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A look at the relationship between industrial dynamics and aggregate fluctuations*

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Abstract

The firmly established evidence of right-skewness of the firms' size distribution is generally modelled recurring to some variant of the Gibrat's Law of Proportional Effects. In spite of its empirical success, this approach has been harshly criticized on a theoretical ground due to its lack of economic contents and its unpleasant long-run implications. In this chapter we show that a right-skewed firms' size distribution, with its upper tail scaling down as a power law, arises naturally from a simple choice-theoretic model based on financial market imperfections and a wage setting relationship. Our results rest on a multi-agent generalization of the prey-predator model, firstly introduced into economics by Richard Goodwin forty years ago.

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1. Introduction

Starting with the pioneering work of Gibrat (1931), the study of the determinants and the shape of the steady-state distribution of firms' size has long fascinated economists. While the conventional view received from the seminal work of e.g. Hart and Prais (1956), Hart (1962) and Mansfield (1962) holds that the firms' size distribution is significantly right-skewed and approximately log-normal, recent empirical research has lent support to the view suggested by H. Simon and his co-author (Ijiri and Simon, 1977), according to whom a Pareto-Levy (or power law) distribution seems to return a better fit to the data for the whole distribution (Axtell, 2001), or at least for its upper tail (Ramsden and Kiss-Haypal, 2000; Gaffeo *et al.*, 2003).[1]

Regardless of the different outcomes obtained from distribution-fitting exercises, the most popular explanation for right-skewness emerged so far in the literature rests on stochastic growth processes, basically because of their satisfactory performance in empirical modelling.[2] From a theoretical point of view, however, random growth models of firms' dynamics have been generally seen as far less satisfactory. On the one hand, several authors have simply discarded purely stochastic models as *ad-hoc* and uninformative, given that a proper theory of firms' growth should be grounded on richer economic contents and maximizing rational behaviour (Sutton, 1997).[3] Alas, the introduction of stochastic elements in standard maximizing, game-theoretic models[4] has shown that their implications as regards the steady-state firms' size distribution are highly dependent on initial assumptions and modelling choices, to the point that we cannot find "[…] *any reason to expect the size distribution of firms to take any particular form for the general run of industries*"

[1] Another challenging stylized fact on the drivers of corporate growth and the resulting industrial structure is the ubiquitous exponential shape of the growth rates density (Bottazzi and Secchi, 2003; Bottazzi *et al.*, 2007).

[2] For recent evidence, see Geroski *et al.* (2001), who point towards a pure random walk model for firms' growth, and Hart and Oulton (2001), who instead suggest a Galtonian, reversion-to-the-mean growth process.

[3] In spite of being very popular among economists, Sutton's critique is not properly established. In fact, the Gibrat's Law may be perfectly consistent with the behaviour of rational, profit-maximizing firms. In a nutshell, consider a model in which firms, *à la* Penrose (1959), are constrained in their growth opportunities only by their internal resources. Since firms' size, at the optimum, depends on current expectations on future conditions, if firms form rational expectations changes in expectations will be unpredictable. This implies that growth rates are realizations of pure random processes. See e.g. Klette and Kortum (2004).

[4] See, for example, Jovanovic (1982) and Ericson and Pakes (1995).

(Sutton, 1997, p. 43). On the other hand, the original Gibrat's random multiplicative model (also known as the *Law of proportionate effect*) and its numerous extensions[5] all share the unpleasant property of possessing either an implosive or an explosive behaviour under rather general conditions, so that their cross-section dynamics tend alternatively towards a degenerate firms' size distribution with zero mean and variance or a degenerate distribution with infinite mean and variance (Richiardi, 2004).[6]

In this chapter, we propose a model which embeds idiosyncratic stochastic influences in a simple imperfect information, rational expectations framework. We show that a right-skewed firms' size distribution emerges as a natural feature of the endogenous cross-section dynamics, and that the upper tail is Pareto distributed. This results from the interplay of the cross-sectional dispersion (i.e., heterogeneity) of firms and the feedback exerted on it by the competitive pressure that individual actions determine through the labour and the equity markets.[7] Furthermore, certain properties of the steady-state distribution are in some sense *universal*, i.e. they are independent of some of the model's parameters. In other terms, our approach does not possess any tendency towards a long-run degenerate behaviour, typical of Gibrat's processes, and it grants the modeller more degrees of freedom than the stochastic-game-theoretic models referred to above. From an analytical point of view, our results are based on the possibility of describing the economy as a Generalized Lotka-Volterra system, that is a multi-agent extension of the prey-predator framework formally introduced to economists by Richard M. Goodwin (1913-1996) back in the 1960s (Goodwin, 1967). While inside the Econophysics community such a formalism has been already successfully employed to explain puzzling statistical regularities regarding

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[5] Among the many variations of the Gibrat's model, one can cite the preferential attachment mechanism introduced by Simon and Bonini (1958), the multiplicative *plus* additive random process due to Kesten (1973), or the lower reflection barrier by Levy and Solomon (1996).

[6] To grasp the argument, consider this simple example reported by Richiardi (2004). Let us presume that the size of a firm increases or decreases, with equal probability, by 10% in each period. Now suppose that, starting from a size of 1, the firm first shrinks and then bounces back. In the first period the firm's size is 0.9, while in the second one it is only 0.99. If we let the dynamics be inverted, so that the firm first grows and then shrinks, then the firms' size is 1.1 in the first period, and 0.99 in the second one. Clearly, this effect is stronger the bigger the variance of the stochastic growth process, and can be contrasted only by increasing the average growth rate. Richiardi (2004), resorting to simulations, shows that a degenerate long-run distribution can be prevented only by a limited number of mean/variance pairs.

[7] Another model in which financing constraints help to explain the skewness of the firms' size distribution is the one recently proposed by Cabral and Mata (2003).

stock market returns and the personal wealth distribution (Solomon and Richmond, 2001), the application of this Generalized Lotka-Volterra approach to the relation between the firms' size distribution and aggregate fluctuations is new.

The organization of this chapter is as follows. In Section 2 a simple macroeconomic model resting on financial market imperfections, maximizing behaviour, rational expectations and heterogeneity is presented. Section 3 contains the characterization of the associated firms' size cross-section dynamics and its long-run attractor or, in other terms, the equilibrium firms' size distribution. Section 4 concludes.

2. The model

The purpose of this section is to create a simple macroeconomic model that, besides explaining aggregate fluctuations, replicates the main features of industrial dynamics and the firm size distribution. At the centre of our approach there are three basic ideas:

i) **The firms' financial position matters**. Firms display heterogeneous unobservable characteristics, so that lenders may not be perfectly informed on firms' ability or willingness to pay back. As a result, the various financial instruments whereby firms can raise means of payment are not perfect substitutes. Decisions about employment and production are conditional on the cost and contractual terms of the financial instruments available to firms (Gertler, 1988). From an empirical viewpoint, it appears that firms tend to be rationed on the market for equity due to both adverse selection and moral hazard effects (Fazzari *et al.*, 1988).

ii) **Agents are heterogeneous as regards how they perceive risk associated to economic decisions.** Numerous experimental studies indicate that heterogeneity in risk perception is pervasive. Such a finding holds both for consumers and insurees (Hammar and Johansson-Stenman, 2004; Lundborg and Lindgren, 2002; Filkestein and Poterba, 2004), and for entrepreneurs (Pennings and Garcia, 2004; van Garderen *et al.*, 2005). While popular in mathematical psychology, the exploitation of these findings is far less common in theoretical economics, the random utility model discussed in Anderson *et al.* (1992) being a relevant exception.

iii) **Firms interact through the labour and equity markets.** While the feedbacks occurring in the labour market between hiring firms are clear enough not to deserve particular emphasis, a few words are in order to discuss interactions in the equity market. In particular, we are referring to the evidence suggesting that the number of initial public offerings (IPO), of additional new stock issues, as well as the proportion of external financing accounted for by private equity, all tend to increase as the aggregate activity expands and the equity market is bullish, and vice-versa (Choe *et al.*, 1993; Brailsford *et al.*, 2000).

All other aspects of the model are kept as simple as possible. [8]

We follow the literature on information imperfections in financial markets, in particular Greenwald and Stiglitz (1993), in assuming that finitely many competitive firms indexed by $i = 1,\ldots, I$ operate in an uncertain environment, in which futures markets do not exist.[9]

Each firm has a constant returns to scale technology which uses only labour as an input, $y_{it} = \phi n_{it}$, where y_{it} is the time t output of firm i, n_{it} its employment, and ϕ the labour productivity, constant and common to all. The production cycle takes one period regardless of the scale of output, implying that firms have to pay for inputs before being able to sell output.

Since capital markets are characterized by informational imperfections, firms' ability to raise risk capital on external stock markets is sub-optimal, and restricted to depend on the average capitalization in the economy. As a result, firms must generally rely upon bank loans to pay for production costs (i.e., the wage bill). In real terms the demand for credit of the i-th firm is $d_{it} = w_t n_{it} - a_{it}$, where w_t is the real wage, determined on an aggregate labour market, while a_{it} is the firm i's real equity position. For simplicity we assume that firms can borrow from banks as much as they want at the market expected real return r, and debt is totally repaid in one period.

The individual demand faced by firms is affected by idiosyncratic real shocks. The individual selling price of the i-th firm is the random outcome of a market process around the average market price of output P_t, according to the law $P_{it} = u_{it}P_t$, with expected value $E(u_{it}) = 1$ and finite variance. Let the I

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[8] For a more complete, agent-based model where aggregate (mean-field) interactions occur through the credit market, see Delli Gatti *et al.* (2005).

[9] The analytical details of the framework used in this paper are discussed at length in Delli Gatti (1999).

random variables $\{u_i\}$ be uniformly, but non-identically, distributed. In particular, support of the i-th firm relative price shock u_{it} is given by $[z_{it}, 2-z_{it}]$, with $\{z_i\} \in (0,1)$ being a *iid* random variable with finite mean and variance. It follows that individual price shocks are characterized by a common and constant expected value equal to 1, but the variance, $V(u_{it}) = \frac{(1-z_{it})^2}{3}$, evolves stochastically. This aims to capture the idea that people usually perceive the same signal in different ways or, alternatively, that people differ as regards the degree of risk they attach to random events to be forecasted.

If a firm cannot meet its debt obligations (in real terms, $\pi_{it+1} < 0$, where π_{it+1} are real profits at the beginning of period t+1), it goes bankrupt. From the assumptions above, it follows that this event happens as the relative price is below a threshold given by:

$$\bar{u}_{it} = R\left(\frac{w_t}{\phi} - \frac{a_{it}}{y_{it}}\right) \tag{1}$$

where $R = (1 + r)$.

Going bankrupt is costly, not only because of direct legal and administrative costs (Altman, 1984; White, 1989), but also because of the indirect costs of bankruptcy-induced disruptions, like asset disappearance, loss of key employees and investment opportunities, and managerial stigma and loss of reputation (Gilson, 1990; Kaplan and Reishus, 1990). Estimates suggests that the costs associated to bankruptcy may be very large (White, 1983; Weiss, 1990). For simplicity, we assume that the real bankruptcy cost C_{it} is an increasing quadratic function of output, so that the expected bankruptcy cost becomes:

$$E(C_{it}) = cy_{it}^2 \Pr(u_{it} < \bar{u}_{it}) = \frac{cy_{it}^2(\bar{u}_{it} - z_{it})}{2(1-z_{it})} = \frac{cy_{it}^2\left[R\left(\frac{w_t}{\phi} - \frac{a_{it}}{y_{it}}\right) - z_{it}\right]}{2(1-z_{it})} \tag{2}$$

where $c > 0$ is a measure of the aversion to bankruptcy on the part of the firm's owners and managers. As recalled above, the magnitude of the parameter measuring aversion to bankruptcy depends on a number of factors, and it can vary with the institutional framework and the economic conditions faced by firms. Clearly, the higher is c, the lower is the incentive to recur to debt in financing production and — as we will see presently — the lower is the level of output maximizing real profits.

While the model could be expanded to more general cases with similar qualitative results - although at the cost of additional remarkable analytical complications – for the sake of tractability we limit the dynamics of the system in a region in which the two following conditions holds true:

C1: The individual demand for credit is always positive, i.e. $d_i > 0, \forall i$.

C2: Expected profits net of expected bankruptcy costs are always positive, so that we are assuming that $\frac{Rw_t}{\phi} < 1$.

The problem of firm i consists in maximizing the expected value of real profits net of real bankruptcy costs:

$$\max_y E(\pi_{it} - C_{it}) = y_{it} - R\left(\frac{w_t y_{it}}{\phi} - a_{it}\right) - \frac{c}{2(1-z_{it})}\left[\left(\frac{Rw_t}{\phi} - z_{it}\right)y_{it}^2 - Ra_{it}y_{it}\right]. \quad (3)$$

From the first order condition it follows that as c grows large[10] the individual supply can be approximated to the linear function:

$$y_{it} \cong \frac{R}{2\left(\frac{Rw}{\phi} - z_{it}\right)} a_{it} = h_{it} a_{it} \quad (4)$$

where h_{it}, being a function of the random variable z_{it}, is a random variable as well, which we assume follows a distribution $\Pi(h_i)$ with finite mean and variance. Individual supply is an increasing linear function of net worth, as a

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[10] Such that the term $\frac{(1-z_{it})\left(1-\frac{Rw_t}{\phi}\right)}{c\left(\frac{Rw_t}{\phi} - z_{it}\right)}$ tends to approach 0. It seem worthwhile to note that this could happen well before c goes to infinity, since both terms at the numerator are lower than 1. In particular, for $c \leq 10{,}000$ the expression above becomes lower than 0.01 but for a negligible number of cases.

higher net worth reduces the marginal bankruptcy costs. In turn, for any given level of net worth higher uncertainty (a realization of the random variable z_{it} close to 0) on the relative price makes production more risky. As firms trying to maximize the concave profit function (3) behaves in a risk-averse manner, higher uncertainty means lower production.

The dynamics of production can be tracked by the evolution of the equity base, which in real terms reads as:

$$a_{it+1} = u_{it} y_{it} - R(w_t n_{it} - a_{it}) + \gamma_i \bar{a}_t \tag{5}$$

where $\bar{a}_t$ is the average capitalization of firms operating at time *t*, and $\gamma_i > 0$. The last term measures the amount of new equity firm *i* can raise on the stock market, assumed to be proportional to the average capitalization of the economy at time *t*. The value of γ_i depends on the level of development of the financial system, that is on the presence of financial instruments, markets and institutions aimed at mitigating the effects of information and transaction costs. Firms operating in economies with larger, more active and more liquid stock markets with a large number of private equity funds are characterized by a higher γ than economies with poorly developed financial systems. From the point of view of each individual firm, however, the *hot market* effect represents an externality arising from a mean-field interaction.

Finally, we assume that at any time *t* the real wage is determined on the labour market according to an aggregate wage setting function, which for simplicity we assume to be linear:

$$w_t = bn_t \tag{6}$$

where n_t is total employment, and $b > 0$. Such a positive relationship between aggregate employment (at given labour supply) and real wage can be alternatively derived from union models, insider-outsider models or efficiency wage models (Lindbeck, 1992).

The parameters γ_i and *b* then capture the interdependence between aggregate outcomes on the financial and the labour markets, respectively, and individual decisions. Their values vary with the institutional context governing both market transactions and non-market interactions. Such an institutionally-constrained microeconomic analysis calls for the search of appropriate macrofoundations of microeconomics (Colander, 1996). The importance of this issue will be further discussed below.

Due to the constant returns technology and the supply function (4), knowledge of a_{it} immediately translates into knowledge, at least in expectational terms, of other traditional measures of firms' size, that is total sales (y_{it}) and employment (n_{it}). By inserting (4) and (6) into (5), and assuming rational expectations so that $u_{it} = 1$ for any i and t, we obtain:

$$a_{it+1} = (h_{it} + R)a_{it} + \gamma_i \overline{a}_t - Rbn_t n_{it}. \tag{7}$$

Heterogeneity enters the model along two margins. First, because of cumulative differences in individual equity bases, which are influenced by past idiosyncratic shocks to relative prices. Second, because of differences in the way agents perceive risk associated to future profits. The dynamical system (7) for $i \in (1, I)$ and its predictions for the firms' size distribution will be analyzed in the next Section.

For the time being we just want to highlight that this economy can display aggregate — i.e., per capita —endogenous fluctuations under rather mild conditions. To do this, let us take the cross-sectional average in both members of (7) to get:

$$\overline{a}_{t+1} = \left(\overline{h}_t + R\right)\overline{a}_t - R\left(\frac{Ib\overline{h}_t^2 \overline{a}_t^2}{\phi^2}\right) + \overline{\gamma}\overline{a}_t . \tag{8}$$

where to simplify calculations we have assumed that the real shock to individual aggregate demand is always equal to its average value, so that $h_{it} = \overline{h}_t$, $\forall\ i$. Simple algebra and a suitable change of variable, i.e. $x_t = R\frac{Ib\overline{h}_t^2}{\phi^2}\overline{a}_t$, allows us to show that equation (8) can be reduced to a logistic map (Iooss, 1979):

$$x_{t+1} = \Gamma x_t \left(1 - x_t\right). \tag{9}$$

where $\Gamma_t = \overline{h}_t + R + \overline{\gamma}$ is the control parameter. It is well known that such a first-order nonlinear map can display deterministic cycles if $3 < \Gamma < 3.57$, and non-periodic — i.e., chaotic — behaviour if $3.57 < \Gamma < 4$ (Baumol and Benhabib, 1989). In this latter case, the time series generated by the onedimensional deterministic difference equation (9) are characterized by irregular fluctuations which can mimic actual data. In particular, the dynamic path of the economy enter a chaotic regions for particular combinations of the average slope of the individual offer functions (itself parameterized by the interest rate and the real product wage), of the interest rate on loans, and of the parameter measuring spillovers in the equity market.

The literature modelling endogenous business fluctuations in terms of non-linear dynamic systems and chaotic attractors is large. See Day (1996) for a nice introduction to methods and applications. In frameworks strictly related to the one we employ here, Gallegati (1994) shows how to derive sudden shifts between the periodic and non-periodic regimes due to stochastic influences on the tuning parameter, while Delli Gatti and Gallegati (1996) introduce technological progress to obtain fluctuating growth.

3. The firms' size distribution

This Section is devoted to analyzing the cross-section dynamics associated to system (7), which amounts to studying the firms' size distribution and its evolution. A natural question is whether such a distribution converges towards a long-run stable (i.e., invariant) distribution, or if it is bound to fluctuate in a random-like manner.

It must be stressed from the start that the aggregate dynamics generated by the aggregate model (8)-(9) resembles the Richard Goodwin's prey-predator growth-cycle model (Goodwin, 1967), which represents the first attempt to adapt the mathematical description of biological evolution due to Alfred Lotka and Vito Volterra to an investigation of the way a modern economy works. In this masterpiece of non-linear economic dynamics, Goodwin re-cast in a new, analytically elegant guise[11] the Marxian analysis of capitalism and its inherent instability. The two classes of capitalist and workers compete for the national income, which at any time period is divided between profits and wages according to their relative strength. During an expansion phase, growing investment opportunities call for an increasing demand for labour. Rising wages and rising employment, in turn, determine an increase in the share of national income going to workers. As the profit

[11] The Nobel Laureate Robert Solow used the following words to comment the 1967 Goodwin's paper: "[It] *is five pages long. It does its business clearly and forcefully and stops. It contains no empty calories*" (Veluppillai, 1990, p.34).

share declines, however, gross investments - and therefore total output - fall. As a result, new job creation shrinks and the unemployed labour force (the so-called *reserve army of the unemployed*) is enlarged. The growth in the unemployed puts downward competitive pressure on wages, and thus provides greater profit opportunities. Larger profits stimulates additional capital accumulation, and economic activity and labour demand increase. The business cycle can begin again.

The story we are telling is basically the same as the Goodwin's one. During an upswing, the increase of output induces higher profits and more private equity funds. Higher production means also rising employment and higher wages, however. The increased wage bill calls for more bank loans which, when repaid, will depress profits and, through equations (4) and (5), the production and the equity level as well. The labour requirement thus decreases, along with the real wage, while profits rise. This restores profitability and the cycle can start again.

Moving from an aggregate to a cross sectional perspective, it seems worthwhile to stress that the predator-prey analogy can be extended to the non-linear dynamical system (7), as it represents a Generalized Lotka-Volterra (GLV) system (Solomon, 2000), whose solution describes the limit (long-run) behaviour of the firms' size distribution as measured by their equity, employment or sales. Such a system is completely defined by: *i*) a stochastic autocatalytic term representing production and how it impacts on equity; *ii*) a drift term representing the influence played – via the *hot market* effect – by aggregate capitalization on the financial position of each firm; and *iii*) a time dependent saturation term capturing the competitive pressure exerted by the labour market.

In what follows we will borrow from the work of Solomon and Richmond (2001), to show that the cross sectional predictions of the GLV system (7) are consistent with the available empirical evidence on how firms are distributed. In particular, the right tail of the firms' size distribution turns out to exhibit a Pareto distribution of the form:

$$P(a) \sim a^{-1-\alpha} \tag{10}$$

over several orders of magnitude. Interesting enough, the GLV model ensures a stable exponent α even in the presence of large fluctuations of the terms parameterizing the economy, namely the random equity productivity term h and the aggregate employment level n.

To see how this result can be obtained, we transform the system (7) from discrete to continuous time to write the time evolution of the equity base of firm *i* as:

$$da_i(t) = a_i(t+\tau) - a_i(t) = [\beta_i(t) - 1]a_i(t) + \gamma_i \overline{a}(t) - \delta(\overline{a},t)a_t(t) \qquad (11)$$

where τ is a (continuous) time interval, $\beta_i = h_i + R$, and $\delta(\overline{a},t) = \frac{Rb\overline{h}^2(t)}{\phi^2}\overline{a}(t)$.

To ensure the limit $\tau \to \infty$ to be meaningful, let $\gamma_i(t)$, $\delta(\overline{a},t)$ and the time averages $s = \langle \beta_i(t) - 1 \rangle$ - equal for all *i* - to be of order τ. Let us define the variance of the random terms β:

$$\sigma^2 = \langle \beta_i^2 \rangle - \overline{\beta}^2 \sim \langle (\beta_i - \overline{\beta})^2 \rangle = \langle [\varepsilon_i(t)]^2 \rangle \qquad (12)$$

where $\varepsilon_i(t) = \beta_i(t) - \overline{\beta}$ represents the stochastic fluctuations of the autocatalytic term. Without any loss of generality, we assume $\langle \varepsilon_i(t) \rangle = 0$.

Note that $\beta_i(t) - 1 = \varepsilon_i(t) + s$, so we can write:

$$da_i(t) = [\varepsilon_i(t) + s]a_i(t) + \gamma_i \overline{a}(t) - \delta(\overline{a},t)a_i(t). \qquad (13)$$

If we introduce the change of variable $\varphi_i(t) = \frac{a_i(t)}{\overline{a}(t)}$, which represents the relative equity of the *i*-th firm, and apply the chain rule for differentials, we obtain:

$$d\varphi_i(t) = [\varepsilon_i(t) - \gamma_i]\varphi_i + \gamma_i. \qquad (14)$$

The system (14) shows that the stochastic dynamics of the relative equity base and, due to the linearity assumptions, of the relative sales and employment as well, reduces to a set of independent, linear equations. If we assume that the variance of the uncertainty associated to the relative price is common to all, so that $\sigma_i^2 = \sigma^2$, and the same is true for the *hot market* influence on individual equity, $\gamma_i = \gamma$, we obtain (Solomon and Richmond, 2001):

$$P(\varphi) \sim \varphi^{-1-\alpha} \exp\left[\frac{-2\gamma}{\sigma^2 \varphi}\right] \tag{15}$$

with $\alpha = 1 + \frac{2\gamma}{\sigma^2}$. The distribution $P(\varphi)$ is unimodal, as it peaks at $\varphi_0 = \frac{1}{1+\frac{\sigma^2}{\gamma}}$. Above φ_0, that is on its upper tail, it behaves like a power law with scaling exponent α, while below φ_0 it vanishes very fast.

The theoretical value of the scaling exponent α is bounded from below to 1,[12] it increases with γ and it decreases with σ^2. In other terms, α depends exclusively on: *i*) how much firms are rationed in issuing new risk capital or, in other terms, on how much capital markets are affected by adverse selection and moral hazard phenomena; *ii*) how much individuals are heterogeneous as regards the perceived risk associated to their final demand. Given that both the degree with which these two factors bites and the array of institutions set up to confront them differ widely among countries, besides predicting that the firms' size distribution scales down as a power law, our model suggests that the degree of industrial concentration should be country-specific.[13]

Both the shape of the distribution $P(\varphi)$ - and therefore of $P(a)$, $P(y)$ and $P(n)$ - and the heterogeneity of scaling among countries are consistent with the empirical evidence. Axtell (2001) reports estimates for the whole universe of the U.S. firms as derived from the Census of Manufactures, suggesting that the firms' size distribution is Pareto distributed with α = 1. Other studies concentrate on large firms in international samples, largely confirming Axtell's results. Gaffeo *et al.* (2003), for instance, show that the upper tail of the firms' size distribution as derived for a pool of quoted large companies from the G7 countries is Pareto, and that the scaling exponent is comprised between 1 (total sales) and 1.15 (total assets) as one changes the proxy used to

[12] The case α = 1 is also known as the Zipf's law.

[13] Naldi (2003) derives analytical relationships between the scaling exponent of a power law distribution and several major concentration indices, like the Hirschman-Herfindahl and the Gini indices.

measure the firms' size. Ramsden and Kiss-Haypál (2000), furthermore, provide evidence on the larger firms for a sample of 20 countries: for 16 of them, the firms' size distribution scales as a power law with α comprised between 1 and 1.25.[14]

Interestingly enough, α does not directly depend on how firms interact in the labour market, which can thus be modelled recurring to alternative assumptions without affecting the shape of the long run firms' size distribution. To put it in another way, our model suggests that a necessary condition for the firms' size to evolve towards a power law distribution is to let firms' decisions be affected by competitive pressures from the labour market, but also that the shape of the distribution is invariant to details. In particular, it is invariant to the precise form of the relation between wages and unemployment, to the way the productivity dynamics or fiscal variables affect it, and consequently to the level of the natural rate of unemployment.

Furthermore, our model does not possess the degenerative asymptotic behaviour affecting both the basic Gibrat's model and its numerous extensions, that is a long run dynamics which implodes or explodes but for a tiny (and empirically implausible) set of values of the average stochastic multiplicative shock. In particular, the industrial dynamics evolves towards a stable distribution with a Pareto-Levy upper tail even if the dynamics of the equity productivity term h is nonstationary, or the parameter b summarizing how the labour market works is subject to sudden, large shifts.

Finally, it is worthwhile to note that the parameter γ — a proxy for the level of development of capital markets — tunes at the same time the dynamic features of aggregate fluctuations and the longitudinal characteristics of microeconomic units. Suppose γ is initially at a low value. The design and implementation of new financial regulations abating moral hazard and adverse selection problems, or the emergence of market-based financial innovations aimed at easing risk diversification, translate into an easier access to new equity financing: γ increases for all firms, regardless of their leverage position. *Ceteris paribus*, this results in a more egalitarian distribution of firms' size (a higher α), due to higher positive externalities in external finance. The increase of γ, in turn, causes an increase of the control parameter of the logistic map Γ, which may move the system's aggregate (i.e., per-capita) dynamics from, say, a periodic oscillation (limit cycle) regime to a

[14] The exceptions are China (α = 0.83; data referred to 1993), Hungary (α = 0.71; data referred to 1992), South-Africa (α = 2.27; data referred to 1994) and U.S.A. (α = 0.8; data referred to 1994).

chaotic one. In our model, reducing the inequality of access to investment opportunities may be a necessary condition for an increase in macroeconomic volatility. The intuition for this result is straightforward. A higher γ implies more powerful spillovers stemming from average capitalization towards individual laws of motion for equity. Higher individual equity bases, however, cause firms to demand more labour and, due to condition C1, to more debt. A higher labour demand, in turn, implies a higher real wage which drives towards lower profits. From an analytical viewpoint, as spillovers raise the curvature of the logistic map (9) increases, and the per-capita dynamics crosses several bifurcations migrating from a stable regime to a chaotic one.

Incidentally, our findings lend some theoretical support to the argument put forth by authoritative commentators (see e.g. ECB, 2007), according to whom the spurt of volatility occurred in international markets during the summer of 2007 has been partly due to the rising leverage of the corporate sector associated to the massive increase of private equity-sponsored leveraged buyout deals occurred since 2003.

4. Conclusions

The relation among the firms' size and growth rates, their longitudinal distribution and macroeconomic activity represents an issue of major importance for the economic profession as a whole. A proper understanding of why firms grow (or do not grow) and how they distribute in the long-run may help in understanding whether microeconomic constraints are important for macroeconomic growth (Kumar *at al.*, 2001). Furthermore, realizing that firms respond differently – and, in particular, assessing how different their responses are – to the business cycle may help in designing policies to reduce firms' vulnerability (Higson *et al.*, 2004) or to foster job creation at an aggregate level (Hart and Oulton, 2001).

In this chapter we show how heterogeneity in the firms' financial position and the competitive pressure exerted by their indirect interactions on the labour and the equity markets can combine to explain both macroeconomic fluctuations and the cross-sectional distribution of microeconomic variables. In particular, a key stylized fact established by empirical studies on longitudinal firm data – the firms' size distribution is right-skewed, and its upper tail scales as a power law – can be interpreted in terms of a stable, steady-state statistical equilibrium associated to microeconomic dynamics in a maximizing, rational expectations model. Our findings suggest that the degree of long-run heterogeneity, measured by the scaling exponent characterizing the right tail of the firms' size distribution, depends on institutional factors, i.e. how financial markets work, and the degree of uncertainty affecting economic agents.

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