The Measurement of Dynamic Poverty With Geographical and Intertemporal Price Variability

Christophe Muller

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Centre for the Study of African Economies
Institute of Economics and Statistics
University of Oxford
St Cross Building
Manor Road
Oxford OX1 3UL

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Abstract: Little attention has been devoted to the effects of absolute and relative prices variability at local and seasonal levels, for the measurement of living standards in LDCs. In particular, it is not known if a substantial share of welfare or poverty indicators may be the consequence of price differences rather than of differences in living standards across households and seasons. With exogenous poverty lines, we show how the directions of effects of accounting for price variability can be theoretically established for popular poverty indices.

With endogenous poverty lines, using data from Rwanda, we show that the composition of the population of the poor can be notably modified by accounting for price variability. The change in aggregate living standards due to price correction is moderate although significant in every quarter, in contrast with the change in poverty which can be considerable. The correction yields generally a larger transient seasonal share of annual poverty. In terms of impact of the price correction on the assessment of poverty, the poverty line or the quarter are generally more influential than the formula of the poverty indicator. Though, poverty indicators giving a high importance to the severity of poverty are more likely to lead to a strong effect of prices. However, the sign of change in poverty indicator is not systematically related with parameters of poverty indices or with poverty lines. These results support the necessity in poverty measurement, of an accurate correction for geographical and seasonal price effects, whose directions cannot be easily predicted. Nonetheless with exogenous poverty lines, for an important class of axiomatically valid poverty indices, the change in poverty with compensation for keeping the harmonic aggregate mean of price indices constant, is always positive.

Résumé

Peu d’attention a été accordé aux effets des variabilités relative et absolue des prix aux niveaux locaux et saisonniers, pour le mesurement des niveaux de vie dans les PVD. En particulier, on ne sait pas si une part substantielle des indicateurs de bien-être social ou de pauvreté peut être la conséquence de différences de prix plutôt que de différences de niveaux de vie entre ménages ou saisons. Avec des lignes de pauvreté exogènes, nous montrons comment la direction des effets de la prise en compte de la correction pour la variabilité des prix peut être théoriquement établi pour des indices de pauvreté populaires.

Avec des lignes de pauvreté endogènes, à partir de données du Rwanda, nous montrons que la composition de la population de pauvres peut-être notablement modifié par la prise en compte de la variabilité des prix. Le changement des niveaux de vie agrégés du à la correction des prix est modéré bien que significatif à chaque trimestre, contrairement au changement de la pauvreté qui peut être considérable. La correction résulte en général en une plus grande part transitoire saisonnière de la pauvreté annuelle. En termes de l’impact de la correction des prix sur l’évaluation de la pauvreté, la ligne de pauvreté ou le trimestre ont généralement plus d’influence que la formule de l’indicateur de pauvreté. Pourtant, les indicateurs de pauvreté accordant une importance élevée à la sévérité de la pauvreté sont plus susceptibles de conduire à un fort effet des prix. Cependant, le signe du changement de l’indicateur de pauvreté n’est pas systématiquement relié aux paramètres des indices de pauvreté ou les lignes de pauvreté. Ces résultats supportent la nécessité pour la mesure de la pauvreté d’une correction précise des effets prix géographiques et saisonniers, dont les directions ne peuvent être facilement prédites. Toutefois, avec des lignes de pauvreté exogènes et pour une classe importante d’indices de pauvreté axiomatiquement valides, le changement de pauvreté avec compensation gardant constant l’indice harmonique agrégé des prix, est toujours positif.

1. Introduction

The design of policies against poverty (see The World Bank (1990)) necessitates an accurate measurement of household living standards. Atkinson (1987), Lipton and Ravallion (1993) and Ravallion (1994) provide general discussions of poverty measures, in which the importance of a careful measurement is a permanent concern. This careful measurement is all the more difficult that due to the high seasonal variability of agricultural output in poor agricultural economics and to the existence of liquidity constraints, prices and living standards of peasants in LDCs fluctuate considerably across seasons. Besides, dynamic poverty analysis is widely considered as at the frontier of the knowledge in poverty analysis. Chaudhuri and Ravallion (1994) show that no static indicator can precisely approximate even the averaged dynamic poverty, which justifies to calculate both chronic and transient poverty indices. In a preceding article (Muller (1997)), we have shown that the seasonal transient component of annual poverty in Rwanda is very substantial and cannot be neglected. Finally, another difficulty arises from the fact that because of high transport and transaction costs as well as deficient information in underdeveloped economies, the price of the consumed goods are generally quite variable across the regions.

The treatment of geographical and temporal price variabilities is crucial. Indeed, if the correction for differences in prices that distinct households face at separate periods, is inaccurate, then apparent welfare fluctuations (or differences between households) might be resulting only from unaccounted large price differences. In that situation, household living standards could be more (or less) stable or heterogeneous than they appear to be.

On the one hand, price indices have been the object of extensive economic analyses, often derived from consumer theory (see for example: Fisher and Shell (1972); Pollak (1978); Diewert (1981); Foss, Manser, Young (1982); Baye (1985); Pollak (1989); Diewert (1990), Selvanathan and Rao (1995)), and have been successfully used in applied welfare analysis, particularly for inequality studies (Muellbauer (1974); Glewwe (1990)). Theoretically, most price indices can be considered as ratios of cost functions representing the preferences of agents. In practice, applied price indices are most of the time ratios of bilinear functions of prices and consumption quantities, such as Laspeyres or Paasche price indices.

Note that using directly Laspeyres and Paasche price indices without availability of local market prices can produce spurious estimates of poverty. Indeed, in that case the household-level prices are generally calculated from ratios of observed values and observed quantities ("unit values"), corresponding to the consumption of categories of goods by the considered household (or group of households). We denote these indices "unit-values" indices. Such procedure entails endogeneity problems similar to those occurring in demand system estimation (Deaton (1988, 1990))[1]. In that situation, a higher level of prices for a specific household (for instance a rich one), might derive from a higher quality of its consumption. Moreover, consumption data is known to incorporate large measurement errors, that can be amplified by the use of unit values instead of exogenous prices. See also INSEE (1987) for a discussion of the necessity of discarding from the price index anything that is not pure variation of prices. Of course, when no price data is available, unit-values Paasche or Laspeyres indices might well be better than no correction at all.

[1] "It is not possible to use unit values as direct substitutes for true market prices in the analysis of demand patterns. Consumers choose the quality of their purchases, and unit values reflect this choice" (Deaton (1988)).

On the other hand, to the best of our knowledge, no poverty analysis with price correction involving local *and* seasonal prices, and no statistical analysis of the impact of such correction is present in the literature. In cross-section poverty measurement, many authors use unit-values Laspeyres and Paasche indices at the national or regional level. Other ones use aggregate Laspeyres and Paasche indices based on regional prices ((Grootaert and Kanbur (1994), Jalan and Ravallion (1996), Grootaert and Kanbur (1996)). In some instance (Grootaert and Kanbur (1994)), it has been noticed that using different such indices can yield different poverty levels, even if no statistical tests of such differences have been formally implemented. In other cases (Slesnick (1993)), the use of different price indices (including true price indices) does not produce very different sets of poverty rates. In practice, the used indices are often rough both in their coverage and their accuracy. However, we suspect that in many poverty studies, correction with local prices might have been implemented without much notice in the articles[2]. In any case, the impact of this correction on poverty has not been systematically analysed which is our intention in this paper.

Finally, scant attention has been paid to the role of price variabilities in the measurement of fluctuations of poverty. The treatment of price variability is very varied in the literature dealing with living standard fluctuations. It refers sometimes to a standard national inflation index (for example in Rodgers and Rodgers (1992), Slesnick (1993)); or is not specified in the article (for example in Bane and Elwood (1986), Stevens (1995), Jalan and Ravallion (1996)). It is well known that inflation, and relative and geographical price variability are only partially positively related (Glezakos and Nugent (1986), Danziger (1987), Domberger (1987)). Moreover, some categories of goods are characterised by much larger price fluctuations than other ones, with these fluctuations having a substantial local component (Riley (1961)). This is particularly true in agricultural context. This implies to control for the price variability with household-level (or local) price indices rather than national or regional inflation indicators, and to account for the seasonal variability of prices as well as the annual one. The price fluctuations may have serious implications in terms of welfare analysis (Jazairy, Alamgir and Panuccio (1992)). Baris and Couty (1981), for example, suggest that in Africa the seasonal variations of prices may aggravate the social differentiation. Slesnick (1993) discusses the distributional impact of relative price changes. However, he uses price indices only to adjust poverty lines over time using an inflation index, and not across households (the prices are assumed to be the same for all households).

Can we obtain theoretical results about the role of prices in poverty measurement ? Is the impact of the correction of price variability, statistically significant for aggregate living standards and poverty measurement ? Can we find systematic effects in dynamic poverty indicators, induced by absolute and relative prices variabilities at local and seasonal levels ? Are these effects different, for different seasons, different poverty indicators or different poverty lines ? Is the share of the transient component in the annual poverty, affected by the price correction ? Does the composition of the population of the poor, depend on this correction ? The aim of this article is to answer these questions by first providing a theoretical framework, secondly by studying the effects of the correction for the price variability on seasonal, transient and chronic poverty indicators, using data for peasants in Rwanda.We define poverty and price indices and we present the sampling estimators in section 2. We analyse the theoretical effects of the price variability on poverty measures with exogenous poverty lines, in section 3. In particular, we provide a decomposition in terms of effects of pure variability at constant aggregate price level, and of

[2] We have heard of a study by Azam and Ravallion using data from Morocco, in which price correction at the village level would have been implemented.

effects of the change in aggregate price level for several popular poverty indicators. We describe the data used in the estimation in section 4. In section 5, we present estimation and test results for aggregate living standards and poverty measures with endogenous poverty lines, and we study the variation of the population of the poor. Finally, we conclude in section 6.

2. Definition and Estimators of Poverty Indices and Price Indices

2.1. Price indices

We calculate elementary price indicators of the different categories of goods, for every season and every cluster of the sample (corresponding to three neighbouring households). The prices of every category are represented by the price of the main good in the category, which allows the comparability of prices across seasons and regions with little quality bias. We obtain optimal price indicators by comparing "market prices" (from price surveys in the market), prices extracted from observations of elementary consumption expenses[3] ("consumption prices"), and prices extracted from observations of elementary output sales ("production prices"), at different geographical and temporal aggregation levels for every good. At each stage of the algorithm of calculus of the price indicators, we account for the number of observations for each type of prices (controlling for the representability of means of recorded prices) and for their plausibility (controlling for measurement errors). Thus, we are able to compute price indicators that are close to market prices and possess good statistical properties.

True price indices could be derived from the estimation of a complete demand system (as in Braithwait (1980)[4] and Slesnick (1993)). However, this would introduce some variability stemming from the inaccuracy of estimates from a small sample. Moreover, because of market imperfections and high own-consumption rates, production and consumption decisions of most agricultural households are likely to be non separable. For this reason, shadow prices (deduced from the separating budget constraint of an agricultural household model (Pollak (1978), Singh, Squire and Strauss (1986)), would be more adapted to the calculus of price indices for this type of household[5]. However, these shadow prices are unobserved and their estimation from a complete agricultural household model would as well produce noisy estimates. Lanot and Muller (1997) is an example of simultaneous estimation of preferences and technology in nonlinear nonseparable behavioural model, which shows that the information requirement for such an estimation is substantial. Despite its theoretical attractiveness, this approach does not seem tractable with the present data set.

In an aggregate annual approach, because of the high own-consumption ratios observed in the sample, these shadow prices are expected to be intermediate between observed consumption prices and observed production prices (de Janvry, Sadoulet, Fafchamps (1991)). An

3 "Elementary" means here calculated as ratios of values and quantities of individual records of consumption expenses.

4 Braithwait found a 1.5 percent bias of the overall Laspeyres price index over the period 1958-1973 in U.S.. Such magnitude seems much below that of the other sources of errors in our problem.

5 The effect of observed prices may imply conflicting effects on the consumer-side and the producer-side of the household behaviour (see for example Besley and Kanbur (1988)), which cannot be properly analysed using a mere consumer model with observed prices. However, such considerations are relevant for the analysis of global household behaviour, but not necessarily for the measurement of living standards. In the latter case, all what we would need are shadow prices associated with the separating hyperplane in the optimisation program representing the household behaviour.

examination of national averages of the different observed prices for several products in the survey shows that differences between consumption and production prices may be substantial: 24.4% for beans; 31.5% for sorghum; 36.8% for potatoes; but only 1.7% for sweet potatoes. However, at the local geographical and temporal level, consumption prices correspond better to the timing of the observed consumption of households, and market prices have been specifically collected to valorise the observed food for consumption. These market prices are close to the consumption prices and are a good approximation of purchase (and even sale) prices at the very moment of consumption, although they do not account for transport and other transaction costs. The average market price minus the average consumption price is : 2.3% for sorghum; 1.1% for potatoes; 2.1% for beans. Because of their temporal proximity with the actual consumption, average observed market and average consumption prices, at the cluster level, are considered as a reasonable approximation for shadow prices, and we use them when possible, in the calculus of price indices.

We approximate the theoretical price index by a Laspeyres price index (I_{it}) specific to each household and each quarter, in which the comparison basis is the annual national average consumption.

$$I_{it} = \sum_j \omega^j \frac{p_{gt}^j}{p_{..}^j} \quad (1)$$

$$where \ \omega^j = \frac{\sum_i \sum_t POND_{it} p_{it}^j q_{it}^j}{\sum_j \sum_i \sum_t POND_{it} p_{it}^j q_{it}^j}$$

is the weight of good j in the price index, $POND_{it}$ is the sampling weight of household I belonging to cluster g at date t, corrected for missing values. p_{gt}^j (resp. p_{it}^j) is the price of good j in cluster g in which household i is observed (resp. for household I); q_{it}^j is the consumed quantity of good j by household I at date t (in cluster g)[6].

The annual national price of good j, $p^j_{..}$ is calculated as follows:

$$p_{..}^j = \frac{\sum_i \sum_t q_{it}^j p_{it}^j POND_{it}}{\sum_i \sum_t q_{it}^j POND_{it}} \quad (2)$$

The living standard indicator for household I at period t is

[6] The Laspeyres price index is an upper bound of the true price index of the consumer theory, when the reference utility level is the basis utility level, u_0. The Laspeyres price index can also be considered as a first order approximation of the true price index at utility level u_0 (Deaton and Muellbauer (1980)).

$$y_{it} = \frac{c_{it}}{S\ I_{it}} \quad (3)$$

where c_{it} is the value of the consumption of household I at period t ; S is the household size and I_{it} is the price index associated with household I and period t . We denote $x_{it} = c_{it}/S$, the living standard indicator non corrected for price variability (nominal living standard)[7].

3.2. Poverty indices

Most of poverty indices used in applications are additively decomposable, and can be written in the following form

$$P = \int_0^z f(y,z)\ d\mu(y) \quad (4)$$

where μ is the probability distribution of living standards y, and z is the poverty line. This formula can be used with quarterly living standards y_t and a quarterly poverty line to yield the quarterly poverty index P_t at period t. We consider the most popular of these indices.

We first define the Foster-Greer-Thorbecke (FGT) poverty indices with a = 0, 1, 2, 3, the poverty aversion parameter of the public planner[8].

$$FGT_a = \int_0^z (1-y/z)^a\ d\mu(y) \quad (5)$$

The Watts' poverty index (Watts (1968)) is defined as

$$W = \int_0^z -\ln(y/z)\ d\mu(y) \quad (6)$$

and the CHU poverty indices (Clark, Hemming and Ulph (1981)) are:

7 We have checked that using other equivalence scales do not substantially change the results of this paper. Eq. (3) means implicitly that different prices has been used for the calculus of the consumption and of the price index, so as to avoid quality effects in the estimation of living standards. Other elements could have been included in the economic welfare such as leisure time (Riddell (1990)), but would have created intractable valorisation difficulties.

8 See Foster, Greer and Thorbecke (1984) for the detailed properties of this family of indices. In particular, they are additively decomposable, and satisfy the monotonocity axiom (for a > 0), the transfer axiom (for a > 1), the transfer sensitivity axiom (for a > 2), and the subgroup monotonocity axiom.

$$CHU_c = \int_0^z 1-(y/z)^c \; d\mu(y) \quad (7)$$

where c is a positive parameter[9].
AP is the mean of the four quarterly poverty indices P_t.

$$AP = \frac{1}{4} \sum_{t=1}^{4} P_t \quad (8)$$

AP is the mean poverty over the year, which we denote "annual poverty". An other way to write eq. 8 is:

$$AP = \frac{1}{NT} \int\int f(y_t,z) \; d\mu_0(y_t)\, d\nu(t) \quad (9)$$

where y_t is the living standard variable at period t, N is the sample size and T is the number of considered periods. μ_0 is here a measure with μ_0 /N the associated probability distribution. ν is the counting measure. Note that μ_0 and ν are not independent because of the temporal dependence of the variable y_t. However, if we consider the living standard observations y_{jgt}, where j is the individual number in the cluster, g the cluster and t the period:

$$AP = \frac{1}{JGT} \int\int\int f(y_{jgt},z) \; d\mu_1(j)\, d\mu_2(g)\, d\nu(t) \quad (10)$$

where J is the average number of households in a cluster and G the number of clusters. μ_1 and μ_2 are measure functions associated with the respective drawing of households in clusters and with the drawing of clusters. The correspondence between (j, g, t) and y_{jgt} shows that j, g and t play similar roles. The temporal variabilities (associated with t) and the geographical variabilities (associated with g), are therefore treated similarly in the specification of annual poverty. This symmetry suggests using the national annual level as a reference for price indices associated with cluster-quarter couples. The same type of considerations could be developed directly with y_{jgt}.

CP denotes the poverty index directly calculated from the sole observation of the annual living standard. It describes the level of annual poverty at stabilised living standard, and represents the "annual chronic poverty". Ravallion (1988) suggests that the “transient poverty”, TP = AP - CP is the poverty increase that can be attributed to the variability of living standards

9 The Watts index satisfies the monotonicity, transfer and transfer sensitivity axioms. The CHU indices satisfy the monotonicity axiom. They satisfy transfer and transfer sensitivity axioms for c < 1.

(when AP > CP)[10]. A natural measure of the proportion of poverty caused by the fluctuations of living standards is the ratio:

$$F = \frac{(AP - CP)}{AP} \quad (9)$$

As we said above, the living standard indicator of household i at date t is the ratio $y_{it} = x_{it}/I_{it}$ where x_{it} is the nominal living standard indicator. In the case of general and proportional inflation, the correction of prices is equivalent to a change in the poverty line, in most of poverty indicators, as it is clear in eqs. 5, 6 and 7.

3.3. Estimators

We estimate the poverty indices at period t, with ratios of Horwitz-Thompson estimators:

$$\hat{P}_t = \frac{\sum_{s=1}^{n} \frac{f(y_{st},z) 1_{[y_{st}<z]}}{\pi_{st}}}{\sum_{s=1}^{n} \frac{1}{\pi_{st}}} \quad where \ \pi_{st} = \frac{m_h r_{hij} q_{hijk}}{M_h N_{hi} R_{hij} Q_{hijk}} \quad (10)$$

π_{st} is the inclusion probability (in the sample) of household s at date t (s = 1,...,n); f is the kernel function associated with the poverty index; M_h is the number of communes in strata h; m_h is the number of communes drawn in strata h; N_{hi} is the number of sectors in commune i of strata h (only one sector is drawn in any drawn commune), R_{hij} is the number of districts in sector j of commune i of strata h, r_{hij} is the number of drawn districts in sector j of commune i of strata h, Q_{hijk} is the number of households in district k of sector j of commune i of strata h, and q_{hijk} is the number of households drawn in district k of sector j of commune i of strata h.

The sampling standard errors of the poverty indicators are less easy to estimate, because of the complexity of the actual sampling scheme.[11] In particular, only one sector was drawn at the second stage of the sampling plan in every primary unit, which does not allow the direct calculus of the inter-strata variance. Another difficulty is the small sample size at several stages of the sampling scheme. This does not enable a robust use of classical sampling variance formula that are based on asymptotic properties at every stage. We use an estimator for sampling standard errors inspired from the method of balanced repeated replications (see Krewski and Rao (1981), Roy (1984) for discussions of the properties of this type of estimator). This estimator is adapted to the actual survey (see appendix 1).

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10 More subtle definition of "chronic" and "transient" poverty could be considered in an analysis of poverty. However, this would imply to distinguish risk-aversion, intertemporal substitution and poverty aversion in poverty indices, and would lead to poverty indices incorporating a too large number of parameters to yield unambiguous results. Here, we reject too general specifications and we prefer using simple but robust definitions.

11 See Gouriéroux (1981) for discussion of estimators in sampling theory. Kakwani (1993) provides an estimator for sampling standard errors of poverty indices, but it is only valid for simple random sample frame, which is not the case in most national surveys.

Calculating sampling errors for poverty indices and differences in poverty indices is particularly important in this context. Indeed, the difference in poverty measures with and without accounting for prices can only be analysed in terms of statistical significance. Moreover, the sample size is small enough to put such a priori significance into question. We shall show that even with this small sample size, significant differences can occur. This has been made possible thanks to the stratification involved in the sampling scheme, which enhances a lot the accuracy of estimators. Finally, we did not consider the sampling error in the consumer price index itself (Wilkerson (1966)), which would be beyond the scope of this paper.

4. Effects of Price Variability on Poverty

4.1. The Problem

The following theoretical analysis is important on several grounds. It provides results for a general question for which very little insight seem to be a priori available. It shows off the structure and the difficulties of the theoretical problem. It suggests that a direct empirical analysis is the best way to overcome the problems met in the theoretical analysis.

We can rewrite eq. (4) in terms of the two following distributions: the distribution of I conditionally to x, and the marginal distribution of x (respectively denoted by their cumulative distribution functions, F_2 and F_1).

The kernel function f(x/I, z) must be integrable with respect to $F_1 \otimes F_2$ over the set $\Omega = \{(x, I) \mid x > 0; I > 0 ; x/I < z \}$. These conditions are satisfied with usual functional forms for f. Conditioning with respect to the value of x, yields

$$P = \int_{0}^{+\infty} \int_{x/z}^{+\infty} f(\frac{x}{I},z) \; dF_2(I|x)dF_1(x) \qquad (11)$$

with F_1 the c.d.f. of the marginal distribution of x, and F_2 the c.d.f. of the distribution of I conditionally to x.

We compare now the poverty index without price correction (I = 1 by convention for every household, the nominal living standard x being directly used in the calculus of poverty indicators), with the poverty index accounting for price variability (using corrected living standard indicators, y = x/I). ΔP is the variation of the poverty index due to the incorporation of the price index I.

Proposition 1:

The variation of the poverty index due to the price correction is

$$\Delta P = \int_0^{+\infty}\int_{x/z}^{+\infty} f(\frac{x}{I},z)\ dF_2(I|x)dF_1(x)$$

$$- \int_0^{z} f(x,z)\ dF_1(x) \qquad (12)$$

We shall follow two stages in the analysis of ΔP. First, we shall condition the formula of P with respect to x and investigate the case of independence of prices and nominal living standard. Second, we shall examine the general case of dependance between x and I.

4.2. Conditioning with respect to x

We show now that under convexity conditions for an appropriate function defined from z and f, using the Jensen's inequality provides information about the direction of the contribution of the price variability in the poverty measure. The discussion can be related to analyses of risk in additive indicators of social welfare (Atkinson (1970), Rotschild-Stiglitz (1970), Ravallion (1988), Lambert (1993)). The price-corrected poverty, P, is given by eq. (11).

Proposition 2:

Let $K_x(I) = f(x/I)$ and $L_x(I) = K_x(I).1_{[x/I < z]}$.

If L_x is a convex function in I, we have

$$P \geq \int_0^{+\infty} f\left(\int_0^{+\infty} \frac{x}{I}dF_2(I|x), z\right)\ 1_{\left[\int_0^{+\infty} \frac{x}{I}dF_2(I|x) < z\right]} dF_1(x) \quad (13)$$

So, in favourable cases, upper or lower bounds of P can be defined in terms of a pseudo-poverty index in which the welfare variable has been replaced by a weighted average welfare, in which the weights depend on the values of I and x.

If moreover I is independent of x, we obtain

$$P \geq \int_0^{+\infty} f(\frac{x}{H}, z)\ 1_{[\frac{x}{H}<z]}\ dF_1(x) = \int_0^{zH} f(\frac{x}{H}, z)\ dF_1(x)$$

$$= \int_0^{z} f(v, z)\ dF_v(v) \quad (14)$$

here H is the harmonic average price index, and (v, F_v) is obtained from (x,F_1) by the change in variable: v = x/H.

Proof: We start from

$$P = \int_0^{+\infty} \int_0^{+\infty} f(\frac{x}{I}, z)\, 1_{[V(x/I)<z]}\ dF_2(I|x)\ dF_1(x)$$

$$= \int_0^{+\infty} \int_0^{+\infty} L_x(I)\ dF_2(I|x)\ dF_1(x)$$

When L_x is convex in I, we can apply Jensen's inequality. If moreover I and x are independent, the integral corresponding to the inverse of the geometric mean of prices, can be isolated as a product by x in function f, in the r.h.s. of the inequality. QED.

Notice that the case L_x concave is not possible since f(V(x/I)) > 0 and L_x (I) = 0 on [0, x/z[. However, consequences of the concavity of K_x can still be explored.

The right-hand-side terms in ineq. (14) can be interpreted as a poverty index non corrected for price variability, associated with a distribution of living standards transformed by the homothety of ratio 1/H, from the original one. When the harmonic mean of price indices, H, is close to 1, it is approximatively equal to non corrected poverty. H = 1 provides a reference to give sense to the notion of "stability in aggregate level" of prices. In that sense, when the "aggregate level of prices is stable" and L_x convex, non-corrected poverty indices underestimate the actual poverty.

Proposition 3:

If prices and living standards are independent, and if K_x is concave in I, we have:

$$P \leq \int_0^{+\infty} f\left(\frac{x}{H_x}, z\right)\ 1\left[\frac{x}{H_x} < z\right] dF_1(x) \quad (15)$$

where H_x is the harmonic average of price indices I ∈ [x/z , +∞] .

Moreover, if x^*, the highest admissible level of nominal living standards, is assumed finite, then there exists an harmonic average of prices in the interval [x^*/z, +∞[,

H^* , such that

$$P \le \int_0^{+\infty} f\left(\frac{x}{H^*} ,z \right) 1\left[\frac{x}{H^*} < z \right] dF_2(x) \qquad (16)$$

Proof: In appendix.

I ∈ [x/z , +∞ [defines a price index H_x which depends on and is increasing in the level of the nominal living standard x. It therefore cannot be extracted from the integral.

Eq. (16) shows that if there exists a maximal living standard, the corrected poverty is majored by a pseudo-poverty index for a distribution of living standards transformed by an homothety of ratio $1/H^*$. Unfortunately, it is unlikely that this price index would be close to 1 and could be attached to a reasonable notion of stability of prices. However, if x^* is not too high, corresponding to a low inequality, eq. (16) may provide a useful upper bounds of corrected poverty by a non-corrected poverty index.

Results established in propositions 2 and 3 can be used to discuss the deviation of the corrected poverty from the poverty measured with a fixed level of prices, I^*. Choosing $I^* =$ H or $I^* = H^*$ yields results with explicit signs, while choosing the arithmetic average does not lead to any insight. Indeed, there is no reason to favour the use of the norm L_2 in the present problem, by contrast with the case of variance analysis or in linear models. From proposition 2, we obtain a decomposition of the bias due to the prices effects, in effects of price variability at constant aggregate level of prices, plus effects of change in aggregate level of prices.

Proposition 4:

If prices and nominal living standards are independent, if L_x is convex, if the aggregate level of prices is defined by $I^* = H$, the harmonic average of price indices. Then:

(a) The variation of poverty due to the correction for the price variability is such that

$$\Delta P = \Delta P_1 + \Delta P_2 \qquad where$$

$$\Delta P_1 = P - \int_0^{z} f\left(\frac{x}{H} ,z \right) dF_1(x) \qquad (17) \ and$$

$$\Delta P_2 = \int_0^{z} f\left(\frac{x}{H} ,z \right) dF_1(x) - \int_0^{z} f(x ,z) dF_1(x) \qquad (18)$$

ΔP_1 is the deviation in poverty due to the variability of prices at constant level.
ΔP_2 is the deviation in poverty due to the change in the level of prices.

(b) $\Delta P_1 \ge 0$,

(c) $\Delta P_2 \ge 0$ if $H >1$, and $\Delta P_2 \le 0$ if $H <1$.

Proof: (a) and (b) are the direct consequences of proposition 2. (c) follows from the fact that for all x, f(x/H, z) > (resp. <) f(x, z) if H > 1 (resp. H < 1). QED.

From proposition 3, we obtain a similar decomposition:

Proposition 5:

If prices and nominal living standards are independent, if K_x is concave, if there exists a maximal living standard x^*, if the aggregate level of prices is defined by $I^* = H^*$, the harmonic average of price indices in $[x^*/z , +\infty[$. Then:

(a) The variation of poverty due to the correction for the price variability is such that

$$\Delta P = \Delta P_3 + \Delta P_4 \qquad \textit{where}$$

$$\Delta P_3 = P - \int_0^z f\left(\frac{x}{H^*}, z\right) dF_1(x) \qquad (19) \ \textit{and}$$

$$\Delta P_4 = \int_0^z f\left(\frac{x}{H^*}, z\right) dF_1(x) - \int_0^z f(x, z)\, dF_1(x) \qquad (20)$$

ΔP_3 is the deviation in poverty due to the variability of prices at constant (high) aggregate level H^*.

ΔP_4 is the deviation in poverty due to the change in the (high) aggregate level of prices between H^* and 1.

(b) $\Delta P_3 \leq 0$,

(c) $\Delta P_4 \geq 0$ if $H^* > 1$, and $\Delta P_4 \leq 0$ if $H^* < 1$.

Proof: Similar to proposition 4.

4.3. Cases of usual poverty indices

When f is of type C^2 , we can study the concavity and convexity of K_x (I) by derivation. $K'(I) = f'(x/I).(-x/I^2)$ is positive for well behaved poverty indicators (f decreasing in y).

$K_x''(I) = f''(x/I).(x^2/I^4) + f'(x/I).(2\, x/I^3)$.

The last term of the r.h.s. terms of the preceding equation is negative, and since the sign of the first term is indeterminate, the sign of K_x'' is generally ambiguous. However, if f is such that $f'' < 0$, then K_x'' is unambiguously negative for all I, which implies that K_x is concave. Unfortunately, for most used indicators, we have $f'' > 0$, which corresponds to desirable axiomatic properties, and a direct examination of special cases is therefore necessary.

Proposition 6: concavity of K_x .

We assume y = x/I < z. Then,

** K_x'' is of the sign of $\xi = y f''(y) + 2f'(y)$.*

** **For poverty indices FGT(a)** : $f(y) = (1-y/z)^a$ with $a \geq 0$. $\xi \leq 0$ iff $y < 2z/(a+1)$ for the poor, or a=0.*
If $0 < a \leq 1$, then K_x is concave.
If $a > 1$ and all $y < 2z/(a+1)$ for the poor (i.e. there are only very poor households), then K_x is concave.
If $a \geq 1$ and all $y > 2z/(a+1)$ for the poor (i.e. there are only moderately poor households), then K_x is convex.
If a = 0, $K_x'' = 0$ and K_x is linear (therefore convex and concave).

** **For the Watts' poverty index, W:** $f(y) = -\ln(y/z)$ and K_x is concave.*

** **For poverty indices CHU(c)**: $f(y) = 1 - (y/z)^c$ with $c > 0$, and K_x is concave.*

Proof:
FGT: for $a \neq 0$, $K_x''(I) = a\, y\, (1 - y/z)^{a-1} [(a + 1) y/z - 2]/(z I^2)$ of the sign of $(a + 1) y/z - 2$.
CHU: $K_x''(I) = - y^2 c\, (y/z)^{c-2} [c+1]/ I^2$ of the sign of $- c -1 < 0$ for $c > 0$.
W : $K_x''(I) = - 1/I^2 < 0$.

Proposition 4 applies to FGT(a) indices, $a \geq 1$ and all $y > 2z/(a+1)$ for the poor.
Proposition 5 applies to indices W, CHU and FGT(a) ($a \leq 1$, or $a \geq 1$ and $y < 2z/(a+1)$) for the poor.

Proposition 7: convexity of L_x .

L_x is convex in I if for all $I > x/z$, $K_x''(I) > 0$; $K_x(x/z) = 0$ and $K'_x(I) > 0$.

These conditions are satisfied for poverty indices: FGT(a) , $a \geq 1$ if all $y > 2z/(a+1)$ for the poor.

Proof: Obvious.

If prices and nominal living standards are independent, and if there are only moderately poor households among the poor, the corrected poverty is therefore underestimated by the non-corrected poverty if H = 1, for indices FGT(a), $a \geq 1$ (which corresponds to axiomatically valid indicators and if the living standards of the poor are not too far from the poverty line.

4.4. With dependence between prices and nominal living standards

Without the simplifying assumptions of the independence of distributions, we can nonetheless consider a poverty index conditional to I, denoted P^I, and calculate its two successive derivatives in I, $P^{I'}$ and $P^{I''}$, to study its concavity, and apply Jensen's inequality directly to P^I.

We start from the definition of the conditional integral P^I.

$$P^I = \int_0^{I.z} f(\frac{x}{I}, z)\ dF(x|I) \qquad (21)$$

Where F is the marginal c.d.f. of the distribution of price indices.

$$P = \int_0^{I.z} P^I\ dF_3(I) = E_I\ P^I \qquad (22)$$

where F_3 is the marginal c.d.f. of the distribution of price indices.

This enables us to investigate the consequences of the dependence between prices and nominal living standards. We examine the condition for P^I to be concave (resp. convex) in I.

If f(x/I) is continuous, P^I is derivable everywhere with respect to the boundary argument of the integral. Even when f(x/I) is not continuous, the set of discontinuity points of P^I with respect to the boundary of the integral is at most countable since P^I can be seen as a cumulative distribution function (Monfort (1980)). A convenient choice of z can avoid the discontinuity points, and we can then derive P^I. Moreover, if the c.d.f. F_4 is absolutely continuous with respect to the Lebesgue measure, its associated p.d.f. can be defined: ϕ (x|I). Under these conditions and if it is possible to derive under the integral (with ϕ differentiable with respect to I), the derivation of eq. (21) yields

$$P^{I'} = z f(z,z)\,\phi(I.z|I) +$$

$$\int_0^{I.z} f_1'(\frac{x}{I}, z)\ (-\frac{x}{I^2}) dF(x|I) +$$

$$\int_0^{I.z} f(\frac{x}{I}, z)\ \frac{\partial\phi}{\partial I}(x|I)\, d\lambda(x) \qquad (23)$$

where λ is the Lebesgue measure.

The first term in the right-hand-side term of eq. (23) is the (positive) change of conditional poverty associated with a marginal increase in the population of poor, due to the increase in the poverty line. The second term in the r.-h.-s. term shows the (positive) change in conditional poverty, due to the increase in the poverty severity assessment following the reduction in real income ($f_1' <0$). The third term in the r.h.s. term shows the change (generally of ambiguous sign) in conditional poverty, due to the change in the conditional distribution of nominal living standards, because of its dependence on prices. $P^{I'}$ is generally of ambiguous sign, although it is positive in the case of independence of prices and living standards.

Note that for many poverty indicators f(z, z) = 0 , although that is not compulsory (for example if the fact of becoming poor is materialised by a discrete jump in welfare like in the case of the head-count index).

If ϕ and f'_1 are differentiable and ϕ twice differentiable with respect to I, we can proceed and obtain the second derivative, whose sign determines the concavity or convexity of P^I.

$$
\begin{aligned}
P^{I''} &= f(z,z)\frac{\partial\phi}{\partial x}(I.z|I) \\
&+ 2z f(z,z)\frac{\partial\phi}{\partial I}(I.z|I) \\
&- \frac{(z)^2}{I} f_1^{/}(z,z)\,\phi(I.z|I) \\
&+ \frac{1}{I^4}\int_0^{I.z} x^2 f_{11}^{//}(\frac{x}{I},z)\ dF(x|I) \\
&+ \frac{2}{I^3}\int_0^{I.z} x f_1^{/}(\frac{x}{I},z)\ dF(x|I) \\
&- \frac{2}{I^2}\int_0^{I.z} x\ f_1^{/}(\frac{x}{I},z)\ \frac{\partial\phi}{\partial I}(x|I)\,d\lambda(x) \\
&+ \int_0^{I.z} f(\frac{x}{I},z)\frac{\partial^2\phi}{\partial I^2}(x|I)\,d\lambda(x) \qquad (24)
\end{aligned}
$$

Of course, due to Jensen’s inequality, we have:

Proposition 8:

If $P^{I''} < 0$, then $P = E(P^I) \leq P^{EI}$.

If $P^{I''} > 0$, then $P = E(P^I) \geq P^{EI}$.

P^{EI} is equal to the poverty without price correction, if EI = 1.

As precedingly, each term in eq. (24) could be commented in terms of changes (and cross changes or accelerations) in the population of the poor, in the distribution of living standards, in the severity of poverty. With axiomatically correct poverty indicators satisfying $f_1' < 0$ and $f_{11}'' > 0$, and with reasonable location of the poverty line in unimodal distribution ($\phi' < 0$), the terms in the r.-h.-s. term of eq. (24) are of various signs, with no obvious simplifications. This implies generally an ambiguous sign for $P^{I''}$. Even if the kernel function f is such that f(z, z) = 0 and $f'_1(z,z) = 0$, the ambiguous sign remains. Therefore, P^I is generally nor convex neither concave in I. Since the involved terms depend on the functional forms of f, F_1 and F_2 , little can be said outside special cases. This suggests to carry out a direct empirical analysis, when simplifying assumptions are not satisfied. Eq. (24) is still interesting as it shows the origin of this ambiguous sign, preventing the use of Jensen's inequality. In a large part the difficulty comes from the fact that the distribution of x and I may be linked, introducing first and second derivatives of the conditional p.d.f. of x with respect to I.

Moreover, in applied work, poverty lines are likely to be endogenous to the type of living standards considered (corrected or not corrected), and not exogenous as in our theoretical analysis. This is a supplementary element of complication, supporting a direct empirical approach.

4. The Data

Rwanda in 1983 is a small rural country in Central Africa. Its population estimated at 5.7 million, nearly half under 15 years of age, and growing at 3.7 percent rate, is a major constraint on development and eradication of poverty. Rwanda is one of the poorest country in the world, with per capita GNP of US $ 270 per annum. More than 95% of the population live in rural areas (Bureau National du Recensement (1984)). Agriculture is the cornerstone of the economy, accounting for 38% of GNP and most of the employment.

Data for the estimation is taken from the Rwandan national budget-consumption survey, conducted by the government of Rwanda and the French Cooperation and Development Ministry, in the rural part of the country from November 1982 to December 1983 (Ministère du Plan (1986a))[12]. 270 households were surveyed about their budget and their consumption. The consumption indicators are of very high quality. Indeed, every household was visited at least once a day (and often twice), during two weeks for every quarter. The consumption has been systematically recorded with daily and retrospective interviews, and with food weighting. Every household had also to register much information in a diary between the quarterly survey rounds. The redundancy of different methods of collection enabled a thorough cleaning of the data, by more than thirty ex-inquirers after the collection, under our supervision. We could as well design sophisticated verification algorithms. Finally, the consumption indicators are based on algorithms reducing measurement errors, from the confrontation of several information sources. The measurement errors of consumption levels should therefore be smaller than usual. This is a major requirement if we want to study price effects in welfare measurement, since these effects may be of moderate size and may be lost among data contamination when the consumption indicators are inaccurate.

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12 The main part of the collection was designed with the help of INSEE (French National Statistical Institute). The author was itself involved in this project and supervised the end of the analysis.

Year 1983 (or rather agricultural year 1982-83) is a fairly normal year in terms of climatic fluctuations (Bulletin Climatique du Rwanda (1982, 1983, 1984)). It is also relatively preserved from extreme economical or political shocks. It is often considered that the agricultural year can be split up into four climatic seasons and two cultural seasons. The large rainy season goes from February to May, and accounts for 41 to 61 percent of annual precipitations. The large dry season extends from June to September. The small rain season occurs in October and November, and the small dry season from December to January. In fact, the two latter seasons constitute an intermediary season not very delimited. Moreover, the climate can be quite different in different years, with slight shifts of seasons.

The first cultural season extends from October (seeding) to January (harvest). It is dominated by the culture of pulses, mostly beans. Corn culture is also concentrated in this season. The second cultural season goes from March (seeding) to August (harvest). Cereals are often cultivated in this season, mostly sorghum. The harvest periods are mostly in end December until April, then from June to July. The fourth period of our survey is a period with more limited harvest. However, such aggregated picture does not describe accurately the extreme variety of cultural contexts in Rwanda. An examination of each specific crop shows first that the high altitude and low altitude areas may have very different cultural rhythms, sometimes organised about different products. Beans are harvested at the end of December or the beginning of January; in April and in July. Sweet potatoes are harvested at the end of February and the beginning of March, in May, September and end of November. The harvest period for the sweet potatoes can also be very variable with altitude. Finally, because of its mountainous character, Rwanda is divided in a large number of microclimates and every family has its own crop decisions in every season, according to the type of land and inputs owned.

The collection of the consumption data was organised in four rounds and their dates are the following:

Round A: 01/11/1982 until 16/01/1983. Median date: 08/12/1982.
Round B: 29/01/1983 until 01/05/1983. Median date: 12/02/1983.
Round C: 08/05/1983 until 07/08/1983. Median date: 23/05/1983.
Round D: 14/08/1983 until 13/11/1983. Median date: 29/08/1983.[13]

The sampling scheme (modelled in Roy (1984) and completed by our own investigations during our stay at the Direction Générale de la Statistique du Rwanda), has four sampling levels (communes, sectors, districts and households). The drawing of the communes was stratified by prefectures, agro-climatic regions and altitude zones. One district was drawn in each drawn commune and one cluster of three households was drawn in each drawn district. From this information, we calculate sampling weights that reflect the probabilities of drawings of units at every stage of the sample scheme.

The average household size has 5.22 members, including 2.67 children or adolescents (less than 18 years old), and 2.55 young or adults (18 years old and more). The average age of members is 24.32 years. The average age of the head is 47.45 years, and 21.8 percent of these heads are women, while 10.9 percent belong to the Tutsi ethnic group. The average education level of the head is very low with 1.81 schooling years.

The mean land area is very small (1.24 ha). Table 1 shows that for the sample used in estimations, it corresponds (in real terms) to an average production of 57 158 Frw (Rwandan

13 Naturally, since the collection method mixes diaries, daily and retrospective interviews, these dates are only approximative.

Francs[14]) of agricultural output, which is close to the 51 176 Frw of average consumption (10613 Frw per capita). Muller (1989) provides a detailed description of the consumption of Rwandan peasants. Eight main categories of goods are defined, with their share in the aggregate value of consumption given in table 1.

The mean and standard deviation of seasonal prices for the main goods used in the price index, are shown in table 2 for the four seasons, jointly with the price index. As show the coefficient of variation across the four seasons, the average prices of soap and palm oil are characterised by relatively moderate quarterly fluctuations. The aggregate fluctuations of prices are more notable for other goods: sweet potatoes (low price in period C, 7.90 Frw, and high price in period A, 10.11 Frw); sweet cassava (14.4 Frw in C, 17.0 Frw in A); banana beer (36.9 Frw in D, 43.0 in C); plantain banana (12.21 Frw in B, 14.78 Frw in C), and beans (24.80 in B, 38.70 Frw in A). The more variable national prices are those of beans and sweet potatoes. The general level of prices shown by the average of the price index across households is relatively high in quarters A (1.109) and D (1.085), and low in quarter B (0.953). However, as reveal the standard deviations, these means hide considerable local differences. Moreover, several studies from price surveys in Rwanda, support the existence of substantial both geographical and seasonal price variability (Niyonteze and Nsengiyumva (1986), O.S.C.E. (1987), Ministère du Plan (1986b), Muller (1988)).

Figures 1 and 2 show the evolution curves for aggregate consumption and aggregate production across quarters, respectively with and without price correction. The price correction enables to better distinguish the poverty crisis during the last quarter, and the regularity of aggregate consumption during the remaining of the year, notably by reevaluating the levels of production and consumption at periods A and B when prices are respectively high and low before and after the large harvests of January.

5. <u>Estimation Results</u>

5.1. Average quarterly living standards

Table 3 (resp. 4) presents the means and standard deviations of per capita consumption, per capita production, total consumption and total production, corrected by the price index (resp. non corrected), in the four quarters, and for the global sample and each quintile of the annual per capita consumption. The evolution of the means at the national level has already been examined in figures 1 and 2. The standard deviations of these variables are substantial and the comparison of means may hide a lot of variability at the household level. To account for that possibility, we implement t-tests of comparisons of means (see Tassi (1984) for a discussion of these tests, and Wang (1971) for the calculus of the P-values). At the national level, corrected mean living standards in periods A, B and D are statistically different from uncorrected mean living standards in the same periods. This is not the case for period C in which the correction with the price index is not significant (P-value = 0.14). Using different equivalence scales leads to similar qualitative results.

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14 In 1983, the average exchange rate was 100.17 Frw for 1 $ (source: IMF, International Finance Statistics).

Inside each quintile of the annual living standard distribution, the effect of the price variability is as well pervasive. The results of t-tests of comparisons of means for variables corrected and non-corrected, generally indicate a strong rejection of the hypothesis of equality of means. However, in a few cases this hypothesis is not rejected at the 5% level: in the second quintile, for period B (P-value = 0.18); and in the fifth quintile: for annual living standard (P-value = 0.90) or period C (P-value = 0.15). Still there, the results are not sensitive to the use of different equivalence scales. Therefore, except in a few cases, the correction of price variability is significant for the estimation of the mean living standards in most quarters and quintiles. This is a first interesting result since most of the time, living standard statistics are published without price correction in the statistical reports of household surveys. A certain care seems advisable when interpreting non corrected results as genuine welfare statistics.

However, close examination of means reveals that the differences in these aggregates, with and without price correction, are always moderate, generally about 10 percent. Period D is unambiguously a period of crisis, which appear with or without price correction, and during which the mean per capita consumption and the mean total consumption are clearly lower. For the first three periods, these averages seem to be more stable when there is correction for price variability, although the fall of consumption at the last period is larger in that case. Note that these features are not necessarily conserved at the quintile level, which suggests that aggregate means might be misleading when fluctuations of living standards are concerned. In contrast, the average total productions seem to be more regular without the correction for price variability. Therefore, noises due to prices in non corrected indicators partially hide not only the relative stability of aggregate consumption in the beginning of the year, and the severity of the welfare crisis at the last quarter, but also the substantial seasonality of the aggregate production level.

5.2. Tests of independence and residual population

Table 5 shows the correlation coefficients between price indices, and nominal and real living standards at several periods. The correlation coefficients with nominal living standards are not significant. This is not the case between price indices and real living standards whose formula includes explicitly the price index[15].

Table 6 shows the results of several tests of independence, using categories based on deciles. χ^2 , γ (difference between conditional probabilities of like and unlike order) and Kendall's τ_b test statistics have been calculated, as well as the Cramer's V association measure[16]. The khi-square, gamma and the tau-b tests suggest that there is independence between price indices and nominal living standards, in every quarter. This corresponds to a favourable situation for the application of our theoretical results. Notice however, that the non rejection of the independence hypothesis may well be attributed to the small sample size. Indeed, the Cramer's V association measure is always above 17 percent (and below 20 percent), and the link between price indices and living standards might appear significant with a larger sample size.

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15 Using unit-values instead of observed prices would lead to misleading results in such tests because of the strong link between the value of consumption and the unit-values, originated in their definition itself.

16 Let be P, the number of concordances of the two classification variables, and Q, the number of discordances, then $\gamma = (P-Q)/(P+Q)$; $\tau b = (P-Q)/((n^2 - \Sigma ni.)(n^2 - \Sigma n.j))^{½}$ and Cramer's $V = (\chi 2/(n.Min(I-1,J-1)))^{½}$ See Goodman and Kruskal (1954, 1959, 1963, 1972) for discussions of measures of association for cross classification.

Table 14 shows the residual population of households such that y > 2 z /(a + 1), with a = 1, 2, 3, 4, and 5, for the six poverty lines. At the yearly level, there is no household left for *a* greater or equal to 5, and 5 times out of 6 for *a* equal to 4. For a = 2 or 3, the residual population is of very small size (between 0 and 10 percent). For a = 1, the residual population corresponds to the population of the poor and is of substantial size (7 to 40 percent). By contrast, at the quarterly level, the residual population is always non negligible for all values of a. Consequently, the hypothesis y > 2 z /(a + 1) is generally not satisfied with our quarterly data, for useful values of parameter a. This is one reason why we shall not insist too much on the exploitation of results of propositions 2 through 5, in the comment of poverty estimates. Another reason is that we want now to explore directly the consequence of the correction for prices, when poverty lines are endogenous, which is a situation for which no theoretical results are available.

5.3. Poverty estimates

Six poverty lines are used and expressed in terms of Rwandan Francs (Frw).
z_1 is the first quintile of the annual living standards;
z_2 is the sum of the first quintiles of the quarterly living standards;
z_3 is four times the minimum of the first quintiles of the quarterly living standards.

We denote the population whose per capita consumption is under these poverty lines, the "very poor". Three remaining poverty lines are as well associated to the set of "poor" (very poor plus moderately poor). They are calculated as above, although from the second quintiles of the living standard distribution, and respectively denoted z_4 , z_5 , z_6. Note that $z_4 > z_5 > z_6 > z_1 > z_2 > z_3$. The same types of poverty lines have been calculated using the nominal living standard distribution (non corrected by price indices). This implies that the poverty lines are endogenous to the type of considered living standard distributions, as it is frequently the case in poverty studies. Here, poverty indices and poverty lines are based uniquely on consumption levels (real or nominal depending on the living standard variable), and the same poverty line is used for all seasons. Other elements could enter in the definition of living standards, such as health status, leisure or the arduousness of work. Accounting for the supplied work would be especially interesting since it may entail different needs at different seasons due to the seasonality of agricultural tasks (Chambers, Longhurst and Pacey (1981)). Another origin of seasonal variation of needs is the climate itself, which implies different biological pressures, mostly due to heat, illness and dust, at different periods. A fundamental difficulty is that the actual nutritional needs of individuals corresponding to different tasks or climatic conditions, are very badly known, even in laboratory biological experiments, and are very variable across individuals. We shall not enter in these extensions that would require very detailed indicators about daily life of peasants and postpone these questions for further investigation.

Tables 7 and 8 show the estimates of FGT poverty indices for the six (endogenous) poverty lines, respectively without and with correction for the price variability. Table 9 shows the percentage of variation in these indices that is induced by the price correction. Tables 10, 11 and 12 show the CHU and Watts' indices[17]. To assess the inaccuracy involved in our inferences, we calculate sampling errors that are shown in the tables.

[17] The poverty gap is both FGT(1) and CHU(1), and is repeated in tables so as to facilitate comparisons for different values of parameters in the same set of indicators.

There exists at least a magnitude order between poverty estimates and their standard errors, which ensures that all poverty indicators are significant. The calculus of sampling errors is crucial since our estimates show that some variations of poverty indicators due to the price correction, are significant and other ones insignificant. Tables 9 and 12 show the relative variations due to the price correction ($\Delta P/P$) and the sampling errors for the absolute variations (ΔP). The ΔCP (for chronic poverty CP) are sometimes non significant for the FGT, CHU and W indices (with line 1), especially in the case of the incidence of poverty (line 1, 3 and 4 for FGT). This is less the case for high poverty lines. The ΔAP (for annual poverty AP) are often non significant (for FGT, CHU and W with lines 2, 3, 5). Thus, averaging living standards or poverty indices across the year, reduces generally the impact of price variability without eliminating it. By contrast, the ΔP_t for quarters A and B are generally significant for all indicators (except for line 6 in period A). Even if they are often significant, the ΔP_t for quarters C and D, may also be non significant (FGT and C&W for lines 2, 3 and 5). The ΔTP (variations of transient poverty due to the seasonal fluctuations of living standards) are almost always statistically significant (except sometimes for the incidence of poverty). On the whole, even with a relatively small sample size, the effect of price correction is significant for most poverty lines, most poverty indicators and most quarters.

There exists a rough agreement between the results for the FGT indices and that of the CHU and Watts' indices. The change in poverty indices due to the price correction has not always the same sign (except for period A and B), whatever the category of considered indices. Let us see if we can detect some systematic influences of the price correction.

The direction of the poverty changes due to price correction is very variable, even if it is more related to poverty lines than to parameters of indicators. Thus, some regularities can be noticed for specific poverty lines, that are not systematically valid for other ones. The sign of the change is often negative for the incidence of poverty. The changes in FGT CP indices are negative for lines z_2 , z_3 , z_5 , z_6, while changes in FGT AP indices are negative for z_5 and z_6 , and positive for z_1 , z_2 , z_4 . CP CHU and Watts' indices (C&W) are positive for z_1 and z_4 , and negative for z_2 , z_3 , z_5 , z_6 . AP C&W indices are positive for z_1 , z_2 and z_4 , and negative for z_5 and z_6 .

The changes in FGT and C&W P_A indices are always positive, although there are always negative for FGT and C&W P_B indices. FGT and C&W P_C indices changes are positive for z_1, z_4 and negative for z_6. This is consistent with a dominance of aggregate shifts of prices, for periods A and B. This illustrates the importance of seasonal aggregate fluctuations of prices, which may sometimes overcome distributional effects. FGT and C&W P_D indices changes are positive for z_1 , z_2 and z_4 . For the non mentioned lines, the signs of changes are contradictory among indicators.

The absolute magnitude of changes are sometimes substantial, especially for seasonal poverty indices in periods A and B (more than 30 percent for some lines). However, this is not systematic and depends on the considered poverty line. The magnitude of changes in CP FGT indices vary a lot (from -0.33 to 0.11), although it is "always"[18] below 10 percent for line z_1 and always above 10 percent for line z_6 . The magnitude of changes in AP FGT indices (-0.12 to 0.14) is always below 10 percent for lines z_2 , z_3 and z_5, and always above 10 percent for line z_6. The magnitude of changes of P_A FGT indices (0.009 to 0.43) is always above 10 percent for lines z_1, z_2, z_3 , z_4, and may be substantial (above 0.30 for z_2 and in many cases) while the magnitude of changes in P_B (-0.39 to -0.14) is always above 14 percent for all lines and often

18 always means: " for all values of parameters of the considered indices"

considerable (over 0.25 for z_3, z_5 and z_6). The magnitude of changes in P_C FGT indices (-0.17 to 0.16) is always below 10 percent for lines z_2, z_3 , z_5 and always above 10 percent for z_6 . The magnitude of changes in P_D FGT indices (-0.07 to 0.18) is always below 10 percent for lines z_2, $z_{3,}$, z_5 , z_6 and always above 10 percent for z_4.

Such a variety in the absolute magnitude of price effects is also true for C&W indices. The magnitude of changes in CP C&W indices (-0.25 to 0.09) is always below 10 percent for z_1, z_2, z_4, and always above 23 percent for z_3 and z_6. The magnitude of changes in AP C&W indices (-0.12 to 0.11) is always below 10 percent for z_1, z_2, z_3, z_5 and always above 10 percent for z_4 and z_6. The magnitude of changes in P_A C&W indices (0.064 to 0.36) is always below 10 percent for z_6 and always above 10 percent for other lines (above 0.30 for z_1 and z_2) , while the magnitude of changes in P_B C&W indices (-0.35 to -0.14) is always above 14 percent for all lines (above 0.30 for z_3 and z_6). The magnitude of changes in P_C C&W indices (-0.11 to 0.11) is always below 10 percent for z_2, z_3 and z_5, and always above 11 percent for z_6. The magnitude of changes in P_D C&W indices (-0.06 to 0.17) is always below 10 percent for z_2, z_3 , z_5 and z_6, and always above 10 percent for z_1 and z_4.

The relative changes in poverty indicators due to the price correction, often increase with parameter a (resp. c) in FGT (resp. CHU) poverty indicators, showing that a higher concern for poverty severity is often associated with a relatively larger impact of prices. This phenomena is observed both for FGT and CHU indices, AP and CP, and seasonal indicators except in periods B and D. It implies more sensitivity to price correction, for indicators with high values of these parameters.

The link of changes in poverty with the poverty line is more complex. There seems to exist two different regimes for the very poor (lines z_1, z_2 and z_3), and the moderately poor (z_4, z_5 and z_6). In every regime, the poverty variation due to price correction decreases when the line diminishes, whatever the type of poverty indicator considered. However, it is no longer true when all poverty lines are considered altogether.

Changes in the share of transient poverty can be sometimes considerable (from -26 percent to 78 percent), although it is generally small (about 5 percent). It is always positive for C&W and FGT indices for lines z_1, z_4, z_5 and z_6, showing that the price variability generally hides part of the influence of the seasonal variability of living standards on the annual poverty. The change in the case of FGT indices is always below 10 percent for z_1 and z_4, and always above 10 percent for z_6. In the case of the C&W indices, it is below 10 percent for z_1, z_2, z_3, z_3, and always above 10 percent for z_5 and z_6.

On the whole, it is generally difficult to predict the direction and the size of the bias due to price variability, for quarterly poverty indicators as well as for annual or chronic poverty indicators. The formula of the poverty indicator is less influential in the explanation of changes than the poverty line or the quarter. Aggregate level of prices at different seasons and the definition of the categories of the poor play a more important role than the choice of the kernel of the poverty indicator. This suggests that the price correction is important whatever the poverty indicator used, and that collection of local and seasonal prices must be encouraged for all types of poverty analysis.

We used a different price index in Muller (1997). This index was weighted by the budget shares of each household and calculated from average prices at the level of geographical zones rather than at the cluster level. In practice, we have observed that this index could have underestimated the seasonality of prices[19]. The comparison shows that these preceding poverty estimates are generally intermediate between the present estimates with and without prices correction, confirming that price correction using regional prices, only partially corrects for the price variability. In all cases, the main results of Muller (1997) are confirmed, which are that the share of transient seasonal poverty in the annual poverty is very substantial in Rwanda. Paasche indices have also been experimented, although they should be avoided since they often give a too large weight to aberrant values and tend to artificially increase the price variability by seriously confusing the effects of price with that of data contamination. Such problem is much less serious with Laspeyres indices.

We present finally an empirical result with exogenous poverty lines. Here, since the harmonic average of price indices, H is inferior to 1, ΔP_2, the poverty change associated to the change in harmonic level of price indices, is negative, and the total change $\Delta P = \Delta P_1 + \Delta P_2$ is of ambiguous sign. Table 15 shows the AP FGT indices corresponding to the distribution of x/H, where H = 0.96607. Despite the fact that the residual population of the poor such that y > 2z/(a+1) are not empty, the estimates show that, accordingly to the theory developed precedingly, ΔP_1, the variation of poverty (at exogenous poverty lines) measured by FGT(a), a >1, associated to the correction for the price variability at constant harmonic level, is always positive. This is particularly interesting since FGT(2) and FGT(3) are very popular poverty indices that satisfy desirable axiomatic properties. For these poverty indices the change in poverty with compensation for keeping the harmonic average of price indices constant, is always positive.

5.4. Variations of populations of the poor

The correction for prices not only may change the estimated level of poverty, but also, even when the poverty is not significantly modified, may change the composition of the population of the poor. In table 13, columns A show the percentages of households that are poor before the price correction, but not after. Columns B show the percentages of households that are poor after the price correction, but not before. The variations of the population of the poor, due to the price correction can sometimes be notable: 0.20 to 7.12 percent of the whole population at the yearly level, although less than 3 percent in most cases; 0 to 11.8 percent for various quarters. There is no systematic tendencies in the comparison of the two columns at the yearly level or for period C and D. In contrast, in period A, the number of the “new” poor is always greater than the number of the “new” non-poor, while it is the opposite in period B. Again, the aggregate movement of prices at the two quarters is determinant.

6. Conclusion

Dynamic poverty indicators are generally imperfectly corrected for the variability of prices across households and seasons. To some extent the poverty deduced from variations in nominal living standards could as well follows from variations in prices across households and periods. To our knowledge, the importance of such price effects for poverty measurement has not been accurately studied in the literature. We provide in this article a theoretical analysis of

[19] This price index was chosen because it is associated with a rather conservative picture of the transient poverty, in order to guaranty that the results of the paper were not overly determined by the price correction.

the variations of poverty indices caused by the price variability, followed by an empirical investigation.

With exogenous poverty lines, we derive the direction of the bias in the measurement of poverty due to neglecting price variability, for popular indices, under convexity assumptions and when prices and nominal living standards are independent. This allows the decomposition of the effect of price variability in effect of harmonic aggregate level of prices and effect of compensated prices variability. The latter effect can be proved positive for an important class of axiomatically valid poverty indices, while the first one is consistent with the direction of aggregate prices.

With endogenous poverty lines, using seasonal panel data from Rwanda, we show the substantial consequences of an accurate price correction using local and seasonal prices. The independence of prices and nominal living standards is not rejected by our tests. In many instances, the price correction changes significantly the levels of average living standards and poverty indicators and cannot be neglected. However, if the change in aggregate living standards is moderate (about 10 percent) in every quarter, it is not the case for poverty for which the magnitude of the change is often around 30 percent in the first two seasons. In terms of impact of the price correction on the assessment of poverty, the poverty line or the quarter are generally more influential than the formula of the poverty indicator, especially when the attention is restricted to axiomatically valid poverty indicators. In the first two quarters the effects of aggregate seasonal fluctuations of prices dominate the effect of price variability and imply substantial unambiguously positive or negative variations of poverty at these periods when correction for prices is implemented. Poverty indicators giving a high importance to the severity of poverty are more likely to lead to a strong effect of prices. Moreover, notable changes in the composition of the population of the poor may occur, due to the price correction. Finally, with exogenous poverty lines, for an important class of poverty indices the change in poverty with compensation for keeping the harmonic average of price indices constant, is always positive.

The importance of the price variability for welfare and poverty measurement, and the major role played by the choice of poverty lines, invites to develop alternative theoretical approaches based on analysis of distributions of both prices and nominal living standards. Similar investigations for inequality are as well under progress.

Once safe measures of poverty, accounting for price variability, are established, the road is open to studies of impact of price fluctuations or price policies, on poverty of peasants, which would otherwise be confounded with measurement problems.

Table 1: Mean and standard deviation of yearly consumption and production, corrected for price variability, and shares of goods in consumption

Total Consumption	51176.15 (24985)
Total Production	57158.02 (38207)
Per Capita Total Consumption	10613.27 (5428.08)
share of beans in consumption	0.20279
share of fruits and vegetables	0.12704
share of sweet potatoes	0.090645
share of other tubers	0.12067
share of traditional beers	0.13890
share of other foods	0.14960
share of fire wood	0.027803
share of other non foods	0.14255

The share of goods in the production are the ratios of values of aggregated consumptions for these goods over the value of the total consumption .For total consumption and total production, the first number is the mean and the number in parenthesis is the standard deviation

Table 2: Local seasonal prices (Frw)

Products \ Period	A	B	C	D	CV for quarters	σ for quarters	average for quarters
beans (kg)	38.7019531 (9.0707652)	24.798828 (6.432865)	31.8125000 (11.4566498)	36.417187 (6.3468888)	0.161	5.31	32.93
plantain (kg)	12.5136719 (3.2773867)	12.214453 (4.9492707)	13.6738281 (5.7032549)	14.7750000 (5.1316702)	0.077	1.017	13.29
sweet potatoes (kg)	10.1121094 (4.2133739)	8.133203 (2.7748363)	7.9093750 (3.5480774)	9.9894531 (5.1228521)	0.113	1.019	9.04
sweet cassava (kg)	17.0023438 (4.9297809)	14.3554687 (3.5128293)	16.1097656 (4.3376354)	15.5742187 (3.9728835)	0.061	0.956	15.76
banana beer (l)	39.1620703 (10.3432217)	38.4128906 (9.7180663)	43.0123047 (11.2116236)	36.8570313 (9.5145210)	0.058	2.265	39.36
palma oil (kg)	181.2312500 (27.7787018)	165.1644531 (20.495641)	178.3183594 (21.8155983)	179.918750 (40.0212458)	0.034	6.431	176.16
soap (kg)	22.5574219 (6.2778933)	22.6726562 (6.5253619)	21.5757813 (4.5804657)	20.8699219 (4.2841424)	0.034	0.741	21.92
price index	1.1087 (0.1294)	0.9534 (0.1015)	1.0476 (0.1316)	1.0847 (0.0978)	0.057	0.0594	1.049

Standard deviations in parentheses.

Table 3 : Mean and standard deviation of variables corrected by price index

Variable	period A	period B	period C	period D
per capita consumption	2750.17 (1701.17)	2702.94 (1620.90)	2850.08 (1968.64)	2310.08 (1511.55)
per capita production	2771.46 (2677.99)	3222.97 (3349.09)	3114.67 (3300.66)	2718.67 (2688.54)
total consumption	13521.52 (9527.40)	13232.20 (8192.52)	13452.85 (8249.68)	10969.57 (6092.44)
total production	13240.50 (12178.27)	15548.30 (16610.28)	15416.63 (18171.03)	12952.59 (10662.06)

By quintiles:

Variable	period A	period B	period C	period D
per capita consumption (Q=1)	1331.18 (487.00)	1547.66 (525.31)	1328.09 (506.20)	1255.12 (417.04)
per capita consumption (Q=2)	2088.66 (766.31)	1984.96 (499.56)	1776.30 (577.33)	1619.03 (558.09)
per capita consumption (Q=3)	2500.45 (1003.68)	2356.20 (792.57)	2529.50 (844.26)	1959.58 (559.89)
per capita consumption (Q=4)	3221.48 (1365.95)	2736.57 (828.58)	3345.54 (1108.39)	2593.75 (1075.51)
per capita consumption (Q=5)	4587.92 (2154.43)	4855.69 (2149.21)	5233.04 (2695.84)	4095.04 (2110.58)

Variable	period A	period B	period C	period D
per capita production (Q=1)	1234.13 (717.02)	1700.39 (1264.44)	1977.46 (2116.22)	1660.90 (2285.94)
per capita production (Q=2)	1774.06 (815.80)	2040.61 (1102.60)	2210.53 (1829.33)	1697.83 (1157.43)
per capita production (Q=3)	2709.89 (1795.04)	3073.22 (3574.42)	2736.91 (3143.57)	2518.85 (1363.99)
per capita production (Q=4)	3471.94 (2638.82)	3336.09 (1745.86)	3410.70 (2834.60)	3234.79 (2905.74)
per capita production (Q=5)	4641.22 (4156.29)	5918.35 (5159.83)	5201.41 (4735.23)	4447.81 (3742.93)

Variable	period A	period B	period C	period D
total consumption (Q=1)	8420.10 (4722.72)	9553.91 (4804.68)	8382.73 (4466.10)	7775.14 (3644.19)
total consumption (Q=2)	12429.25 (6129.59)	11723.87 (5278.01)	10513.80 (5214.27)	9473.42 (4614.28)
total consumption (Q=3)	15415.12 (9520.31)	14784.15 (8058.68)	15609.07 (7966.09)	11980.88 (5781.73)
total consumption (Q=4)	14092.45 (9452.79)	12299.07 (7380.43)	14623.30 (8352.31)	11594.39 (7033.33)
total consumption (Q=5)	17256.06 (13158.87)	17753.90 (11336.40)	18086.31 (10050.98)	13997.95 (6871.58)

Variable	period A	period B	period C	period D
total production (Q=1)	7609.15 (5356.44)	10678.95 (9021.30)	12946.57 (16071.97)	10069.08 (10981.98)
total production (Q=2)	10400.97 (6404.81)	11616.16 (6704.62)	14039.39 (16958.83)	10474.88 (8742.95)
total production (Q=3)	15258.41 (8497.64)	19483.80 (25410.29)	17375.47 (23995.45)	15092.70 (9518.95)
total production (Q=4)	15667.33 (16620.56)	14877.30 (10021.29)	14739.40 (16361.38)	13757.41 (11587.49)
total production (Q=5)	17242.90 (16287.82)	20996.83 (20926.40)	17954.00 (16444.49)	15330.23 (11331.07)

Table 4: Mean and standard deviation of variables not corrected by new price index

Variable	period A	period B	period C	period D
per capita consumption	2995.40 (1826.01)	2539.35 (1475.74)	2902.02 (1834.13)	2468.42 (1524.95)
per capita production	3013.80 (2926.56)	3012.62 (3040.90)	3152.83 (2987.24)	2882.42 (2632.76)
total consumption	14681.44 (10396.15)	12431.20 (7451.70)	13755.75 (7995.07)	11764.18 (6274.36)
total production	14345.32 (13265.64)	14476.60 (15191.76)	15536.02 (15716.16)	13853.77 (11106.24)

By quintiles:

Variable	period A	period B	period C	period D
per capita consumption (Q=1)	1492.56 (512.86)	1475.18 (502.29)	1408.84 (558.29)	1377.34 (469.94)
per capita consumption (Q=2)	2225.96 (691.72)	1946.50 (504.13)	1865.80 (554.96)	1758.66 (607.35)
per capita consumption (Q=3)	2748.04 (1166.79)	2227.50 (712.06)	2663.65 (960.39)	2119.54 (571.27)
per capita consumption (Q=4)	3473.67 (1408.84)	2586.60 (780.30)	3510.77 (1168.28)	2729.44 (1011.97)
per capita consumption (Q=5)	5011.61 (2313.03)	4433.07 (2010.81)	5028.32 (2367.88)	4328.12 (2074.68)

Variable	period A	period B	period C	period D
per capita production (Q=1)	1366.05 (768.93)	1613.29 (1218.61)	2101.19 (1978.88)	1819.67 (2476.27)
per capita production (Q=2)	1905.32 (819.10)	2018.27 (1142.27)	2284.13 (1616.01)	1858.76 (1303.99)
per capita production (Q=3)	2953.51 (2065.50)	2878.40 (3477.98)	2735.80 (2457.56)	2743.72 (1535.40)
per capita production (Q=4)	3801.03 (2933.70)	3154.36 (1694.81)	3596.94 (2790.03)	3412.27 (2741.66)
per capita production (Q=5)	5014.42 (4504.47)	5360.67 (4494.00)	5013.21 (4344.26)	4545.84 (3451.21)

Variable	period A	period B	period C	period D
total consumption (Q=1)	9440.48 (4978.95)	9110.40 (4641.24)	8901.63 (4987.36)	8574.25 (4191.22)
total consumption (Q=2)	13215.92 (6095.80)	11480.24 (5230.93)	11079.22 (5724.83)	10256.16 (5056.32)
total consumption (Q=3)	17034.47 (11230.03)	13954.29 (7642.12)	16103.69 (7872.16)	12864.47 (5937.07)
total consumption (Q=4)	15145.49 (10063.10)	11572.96 (6904.90)	15137.78 (7809.90)	12200.63 (6888.46)
total consumption (Q=5)	18568.65 (14342.80)	16014.29 (9875.80)	17525.21 (9472.64)	14896.94 (7043.94)

Variable	period A	period B	period C	period D
total production (Q=1)	8437.25 (5777.85)	10165.06 (8678.46)	13840.37 (15869.17)	11089.65 (11964.15)
total production (Q=2)	11117.96 (6597.44)	11433.92 (6566.02)	14278.04 (14546.67)	11461.50 (10002.06)
total production (Q=3)	16609.78 (9249.14)	18052.24 (24534.58)	16710.53 (16589.75)	16224.55 (9828.41)
total production (Q=4)	17068.06 (18137.49)	13912.61 (9026.72)	15230.59 (14805.93)	14691.39 (12436.48)
total production (Q=5)	18465.31 (17962.35)	18760.06 (17531.17)	17588.92 (16877.72)	15771.46 (10376.58)

Table 5 : <u>Correlation coefficients between price indices and living standards</u>

Quarters	Nominal living standards	Real living standards
A	-0.0448 (0.48)	-0.246* (0.0001)
B	-0.0442 (0.48)	-0.2298* (0.0002)
C	-0.1103 (0.0782)	-0.3245* (0.0000)
D	-0.1124 (0.0726)	-0.2530* (0.0000)

The table shows for each quarter, the correlation coefficients between the price index, and nominal and real living standards.
P-values in parenthesis. * = significant at 5 percent.

Table 6: <u>Tests of independence</u>

Quarter	P-value of the Khi-square test	Cramer's V	Result of the Gamma test at 5 percent level	Result at the tau-b test at the 5 percent level
A	0.340	0.1928	A	A
B	0.701	0.1784	A	A
C	0.304	0.1943	A	A
D	0.287	0.1951	A	A

R = rejected. A = non rejected.

Table 7 : FGT's Poverty indices

(without correction for price variability)

poverty lines based on first quintile												
	z_1 >				z_2 >				z_3			
a	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**
A	0.22724 (0.032372)	0.056450 (.0085433)	0.022917 (.0045108)	0.011500 (.0028381)	0.14289 (0.018457)	0.038136 (.0067978)	0.015247 (.0036458)	.0076064 (.0022485)	0.13108 (0.017001)	0.031783 (.0064083)	0.012588 (.0032845)	.0062881 (.0020110)
B	0.29033 (0.047001)	0.074961 (0.010376)	0.02768 (.0049961)	0.012475 (.0029501)	0.21468 (0.029169)	0.049022 (.0079844)	0.016791 (.0038721)	.0074576 (.0021871)	0.19148 (0.020691)	0.038836 (.0071677)	0.013126 (.0033963)	.0058939 (.0018945)
C	0.24022 (0.026413)	0.07058 (.0090653)	0.028323 (.0041660)	0.013556 (.0022386)	0.20399 (0.025220)	0.049092 (.0069807)	0.018231 (.0029885)	.0085290 (.0015983)	0.17188 (0.015691)	0.039633 (.0060750)	0.014707 (.0025337)	.0068729 (.0013912)
D	0.36998 (0.047373)	0.10197 (0.015113)	0.04517 (.0097907)	0.026636 (.0077450)	0.27800 (0.037547)	0.069851 (0.011783)	0.032198 (.0087118)	0.020456 (.0070165)	0.22606 (0.028832)	0.057922 (0.011177)	0.027927 (.0083579)	0.018454 (.0067160)
AP	0.28194 (0.027264)	0.075991 (.0058738)	0.031027 (.0025860)	0.016042 (.0016952)	0.20989 (0.017109)	0.051525 (0.043290)	0.020617 (.0018790)	0.011012 (.0014845)	0.18013 (0.013796)	0.042043 (.0033971)	0.017087 (.0016690)	.0093772 (.0014480)
CP	0.20021 (0.016454)	0.037374 (.0043412)	0.011192 (.0019041)	.0041173 (.00081049)	0.09944 (0.011849)	0.08184 (.0038821)	0.092405 (.0012062)	0.12307 (.00045115)	0.084648 (0.012021)	0.015858 (.0033597)	.0041071 (.00090929)	.0012937 (.00034497)
F	0.28989 (0.017495)	0.50817 (.0063319)	0.6392 (.0029933)	0.74334 (.0020730)	0.5261 (0.021774)	-0.58834 (.0043939)	-3.48207 (.0022769)	-10.1753 (.0017976)	0.53006 (0.015950)	0.62282 (.0033511)	0.75963 (.0020865)	0.86204 (.0017198)

poverty lines based on second quintile												
	z_4 >				z_5 >				z_6			
a	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**
A	0.36668) (0.035531)	0.099308 (0.012818)	0.040904 (.0064911)	0.020728 (.0040157)	0.35845 (0.042983)	0.094143 (0.01231)	0.038675 (.0062558)	0.019575 (.0038773)	0.29763 (0.032514)	0.077667 (0.010613)	0.031683 (.0055180)	0.015981 (.0034310)
B	0.48185) (0.038625)	0.12870 (0.013654)	0.052094 (.0069722)	0.024941 (.0042782)	0.43361 (0.031864)	0.12215 (0.013201)	0.049168 (.0067569)	0.023387 (.0041356)	0.36878 (0.02820)	0.10231 (0.012427)	0.039825 (.0060434)	0.018519 (.0036554)
C	0.38567 (0.033330)	0.11798 (0.011634)	0.050013 (.0062033)	0.025184 (.0035772)	0.38125 (0.036816)	0.11274 (0.011430)	0.047402 (.0060022)	0.023761 (.0034272)	0.3497 (0.037521)	0.094562 (0.010645)	0.039051 (.0052793)	0.019273 (.0029275)
D	0.52273 (0.040554)	0.16301 (0.019096)	0.074119 (0.011956)	0.041691 (.0091169)	0.49905 (0.041372)	0.15609 (0.018742)	0.070708 (0.011713)	0.039832 (.0089635)	0.43212 (0.041442)	0.13352 (0.017250)	0.059728 (0.010914)	0.033976 (.0084571)
AP	0.43923 (0.029367)	0.12725 (.0086872)	0.054283 (.0039786)	0.028136 (.0023664)	0.41809 (0.028774)	0.12128 (.0083188)	0.051488 (.0038194)	0.026638 (.0022824)	0.36207 (0.023873)	0.10201 (.0073173)	0.042572 (.0033043)	0.021937 (.0020179)
CP	0.40021) (0.027372)	0.084134 (.0047054)	0.027167 (.0024822)	0.010799 (.0014396)	0.37183 (0.028127)	0.078213 (.0044218)	0.025011 (.0024373)	.0098743 (.0013817)	0.30756 (0.024802)	0.059477 (.0039451)	0.018536 (.0022830)	.0071423 (.0011672)
F	0.088836 (0.013333)	0.33882 (.0056222)	0.49952 (.0039393)	0.61618 (.0027077)	0.11064 (0.017356)	0.35511 (.0057317)	0.51424 (.0038818)	0.62932 (.0026404)	0.15057 (.0062085)	0.41698 (.0064543)	0.56459 (.0035949)	0.67442 (.0024045)

(Sampling errors in parenthesis. The number in parenthesis in the F line is the sampling error for TP, not for F)

Table 8 : FGT's Poverty indices

(Indicator par capita corrected by price index)

poverty lines based on the first quintile												
	z_1 >				z_2 >				z_3			
a	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**
A	0.27189 (0.030734)	0.075877 (.0099346)	0.031568 (.0052653)	0.016138 (.0034780)	0.19746 (0.030005)	0.048906 (.0074624)	0.019924 (.0040855)	0.010223 (.0027987)	0.15348 (0.022306)	0.039262 (.0063292)	0.016126 (.0036951)	.0083063 (.0025537)
B	0.23740 (0.051143)	0.060845 (0.010223)	0.022156 (.0048516)	0.010036 (.0028479)	0.16945 (0.023375)	0.035544 (.0071378)	0.012353 (.0036807)	.0056358 (.0019884)	0.13475 (0.013544)	0.026476 (.0066739)	.0094194 (.0032016)	.0043511 (.0016486)
C	0.27967 (0.016900)	0.077589 (.0085844)	0.031239 (.0044593)	0.015304 (.0025955)	0.19695 (0.030958)	0.049681 (.0068602)	0.019153 (.0031984)	.0092182 (.0018683)	0.16626 (0.028095)	0.039814 (.0055685)	0.015127 (.0027770)	.0072705 (.0016188)
D	0.39871 (0.069129)	0.11607 (0.019727)	0.051120 (0.012408)	0.029425 (.0092638)	0.29491 (0.040729)	0.075526 (0.015837)	0.034325 (0.010597)	0.021273 (.0078982)	0.22351 (0.032612)	0.061999 (0.014641)	0.028930 (.0098712)	0.018728 (.0073296)
AP	0.29692 (0.032948)	0.082595 (.0073364)	0.034021 (.0033337)	0.017726 (.0021177)	0.21469 (0.022837)	0.052414 (.0047676)	0.021439 (.0023776)	0.011587 (.0017009)	0.16950 (0.016074)	0.041888 (.0039802)	0.017401 (.0020882)	.0096639 (.0015855)
CP	0.20748 (0.023314)	0.039374 (.0044385)	0.01 136 (.0017709)	.0040092 (.00073657)	0.099200 (.0096705)	0.075386 (.0034102)	0.079204 (.0010625)	0.099105 (.00035211)	0.080841 (.0093056)	0.013300 (.0030093)	.0032002 (.00076423)	.00093121 (.00023376)
F	0.30122 (0.015865)	0.52329 (.0059729)	0.66590 (.0032576)	0.77382 (.0022977)	0.53794 (0.019062)	-0.43827 (.0044953)	-2.69449 (.0026387)	-7.55279 (.0019059)	0.52307 (0.012722)	0.68248 (.0039738)	0.81609 (.0024122)	0.90364 (.0017542)

poverty lines based on the second quintile												
	z_4 >				z_5 >				z_6			
a	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**
A	0.43764 (0.049020)	0.13182 (0.014818)	0.057000 (.0076640)	0.029727 (.0048762)	0.39108 (0.035283)	0.11038 (0.012988)	0.046880 (.0067157)	0.02425 (.0043267)	0.30032 (0.024550)	0.082611 (0.010378)	0.034563 (.0055528)	0.017709 (.0036483)
B	0.41365 (0.052073)	0.10961 (0.016335)	0.044291 (.0075299)	0.021253 (.0043868)	0.31763 (0.040284)	0.090437 (0.014440)	0.035551 (.0064653)	0.016660 (.0037985)	0.25984 (0.054530)	0.066808 (0.011252)	0.024774 (.0051549)	0.011283 (0.0030443)
C	0.41846 (0.042373)	0.12940 (0.012040)	0.056625 (.0062941)	0.029208 (.0039252)	0.35536 (0.037370)	0.10883 (.0099240)	0.046770 (.0055811)	0.023673 (.0034401)	0.29025 (0.017897)	0.084144 (.0087407)	0.034360 (.0047169)	0.016928 (.0027743)
D	0.60316 (0.056989)	0.19155 (0.025805)	0.087302 (0.015705)	0.048714 (0.011503)	0.54383 (0.055796)	0.16261 (0.023978)	0.073130 (0.014470)	0.040957 (0.010686)	0.41977 (0.065537)	0.12556 (0.020709)	0.055501 (0.012836)	0.031642 (.0095678)
AP	0.46823 (0.038324)	0.14060 (0.011728)	0.061305 (.0053892)	0.032225 (.0031634)	0.40198 (0.031797)	0.11807 (0.010249)	0.050583 (.0045829)	0.026387 (.0027419)	0.31755 (0.031250)	0.089780 (.0079631)	0.037300 (0.037300)	0.019390 (.0022372)
CP	0.39403 (0.028823)	0.091759 (.0062360)	0.030223 (.0027097)	0.011939 (.0014183)	0.33501 (0.016019)	0.070677 (0.005298)	0.022161 (.0023650)	.0084418 (.0011772)	0.23925 (0.023817)	0.045216 (.0045317)	0.013308 (.0019074)	.0047814 (.00083111)
F	0.15846 (0.017412)	0.34735 (.0070198)	0.50701 (.0043154)	0.62951 (.0029991)	0.16658 (0.025985)	0.40138 (.0069776)	0.56188 (.0039706)	0.68008 (.0027407)	0.24657 (0.015171)	0.49637 (.0062677)	0.64322 (.0034104)	0.75341 (.0023886)

(Sampling errors in parenthesis. The number in parenthesis in the F line is the sampling error for TP, not for F)

Table 9: <u>proportion of changes in FGT poverty indices, due to the price correction</u>

poverty lines based on the first quintile

	z_1			>	z_2			>	z_3			
a	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**
A	0.19649** (0.023858)	0.34415* (.0046442)	0.37749* (.0019527)	0.40330* (.0010873)	0.38190* (0.025451)	0.28241* (.0031295)	0.30675* (.0012457)	0.34400* (.0008014)	0.17089 (0.020485)	0.23531* (.0024399)	0.28058* (.0010489)	0.32098* (.00073014)
B	-0.18231* (0.025483)	-0.18831* (.0016063)	-0.19983* (.0003868)	-0.19551* (.00014697)	-0.21069* (0.011822)	-0.27494* (.0016692)	-0.26431* (.00029671)	-0.24429* (.0002133)	-0.29653 (0.011549)	-0.31826* (.0013447)	-0.28239* (.00026636)	-0.2618* (.00027169)
C	0.16422* (0.011661)	0.099259* (.0017733)	0.10296* (.0010298)	0.12895* (.00070888)	-0.03451 (.0060296)	0.011998 (.0019987)	0.050409 (.00071220)	0.080807 (.00056326)	-0.0327 (0.019413)	0.00457 (.0019967)	0.0286 (.00057141)	0.057850 (.00052997)
D	0.077653* (0.023707)	0.13828* (.0051992)	0.13150** (.0030082)	0.10471** (.0016792)	0.060827 (0.014360)	0.081244 (.0049327)	0.066060 (.0022128)	0.039939 (.0010082)	-0.01132 (.0096882)	0.070388 (.0048803)	0.035915 (.0017854)	0.014848 (.00074811)
AP	0.053132 (0.013300)	0.086905* (.0023388)	0.096497* (.0009962)	0.10497* (.00047937)	0.022869 (.0083444)	0.017254 (.0015222)	0.039870 (.00063842)	0.052216* (.00025861)	-0.05901 (.0045181)	-.003686 (.0014440)	0.018318 (.00050899)	0.030574 (.00018728)
CP	0.036312 (0.010227)	0.053406 (.0026896)	0.015011 (.0007148)	-0.026255 (.00026352)	-.002473* (.0081068)	-0.07886* (.0011051)	-0.14286* (.00028498)	-0.19477* (.00015577)	-0.04498 (.0068518)	-0.16131* (.0006901)	-0.22086 (.00022265)	-0.28020 (.00014493)
F	0.039084 (0.015253)	0.029754* (.0010057)	0.041641* (.0003486)	0.041004* (.00024772)	0.022330* (.0057855)	-0.25507* (.0006904)	-0.22618* (.00041716)	-0.25773* (.00013640)	-0.01318 (.0045066)	0.095790* (.0008583)	0.074326* (.00036152)	0.048258* (.000083998)

poverty lines based on the second quintile

	z_4			>	z_5			>	z_6			
a	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**	**0**	**1**	**2**	**3**
A	0.19352* (0.013838)	0.32739* (.0054379)	0.39351* (.0029346)	0.43415 (.0017476)	0.091031* (0.017698)	0.17162* (.0045340)	0.21215* (.0025028)	0.23928* (.0014859)	.0091398 (0.011377)	0.063656 (.0040950)	0.090900 (.0021133)	0.10813 (.0012305)
B	-0.14154* (0.020583)	-0.14833* (.0051404)	-0.14979* (.0013416)	-0.14787* (.00035497)	-0.26748* (0.016944)	-0.25962* (.0046522)	-0.27695* (.0011972)	-0.28764* (.00052006)	-0.29541* (0.034528)	-0.34700* (.0039181)	-0.37793* (.0015214)	-0.39044* (.00071117)
C	0.085021 (0.037852)	0.096796* (.0035301)	0.13221* (.0012685)	0.15978* (.00088391)	-0.06790 (0.036075)	-0.03468 (.0031442)	-0.013333 (.0012623)	-.0037035 (.00069958)	-0.17015* (0.020343)	-0.11017* (.0030949)	-0.12012* (.0011552)	-0.12167* (.00055386)
D	0.15387* (0.017243)	0.17508* (.0079112)	0.17786* (.0043152)	0.16845* (.0026658)	0.089730* (0.020486)	0.041771 (.0060684)	0.034254* (.0032413)	0.028244* (.0020153)	-0.02858 (0.030013)	-0.05961** (.0039857)	-0.070771** (.0025743)	-0.068696** (.0014624)
AP	0.066025* (0.013996)	0.10491* (.0039257)	0.12936* (.0018363)	0.14533* (.00095095)	-0.03853 (0.012343)	-0.02646 (.0033498)	-0.017577 (.0013989)	-.0094226 (.00069028)	-0.12296* (0.013080)	-0.11989* (.0027424)	-0.12384* (.0010506)	-0.11611* (.00048370)
CP	-0.01544 (0.016140)	0.090629* (.0034957)	0.11249* (.0015513)	0.10557* (.00069238)	-0.09902* (0.024106)	-0.09635* (.0027577)	-0.11395** (.0011509)	-0.14507* (.00051911)	-0.22210* (0.015362)	-0.23977* (.0022373)	-0.28205* (.00081065)	-0.33055* (.00041112)
F	0.78374* (0.020539)	0.025176* (.0018006)	0.014994* (.0005333)	0.021633* (.00035170)	0.50560* (0.028051)	0.13030* (.0015000)	0.092642* (.00055199)	0.080658* (.00029433)	0.63758* (0.019936)	0.19039* (.0016612)	0.13927* (.00064138)	0.11712* (.00029162)

The first line of each cell is: 1 - (result in table 7)/(result in table 8).

(Sampling errors for the differences in parenthesis)

* = difference significant at 5 % level.

** = difference significant at 10 % level.

Table 10 : CHU's and Watts' Poverty indices

(with correction for price variability)

poverty lines based on the first quintile												
	z_1			>	z_2			>	z_3			
a	W	1	1/2	1/3	W	1	½	1/3	W	1	½	1/3
A	0.10248	0.075877	0.043652	0.030363	0.06570	0.048906	0.028068	0.01950	0.052879	0.039262	0.022562	0.015688
	(0.015417)	(.0099346)	(.0060365)	(.0043034)	(0.011637)	(.0074624)	(.0045493)	(.0032465)	(0.010081)	(.0063292)	(.0038989)	(.0027926)
B	0.077832	0.060845	0.034201	0.023564	0.044994	0.035544	0.019883	0.013677	0.033694	0.026476	0.01485	0.010227
	(0.014248)	(0.010223)	(.0059550)	(.0041665)	(0.010088)	(.0071378)	(.0042136)	(.0029529)	(.0090910)	(.0066739)	(.0038717)	(.0026957)
C	0.10266	0.077589	0.044286	0.030692	0.064816	0.049681	0.028181	0.019485	0.051707	0.039814	0.022540	0.015574
	(0.012468)	(.0085844)	(.0051129)	(.0036007)	(.0094726)	(.0068602)	(.0039895)	(.0027861)	(.0077899)	(.0055685)	(.0032637)	(.0022838)
D	0.17637	0.11607	0.068887	0.048895	0.12081	0.075526	0.045576	0.032655	0.10232	0.061999	0.037828	0.027259
	(0.035152)	(0.019727)	(0.012604)	(.0091921)	(0.029908)	(0.015837)	(0.010385)	(.0076466)	(0.027801)	(0.014641)	(.0096173)	(.0070867)
AP	0.11484	0.082595	0.047756	0.033378	0.074080	0.052414	0.030427	0.021331	0.060151	0.041888	0.024446	0.017187
	(0.011726)	(.0073364)	(.0043977)	(.0031484)	(.0084576)	(.0047676)	(.0029602)	(.0021594)	(.0073807)	(.0039802)	(.0025092)	(.0018455)
CP	0.046966	0.039374	0.021448	0.014593	0.022011	0.018846	0.010168	.0068928	0.015298	0.013300	.0071240	.0048169
	(.0054932)	(.0044385)	(.0024613)	(.0016852)	(.0040220)	(.0034102)	(.0018498)	(.0012559)	(.0034132)	(.0030093)	(.0016012)	(.0010799)
F	0.59102	0.52329	0.55089	0.56280	0.70288	0.64043	0.66583	0.67687	0.74568	0.68248	0.70858	0.71974
	(0.010229)	(.0059729)	(.0036951)	(.0026787)	(.0083193)	(.0044953)	(.0028800)	(.0021161)	(.0075223)	(.0039738)	(.0025720)	(.0018970)

poverty lines based on the second quintile												
	z_4			>	z_5			>	z_6			
a	W	1	½	1/3	W	1	½	1/3	W	1	½	1/3
A	0.18042	0.13182	0.076296	0.053185	0.15015	0.11038	0.063708	0.044365	0.11177	0.08261	0.047565	0.033094
	(0.022870)	(0.014818)	(.0089861)	(.0063989)	(0.020047)	(0.012988)	(0.007858)	(.0056085)	(0.016153)	(0.010378)	(.0063146)	(.0045037)
B	0.14458	0.10961	0.062498	0.043285	0.11817	0.090437	0.051343	0.035501	0.085852	0.066808	0.037635	0.025950
	(0.022697)	(0.016335)	(.0094919)	(.0066354)	(0.019778)	(0.014440)	(.0083266)	(.0058070)	(0.015495)	(0.011252)	(.0065098)	(.0045456)
C	0.17641	0.12940	0.074867	0.052150	0.14718	0.10883	0.062746	0.043646	0.11183	0.084144	0.048126	0.033378
	(0.017939)	(0.012040)	(.0072369)	(.0051208)	(0.015105)	(.0099240)	(.0060381)	(.0042877)	(0.012917)	(.0087407)	(.0052499)	(.0037081)
D	0.28455	0.19155	0.11332	0.080092	0.24269	0.16261	0.096205	0.068057	0.18977	0.12556	0.074428	0.052776
	(0.044618)	(0.025805)	(0.016275)	(0.011813)	(0.041246)	(0.023978)	(0.015081)	(0.010935)	(0.036369)	(0.020709)	(0.01314)	(.0095614)
AP	0.19649	0.14060	0.081745	0.057178	0.16455	0.11807	0.068501	0.047892	0.12481	0.089780	0.051938	0.036299
	(0.017685)	(0.011728)	(.0069294)	(.0049087)	(0.015498)	(0.010249)	(.0060429)	(.0042833)	(0.012503)	(.0079631)	(.0047444)	(.0033858)
CP	0.11300	0.091759	0.050724	0.034702	0.085989	0.070677	0.038850	0.026524	0.054174	0.045216	0.024681	0.016805
	(.0079227)	(.0062360)	(.0034921)	(.0024023)	(.0067538)	(.0052982)	(.0029736)	(.0020467)	(.0056835)	(.0045317)	(.0025280)	(.0017349)
F	0.42491	0.34735	0.37949	0.39309	0.47742	0.40138	0.43285	0.44618	0.56593	0.49637	0.52481	0.53703
	(0.012445)	(.0070198)	(.0044146)	(.0032186)	(0.012049)	(.0069776)	(.0043413)	(.0031508)	(0.010753)	(.0062677)	(.0038860)	(.0028175)

(Sampling errors in parenthesis. The number in parenthesis in the F line is the sampling error for TP, not for F)

Table 11 : CHU's and Watts' Poverty indices

(Indicator par capita not corrected by price index)

poverty lines based on the first quintile												
	z_1 >				z_2 >				z_3			
a	**W**	**1**	**1/2**	**1/3**	**W**	**1**	**1/2**	**1/3**	**W**	**1**	**1/2**	**1/3**
A	0.075291 (0.012701)	0.056450 (.0085433)	0.032316 (.0051262)	0.022429 (.0036259)	0.050596 (0.010033)	0.038136 (.0067978)	0.021782 (.0040646)	0.015105 (.0028711)	0.042043 (.0093312)	0.03178 (.0064083)	0.018132 (.0038079)	0.012568 (.0026836)
B	0.096194 (0.014627)	0.074961 (0.010376)	0.042197 (.0060616)	0.029088 (.0042496)	0.061833 (0.011300)	0.049022 (.0079844)	0.027367 (.0046987)	0.018812 (.0032950)	0.048890 (.0099857)	0.03883 (.0071677)	0.021661 (.0041915)	0.014886 (.0029308)
C	0.093029 (0.012464)	0.070583 (.0090653)	0.040223 (.0052635)	0.027855 (.0036715)	0.063339 (.0093685)	0.04909 (.0069807)	0.027697 (.0040049)	0.019113 (.0027822)	0.051103 (.0080968)	0.039633 (.0060750)	0.022357 (.0034729)	0.015428 (.0024095)
D	0.15738 (0.028261)	0.10197 (0.015113)	0.060806 (.0098302)	0.043281 (.0072283)	0.11355 (0.023963)	0.069851 (0.011783)	0.042396 (0.007976)	0.030466 (.0059468)	0.097625 (0.022629)	0.057922 (0.011177)	0.035647 (.0075486)	0.025784 (.0056214)
AP	0.10547 (.0095695)	0.075991 (.0058738)	0.043885 (.0035132)	0.030663 (.0025226)	0.072329 (.0074021)	0.051525 (.0042015)	0.029811 (.0079849)	0.020874 (.0018666)	0.059915 (.0063575)	0.042043 (.0033971)	0.024449 (.0021069)	0.017167 (.0015537)
CP	0.044979 (.0056163)	0.037374 (.0043412)	0.020442 (.0024614)	0.013930 (.0016979)	0.024239 (.0046260)	0.020460 (.0038821)	0.011112 (.0021145)	.0075514 (.0014383)	0.018495 (.0038818)	0.015858 (.0033597)	.0085484 (.0018022)	.0057938 (0012194)
F	0.57355 (0.010399)	0.50817 (.0063319)	0.53420 (.0038369)	0.54572 (.0027611)	0.66489 (.0079849)	0.60291 (.0043939)	0.62724 (.0027727)	0.63824 (.0020324)	0.69132 (.0067474)	0.62282 (.0033511)	0.65036 (.0022188)	0.66250 (.0016555)

poverty lines based on the second quintile												
	z_4 >				z_5 >				z_6			
a	**W**	**1**	**1/2**	**1/3**	**W**	**1**	**1/2**	**1/3**	**W**	**1**	**1/2**	**1/3**
A	0.13328 (0.019050)	0.099308 (0.012818)	0.056989 (.0076762)	0.039592 (.0054292)	0.12622 (0.018292)	0.094143 (0.012312)	0.054003 (.0073742)	0.037511 (.0052155)	0.10386 (0.015869)	0.077667 (0.010613)	0.044503 (.0063802)	0.030899 (.0045172)
B	0.16981 (0.019741)	0.12870 (0.013654)	0.073391 (.0080642)	0.05083 (.005677)	0.16082 (0.019038)	0.12215 (0.013201)	0.069587 (.0077851)	0.048178 (.0054783)	0.13329 (.017559)	0.10231 (0.012427)	0.058001 (0.0072561)	0.040084 (.0050884)
C	0.15884 (0.017048)	0.11798 (0.011634)	0.067872 (.0069534)	0.047173 (.0049041)	0.15134 (0.016632)	0.11274 (0.011430)	0.064772 (.0068117)	0.044997 (.0047982)	0.12602 (.015114)	0.094562 (0.010645)	0.054156 (.0062757)	0.037576 (.0044018)
D	0.24450 (0.034537)	0.16301 (0.019096)	0.09664 (0.012245)	0.068423 (.0089566)	0.23448 (0.033872)	0.15609 (0.018742)	0.092548 (0.012012)	0.065543 (.0087848)	0.20195 (0.031512)	0.13352 (0.017250)	0.079230 (0.011107)	0.056184 (.0081369)
AP	0.17661 (0.013216)	0.12725 (.0086872)	0.073723 (.0051137)	0.05150 (.0036259)	0.16822 (0.012737)	0.12128 (.0083188)	0.070228 (.0049041)	0.049057 (.0034812)	0.14128 (0.011403)	0.10201 (.0073173)	0.058973 (.0043264)	0.041186 (.0030810)
CP	0.10332 (.0063066)	0.084134 (.0047054)	0.046436 (.0026931)	0.03175 (.0018707)	0.095817 (.0060175)	0.078213 (.0044218)	0.043119 (.0025520)	0.029474 (.0017772)	0.072359 (.0055395)	0.059477 (.0039451)	0.032688 (.0023243)	0.022317 (.0016263)
F	0.41494 (0.010650)	0.33882 (.0056222)	0.37013 (.0036544)	0.38347 (.0026940)	0.43039 (0.010747)	0.35511 (.0057317)	0.38601 (.0037080)	0.39920 (.0027287)	0.48784 (0.011153)	0.41698 (.0064543)	0.44571 (.0040123)	0.45815 (.0029119)

(Sampling errors in parenthesis. The number in parenthesis in the F line is the sampling error for TP, not for F)

Table 12: <u>Proportions of change in CHU and Watts' poverty indices, due to the price correction</u>

poverty lines based on the first quintile												
	z_1 >				z_2 >				z_3			
a	W	1	1/2	1/3	W	1	1/2	1/3	W	1	1/2	1/3
A	0.36112* (.0065501)	0.34415* (.0046442)	0.35079* (.0026915)	0.35374* (.0018867)	0.29862* (.0043702)	0.28241* (.0031295)	0.28859* (.0018059)	0.29149* (.0012641)	0.25774* (.0033861)	0.23531* (.0024399)	0.24432* (.0014001)	0.24825* (.00097919)
B	-0.19089* (.0018004)	-0.18831* (.0016063)	-0.18949* (.0008430)	-0.18991* (.00056757)	-0.27233* (.0019542)	-0.27494* (.0016692)	-0.27347* (.00088512)	-0.27296* (.00060087)	-0.31082* (.0015599)	-0.31826* (0.003344)	-0.31430* (.00070535)	-0.3129* (.00047840)
C	0.10353* (.0025094)	0.099259* (.0017733)	0.10101* (.0010235)	0.10185* (.00071774)	0.023319 (.0025823)	0.011998* (.0019987)	0.017475 (.0011085)	0.019463 (.00076528)	0.011819* (.0023592)	.0045669* (.0019967)	.0081854* (.0010609)	.0094633* (.00072139)
D	0.12066* (.0074118)	0.13828* (.0051992)	0.13290* (.0030843)	0.12974* (.0021634)	0.06393 (.0064498)	0.081244 (.0049327)	0.075007 (.0028032)	0.071851 (.0019355)	0.048092 (.0059828)	0.070388 (.0048803)	0.060959 (.0026852)	0.057206 (.0018339)
AP	0.089564* (.0030070)	0.086905* (.0023388)	0.088208* (.0013246)	0.088543* (.00091336)	0.024209 (.0018638)	0.017254 (.0015222)	0.020664 (.00084788)	0.021893 (.00057910)	.0039222 (.0016524)	-.003877 (0.001440)	-.0001227 (.00078017)	.0011650 (.00052677)
CP	0.044176 (.0031820)	0.053513 (.0026896)	0.049212 (.0014558)	0.047595 (.00098916)	-0.09191* (.0013243)	-0.07888** (.0011051)	-0.084953 (.00060018)	-0.087216 (.00040888)	-0.17263* (.0008484)	-0.16131* (.0006901)	-0.16663* (.00037856)	-0.16861* (.00025910)
F	0.030459* (.0009981)	0.029754* (0.0010057)	0.031243* (.0005079)	0.031298* (.00033506)	0.057137* (.0008498)	0.062232* (.0006904)	0.061523* (.00038497)	0.060526* (.00026244)	0.078632* (.0009772)	0.09579* (.0008583)	0.08952* (.00046377)	0.08952* (.00031223)

poverty lines based on the second quintile												
	z_4 >				z_5 >				z_6			
a	W	1	1/2	1/3	W	1	1/2	1/3	W	1	1/2	1/3
A	0.35369* (.0084444)	0.32739* (.0054379)	0.33878* (.0033070)	0.34333* (.0023564)	0.18959* (.0070398)	0.17247* (.0045340)	0.17971* (.0027648)	0.18272* (.0019699)	0.076160 (.0060642)	0.063656 (.0040950)	0.068804 (.0024449)	0.071350 (.0017277)
B	-0.14858* (.0058664)	-0.14833* (.0051404)	-0.14842* (.0027511)	-0.14844* (.0018574)	-0.26564 * (.0050605)	-0.25968* (.0046522)	-0.26218* (.0024292)	-0.26313* (.0016264)	-0.35592* (.0046450)	-0.34700* (.0039181)	-0.35113* (.0021112)	-0.35261* (.0014343)
C	0.11061* (.0048347)	0.096796* (.0035301)	0.10296* (.0020228)	0.10551* (.0014115)	-0.02748 (.0040658)	-0.03468 (.0031442)	-0.031279 (.0017692)	-0.030158 (.0012207)	-0.11284* (.0038431)	-0.11017* (.0030949)	-0.11135* (.0017111)	-0.11172* (.0011722)
D	0.16380* (0.011533)	0.17508* (.0079112)	0.17259* (.0047220)	0.17054* (.0033268)	0.035014 (.0084972)	0.041771 (.0060684)	0.0396 (.0035713)	0.0384 (.0024993)	-0.06031* (.0056185)	-0.05961* (.0039857)	-0.060608 (.0023781)	-0.060658* (.0016646)
AP	0.11256* (.0053990)	0.10491* (.0039257)	0.10881* (.0022826)	0.11014* (0.001591)	-0.0218 (.0043379)	-0.02646 (.0033498)	-0.024591 (.0019048)	-0.023748 (.0013150)	-0.11665* (.0033562)	-0.11989* (.0027424)	-0.11929* (.0015295)	-0.11866* (.0010456)
CP	0.093690* (.0046170)	0.090629* (.0034957)	0.092342* (.0019977)	0.092839* (.0013832)	-0.10257* (.0035865)	-0.09635* (.0027577)	-0.099005* (.0015601)	-0.10009* (.0010776)	-0.25132* (.0028234)	-0.23977* (.0022373)	-0.24477* (.0012443)	-0.24699* (.00085514)
F	0.024126* (.0021021)	0.025176* (.0018006)	0.025288* (.0009499)	0.025274* (.00064380)	0.10927* (.0016095)	0.13030* (.0015000)	0.12134* (.00077834)	0.11769* (.00052054)	0.16007* (.0019083)	0.19039* (.0016612)	0.17747* (.00089808)	0.17217* (.00060692)

* = difference significant at 5 % level.

** = difference significant at 10 % level.

Table 13: Variation of the population of the poor, due to the price correction

Lines Periods	1 A	1 B	2 A	2 B	3 A	3 B	4 A	4 B	5 A	5 B	6 A	6 B
Year	1.45	2.18	0.63	0.61	0.58	0.20	2.90	2.29	4.94	1.26	7.12	0.29
Quarter A	0.67	5.13	0.00	5.46	0.93	3.17	0.36	7.46	0.93	4.20	3.66	3.93
Quarter B	5.59	0.30	4.52	0.00	5.67	0.00	8.60	1.78	11.8	0.29	10.8	0.00
Quarter C	2.57	6.52	2.23	1.53	2.35	1.78	2.36	5.64	4.43	1.84	6.38	0.43
Quarter D	1.57	4.44	1.85	3.55	3.52	3.27	0.00	8.04	0.89	5.37	2.94	1.70

The first column (A) for each line corresponds to the percentage households that are poor before the price correction but not after. The second column (B) for each line corresponds to the percentage households that are poor after the price correction but not before.The lines are those calculated from corrected living standard distributions.

Table 15: AP FGT indices for distributions x/H

poverty lines based on the first quintile												
	z_1 >				z_2 >				z_3			
a	0	1	2	3	0	1	2	3	0	1	2	3
AP	0.24152	0.063711	0.025632	0.013389	0.16460	0.039040	0.015976	0.0088629	0.13122	0.030625	0.013007	0.0074826
	z_4 >				z_5 >				z_6			
a	0	1	2	3	0	1	2	3	0	1	2	3
AP	0.39445	0.11318	0.047675	0.024614	0.34075	0.093825	0.038872	0.020023	0.25557	0.069590	0.028211	0.014644

Table 14: **Residual populations (y > 2 z /(a + 1))**

period	a \ line	1	2	3	4	5	6
year	1	20.3	10.2	7.4	39.8	34.0	23.4
year	2	3.9	0.8	0.8	9.8	7.0	4.7
year	3	0.8	0	0	2.0	1.6	0.8
year	4	0	0	0	0.8	0	0
year	5	0	0	0	0	0	0
quarter A	1	27.0	19.5	15.6	43.4	38.7	29.7
quarter A	2	10.2	6.3	5.1	18.4	14.5	10.9
quarter A	3	4.7	3.1	2.3	8.6	6.3	5.1
quarter A	4	2.3	1.2	1.2	4.7	3.5	2.7
quarter A	5	1.2	1.2	0.8	2.7	1.6	1.2
quarter B	1	23.0	16.4	12.9	41.4	32.4	25.8
quarter B	2	6.3	3.5	3.1	15.2	10.9	7.4
quarter B	3	2.7	0.8	0.8	3.9	3.9	2.7
quarter B	4	0.8	0.8	0.8	2.7	2.0	0.8
quarter B	5	0.8	0.4	0.4	0.8	0.8	0.8
quarter C	1	29.3	20.3	16.8	42.2	35.9	30.1
quarter C	2	10.9	5.1	4.7	18.0	16.4	10.9
quarter C	3	4.3	3.1	2.7	8.2	5.9	4.7
quarter C	4	2.7	1.2	1.2	4.3	3.5	2.7
quarter C	5	1.2	0.4	0.4	2.7	2.0	1.2
quarter D	1	37.5	27.7	20.3	59.0	53.1	39.8
quarter D	2	12.9	7.8	5.5	24.2	18.8	15.6
quarter D	3	3.9	2.7	2.7	10.5	8.2	3.9
quarter D	4	2.7	2.3	2.3	3.9	3.9	2.7
quarter D	5	2.3	2.0	2.0	2.7	2.7	2.3

Appendix 1: Sampling standard-errors estimators

The poverty indicator of a sub-population is estimated by a ratio of the type

$$\overline{y_x}' = \frac{z'}{x'}$$

where ' denotes the Horwitz-Thompson estimator for a total (sum of values for the variable of interest weighted by the inverse of inclusion probability). z is the sum of the poverty in the sub-population and x is the size of the sub-population. The variance associated with the sampling error is then approximated by:

$$V(\overline{y_x}') = [V(z') - 2\overline{y_x}' Cov(z',x') + (\overline{y_x}')^2 V(x')] \;/\; (x')^2$$

obtained from a Taylor expansion at the first order from function Y = f(Z/X) around (E y', Ex') and because E z' ≠ 0 and x' does not cancel, where the appropriate expectancies are estimated by x' and $\overline{y_x}'$.

We divide the sample of communes (first actual stage of the sampling since all the prefectures are drawn) in five super-strata (α = 1 to 5) so as to group together the communes sharing similar characteristics, and to reduce a priori the variance intra-strata. Several sectors are assumed to have been drawn in each strata. This allow the estimation of the variance intra-strata, while the calculus of the variance intra-commune was impossible since in fact only one sector had been drawn in each commune. Then, the Horwitz-Thompson formula for superstrata α is:

$$z'_\alpha = \sum_h \frac{M_h}{m_{h_\alpha}} \sum_{i=1}^{m_{h_\alpha}} \frac{N_{hi}}{n_{hi}} \sum_{j=1}^{n_{hi}} \frac{Q_{hij}}{q_{hij}} \sum_{k=1}^{q_{hij}} z_{hijk}$$

and

$$x'_\alpha = \sum_h \frac{M_h}{m_{h_\alpha}} \sum_{i=1}^{m_{h_\alpha}} \frac{N_{hi}}{n_{hi}} \sum_{j=1}^{n_{hi}} \frac{Q_{hij}}{q_{hij}} \sum_{k=1}^{q_{hij}} x_{hijk}$$

where M_h is the number of communes in prefecture h; $m_{h\alpha}$ is the number of communes in prefecture h and drawn in superstrata α; N_{hi} is the number of sectors in commune i of prefecture h and superstrata α; n_{hi} is the number of sectors drawn in commune i of prefecture h and superstrata α; Q_{hij} is the number of households in sector j of commune i of prefecture h; q_{hij} is the number of households drawn in sector j of commune i of prefecture h and superstrata α. A similar formula can also be used to account for the intermediary drawing of several districts in every sector.

Cov(z',x') is estimated by:

$$\hat{Cov}(z',x') = \frac{1}{20} \sum_{\alpha=1}^{5} (z'_\alpha - z').(x'_\alpha - x')$$

and similar formula for V(x) and V(z) are obtained by making x=z.

More accurate formula could be used (with resampling, post-stratification, optimal definition of strata), but the present one is believed to be enough to obtain useful assessments of the standard-errors. Indeed, the existence of remaining measurement errors makes illusory the accuracy of inferences based only on sampling errors.

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Appendix

Proof of proposition 5.

$$P = \int_0^{+\infty} \int_0^{x/z} f\left(\frac{x}{I}, z\right) 1_{\left[\frac{x}{I} < z\right]} dF_2(I|x)\; dF_1(x)$$

$$+ \int_0^{+\infty} \int_{x/z}^{+\infty} f\left(\frac{x}{I}, z\right) 1_{\left[\frac{x}{I} < z\right]} dF_2(I|x) dF_1(x)$$

$$= \int_0^{+\infty} \int_{x/z}^{+\infty} f\left(\frac{x}{I}, z\right) 1_{\left[\frac{x}{I} < z\right]} dF_2(I|x) dF_1(x)$$

$$\leq \int_0^{+\infty} f\left(x \int_{x/z}^{+\infty} \frac{1}{I} dF_2(I|x), z\right) 1_{\left[x \int_{x/z}^{+\infty} \frac{1}{I} dF_2(I|x) < z\right]} dF_1(x)$$

$$= \int_0^{+\infty} f\left(\frac{x}{H_x}, z\right) 1_{\left[\frac{x}{H_x} < z\right]} dF_1(x)$$

Moreover, if there exists a finite maximal living standard x^*, that will be assimilate to the observed maximum, then we have

$$\frac{1}{H_x} < \int_{(\max\ x)/z}^{+\infty} \frac{1}{I} dF_1(I) = \frac{1}{\bar{H}}$$

Then,

$$P \leq \int_0^{+\infty} f\left(\frac{x}{\bar{H}}, z\right) 1_{\left[\frac{x}{\bar{H}} < z\right]} dF_2(x) = \bar{P}$$

QED.