Supply Flexibility and Insurance Under Commodity Market Instability

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Abstract: The paper integrates the analysis of price volatility and supply instability for a commodity exporting country, adding consideration of the role of the non-tradables sector to the literature. The non-tradables sector plays a potentially significant role in changing the economy's response to instability and increases the returns from access to risk markets.

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Introduction

Instability in primary commodity markets has given rise to sustained concern amongst policy makers and a substantial academic literature. Turnovsky (1974), Anderson and Riley (1976) and Eaton (1979) amongst others have analysed the consequences of world price volatility for small open economies while Svedberg (1990) considered the welfare effects of supply variability. Gilbert (1985) considers both types of volatility together with insurance by means of trading on futures markets. A literature has developed in parallel on commodity price stabilisation schemes including Newbery and Stiglitz (1981) and Ghosh, Gilbert and Hughes Hallett (1987).

The current paper follows the first set in taking instability as given and analysing its consequences together with some preliminary results for domestic policy. A country level analysis, within a world market setting, appears to be the most appropriate since nearly all the International Commodity Agreements that sought to influence prices at the world level have now come to an end. Hence price volatility is an ongoing phenomenon that is more likely to be dealt with by domestic rather than international policy. A further institutional change of note is the widespread end of attempts to stabilise producer prices at the national level through the activities of marketing boards in exporting countries. Many of these were abolished in the context of structural adjustment programmes in the 1980s.

The paper extends the literature by including a non-tradables sector together with a systematic treatment of the consequences of different sources of commodity market instability (domestic supply shocks, foreign supply and demand shocks) within a simple model of a world commodity market. The non-tradables sector is missing from the current literature,[1] despite its well understood role in modifying the response of an economy to permanent shocks of the Dutch Disease variety, and is significant when considering volatility or sequences of temporary shocks also.

While a comprehensive analysis is presented, the main theme that is brought out is the potential value of flexibility in response to unstable prices or production conditions. As is well known, the revenue function of a small open economy is convex in relative prices if sectoral outputs can adjust to price changes and on the consumer side the indirect utility function is also convex in prices. These convexities contribute a benefit under price instability even though in practice these effects may be offset by the more intuitive utility costs to risk averse agents from the fluctuations in income caused by price or output instability in the absence of insurance. The benefits of pure price or supply variability, considered separately from income variability, are of interest however, both because they have received less attention in the literature and because the flexibility of supply in response to world prices is likely to be greatly enhanced by the end of producer price stabilisation. The separation between beneficial pure price effects and income effects is brought out by giving the economy access to perfect risk markets so that the link between price and income changes can be broken and the benefits of price or supply variability retained while income is smoothed. Thus in principle insurance will be superior to price stabilisation which removes both effects.

The idea that instability may give rise to benefits is to some extent counter-intuitive. Samuelson (1972) for example shows that in an otherwise stable market artificially created instability reduces expected welfare. The key difference between that model and the literature on price volatility is that the latter considers exogenous instability in which the economy may gain through a flexible response to changing conditions even if it would lose from moving away from a Pareto Optimum

[1]Though Gilbert (1985) notes that it may be important.

in the absence of exogenous disturbances.

The paper is structured as follows. Section 1 sets out the model and methodology used in the rest of the paper. The following three sections present the core analysis of the effects of supply and demand shocks in or on an open economy in the absence of risk markets. Section 2 considers price volatility on its own, caused by supply and/or demand shocks outside the country of interest. This section builds on Turnovsky (1974), Anderson and Riley (1976) and Eaton (1979), the major innovation being the introduction of non-tradables. The Eaton paper is of particular interest since it is the first to allow for some ex post production flexibility and hence make the revenue function potentially convex. Making use of modern methodology this paper shows the benefits of supply flexibility first noted by Oi (1961) in a more limited framework. Section 3 considers supply variability on its own, resulting from domestic supply shocks in the country of interest, following Svedberg (1990) who demonstrated the benefits of supply flexibility in this context though without non-tradables or insurance, and only for a small economy. Section 4 completes the core results by considering cross effects between domestic supply and foreign demand shocks and correlated supply shocks across producing countries. Section 5 brings this material together to show the impact of the different kinds of shocks occurring simultaneously.

Section 6 allows the economy access to a perfect risk market and analyses the optimal insurance or hedging choices for the representative agent and the impact of optimal risk market use on the change in welfare from volatility.

It is important to note the limitations of the paper and possible extensions for later research. Within the scope of the model the most important of these are that there is no consideration of possible changes in resource allocation or the appropriate role for government policy, and empirical estimates of the results are not presented. These are analysed in two accompanying papers.[2]

There are also a number of desirable extensions of scope. Each of these involves some form of investment decision that is at least partly irreversible and hence would represent a considerable change in the nature of the modelling exercise. The first of these is that the current paper does not consider commodity storage though it is unlikely that including storage would significantly affect the main results about the effects of volatility. The second limitation is that the paper makes use of a standard real international trade model and hence does not consider macroeconomic effects. Kanbur (1984) argues forcefully that a complete account of the consequences of volatility must contain a macroeconomic analysis. In addition there is no consideration of possible links between volatility and investment or growth. The analysis of the investment response to single temporary shocks in Bevan, Collier and Gunning (1990) provides a useful basis for future work in this area.

The final point is that the model assumes that factors of production are either costlessly mobile or entirely immobile. This makes the structure of the model reversible and avoids difficulties arising from the investment nature of a decision to incur a cost and move between sectors when agents may have an incentive to move back again in the future. This situation is analysed by Dixit (1989). Grossman (1983) and Grossman and Shapiro (1982) analyse the determinants of factor mobility, offering a richer framework to which the results of this paper could easily be extended.

1. Model and Methodology

[2] Mash (1997a) and (1997b).

The paper considers the impact of multiplicative demand and supply shocks in a world commodity market. Demand is characterised by (1) where Θ^D is a random variable that may be taken to represent random preferences or fluctuating incomes on the consumption side of the market.

$$D = 2^D D(P) \tag{1}$$

D is world demand for the commodity and P its price. Θ^D is taken to be exogenous to the country of interest whose consumers are assumed to have stable tastes. Making Θ^D exogenous implicitly assumes that commodity consumption in commodity exporting countries is an insignificant fraction of world consumption such that changes in incomes and consumption in those countries as a result of commodity price changes do not have a significant feedback effect on total consumption. This is a reasonable assumption since the level of commodity consumption in the majority of commodity exporters is very small both absolutely and compared with production. Most of these countries are also small on the production side of world markets.

In addition the assumed exogeneity of Θ^D means that the model does not consider possible North-South interactions of the kind examined by Kanbur and Vines (1986) whereby fluctuations in commodity markets affect incomes and hence consumption in the major consuming countries.

On the supply side we consider two countries, identified by subscript i, of which without loss of generality country 1 is taken to be the country of interest and 'country' 2 all other exporting countries aggregated together. It would be straightforward to disaggregate the latter but nothing would be gained by doing so. Supply in the two countries is given by (2) where x_i is the level of output of the commodity in country i, Θ_i^S is a random variable representing multiplicative supply variability, as a result of fluctuations in the weather for example, and $S_i(.)$ is the supply function for given Θ_i^S. This function could be written with the amount of capital and labour allocated to the x sector as its arguments but it is clearer to keep it as shown since the world commodity price, P, the price of non-tradables in country i, $P_{ni,}$ and the value of Θ_i^S are the determinants of the allocation of mobile factors and are of more immediate interest.

$$x_i = 2_i^S S_i(P, P_{ni}, 2_i^S) \qquad i=1,2. \tag{2}$$

The inclusion of prices and Θ_i^S in the function $S_i(.)$, rather than simply making S a constant, allows for possible ex post flexibility in production such that the allocation of mobile factors between sectors may respond to changes in those arguments. Both Θ_1^S, P and P_n will affect factor marginal products across the different sectors so in general it will be optimal for some factor reallocation to take place. Whether factor mobility, and hence production flexibility, is feasible will depend in part on the timing of supply and price shocks. For example if both price and the value of Θ are known only after resources have been committed to production and there is no further feasible flexibility, S will be a constant. Alternatively one or both shocks may be known while factors are still mobile in which case a response to the shock parameter(s) becomes possible. Which applies will depend (for commodities) on the particular good; annual crops for example may be less flexible within a production year than tree crops or mining where harvesting/extraction is closer to a continuous process (subject to a usually inflexible stock of trees or capital equipment in a mine). We leave this as an empirical issue and assume that there may be flexibility with respect to both price and supply conditions since it is desirable for the theoretical results to be as general as possible. This approach does not preclude the possibility that the supply elasticities with respect to price and Θ may be small or even zero.

Turning to the effect of the supply and demand shocks on price, a change in the price, denoted dP, is given by:

$$dP = \frac{dP}{d2^D}\, d2^D + \frac{dP}{d2^S_1}\, d2^S_1 + \frac{dP}{d2^S_2}\, d2^S_2 \qquad (3)$$

In which the price derivatives are to be evaluated with the other two shock parameters constant. The same interpretation is assumed for these price derivatives in the rest of the paper.

Equation (3) allows us to derive approximate expressions for the variance of the world price ($E[(dP)^2]$) and the covariance between price and any other variable, say z, which is E[dPdz] with dz given by substituting z for P in (3). In expressions of this kind the derivatives are to be evaluated with the shock parameters at their expected values.

The model and methodology adopted are both standard. The model is a real trade model with Walrasian microfoundations incorporating non-tradables. National income in either exporting country is given by the revenue function (4) where as before P is the commodity price and P_n the price of non-tradables which are treated as a single aggregate. The i subscript is dropped for simplicity in this expression and the one following.

$$m = m(P, P_n) \qquad (4)$$

We consider three sectors of which one is the non-tradable. The other two comprise good y whose price is used as the numeraire and normalised to unity (making P and P_n relative prices) and good x which is the commodity export. There is, of course, nothing in the formal analysis which means that good x has to be a primary commodity but it is convenient to refer to it in this way since market instability is chiefly of interest in commodities markets given their susceptibility to weather conditions and the concentration of many developing countries exports in these goods.

A further point is that good y is defined as tradable goods with stable prices with no restriction on whether they are exported or imported. The early literature tended to assume that the commodity comprises all of the economy's exports, in which case P is also the terms of trade, but this obscures the role of the share of exports of the commodity in national income which determines the exposure of the economy to price risk. In addition the paper assumes that the country of interest only produces one commodity.

We also make use of the indirect utility function which, for the representative agent of a country, is given by:

$$V = V(P, P_n, m(P, P_n)) \qquad (5)$$

The methodology used for assessing the effects of instability is the standard approach based on a second order Taylor expansion around the counterfactual for the variable of interest. We assume that the distribution of the Θ parameters is such that their volatility constitutes an arithmetic mean preserving spread and hence their counterfactual value is their expected value. This is chiefly for convenience because we are more interested in the impact of their volatility rather than level effects though the latter are also discussed. Allowing for symmetry between the cross derivatives, this approach gives the change in the expected utility of the representative agent by:

$$\Delta E[V] = E[V] - V(\bar{\Theta}^D, \bar{\Theta}_1^S, \bar{\Theta}_2^S) \approx$$

(6) $$\frac{1}{2}Var(\Theta^D)\frac{d^2V}{d\Theta^{D^2}} + \frac{1}{2}Var(\Theta_1^S)\frac{d^2V}{d\Theta_1^{S^2}} + \frac{1}{2}Var(\Theta_2^S)\frac{d}{d\Theta}$$

$$+Cov(\Theta^D, \Theta_1^S)\frac{d^2V}{d\Theta^D d\Theta_1^S} + Cov(\Theta^D, \Theta_2^S)\frac{d^2V}{d\Theta^D d\Theta_2^S} + Cov(\Theta_1^S, \Theta_2^S)\frac{d^2V}{d\Theta_1^S d\Theta_2^S}$$

The expressions above make two further assumptions. The first is that the methodology establishes the change in the expected value of the function of interest as a result of fluctuations in one or more of its arguments, thus ruling out possible changes in the function itself. This type of effect was shown by Newbery and Stiglitz (1981) when discussing supply responses to price stabilisation. A similar situation is examined in Mash (1997a) which asks the reverse question of whether the allocation of resources between sectors may change as prices and/or supply conditions become volatile. If a change does take place the revenue and indirect utility functions both shift at any given price. This paper ignores this possibility and the supply functions given by (2) are assumed stable.

The second assumption is that the relevant function is monotonic in the range of interest since, if it is not, first order effects may dominate the second order effects shown. A well known example is that price volatility around the autarkic price ratio will raise the expected welfare of the representative agent since utility is increasing in relative prices in both directions away from that point. It is assumed below that any price fluctuations are over a range solely to one side of the autarkic price ratio in the country of interest. As a result tradable goods are either always exported or always imported.

2. Price Volatility Alone

This section examines the consequences of price volatility for the representative agent of the model outlined above assuming that the only source of instability is through price volatility. This coincides with the scope of Turnovsky (1974), Anderson and Riley (1976) and Eaton (1979). Turnovsky et.al. (1980) give a through analysis of the case of a pure consumer. The main innovations are the addition of non-tradables to the model and clarification of the results by examining expected income before turning to expected utility. Risk markets are assumed absent.

Considering only the impact of foreign shocks amounts to setting Θ_1^S equal to its expected value, hence leaving only those terms in (6) that involve Θ^D and Θ_2^S which gives:

(7) $$\approx \frac{1}{2}Var(\Theta^D)\frac{d^2V}{d\Theta^{D^2}} + \frac{1}{2}Var(\Theta_2^S)\frac{d^2V}{d\Theta_2^{S^2}} + Cov(\Theta^D, \Theta_2^S$$

Before exploring (7) it is useful to consider the change in expected national income which is given by (7) with m in place of V. Hence we need to derive the second derivatives of m with respect to Θ^D and Θ_2^S plus the second cross derivative. With domestic supply conditions stable, Θ^D and Θ_2^S impact on national income solely through price so we have:

$$\frac{dm}{d\Theta^D} = \frac{dm}{dP}\frac{dP}{d\Theta^D} \qquad \frac{dm}{d\Theta_2^S} = \frac{dm}{dP}\frac{dP}{d\Theta_2^S}$$

$$\frac{d^2m}{d\Theta^{D^2}} = \frac{dm}{dP}\frac{d^2P}{d\Theta^{D^2}} + \left(\frac{dP}{d\Theta^D}\right)^2\frac{d^2m}{dP^2}$$

$$\frac{d^2m}{d\Theta_2^{S^2}} = \frac{dm}{dP}\frac{d^2P}{d\Theta_2^{S^2}} + \left(\frac{dP}{d\Theta_2^S}\right)^2\frac{d^2m}{dP^2}$$

$$\frac{d^2m}{d\Theta^D d\Theta_2^S} = \frac{dm}{dP}\frac{d^2P}{d\Theta^D d\Theta_2^S} + \frac{dP}{d\Theta^D}\frac{dP}{d\Theta_2^S}\frac{d^2m}{dP^2}$$

Substituting these into (7) (with m instead of V) and rearranging gives the change in expected income from price volatility caused by supply and demand shocks outside the country by:

$$\Delta E[m] \approx \frac{dm}{dP}\left[\frac{1}{2}Var(\Theta^D)\frac{d^2P}{d\Theta^{D^2}} + \frac{1}{2}Var(\Theta_2^S)\frac{d^2P}{d\Theta_2^{S^2}} + Cov(\Theta^D,\Theta_2^S)\frac{d^2P}{d\Theta^D\Theta_2^S}\right]$$

(8) $$\frac{1}{2}Var(\Theta^D)\left(\frac{dP}{d\Theta^D}\right)^2 + \frac{1}{2}Var(\Theta_2^S)\left(\frac{dP}{d\Theta_2^S}\right)^2 + Cov(\Theta^D,\Theta_2^S)$$

The first line of (8) gives the change in expected income as a result of a possible change in expected price. The first part of the line is the first derivative of national income with respect to price while the contents of the square bracket give the change in expected price. The second line is the impact of volatility in the commodity price separate from any change in the average level of the price. This comprises the product of the second derivative of income with respect to the price and the square bracket. The latter is in fact the variance of the price from (3) and the discussion beneath it.

The structure of (8) shows that when looking at foreign supply and demand shocks there is in fact no need to model the impact of the two kinds of shocks separately. This is because both impact on the economy of interest solely through the commodity price and hence nothing would be lost by considering the effect of price variation and possible change in expected price alone. This was done by Anderson and Riley (1976) and Eaton (1979) but it adds clarity to model the role of Θ^D and Θ_2^S explicitly when domestic supply shocks are added to the analysis in the following section.

The structure also confirms that the impact of price volatility, the second line in (8), may be considered separately from the changes in the expected price given by the first line. This puts into perspective a debate in the literature about the appropriate choice of a counterfactual relative price to use for comparison purposes when prices are volatile.[3] The early literature, including Anderson and Riley (1976), assumed an arithmetic mean counterfactual for the volatile relative price whereas Eaton (1979) uses a geometric mean counterfactual. The change was made because the arithmetic mean had been shown to be numeraire dependent in the sense that it makes a difference if P_y or P_x is taken as the numeraire or stable price with the other being volatile. This is because the value of $E[P_x/P_y]$ depends on which price is considered volatile, essentially because $P/1$ is linear in P whereas $1/P$ is not. It is straightforward to show that the geometric mean of P_x/P_y, assuming a geometric mean preserving spread in either of the prices, does not depend on which price is volatile. This is a desirable property of a geometric mean preserving spread as a counterfactual in the narrow context of modelling price volatility directly. It is clear, however, that the true counterfactual (the stable price that would occur in the absence of any shocks) depends on the value of the square bracket in the first line of (8) which will in turn depend on the particular circumstances of an individual commodity market and is also contingent on the assumption made that the shock parameters are themselves arithmetic mean preserving spreads. A priori an arithmetic or geometric mean might be a better approximation to the true counterfactual but neither is likely to be accurate. A more promising approach, given that the supply and demand shocks are likely to continue, is to focus on the volatility effects rather than the level effects which is the approach adopted below. It may also be argued that a more pragmatic counterfactual is the feasible stable price that could be achieved by a price stabilisation scheme. This is not considered explicitly since the focus of the paper is on the effects of ongoing volatility but it may be noted that Newbery and Stiglitz (1981) and Ghosh et.al. (1987) find that this price is likely to be lower than the arithmetic mean of the pre-stabilisation price distribution.

Turning to the actual value of (8), particularly the volatility effect given by the second line, it is useful to consider the case where non-tradables are excluded. This is because the expression refers to income in terms of good y and changes in this variable due to changes in the price of non-tradables do not correspond to real income since output and consumption of the non-tradable are equal by definition.

Without non-tradables the revenue function (4) and its price derivatives are simply:

$$m = Px + y \qquad \frac{dm}{dP} = x \qquad \frac{d^2m}{dP^2} = \frac{dx}{dP} \tag{9}$$

Substituting the second derivative in (9) into the second line of (8) shows that the volatility effect on expected national income is positive if sectoral outputs can respond to relative price changes such that sector y shrinks and sector x expands when P rises and vice versa. This is of course also the condition for the revenue function to become strictly convex.

Intuitively one may think of expected income (or the expected value of output) as a weighted average of the volume of sectoral outputs with prices as the weights. If sectoral outputs were fixed we would expect no increase in expected output because with symmetric price fluctuations the gains to the value of output from price increases for each sector would be exactly offset by losses from

[3]See Flemming, Turnovsky and Kemp (1977) and Boonekamp and Donaldson (1979).

price falls. If sectoral outputs do respond to relative prices, however, the weighted average would be higher as a result of price fluctuations because physical output for a sector is high when its "weight" is large and vice versa.

The possible benefit to expected income from production flexibility was first shown by Oi (1961) and is implicit in the welfare results in Eaton (1979) but not in Turnovsky (1974) or Anderson and Riley (1976). The latter papers assumed that factors must be allocated to sectors ex ante before relative prices are known, thus removing the possibility of production flexibility. Newbery and Stiglitz (1981) and Ghosh et.al. (1987) both mention Oi's result but do not include the flexibility effect in their assessments of the costs and benefits of price stabilisation. This may be due to a feeling that the relevant supply elasticity is likely to be small and perhaps also because the benefit from flexibility may not be so clear when considering all producers in a world market.

We turn to the effect of price volatility on the expected utility of the representative agent. As before we focus primarily on the second line of (8) (with V instead of m) which isolates the effect of volatility rather than the level effect of the first line. It is useful to first derive the impact effect of price on national income with non-tradables present which is given by:

$$\frac{dm}{dP} = x + n\frac{dP_n}{dP} \tag{10}$$

Where the presence of non-tradables leads to the addition of the second term compared with the first derivative in (9). This term reflects the change in the value of output of the non-tradable as its price changes in response to a change in the tradable price.

Given the indirect utility function (5) we have:

$$\frac{dV}{dP} = \frac{\partial V}{\partial P} + \frac{\partial V}{\partial P_n}\frac{dP_n}{dP} + \frac{\partial V}{\partial m}\frac{dm}{dP}$$

From which, using (10) and Roy's Identity for both the commodity and non-tradables we have:

$$\frac{dV}{dP} = \frac{\partial V}{\partial m}[x - c_x + \frac{dP_n}{dP}(n - c_n)]$$

where c_x is the level of consumption of the commodity, x its level of output and c_n and n the consumption and output of the non-tradable. Output must equal consumption for the non-tradable so $n=c_n$, the first order effect on non-tradables nets out, and the derivative reduces to:

$$\frac{dV}{dP} = =(x - c_x)\frac{\partial V}{\partial m} \tag{11}$$

Hence the impact effect of price on utility is equal to the product of exports of good x and the marginal utility of income. From (11) the second derivative is given by:

(12) $$\frac{d^2V}{dP^2} = (x - c_x)\frac{d}{dP}\left[\frac{\partial V}{\partial m}\right] + \left(\frac{dx}{dP} - \frac{dc_x}{dP}\right)\frac{\partial V}{\partial m}$$

where:

$$\frac{d}{dP}\left[\frac{\partial V}{\partial m}\right] = \frac{\partial}{\partial P}\left[\frac{\partial V}{\partial m}\right] + \frac{\partial}{\partial P_n}\left[\frac{\partial V}{\partial m}\right]\frac{dP_n}{dP} + \frac{\partial}{\partial m}\left[\frac{\partial V}{\partial m}\right]\frac{dm}{dP}$$

$$= \frac{\partial}{\partial m}\left[\frac{\partial V}{\partial P}\right] + \frac{\partial}{\partial m}\left[\frac{\partial V}{\partial P_n}\right]\frac{dP_n}{dP} + \left(x + n\frac{dP_n}{dP}\right)\frac{\partial^2 V}{\partial m^2}$$

$$= \frac{\partial}{\partial m}\left[-c_x\frac{\partial V}{\partial m}\right] + \frac{\partial}{\partial m}\left[-c_n\frac{\partial V}{\partial m}\right]\frac{dP_n}{dP} + \left(x + n\frac{dP_n}{dP}\right)\frac{\partial^2 V}{\partial m^2}$$

(13) $$= \frac{\partial V}{\partial m}\left[-\frac{\partial c_x}{\partial m} - \frac{\partial c_n}{\partial m}\frac{dP_n}{dP} - (x - c_x)A\right]$$

A is the coefficient of absolute risk aversion. We also have:

(14) $$= \frac{\partial x}{\partial P} + \frac{\partial x}{\partial P_n}\frac{dP_n}{dP} \qquad \frac{dc_x}{dP} = \frac{\partial c_x}{\partial P} + \frac{\partial c_x}{\partial P_n}\frac{dP_n}{dP} +$$

Substituting (14) and (13) into (12) and simplifying gives:

$$\frac{d^2V}{dP^2} = \frac{\partial V}{\partial m}\left[\frac{\partial x}{\partial P} - \frac{\partial c_x}{\partial P}\Big|_c - 2(x - c_x)\frac{\partial c_x}{\partial m} - (x - c_x)^2 A\right.$$

(15) $$\left. - \frac{dP_n}{dP}\left(\frac{\partial c_x}{\partial P_n}\Big|_c - \frac{\partial x}{\partial P_n} + (x - c_x)\frac{\partial c_n}{\partial m}\right)\right]$$

Where the top line is all that would apply if non-tradables were absent and the second line shows the impact of including them.

We first consider the top line of (15), leaving non-tradables to one side for the time being. The first term in the bracket is familiar from the second derivative in (9)[4] and represents the potential benefit from production flexibility for expected income. The second term is the equivalent term for consumption flexibility which will be positive allowing for its sign since this term reflects the convexity of the indirect utility function with respect to price.

The first two terms are pure price effects and both positive, contrasting with the second pair of terms which are income effects and both negative if the commodity is normal in consumption. The third term involves the income elasticity of demand and shows that the value of the expression is higher if the income response of consumption of the commodity is small or even for the good to be inferior. Intuitively this is because income rises when the commodity price rises and hence if the good is normal, consumption of it will tend to be high when its price is high and low when its price is low. More formally the term is present because a higher level of consumption of c_x means that the marginal utility of income is reduced more by the rise in the commodity price. The final term shows the cost to expected utility from the income fluctuations caused by price volatility if the representative agent is risk averse and risk markets are absent.

The contrast between the positive price effects and negative income effects has two key implications. First, and most obviously, the overall change in expected utility from volatility without non-tradables is ambiguous in sign. This contrasts with the view sometimes heard that price volatility must always be harmful and was demonstrated by both Anderson and Riley (1976) and Eaton (1979)[5]. These papers also note that the expression is more likely to be positive if the volume of exports of the commodity is small. In the equations above this is because this factor appears as a coefficient on each of the (negative) income effects. Intuitively if the volume of exports is small, price changes lead to only small income changes and hence the income effects are muted while the benefits from the pure price effects remain.

The second implication is that if the economy has access to a perfect risk market the income effects may be removed by smoothing income while the positive price effects remain. Leaving any level effects to one side, this in turn implies that in principle access to a perfect risk market is superior to price stabilisation.

Turning to the second line, which is made up of the additional terms arising from the inclusion of non-tradables, we may first derive the response of the non-tradable price to the commodity price. Since $n=c_n$ by definition it must be the case that:

(16) $$= \frac{\partial n}{\partial P} + \frac{\partial n}{\partial P_n}\frac{dP_n}{dP} = \frac{dc_n}{dP} = \frac{\partial c_n}{\partial P_n}\Big|_c \frac{dP_n}{dP} + \frac{\partial c_n}{\partial P}\Big|_c + (x-c$$

From which:

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[4]It is a partial derivative in (15) since P_n also changes whereas in (9) only the commodity price changes since non-tradables were excluded.

[5]See Anderson and Riley's equation (10), which omits the first term above since they assumed zero production flexibility ex post, and Eaton's equation (3.3) which includes the flexibility term. The latter differs slightly because it also includes the level effect that arises from Eaton's use of a geometric mean counterfactual.

(17) $$\frac{dP_n}{dP} = \frac{\frac{\partial c_n}{\partial P}\Big|_c - \frac{\partial n}{\partial P} + (x - c_x)\frac{\partial c_n}{\partial m}}{\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}\Big|_c}$$

All four partial price derivatives in (17) are positive, allowing for their signs, and the third term of the numerator will be positive if non-tradables are normal. Hence the overall expression is likely to be positive. In the language of Corden's (1984) paper on Dutch Disease, the partial price response of n in the numerator is part of the resource movement effect while the tendency of increased income to raise P_n shown by the third term is part of the spending effect.

Turning back to (15), noting that (17) will be positive in the standard case, the bracket shows that P_n increasing at the same time as P increases gives rise to a negative cross price effect of P_n on $(x-c_x)$. Welfare is increasing in the extent to which exports respond to increases in the commodity price, P and this is dampened by a simultaneous increase in P_n. On the production side the non-tradables sector will tend to absorb or release mobile factors at the same time as the commodity sector, thus reducing the flexibility of the economy in response to the commodity price.

The final term in the bracket of the second line of (15) shows a negative income effect whereby a rise in P, which gives rise to an increase in real income scaled by $(x-c_x)$, has a harmful effect on expected utility if accompanied by a rise in P_n when non-tradables are normal in consumption. This is similar to the income effect on c_x discussed above. Income in the model is measured in units of good y, the numeraire, and hence an expression for expected utility must reflect the fact that changes in P and P_n will affect the marginal utility of that income. A positive response of c_x or c_n to income gives rise to an asymmetry as the commodity price fluctuates since increases in P or P_n will reduce the marginal utility of income more if c_x or c_n increase.

It may also be noted that while these negative cross price effects result from a price change, they may be thought of as partly income effects since the rise in income with a rise in the commodity price helps to determine the rise in the non-tradables price through the last term in the numerator of (17). This is suggestive of a further benefit from access to risk markets since smoothing income will tend to smooth the price of non-tradables and hence reduce their negative cross price effects.

A final point is that while the discussion above has been in terms of harmful effects from the price of non-tradables it is not necessarily the case that, in the absence of insurance and for given taste parameters, the response of the economy is sub-optimal. There is a parallel here with Dutch Disease theory where the non-tradables sector plays an important but not necessarily sub-optimal role in withdrawing resources from the x and y sectors as the price of non-tradables rises.

The links between Dutch Disease theory and the results concerning volatility above may be further clarified by noting that the welfare effect of a permanent change in the commodity price, ΔP, the Dutch Disease case, may be given to a second order approximation by:

$$\Delta E[V] \approx \frac{dV}{dP}\Delta P + \frac{1}{2}\frac{d^2V}{dP^2}(\Delta P)^2$$

The second term of which is the same as the welfare effect of (symmetric) price volatility. Since Dutch Disease theory deals with permanent price changes, however, there is clearly no role for insurance.

3. Supply Volatility Alone

Having analysed the welfare impact from price volatility caused by foreign supply and demand shocks we turn now to the case where the only source of instability is the domestic supply shocks denoted Θ_1^S. These will be felt in the first instance through changed output in the commodity sector and from there to changed national income. If non-tradables are present there will also be an effect from a change in their price and, for large economies only, a further feedback effect from changing output of the commodity to a change in the world price. The section follows the work of Svedberg (1990) though that paper does not include non-tradables and does not consider the large economy case.

Following the pattern of the previous section we examine first the response of expected income to volatility, in this case supply volatility, without non-tradables and subsequently the impact on expected welfare with non-tradables. All the results in this section refer to a single exporting country and hence for simplicity we drop the country subscripts and use the notation Θ for the supply shock parameter Θ_1^S in (2).

Without non-tradables the first derivative of national income with respect to Θ is given by:

(18) $$\frac{dm}{d2} = P\frac{dx}{d2} + \frac{dy}{d2} + x\frac{dP}{d2}$$

Where the last term will take a value significantly different from zero only for large economies. Since the country of interest can only affect the world price through a change in output (given the previous assumption that consumption within the country is an insignificant fraction of world consumption) the derivative may also be expressed by (19).

(19) $$\frac{dP}{d2} = \frac{dP}{dx}\frac{dx}{d2} = \frac{dP}{dx}\left(\frac{\partial x}{\partial 2} + \frac{\partial x}{\partial P}\frac{dP}{d2}\right) = \frac{\frac{dP}{dx}\frac{\partial x}{\partial 2}}{1 - \frac{\partial x}{\partial P}\frac{dP}{dx}}$$

Where dP/dx is the component truly exogenous to the country of interest.

Given that small changes in production from the static optimum (that result from the change in Θ or an induced change in P in the large economy case) do not affect national income, (18) may be simplified to:

(20) $$\frac{dm}{d2} = PS + x\frac{dP}{d2}$$

In which the first term is the impact effect of a rise in Θ on the value of output of x (given that x=ΘS from (2)) and the second term a valuation effect on x from the possible feedback effect on P in the large economy case. Differentiating (20) again gives:

(21) $$\frac{d^2m}{d2^2} = P\frac{dS}{d2} + S\frac{dP}{d2} + \frac{dx}{d2}\frac{dP}{d2} + x\frac{d^2P}{d2^2}$$

The first term of this expression shows the benefit of supply flexibility under supply risk first noted by Svedberg[6]. Recalling that x=ΘS(P,Θ) it is clear that if there was no flexibility ex post, such that dS/dΘ=0, x would be linear in Θ and the effects of symmetric fluctuations in Θ would cancel out. If there is flexibility ex post, such that dS/dΘ>0 as the economy moves resources in response to fluctuating supply conditions, x will be convex in Θ and there will be a gain to the expected level of output of x and to expected income unless there is a very large feedback effect on the world price.

Svedberg (1990) finds a gain from supply flexibility when supply shocks are product specific as above, but a neutral effect of supply fluctuations on expected income and welfare when the supply shocks impact on production conditions in all sectors simultaneously. This is because in the latter case the different goods in effect form a composite commodity with no scope for factor movements between sectors when their relative (as opposed to absolute) marginal productivity does not change.

The flexibility benefit under supply risk closely resembles that under price risk analysed in the previous section. The similarity is clear if it is recalled that P and Θ have a symmetric role in determining the value of output from the commodity sector since Px=PΘS(P,Θ).

The second and third terms arise from the possible large economy feedback effect reducing price when there is a positive supply shock that devalues existing and extra output. The final term is a simple valuation effect on x from a change in its expected price.

We may further simplify (21) by taking into account the separate effects of changes in Θ alone and the changes to P that it may cause, a distinction obscured by the full derivatives used above.

(22) $$\frac{d^2m}{d2^2} = P\frac{\partial S}{\partial 2} + \frac{dP}{d2}\left(S + \frac{\partial x}{\partial 2} + \frac{P}{2}\frac{\partial x}{\partial P}\right) + \frac{\partial x}{\partial P}\left(\frac{dP}{d2}\right)^2 + x\cdot$$

In which the first term is the potential flexibility benefit and the second the change in value of existing output plus any change in output (the latter being the sum of increased output through Θ and reduced output in response to the world price fall. The third term demonstrates again the benefit of flexibility in response to price volatility, this time world price fluctuations caused by supply shocks within the country itself. The fourth term reflects a possible change in expected price as before.

Turning to the impact of supply volatility on expected welfare when non-tradables are present, we have first that:

[6]See equations (16)-(22) of Svedberg (1990).

(23) $$\frac{dm}{d2} = PS + x\frac{dP}{d2} + n\frac{dP_n}{d2}$$

Which shows that including non-tradables adds the third term above to (20), the same comparison as between (10) and (9). In addition:

$$\frac{dV}{d2} = \frac{\partial V}{\partial m}\frac{dm}{d2} + \frac{\partial V}{\partial P}\frac{dP}{d2} + \frac{\partial V}{\partial P_n}\frac{dP_n}{d2}$$

Which simplifies, using (23) and Roy's Identity, to:

(24) $$\frac{dV}{d2} = \frac{\partial V}{\partial m}[PS + (x-c_x)\frac{dP}{d2}]$$

The first term of (24), involving PS, is simply the product of the impact effect of a change in Θ on income and the marginal utility of income. The second term reflects the large economy feedback on the world price which in turn affects the economy through the level of exports and the marginal utility of income as in the case of an exogenous price change.

Differentiating (24) once more gives:

$$\frac{d^2V}{d2^2} = [PS+(x-c_x)\frac{dP}{d2}]\frac{d}{d2}[\frac{\partial V}{\partial m}] + \frac{\partial V}{\partial m}[P\frac{dS}{d2} + S\frac{dP}{d2} + (\frac{dx}{d2} - \frac{dc_x}{d2})\frac{dP}{d2} + (x-c_x)\frac{d^2}{d2}$$

Which may be expanded along the same lines as (12)-(15) to yield equation (25).

We consider each line in turn. The first two terms of the first line would be the only ones for a small economy without non-tradables and comprise the Svedberg flexibility effect followed by the utility costs of fluctuating incomes with risk aversion (in the absence of insurance). PS represents the direct exposure of the economy to supply shocks and its square, together with the degree of risk aversion, determines the welfare cost of those fluctuations.

$$\frac{d^2V}{d2^2} = \frac{\partial V}{\partial m}[P\frac{\partial S}{\partial 2} - P^2S^2A + (x-c_x)\frac{d^2P}{d2^2}$$

$$+\frac{dP_n}{d2}(P\frac{\partial S}{\partial P_n} - PS\frac{\partial c_n}{\partial m})$$

(25) $$+\frac{dP}{d2}[S + \frac{\partial x}{\partial 2} + P\frac{\partial S}{\partial P} - 2PS[\frac{\partial c_x}{\partial m} + (x-c_x)A]]$$

$$+\left(\frac{dP}{d\Theta}\right)^2\left[\frac{\partial x}{\partial P}-\frac{\partial c_x}{\partial P}\Big|_c-2(x-c_x)\frac{\partial c_x}{\partial m}-(x-c_x)^2A\right]$$

$$+\frac{dP}{d\Theta}\frac{dP_n}{d\Theta}\left(\frac{\partial x}{\partial P_n}-\frac{\partial c_x}{\partial P_n}\Big|_c-(x-c_x)\frac{\partial c_n}{\partial m}\right)]$$

The second line also applies to a small economy and comprises the additional effect of including non-tradables. The derivative $dP_n/d\Theta$ will be analysed below but, taking it to be positive for the time being, it may be seen that the terms within the bracket are similar to the cross effects from the price volatility case of the previous section. The first term shows that the economy's production response to the supply shock, which of itself is to move resources into the x sector (for a positive shock), will be dampened by a simultaneous increase in P_n that will tend to move resources into the non-tradables sector. The second term is a marginal utility of income effect whereby a positive supply shock raises income (with weight PS) which will raise c_n if it is normal at a time when its price has risen and hence reduce the marginal utility of income in terms of the y good. More loosely we may think of this combination leading to extra consumption of the non-tradable at times when its price is high.

The following three lines apply to a large economy such that $dP/d\Theta$ is non-zero.[7] Taking this derivative to be negative for the time being (the typical case), the third line shows the consequences of a fall in the world price as commodity output expands in response to the supply shock. The first three terms within the bracket are part of the response of expected income discussed above. The rest of this line comprises an insurance effect from the world price falling, which reduces income, at the same time as a supply shock tends to raise income. The two terms within the sub-bracket comprise the income response of c_x and the degree of risk aversion weighted by the economy's exposure to a price change, $x-c_x$. The first of these is present because while it is costly for c_x to rise when its price rises, the large economy feedback effect on the world price reduces that increase. The second simply reflects the income insurance provided by a downward sloping demand curve for a large economy discussed by Newbery and Stiglitz (1981).

The fourth line is familiar from the previous section. The square of $dP/d\Theta$ is the world price volatility caused by supply variability within a large economy. The contents of the bracket reflect the costs and benefits of the economy's response to the price volatility and are the same as the first line of (15). Hence the source of price volatility, taken on its own, does not affect its impact on expected utility.

The fifth and final line reflects cross price effects between P and P_n. On the same assumptions that $dP_n/d\Theta$ is positive and $dP/d\Theta$ negative (such that their product outside the bracket is negative) these cross price effects are beneficial. The first two terms within the bracket shows that changes in P_n will reduce $(x-c_x)$ at times when the commodity price is low. The third term is similar to the second term of the second line except that this time the change in income arises from a price change, so it is weighted by $(x-c_x)$, and will be beneficial under the assumptions made since income and consumption of the non-tradable fall due to the commodity price fall at a time when the price of non-tradables is high.

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[7]As is the last term of the first line which is a level effect due to a possible change in expected price.

Having discussed the interpretation of (25) we turn to the determinants of $dP_n/d\Theta$ and $dP/d\Theta$. For the first of these:

(26) $$\frac{dP_n}{d2} = \frac{\partial P_n}{\partial 2} + \frac{\partial P_n}{\partial P}\frac{dP}{d2}$$

For a small economy only the first term is relevant in which case, following similar steps to the derivation of (17), (26) is given solely by:

(27) $$\frac{\partial P_n}{\partial 2} = \frac{-\frac{\partial n}{\partial 2} + PS\frac{\partial c_n}{\partial m}}{\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}|_c}$$

Which is very similar to (17), the differences being that there is no direct effect of Θ on c_x and the income effect on c_n is weighted by PS rather than $(x\text{-}c_x)$. If non-tradables are normal both (17) and (27) are positive.

For a large economy we may substitute (17) into (26) (there being no difference between the partial derivative in (26) and the whole derivative in (17)), and also make use of:

(28) $$\frac{dP}{dx}\frac{dx}{d2} = \frac{dP}{dx}\left(\frac{\partial x}{\partial 2} + \frac{\partial x}{\partial P}\frac{dP}{d2} + \frac{\partial x}{\partial P_n}\frac{dP_n}{d2}\right) = \frac{\frac{dP}{dx}\left(\frac{\partial x}{\partial 2} + \ldots\right.}{1 - \frac{\partial x}{\partial P}\ldots}$$

Such that (26) may be expanded to:

(29) $$\frac{dP_n}{d2} = \frac{-\frac{\partial n}{\partial 2} + PS\frac{\partial c_n}{\partial m} + \frac{\partial x}{\partial 2}\frac{dP}{dx}\left(\frac{\frac{\partial c_n}{\partial P}|_c - \frac{\partial n}{\partial P} + (x-c_x)\frac{\partial c_x}{\partial m}}{1 - \frac{\partial x}{\partial P}\frac{dP}{dx}}\right.}{\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}|_c - \frac{\partial x}{\partial P_n}\frac{dP}{dx}\left(\frac{\frac{\partial c_n}{\partial P}|_c - \frac{\partial n}{\partial P} + (x-c_x)\frac{\partial c_x}{\partial m}}{1 - \frac{\partial x}{\partial P}\frac{dP}{dx}}\right.}$$

For a small economy $dP/dx=0$ and the expression collapses to (27). For a large economy dP/dx becomes negative, making the last term on each line negative and the impact of dP/dx on $dP_n/d\Theta$ ambiguous. It may be shown, however, that the latter becomes smaller (and hence may potentially become negative overall) if the following condition holds:

(30) $$\frac{\partial x}{\partial \Theta} + \frac{\partial x}{\partial P_n}\left(\frac{-\frac{\partial n}{\partial \Theta} + PS\frac{\partial c_n}{\partial m}}{\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}\Big|_c}\right) > 0$$

Which is the same condition as that for $dx/d\Theta>0$ in a small economy which shows that $dP_n/d\Theta$ may be thought of as follows. In a small economy the derivative is positive if c_n is normal by (27). At this value of $dP_n/d\Theta$, $dx/d\Theta$ is given by the left hand side of (30). If this is positive x will increase as Θ rises and hence P will fall since dP/dx is negative. We know from the previous section that dP_n/dP is positive (if c_n is normal) and hence the fall in P will tend to reduce the rise in P_n caused by the rise in Θ and may even reverse it. If (30) is not satisfied, $dx/d\Theta$ with P constant is negative and the fall in x will increase P and hence further raise P_n. Substituting (29) into (28) and simplifying gives:

(31) $$\frac{dP}{d\Theta} = \frac{dP}{dx}\left[\frac{\frac{\partial x}{\partial \Theta} + \frac{\partial x}{\partial P_n}\frac{\partial P_n}{\partial \Theta}}{1 - \frac{\partial x}{\partial P}\frac{dP}{dx} - \frac{\partial x}{\partial P_n}\frac{dP}{dx}\frac{dP_n}{dP}}\right]$$

Where $\partial P_n/\partial\Theta$ is given by (27) and dP_n/dP by (17) and both are positive. From (31) it is clear that without non-tradables, $dP/d\Theta$ is negative since dP/dx is negative but including non-tradables makes the sign ambiguous.

The discussion of (25) was based for convenience on the assumptions that $dP_n/d\Theta$ was positive and $dP/d\Theta$ negative whereas it may now be seen from (29) and (31) that while these are perhaps the most likely outcomes any combination of signs is possible if large economy effects and/or cross effects from non-tradables on x are significant. If the signs do change the intuition behind the relevant terms in (25) remains the same but costs may become benefits and vice versa.

4. Cross Effects Between Price and Supply Shocks

We conclude the core results by examining the remaining cross effects between price and supply shocks. The cross effect between Θ^D and Θ_2^S was covered in Section 3 which leaves the first and third terms of the last line of (6). These cross or covariance effects may be small in practice but not necessarily insignificant. The term in $Cov(\Theta_1^S,\Theta_2^S)$ for example would apply to an agricultural commodity where neighbouring countries experience similar weather conditions. The term in $Cov(\Theta^D,\Theta_1^S)$ may also yield information about the possible effect of North-South interactions whereby changes in commodity prices affect the business cycle in the major consuming countries though as noted in the introduction the paper does not provide an explicit model of this possibility.

Both the terms considered cover the effect of a simultaneous domestic supply shock and foreign price shock, the latter arising from a demand or supply shock. Considering a foreign demand shock first, and following similar steps to the earlier sections, we may derive:

$$Cov(2^D, 2_1^S)\frac{d^2V}{d2^D d2_1^S} = Cov(2^D, 2_1^S)\frac{\partial V}{\partial m}[(x-c_x)\frac{d^2P}{d2^D d2_1^S}$$

$$+ \frac{dp}{d2^D}\ [S+\frac{\partial S}{\partial 2_1^S} - PS[\frac{\partial c_x}{\partial m} + (x-c_x)A]]$$

(32)
$$+ \frac{dP}{d2^D}\frac{dP_n}{d2_1^S}\ [\frac{\partial x}{\partial P_n} - \frac{\partial c_x}{\partial P_n}|_c - (x-c_x)\frac{\partial c_n}{\partial m}]$$

$$+ \frac{dP}{d2^D}\frac{dP}{d2_1^S}\ [\frac{\partial x}{\partial P} - \frac{\partial c_x}{\partial P}|_c - 2(x-c_x)\frac{\partial c_x}{\partial m} - (x-c_x)^2A]]$$

We discuss the expression assuming that the covariance is positive such that positive foreign demand and domestic supply shocks occur at the same time though of course a negative covariance is also possible. Given this assumption the economy faces a rising world price (unless there is a strong large country effect) at the same time as domestic supply conditions in the commodity sector are favourable.

The first term in the square bracket reflects a possible change in expected price as a result of the simultaneous occurrence of the two shocks. The second line is prefaced by the extent to which the foreign demand shock raises the world price and hence the terms within it reflect the costs or benefits from the simultaneous price rise and supply shock. The first term within the bracket is present because the price rise at the same time as an output increase raises the value of the latter. The other terms are familiar from the earlier analysis and represent the costs of fluctuating income due to the simultaneous positive price and supply shocks.

The third line is also familiar, showing negative cross price and income effects from simultaneous increases in the commodity and non-tradables prices (assuming that large country effects are not too large), including a dampening of the response of x to Θ_1^S and P as a result of P_n increasing also. The fourth line repeats the terms from the price volatility case, the price derivatives at the front indicating the extent to which simultaneous demand and supply shocks reduce the variability of the commodity price if their covariance is positive or increase it if the covariance is negative.

Given the supply function (2), the expression above in $d^2V/d\Theta^D\Theta_1^S$ is equal to $d^2V/\Theta_1^S d\Theta^D$ but it is convenient to derive the latter also:

$$Cov(2^D, 2_1^S)\frac{d^2V}{d2_1^S d2^D} = Cov(2^D, 2_1^S)\frac{\partial V}{\partial m}[(x-c_x)\frac{d^2P}{d2_1^S d2^D}$$

$$+\ \frac{dp}{d2^{D}}\ [S+P\frac{\partial S}{\partial P}-PS[\frac{\partial c_x}{\partial m}+(x-c_x)A]]$$

$$(33) \qquad +\ \frac{dP}{d2^{D}}\frac{dP_n}{dP}\frac{dP}{d2^{S}_{1}}\ [\frac{\partial x}{\partial P_n}-\frac{\partial c_x}{\partial P_n}|_c-(x-c_x)\frac{\partial c_n}{\partial m}]$$

$$+\frac{dP}{d2^{D}}\frac{dP_n}{dP}[P\frac{\partial S}{\partial P_n}-PS\frac{\partial c_n}{\partial m}]$$

$$+\ \frac{dP}{d2^{D}}\frac{dP}{d2^{S}_{1}}\ [\frac{\partial x}{\partial P}-\frac{\partial c_x}{\partial P}|_c-2(x-c_x)\frac{\partial c_x}{\partial m}-(x-c_x)^2A]]$$

Which may be shown to be equal to (32) using (17), (29) and (31) together with symmetry between the cross effects on S and n.[8]

There is also symmetry between the cross derivatives above and the same expressions involving $Cov(\Theta_1^S,\Theta_2^S)$ in which $dP/d\Theta^D$ is simply replaced by $dP/d\Theta_2^S$, all other terms remaining the same. This is to be expected since both Θ^D and Θ_2^S are foreign shocks that impact on the country of interest solely through price. The difference between the two shocks is that the demand shock raises price while the supply shock lowers it but this does not affect the structure of the expression or the interpretation of its terms.

5. All Shocks Together

This section combines the results above to examine the effect on expected utility of all three shock parameters being volatile simultaneously, thus permitting a more rounded assessment of the impact of commodity market instability. This is done for the general case of a large economy with non-tradables present, and with potential flexibility in response to changes in both price and supply conditions, though of course any of the terms may be set to zero to correspond to a particular special case.

As part of the simplification we have the following expressions, firstly from (6) (but for price instead of indirect utility), and from (3) noting the discussion that follows it.

[8]This symmetry presupposes that the economy can respond in equal measure to price and supply shocks which, as noted earlier, may not hold if one precedes the other and at least one potentially mobile factor must commit to a sectoral allocation between those times. We make use of both (32) and (33) below so a symmetric response is not imposed.

$$)E[P] \approx \frac{1}{2}Var(2^d)\frac{d^2P}{d2^{D^2}} + \frac{1}{2}Var(2_1^S)\frac{d^2P}{d2_1^{S^2}} + \frac{1}{2}Var(2_2^S)\frac{d^2P}{d2_2^{S^2}} + Cov(2^D, 2_1^S)\frac{d^2P}{d2^D 2_1^S} + Cov(2^D 2_2^S)\frac{d^2P}{d2^D 2_2^S} + Cov(2_1^S, 2_2^S)\frac{d^2P}{d2_1^S 2_2^S}$$

$$Var(P) \approx Var(2^D)\left(\frac{dP}{d2^D}\right)^2 + Var(2_1^S)\left(\frac{dP}{d2_1^S}\right)^2 + Var(2_2^S)\left(\frac{dP}{d2_2^S}\right)^2 + 2Cov(2^D, 2_1^S)\frac{dP}{d2^D}\frac{dP}{d2_1^S} + 2Cov(2^D, 2_2^S)\frac{dP}{d2^D}\frac{dP}{d2_2^S} + 2Cov(2_1^S, 2_2^S)\frac{dP}{d2_1^S}\frac{dP}{d2_2^S}$$

$$ov(P, P_n) \approx Var(2^D)\left(\frac{dP}{d2^D}\right)^2\frac{dP_n}{dP} + Var(2_1^S)\frac{dP}{d2_1^S}\frac{dP_n}{d2_1^S} + Var(2_2^S)\left(\frac{dP}{d2_2^S}\right)^2\frac{dP_n}{dP} + Cov(2^D, 2_1^S)\left(\frac{dP}{d2^D}\frac{dP_n}{d2_1^S} + \frac{dP}{d2^D}\frac{dP_n}{dP}\frac{dP}{d2_1^S}\right) + 2Cov(2^D, 2_2^S)\frac{dP}{d2^D}\frac{dP}{d2_2^S}\frac{dP_n}{dP} + Cov(2_1^S, 2_2^S)\left(\frac{dP}{d2_2^S}\frac{dP_n}{d2_1^S} + \frac{dP}{d2_2^S}\frac{dP_n}{dP}\frac{dI}{d2}\right.$$

$$Cov(2_1^S, P) \approx Var(2_1^S)\frac{dP}{d2_1^S} + Cov(2^D, 2_1^S)\frac{dP}{d2^D} + Cov(2_1^S, 2_2^S)\frac{dP}{d2_2^S}$$

$$Cov(2_1^S, P_n) \approx Var(2_1^S)\frac{dP_n}{d2_1^S} + Cov(2^D, 2_1^S)\frac{dP}{d2^D}\frac{dP_n}{dP} + Cov(2_1^S, 2_2^S)\frac{dP}{d2_2^S}\frac{dP_n}{dP}$$

Substituting these approximations, plus (15) and (25) together with (32) and (33) for both $\mathrm{Cov}(\Theta^D,\Theta^S_1)$ and $\mathrm{Cov}(\Theta^S_1,\Theta^S_2)$, into (6) and allowing for a split between the second cross shock derivatives in the latter gives:

$$
\begin{aligned}
&)E[P](x-c_x) \\
& +\tfrac{1}{2}Var(P)[\frac{\partial x}{\partial P}-\frac{\partial c_x}{\partial P}|_c-2(x-c_x)\frac{\partial c_x}{\partial m}-(x-c_x \\
& +\tfrac{1}{2}Var(2^S_1)[P\frac{\partial S}{\partial 2^S_1}-(PS)^2A] \\
- \approx\ & +\tfrac{1}{2}Cov(P,2^S_1)[2S+2^S_1\frac{\partial S}{\partial 2^S_1}+P\frac{\partial S}{\partial P}-2PS[\frac{\partial c_x}{\partial m}+(x- \\
& +\tfrac{1}{2}Cov(P,P_n)[\frac{\partial x}{\partial P_n}-\frac{\partial c_x}{\partial P_n}|_c-(x-c_x)\frac{\partial c_n}{\partial m} \\
& +\tfrac{1}{2}Cov(2^S_1,P_n)[P\frac{\partial S}{\partial P_n}-PS\frac{\partial c_n}{\partial m}]
\end{aligned}
\tag{34}
$$

The terms in (34) are familiar from the previous sections: the first line simply reflects any change in expected price; the second the economy's response to price volatility. The third line is the impact of supply variability taken on its own and comprises the degree of production flexibility with respect to the supply shock parameter and the risk aversion costs of the income fluctuations that result from it. The fourth line reflects cross effects between price and supply conditions, the first three terms being part of expected income while the latter two (in the square bracket) show the benefits (or costs) from smoothing (destabilising) income that results from positive (negative) covariance between price and the supply shock parameter. The last two lines are cross effects between P and P_n and Θ^S_1 and P_n respectively. Each comprises the extent to which a positive covariance limits the economy's production response to the price or supply shock followed by a negative income effect.

While (34) is the central result of the paper it is of interest from a quantitative point of view to change it into a dimensionless form in which the derivatives become elasticities.

$$(35)\qquad \frac{E[V]}{m\frac{\partial V}{\partial m}} \approx \begin{array}{l} \Delta E[lnP]\gamma_x \\ +\frac{1}{2}Var(lnP)[\alpha_x\varepsilon_x^{\partial P}-\beta_x\varepsilon_{c_x}^{\partial P}|_c-2\gamma_x\beta_x\varepsilon_{c_x}^{\partial m}-\gamma_x^2 R \ldots \\ +\frac{1}{2}Var(\ln\theta_1^S)\alpha_x[\varepsilon_S^{\partial\theta_1^S}-\alpha_x R] \\ +\frac{1}{2}Cov(lnP,\ln\theta_1^S)\alpha_x[2+\varepsilon_x^{\partial P}+\varepsilon_S^{\partial\theta_1^S}-2[\beta_x\varepsilon_{c_x}^{\partial m}+\gamma \ldots \\ +\frac{1}{2}Cov(lnP,\ln P_n)[\alpha_x\varepsilon_x^{\partial P_n}-\beta_x\varepsilon_{c_x}^{\partial P_n}|_c-\gamma_x\beta_n\varepsilon_c^{\partial}\ldots \\ +\frac{1}{2}Cov(\ln\theta_1^S,\ln P_n)\alpha_x[\varepsilon_x^{\partial P_n}-\beta_n\varepsilon_{c_n}^{\partial m}] \end{array}$$

Where the elasticity notation indicates that they are all correspond to partial derivatives, R is the coefficient of relative risk aversion and α_x, $ß_x$ and γ_x are the shares of output, consumption and exports of good x respectively:

$$\alpha_x = \frac{Px}{m} \qquad \beta_x = \frac{Pc_x}{m} \qquad \gamma_x = \frac{P(x-c_x)}{m} = \alpha_x - \beta_x$$

The parameter for the non-tradable, $ß_n$ (=α_n) is defined in the same way. It may also be noted that the left hand side of (35) is a first order approximation of the equivalent monetary value to the representative agent of the whole effect of instability in the commodity market as a share of national income.

In (34) and (35) there is an asymmetry between some of the terms arising from the fact that a price shock impacts through (x-c_x) or γ_x whereas the supply shock operates through PS or α_x. It will often be the case in practice that consumption of the commodity (and hence $ß_x$) will be of negligible size in which case, setting $ß_x$ to zero, the asymmetry between the two shocks is removed and we have:

$$(36)\qquad \frac{E[V]}{m\frac{\partial V}{\partial m}} \approx \alpha_x\left[\begin{array}{l} \Delta E[lnP] \\ +\frac{1}{2}Var(lnP)[\varepsilon_x^{\partial P}-\alpha_x R] \\ +\frac{1}{2}Var(\ln\theta_1^S)[\varepsilon_S^{\partial\theta_1^S}-\alpha_x R] \\ +\frac{1}{2}Cov(lnP,\ln\theta_1^S)[2+\varepsilon_x^{\partial P}+\varepsilon_S^{\partial\theta_1^S}-2\gamma_x R] \\ +\frac{1}{2}Cov(lnP,\ln P_n)[\varepsilon_x^{\partial P_n}-\beta_n\varepsilon_{c_n}^{\partial m}] \\ +\frac{1}{2}Cov(\ln\theta_1^S,\ln P_n)[\varepsilon_x^{\partial P_n}-\beta_n\varepsilon_{c_n}^{\partial m}] \end{array}\right.$$

Equation (36) is a generalisation of equation (24) of Gilbert (1985) in conjunction with his equation (38) (which is necessary to make his (24) consistent with the multiplicative shocks considered here). Gilbert's results are the same as the first, second and fourth lines of (36) except that he considers the welfare effect of price stabilisation. As a result the sign of the expression is different (since (36) is the welfare change going from stability to instability), and the elasticity of supply with respect to Θ_1^S in the fourth line (together with the third line) is not present in Gilbert's (24) because supply variability will continue even if prices are stabilised. The last two lines of (36) are also new because Gilbert does not include non-tradables in his model.

If the economy can respond with equal flexibility to both price and the supply shock Θ_1^S, in other words setting to one side possible differences in the timing of the two shocks relative to factor allocation, we may further simplify (36) since in these circumstances the supply elasticities with respect to price and the supply shock will be equal. Using the notation ϵ^F for both of these (36) becomes:

$$(37) \qquad)E[lnP] + \frac{1}{2}Var[\ln(P2_1^S)](,^F - "_x R) \approx "_x[\; + Cov(\ln P, \ln 2_1^S) + \frac{1}{2}Cov[\ln(P2_1^S), P_n](,_x^{\partial P_n} -$$

The product $P\Theta_1^S$ appears here due to the symmetry between the impact of the two kinds of shock with $ß_x$ set to zero together with the symmetry between P and Θ_1^S in determining the value of output of the x sector and the marginal conditions in the allocation of mobile factors given that $Px=P\Theta_1^S S(P,\Theta_1^S)$.

The top line of (37) gives the familiar expected price term together with the product of the variance of $P\Theta_1^S$ with a term that compares production flexibility with the cost of income fluctuations due to risk aversion. The second line shows the mutually reinforcing benefits (costs) for expected income of a positive (negative) covariance between price and the supply shock followed by the usually negative effect of induced changes in the price of non-tradables on both the supply response and the marginal utility of income via the income response of consumption of non-tradables.

6. Risk Markets

Having established the costs and benefits of commodity market instability when risk markets are missing we now permit the representative agent access to such markets. The section analyses, for a small economy, both the optimal use of perfect risk markets and the change in expected utility from instability given access to them.

We consider perfect insurance markets for simplicity and without loss of generality since it may be shown that such a market is equivalent to a perfect capital market or an unbiased futures market.[9] Perfect risk markets are of course rare but our motivation here is to establish a theoretical benchmark rather than to address the important question of the use that commodity exporters should make of the markets that do exist.

The economy faces three random variables, the commodity price, the supply parameter, Θ_1^S, and

[9]See, for example, Mash (1995) chapter 3.

the price of non-tradables. We do not consider insurance with respect to the latter because there is a deterministic relationship between each of the first two and the price of non-tradables so optimal insurance behaviour with respect to the commodity price and Θ_1^S will take into account their effect on P_n. We assume initially that the agent has access to zero cost insurance with respect to both the commodity price and the supply parameter followed by the case where insurance against supply shocks is not possible. Insurance against price fluctuations is to an extent feasible due to the availability of futures markets in many commodities whereas insurance against supply instability, as noted by Gilbert (1985), is subject to moral hazard problems.[10]

The assumption of a perfect risk market with respect to the commodity price implies that the representative agent has access to a contract giving a payout, I_p, conditioned on the deviation of the relative price from its mean value, and specifying a quantity of insurance q_p such that:[11]

$$I_p = (\bar{P} - P)q_p \qquad \frac{dI_p}{dP} = -q_p$$

If the agent is also allowed to insure at zero cost with respect to the supply shock parameter we also have:

$$I_2 = P_c(\bar{2} - 2)q_2 \qquad \frac{dI_2}{d2_1^S} = -P_c q_2$$

In which q_Θ is the quantity of insurance, in units of commodity output, with respect to the supply variable Θ_1^S and the payout I_Θ is conditioned on the value of deviations of Θ_1^S from its mean using P_c which is the commodity price when Θ_1^S takes its expected value. The expressions below are to be evaluated when $P=P_c$ and hence the subscript is dropped for simplicity.

Both the above contracts leave income (and its marginal utility) unchanged when P and Θ_1^S take their expected values. They may be integrated into the derivation of (34) by noting that their derivatives with respect to P or Θ_1^S, given above, must be added to dm/dP or $dm/d\Theta_1^S$ when the latter appear, including in the determination of the response of the price of non-tradables to either variable. If this is done (34) may be expressed as follows, in which large economy effects are grouped in the final three lines:

[10] Which imply that any insurance payout would need to be conditioned on a variable such as the weather which is exogenous to the producer rather than output which is not. Since this section considers a small economy there is no moral hazard issue with insurance against the world price.

[11] We also assume that there are no difficulties arising from the need for the agent to commit to such a contract.

$$
\begin{aligned}
&)E[P](x-c_x) \\
&+\tfrac{1}{2}D\Big[\frac{\partial x}{\partial P}-\frac{\partial c_x}{\partial P}\Big|_c-2(x-q_p-c_x)\frac{\partial c_x}{\partial m}-(x-q_p-c_x \\
&+\tfrac{1}{2}\Big[D\frac{dP_n}{dP}+C\frac{\partial P_n}{\partial 2^S_1}\Big].\Big[\frac{\partial x}{\partial P_n}-\frac{\partial c_x}{\partial P_n}\Big|_c-(x-q_p-c_x) \\
(38)\qquad l \approx\; &+\tfrac{1}{2}\Big[Var(2^S_1)\frac{\partial P_n}{\partial 2^S_1}+C\frac{dP_n}{dP}\Big].\Big[P\frac{\partial S}{\partial P_n}-P(S-q_2) \\
&+\tfrac{1}{2}C\Big[2S+2^S_1\frac{\partial S}{\partial 2^S_1}+P\frac{\partial S}{\partial P}-2P(S-q_2)\Big[\frac{\partial c_x}{\partial m}+(x-q_p- \\
&+\tfrac{1}{2}Var(2^S_1)\Big[P\frac{\partial S}{\partial 2^S_1}-P^2(S-q_2)^2A\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+\tfrac{1}{2}\frac{dP}{d2^S_1}\Big[2C+Var(2^S_1)\frac{dP}{d2^S_1}\Big]\Big[\frac{\partial x}{\partial P}-\frac{\partial c_x}{\partial P}\Big|_c-2(x-q_p-c_x)\frac{\partial c_x}{\partial m}-(x-q_p-c_x)^2A\Big] \\
&+\tfrac{1}{2}\frac{dP}{d2^S_1}\Big[Var(2^S_1)\frac{dP_n}{d2^S_1}+2C\frac{dP_n}{dP}\Big]\Big[P\frac{\partial S}{\partial P_n}-P(S-q_2)\frac{\partial c_n}{\partial m}\Big] \\
&+\tfrac{1}{2}\frac{dP}{d2^S_1}Var(2^S_1)\Big[2S+2^S_1\frac{\partial S}{\partial 2^S_1}+P\frac{\partial S}{\partial P}-2P(S-q_2)\Big[\frac{\partial c_x}{\partial m}+(x-q_p-c_x)A\Big]\Big]
\end{aligned}
$$

Where, using (17):

$$
(39)\qquad \frac{dP_n}{dP}=\frac{\dfrac{\partial c_n}{\partial P}\Big|_c-\dfrac{\partial n}{\partial P}+(x-q_p-c_x)\dfrac{\partial c_n}{\partial m}}{\dfrac{\partial n}{\partial P_n}-\dfrac{\partial c_n}{\partial P_n}\Big|_c}
$$

and from (27):

$$
(40)\qquad \frac{\partial P_n}{\partial 2^S_1}=\frac{-\dfrac{\partial n}{\partial 2^S_1}+(PS-Pq_2)\dfrac{\partial c_n}{\partial m}}{\dfrac{\partial n}{\partial P_n}-\dfrac{\partial c_n}{\partial P_n}\Big|_c}
$$

There is no q_p in the first line of (38) since insurance is conditioned on movements in the commodity price away from its expected value whereas this term reflects a possible change in expected price compared with a situation in which there were no shocks. As noted the last three lines, involving $dP/d\Theta_1^S$, are only relevant for a large economy so if $q_p = q_\Theta = 0$, (38) would simply be a different way of expressing (34) in which small and large economy components are separated. The latter are not considered in the rest of this section.

In addition the parameters C and D in (38) are given by the following expressions:

$$C \approx Cov(2_1^S, P)|_{\frac{dP}{d2_1^S}=0} \approx Cov(2^D, 2_1^S)\frac{dP}{d2^D} + Cov(2_1^S, 2_2^S)\frac{dP}{d2_2^S}$$

$$D \approx Var(P)|_{\frac{dP}{d2_1^S}=0} \approx Var(2^D)(\frac{dP}{d2^D})^2 + Var(2_2^S)(\frac{dP}{d2_2^S})^2 + 2Cov(2^D, 2_2^S)\frac{dP}{d2^D}\frac{dP}{d2_2^S}$$

Where the notation indicates that for a small economy (such that $dP/d\Theta_1^S=0$) the right hand side of the two expressions correspond to the covariance between price and supply shocks and the variance of price respectively. In a large economy they refer to the foreign or exogenous component of these two parameters.

The next step is to determine the optimal insurance quantities, denoted q_p^* and q_Θ^*, which are derived from differentiating the small economy component of (38) with respect to q_p and q_Θ separately and solving the simultaneous equations that result:

(41)
$$q_p^* = x - c_x + \frac{\frac{\partial c_x}{\partial m} + \frac{\partial c_n}{\partial m}\left(\frac{\frac{\partial c_x}{\partial P_n}|_c - \frac{\partial x}{\partial P_n}}{\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}|_c}\right)}{A + \frac{\left(\frac{\partial c_n}{\partial m}\right)^2}{\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}|_c}}$$

(42) $$Pq_2^* = PS + \frac{\frac{\partial c_n}{\partial m}\left(\frac{-\frac{1}{2}\frac{\partial n}{\partial 2_1^S} - \frac{1}{2}P\frac{\partial S}{\partial P_n}}{\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}|_c}\right)}{A + \frac{\left(\frac{\partial c_n}{\partial m}\right)^2}{\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}|_c}}$$

Expressions (41) and (42) show that for insurance against both stochastic variables the optimal quantity is greater than the impact of the variable on real income. In other words the optimal insurance quantity against P is larger than ($x-c_x$) and that against supply shocks greater than S. If insurance was intended simply to stabilise real income the quantities would be set equal to these amounts. The expressions reveal that optimal insurance behaviour involves making real income move in the opposite direction to the change in P or Θ_1^S if non-tradables are present. The reason for this is that it is advantageous from the point of view of the cross price effects from P_n for the price of non-tradables to move as little as possible in relation to the commodity price. The positive relationship between them is dampened by insurance since a part of the rise in P_n when P or Θ_1^S increases in the absence of insurance comes from the increase in income that the latter variables cause. Hence the last term in (41) and (42) has the cross price effects in the numerator. Expression (41) also has the income response of x in the numerator since from the earlier discussion it is also advantageous for consumption of the commodity to fall when its price rises. The denominator of each expression shows that the cross price benefits of a large quantity of insurance must be balanced against the degree of risk aversion and the income response of non-tradables. These appear because a highly risk averse agent would seek to stabilise real income and a large income response of non-tradables consumption implies that there is a cost to the marginal utility of income from large swings in P_n given that consumption of the non-tradable will tend to move in the same direction as its price.

Hence in summary, the optimal quantity of insurance over and above that required to stabilise real income represents a balance between two factors. The first is the opportunity to reduce the previously harmful cross price effects from the non-tradables sector, the second is the utility costs of creating large swings in real income even if they are now negatively related to the commodity price and supply shock parameter.

A final point in relation to (41) and (42) is that neither expression includes any of the variances or covariances between the stochastic variables. Variances do not appear because (38) shows that the change in expected utility from volatility is linear in the variances. The covariance term, C in (38), is not present because the representative agent has been given the opportunity to insure at zero cost against both stochastic variables. In doing so the agent optimises the relationship between each of them and income and hence the covariance is not relevant.

This changes if we now suppose that the agent can insure with respect to the commodity price only so q_Θ is constrained to zero by assumption. In this case the optimal insurance quantity with respect to the commodity price becomes:

(43) $$q_p^*|_{q_2=0} = q_p^*|_{q_2=q_2^*} + \frac{Cov(P,2_1^S)}{Var(P)} q_2^*$$

Where the notation indicates that the optimal insurance quantity against price fluctuations, when insurance against supply shocks is not possible, is equal to its value when the latter is possible (given by (41)) plus the product of the covariance-variance ratio shown and the optimum insurance quantity against supply fluctuations when they can be insured against given by (42). In effect forcing q_Θ to zero results in q_p^* comprising both the direct benefit of insuring against price fluctuations and the indirect benefit of quasi-insuring against Θ_1^S to the extent that the latter has a non-zero covariance with price.

Expression (43) may also be expanded, using (41) and (42), and made dimensionless to yield:

(44) $$[1+\frac{Cov(lnP,\ln 2_1^S)}{Var(lnP)}]"_x - [1-\frac{,_{c_x}^{\partial m}}{R+(\frac{\$_n(,_{c_x}^{\partial m})^2}{,_n^{\partial P_n}\_ ,_{c_n}^{\partial P_n}|_c}}$$
$$+\frac{,_{c_x}^{\partial m}[\frac{\$_x,_{c_x}^{\partial P_n}|_c - "_x,_x^{\partial P_n}}{,_n^{\partial P_n}\_ ,_{c_n}^{\partial P_n}|_c} - \frac{1}{2}\frac{Cov(lnP,\ln 2_1^S)}{Var(lnP}(\frac{\$_n,_n^{\partial 2_1^S}+'}{,_n^{\partial P_n}\_ ,}}{R+\frac{\$_n(,_{c_n}^{\partial m})^2}{,_n^{\partial P_n}\_ ,_{c_n}^{\partial P_n}|_c}}$$

If non-tradables are removed (44) reduces to equation (14) of Gilbert (1985).

We now return to the case where the representative agent may insure against both the commodity price and Θ_1^S and substitute (41) and (42) into (38) excluding the large economy terms of the last three lines:[12]

(45) $$\approx \frac{1}{2}Var(P)[\frac{\partial x}{\partial P}-\frac{\partial c_x}{\partial P}+\frac{dP_n}{dP}(\frac{\partial x}{\partial P_n}-\frac{\partial c_x}{\partial P_n}|_c)$$
$$+\frac{1}{2}Var(2_1^S)[P\frac{\partial S}{\partial 2_1^S}+P\frac{\partial S}{\partial P_n}\frac{\partial P_n}{\partial 2_1^S}]$$
$$+\frac{1}{2}Cov(P,2_1^S)[2S+2_1^S\frac{\partial S}{\partial 2_1^S}+\frac{P}{2_1^S}\frac{\partial x}{\partial P}+(\frac{P}{2_1^S}\frac{\partial x}{\partial P_n}+\frac{\partial}{\partial 2}$$

[12]We assume that the economy can respond to both price and supply shocks with the same degree of factor mobility.

In which, given optimal insurance behaviour from (39)-(42):

$$\frac{\partial P_n}{\partial \Theta_1^S} = -\frac{A\frac{\partial n}{\partial \Theta_1^S}}{A(\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}|_c) + (\frac{\partial c_n}{\partial m})^2} \qquad (46)$$

$$\frac{dP_n}{dP} = \frac{A(\frac{\partial c_x}{\partial P_n}|_c - \frac{\partial x}{\partial P_n}) - \frac{\partial c_x}{\partial m}\frac{\partial c_n}{\partial m}}{A(\frac{\partial n}{\partial P_n} - \frac{\partial c_n}{\partial P_n}|_c) + (\frac{\partial c_n}{\partial m})^2} \qquad (47)$$

Comparing (45) with (34) it may be seen that access to a complete set of perfect risk markets removes the negative income effects present without insurance, as expected, leaving the own price/Θ_1^S flexibility terms and some cross price effects from the price of non-tradables. Insurance reduces the response of P_n to both P and Θ_1^S so these will be smaller than before. It may be recalled that the choice of optimal insurance quantities involved a tradeoff between manipulating those responses and the costs due to risk aversion of destabilising real income in the other direction. From (46) and (47) it may be seen that if the agent is risk neutral (and c_x is not significant) the price of non-tradables (given optimal insurance choices) is stabilised in which case the terms involving non-tradables in (45) would drop out. In the general case where A>0 it is not optimal to completely stabilise the price of non-tradables and hence the cross price effects remain in (45) though in muted form compared with (34).

If we adopt the same assumptions as in the derivation of (37), that c_x=0 (which implies that the elasticities of the price of non-tradables with respect P and Θ_1^S are equal), and the economy can respond to shocks symmetrically, we have:

$$\frac{\Delta E[V]}{m\frac{\partial V}{\partial m}} \approx \beta_x\left[\begin{array}{c}\Delta E[lnP] \\ +\frac{1}{2}Var[\ln(P\Theta_1^S)](,^F + ,_x^{\partial P_n} ,_{P_n}^P) \\ +Cov(\ln P, \ln\Theta_1^S)\end{array}\right] \qquad (48)$$

The first and third terms of (48) are common with (37) and are exogenous for the small economy considered in this section. The middle term above has greater economic content and replaces the second and fourth terms of (37). It shows that with optimal and costless insurance (with $ß_x$ insignificant) the change in expected welfare from market instability is reduced to a tradeoff between the own price/supply parameter flexibility term on the one hand and the product of the cross price effect and the residual response of the price of non-tradables to the stochastic parameters on the other.

7. Conclusion

The paper has integrated the analysis of price and supply shocks for an open economy, added a non-tradables sector to the models considered in the literature and analysed the impact of access to perfect risk markets.

Excluding non-tradables the paper confirms that the change in expected utility from either source of instability is ambiguous in general. The total change depends on the balance between positive pure price (or supply shock) flexibility effects against the negative income effects that result from price fluctuations or supply instability. Perfect risk markets allow the representative agent to remove the income effects while keeping the flexibility benefits.

Including non-tradables adds negative cross price effects from the price of non-tradables. This price will rise when positive shocks occur, tending to draw resources out of the commodity sector at a time when its output would otherwise increase in a manner familiar from Dutch Disease theory. Optimal insurance behaviour with non-tradables removes the income effects as before but also enhances the efficiency of the economy's response to shocks by reducing the rise of the non-tradables price (and the strength of the negative cross price effects) when the shock variables increase.

The paper leaves a rich set of issues for future research including possible changes in the allocation of resources as a result of market instability (examined in part in the literature), the empirical magnitudes of the supply elasticities and the role of government policy. Policies such as producer price stabilisation and the extent to which higher tax revenues in a boom are spent on non-tradables are likely to have a significant impact on the net cost or benefit from market instability.

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