is also perceived as a means for boosting national power and prestige.

Our paper contributes to this debate by providing an analytical setup able to reproduce several aspects of the “Sino-American co-dependency” story and broadly consistent with the available evidence. Indeed, we present a two-stage growth model that captures some important features of what happened in the recent past and that allows the evaluation of alternative scenarios of the evolution of the Sino-American relationship in the medium and longer-term. Accordingly, the first stage of the model is characterized by an interaction similar to that described by Dooley and co-authors: a developed country (the United States) runs current account deficits against a large and fast-growing developing country (China), whose monetary authorities accumulate foreign reserves so as to set the exchange rate at a level that guarantees the continuous growth of external demand and the absorption into the most productive sectors of the working population employed in the least productive ones. The second stage of the model, instead, reflects the possible scenarios that can materialize, depending on whether China liberalizes the capital account and floats the currency, on its fiscal policies and on the timing of the regime switch. Therefore, we focus on the policy options of the Chinese authorities, while assuming the existence of a strong incentive for the US policymakers to exploit the possibility of running a persistent external deficit in order to guarantee a high level of domestic consumption. Under this respect, the model allows to discuss the possible reasons that may suggest to the Chinese authorities to keep on financing a limited US external deficit, thus relaxing the intertemporal budget constraint of the US economy. In other words, our formal setup

3 With regard to this, those remarking the fact that consumption represents a strikingly small share of China’s total expenditure, and thus invoking a drastic rebalancing of growth toward consumption in order to let the households fully enjoy the fruits of their country’s economic success, should consider that consumption has been increasing in China at very high rates, which is probably what really matters in terms of people’s perception of well-being.

4 In our setting, the RMB is allowed to float after the regime switch: this is clearly different from the one-off appreciation envisaged by Devereux and Genberg (2007).
accounts for the possibility that the US faces a “soft” intertemporal budget constraint, while—in the long run perspective of the model—it is not relevant how the US external debt is divided up according to government and private sector net liabilities.

We explicitly consider three alternative policy scenarios. In Scenario A, the composition of China’s government expenditures is such that fiscal policy is effective in shifting domestic demand toward tradables, thus raising the profitability of investment in the tradable sector. Hence, the Chinese authorities are able to adopt a mix of fiscal policy and exchange rate pegging that allows in time to absorb all domestic manpower in the modern sectors of the economy, and—as soon as this objective is reached—to liberalize the capital account and let the exchange rate float. Notably, even when the exchange rate is permitted to float, the Chinese authorities may still be willing to finance persistent US trade deficits in order to sustain long-run growth. In Scenario B, the composition of China’s government expenditures is such that fiscal policy is less effective in substituting exports as a source of demand for Chinese tradables, and China’s authorities end up liberalizing the capital account and floating the currency before all Chinese manpower is absorbed in the modern sectors of the economy. By doing so, they do allow for larger domestic consumption in the short run, but they depress long-run growth and maintain some of the labor force entrapped into the traditional sector of the economy. In Scenario C, the Chinese authorities neither liberalize the capital account nor make the currency float; rather, they implement the same strategy which has characterized the recent past: the renminbi (RMB) is maintained undervalued so as to let the Chinese economy grow asymptotically faster than the US, consistently with an alleged mercantilist political objective. On the other hand, Chinese domestic consumption remains compressed and reserves expand further.

Our work shares some features with other recent contributions on structural change in China, but it differs along several dimensions, thus contributing to this strand of the literature. While Lipschitz et al. (2009) encompasses a real neoclassical growth model, treats China as a small open economy and, by mainly focusing on FDI-related capital flows, is silent on the current account and on the exchange rate, our model adopts a two-country growth framework and addresses the evolution of the current account, the exchange rate regime and the government policy mix. In doing so, our work contributes to the literature exploring the relationship between exchange rate policy, capital account management and growth (see Levy-Yayati and Sturzenegger, 2009, and Montiel and Serven, 2008). The nature of the structural change, instead, distinguishes Song et al. (2009) from our work: while we look at the transition of part of the labor force from the traditional to the advanced sectors of the economy, Song and co-authors study the reallocation of resources among heterogeneous firms (different in terms of ownership, productivity and access to credit) within the manufacturing sector.
The remainder of the paper proceeds as follows. Section 2 briefly discusses the “Sino-American co-dependency view” as one among the strands of the literature on global imbalances and then presents some stylized facts regarding the relationship between the US and China. The building blocks and the derivation of the model are discussed in section 3, while the characterization of the equilibrium path is presented in section 4. Section 5 is dedicated to growth dynamics under the abovementioned three policy scenarios. Section 6 concludes.

2. Global imbalances and the Sino-American co-dependency

2.1 Relevant literature

Global imbalances, that is the accumulation of large current account deficits by the US vis-à-vis the rest of the world over time, have engaged many economists in a lively debate since the early 2000s. The phenomenon, albeit ongoing during the last 30 years, has accelerated remarkably in the last decade. At the time of Bretton Woods, these imbalances were the by-product of the catching up process of industrializing nations and, accordingly, US deficits were mainly financed by European and Japanese saving. Since the early 2000s, the role once played by Europe and Japan has been played by the fast-growing Asian economies and some oil-producing nations. These economies have run large current account surpluses and accumulated massive official foreign reserves, mainly US Treasury bonds denominated in US dollars.

Several are the rationales for the accumulation of foreign reserves by emerging economies running current account and financial account surpluses: these countries i) have been engaged in fixed (or highly managed) exchange rate regimes to ensure a steady growth of exports and GDP; ii) have been motivated by a precautionary motive (in light of the high costs connected to a potential reversal of private capital inflows or to the occurrence of foreign exogenous shocks); iii) have used reserves as a form of collateral to attract steady foreign investment. Remarkably, this reserve accumulation has continued over time despite the increased cost opportunity of allocating resources to low-yield foreign assets and notwithstanding the risks connected to the valuation effects stemming from the variations in the exchange rates of their currencies with respect to the USD.

The US, in its turn, has played the role of a financial intermediary, which gathers foreign savings by issuing safe, liquid and low-yield securities and reinvests a part of them as domestic (high risk)
investments, as well as foreign direct investments in the very same countries from which the funds had come.  

While there is not much disagreement on this characterization of the phenomenon of global imbalances\(^9\), it has remained highly debated whether this implicit international arrangement can be conceptualized as an equilibrium, and what are its main underlying forces. Some rationalizations accounting for the global imbalances focus on specific issues: diverging saving patterns in the US and in most emerging countries; differences in the relative quality of and expected returns from US and foreign assets; heterogeneous degrees of financial development of the various countries.  

The “Sino-American co-dependency” view (put forward by Dooley et al., 2003), instead, has more a systemic flavor.

Dooley and co-authors argued that the abovementioned pattern of US external deficits is consistent with a revived Bretton Woods regime (called “Bretton Woods II” or BWII), based on a core-periphery division of the world. The heart of the idea is that developing countries (the periphery) aim at maintaining competitive (i.e., undervalued) exchange rates so as to sustain the highly productive exporting sectors and to foster growth in a measure sufficient to absorb the large amount of rural workers into the industrial sector. These countries, in addition, employ capital controls to avoid the speculative inflows generated by the expected appreciation of their currencies and maintain a high degree of domestic financial repression in order to control the allocation among competing uses of the capital blocked in the countries.  

The US (the core), instead, exploits the privileged position of the dollar and the degree of development of its financial sector to enjoy high consumption and income growth, while maintaining low interest rates and subdued inflation.

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\(^10\) As in Eichengreen (2006a,b) and Fracasso (2007), these rationalizations could be summarized in some of the following terms: i) the “deficient US savings view” (Roubini and Setser, 2004; Krugman, 2007); ii) the “new economy view” (Cooper, 2006, and Engel and Rogers, 2006); iii) the “global savings glut view” (Bernanke, 2005; Calvo and Talvi, 2006); iv) the “investment drought view” (see Rajan, 2006a,b; Makin, 2006).

\(^11\) In China, an additional reason for favoring the exporting sector can be identified in the weak conditions of the domestic banking system. High domestic saving, capital controls and politically-driven allocation of credit to state-owned companies would lead to a misallocation of capital across sectors and to increasing nonperforming banking loans. To avoid excessive overinvestment in the nontradable sector without raising interest rates much (that would attract capital inflows), the Chinese authorities adopted a twofold approach: on the one hand, they restricted credit creation through administrative measures (as we will show in the next section) and, on the other hand, they secured an undervalued exchange rate to stimulate the external demand of tradable goods. See Clarida (2005), Makin (2006), Prasad and Rajan (2006), Prasad (2009), Rajan (2006b), and Zheng et al. (2009) on this account.
Leaving aside the issue of whether the recent situation does or does not (as contended for instance by Eichengreen, 2004, and Rose, 2007) resemble the original Bretton Woods system\textsuperscript{12}, the mechanism proposed by Dooley and co-authors seems a reasonable explanation for the forces underlying the formation of global imbalances, in particular with respect to the Chinese and US relationship. Even though this rationalization has been questioned, criticisms have mainly focused on the sustainability of the proposed co-dependency, on the expected duration of the imbalances, and on the mounting costs of the future reversal of ever-increasing imbalances, rather than on its plausibility.\textsuperscript{13} Indeed, this account of the Sino-American co-dependency well reflects the different objectives that the Chinese and US policy-makers have pursued in the last decade and may continue to pursue in the future.\textsuperscript{14}

Dooley and co-authors argued that this arrangement was a reasonable development strategy for several Asian economies (China \textit{in primis}) and, accordingly, that it could continue as long as the policymakers of these emerging markets would aim at fostering the growth of GDP. While these authors reckoned that sooner or later the Asian periphery would reach a developmental stage allowing it to join the core (and thus float the currency, liberalize the capital account, and decumulate foreign reserves), they also maintained that the relationship between the US and the periphery would remain sustainable in an unspecified “near term” given that China and other Asian economies were still far from graduating to the center.\textsuperscript{15}

As the continuation of the Chinese growth strategy entails growing costs, many have argued that a regime change is likely to occur in the future. Accumulating foreign reserves, raising the Chinese rate of growth and fostering mobilization of the labor force in the transitional path require the maintenance of a permanently subdued level of domestic consumption, the imposition of capital

\textsuperscript{12} In a nutshell, Eichengreen (2004) argues that the current situation and Bretton Woods I differ in that: 1) the US run current account surpluses and financial account deficits (due to high internal saving rates) in the 60s, while it runs current and financial account deficits (due to low internal saving rates) now; 2) peripheral countries today are less organized and share less homogenous priorities than the surplus countries (the Gold Pool) in the ’60s; 3) the euro represents an alternative reserve currency that was absent in the past; 4) managing the exchange rate is more difficult today, since sterilization costs are proportional to the international degree of capital mobility; 5) managed exchange rates and low interest rates in emerging markets tend to end up in asset bubbles (rather than in productive investments in traded sector) today than in the past, when financial regulations were tighter.


\textsuperscript{14} These different objectives, in turn, can be explained by the remarkable differences in the two political systems and by the inherently diverse priorities connected to the countries being at different stages of development.

\textsuperscript{15} As argued by Jan Kregel (2008), China and other developing countries’ strategy of supporting demand for domestic resource mobilization through external demand rendered traditional balance of payments adjustment mechanisms ineffective. This contributed to the persistent expansion of global imbalances.
restrictions, the accumulation of currency exposure towards the dollar, and the subjugation of the domestic banking sector to the political leadership. As pointed out by Prasad (2009), the Chinese authorities face a trade-off between keeping on with their growth-enhancing, yet costly and risky, strategy and shifting to a stage characterized by different internal and external policies. While in the past, as argued by Kregel (2008), the opportunity costs of accumulating reserves were high, but still lower than the gains obtained from transferring part of the rural population to urban employment thanks to an export-led growth strategy, in the future this may not be the case. If China switches to a more democratic political framework or if its political authorities attempt to gain more internal support by reducing the costs and risks associated with the persistence of the Sino-American co-dependency, a different international order may emerge and Chinese internal growth may be supported by means of other policies.

As a matter of fact, there are numerous signs that the Chinese authorities are pondering on the strategy to undertake in the future. In the second half of 2008, for instance, the monetary authorities reduced the purchases of long-term US Treasury securities (besides selling US agency bonds) and increased those of the short-term notes. In addition, a massive fiscal stimulus, accompanied by restrictive trade and procurement measures, has been put in place to foster domestic demand. During the first half of 2009, the authorities in China took also actions (such as the establishment of bilateral local currency swaps worth more than RMB600bn with a few commercial partners, the acceleration of pilot programs using the RMB in cross-border settlements, the promotion of investment opportunities in Hong Kong for foreign RMB holders, the advances in regional monetary cooperation and reserve pooling arrangement among ASEAN+3 parties) to encourage the international use of the renminbi and put forward the proposal of introducing a “super-sovereign” reserve currency in place of the US dollar.

While some sections of the press have interpreted this proposal as a sign of China trying to exit the Sino-American co-dependency, this arrangement would in fact allow China to differentiate the denomination of its reserve holdings while simultaneously maintaining an export-led growth

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16 The US authorities are clearly confronted with a similar trade-off. On the one hand, the persistence of the existing Sino-American relationship in the long run is likely to reduce the relative size of the US economy and increase its dependency on foreign capital inflows. On the other hand, the maintenance of such relationship might ensure a high net present value of US consumption which, thanks to the Chinese reserve accumulation, can expand beyond what would otherwise be possible.


strategy in line with the continuation of the current situation.\textsuperscript{19} Similarly, and notwithstanding the crisis, US-dollar denominated Chinese reserves have kept on growing during the first quarters of 2009. These controversial pieces of evidence suggest that the issues discussed in this work are still open and that further theoretical and empirical research is warranted to inform the lively political discussion. Accordingly, our work aims at providing an analytical tool to understand and interpret the Sino-American co-dependency and to evaluate its alternative future scenarios.

2.2 Stylized facts on the Sino-American co-dependency.

Historically, current account and trade imbalances are recurrent phenomena. The global imbalances to which we refer here, instead, are those characterizing the period after the early 2000s. The imbalances characterizing the most recent period, in particular, mostly depend on the bilateral relationship between the US and China, called “Sino-American co-dependency” or “Chimerica”.

Current and trade account balances. Since mid-90s, both China and the US have run persistent overall trade and current account imbalances, whose size has rapidly increased since the early 2003-2004 (Figure 1). In 2007, the Chinese current account surplus exceeded 10% of the GDP whereas the US deficits almost reached 6% over the GDP in 2005 and 2006. The diverging patterns of the overall balances for the two countries are not independent: the bilateral relationship between the US and China heavily affected their positions against the rest of the world (Figure 2). While China accounted for about 20% of the US deficits in early 2000s, the percentage reached almost 40 in more recent years. Notably, notwithstanding the surge in the value of US oil imports since 2004, the share of the US deficits ascribable to China has been larger than that of the oil producing countries.\textsuperscript{20}

\textsuperscript{19} It is not a case that several economists at the Peterson Institute for International Economics supported this last version of the Chinese proposal, which would allow the continuation of the Sino-American co-dependency without China bearing the valuation risks connect to a devaluation of the dollar against the renminbi. While US financial institutions in the past took lots of credit risk and misallocated credit among alternative uses, the Chinese central bank took currency risk. As observed by Brad Setser on his CFR blog, Chinese authorities seem more concerned with this than with the idea of keeping on with accumulating financial claims on the rest of the world.

\textsuperscript{20} It is worth noticing that Chinese and US estimates of the US trade deficit differ considerably because of different ways the two countries treat trade flows passing through Hong Kong.
**International investment positions.** The persistence accumulation of current account deficits (surpluses) by the US (China) portends a negative (positive) international investment position that has grown in size over time. The US is now the largest debtor in the world, and China and Japan are its largest creditors.

It is worth noting that, while both the public and private sectors in the US have recorded deficits financed by borrowing in the international markets, only the official Chinese sector was allowed to intermediate capital abroad. Hence, monetary authorities in China have accumulated high international reserves, most of which denominated in US dollars, and have thus contributed to finance the US external deficits (Figure 3).\(^{21}\) According to Prasad and Sorkin (2009), the current account surplus accounted for 91% of the accumulation of reserves from 2004 to 2008.

\(^{21}\) With official reserves we refer to total reserves minus gold holdings. In the calculations proposed in this section we assume, as commonly done, that 70% of the Chinese foreign reserves is denominated in US dollars.
In 2008, China overcame Japan in terms of total reserves and became the largest official creditor of the US. In April 2009, the Chinese authorities announced that foreign exchange reserves touched $2 trillion, accounting for about 30% of global reserves.\textsuperscript{22} As shown in Figure 4, Chinese official reserves have grown both with respect to the rapidly expanding GDP and in relation to the (steadily increasing) world foreign reserves.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Chinese foreign reserves and US Treasury holdings. Sources: IFS and US Treasury (TIC)}
\end{figure}

The relative importance of Chinese official reserves may be in fact even greater than these official statistics suggest. The data reported here might underestimate the actual amount of reserves directly or indirectly in the hands of the Chinese authorities, because of i) a custodial center bias (which, however, is probably more important in the case of Middle East exporters), ii) the People’s

\textsuperscript{22} According to Brad Setser, total China’s foreign portfolio reached $2bn in June 2008. It is worth noting in passing that although oil exporting countries did accumulate reserves too, they diversified their assets to a greater extent, particularly through the intermediation of national sovereign wealth funds (Alberola and Serena, 2008).
Bank of China (PBC)’s control over Chinese banks’ external asset purchases, and iii) the recent creation of a Sovereign Wealth Fund (CIC) financed with $200 billion of official reserves.

The US international investment position (IIP) has worsened by less than what implied by the sum of its current and capital account deficits (Figure 3). This evolution of the US net foreign asset position is mainly due to favorable valuation effects and capital gains that the US has enjoyed because of the composition and the currency denomination of its gross positions. This can be better appreciated in Figures 3 and 5, where the relative importance of the annual accumulation of dollar denominated assets by the Chinese authorities is compared to the US current account imbalances and to the actual change in the US net IIP.

The relative importance of the Chinese accumulation of reserves is even clearer if one looks at the distribution of US Treasury (UST) securities across domestic and foreign bond holders. Though overall international reserves are far larger than the Chinese ones (since oil producing countries, Japan and other Asian countries hold large stocks too), these latter are largely concentrated on UST securities. At the end of 2008, $3 trillion securities were in the hands of non-US residents, of which $2 trillion were held by foreign official institutions (Figure 6). According to our calculations based on UST TIC data, China’s holdings of short and long-term UST securities passed from $60 billion in 2000 up to $400 in 2006, and reached $800 billion in August 2009.

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23 See Tille (2003, 2008a,b), Higgins et al (2005), Gourinchas and Rey (2007a,b), and Xafa (2007) on return differentials and on valuation effects.

24 The recent valuation effects were favorable to the US (in part because of the steady depreciation of the dollar since 2001) and this contributed to postpone the creation of a stabilizing feedback in the US (see Warnock, 2008). The wealth losses accrued to the foreign holders of US assets did not speed up the rebalancing process either. The fact that reserves are not marked-to-market in emerging markets has favored the preservation of their reserve accumulation strategy.
The importance of the Chinese acquisitions of UST securities is more evident if one considers flows rather than stock holdings: China’s net purchases of long-term UST bonds and notes accounted for 27% and 33% ($847 billion and $861 billion) of all countries’ purchases in 2008 and in the first eight months of 2009, respectively. The large accumulation of long-term UST securities over time can in part account for the Greenspan’s conundrum during 2003-2005.

Large purchases of short-term UST and federal agency securities contributed to expand the US assets in Chinese hands. In the second half of 2008, in particular, China heavily invested in short-term UST securities: its holdings passed from $13 billion in June 2008 to $165 billion in December of the same year. At least until June 2008, China did also purchase billions of US federal agency and private sector bonds. According to the BEA, in mid-2008 China held more than 30% of all federal agencies debt in foreign hands.

The possibility that China could stop buying UST securities has represented a serious concern in the press and among policymakers, and this is likely to continue in the light of the prospective negative conditions of the US fiscal balances. In order to support the financial system and tackle the credit crunch, the US political authorities put in place a large fiscal stimulus package and bailed out several financial institutions. The US fiscal deficit jumped from $0.46 in 2008 to $1.42 trillion in 2009 and the federal debt held by the public increased from 41% up to 55% of GDP. Considering also the debt to be rolled over, the US Treasury might end up issuing more than $2 trillion in 2009, entering in direct competition with a corporate sector unable to tap bank credit. China’s commitment to purchase the newly issued securities and to roll over the debt due for payment is thus crucial for the US strategy to redress domestic growth. Notwithstanding recent covert threats of diversifying the official reserves, the Chinese authorities repeatedly pledged to

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25 This was a global trend. Official and private foreign holdings of short-term UST securities doubled (from $376 to $756 billion) in the second half of 2008. Foreign official institutions increased their holdings from $226 to $460 billion. Though short-term UST securities holdings grew also in the first half of 2009, the trend recently abated worldwide.
keep on with the acquisitions. This suggests that, at least in the medium term, the Sino-American co-dependency will persist.

**Exchange rates.** In 1994 and 1995, the Chinese authorities radically modified exchange rate policy: they abolished exchange rate controls on current account transactions, unified the exchange rate and started to peg the renminbi to the US dollar. The peg, initially set at 8.7 RMB per dollar, was kept for ten years at 8.28 RMB per dollar: this strategy helped China to expand its tradable sector while anchoring the domestic price level. In 2005, however, the authorities chose to transform this regime into a managed float with reference to a basket of 11 currencies, with unannounced weights.\(^{26}\) Although authorities proceeded with a *de facto* peg until the end of 2005, the new regime became officially effective on July 21, when the RMB was revalued against the USD by 2.1% (Figure 7).

The regime switch in 2005 did not take market participants by surprise: since 2003 an appreciation of the RMB was anticipated in forward markets and expectations of appreciation were feeding large speculative financial inflows in China. Three main facts may account for the exchange rate regime switch: the first is the very large size of Chinese current account surpluses; the second is the growing problems encountered by the authorities in managing (i.e., sterilizing) the large capital inflows stimulated by the expectations of the appreciation of the currency (net financial inflows doubled in 2004 and drastically fell in 2005 and 2006 after the regime switch); the third is the

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\(^{26}\) The new regime allowed for a daily ±0.3% movement of RMB against each currency of the basket, which implies a maximum 6% upward or downward trend per month. This arrangement was thus consistent with the implementation of a *de facto* crawling peg to the dollar.
international pressure in favor of a more flexible RMB, whose alleged undervaluation was considered as responsible for the difficulties of US and European exporting sectors.

The Chinese regime switch in 2005 has been read either as a sign of recognition of some RMB fundamental misalignment, or as a Chinese concession to the anxious American Congressmen. Most likely, however, the policy shift represents an attempt of the monetary authorities to gradually increase exchange rate flexibility and recover monetary policy independence. The Chinese authorities, committed to favoring the absorption of the rural population into the tradable sectors of the economy and concerned about the negative valuation effects linked to an abrupt depreciation of the dollar on the reserves and the banking system, believed that a more radical regime switch (i.e. floating the exchange rate) was still premature.²⁷ It is worth noticing that while many US-based economists and politicians have repeatedly argued that the USD-RMB exchange rate was overvalued, notwithstanding the gradual adjustment following the adoption of a managed float, there is not clear empirical evidence on the degree of undervaluation of the renminbi²⁸, nor consensus on the desirability of accelerating its appreciation.

As a matter of fact, Chinese and American nominal and real effective exchange rates appreciated hand in hand on a trade weighted basis until 2002 (Figure 8). This pattern changed in 2002: the depreciating trend stopped in 2005 for the RMB, whereas it continued for the USD until 2008.²⁹ The inception of the financial crisis led to an only temporary stabilization of the dollar in the second half of 2008, but the dollar went back to its depreciating trend in 2009. We will not embark here on a discussion on the degree of undervaluation of the RMB: the Chinese development strategy is qualified by the Chinese fiscal policy and by the feature of the exchange rate regime, and not by the exchange rate level against the USD per se. As we shall discuss in greater detail in section 5, China may preserve its positive trade balance by modulating its fiscal policy, not just controlling the exchange rate.

**Saving and investment.** National accounting identities imply that trade imbalances correspond to saving-investment domestic imbalances: accordingly, the Sino-American current account imbalances are reflected in Chinese (US) investment lower (higher) than domestic saving (Figure 8). The US net borrowing needs have risen since 2000 and, after reaching 6% of gross national income

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²⁷ As argued by McKinnon (2006 and 2007) and McKinnon and Schnabl (2004 and 2006), the Chinese authorities were also concerned about the deflationary risks connected to an excessive appreciation of the RMB and the weak financial conditions of the domestic banking sector.


²⁹ The declining trend of the USD was more substantial against the currencies of the major commercial partners, though not necessarily against the countries with the largest bilateral surpluses. See Fracasso and Schiavo (2008,2009).
(GNI) in 2006, stabilized around 5%. China, on the contrary, has kept on recording positive and growing aggregate net saving.

While US gross domestic investment (as well as gross capital formation) remained stable in the last decade in terms of GNI, domestic saving has steadily fallen since 2000: net government saving turned negative in 2002 (and remained so afterwards), net private saving halved, and net corporate savings did not increase enough to compensate for the reduction in the other two.

The decomposition of the US net saving (Figure 9) reveals that its progressive decline owes both to a shift in the sign (from positive to negative) of the government fiscal balance after 2002 and to a sharp reduction of personal saving.\(^\text{30}\) With the benefit of the hindsight, the reduction in personal

\(^{30}\) Cooper (2006) argues that US saving is underestimated because expenditures on R&D, durables and education (all typically high in the US) represent forms of investment rather than consumption.
saving and the stability of investment may be partially accounted for by the presence of a housing and credit bubble. In the first quarters of 2009, in the attempt of rebuilding part of the financial wealth slashed during the crisis, private saving increased. Such additional saving, however, has been relatively modest and anyway smaller than the surge in public borrowing needs: in 2009, as in the previous years, the US is going to be a net borrower from the rest of the world.

China, on the contrary, has historically enjoyed very high national saving, even when compared to other countries at the same stage of development. Gross saving, already above 40% of GNI in early 2000, reached 50% in 2007. Investment, notwithstanding a steady growth in China in the last decade (it passed from less than 40% of GNI in the late 90s to almost 45% in 2008), has always fallen short of domestic saving. This and the large inflows of FDI have made China a net lender to foreign countries.

![Figure 10. Breakdown of gross domestic saving in China. Source: Asian Development Bank](image)

In the period under investigation, China always recorded positive saving in the all (i.e., household, corporate and government) domestic sectors. Although personal saving tend to be high in any country maintaining an undervalued exchange rate and implementing an export-led growth strategy, extraordinary Chinese personal saving owes also to some country-specific factors: i) the demographic developments of the population; ii) the accumulation of precautionary saving due to high uncertainty about future economic, social, health, pension and education issues (due to the so-called “breaking of the iron rice bowl”); iii) the declining share of GDP accrued to disposable income (as wages grew less than labor productivity); iv) the high expectations of future house
purchases after the privatization of the housing stock; and v) the permanence of a fragile domestic financial system and of borrowing constraints for the households.\(^3\)

Chinese enterprises, in their turn, kept on accumulating large saving through retained earnings: this was made possible by the remarkable growth of the economy and of firms’ profits, the large investment in capital intensive sectors, the steady labor productivity growth, the high precautionary saving of private entrepreneurial firms – at risk of being financially constrained because of the limited access to the political-driven banking system (see Song et al., 2009) -, and the scant incentives for state-owned firms to distribute dividends. Notwithstanding such large values, as shown in Figure 10, household saving has been recently falling in relative term as disposable income has grown less than GDP. This is probably due to large investment in capital intensive sectors and to steady labor productivity growth.

It has been correctly argued that the development and the liberalization of the domestic financial sector, together with the privatization of state-owned enterprises and a reduced uncertainty about government’s plans on the social safety net, might negatively affect the Chinese saving rate in the future. If investment will not decrease as much as saving\(^3\), this may help China to slow down the accumulation of current account surpluses. As argued by the PBC Governor, Mr Zhou Xiaouchuan, however, the reduction in the Chinese private saving will be only a gradual process.

**Monetary aggregates.** Maintaining the peg to the US dollar in the face of increasing current account surpluses, abundant inwards FDI and speculative capital inflows (“twin surpluses”) represents a challenge for the Chinese authorities.\(^3\) The accumulation of foreign exchange reserves would naturally fuel strong liquidity growth, which would cause a credit boom, the overheating of the economy in the short-term, and overinvestment (which instead entails a risk of deflation and of growing nonperforming banking loans\(^3\) in the medium term).

To hold down liquidity growth while continuing to accumulate foreign reserves, Chinese authorities have undertaken a massive sterilization effort. Since 2002, when the PBC first issued the

---

\(^3\) See Chamon and Prasad (2008) and Cappiello and Ferrucci (2008), among numerous contributions, on the persistence of large private (urban) household saving in the presence of high expected income growth. Lane and Schmukler (2007) discuss the origins of high corporate saving and investment in China.

\(^3\) Dollar and Wei (2007) argue that a more efficient allocation could reduce investment intensity by 5% of GDP without denting economic growth. This, however, needs not to occur as investment continues to appear profitable. Bai et al. (2006), for instance, show that the returns to capital in China have remained high despite steady investment growth. The efficiency of sectoral allocation, moreover, has improved over time as the share of investment in the manufacturing sector has grown since 2000 (Song et al., 2009).

\(^3\) FDI are important determinants of the “twin surpluses” and help to account for the size of exchange rate and sterilization interventions.

\(^3\) The nonperforming loans of the major commercial banks amounted to $150 bn in 2007, approximately equal to 10% of the total PBC reserves.
RMB-denominated sterilization bills, commercial banks and other financial institutions have been forced to accumulate these assets, which have expanded fast since 2004 and reached the equivalent 40% of total reserves in 2007. Banks have been also asked to hold part of their reserves in dollars.

![Figure 11. PBC interventions to drain liquidity. Source: CEIC](image)

In addition, as can be seen in Figure 11, the monetary authorities repeatedly tightened the administrative measures in place (such as reserve requirements against deposits) and used their direct control on the lending activity of the domestic banking system so as to limit the expansion of domestic credit. Restrictive administrative measures to control liquidity and pressures on the domestic banks to buy sterilization bills were the only instruments in PBC hands: interest rates could not be used to check on the credit boom as an increase in the rates would have attracted further speculative capital inflows and increased the costs of sterilization.\(^{35}\)

In 2006, not only almost half of banks’ liquid assets were in low-yield PBC bills, but the other half took the form of (mandatory and voluntary) banks’ deposits at the PBC. Almost 25% of Chinese commercial banks deposits were invested in either way.\(^{36}\) Thus, notwithstanding a rapid growth of loans and M2 (the average annual growth of 16% repeatedly overshot the official target), the loan-to-deposit ratio remained under control and stable with respect to the GDP (Figure 12).

---

\(^{35}\) Sterilization has been facilitated by the high return differential between US bonds and the sterilization bills. The direct costs of sterilization remained relatively low also because of the high degree of financial repression: commercial banks have been forced to hold sterilization bills and reserves remunerated at rates lower than the market ones. This suggests that the liberalisation of the financial system will have to go hand in hand with that of the capital account.

Though working, the maintenance of this strategy has some inherent shortcomings: a) the mounting size of sterilization debt required to offset the reserve accumulation; b) the liquidity overhang in the banking system induced by the domestic financial repression and asset portfolio distortions; c) the currency mismatch between PBC assets and liabilities; d) the growing social and quasi-fiscal costs of reserve accumulation; and e) the inability of the monetary authorities to liberalize the domestic interest rates in this context. This suggests that PBC reserve accumulation *cum* sterilization may continue as long as three conditions are met: interest rate differentials between UST securities and domestic sterilization bills remain favorable, commercial banks’ claims on the central bank do not expand too fast and the gains due to export-led growth more than compensate the implications of financial distortions.

In the US, on the contrary, money supply and broad money indicators appear as almost stable over time (Figure 13). The credit boom, whose burst led to the current financial turmoil, in fact, is
mainly linked to the liquidity created by the “shadow” banking system (see Brunnermeier, 2009) and is not detected by typical monetary aggregate statistics.37

**Growth.** In the period under scrutiny, the US managed to secure satisfactory rates of growth. Certainly, part of this result after the burst of the dot.com bubble was due to the expansion of the housing sector, which absorbed most of the new employment, and the increase of house prices, which strengthened the perceived wealth of the households. Net exports fell over the period and investment remained constant: most of the increase in output, thus, was due to an increase in domestic consumption financed by a growing share of US liabilities held in foreign hands. The evolution of the Chinese growth is more complex. Figure 14 shows that overall real GDP growth has remained above 8% in the 2000s and, remarkably, above 10% from 2004 to 2007.

![Figure 14. Contribution to Chinese GDP growth by expenditure. Sources: CEIC data](image)

As other emerging economies, China has expanded faster than the US: real GDP growth exceeded 8% since 2000, remained above 10% from 2004 to 2007, and lowered to 9% in 2008 because of the financial crisis. The decomposition of growth shows that China increasingly depended on investment and foreign demand: besides adding to domestic demand in a purely statistical term, exports also drove investment in the tradable sector up and generated positive spillovers on business and consumer confidence. Guo and N’Diaye (2009) estimate that export and investment linked to the tradable sector accounted for 60% of GDP growth during 2001-2008, up from 40% in the 1990s. The contribution of final consumption to growth, instead, has been somehow muted: the increase in domestic saving discussed above has its counterpart in a steady reduction of the share of final

37The increase in the total reserves of depository institutions and the monetary base in 2008 is the consequence of both Fed interventions to reactivate credit and of precautionary reserve hoarding by the depository institutions.
consumption in GDP. The ratio of per capita living expenditure over disposable income in the urban areas fell from above 80% in the late 90s to 70% in 2006-2007.

Diverse sectors contributed to a different extent to the high Chinese GDP growth (Figure 15). The bulk of Chinese growth was and remains concentrated in the secondary sector even though the relative importance of the tertiary sector has increased over time. This composition is also reflected in the evolution of the sectoral contribution to GDP represented in Figure 16. As in most emerging markets, the share of the primary sector in terms of GDP halved from 1995 to 2007, while the share of the manufacturing sector remained constant and the relative size of the tertiary sector increased.

The evolution of employment across sectors gives a similar picture (Table 1). Despite a steadily growing active population, employment in the primary sector has contracted over the last decade, while that in the secondary sector and (even more) the tertiary sector expanded.³⁸

³⁸ It is worth noting that figures at this level of aggregation likely underestimate the actual dimensions of the trend driving employment out of traditional sectors into more advanced activities.
Although urban areas employed less than 40% of the Chinese labor force in 2007, almost the entire growth in employment materialized in these areas and employment contracted in the rural ones.

Household consumption followed a similar pattern: the share of expenditure in rural areas passed from 30% of total domestic expenditure in 1997 to 21% in 2003 and 18.6% in 2007, whereas the share of urban households went from 46% in 1997 to 52% in 2003, and to 54% in 2007 (Table 2).

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Consumption growth % rate</td>
<td>9.61</td>
<td>10.57</td>
<td>8.03</td>
<td>12.37</td>
<td>12.40</td>
<td>13.06</td>
<td>16.13</td>
</tr>
<tr>
<td>Share of household expenditure</td>
<td>76.69</td>
<td>74.54</td>
<td>73.38</td>
<td>73.34</td>
<td>72.80</td>
<td>72.44</td>
<td>71.99</td>
</tr>
<tr>
<td>of which Urban household</td>
<td>46.42</td>
<td>49.92</td>
<td>52.33</td>
<td>53.18</td>
<td>53.15</td>
<td>53.68</td>
<td>54.04</td>
</tr>
<tr>
<td>of which Rural household</td>
<td>30.28</td>
<td>24.62</td>
<td>21.05</td>
<td>20.17</td>
<td>19.66</td>
<td>19.08</td>
<td>18.62</td>
</tr>
<tr>
<td>Share of government expenditure</td>
<td>23.30</td>
<td>25.46</td>
<td>26.62</td>
<td>26.66</td>
<td>27.20</td>
<td>27.39</td>
<td>27.93</td>
</tr>
</tbody>
</table>

These figures help to appreciate why the Chinese policymakers aim at mobilizing labor from the low productive activities in the rural areas into the most productive ones in the urban areas.

**Wages.** The evolution of nominal wages in China reflects the growth in the economy, the abundant supply of unskilled labor and the productivity differentials across sectors. Nominal wages increased over time in all sectors, yet less in the primary one (Figure 17). The average rate of growth of wages in the whole economy was closely followed by that in manufacturing, while wages in the tertiary sector grew at higher rates. As wages in the primary sector lagged behind, they can be treated as reservation wages for the Chinese workers.
Nominal wages are mirrored by real wages due to rapid productivity growth (Figure 18).

3. The model

The world economy includes two countries, the US and China. Three market goods are produced in this world economy: an internationally tradable good that is produced in both countries, an (internationally) nontradable good that is produced and sold in the US, and an (internationally) nontradable good that is produced and sold in China. Hence, in both countries there are firms specialized in the production of tradable goods and firms specialized in the production of nontradable goods. The tradable good is used as capital in the production of both goods and as consumption good, while the nontradable good can be only consumed.\(^{39}\) Labor is internationally immobile but can freely move across sectors within each country. Labor that is not employed in the

\(^{39}\)As argued by Turnovsky (1997), there is no agreed conclusion on the share of tradables and nontradables in total investment. For some evidence on the issue, see Bems (2008).
two market sectors is employed in the non-market sector of each country. One can interpret this non-market sector as consisting of low-productive activities that people undertake if they cannot be employed profitably in the market economy. Goods and labor markets are perfectly competitive. Both countries are populated by households that supply labor, buy the consumer goods, accumulate financial assets and hold money. Moreover, each country has its own government sector. Two policy regimes governing the world financial markets are considered. In phase 1, the Chinese authorities fix the nominal exchange rate and only official transactions in financial assets are permitted. The world economy may enter a phase 2 if the Chinese authorities decide to liberalize the capital account and to let the nominal exchange rate float consistently with the two countries’ policies and market fundamentals.

Finally, time is discrete and the time horizon is infinite. There is no source of random disturbances and agents’ expectations are rational (in the sense that they are consistent with the true processes followed by the relevant variables), thus implying perfect foresight.40

**Firms producing the (internationally) nontradable good**

In each country j, j=us,ch, there is a large number (normalized to be one) of identical firms, which—in each period t—produce the nontradable good \( Y_{jNt} \). This good is not storable and must be immediately consumed.41 Firms produce \( Y_{jNt} \) according to the following technology:

\[
Y_{jNt} = A_{jNt} K_{jNt}^{1-\gamma_j} L_{jNt}^{\gamma_j}, \quad 0 < \gamma_j < 1, \tag{1}
\]

where \( K_{jNt} \) and \( L_{jNt} \) are, respectively, the capital stock and the labor input used in country j to produce the (internationally) nontradable market good \( Y_{jNt} \), and \( A_{jNt} \) is a variable measuring the state of technology of the firms operating in that sector of country j which produces the (internationally) nontradable good \( Y_{jNt} \). It is assumed that \( A_{jNt} \) is a positive function of the capital installed in the sector of j which produces \( Y_{jNt} \): \( A_{jNt} = K_{jNt}^{\gamma_j} \).42 This assumption combines the idea that learning-by-doing works through each firm’s capital investment and the idea that knowledge and productivity gains spill over instantly across all firms (see Barro and Sala-i-Martin, 1995). Therefore, in accordance with Frankel (1962), it is supposed that although \( A_{jNt} \) is endogenous to the

---

40 The distinction between two main sectors (tradables and nontradables) and the assumption that labor is mobile across sectors but not across countries while the capital good is mobile both across sector and countries are consistent with the standard trade model developed by Obstfeld and Rogoff (1996), Chapter 4. We extend this framework by introducing a technological spillover in both sectors. The latter replaces the assumption of exogenous productivity improvements and generates endogenous growth.

41 Typically, consumer services are consumed while they are produced.

42 Consistently with this formal set-up, one can interpret technological progress as labor augmenting.
economy, each firm takes it as given, since a single firm’s decisions have only a negligible impact on the aggregate stock of capital of the nontradable sector.\(^4\)

In each \(t\), the net profit (cash flow) \(\pi_{jNt}\) of the representative firm producing nontradables is given by:

\[
\pi_{jNt} = P_{jNt}Y_{jNt} - W_{jt}L_{jNt} - P_{jTt}I_{jNt}, \quad I_{jNt} \geq 0,
\]

where \(P_{jNt}\) and \(P_{jTt}\) are, respectively, the price of the nontradable good and the price of the tradable good in country \(j\) at time \(t\), \(W_{jt}\) is the nominal wage in country \(j\) at time \(t\), and \(I_{jNt}\) is capital investment by the representative firm producing nontradables in country \(j\) at time \(t\).

The capital stock installed in the nontradable sector evolves according to

\[
K_{jNt+1} = I_{jNt} + (1 - \delta_j)K_{jNt}, \quad 0 \leq \delta_j \leq 1, \quad K_{jN0} \text{ given.} 
\]

In each \(t\), firms decide on \(\{L_{jNt+n}\}_{n=0}^{\infty}\) and \(\{I_{jNt+n}\}_{n=0}^{\infty}\) subject to (3) in order to maximize their discounted sequence of net profits

\[
\sum_{v=0}^{\infty} \frac{\pi_{jNt+v}}{\prod_{s=1}^{v} (1 + i_{jt+s})},
\]

where \(\prod_{s=1}^{0} (1 + i_{jt+s}) = 1\), and \(i_{jt}\) is the nominal interest rate in country \(j\) at time \(t\).

**Firms producing the (internationally) tradable good**

In each country \(j\), there is a large number (normalized to be one) of identical firms producing the (internationally) tradable good \(Y_{jTt}\). In each period \(t\), these firms produce \(Y_{jTt}\) according to the following technology:

\[
Y_{jTt} = A_{jTt}^{1 - \alpha_j} K_{jTt}^{\alpha_j} L_{jTt}^{\alpha_j}, \quad 0 < \alpha_j < 1,
\]

where \(K_{jTt}\) and \(L_{jTt}\) are, respectively, the capital stock and the labor input used in country \(j\) to produce the (internationally) tradable market good \(Y_{jTt}\), and \(A_{jTt}\) is a variable measuring the state of technology of the firms operating in that sector of country \(j\) which produces the (internationally) tradable good \(Y_{jTt}\). It is assumed that \(A_{jTt}\) is a positive function of the capital installed in the sector of \(j\) which produces \(Y_{jTt}\): \(A_{jTt} = K_{jTt}^{\alpha_j}\).

In each \(t\), the net profit \(\pi_{jTt}\) of the representative firm producing tradables is given by

\[
\pi_{jTt} = P_{jTt}Y_{jTt} - W_{jt}L_{jTt} - P_{jTt}I_{jTt}, \quad I_{jTt} \geq 0,
\]

\(^4\)This amounts to say that technological progress is endogenous to the economy, although it is an unintended by-products of firms’ capital investment rather than the result of purposive R&D efforts.
where \( I_{jTt} \) is capital investment by the representative firm producing tradables in country \( j \) at time \( t \).

The capital stock installed in the tradable sector evolves according to
\[
K_{jTt+1} = I_{jTt} + (1 - \delta_j) K_{jTt}, \quad 0 \leq \delta_j \leq 1, \quad K_{jT0} \text{ given.} \tag{7}
\]

In each \( t \), firms decide on \( \{I_{jTt+n}\}_{n=0}^\infty \) and \( \{L_{jTt+n}\}_{n=0}^\infty \) subject to (7) in order to maximize their discounted sequence of net profits
\[
\sum_{\nu=0}^\infty \frac{\pi_{jTt+\nu}}{\prod_{s=1}^{\nu} (1 + i_{jTt+s})}. \tag{8}
\]

**Households**

Households are infinitely lived. Their large number living in country \( j \) is normalized to be one. Consumption, real money balances providing liquidity services and a public good provided by the government enter the period utility function of the representative household of country \( j \), \( u_{jt} \):
\[
u_j = \ln(C_{jt}) + \chi_j \ln \left( \frac{M_{jt}}{P_{jt}} \right) + \nu(G_{jt}), \quad \chi_j > 0, \nu' > 0, \tag{9}
\]
where \( M_{jt} \) and \( P_{jt} \) are, respectively, the household’s nominal money holdings and the consumer price index in country \( j \) at time \( t \), \( C_{jt} \) is the consumption index for the households located in country \( j \) at time \( t \), and \( G_{jt} \) is the amount of public good provided by the government of country \( j \) in \( t \). The consumption index is defined as
\[
C_{jt} = C_{jNt}^\eta_j C_{jTt}^{1-\eta_j}, \quad 0 < \eta_j < 1, \tag{10}
\]
where \( C_{jNt} \) and \( C_{jTt} \) are, respectively, the consumption of nontradables and the consumption of tradables by the representative household located in country \( j \) at time \( t \). Notice that \( C_{jt} \) can be interpreted as a composite good. Given (10), \( P_{jNt} \) and \( P_{jTt} \), the consumer price index \( P_{jt} \) is obtained by minimizing the expenditure necessary to buy one unit of \( C_{jt} \):
\[
P_{jt} = \frac{P_{jNt}^{\eta_j} P_{jTt}^{1-\eta_j}}{D_j}, \quad D_j \equiv \eta_j^{\eta_j} (1-\eta_j)^{1-\eta_j}. \tag{11}
\]

The representative household’s period budget constraint is:
\[
B_{jHt+1} + E_{jt} F_{jHt+1} + M_{jt} + P_{jNt} C_{jNt} + P_{jTt} C_{jTt} \leq (1+i_{jt}) B_{jHt} + (1+i_{jt}) E_{jt} F_{jHt} + M_{jt-1} + \pi_{jNt} + \pi_{jTt} + L_{jt} W_{jt}\cdot T_{jt},
\]
\[
B_{jH0}, F_{jH0} \text{ and } M_{j-1} \text{ given, } i \neq j, \tag{12}
\]
where \( B_{jHt} \) are the domestic financial assets accumulated during period \( t-1 \) by the representative household of country \( j \) and carried over into period \( t \) with nominal yield \( i_{jt} \), \( E_{jt} \) (\( E_{jt}=1/E_{it} \)) is the nominal exchange rate of country \( j \) at time \( t \) (the price in units of the \( j \)-country’s currency of one
unit of the i-country currency at time t), $F_{jHt}$ are the foreign financial assets (denominated in foreign currency) accumulated during period t-1 by the representative household of country j and carried over into period t with nominal yield $i_{jt}$, $L_{jt}$ is the amount of labor supplied by the representative household of country j in period t, and $T_{jt}$ are the net monetary transfers (“net taxes”) from the representative household of country j to its government in t. Notice that in each period the representative household of country j is entitled to receive the net profits earned by the firms located in its own country as dividend payments. It should be also apparent that nominal balances (no-interest bearing financial assets) $M_{jt}$ are accumulated during period t and carried over into period t+1 because of the liquidity services that they provide to the households.

To rule out the possibility that households borrow arbitrary large sums, we impose the usual no-Ponzi condition

$$
\sum_{v=0}^{\infty} (E_{jt+v} - E_{jt+v+1})F_{jHt+v+1} + i_{jt+v}M_{jt+v+1} + T_{jt} + P_{jNt+v}C_{jNt+v} + P_{jTt+v}C_{jTt+v} \leq B_{jHt} + \prod_{s=0}^{\infty} (1 + i_{jt+s})
$$

$$
+ E_{jt}F_{jHt} + M_{jt-1} + \sum_{v=0}^{\infty} \pi_{jTt+v} + \pi_{jNt+v} + W_{jt+v}L_{jt+v} + \frac{(i_{jt+v} - i_{jt+v})E_{jt+v}F_{jHt+v}}{i_{jt+v}} \prod_{s=0}^{\infty} (1 + i_{jt+s})
$$

The amount of labor supplied by the representative household of country j in period t is determined as follows:

$$
L_{jt} = \begin{cases} 
H_j & \text{if } \frac{W_{jt}}{P_{jt}} > V_{jt} \\
0 & \text{if } \frac{W_{jt}}{P_{jt}} < V_{jt} \\
\text{indeterminate, otherwise,} & 
\end{cases}
$$

where $H_j$ is the fixed time endowment of each household located in country j, and $V_{jt}$ is the reservation wage for households located in j at time t. One could argue that this reservation wage depends on labor productivity in the non-market sector of the economy, which may be interpreted as a traditional sector where low-productive technologies are utilized for subsistence consumption\(^{44}\) (it can be considered a proxy of China’s primary sector). Thus, it is plausible to assume that $V_{jt}$ evolves according to

\(^{44}\) The net utility that the representative household gets by undertaking the non-market activities is assumed to be zero.
This amounts to assume that, in any period in which households devote some time to non-market activities, some technological progress occurs in the non-market sector because of learning by doing (the rate at which labor productivity increases in this sector is exogenously given), while labor productivity is stagnant in this sector whenever households devote all their time to market activities.

In each period \( t \), households located in country \( j \) decide on

\[
\begin{align*}
L_{jNt} + L_{jTt} &= H_j, \\
(1+\omega_j)V_{jt} + \omega_j > 0, \\
V_{jt+1} &= \begin{cases} 
  V_{jt} & \text{if } L_{jNt} + L_{jTt} = H_j, \\
  (1+\omega_j)V_{jt} & \text{if } L_{jNt} + L_{jTt} < H_j.
\end{cases}
\end{align*}
\]

(15)

subject to (12), (13) and (14) in order to maximize their discounted sequence of utilities

\[
\sum_{v=0}^{\infty} \theta_j^v u_{jt+v}, \ 0 < \theta_j < 1,
\]

(16)

where \( \theta_j \) represents the subjective discount factor of country \( j \)'s households.

**Government sectors**

In each period \( t \), the government of country \( j \) produces the public good \( G_{jt} \) combining nontradable and tradable goods according to:

\[
G_{jt} = \min(G_{jNt}, \zeta_j G_{jTt}), \quad \zeta_j > 0,
\]

(17)

where \( G_{jNt} \) and \( G_{jTt} \) are, respectively, the quantity of nontradable good and the quantity of tradable good that the government of country \( j \) buys in \( t \) to produce the public good. Since it is assumed that the government produces efficiently, (17) implies that \( G_{jNt} = \zeta_j G_{jTt} \) (the parameter \( \zeta_j \) can be interpreted as a purely technological parameter or as a parameter reflecting the choice that the government of country \( j \) does concerning the characteristics of the public good that it intends to provide).

Hence, in each period \( t \), the government of country \( j \) has to decide the fraction \( g_{jt} \) of the country’s GDP to be spent for the production of the public good:

\[
P_{jNt} G_{jNt} + P_{jTt} G_{jTt} = g_{jt} (P_{jNt} Y_{jNt} + P_{jTt} Y_{jTt}), \quad 0 \leq g_{jt} < 1.
\]

(18)

In each period, the government of country \( j \) must satisfy its period budget constraint:

\[
B_{jGt+1} + E_{jt} F_{jGt+1} + g_{jt} (P_{jNt} Y_{jNt} + P_{jTt} Y_{jTt}) \leq M_{jt} - M_{jt-1} + T_{jt} + (1+i_{jt}) B_{jGt} + E_{jt} (1+i_{jt}) F_{jGt},
\]

\[
B_{jG0}, F_{jG0} \text{ and } M_{j-1} \text{ given, } i \neq j.
\]

(19)

where \( B_{jGt} \) are the domestic financial assets accumulated during period \( t-1 \) by the \( j \)-country’s government sector and carried over into period \( t \) with nominal yield \( i_{jt} \), and \( F_{jGt} \) are the foreign
financial assets (denominated in foreign currency) accumulated during period t-1 by the j-country’s government sector and carried over into period t with nominal yield $i_{jt}$.

The no-Ponzi condition of the j-country’s government sector is

$$M_{jt-1} + \sum_{v=0}^{\infty} (E_{jt+v} - E_{jt+v+1})F_{jt+v+1} + g_{jt+v}(P_{jNt+v}Y_{jNt+v} + P_{jTt+v}Y_{jTt+v}) \leq \prod_{s=0}^{v} (1 + i_{jt+s})$$

$$\leq B_{jGt} + E_{jT}F_{jT} + \sum_{v=0}^{\infty} i_{jt+v}M_{jt+v+1} + T_{jt+v} + (i_{jt+v} - i_{jt+v})E_{jt+v}F_{jT+v} , i \neq j. \quad (20)$$

**Markets equilibrium conditions**

Markets for labor and for the nontradable good are purely domestic. Hence, equilibrium in these markets requires:

$$L_{jt} = L_{jNt} + L_{jTt} \quad (21)$$

and

$$Y_{jNt} = C_{jNt} + G_{jNt} \quad (22)$$

The market for the tradable good is internationally integrated. Equilibrium in this market requires:

$$Y_{usTt} + Y_{chTt} = C_{usTt} + C_{chTt} + G_{usTt} + G_{chTt} + I_{usNt} + I_{usTt} + I_{chNt} + I_{chTt} \quad (23)$$

In this internationally integrated market, the one-price law must hold:

$$P_{jTt} = E_{jt}P_{iTt}, \quad i \neq j, \quad (24)$$

Money market equilibrium in country j requires that in each t money supply is equal to money demand:

$$M_{jt}^s = M_{jt}^d \quad (25)$$

Equilibrium in the world markets for financial assets requires

$$B_{usHt} + B_{usGt} + F_{chHt} + F_{chGt} = 0, \quad (26)$$

and

$$B_{chHt} + B_{chGt} + F_{usHt} + F_{usGt} = 0. \quad (27)$$

**Policy regimes governing the world financial markets**

Two phases in the history of the world economy—corresponding to different policy regimes governing the world financial markets—are considered.
Under both regimes, the US authorities decide on fiscal policy and on monetary policy by setting \( \{E_{ust}\}_{t=0}^{\infty} \) and the fixed rate of growth of money supply \( \bar{\mu}_t \), \( \mu_jt \equiv \frac{M_{jt+1} - M_{jt}}{M_{jt}} \), \( \bar{\mu}_j > \theta_j - 1 \). Similarly, the Chinese authorities set \( \{E_{cht}\}_{t=0}^{\infty} \) and \( \bar{\mu}_c \).

In phase 1, the Chinese capital account is not liberalized: the only international transactions in financial assets that take place are those operated by the Chinese authorities, which decide on \( \{E_{cht}\}_{t=0}^{t^*} \), where \( t^* (t^* > 0) \) is the period in which an irreversible regime switch occurs and phase 2 begins. Consistently, in phase 1, the Chinese authorities let their foreign asset holdings (“foreign reserves”) adjust so as to accommodate the flows of funds generated by this mix of policies. In other words, phase 1 is characterized by (26), (27),

\[
F_{ustHt} = F_{ustGt} = F_{chtHt} = 0, \quad t < t^*, \tag{28}
\]

and

\[
E_{cht} = \bar{E}_{cht} = E_{cht0} \prod_{s=1}^{t-1} (1 + \varepsilon_s^{c}), \quad \prod_{s=1}^{t} (1 + \varepsilon_s^{c}) = 1, \quad t < t^*, \tag{29}
\]

where \( E_{cht0} \) and \( \{\varepsilon_t^{c}\}_{t=0}^{t^*-2} \) are both decided by the Chinese authorities and are, respectively, the level of the nominal exchange rate in period 0 and the time profile of the crawl rate of the exchange rate. Notice that (28)—together with (26) and (27)—entails \( B_{ustHt} + B_{ustGt} + F_{chtHt} = 0 \) and \( B_{chtHt} + B_{chtGt} = 0, \quad t < t^* \): the Chinese accumulation of foreign reserves is the counterpart of the US negative net foreign asset position, and in phase 1 it is assumed that the Chinese net holdings of domestic assets are equal to zero. Hence, in phase 1, China’s foreign reserves evolve according to

\[
F_{chtGt+1} - F_{chtGt} = \varepsilon_{ust}^{c} F_{chtGt} - T_A_{ust}, \quad t < t^*, \tag{30}
\]

where \( T_A_{jt} = P_{jtT}(Y_{jtT} - C_{jtT} - G_{jtT} - I_{jNtT} - I_{jTt}) \) is the trade account of country \( j \) (denominated in \( j \) currency) at time \( t \). By considering (26) and (28), one can see that (30) can be written as

---

\(^{45}\) The condition \( \bar{\mu}_j > \theta_j - 1 \) is necessary for insuring that real money holdings in country \( j \) increase asymptotically at the same rate as \( K_{jTt} \) and \( K_{jNt} \).

\(^{46}\) By the end of 2007 China almost eliminated controls on capital outflows by industrial corporations and financial institutions. These latter, however, did not diversify by investing outside China because of the expectations of revaluation of the renmbinbi. It follows that, by keeping the exchange rate at an undervalued level, the Chinese authorities have de facto preserved a situation of very limited capital flows. Hence, it is reasonable to treat the financial account as closed.

\(^{47}\) Typically, the People’s Bank of China seeks to compensate the accumulation of foreign reserves by selling sterilization bills to domestic agents, so as to keep control over money supply. As a result of this kind of operations, it is normally the case that the government sector reduces its holdings of domestic assets, while private agents increase theirs. However, for our purposes, it is not necessary to model the specific modalities whereby the Chinese central bank controls the supply of money while accumulating foreign reserves. What is essential for us is that an increase in the government sector’s holdings of foreign assets has its counterpart in an improvement of the country’s trade account.
\[ B_{usHt+1} + B_{usGt+1} = (1 + i_{ust})(B_{usHt} + B_{usGt}) + TA_{ust}, \quad t < t^*, \] which is the consolidated (government+private sector) balance sheet of the US economy in phase 1: given the Chinese authorities’ willingness to accumulate foreign reserves, it is immaterial how the US external debt is divided up according to government and private sector net liabilities.

In period \( t^* \), the Chinese authorities liberalize the capital account and let the nominal exchange rate float consistently with the two countries’ policies and market fundamentals. Hence, under this regime, one has the interest-parity condition

\[ (1 + i_{cht}) = \frac{E_{cht}}{E_{cht-1}}, \quad t \geq t^*. \quad (31) \]

Moreover, in phase 2, the Chinese authorities set the maximum amount of US trade deficit—as a fraction \( \xi \) of US GDP—that they are willing to finance in each period by maneuvering their foreign reserves.\(^{48}\) Therefore, China’s net foreign asset position (denominated in US currency) evolves in phase 2 according to

\[ F_{chHt+1} + F_{chGt+1} - E_{ust}(F_{usHt+1} + F_{usGt+1}) - \left[ F_{chHt} + F_{chGt} - E_{ust-1}(F_{usHt} + F_{usGt}) \right] = i_{ust}\left[ F_{chHt} + F_{chGt} - E_{ust-1}(F_{usHt} + F_{usGt}) \right] - TA_{ust}, \quad t \geq t^*, \quad (32) \]

where \( TA_{ust} \geq -\xi(P_{usNt}Y_{usNt} + P_{usTt}Y_{usTt}), \quad \xi \geq 0, \quad t \geq t^*. \)

Summarizing, phase 2 is characterized by (26), (27), (31) and (32), and it is worth to emphasize that also in this phase the possibility for the US to run a persistent external deficit rests ultimately on the Chinese authorities’ willingness to finance it.

Finally, it should be stressed that it is up to the Chinese authorities to decide when the regime switch has to take place: \( t^* \) is decided by the Chinese government and known to everybody. It is even possible that the regime switch is postponed forever (\( t^* \to \infty \)): in this case the Chinese authorities never liberalize the capital account and never let the nominal exchange rate float.

### 4. Characterization of an equilibrium path

Using the market equilibrium conditions and solving the agents’ optimization problems (see the Appendix), we obtain the system of equations governing the equilibrium path of the economy:

\[ Z_t \left[ \frac{G_{atTt}}{K_{atTt}} - I_{aTt} + \left( 1 + \frac{K_{mNt} + \zeta}{K_{atTt+1}} \right) (1 + \rho_{at}) + \frac{C_{atTt}}{K_{atTt}} - (1 - \delta_{at}) \left( 1 + \frac{K_{mNt}}{K_{atTt}} \right) \right] + G_{chTt} - I_{aTt} = \]

\(^{48}\) There are alternative ways for setting the limit to the size of the US external deficit that the Chinese authorities are willing to finance (for instance, by setting a limit to the US current account deficit as a fraction of China’s GDP). However, in a two-country setup it is not relevant how this external constraint imposed on the US is formulated: for simplicity and analytical convenience we opt for the formulation contained in the text.
\[ + \left(1 + \frac{K_{\text{chN}+1}}{K_{\text{chT}+1}}\right) \left(1 + \rho_{\text{ch}}\right) + \frac{C_{\text{chN}}}{K_{\text{chT}}} - (1 - \delta_{\text{ch}}) \left(1 + \frac{K_{\text{chN}}}{K_{\text{chT}}}\right) = 0, \]
\[ Z_i = \frac{K_{\text{chN},T}}{K_{\text{chT}}}, \quad \rho_{ji} = \frac{K_{ji+1}}{K_{ji}}, \] (33)

\[ \frac{C_{ji}}{K_{ji}} = C \left(\frac{K_{jN_i}}{K_{ji}}, L_{jN_i}, L_{ji}, g_{ji}\right) = \frac{(1 - \eta_j) \alpha_j L_{jN_i}}{\eta_j \gamma_j L_{ji}^{\frac{1}{\gamma_j}}} \left(1 - \frac{\frac{K_{ji}}{K_{ji}} L_{jN_i}^{\frac{1}{\gamma_j}}}{\frac{K_{ji}}{K_{ji}} L_{jN_i}^{\frac{1}{\gamma_j}}} \frac{\alpha_j L_{jN_i}}{\gamma_j L_{ji}^{\frac{1}{\gamma_j}}} + 1\right), \] (34)

\[ \frac{G_{ji}}{K_{ji}} = G \left(\frac{K_{jN_i}}{K_{ji}}, L_{jN_i}, L_{ji}, g_{ji}\right) = \frac{\alpha_j L_{jN_i} g_{ji}}{\gamma_j L_{ji}^{\frac{1}{\gamma_j}}} \left(\frac{\alpha_j L_{jN_i}}{\gamma_j L_{ji}^{\frac{1}{\gamma_j}}} + 1\right), \] (35)

\[ Z_{i+1} = \left(1 + \frac{\rho_{\text{ust}}}{1 + \rho_{\text{ch}}\right)} Z_i, \] (36)

\[ \theta_j [(1 - \alpha_j) L_{jN_i+1}^{\frac{1}{\gamma_j}} + 1 - \delta_j] C \left(\frac{K_{jN_i}}{K_{ji}}, L_{jN_i}, L_{ji}, g_{ji}\right) \]
\[ \rho_{ji} = -1 - \frac{C \left(\frac{K_{jN_i+1}}{K_{ji+1}}, L_{jN_i+1}, L_{ji+1}, g_{ji+1}\right)}{\frac{K_{jN_i}}{K_{ji}}} \] (37)

\[ K_{ji} = \begin{cases} K(L_{jN_i}, L_{ji}) = \frac{\alpha_j (1 - \gamma_j) L_{jN_i}}{\gamma_j (1 - \alpha_j) L_{ji}} & \text{if } t > 0 \\ \frac{K_{jN_0}}{K_{ji}} & \text{otherwise,} \end{cases} \] (38)

\[ L_{jN_i} = \begin{cases} H_j - L_{ji} & \text{if } \frac{\alpha_j D_{ji}}{L_{ji}^{\frac{1}{\gamma_j}}} \left[\frac{\gamma_j K_{ji} L_{ji}^{\frac{1}{\gamma_j}}}{\alpha_j K_{ji} (H_j - L_{ji})^{\frac{1}{\gamma_j}}}\right]^{\eta_j} > N_j \\ L \left(\frac{K_{jN_i}}{K_{ji}}, N_j, L_{ji}\right) = \left[\frac{\gamma_j K_{ji}^{\frac{1}{\gamma_j}}}{\alpha_j K_{ji}}\right]^{\eta_j} \left[\frac{\alpha_j D_{ji}}{N_j L_{ji}^{\frac{1}{\gamma_j}}}\right] \frac{1}{(1 - \gamma_j) N_j} & \text{otherwise,} \end{cases} \] (39)

\[ N_{j+1} = \begin{cases} V_{ji+1} = \left(\frac{1 + \omega_j}{1 + \rho_{ji}}\right) N_j & \text{if } L_{jN_i} + L_{ji} < H_j \\ \left(\frac{1}{1 + \rho_{ji}}\right) N_j & \text{otherwise,} \end{cases} \] (40)

\[ E_{ji} = \frac{(1 + \bar{\rho}_j - \theta_j)(1 - \eta_j) \chi_i(1 + \bar{\rho}_i)^4 M_{ji} C_{ji}}{(1 + \bar{\rho}_i - \theta_i)(1 - \eta_i) \chi_j(1 + \bar{\rho}_j)^4 M_{ji} C_{ji}}, i \neq j, \] (41)
It is easy to verify that equation (33) is derived from the equilibrium condition of the world market for the tradable good (23) by using (5) and (7). Equations (34) and (35) give us the amounts of tradables that are purchased in equilibrium, respectively, by the households and by the government of country \( j \). Equation (36) governs the equilibrium trajectory of the ratio between the capital installed in the US tradable sector and that installed in the Chinese tradable sector (consistently with the stylized facts, it is reasonable to assume that at time 0 this ratio is relatively large, surely larger than one). Equation (37) shows—together with (38)—that the rate of growth of the capital installed in the tradable sector of country \( j \) depends in any \( t>0 \) on the quantities of labor that \( j \) devotes to the production of tradables and nontradables both in \( t \) and in \( t+1 \). In (38), one can see the relationship linking, in each country \( j \), the evolution of the capital installed in the nontradable sector to that of the capital installed in the tradable sector. Notice that the rate of growth of the capital installed in the nontradable sector of country \( j \) in any \( t>0 \) can be easily derived from (37) and (38): 

\[
\frac{K_{jNt+1}}{K_{jNt}} = (1 + \rho_j \frac{L_{jNt+1} \gamma_j}{L_{jNt} \gamma_{jNt+1}}) \left(1 - \frac{L_{jNt} \nu_j}{L_{jNt+1} \nu_{jNt+1}}\right). 
\]

In (39), one can check that the possibility for country \( j \) to employ all its labor in the two market sectors of the economy depends crucially on its endowments of capital in both sectors relatively to its reservation wage (again, consistently with the stylized facts, it is reasonable to assume that at time 0 China employs some of its labor in the traditional sector of the economy, while in no period this is the case for the US). The law of motion of the ratio in country \( j \) between the reservation wage and the capital installed in the tradable sector is given by (40). Equation (41) is derived from the one-price law (24) and gives the equilibrium level of the nominal interest rate.
exchange rate of country j. In equation (42), one has the equilibrium level of the nominal interest rate in country j, which is constant since the rate of money growth is fixed in both countries. Finally, equation (43) gives the equilibrium level of the trade account of country j.

By using (29), equation (41) can be rewritten in phase 1 as

$$
\frac{(1 + \bar{\rho}_m - \theta_m)(1 - \eta_m) \chi_m (1 + \bar{\rho}_m)^{\frac{1}{2}} M_{us,1}}{(1 + \bar{\rho}_ch - \theta_ch)(1 - \eta_ch) \chi_ch (1 + \bar{\rho}_ch)^{\frac{1}{2}} M_{ch,1}} \frac{C_{us,T}}{C_{ch,T}}, \quad 0 \leq t < t^*.
$$

(44)

One can easily see from (44) that in phase 1—by keeping their currency undervalued with respect to the US currency—the Chinese authorities compress the Chinese consumption of tradables relatively to that of the US (this compression of the Chinese consumption of tradables is consistent with the stylized facts documented in section 2). By using (34), (38) and (39) for substituting $C_{jTt}$, $K_{jNt}$ and $L_{jNt}$, one can also verify that in phase 1 equation (44) defines implicitly the level of employment in the US tradable sector as a function of $L_{chTt}$, $L_{chNt}$, $Z_t$, $\varepsilon_{ch}$, $\varepsilon_{ust}$, $\bar{E}_{cht}$, $\bar{\rho}_us$ and $\bar{\rho}_{ch}$.

$$
L_{usTt} = e(L_{chTt}, L_{chNt}, Z_t, \varepsilon_{ch}, \varepsilon_{ust}, \bar{E}_{cht}, \bar{\rho}_us, \bar{\rho}_{ch}), \quad 0 < t < t^*.
$$

(45)

Given the time profile of the nominal exchange rate set by the Chinese authorities, in phase 1 monetary policies in US and in China can affect the dynamics of the real variables. This is not the case in phase 2: monetary policies have no effect on real variables. In phase 2, indeed, the relation between $L_{usTt}$ and $L_{chTt}$ is given by (see the Appendix)

$$
L_{usTt} = l(L_{chTt}) = \left[ \frac{(1 - \alpha_ch) L_{chTt}^{\alpha_ch} + \delta_{us} - \delta_{ch}}{(1 - \alpha_us)} \right]^{\frac{1}{\alpha_us}}, \quad t \geq t^*.
$$

(46)

5. Growth dynamics under different policy regimes

We examine the growth dynamics of the world economy under the hypothesis that at time 0 China—differently than the US—employs some of its labor in the low-productive sector of the economy. This amounts to assume that the initial endowments $K_{chT0}$ and $K_{chN0}$ are relatively low with respect to $V_{ch0}$, while the US has larger initial stocks of capital per household with respect to $V_{us0}$ (see (37)).

We consider three possible scenarios for the world economy depending on the policies pursued by the Chinese authorities:

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51 At time 0, the level of employment in the US tradable sector depends also on the initial endowments of capital $K_{chT0}$, $K_{usT0}$, $K_{chN0}$ and $K_{usN0}$. 
(A) The Chinese authorities adopt a combination of fiscal policy and exchange-rate pegging which allows China to absorb all its manpower in the market sectors of the economy. As soon as this objective is reached, they fully liberalize the capital account and let the exchange rate float.

(B) The Chinese authorities fully liberalize the capital account and let the exchange rate float even if China has not yet succeeded in absorbing all its manpower in the market sectors of the economy.

(C) The Chinese authorities never liberalize the capital account and never let the exchange rate float \((t^* \to \infty)\). As a result, in period \(t^* > 0\) China succeeds in absorbing all its manpower in the market sectors of the economy.

5.1 Scenario A

In this case, the equilibrium trajectory of the real variables of the world economy is governed for \(t \geq t^*\) by two difference equations in \(L_{chTt}\) and \(Z_t\) (see the Appendix):

\[
\Psi(L_{chTt+1}, L_{chTt}, Z_t, \overline{g}_{us}, \overline{g}_{ch}) = Z_t b(L_{chTt+1}, L_{chTt}, \overline{g}_{us}) + y(L_{chTt+1}, L_{chTt}, \overline{g}_{ch}) = 0, \quad t \geq t^*,
\]

\[
\Lambda(L_{chTt+1}, Z_{t+1}, L_{chTt}, Z_t, \overline{g}_{us}, \overline{g}_{ch}) = 0, \quad t \geq t^*,
\]

where (see the Appendix)

\[
b(L_{chTt+1}, L_{chTt}, \overline{g}_{us}) \leq \xi[\xi[(L_{chTt})}]^n_{us} \left[\alpha_{us} [H_{us}-l(L_{chTt})]+1\right], \quad t \geq t^*,
\]

Equations (47) and (48) are obtained, respectively, from (33) and (36) by using (34), (35), (37), (38), (39) (with \(L_{usNt} = H_{us} - L_{usTt}\) and \(L_{chNt} = H_{ch} - L_{chTt}\)) and (46), where for simplicity and without loss of generality it is assumed that \(g_{ust} = \overline{g}_{us} \) and \(g_{cht} = \overline{g}_{ch} \forall t \geq t^*\). The inequality (49) reflects the limit imposed on US policies by the Chinese willingness to finance the US external deficit, where

\[
b(L_{chTt+1}, L_{chTt}, \overline{g}_{us}) = \frac{TA_{us}}{P_{usTt} K_{usTt}} \text{ and } \xi[(L_{chTt})]^n_{us} \left[\alpha_{us} [H_{us}-l(L_{chTt})]+1\right] = \frac{\xi[P_{usNt} Y_{usNt} + P_{usTt} Y_{usTt}]}{P_{usTt} K_{usTt}}.
\]

Some propositions concerning long-run growth hold in Scenario A.

**Proposition 1** The asymptotic rate of real GDP growth of country \(j\) increases with \(L_{jT}\), where \(L_{jT} = \lim_{t\to\infty} L_{jTt}\) is the asymptotic equilibrium level of employment in the tradable sector of country \(j\).

**Proof:** If \(L_{jTt} \to L_{jT}\) as \(t \to \infty\), then the country \(j\)’s rate of real GDP growth approaches

\[
\rho_j = \theta_j [(1 - \alpha_j)L_{jT}^\theta_j + 1 - \delta_j], \quad \rho_j = \lim_{t \to \infty} \rho_{jT} (\text{see the Appendix}), \text{ thus entailing } \frac{\partial \rho_j}{\partial L_{jT}} > 0.
\]

\(^{52}\) As shown in the Appendix, \(L_{jTt} \to L_{jT}\) as \(t \to \infty\) implies that \(\lim_{t \to \infty} \rho_{jGDPT} = \lim_{t \to \infty} \rho_{jT} \).

\(^{52}\) As shown in the Appendix, \(L_{jTt} \to L_{jT}\) as \(t \to \infty\) implies that \(\lim_{t \to \infty} \rho_{jGDPT} = \lim_{t \to \infty} \rho_{jT} \).
Proposition 1 is a consequence of the fact that the long-run rate of real GDP growth is a function of the marginal productivity of capital in the production of tradables, since the production process of all market sectors of the economy requires capital goods that are typically tradables (e.g. equipment and machinery), and technological progress is driven by the accumulation and installment of capital.

**Proposition 2** Asymptotically, the real GDP of the country whose households are less impatient, say China \(\theta_{ch} > \theta_{us}\), grows at a higher rate. Moreover, the asymptotic rate of real growth of both countries depends on the fiscal policy of the country that tends to grow faster in the long run (say China), while it is independent of the fiscal policy of the country growing slower in the long run (say the US).

**Proof:** Since (46) must hold \(\forall t \geq t^*\), it is trivial to see that

\[
(1 - \alpha_{us})L_{usT}^{\alpha_{us}} + 1 - \delta_{us} = (1 - \alpha_{ch})L_{chT}^{\alpha_{ch}} + 1 - \delta_{ch},
\]

thus implying that \(\rho_{ch} > \rho_{us}\) since \(\theta_{ch} > \theta_{us}\). Moreover, if \(\theta_{ch} > \theta_{us}\), then \(L_{chT}\) is a function of \(\alpha_{ch}, \gamma_{ch}, \eta_{ch}, \theta_{ch}, \delta_{ch}, H_{ch}, \zeta_{ch}\) and \(\overline{\rho}_{ch}\) (see the Appendix), thus making \(\rho_{ch} = \theta_{ch}[(1 - \alpha_{ch})L_{chT}^{\alpha_{ch}} + 1 - \delta_{ch}]\) dependent on \(\overline{\rho}_{ch}\) but independent of the structural and policy parameters of the US. Finally, given that \((1 - \alpha_{us})L_{usT}^{\alpha_{us}} + 1 - \delta_{us} = (1 - \alpha_{ch})L_{chT}^{\alpha_{ch}} + 1 - \delta_{ch}\), one has that \(L_{usT}\) is a function of \(\alpha_{us}, \delta_{us}, \alpha_{ch}, \gamma_{ch}, \eta_{ch}, \theta_{ch}, \delta_{ch}, H_{ch}, \zeta_{ch}\) and \(\overline{\rho}_{ch}\), thus making \(\rho_{us} = \theta_{us}[(1 - \alpha_{us})L_{usT}^{\alpha_{us}} + 1 - \delta_{us}]\) dependent on \(\overline{\rho}_{us}\) but independent of \(\overline{\rho}_{us}\).

Proposition 2 implies that if we treat the US as the relatively impatient country \(\theta_{us} < \theta_{ch}\), consistently with the evidence in favor of a lower propensity to save for US households relative to their European and Asian counterparts (see Ghironi et al., 2008), we should expect higher long-run real growth in China than in the US and \(Z=0\), where \(Z = \lim_{t \to \infty} Z_t\). Moreover, if \(\theta_{us} < \theta_{ch}\), we should expect that in the long run the performance of the Chinese economy will not be affected by the performance of the US economy, while the latter will be affected by the Chinese structural and policy parameters, since the size of the US economy will become negligible as \(t \to \infty\) relative to the size of the Chinese economy.

**Proposition 3** The asymptotic rate of real growth increases in both countries with the fraction of GDP devoted to the provision of the public good by the country whose households are less impatient if the public good is produced in this country (say China) by using a relatively small proportion of nontradable good, i.e., if its \(\zeta_{ch}\) is below a critical threshold \(\overline{\zeta}_{ch}\) depending on \(\alpha_{ch}, \gamma_{ch}, \eta_{ch}, \theta_{ch}, \delta_{ch}\) and \(H_{ch}\). The opposite is true if \(\zeta_{ch}\) is relatively large: if \(\zeta_{ch} > \overline{\zeta}_{ch}\), a larger fraction of GDP spent for producing the public good in China (that is a larger \(\overline{\rho}_{ch}\)) depresses long-run real
growth in both countries. If $\zeta_{ch}$ is close to $\overline{\tau}_{ch}$, a change in $\overline{g}_{ch}$ has little effect on long-run real growth (in the special case in which $\zeta_{ch} = \overline{\tau}_{ch}$, a change in $\overline{g}_{ch}$ does not affect long-run real growth in any country).

Proof: See the Appendix.

Long-run real growth in the two countries is sensitive to both the fraction of GDP devoted to public expenditures and the composition of public expenditures (the mix of tradables and nontradables purchased by the government) in the country whose households are less impatient (say China). In particular, if $\theta_{us} < \theta_{ch}$, we should expect that both $\rho_{us}$ and $\rho_{ch}$ are boosted by a larger $\overline{g}_{ch}$ if and only if the Chinese government dedicates a relatively large fraction of its expenditure to the purchase of tradables. This result reflects the fact that fiscal policy can affect the composition of aggregate demand and shift domestic production towards the sector producing tradables, thus feeding long-run growth (see Proposition 1). Finally, notice that—as $\zeta_{ch}$ is very close to the threshold $\overline{\tau}_{ch}$—changes in China’s fiscal policy have very little effect on long-run real growth.

**Proposition 4** If $\theta_{ch} > \theta_{us}$, one has $\lim_{t \to \infty} ta_{cht} = ta_{ch} = 0$, where $ta_{ji} \equiv \frac{TA_{ji}}{(P_{ji} Y_{jtN} + P_{jt} Y_{jtT})}$; asymptotically, the trade account becomes a negligible component of the GDP of China, namely of the country that exhibits higher real growth in the long run.

Proof: By evaluating equation (47) as $t \to \infty$, one can easily verify that $\lim_{t \to \infty} Z_t = 0$ entails $\lim_{t \to \infty} ta_{cht} = 0$.

The intuition behind this result should be straightforward: as $t \to \infty$, the size of the economy growing slower in the long run—which is (in the model!) the only trading partner of China—becomes negligible relative to the size of the country growing faster. Hence, any trade between them tends to become insignificant relative to China’s GDP. This is not necessarily the case for the US, i.e., the country growing slower in the long run: $ta_{us} = \lim_{t \to \infty} ta_{ust} \geq 0$.

It is worth to emphasize that the model leaves open the possibility that China’s authorities are willing to finance a permanent US current account deficit ($\xi > 0$), thus accumulating foreign reserves forever. In this way, our model captures an important feature of the world economy under analysis, namely that if the authorities of one country are willing to let their country increase its net foreign asset position forever, the other country can face a softened intertemporal budget constraint. In other words, it is possible that the world economy moves along an equilibrium path such that $nfa_{ust} \to -\infty$ as $t \to \infty$, where $nfa_{ji} = \frac{E_{ji} (F_{ijH} + F_{ijG}) - (F_{jiH} + F_{ijG})}{P_{ji} Y_{jtN} + P_{jt} Y_{jtT}}$, $i \neq j$, is the country j’s ratio.
between its net foreign asset position (denominated in domestic currency and evaluated at the beginning of $t$) and its nominal GDP.\footnote{Country j’s net foreign asset position-GDP ratio evolves according to
\[
\frac{\text{nfa}_{jt}}{P_{jt} Y_{jt} + P_{jt} Y_{jt+1}} = (1 + i_{jt}) \text{nfa}_{jt} + \text{ta}_{jt}, \quad t \geq t^*,
\]
where
\[
\lim_{t \to \infty} \frac{P_{jt+1} Y_{jt+1} + P_{jt+1} Y_{jt+1}}{P_{jt} Y_{jt} + P_{jt} Y_{jt+1}} = 1 + \bar{P}_{jt} < 1 + i_{jt} = \frac{1 + \bar{P}_{jt}}{\theta_j};
\]
since the long-run rate of growth of country j’s nominal GDP is lower than its nominal rate of interest, country j’s external debt (or, possibly, its positive net foreign asset position) tends in the long run to increase faster that its nominal GDP even if \(\lim_{t \to \infty} \text{ta}_{jt} = 0\).}

Even if the net flow of resources that would be necessary to finance the US current account deficit approaches asymptotically zero as a proportion of China’s GDP, one may wonder why the Chinese authorities should possibly let their country’s net foreign asset position increase forever, thus allowing the US to undertake policies conducive to a permanent external deficit. As a matter of fact, indeed, the asymptotic performance of the Chinese economy will not be affected by the policies undertaken by the US authorities (recall our comment to Proposition 2). However, a possible rationale for a Chinese benign attitude towards a persistent US external deficit can be found in the performance of the Chinese economy along the transitional path, which is affected by the US policies.

For studying the transitional path along which the world economy moves from period $t^*$ onwards in Scenario A, we linearize the system (47)-(48) around $(L_{chT}, Z=0)$ under the assumption that $\theta_{ch} > \theta_{us}$. The linearized system thus obtained has only one path converging to $(L_{chT}, Z=0)$, which is governed by

\[
L_{chT} = L_{chT} - L_{chT} = \frac{Z_1 \Psi_{Z_1}}{\Psi_{L_{chT+1}} + \Lambda_{Z_1}}, \quad t \geq t^*,
\]

\[
Z_t = Z_t \left( \frac{\Lambda_{Z_{t+1}}}{\Lambda_{Z_{t+1}}} \right)^{t-t^*}, \quad t \geq t^*,
\]

where all the partial derivatives $\Psi_{Z_1}, \Psi_{L_{chT}}, \Lambda_{Z_1}$ and $\Lambda_{Z_{t+1}}$ are evaluated at $(L_{chT}, Z=0)$ and are such that $0 < \frac{\Lambda_{Z_{t+1}}}{\Lambda_{Z_{t+1}}} < 1, \frac{\Psi_{L_{chT}}}{\Psi_{L_{chT+1}}} > 1, \text{ and } \frac{\Psi_{Z_1}}{\Psi_{Z_{t+1}}} \begin{cases} < & \text{whenever } \text{ta}_{us} \leq 0 \end{cases}$ (see the Appendix). Considering (50), this implies that—along the transitional path—$L_{chT} > L_{chT}$ if and only if $\text{ta}_{us} < 0$. In particular, if $\text{ta}_{us} < 0$, then $L_{chT} > L_{chT}$, thus favoring the accumulation of capital and the
employment of the entire Chinese manpower in the high-productive sectors of the economy. In this situation, the Chinese authorities should be interested in convincing market participants of their willingness to finance the US external deficit also in the future. Notice that this implicit commitment can be considered credible, in the light of the fact that if at some $t^*>t^*$ the Chinese authorities abruptly ceased to accumulate US assets, thus forcing the US authorities to implement an unanticipated permanent change in fiscal policy aimed at reducing absorption, there would be a remarkable fall in the labor employed in the Chinese tradable sector. As a likely consequence of this fall, a fraction of the Chinese workforce would be again expelled from the high-productive sectors of the economy. One could argue that under these circumstances the Chinese authorities might well change the fraction of GDP devoted to public expenditure so as to compensate the permanent decrease in foreign demand for tradables. However, as one can conclude from our comment to Proposition 3, fiscal policy could be very ineffective in doing so.

In phase 1, the Chinese nominal exchange rate is kept undervalued so as to maintain the Chinese tradables relative cheap with respect to the US tradables. As $Z_t$ decreases, that is as China reduces its gap relatively to the US in terms of capital per household in the tradable sector, the Chinese authorities let their currency gradually appreciate, but preserving the price competitiveness of the Chinese tradables relatively to the US tradables. This policy amounts to set

$$E_{t^0} = \frac{Q(1 + \mu_{ch} - \theta_{ch})(1 - \eta_{ch})\chi_{as}M_{ch}^{-1}Z_0}{(1 + \mu_{as} - \theta_{as})(1 - \eta_{as})\chi_{as}M_{as}^{-1}}, \quad Q>0$$

and

$$\epsilon_t = \frac{(1 + \mu_{ch})(1 + \rho_{ast})}{(1 + \rho_{as})(1 + \rho_{ch})}, \quad 0\leq t < t^*,$$

where $Q$ is a constant whose value is decided by the Chinese authorities (it measures the degree of “aggressiveness” of the mercantilist strategy adopted by the Chinese authorities in phase 1: a larger $Q$ means that—other things being equal—the Chinese currency is maintained more undervalued with respect to the US currency). Given (36) and (44), the policy rule (52)-(53) allows us to write (45) as

54 With regard to period $t^*-1$, one can check by considering (34) and (37)-(40) that $\frac{\partial \rho_{cht^*}}{\partial \tau_{cht^*}} > 0$ and $\frac{\partial N_{cht^*}}{\partial \tau_{cht^*}} < 0$.

55 This can be seen by considering that, if the world economy were moving along an equilibrium path such that $\lim_{t \to \infty} \tau_{ast} = \tau_{ast} < 0$ and at $t^* > t^*$ the US authorities implemented an unanticipated fiscal adjustment consistent with $\lim_{t \to \infty} \tau_{ast} = \tau_{ast} = 0$, the Chinese workforce employed in the tradable sector—which was $L_{cht^*} > L_{cht}$ (see equation (50))—would fall immediately at $L_{cht^*} = L_{cht}$.
\[ L_{\text{us}Tt} = f(L_{\text{ch}Tt}, L_{\text{chNt}}, \hat{g}_{\text{ch}}, \hat{g}_{\text{us}}, Q), \quad f_Q < 0, \quad 0 < t < t^*, \]

where also in phase 1 we assume for simplicity and without loss of generality that fiscal policies do not change \((\hat{g}_{\text{cht}} = \hat{g}_{\text{ch}} \quad \text{and} \quad \hat{g}_{\text{ust}} = \hat{g}_{\text{us}} \quad \forall t < t^*).\)

The equilibrium trajectory of the real variables is governed in phase 1 by three difference equations in \(L_{\text{ch}Tt}, N_{\text{cht}} \quad \text{and} \quad Z_t\) (see the Appendix):

\[
\begin{align*}
\Omega(L_{\text{ch}Tt+1}, N_{\text{cht+1}}, L_{\text{ch}Tt}, N_{\text{cht}}, Z_t, \hat{\rho}_{\text{us}t}, \hat{g}_{\text{ch}}) &= Z_t \Omega(L_{\text{ch}Tt+1}, N_{\text{cht+1}}, L_{\text{ch}Tt}, N_{\text{cht}}, \hat{\rho}_{\text{us}t}, \hat{g}_{\text{ch}}) + \\
&+ \hat{\theta}(L_{\text{ch}Tt+1}, N_{\text{cht+1}}, L_{\text{ch}Tt}, N_{\text{cht}}, \hat{g}_{\text{ch}}) = 0, \quad 0 < t < t^*, \\
\Phi(L_{\text{ch}Tt+1}, Z_{t+1}, N_{\text{cht+1}}, L_{\text{ch}Tt}, Z_t, N_{\text{cht}}, \hat{\rho}_{\text{us}t}, \hat{g}_{\text{ch}}) &= 0, \quad 0 < t < t^*, \\
\Theta(L_{\text{ch}Tt+1}, N_{\text{cht+1}}, L_{\text{ch}Tt}, N_{\text{cht}}, \hat{g}_{\text{ch}}) &= 0, \quad 0 < t < t^*.
\end{align*}
\]

Equations (55) and (56) are derived, respectively, from (33) and (36) by using (34), (35), (37), (38), (39) (with \(L_{\text{us}Nt} = H_{\text{us}} - L_{\text{us}Tt}\) and \(L_{\text{ch}Nt} = L\left(\frac{K_{\text{chNt}}}{K_{\text{ch}Tt}}, N_{\text{cht}}, L_{\text{ch}Tt}\right) < H_{\text{ch}} - L_{\text{ch}Tt}\)) and (54). Equation (57) is derived from (40) by using (34), (35), (37), (38) and (39) (again, with \(L_{\text{ch}Nt} = L\left(\frac{K_{\text{chNt}}}{K_{\text{ch}Tt}}, N_{\text{cht}}, L_{\text{ch}Tt}\right) < H_{\text{ch}} - L_{\text{ch}Tt}\)).

From (55)-(57) we have that \(Q\) (the degree of aggressiveness of the mercantilist policy undertaken by the Chinese authorities) affects the dynamics of the real variables. Moreover, equations (42) and (52)-(53) show that having decided on \(Q\) the Chinese authorities can still choose their preferred combination of (equilibrium) level of the nominal interest rate and level (and time profile) of the nominal exchange rate: given \(\bar{\rho}_{\text{us}}\) (the US rate of nominal money growth), there is a continuum of combinations of \(\bar{\rho}_{\text{ch}}\) and \(\bar{E}_{\text{cht}}\) that are consistent with a given \(Q\). Similarly, if the US authorities implement a more (less) inflationary monetary policy by setting a higher (lower) \(\bar{\rho}_{\text{us}}\), the Chinese authorities may keep the dynamics of the real variables and their nominal interest rate unchanged by fixing their nominal exchange rate at a lower (higher) level and letting it appreciate at a higher (lower) rate.

The fact that in phase 1 the dynamics of the world economy depends also on \(N_{\text{ch}t}\) reflects the presence in China during this phase of some labor which is not employed in the advanced sectors of the economy. In Scenario A, however, the combination of exchange-rate pegging and fiscal policy

\[56\text{ At time 0, the level of employment in the US tradable sector depends also on the initial endowments of capital } K_{\text{ch}T0}, K_{\text{us}T0}, K_{\text{ch}N0}, \text{ and } K_{\text{us}N0}.\]

\[57\text{ At time 0, the dynamics of the economy depends also on the initial endowments of capital } K_{\text{ch}T0}, K_{\text{us}T0}, K_{\text{ch}N0}, \text{ and } K_{\text{us}N0} \text{ (see the Appendix).}\]
adopted by the Chinese authorities manages to raise $L_{chT_t}$ so as to reach in period $t^*$ the objective to employ the entire Chinese manpower in the two advanced sectors of the economy. Entering phase 2 and abandoning the exchange-rate pegging, it is crucial for the Chinese authorities that fiscal policy be an effective instrument for sustaining growth and maintaining $L_{chN_t} + L_{chT_t} = H_{ch}$.

5.2 Scenario B

In this case, the equilibrium trajectory of the real variables of the world economy is governed for $t \geq t^*$ by three difference equations in $L_{chT_t}$, $N_{cht}$ and $Z_t$ (see the Appendix):

$$\Xi(L_{chT_{t+1}}, N_{cht+1}, L_{chT_t}, N_{cht}, Z_t, \bar{\varepsilon}_{us}, \bar{\varepsilon}_{ch}) = Z_t b(L_{chT_{t+1}}, L_{chT_t}, \bar{\varepsilon}_{us}) + m(L_{chT_{t+1}}, N_{cht+1}, L_{chT_t}, N_{cht}, \bar{\varepsilon}_{ch}) = 0, \ t \geq t^*, \quad (58)$$

$$\Gamma(L_{chT_{t+1}}, N_{cht+1}, L_{chT_t}, N_{cht}, \bar{\varepsilon}_{ch}) = 0, \ t \geq t^*, \quad (59)$$

$$\Sigma(L_{chT_{t+1}}, Z_{t+1}, N_{cht+1}, L_{chT_t}, Z_t, N_{cht}, \bar{\varepsilon}_{us}, \bar{\varepsilon}_{ch}) = 0, \ t \geq t^*, \quad (60)$$

where $b(L_{chT_{t+1}}, L_{chT_t}, \bar{\varepsilon}_{us})$ must satisfy (49).

Equations (58) and (60) are obtained, respectively, from (33) and (36) by using (34), (35), (37), (38), (39) (with $L_{usN_t} = H_{us} - L_{usT_t}$ and $L_{chN_t} = L\left(\frac{K_{chN_t}}{K_{chT_t}}, \frac{N_{cht}}{L_{chT_t}}\right) < H_{ch} - L_{chT_t}$) and (46). Equation (59) is derived from (40) by using (34), (35), (37), (38) and (39) (again, with $L_{chN_t} = L\left(\frac{K_{chN_t}}{K_{chT_t}}, \frac{N_{cht}}{L_{chT_t}}\right) < H_{ch} - L_{chT_t}$).

If we keep treating the US as the relatively impatient country ($\theta_{us} < \theta_{ch}$), the following proposition concerning long-run growth holds in Scenario B:

**Proposition 5** If $\theta_{ch} > \theta_{us}$, the asymptotic growth rate of China’s real GDP is equal to $\omega_{ch}$, i.e., the rate at which labor productivity increases in the Chinese traditional sector. Moreover, the US real GDP grows asymptotically at a rate lower than $\omega_{ch}$.

**Proof:** If $L_{jT_t} \to L_{jT}$ and $N_{cht} \to N_{ch}$ as $t \to \infty$, then the country j’s rate of real GDP growth approaches

$$\rho_j = \theta_j [(1 - \alpha_j) L_{jT}^{\gamma_j} + 1 - \delta_j],$$

where $N_j = \lim_{t \to \infty} N_{jt}$ (see the Appendix). Moreover, by inspecting (40) one can easily verify that $N_{cht} \to N_{ch}$ as $t \to \infty$ entails $\rho_{ch} = \omega_{ch}$ . Finally, from (46) one has that

$$\rho_{us} = \frac{\rho_{ch}}{\theta_{us}} = \frac{\omega_{ch}}{\theta_{ch}},$$

thus entailing $\rho_{us} < \rho_{ch} = \omega_{ch}$ and $Z_t \to 0$ as $t \to \infty$.

According to Proposition 5, in this scenario in which a portion of China’s manpower is entrapped in the low-productive sector of the economy, the long-run rate of growth of both
countries depends on the pace at which productivity increases in the Chinese low-productive sector. This is a consequence of the fact that in Scenario B the long-run evolution of the real wages in the advanced sectors of China—the country that tends to grow faster—reflects the dynamics of labor productivity in the low-productive sector of the economy.

To verify whether this scenario can be plausible, we study the transitional path along which the world economy moves from period $t^*$ onwards in Scenario B. By linearizing the system (58)-(60) around $(L_{chT}, N_{ch}, Z=0)$ under the assumption that $\theta_{ch} > \theta_{us}$, one can verify for reasonable sets of structural and policy parameter values that $(L_{chT}, N_{ch}, Z=0)$ is unstable (see the Appendix). Only in the case in which the policy makers manage to control the economy so as to enter phase 2 with $N_{cht} - N_{ch} = q_{23}Z_{t^*}$, the system can converge asymptotically to $(L_{chT}, N_{ch}, Z=0)$ along the path governed by

$$
\bar{L}_{chT} = q_{13}Z_{t^*}q_3^{l-t^*}, \quad t \geq t^*, \quad (61)
$$

$$
\bar{N}_{cht} = N_{cht} - N_{ch} = q_{23}Z_{t^*}q_3^{l-t^*}, \quad t \geq t^*, \quad (62)
$$

$$
Z_t = Z_{t^*}q_3^{l-t^*}, \quad t \geq t^*, \quad (63)
$$

where $q_{13}$ and $q_{23}$ are eigenvectors—and $\varphi_3$ ($0 < \varphi_3 < 1$) is the only stable eigenvalue—of the linearized system (see the Appendix). However, even in this case, a perturbation may easily lead the economy to diverge from $(L_{chT}, N_{ch}, Z=0)$, if it causes $N_{cht} - N_{ch} \neq q_{23}Z_t$ at some $t > t^*$, thus moving the economy away from the trajectory governed by (61)-(63). This intrinsic instability reflects the fact that a shock determining an increase (decrease) in the Chinese reservation wage relatively to the stocks of capital accumulated in the Chinese and in the US tradable sector can make the Chinese tradable sector less (more) profitable, thus depressing (boosting) employment and capital investment in this sector. As a result, capital accumulation and growth in China’s tradable sector decelerates (accelerates), and the effects of the initial shock are amplified.

It is legitimate to wonder why the Chinese policy makers may end up in this situation, where the economy is intrinsically unstable and a portion of the workforce is entrapped in the low-productive sectors of the economy. Indeed, this scenario is the consequence of a regime switch which takes place too early, or in a context where the fiscal policy cannot adequately offset the decrease in the demand for tradables brought about by the appreciation of the Chinese currency that follows the full liberalization of the capital account and the end of the exchange-rate pegging. Possibly under pressure of the US or to alleviate the excessive compression of domestic consumption due to the persistent undervaluation of the exchange rate, the Chinese policy makers may decide to abandon the export-led growth strategy pursued in phase 1 without having a valid alternative strategy, especially if fiscal policy has little effect on growth because of its composition.
Finally, note that also in Scenario B—as in Scenario A—the equilibrium path of the world economy is characterized for \(0 < t < t^*\) by (55)-(57).

5.3 Scenario C

In this case, the equilibrium trajectory of the real variables of the world economy is governed for \(t \geq t^*\) by two difference equations in \(L_{cht}^T\) and \(Z_t\) (see the Appendix):

\[
\Pi(L_{cht}^T, L_{cht^*}, Z_{t+1}, \hat{g}_{cht}, \hat{g}_{cht^*}, Q) = Z_t \sigma(L_{cht}^T, L_{cht^*}, \hat{g}_{cht}, Q) + \rho(L_{cht}^T, L_{cht^*}, \hat{g}_{cht}) = 0, \quad t \geq t^*,
\]

(64)

\[
\chi(L_{cht}^T, Z_{t+1}, L_{cht^*}, Z_{t}, \hat{g}_{cht}, \hat{g}_{cht^*}, Q) = 0, \quad t \geq t^*,
\]

(65)

where from period \(t^*>0\) onwards China employs its entire manpower in the advanced sectors of the economy. Equations (64) and (65) are obtained, respectively, from (33) and (36) by using (34), (35), (37), (38), (39) (with \(L_{cht}^N = H_{cht} - L_{cht^*}\) and \(L_{cht^*} = H_{cht} - L_{cht}\)) and (54). It is significant that—in this scenario—long-run growth depends also on \(Q\), namely on the exchange-rate policy conducted by the Chinese authorities.

Proposition 1 holds even in Scenario C. In contrast, it is not necessarily the case in Scenario C that the country whose households are less impatient exhibits the higher asymptotic rate of real GDP growth. However, the following propositions concerning long-run growth hold in Scenario C:

**Proposition 6** The asymptotic rate of China’s real GDP growth is higher than the US asymptotic rate of real GDP growth if China’s exchange rate is maintained sufficiently undervalued, i.e., if \(Q > \bar{Q}\), where the threshold \(\bar{Q}\) depends on the structural and policy parameters of the two countries \((\alpha_{cht}, \alpha_{cht^*}, \gamma_{cht}, \gamma_{cht^*}, \eta_{cht}, \eta_{cht^*}, \theta_{cht}, \theta_{cht^*}, \delta_{cht}, \delta_{cht^*}, H_{cht}, H_{cht^*}, \zeta_{cht}, \zeta_{cht^*}, \hat{g}_{cht}, \hat{g}_{cht^*})\).

**Proof:** See the Appendix.

**Proposition 7** If the asymptotic rate of real GDP growth is higher in China than in the US, i.e., if \(Q > \bar{Q}\), a higher \(Q\) depresses US long-run growth without increasing China’s long-run growth.

**Proof:** If \(Q > \bar{Q}\), then \(L_{cht}^T\) is a function of \(\alpha_{cht}, \gamma_{cht}, \eta_{cht}, \theta_{cht}, \delta_{cht}, \zeta_{cht}\) and \(\hat{g}_{cht}\) (see the proof of Proposition 6 in the Appendix), thus making \(\rho_{cht} = \theta_{cht} [1 - \alpha_{cht}] L_{cht}^T + 1 - \delta_{cht}\) independent of \(Q\), where \(L_{cht}^T = \lim_{t \to \infty} L_{cht}^T\). Moreover, given (54), one has \(\frac{\partial L_{cht}^T}{\partial Q} < 0\), thus implying that \(\rho_{cht} = \theta_{cht} [(1 - \alpha_{cht}) L_{cht}^T + 1 - \delta_{cht}]\) is decreasing in \(Q\).

Two points should be stressed regarding the previous propositions. First, the pegging of the exchange rate by the Chinese authorities may not be necessary to insure that the asymptotic rate of real GDP growth is higher in China than in the US: the structural and policy parameters of the two countries may be sufficient to guarantee higher long-run growth in China, without the need of
keeping its exchange rate artificially undervalued. Second, once that the asymptotic rate of growth is higher in China than in the US, a more aggressive exchange-rate policy by the Chinese authorities has no effect on China’s long-run growth: again, the contribution of the US demand for tradables to China’s growth tends to become irrelevant as the size of the US economy tends to become negligible relative to the size of the Chinese economy. Also in Scenario C, however, the performance of the Chinese economy along the transitional path is affected by \( t_{as} = \lim_{t \to \infty} t_{as} \).

For studying the transitional path along which the world economy moves from period \( t^o \) onwards in Scenario C, we linearize the system (64)-(65) around \( (L_{chT}, Z=0) \) under the assumption that \( Q > \overline{Q} \). The linearized system thus obtained has only one path converging to \( (L_{chT}, Z=0) \), which is governed by

\[
L_{chT}^t = \frac{Z_t \Pi_{Z_t}}{\Pi_{\Pi_{Z_t}^t + \frac{X_{Z_t}}{X_{Z_t+1}}}}, \quad t \geq t^o, \tag{66}
\]

\[
Z_t = Z_t^o \left( \frac{X_{Z_t}}{X_{Z_t+1}} \right)^{t-t^o}, \quad t \geq t^o, \tag{67}
\]

where all the partial derivatives \( \Pi_{Z_t}, \Pi_{\Pi_{Z_t}^t}, \Pi_{\Pi_{Z_t}^t+1}, X_{Z_t} \) and \( X_{Z_t+1} \) are evaluated at \( (L_{chT}, Z=0) \) and are such that

\[
0 < \frac{X_{Z_t}}{X_{Z_t+1}} < 1, \quad \frac{\Pi_{\Pi_{Z_t}^t+1}}{\Pi_{\Pi_{Z_t}^t+1}} > 1, \quad \text{and} \quad \frac{\Pi_{Z_t}}{\Pi_{\Pi_{Z_t}^t+1}} \begin{cases} < 0 & \text{whenever } t_{as} = \lim_{t \to \infty} t_{as} \end{cases} \begin{cases} > 0 \end{cases}
\]

(see the Appendix). Considering (66), this implies that—along the transitional path—\( L_{chT}^t > L_{chT} \) if and only if \( t_{as} < 0 \). The same remarks made while commenting Scenario A on the reasons that may motivate the Chinese authorities to finance a permanent US trade deficit applies here.

Finally, note that in Scenario C the equilibrium path of the world economy is characterized for \( 0 < t^o \) by (55)-(57).

6. CONCLUSIONS

In this work, we develop a two-country two-stage growth model capturing the relationship that has emerged in the last few years between the US and China and the different policy objectives of the authorities in these countries. The Chinese leaders maintain a competitive (i.e., undervalued) exchange rate so as to sustain China’s exporting sectors and to absorb part of rural workers into the industrial sectors. The US policy-makers, instead, are supposedly more concerned with keeping
high the consumption possibilities of the population. As the continuation of the Chinese growth strategy entails some costs, a regime change may occur in the future.

Our paper models the Sino-American relationship and allows for a comparison among alternative policies that the Chinese authorities may adopt to modify their growth strategy in the medium and long-term. We envisage three possible scenarios for the evolution of the Sino-American relationship. All the scenarios share phase 1, resembling what has actually occurred in recent years, but differ in accordance with what fiscal policy the Chinese authorities adopt, and whether and when China fully liberalizes its capital account and floats the currency (thus starting phase 2).

Scenario A is quite optimistic because it assumes that the structural parameters of the Chinese economy are such that fiscal policy can be effective in partially substituting the mercantilist policy undertaken in phase 1 as a fundamental source of demand for tradables and as an engine of growth. The evolution of the US current account turns out to be determined by the structural features of the economies and, more interestingly, by the ultimate willingness of the Chinese authorities to finance its external deficits. Scenario B emphasizes the risks for the Chinese authorities of abandoning the pegging too early: although allowing for larger domestic consumption in the short run, they depress long-run growth and maintain some of the labor force entrapped in the traditional sector of the economy if their fiscal policy is little effective in substituting exports as a source of demand for Chinese tradables. Finally, Scenario C shows that a Chinese continuation of the export-led growth strategy based on the exchange rate pegging can be economically feasible, but has both pros and cons: on the one hand, it would be conducive to high rates of growth and to the absorption of the Chinese manpower into the advanced sectors of the economy; on the other hand, Chinese domestic consumption would remain compressed and reserves would expand further.

A peculiar feature of our model is its ability to account both for the possible differential between the asymptotic rates of real GDP growth of the two countries and for the possibility that the country growing slower in the long run (the US in the model) faces a “soft” intertemporal budget constraint because of the willingness of the other country’s authorities to keep on financing its current account deficits. Under this respect, what really matters, from the model’s standpoint, is the consolidated (government sector + private sector) balance sheet of the US economy, which has to remain within the limits imposed by the Chinese authorities’ willingness to finance the US deficits. This has important policy implications: the model suggests that the current US policies seeking to support domestic demand by directly or indirectly transforming private debt into government debt, do not affect the fundamental constraint faced by the US economy. This
constraint depends, rather, on the extent to which China (as well as the other countries running a current account surplus) is willing to continue financing the US deficits.

At the Global Think-tank Summit held in Beijing on July 3, 2009, the PBC Governor, Mr Zhou Xiaouchuan, discussed a number of plausible ways for rebalancing the global economy and called for further research on the issue. Our work is a contribution in this direction as we discuss plausible prospective paths of the US-China co-dependency as well as their implications. In particular, we show that not all the scenarios may comply with the desiderata of the Chinese authorities. The maintenance of the current Sino-American co-dependency (our scenario C) does ensure the absorption of the entire Chinese labor force into the highly productive sectors of the economy, but it requires a persistently subdued level of consumption and the sterilization of growing amounts of foreign reserves. In its turn, to achieve the authorities’ objectives the liberalization of the exchange rate and of the capital account in China needs to be timely and to be accompanied by an adequate change in the government’s fiscal policy (like in scenario A); either a premature switch or an inadequate fiscal policy after the regime change (as in scenario B) may prevent the absorption of all Chinese workers into the most advanced sectors of the economy.

Some general questions are raised by our analysis. To what extent can China’s public expenditure be considered a plausible candidate for substituting export as a source of demand for the tradable sector of the economy? For how long will the Chinese people accept the compression of domestic consumption brought about by the mercantilist policy undertaken by their leadership? Would the US accept to reduce its relevance in the world economy by allowing China to overtake it by size thanks to the fast growth made possible by this export-led strategy?

For the sake of clarity and to keep the model more tractable, we deliberately neglect three aspects which represent avenues for future research. First, we do not model the private financial sector in the US and the financially repressed banking system in China (see Song et al., 2009). Second, we focus our attention on two countries, thereby leaving aside i) the interconnections between China and other Asian countries; ii) the role of the Euro area (see Bonatti, 2006 on the US-EU relationship); iii) the role of the oil exporting countries and of oil prices; iv) the problems linked to geographical distribution of output and consumption within China (tackled for instance by Blanchard and Giavazzi, 2006); and v) the growth of global imbalances in the 1990s, when China was still a marginal player in the world economy (see Hunt and Rebucci, 2005 on this). Finally, our

58 While some emerging Asian economies can be reasonably treated as similar and assimilated to China, one should recall that most of Chinese exports are due to processing trade and appear as the result of the development of international production networks.
model does not address FDI and portfolio flows\textsuperscript{59}: modeling both of them would be useful to reflect the whole amount of resources intermediated by the Chinese central bank, but would also require a fully-fledged portfolio model at the cost of reducing our ability to highlight the real (i.e. non financial) aspects of the global imbalances.\textsuperscript{60}

We also reckon that the uninterrupted expansion of global imbalances was strictly related to two other US-specific ‘pathological’ phenomena, that is the housing and credit booms.\textsuperscript{61} These latter, though certainly not independent from global imbalances\textsuperscript{62}, depended also on other factors internal to the US, such as the unusually expansionary stance of US economic authorities and the unprecedented level of leverage of the private sector. In evaluating the merits and the prospects of the Sino-American co-dependency, we abstracted from these and other US “excesses”, as well as from the extraordinary efforts the US authorities have endured to push internal demand and bail out the US financial system after the crisis erupted. As to China, some of the explanations offered to account for the large Chinese current account surpluses are not modeled either; for instance, the demographic evolution of the Chinese population and the precautionary private saving motive linked to the alleged absence of adequate social security are not directly considered.\textsuperscript{63}

Finally, we maintain the existence of a positive impact of export-led production growth on employment in the tradable sector, even though we acknowledge that this impact may be decreasing over time because of productivity gains (such as those due to capital accumulation and to the Denison effect).\textsuperscript{64}

\textsuperscript{59} Lipschitz et al. (2009), on the contrary, develop a model to account for the Chinese transitional growth which focuses on FDI-related capital flows and neglects the factors explaining the accumulation of reserves.

\textsuperscript{60} For papers modeling international capital flows, see, \textit{inter alia}, Albuquerque et al. (2007), Blanchard et al. (2005), Caballero et al. (2008), Courdacier et al. (2008), Dedola and Straub (2007), Devereux and Sutherland (2007, 2008, 2009), Evans and Hnatkovska (2007), and Tille and Van Wincoop (2007).

\textsuperscript{61} Palley (2006) and Ferguson and Schularick (2007) identify the limits of the BWII model in the growing US financial fragility and in the undermining of its manufacturing (tradable) sector. Forbes (2008), while finding empirical support for the primary role of the US financial system for developing countries in the past, argues that this is likely to fade in the future when emerging countries will strengthen their financial systems, and the US assets and markets will lose part of their perceived advantages (in terms of liquidity, safety and depth). The crucial issue behind this plausible scenario, as Calvo and Talvi (2006) and Krugman (2007) pointed out, remains ‘when’ the US will lose its attractiveness.

\textsuperscript{62} Ferguson and Schularick (2007) agree on the BWII characterization proposed by Dooley and co-authors (and dub US-China relationship as Chimerica), yet warn that the acceleration, the duration and the extent of the imbalances have fed global asset price imbalances that in turn undermine the macro-financial mechanism underlying their formation.

\textsuperscript{63} See Ma and Zhou (2009) on the impact of demographics on the Chinese net foreign asset position.

\textsuperscript{64} Feenstra and Hong (2007) argue that, between 1997 and 2005, export-led growth may explain at most 30% of employment gains in China while the majority is due to gains in the nontradable sector. This is because a great share of Chinese exports involves assembled products (i.e., processing trade). While Rodrik (2006b) and Schott (2008) argue that Chinese exports exhibit a higher level of sophistication with respect to those of similar emerging markets, Branstetter and Lardy (2006), Amiti and Freund (2009) and Athukorala (2009) show that this is due to the processing exports even in the high-tech sectors. Being the relative importance of processing exports as it may, we argue that there
These omissions do not certainly imply that we underestimate the importance of such aspects. Rather, by keeping the model focused and tractable, we managed to analyze the possible policy shifts that may mark Chinese development and monetary strategies in the future and to investigate their international and internal economic consequences.

References


would not have been a fast urbanisation process without export-led growth: exports have most likely had also an indirect effect on Chinese employment in all the productive sectors.


Cappiello L. and G. Ferrucci, (2008), The sustainability of China’ exchange rate policy and capital account liberalisation, ECB Occasional paper 82.


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**APPENDIX**

1 Derivation of the equations characterizing an equilibrium path

1.1 From firms’ first-order conditions with respect to labor, we get:

\[
L_{jN_t} = \left( \frac{\gamma_j y_{jN_t} A_{jN_t} K_{jN_t}^{1 - \gamma_j}}{W_{jt}} \right)^{\frac{1}{1 - \gamma_j}},
\]

(A1)

\[
L_{jT_t} = \left( \frac{\alpha_j y_{jT_t} A_{jT_t} K_{jT_t}^{1 - \alpha_j}}{W_{jt}} \right)^{\frac{1}{1 - \alpha_j}}.
\]

(A2)

By using (A1) to obtain the labor demanded by each firm producing \(Y_{jN_t}\), the intertemporal problem of the representative firm producing nontradables can be solved by maximizing
\[ \sum_{v=0}^{\infty} \left( 1 - \gamma_j \right) \left( \frac{\gamma_j^\prime}{\gamma_j} \right) \frac{P_{jNt+v} A_{jNt+v} K_{jNt+v}^{1-\gamma_j}}{W_{j+1}^{\gamma_j}} \frac{1}{1-\gamma_j} \left( 1 - \delta_j \right) P_{Tt+v} I_{jNt+v} - \lambda_{jNt+v} [K_{jNt+v+1} - I_{jNt+v} - (1 - \delta_j) K_{jNt+v+1}] \right) \]

with

\[ \prod_{s=1}^{v} (1 + i_{j+s}) \]

respect to \( I_{jNt} \), \( K_{jNt+1} \) and the Lagrange multiplier \( \lambda_{jNt} \), and then by eliminating \( \lambda_{jNt} \), thus obtaining:

\[ \frac{\gamma_j}{1+i_{j+1}} \left( \frac{\gamma_j^\prime}{\gamma_j} \right) \frac{P_{jNt+v} A_{jNt+v} K_{jNt+v}^{1-\gamma_j}}{W_{j+1}^{\gamma_j}} \frac{1}{1-\gamma_j} \frac{(1 - \delta_j) P_{Tt+1}}{(1 + i_{j+1})} = P_{Tt}, \]  

(A3)

\[ K_{jNt+1} = I_{jNt} + (1 - \delta_j) K_{jNt}. \]  

(A4)

An optimal path must also satisfy the transversality condition

\[ \lim_{v \to \infty} \frac{P_{jTt+v} K_{jNt+v}}{\prod_{s=1}^{v} (1 + i_{j+s})} = 0. \]  

(A5)

1.2 Similarly, one can solve the intertemporal problem of the representative firm producing tradables, thus obtaining

\[ \frac{(1 - \alpha_j)}{(1 + i_{j+1})} \left( \frac{\alpha_j^\prime}{\alpha_j} \right) \frac{P_{jTt+v} A_{jTt+1}}{W_{j+1}^{\alpha_j}} \frac{1}{1-\alpha_j} \frac{(1 - \delta_j) P_{Tt+1}}{(1 + i_{j+1})} = P_{Tt}, \]  

(A6)

\[ K_{jTt+1} = I_{jTt} + (1 - \delta_j) K_{jTt}. \]  

(A7)

\[ \lim_{v \to \infty} \frac{P_{jTt+v} K_{jNt+v}}{\prod_{s=1}^{v} (1 + i_{j+s})} = 0. \]  

(A8)

1.3 By using (14) to obtain the labor supplied by each household, the intertemporal problem of the representative household can be solved by maximizing

\[ \sum_{v=0}^{\infty} \theta_j^v \left[ \ln \left( \frac{C_{jNt+v}^{\eta}}{C_{jTt+v}^{\eta-1}} \right) + \lambda_j^v \ln \left( \frac{M_{jTt+v}}{P_{jTt+v}} \right) + v(G_{jTt+v}) + \lambda_{jHt+v} \left[ (1 + i_{jTt+v}) B_{jHt+v} + E_{jTt+v} (1 + i_{jTt+v}) F_{jHt+v} + \pi_{jNt+v} + \pi_{jTt+v} + L_{jTt+v} W_{jTt+v} + M_{jTt+v-1} T_{jTt+v} - B_{jHt+v+1} - E_{jTt+v} F_{jHt+v+1} - M_{jTt+v} - P_{jNt+v} C_{jNt+v} - P_{jTt+v} C_{jTt+v} + 1 \right] \right] \]  

with respect to \( C_{jNt} \), \( C_{jTt} \), \( M_{jTt} \), \( B_{jHt+1} \), \( F_{jHt+1} \) and the Lagrange multiplier \( \lambda_{jHt} \), and then by eliminating \( \lambda_{jHt} \), thus obtaining:

\[ \eta_j P_{jTt} C_{jTt} = (1 - \eta_j) P_{jNt} C_{jNt}, \]  

(A9)

\[ \chi_j^v (1 - \eta_j) M_{jTt}^{-1} = (P_{jTt} C_{jTt})^{-1} \theta_j (P_{jTt+1} C_{jTt+1})^{-1}, \]  

(A10)

\[ P_{jTt+1} C_{jTt+1} = \theta_j P_{jTt} C_{jTt} (1 + i_{jTt+1}), \]  

(A11)

\[ E_{jTt} P_{jTt+1} C_{jTt+1} = \theta_j P_{jTt} C_{jTt} E_{jTt} (1 + i_{jTt+1}), \quad i \neq j, \quad t \geq t^*, \]  

(A12)
Notice that (A13) is obtained by using (19) (the government’s budget constraint) for substituting $T_{jt}$ in the household’s budget constraint, and by using (2), (3), (6), (7), (18), (21) and (22).

The household’s optimal path must also satisfy the transversality conditions

\[
\lim_{v \to \infty} \frac{\theta_j^v (1 - \eta_j) B_{jtHt+v+1}}{C_{jtHt+v} P_{jtHt+v}} = 0, \quad (A14)
\]

\[
\lim_{v \to \infty} \frac{\theta_j^v (1 - \eta_j) E_{jtHt+v+1} F_{jtHt+v+1}}{C_{jtHt+v} P_{jtHt+v}} = 0. \quad (A15)
\]

1.4 To derive (35), one can use (18) and the fact that the government produces efficiently ($G_{jNt}=\zeta_j G_{jTt}$) to obtain

\[
G_{jTt} = \frac{g_{jt}(P_{jNt} Y_{jNt} + P_{jtHt} Y_{jtHt})}{P_{jNt} \zeta_j + P_{jtHt}} = \frac{g_{jt} \left( \frac{P_{jNt} Y_{jNt}}{P_{jtHt}} + Y_{jtHt} \right)}{\left( \frac{P_{jNt} \zeta_j}{P_{jtHt}} + 1 \right)}. \quad (A16)
\]

Moreover, one can use (A1)-(A2) to obtain

\[
\frac{P_{jNt}}{P_{jtHt}} = \frac{\alpha_j K_{jNt} L_{jNt}^{1-\gamma_j}}{\gamma_j K_{jNt} L_{jNt}^{1-\alpha_j}}. \quad (A17)
\]

Finally, one can use (A17) and the production functions (1) and (5) to rewrite (A16) as (35).

1.5 To derive (34), one can use $G_{jNt}=\zeta_j G_{jTt}$, the equilibrium condition (22), the production function (1) and (35) to obtain

\[
C_{jNt} = K_{jNt} L_{jNt}^{1-\gamma_j} \frac{\zeta_j \alpha_j K_{jNt} L_{jNt}^{1-\gamma_j} + 1}{\gamma_j K_{jNt} L_{jNt}^{1-\alpha_j} + 1}. \quad (A18)
\]

Moreover, one can use (A9) and (A17) to obtain

\[
C_{jTt} = \frac{(1-\eta_j) \alpha_j K_{jTt} L_{jNt}^{1-\gamma_j} C_{jNt}}{\eta_j \gamma_j K_{jNt} L_{jNt}^{1-\alpha_j}}. \quad (A19)
\]

Finally, one can use (A18) to rewrite (A19) as (34).

1.6 To derive (38), one can use (A1) and the fact that $A_{jNt}=K_{jNt}^{\gamma_j}$ to rewrite (A3) as

\[
\frac{(1-\gamma_j) L_{jNt+1} P_{jNt+1}}{(1+i_{jt+1})} + \frac{(1-\delta_j) P_{jTt+1}}{(1+i_{jt+1})} = P_{jtHt}. \quad (A20)
\]
Similarly, one can use (A2) and the fact that $A_j^{\alpha_j} = K_{jTt}^{\alpha_j}$ to rewrite (A6) as
\[
\frac{(1 - \alpha_j)L_{jTt+1}^{\alpha_j}P_{jTt+1}}{(1 + 1_{jTt+1})} + \frac{(1 - \delta_j)P_{jTt+1}}{(1 + 1_{jTt+1})} = P_{jTt}.
\]

Finally, one can use (A17), (A20) and (A21) to obtain (36).

1.7 To derive (37), one can use (A11) to rewrite (A21) as
\[
\theta_j[(1 - \alpha_j)L_{jTt+1}^{\alpha_j} + 1 - \delta_j] \frac{C_{jTt}}{C_{jTt+1}} = 1
\]

Finally, one can use (34) to rewrite (A22) as (37).

1.8 To derive (39), one should consider that (14), (21), (A1) and (A2)—together—rule out the possibility that $\frac{W_j}{P_j} < V_j$ along an equilibrium path. Hence, labor market equilibrium requires $\frac{W_j}{P_j} \geq V_j$. Furthermore, by inspecting (14) and (21), one can verify that $\frac{W_j}{P_j} > V_j$ entails $L_{jTt} + L_{jNi} = H_j$, which—in its turn—implies (consider (11), (A2), (A17) and (A18)) that
\[
\frac{\alpha_j D_j}{\alpha_j K_{jTt}} \left[ \frac{\gamma_j K_{jNi} L_{jTt}^{1 - \alpha_j}}{\alpha_j K_{jNi} (H_j - L_{jTt})^{1 - \gamma_j}} \right] \geq N_{jt} \text{ entails } L_{jNi} = H_j - L_{jTt}. \]

Thus,
\[
L_{jNi} = H_j - L_{jTt} \text{ if } \frac{\alpha_j D_j}{\alpha_j K_{jTt}} \left[ \frac{\gamma_j K_{jNi} L_{jTt}^{1 - \alpha_j}}{\alpha_j K_{jNi} (H_j - L_{jTt})^{1 - \gamma_j}} \right] \geq N_{jt}. \quad (A23)
\]

Finally,
\[
\frac{\alpha_j D_j}{\alpha_j K_{jTt}} \left[ \frac{\gamma_j K_{jNi} L_{jTt}^{1 - \alpha_j}}{\alpha_j K_{jNi} (H_j - L_{jTt})^{1 - \gamma_j}} \right] \leq N_{jt} \text{ entails } \frac{W_j}{P_j} = V_j \text{ (again, consider (11), (14), (21), (A2) and (A17)). Thus, one can use (A1) and (A17) to obtain}
\]
\[
L_{jNi} = \left[ \frac{\gamma_j K_{jNi}}{\alpha_j K_{jTt}} \right]^{\eta_j} \frac{\alpha_j D_j}{\alpha_j K_{jTt}} \left[ \frac{\gamma_j K_{jNi} L_{jTt}^{1 - \alpha_j}}{\alpha_j K_{jNi} (H_j - L_{jTt})^{1 - \gamma_j}} \right]^{\eta_j} \leq N_{jt}. \quad (A24)
\]

1.9 To derive (41), rewrite (A10) as
\[
\chi_j(1 - \eta_j)^{-1} = x_j - x_{jt + 1} \theta_j (1 + \theta_j)^{-1}. \quad x_j = M_j (P_{jTt} C_{jTt})^{-1}, \quad \chi_j(1 - \eta_j)^{-1} = x_j - x_{jt + 1} \theta_j (1 + \theta_j)^{-1}. \quad x_j = M_j (P_{jTt} C_{jTt})^{-1},
\]

Since $\theta(1 + \theta_j)^{-1} < 1$, equation (A25) is such that if $x_{jt} > x_j$ then $x_{jt} \rightarrow \infty$ as $t \rightarrow \infty$, if $x_{jt} < x_j$ then $x_{jt} \rightarrow \infty$ as $t \rightarrow \infty$, if $x_{jt} = x_j$ then $x_{jt} = x_j$ for all $t$, where $x_j = \frac{\chi_j(1 + \theta_j)}{(1 - \eta_j)(1 + \theta_j - \theta_j)}$. Therefore, the only value of $x_j$ that is consistent with the optimality and boundary conditions is $x_{jt} = x_j$ for all $t$. This implies that along an equilibrium path one has...
\[ P_{jt} = \frac{M_{jt}}{C_{jt}X_j} = \frac{(1-\eta_j)(1+\bar{\mu}_j - \theta_j)(1+\bar{\mu}_j)^{i} M_{j-1}}{C_{jt}X_j}. \quad (A26) \]

Considering (A26), one can use the one-price law (24) to obtain (41).

2 Derivation of equation (46)

Considering (31) and (A11), one can check that

\[ \frac{E_{jt}}{E_{j-1}} = \frac{P_{jt} P_{jt-1} \theta_j C_{jt-1} C_{jt}}{P_{jt} P_{jt-1} \theta_j C_{jt-1} C_{jt}}, \quad i \neq j, t \geq t^*. \quad (A27) \]

Considering (24), one has

\[ \frac{E_{jt}}{E_{j-1}} = \frac{P_{jt} P_{jt-1}}{P_{jt} P_{jt-1}}, \quad i \neq j. \quad (A28) \]

Thus, (A27) and (A28)—together—imply that in phase 2 one has \( \frac{\theta_j C_{jt-1} C_{jt}}{C_{jt-1} C_{jt}} = 1, \quad i \neq j, t \geq t^* \), which in its turn entails (46) (see equation (A22)).

3 Derivation of equations (47)-(48)

Equation (47) contains \( b(L_{chTt+1}, L_{chTt}, \bar{e}_{us}) \) and \( y(L_{chTt+1}, L_{chTt}, \bar{e}_{ch}) \), where

\[ b(L_{chTt+1}, L_{chTt}, \bar{e}_{us}) = \frac{TA_{ust}}{P_{ust} K_{ust}} \]

is obtained by setting \( L_{ustTt} = l(L_{chTt}) \) (see equation (46)) and

\[ g_{ust} = g_{ust+1} = \bar{e}_{us} \quad \forall t \geq t^* \]

\[ \left[ 1 + K(H_{us} - \bar{L}_{usTt+1} + \bar{L}_{usTt+1}) \right] \theta_{us} C(K(H_{us} - \bar{L}_{usTt+1} + \bar{L}_{usTt+1}), H_{us} - \bar{L}_{usTt+1} + \bar{L}_{usTt+1}, g_{ust+1}) \]

\[ \left[ (1 - \alpha_{us}) L_{ustTt} - \alpha_{us} \right] C(K(H_{us} - \bar{L}_{usTt+1} + \bar{L}_{usTt+1}), H_{us} - \bar{L}_{usTt+1} + \bar{L}_{usTt+1}, g_{ust+1}) \]

while \( y(L_{chTt+1}, L_{chTt}, \bar{e}_{ch}) = \frac{TA_{cht}}{P_{cht} K_{cht}} \) is obtained by setting \( L_{chNt} = H_{ch} \cdot L_{chTt} \) (see equation (39)) and

\[ g_{cht} = g_{cht+1} = \bar{e}_{ch} \quad \forall t \geq t^* \]

\[ \left[ 1 + K(L_{chNt+1} - L_{chTt+1}) \right] \theta_{cht} C(K(L_{chNt+1} - L_{chTt+1}), L_{chNt+1} - L_{chNt+1}, g_{cht}) \]

\[ \left[ (1 - \alpha_{cht}) L_{acht+1} - \alpha_{cht} + \delta_{cht} \right] C(K(L_{chNt+1} - L_{chTt+1}), L_{chNt+1} - L_{chNt+1}, g_{cht}) \]

while \( L_{achNt+1} = H_{ach} \cdot L_{achTt} \).

In their turn, equations (A29) and (A30) are derived by using (34), (35), (37), (38) and (39) (with \( L_{achNt+1} = H_{ach} \cdot L_{achTt} \)).

Equation (48) is obtained by setting \( L_{chNt+1} = H_{ch} \cdot L_{chTt} \), \( L_{achNt+1} = l(L_{chTt}) \), \( g_{ust} = g_{ucht+1} = \bar{e}_{us} \) and

\[ g_{cht} = g_{acht+1} = \bar{e}_{ch} \quad \forall t \geq t^* \]

\[ Z_{acht+1} = \frac{Z_{acht} C(K(H_{ach} - \bar{L}_{acht+1} + \bar{L}_{acht+1}), H_{ach} - \bar{L}_{acht+1} + \bar{L}_{acht+1}, g_{acht+1})}{\left[ (1 - \alpha_{acht}) L_{acht+1} - \alpha_{acht} + \delta_{acht} \right] C(K(L_{chNt+1} - L_{chtTt+1}), L_{chNt+1} - L_{chNt+1}, g_{cht})} = 0, \quad \forall t > 0. \quad (A31) \]
where (A31) is derived from (36) by using (34), (35), (37), (38), and by setting \( L_{uNt} = H_{us} \cdot L_{usTt} \) in equation (39).

4 Derivation of inequality (49)

Since \( b(L_{chTti+1}, L_{chTi}, \overline{u}_{us} \) = \( \frac{TA_{mt}}{P_{usTi}K_{usTi}} \leq \frac{2|P_{usNt}Y_{usNt} + P_{usTi}Y_{usTi}|}{P_{usTi}K_{usTi}} \), one can use (1), (5), (38), (A17), and (A31) to obtain (49).

5 Proof that if \( L_{jTi} \to L_{jt} \) as \( t \to \infty \), then in Scenario A the country \( j \)'s rate of real GDP growth approaches \( \rho_j = \theta_j(1 - \alpha_j)L_{jt}^{\alpha_j} + 1 - \delta_j \), where \( \rho_j = \lim_{t \to \infty} \rho_{jt} \).

Considering (1), (5), (11), (37), (38), (39) (with \( L_{jNt} = H_j \cdot L_{jTi} \)) and (A17), one can verify that the country \( j \)'s rate of real GDP growth is given in Scenario A by

\[
\rho_{GDP, jt} = \left( 1 + \rho_{jt} \right) \frac{P_{jTi+1} Y_{jTi+1} + P_{jNt+1} Y_{jNt+1}}{P_{jt+1} Y_{jtTi} + P_{jNt} Y_{jNt}} \cdot \left( \frac{P_{jTi} Y_{jTi} + P_{jNt} Y_{jNt}}{P_{jt}} \right),
\]

where \( \rho_{GDP, jt} = \frac{P_{jTi+1} Y_{jTi+1} + P_{jNt+1} Y_{jNt+1}}{P_{jt+1} Y_{jTi} + P_{jNt} Y_{jNt}} \cdot \frac{P_{jTi} Y_{jTi} + P_{jNt} Y_{jNt}}{P_{jt}} \).

By inspecting (A32), one can easily check that \( L_{jTi} \to L_{jt} \) as \( t \to \infty \) implies that \( \lim_{t \to \infty} \rho_{GDP, jt} = \lim_{t \to \infty} \rho_{jt} \).

Finally, by considering (37) and (38), one can also check that \( L_{jTi} \to L_{jt} \) as \( t \to \infty \) implies that \( \lim_{t \to \infty} \rho_j = \rho_j = \theta_j(1 - \alpha_j) L_{jt}^{\alpha_j} + 1 - \delta_j \). Thus, \( L_{jTi} \to L_{jt} \) as \( t \to \infty \) entails \( \lim_{t \to \infty} \rho_{GDP, jt} = \theta_j(1 - \alpha_j) L_{jt}^{\alpha_j} + 1 - \delta_j \).

6 Proof that if \( \theta_j > \theta_i \), \( j \neq i \), then in Scenario A \( L_{jt} \) is a function of \( \alpha_j, \gamma_j, \eta_j, \theta_j, \delta_j, H_j, \zeta_j \) and \( g_j \).

Since \( \theta_j > \theta_i \) entails \( \rho_j > \rho_i \), \( \theta_j > \theta_i \) also implies that \( \frac{K_{jTi}}{K_{jTi}} \to 0 \) whenever \( L_{jTi} \to L_{jt} \) and \( L_{jTi} \to L_{jt} \) as \( t \to \infty \), \( j \neq i \). In its turn, this implies that as \( t \to \infty \) equation (47) becomes

\[
G(K(H_j \cdot L_{jT}, L_{jT}), H_j \cdot L_{jT}, L_{jT}, \overline{g}_j) + C(K(H_j \cdot L_{jT}, L_{jT}), H_j \cdot L_{jT}, L_{jT}, \overline{g}_j) - (1 - \delta_j)(1 + K(H_j \cdot L_{jT}, L_{jT})) + \frac{1 + K(H_j \cdot L_{jT}, L_{jT})}{(1 - \alpha_j) L_{jt}^{\alpha_j} + 1 - \delta_j} = 0,
\]

where \( w(L_{jT}, \alpha_j, \gamma_j, \alpha_j, \delta_j, H_j, \zeta_j, \overline{g}_j) = \frac{\theta_j(1 - \alpha_j) L_{jt}^{\alpha_j} - (1 - \delta_j)(1 - \theta_j)}{\theta_j(1 - \alpha_j) L_{jt}^{\alpha_j} + 1 - \delta_j} + \frac{(1 - \eta_j) \alpha_j(H_j \cdot L_{jT})}{\eta_j L_{jt}^{\alpha_j}} - L_{jt}^{\alpha_j} - \frac{(1 - \alpha_j) L_{jt}^{\alpha_j}}{\theta_j(1 - \alpha_j) L_{jt}^{\alpha_j} + 1 - \delta_j} \cdot \frac{(1 - \eta_j) \alpha_j(H_j \cdot L_{jT})}{\eta_j L_{jt}^{\alpha_j}}. \]
country j’s tradable sector is a value of \( L_j \) that satisfies (A33). If it exists, this asymptotic equilibrium level is unique. Indeed, in the special case in which \( \delta_j = 1 \), there is at most one value of \( L_j \) satisfying (A33): in this case, the equilibrium level of employment in country j’s tradable sector is this unique value of \( L_j \). In the case in which \( \delta_j < 1 \), there are at most two values of \( L_j \) satisfying (A33) and the asymptotic equilibrium level of employment in country j’s tradable sector exists if the values of \( L_j \) are two. In this case, the equilibrium level of employment is the largest of these two values and it is unique, since the smallest value cannot be an equilibrium because it is inconsistent with \( \sum j_i I_j + I_{jT} \geq 0 \). Thus, given that the asymptotic equilibrium level of employment in country j’s tradable sector is a value of \( L_j \) satisfying (A33) and it is unique, it is a function of \( \alpha_j, \gamma_j, \eta_j, \theta_j, \delta_j, H_j, \zeta_j \) and \( \bar{g}_j \):

\[
L_j = p(\alpha_j, \gamma_j, \eta_j, \theta_j, \delta_j, H_j, \zeta_j, \bar{g}_j).
\]  
(A34)

7 Proof of Proposition 3

Since \([(1 - \alpha_j)I_{jT} + 1 - \delta_j] = [(1 - \alpha_j)I_{jT} + 1 - \delta_j] \), one can immediately see that \( L_{jT} \) is a positive function of \( L_{jT} = p(\alpha_j, \gamma_j, \eta_j, \theta_j, \delta_j, H_j, \zeta_j, \bar{g}_j) \), so that \( \rho_i = \theta_i [(1 - \alpha_j)I_{jT} + 1 - \delta_j] \) (i.e., the asymptotic rate of real GDP growth of country i) increases with \( L_{jT} \), \( j \neq i \).

To check that \( \frac{\partial p(.)}{\partial \bar{g}_j} \) whenever \( \zeta_j > \bar{\zeta}_j \), consider \( L_{jT} = p(\alpha_j, \gamma_j, \eta_j, \theta_j, \delta_j, H_j, \zeta_j, \bar{g}_j) \) that is the asymptotic equilibrium level of employment in the tradable sector of country j conditional on \( \bar{g}_j = 0 \), and

define \( \bar{\zeta}_j = \frac{\eta_j (1 - \gamma_j) (I_{jT} = 0) \gamma_j}{(1 - \eta_j) (1 - \alpha_j) (I_{jT} = 0) \gamma_j} \). One can easily verify that

\[
\text{if } \zeta_j > \bar{\zeta}_j, \text{ then } \frac{\partial w(.)}{\partial \bar{g}_j} \bigg|_{L_{jT} \geq L_{jT} = 0} < 0 \tag{A35}
\]

and

\[
\text{if } \zeta_j < \bar{\zeta}_j, \text{ then } \frac{\partial w(.)}{\partial \bar{g}_j} \bigg|_{L_{jT} \leq L_{jT} = 0} > 0, \tag{A36}
\]
Furthermore, consider that

\[
\frac{\partial w(\cdot)}{\partial \bar{g}_j} \bigg|_{\bar{L}_{jt}} = \frac{(1 - \eta_j)(1 - \alpha_j)\bar{L}^{\alpha_j}_{jt} \zeta_j}{\eta_j(1 - \gamma_j)(H_j - \bar{L}_{jt})^{\gamma_j}} - 1 \left( \frac{\alpha_j(H_j - \bar{L}_{jt})}{\gamma_j \bar{L}_{jt}} + 1 \right) \bigg|_{\bar{L}_{jt}}.
\]

Moreover, consider that

\[
\frac{\partial w(\cdot)}{\partial \bar{L}_{jt}} \bigg|_{\bar{L}_{jt}} = p(\alpha_j, \gamma_j, \eta_j, \bar{L}_{jt}, \zeta_j, \bar{g}_j)
\]

thus implying that in a neighbourhood of \( \bar{L}_{jt} = 0 \) one has

\[
\begin{align*}
 w(L_{jt}, \alpha_j, \gamma_j, \eta_j, \delta_j, H_j, \zeta_j, \bar{g}_j) \bigg|_{\bar{g}_j=0} & \begin{cases} < 0 & \text{whenever } L_{jt} > 0 \end{cases} \\
 & \begin{cases} > 0 & \text{whenever } L_{jt} < 0 \end{cases}
\end{align*}
\]

where

\[
\begin{align*}
 w(L_{jt}, \alpha_j, \gamma_j, \eta_j, \delta_j, H_j, \zeta_j, \bar{g}_j) \bigg|_{\bar{g}_j=0} &= \frac{1 + (1 - \gamma_j)\alpha_j(H_j - \bar{L}_{jt})}{\eta_j(1 - \gamma_j)(1 - \delta_j)(1 - \theta_j)\bar{L}_{jt}^{\alpha_j}} - \frac{(1 - \eta_j)\alpha_j(H_j - \bar{L}_{jt})}{\eta_j \gamma_j \bar{L}_{jt}^{\alpha_j}} \cdot \bar{L}_{jt}^{\alpha_j}.
\end{align*}
\]

From (A35), (A36) and (A38)—together—one can conclude that

\[
L_{jt} = \frac{p(\alpha_j, \gamma_j, \eta_j, \delta_j, H_j, \zeta_j, \bar{g}_j)}{\bar{g}_j} \bigg|_{\bar{g}_j>0} = \begin{cases} < 0 & \text{whenever } \zeta_j > \bar{\zeta}_j \\
 > 0 & \text{whenever } \zeta_j < \bar{\zeta}_j \end{cases}
\]

Furthermore, consider that

\[
\begin{align*}
 w(L_{jt}, \alpha_j, \gamma_j, \eta_j, \delta_j, H_j, \zeta_j, \bar{g}_j) \bigg|_{\bar{g}_j=0} + \bar{g}_j \frac{\partial w(\cdot)}{\partial \bar{g}_j} = 0. 
\end{align*}
\]

This allows us to conclude from (A37), (A38) and (A39) that

\[
\begin{align*}
 \frac{\partial p(\cdot)}{\partial \bar{g}_j} & \begin{cases} < 0 & \text{whenever } \zeta_j < \bar{\zeta}_j \\
 > 0 & \text{whenever } \zeta_j > \bar{\zeta}_j \end{cases} \\
 \frac{\partial w(\cdot)}{\partial \bar{L}_{jt}} & \begin{cases} < 0 & \text{whenever } \zeta_j < \bar{\zeta}_j \\
 > 0 & \text{whenever } \zeta_j > \bar{\zeta}_j \end{cases}
\end{align*}
\]

Finally, one should observe that the absolute value of \( \frac{\partial w(\cdot)}{\partial \bar{g}_j} \) becomes smaller as \( \zeta_j \) becomes closer to \( \bar{\zeta}_j \), thus reducing the effect of a change in \( \bar{g}_j \) on the asymptotic rate of real GDP growth.

### 8 Transitional path of the economy in Scenario A

By solving the characteristic equation of the system obtained by linearizing (47)-(48) around \( (L_{chT}, Z=0) \), one can find the eigenvalues \( \kappa_1 = \frac{\Psi_{chT}^{L_{chT}}}{\Psi_{chT}^{L_{chT}+1}} \) and \( \kappa_2 = \frac{\Lambda_{Z_t}}{\Lambda_{Z_t+1}} \), where \( \kappa_1 > 1 \) and \( 0 < \kappa_2 < 1 \), since

\[
-\Psi_{chT}^{L_{chT}} > \Psi_{chT}^{L_{chT}+1} > 0 \quad \text{and} \quad \frac{\Lambda_{Z_t}}{\Lambda_{Z_t+1}} = \frac{1 + \rho_{as}}{1 + \rho_{ch}}
\]

(notice that all derivatives must be evaluated at \( (L_{chT}, Z=0) \)).

Having only one initial condition (solely \( Z_t \) is given at time \( t^* \)), \( \kappa_1 > 1 \) and \( 0 < \kappa_2 < 1 \) imply that the linearized system is saddle-path stable.
By using the eigenvector $\frac{\Psi_{1}}{(\kappa_{1} - \kappa_{2})\Psi_{1}}$, one can derive the system (50)-(51) governing the saddle path.

Since $\Psi_{1} = \frac{C_{usT}}{K_{usT} + G_{usT}} + (\rho_{us} + \delta_{us})\left[1 + \frac{\alpha_{us}(1 - \gamma_{us})(H_{us} - L_{usT})}{\gamma_{us}(1 - \alpha_{us})L_{usT}}\right] - \frac{\alpha_{us}}{L_{usT}} = -\frac{TA_{us}}{K_{usT} - P_{usT}}$ and $\Psi_{1} > 0$, one can see that $\frac{\Psi_{1}}{\Psi_{1}^{T}} = 0$ whenever $t_{au} = 0$. Given $\kappa_{1} - \kappa_{2} > 0$, this implies that—along the transitional path—$L_{chT} > L_{chT}$ if and only if $t_{au} < 0$.

9 Derivation of equations (55), (56) and (57)

To derive the system (55)-(57), consider that $L_{chNt} = L(K(L_{chNt}, L_{chT}), N_{ch}, L_{chT}) < H_{ch} - L_{chT}, 0 < t < t^*$ (see equations (38) and (39)), from which one obtains

$$L_{chNt} = n(N_{ch}, L_{chT}) = \left[\frac{1}{1 - \alpha_{ch}}\right]^{n_{ch}} N_{ch}^{1 - \alpha_{ch}} \frac{1}{\alpha_{ch}D_{ch}} = H_{ch} - L_{chT}. \quad (A40)$$

Equation (55) contains $\zeta(L_{chT}+1, N_{ch}+1, L_{chT}, \hat{g}_{ch}, \hat{g}_{ch}, Q)$ and $\varrho(L_{chN}+1, N_{ch}+1, L_{chN}, \hat{g}_{ch}, \hat{g}_{ch})$, where $\zeta(L_{chT}+1, N_{ch}+1, L_{chT}, \hat{g}_{ch}, \hat{g}_{ch}, Q)$ is obtained by setting $L_{usT} = f(L_{chT}, n(N_{ch}, L_{chT}), \hat{g}_{ch}, \hat{g}_{ch}, Q)$ and $g_{us} = g_{us+1} = \hat{g}_{us} \forall t$ such that $0 < t < t^*$ in (A29), while $\varrho(L_{chN}+1, N_{ch}+1, L_{chN}, \hat{g}_{ch}, \hat{g}_{ch})$ is obtained by setting $L_{chN} = n(N_{ch}, L_{chT})$ (see equation (54)) and $g_{ch} = g_{ch+1} = \hat{g}_{ch} \forall t$ such that $0 < t < t^*$ in (A30).

Equation (56) is obtained by setting $L_{chN} = n(N_{ch}, L_{chT})$, $L_{usT} = f(L_{chT}, n(N_{ch}, L_{chT}), \hat{g}_{ch}, \hat{g}_{ch}, Q)$, $g_{us} = g_{us+1} = \hat{g}_{us}$ and $g_{ch} = g_{ch+1} = \hat{g}_{ch} \forall t$ such that $0 < t < t^*$ in (A31).

Equation (57) is obtained by setting $L_{chN} = n(N_{ch}, L_{chT})$ and $g_{ch} = g_{ch+1} = \hat{g}_{ch} \forall t$ such that $0 < t < t^*$ in

$$N_{ch} - \frac{N_{ch}}{\theta_{ch}[(1 - \alpha_{ch})L_{chT} + 1 - \delta_{ch}]C(K(L_{chN}+1, L_{chT}, L_{chN}+1, \hat{g}_{ch}+1))} = 0, \ t > 0. \quad (A41)$$

In its turn, equation (A41) is derived from (40) by using (34), (35), (37) and (38).
\[ Z_1 = \frac{\theta_{us} C \left( \frac{K_{usN0}}{K_{usT0}}, L_{usT0} - L_{usT1}, L_{usT1} - \hat{g}_{us} \right)}{\left( 1 - \alpha_{ch} \right) \left( L_{chT1} - 1 \right) \left( \delta_{ch} \right) \left( \theta_{ch} C \left( \frac{K_{chN0}}{K_{chT0}}, L_{chN0} - L_{chT0}, \hat{g}_{ch} \right) \right)} \]

and

\[ N_{ch1} = \frac{N_{ch0} \left( 1 + \alpha_{ch} \right) C \left( K_{chN0}, L_{chT1}, L_{chN1} - L_{chT1}, \hat{g}_{ch} \right)}{\theta_{ch} \left( 1 - \alpha_{ch} \right) \left( L_{chT1} - 1 \right) \left( \delta_{us} \right) \left( K_{chN0}, L_{chN0} - L_{chT0}, \hat{g}_{ch} \right)} \]

where \( L_{chN0} = L \left( \frac{K_{chN0}}{K_{chT0}}, N_{ch0}, L_{chT0} \right) < H_{ch} - L_{chT0}, L_{usT0} = f \left( L_{chT0}, L \left( \frac{K_{chN0}}{K_{chT0}}, N_{ch0}, L_{chT0} \right), \hat{g}_{ch}, \hat{g}_{us}, Q \right), \) and

\[ Z_0, \frac{K_{usN0}}{K_{usT0}}, \frac{K_{usN0}}{K_{usT0}}, \frac{K_{usN0}}{K_{usT0}}, N_{ch0} \) are given.

10 Derivation of equations (58), (59) and (60)

Equation (58) contains \( b(L_{chT1+1}, L_{chT1}, \bar{g}_{us}) \) and \( m(L_{chT1+1}, N_{chT1}, L_{chT1}, N_{chT1}, \bar{g}_{ch}) \), where

\[ b(L_{chT1+1}, L_{chT1}, \bar{g}_{us}) \]

is obtained by setting \( L_{usT1} = l(L_{chT1}) \) and \( g_{ust} = g_{ust+1} = \bar{g}_{us} \forall t \geq t^* \) in (A29), while

\[ m(L_{chT1+1}, N_{chT1}, L_{chT1}, N_{chT1}, \bar{g}_{ch}) \]

is obtained by setting \( L_{chN1} = n(N_{chT1}, L_{chT1}) \) and \( g_{ch} = g_{ch+1} = \bar{g}_{ch} \forall t \geq t^* \) in (A30).

Equation (59) is obtained by setting \( L_{chN1} = n(N_{chT1}, L_{chT1}) \) and \( g_{ch} = \bar{g}_{ch} \forall t \geq t^* \) in (A41).

Equation (60) is obtained by setting \( L_{chN1} = n(N_{chT1}, L_{chT1}) \), \( L_{usT1} = l(L_{chT1}) \), \( g_{ust} = \bar{g}_{us} \) and \( g_{ch} = \bar{g}_{ch} \forall t \geq t^* \) in (A31).

11 Proof that if \( L_{chT1} \rightarrow L_{chT} \) and \( N_{chT} \rightarrow N_{chT} \) as \( t \rightarrow \infty \), then in Scenario B the country \( j \)’s rate of real GDP growth approaches \( \rho_j = \theta_j \left[ (1 - \alpha_j) L_{chT1+1} + \delta_j \right] \), where \( \rho_j = \lim_{t \rightarrow \infty} \rho_{jt} \)

Considering (1), (5), (11), (37), (38), (A17), \( L_{usT1} = H_{us} - L_{usT1} \) and \( L_{chN1} = n(N_{chT1}, L_{chT1}) \), one can verify that the US rate of real GDP growth is given also in Scenario B by (A32), while China’s rate of real GDP growth is given by

\[ \rho_{GDP, ch} = (1 + \rho_{ch}) \frac{\left[ n(N_{chT1+1}, L_{chT1+1}) \right]^{\rho_{ch} - 1} \left[ L_{chT1+1} \right]^{\alpha_{ch}(1 - \eta_{ch}) - 1}}{\left[ n(N_{chT1}, L_{chT1}) \right]^{\rho_{ch} - 1} \left[ L_{chT1} \right]^{\alpha_{ch}(1 - \eta_{ch}) - 1}} - 1. \]

\[ (A45) \]
By inspecting (A45), one can easily check that \( L_{chT} \rightarrow L_{chT} \) and \( N_{chT} \rightarrow N_{ch} \) as \( t \rightarrow \infty \) imply that \( \lim_{t \rightarrow \infty} \rho_{GDP_{ch}} = \lim_{t \rightarrow \infty} \rho_{ch} \). Finally, by considering (37) and (38), one can also check that \( L_{jT} \rightarrow L_{jT} \) and \( N_{chT} \rightarrow N_{ch} \) as \( t \rightarrow \infty \) imply that \( \lim_{t \rightarrow \infty} \rho_{jT} = \rho_j = \theta_j [ (1 - \alpha_j) L_{jT}^{\alpha_j} + 1 - \delta_j ] \), \( j = us, ch \). Thus, \( L_{jT} \rightarrow L_{jT} \) and \( N_{chT} \rightarrow N_{ch} \) as \( t \rightarrow \infty \) entail \( \lim_{t \rightarrow \infty} \rho_{GDP_{ch}} = \theta_j [ (1 - \alpha_j) L_{jT}^{\alpha_j} + 1 - \delta_j ] \), \( j = us, ch \).

12 Transitional path of the economy in Scenario B

By solving the characteristic equation of the system obtained by linearizing (58)-(60) around \( (L_{chT}, N_{chT}, Z=0) \), one can find the eigenvalues \( \phi_1, \phi_2 = \frac{a_{11} + a_{22}}{2} \pm \sqrt{\left( \frac{a_{11} + a_{22}}{2} \right)^2 - a_{11}a_{22} + a_{12}a_2} \) and \( \phi_3 = a_{33} \), where
\[
\begin{align*}
a_{11} &= \frac{2}{2} \left[ a_{11} + a_{22} \pm \sqrt{\left( \frac{a_{11} + a_{22}}{2} \right)^2 - a_{11}a_{22} + a_{12}a_2} \right]
\end{align*}
\]
Moreover, for admissible sets of parameter values one can show that \( \phi_1 > 1 \) and \( \phi_2 > 1 \) (for instance, setting \( \alpha_{ch} = \gamma_{ch} = 2/3 \), \( \eta_{ch} = 0.5 \), \( \delta_{ch} = 0.05 \), \( \theta_{us} = 0.95 \), \( \theta_{us} = 0.945 \), \( H_{ch} = 0.3552635 \), \( \omega_{ch} = 0.01 \), \( \phi_{ch} = 0 \), one obtains: \( L_{chT} = 0.1977922 \), \( L_{chN} = 0.1383763 \), \( N_{ch} = 0.5078785 \), \( Z = 0 \), \( \rho_{us} = 0.0046842 \), \( \phi_1 = 1.0715746 \), \( \phi_2 = 2.0797334 \), \( \phi_3 = 0.9947368 \). Having two endogenous variables whose value is pre-determined at time \( t \) (\( Z_t \) and \( N_{chT} \) are given), \( \phi_1 > 1, \phi_2 > 1 \) and \( 0 < \phi_3 < 1 \) imply that the linearized system is unstable. In the special case in which the policy makers manage to control the economy so as to enter phase 2 with \( (N_{chT} - N_{chT*}) = q_{23} Z_t \), the linearized system can converge to \( (L_{chT}, N_{chT}, Z=0) \) along the path governed by (61)-(63), where the eigenvectors
\[
q_{13} = \frac{a_{12}a_{23} + a_{13}(\phi_3 - a_{22})}{[a_{21}a_{22} - (\phi_3 - a_{11})(\phi_3 - a_{22})]} \quad \text{and} \quad q_{23} = \frac{a_{13}a_{21} + a_{22}(\phi_3 - a_{11})}{[a_{12}a_{21} - (\phi_3 - a_{11})(\phi_3 - a_{22})]} \quad \text{can be computed by using} \ a_{11}, a_{12}, a_{21}, a_{22},
\]
\[
a_{13} = \frac{\Sigma Z_{t+1} \Gamma_{N_{chT}}^{N_{chT+1}} \Gamma_{L_{chT}}^{L_{chT+1}} \Gamma_{L_{chT}}^{L_{chT+1}}}{\Sigma Z_{t+1} \Gamma_{N_{chT}}^{N_{chT+1}} \Gamma_{L_{chT}}^{L_{chT+1}} \Gamma_{L_{chT}}^{L_{chT+1}}} \quad \text{and} \quad a_{23} = \frac{\Sigma Z_{t+1} \Gamma_{L_{chT}}^{L_{chT+1}} \Gamma_{N_{chT}}^{N_{chT+1}} \Gamma_{L_{chT}}^{L_{chT+1}}}{\Sigma Z_{t+1} \Gamma_{N_{chT}}^{N_{chT+1}} \Gamma_{L_{chT}}^{L_{chT+1}} \Gamma_{L_{chT}}^{L_{chT+1}}}.
\]

13 Derivation of equations (64) and (65)

Equation (64) contains \( \sigma(L_{chT}, L_{chT}, L_{chT}, L_{usT}, Q) \) and \( \sigma(L_{chT}, L_{chT}, L_{chT}, L_{usT}, Q) \), where \( \sigma(L_{chT}, L_{chT}, L_{chT}, L_{usT}, Q) \) is obtained by setting \( L_{usT} = f(L_{chT}, H_{ch} - L_{chT}, L_{chT}, L_{chT}, L_{usT}, Q) \) and \( g_{us} = g_{usT+1} = \hat{g}_{us} \ \forall t \geq t^0 \) in (A29), while \( \rho(L_{chT}, L_{chT}, L_{chT}, L_{usT}, Q) \) is obtained by setting \( L_{chT} = f(L_{chT}, H_{ch} - L_{chT}, L_{chT}, L_{chT}, L_{usT}, Q) \) and \( g_{ch} = g_{chT+1} = \hat{g}_{ch} \ \forall t \geq t^0 \) in (A30).

Equation (65) is obtained by setting \( L_{chT} = f(L_{chT}, H_{ch} - L_{chT}, L_{chT}, L_{chT}, L_{chT}, Q) \), \( g_{us} = g_{usT+1} = \hat{g}_{us} \) and \( g_{ch} = g_{chT+1} = \hat{g}_{ch} \ \forall t \geq t^0 \) in (A31).
14 Proof of Proposition 6

Consider that—by using (29), (52) and (53) to substitute for $E_{cht}$ in (44)—one obtains

$$\frac{C_{usTt}}{K_{usTt}} = \frac{Q}{C_{cht}}. \quad (A46)$$

By using (38), $L_{jNTT} = H_j - L_{jTt}$ and $g_j = g_{j+1} = \hat{g}_j$, one can rewrite (34) as

$$C_{jTt} = c(L_{jTt}) = \frac{(1 - \eta_j)\alpha_j(H_j - L_{jTt})}{\eta_j} \left( 1 + \frac{\gamma_j(1 - \alpha_j)L_{jTt}^{-1/\gamma_j} + 1}{\alpha_j(1 - \gamma_j)(H_j - L_{jTt})^{-1/\gamma_j} + 1} \right) \left( \frac{\zeta_j(1 - \alpha_j)L_{jTt}^{-1/\gamma_j} + 1}{(1 - \gamma_j)(H_j - L_{jTt})^{-1/\gamma_j} + 1} \right). \quad (A47)$$

Given (A47) and the fact that $\frac{dc[L_{jTt}]}{dL_{jTt}} < 0$, (A46) implies that

$$L_{usTt} = h(L_{chtTt}, Q), \quad (A48)$$

where $\frac{\partial h(L_{chtTt}, Q)}{\partial L_{chtTt}} > 0$ and $\frac{\partial h(L_{chtTt}, Q)}{\partial Q} < 0$.

Since the asymptotic rate of real GDP growth of country $j$ is given by $\rho_j = \theta_j[(1 - \alpha_j)L_{jTt}^{-1/\gamma_j} + 1 - \delta_j]$ (see Proposition 1 and its proof), one can easily verify that $\rho_{ch} > \rho_{us}$ if and only if

$$\left[ \frac{\theta_j[(1 - \alpha_{ch})L_{chtTt}^{-1/\gamma_j} + 1 - \delta_{ch}] - \theta_{us}(1 - \delta_{us})}{\theta_{us}(1 - \alpha_{us})} \right]^{1/\alpha_{us}} > L_{usTt}. \quad (A49)$$

Moreover, $\rho_{ch} > \rho_{us}$ implies that as $t \to \infty$ equation (64) becomes $w(L_{chtTt}, \alpha_{chtTt}, \gamma_{chtTt}, \eta_{chtTt}, \theta_{chtTt}, \delta_{chtTt}, H_{chtTt}, \zeta_{chtTt}, \hat{g}_{chtTt}) = 0$ (see equation (A33)), from which one can derive the asymptotic equilibrium level of employment in the Chinese tradable sector, $L_{chtTt} = p(\alpha_{cht}, \gamma_{cht}, \eta_{cht}, \theta_{cht}, \delta_{cht}, H_{cht}, \zeta_{cht}, \hat{g}_{cht})$. Hence, one can rewrite the inequality (A49) as

$$\left[ \frac{\theta_j[(1 - \alpha_{cht})L_{chtTt}^{-1/\gamma_j} + 1 - \delta_{cht}] - \theta_{us}(1 - \delta_{us})}{\theta_{us}(1 - \alpha_{us})} \right]^{1/\alpha_{us}} > h(p(\alpha_{cht}, \gamma_{cht}, \eta_{cht}, \theta_{cht}, \delta_{cht}, H_{cht}, \zeta_{cht}, \hat{g}_{cht}), Q). \quad (A50)$$

Since $\frac{\partial h(L_{chtTt}, Q)}{\partial Q} < 0$, the inequality (A50) holds for all $Q > \bar{Q}$, where $\bar{Q}$ is that value of $Q$ satisfying...
Considering (A47), (A48) and (A51), one can see that $Q$ depends on $\alpha_{ch}$, $\gamma_{ch}$, $\eta_{ch}$, $\theta_{ch}$, $\delta_{ch}$, $H_{ch}$, $\zeta_{ch}$, $\hat{g}_{ch}$, $\alpha_{us}$, $\gamma_{us}$, $\eta_{us}$, $\theta_{us}$, $\delta_{us}$, $H_{us}$, $\zeta_{us}$ and $\hat{g}_{us}$.

Finally, notice that if $Q > \overline{Q}$, one has $\rho_{ch} > \rho_{us}$, implying that $L_{chT} = p(\alpha_{ch}, \gamma_{ch}, \eta_{ch}, \theta_{ch}, \delta_{ch}, H_{ch}, \zeta_{ch}, \hat{g}_{ch})$.

15 Transitional path of the economy in Scenario C

By solving the characteristic equation of the system obtained by linearizing (64)-(65) around $(L_{chT}, Z=0)$, one can find the eigenvalues $\beta_1 = \frac{\Pi_{t,CH+1}}{\Pi_{t,CH+1}}$ and $\beta_2 = -\frac{X_{Z_1}}{X_{Z_{t+1}}}$, where $\beta_1 > 1$ and $0 < \beta_2 < 1$, since $-\Pi_{t,CH+1} > \Pi_{t,CH+1} > 0$ and $-\frac{X_{Z_1}}{X_{Z_{t+1}}} = 1 + \rho_{as}$ (notice that all derivatives must be evaluated at $(L_{chT}, Z=0)$).

Having only one initial condition (solely $Z_{t^0}$ is given at time $t^0$), $\beta_1 > 1$ and $0 < \beta_2 < 1$ imply that the linearized system is saddle-path stable.

By using the eigenvector $\frac{\Pi_{t,CH+1}}{(\beta_1 - \beta_2)\Pi_{t,CH+1}}$, one can derive the system (66)-(67) governing the saddle path.

Since $\Pi_{Z_1} = \frac{C_{usT}}{K_{usT}} + \frac{G_{usT}}{K_{usT}} + (\rho_{us} + \delta_{us}) \left[ 1 + \frac{\alpha_{us}(\gamma_{us})(H_{us} - L_{usT})}{\gamma_{us}(1 - \alpha_{us})L_{usT}} \right] - L_{usT} = \frac{-\alpha_{us} - \beta_{us} - \alpha_{us}L_{usT}}{K_{usT}P_{usT}}$ and $\Pi_{t,CH+1+1} > 0$, one can see that $\frac{\Pi_{Z_1}}{\Pi_{t,CH+1}} > 0$ whenever $ta_{us} < 0$. Given $\beta_1 - \beta_2 > 0$, this implies that—along the transitional path—$L_{chT} > L_{chT}$ if and only if $ta_{us} < 0$. 
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