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ANTICIPATION EFFECTS IN ENDOGENOUS PROBABILITY-MIGRATION MODELS

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ABSTRACT. We analyze a probability-migration model based on the threshold of average human capital as in H.-J. Chen [1]. The difficult and interesting case is the one where the probability of migration is dependent on current average human capital (the anticipative case). Here, indeterminacy occurs, and one has to study a lot of subcases. In the present article we deeply study new interesting cases and we give a global answer.

1. INTRODUCTION

The probability of migration on the economic growth of a developing country is an essential factor, since that people living in a source country with higher average human capital are traditionally more incited to emigrate in the future to a foreign country than those living in a source country with lower average human capital.
By endogenizing the probability of migration, a lot of authors (see Chen [1], Vidal [6] an the references therein) found that there is a possibility of club convergence occurring in the short run, and conditional convergence occurring in the long run following the two following possible scenarios:

The first scenario is when the probability of migration is dependent on prior average human capital; we will call it in this paper the *traditional* case. Here, the threshold level will affect economic behavior in the long run. Thus, if the average human capital threshold is sufficiently low (respectively high), the economy will converge to a high (respectively low) steady state level. However, if the average human capital threshold is at the median level, club convergence

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may occur and the initial condition matters.
In the second scenario, the probability of migration can dependent on current average human capital (we will call it the *anticipative* case). Here, the dynamic transition of the economy will be determined by perceptions of the future. In [1] it is found that a belief in the higher probability of migration in the future will provide an incentive for agents to invest more in their education, thereby raising their accumulation of human capital, which will in turn lead to a higher probability of migration indicating that the problem can be a source of indeterminacy.

Our study demonstrates that migration can be used to explain some important economic growth phenomena, with the two scenarios considered in this paper contributing to two distinct lines of research in the literature on economic growth. The occurrence of multiple steady states in the first scenario can help to explain the findings of club convergence in the empirical studies.

The second scenario indicates that migration can be a source of indeterminacy, and therefore emphasizes the role of beliefs. This implies that when embracing migration, economies with similar backgrounds may well follow different equilibrium paths simply because they have different beliefs about their future probability of migration.
We here give a more precise analysis.

2. Position of the problem

In a small open economy characterized by an infinite horizon, Chen [1] considers a no-growth overlapping generations model, where agents live for two successive periods. In each period a new generation is born, agents born in period t are endowed with parental human capital h_t, and are supposed to allocate their time between gaining education e_t and engaging in leisure $1 - e_t$ in the first period of life. In the second period, agents can migrate to a foreign country (country B) with probability $p_{t+1} \in [0, 1]$ or remain into the home country (country A) with probability $1 - p_{t+1}$. During this second period of life, agents spend all of their time working to earn income for consumption.

Moreover, if w_A and w_B represent the respective real wage per unit of human capital in countries A and B, the earnings of agents are equal to their level of human capital h_{t+1} multiplied by the real wage per unit of human capital of the country in which they live. That is, the expected utility function, which is identical for all agents, is defined for $\beta > 0$ and $\theta > 1$ by:

$$u_t = \ln(1 - e_t) + \beta \left[(1 - p_{t+1}) \ln(w_A h_{t+1}) + p_{t+1}\theta \ \ln(w_B h_{t+1})\right]. \tag{1}$$

As in [1], from period t to period $t + 1$ the human capital evolves following the relation

$$h_{t+1} = A e_t^{\gamma} h_t^{\delta}, \qquad \gamma, \delta \in (0, 1). \tag{2}$$

We distinguish two migration processes: the traditional process of migration where the probability of migration is defined as $p_{t+1} = \mathcal{P}(h_t)$ and the anticipative one given by $\mathcal{P}(h_{t+1})$, where $\mathcal{P}$ is an increasing function. In the traditional process the probability of migration of the young adults p_{t+1} is determined by the human capital of the parents h_t. In the anticipative process the probability of migration of the young adults p_{t+1} is determined by the human capital of the young adults at the end of their first period h_{t+1}. As we will see indeterminacy can occur in this anticipative situation, since the time spent in education e_t, and therefore the human capital of the young adults at the end of their first period h_{t+1}, then depend on the probability of mutation p_{t+1}. Indeed, the variation of the utility function u_t with respect of the education function e_t is given by

$$\frac{\partial u_t}{\partial e_t} = \frac{-1}{1 - e_t} + \beta \left[(1 - p_{t+1})\frac{\gamma}{e_t} + \theta p_{t+1}\frac{\gamma}{e_t}\right], \tag{3}$$

and the optimal decision of e_t^* which is reached at $(\partial u_t)/(\partial e_t) = 0$ is given by

$$e_t^* = \frac{\gamma\beta\left[1 + (\theta - 1)p_{t+1}\right]}{1 + \gamma\beta\left[1 + (\theta - 1)p_{t+1}\right]}. \tag{4}$$

2.1. **The traditional model.** The probability of migration is assumed to be dependent on average human capital H_t. We suppose that the agents are homogeneous, then the average human capital is equal to the personal human capital in each period $H_t = h_t$. In this subsection, we consider the traditional model of migration, that is:

$$p_{t+1} = \mathcal{P}(h_t) \tag{5}$$

which means that the probability of migration is dependent on average human capital lagged by one period (i.e. the average human capital of the parents). We also suppose that

$$\mathcal{P}(h) = \begin{cases} p_1 & \text{if} \quad h < h^{\#} \\ p_2 & \text{if} \quad h \geq h^{\#} \end{cases} \tag{6}$$

for some probability constants $0 \leq p_1 < p_2 \leq 1$, where $h^{\#}$ is a nominative threshold human capital as in [1]. For $j = 1, 2$, we finally denote by

$$e_j = \frac{\gamma\beta\left[1 + (\theta - 1)p_j\right]}{1 + \gamma\beta\left[1 + (\theta - 1)p_j\right]}. \tag{7}$$

Note that we have $e_1 < e_2$.

Proposition 1. *The sequence of human capitals $(h_t)_t$ converge to a fixed point.*
The two possible fixed points are $\bar{h}_1$ and $\bar{h}_2$ (with $\bar{h}_1 < \bar{h}_2$) defined by

$$\bar{h}_j = \left(A\, e_j^{\gamma}\right)^{\frac{1}{1-\delta}}, \quad j = 1, 2. \tag{8}$$

We have the following:
- *If $\bar{h}^1 > h^{\#}$, then the sequence $(h_t)_t$ converges to $\bar{h}_2$ for every h_0.*
- *If $\bar{h}^2 < h^{\#}$, then the sequence $(h_t)_t$ converges to $\bar{h}_1$ for every h_0.*
- *If $\bar{h}^1 < h^{\#} < \bar{h}^2$, then*

(a) if $h_0 < h^{\#}$, the sequence $(h_t)_t$ converges to $\bar{h}_1$,
(b) if $h_0 > h^{\#}$, the sequence $(h_t)_t$ converges to $\bar{h}_2$.

The recurrent sequence $(h_t)_t$ is monotonic and we have

$$h_{t+1} = \begin{cases} A e_1^{\gamma} h_t^{\delta} & \text{if} \quad h_t < h^{\#}, \\ A e_2^{\gamma} h_t^{\delta} & \text{if} \quad h_t \geq h^{\#}. \end{cases} \tag{9}$$

We obtain the two fixed points $\bar{h}_1$ and $\bar{h}_2$ given by (8).
$-$ Case $\bar{h}^1 > h^{\#}$: here, an economy with low initial human capital $h_0 < h^{\#}$ will first converge towards $h^{\#}$, then it jumps for converging to the highest economy $\bar{h}^2$ (the only fixed point), and an economy with high initial human capital $h_0 > h^{\#}$ will naturally converge to $\bar{h}^2$.
$-$ In the case $\bar{h}^2 < h^{\#}$ (i.e case where $h^{\#}$ is high), the economy will converge to the smallest steady state $\bar{h}^1$ regardless of its initial condition.
$-$ In the case $\bar{h}^1 < h^{\#} < \bar{h}^2$, we obtain the two scenarios (a) where the economy will converge to the smallest steady state (underdevelopment trap) $\bar{h}^1$, and in

the scenario (b) the economy will converge to a highest economy $\bar{h}^2$.

Remark 2. *With this traditional migration model (i.e when the probability of migration is dependent on the human capital of the parents), the human capital threshold $h^{\#}$ determines the growth of the economy which will converge to one of the two fixed points $\bar{h}_1$ and $\bar{h}_2$ given by (8).*

3. The main result

3.1. **The anticipative model.** In this section we assume that the probability of migration is dependent on the average human capital in period $t+1$ (see [1] and also the work by Cipriani et al. [3]). Here, the dynamics of human capital are dependent on households perceptions and beliefs about the future:

$$p_{t+1} = \mathcal{P}(h_{t+1}) \tag{10}$$

with $\mathcal{P}$ defined by (6). Then (9) becomes

$$h_{t+1} \quad = \quad \begin{cases} A\, e_1^{\gamma}\, h_t^{\delta} & \text{if} \quad h_{t+1} < h^{\#}, \\ A\, e_2^{\gamma}\, h_t^{\delta} & \text{if} \quad h_{t+1} \geq h^{\#}. \end{cases} \tag{11}$$

Let us define

$$h_o^{\#} = \left(\frac{h^{\#}}{Ae_2^{\gamma}}\right)^{\frac{1}{\delta}} \qquad \text{and} \qquad h_p^{\#} = \left(\frac{h^{\#}}{Ae_1^{\gamma}}\right)^{\frac{1}{\delta}}. \tag{12}$$

Note that we have $h_o^{\#} < h_p^{\#}$.

The equation (11) is implicit and, given the value h_t, there may be several possible values for h_{t+1}. We have the:

Lemma 3. *Let h_t be the human capital at period t. The human capital h_{t+1} at period $t+1$ must satisfy equation (11). Then we have the following:*
1) If $h_t < h_o^{\#}$ then there exists a unique possible value $h_{t+1} = Ae_1^{\gamma}h_t^{\delta}$.
2) If $h_t > h_p^{\#}$ then there exists a unique possible value $h_{t+1} = Ae_2^{\gamma}h_t^{\delta}$.

Lemma 4. *The following inequality is satisfied:*

$$e_1^{\gamma(\delta-1)} e_2^{-\gamma\delta} < A\big(h^{\#}\big)^{\delta-1} < e_1^{-\gamma\delta} e_2^{\gamma(\delta-1)} \tag{13}$$

if and only if

$$[0, h_o^{\#}) \text{ and } (h_p^{\#}, +\infty) \text{ are stable} \tag{14}$$

if and only if

$$\bar{h}_1 \in [0, h_o^{\#}) \text{ and } \bar{h}_2 \in (h_p^{\#}, +\infty). \tag{15}$$

Proposition 5. *Under the hypothesis (13), we have the following assertions:*
(1) If $h_0 < h_o^{\#}$ *then the sequence* $(h_t)_t$ *converges to the fixed point* $\bar{h}_1$*.*
(2) If $h_0 > h_p^{\#}$ *then the sequence* $(h_t)_t$ *converges to the fixed point* $\bar{h}_2$*.*

Lemma 6. *Let* h_t *be the human capital at period* t*. Then, if* $h_o^{\#} < h_t < h_p^{\#}$ *there exist two different possible values:*

$$h_{t+1,j} = A\, e_j^{\gamma}\, h_t^{\delta} \qquad \text{for} \quad j = 1, 2. \tag{16}$$

Remark 7. *The proof of Proposition 5 will be given in the Appendix. For Lemma 6: if* $\frac{e_1^{\gamma}}{h^{\#}} < \frac{1}{A} h_t^{-\delta} < \frac{e_2^{\gamma}}{h^{\#}}$ *then we obtain the two solutions defined by (16). The lemma shows that it is necessary to give a mechanism to select between the two possible solutions for* h_{t+1} *in the case in which* $h_o^{\#} < h_t < h_p^{\#}$*. We will address different selection mechanisms in Subsection 3.2 and Subsection 3.3 below.*

3.2. **Optimistic and pessimistic selection mechanisms.** The pessimistic selection mechanism consists in choosing the smallest possible value for the human capital when there are two possible choices. The optimistic selection mechanism consists in choosing the largest possible value for the human capital when there are two possible choices. These are the two extremal selection mechanisms. We will consider an intermediate mechanism later in Subsection 3.3.

The following proposition gives the main result in the case of the two selection mechanisms.

Proposition 8. *Under the hypothesis (13), we have the following assertions:*
(1) With the pessimistic selection mechanism, the sequence $(h_t)_t$ *converges to the fixed point* $\bar{h}_1$ *if* $h_0 < h_p^{\#}$*, and converges to the fixed point* $\bar{h}_2$ *if* $h_0 > h_p^{\#}$*.*
(2) with the optimistic selection mechanism, the sequence $(h_t)_t$ *converges to the fixed point* $\bar{h}_1$ *if* $h_0 < h_o^{\#}$*, and converges to the fixed point* $\bar{h}_2$ *if* $h_0 > h_o^{\#}$*.*

Proof - Indeed, for $j = 1, 2$, the fixed point $\bar{h}_j$ satisfies to $\bar{h}_j = Ae_j^\gamma(\bar{h}_j)^\delta$ (i.e (8). Now, if initially $\frac{1}{A}h_0^{-\delta} > \frac{e_1^\gamma}{h^\#}$ then $Ah_0^\delta < \frac{h^\#}{e_1^\gamma}$ which is equivalent to $h_0 < \left(h^\#/(Ae_1^\gamma)\right)^{\frac{1}{\delta}}$. So in this case $h_{t+1} = Ae_1^\gamma h_t^\delta$ converges to a unique $\bar{h}_1$. And if $h_0 < \left(h^\#/(Ae_1^\gamma)\right)^{\frac{1}{\delta}}$, then the sequence $(h_t)_t$ converges to $\bar{h}_2$. With the same arguments, attaining the point $\bar{h}_2$, means $h_0^\delta > \frac{h^\#}{e_2^\gamma}$ which is equivalent to $h_0 < \left(\frac{h^\#}{Ae_2^\gamma}\right)^{\frac{1}{\delta}}$. ■

3.3. **Conservative selection mechanism.** We still use the hypothesis (13). The conservative selection mechanism consists in choosing for the human capital h_{t+1} at period $t + 1$ the value that is the closest from h_t when there are two possible choices.
Let us define

$$h_c^\# = \frac{e_1^\gamma + e_2^\gamma}{2} \tag{17}$$

We have the following result:

Proposition 9. *Under the hypothesis (13), with the conservative selection mechanism, the sequence $(h_t)_t$ converges to the fixed point $\bar{h}_1$ (resp. $\bar{h}_2$) iff $h_0 < H$ (resp. $h_0 > H$) where we have:*
(a) $H = h_p^\#$ if $h_c^\# > h_p^\#$,
(b) $H = h_o^\#$ if $h_c^\# < h_o^\#$,
(c) $H = h_c^\#$ if $h_o^\# < h_c^\# < h_p^\#$.

Discussion. The threshold value in the traditional case is $h^\#$. In the anticipative case with the optimistic (resp. pessimistic) selection mechanism it is $h_o^\#$ (resp. $h_p^\#$). We have

$$h_o^\# < h^\# < h_p^\#. \tag{18}$$

In the goal to have a high economy level, we notice that the optimistic anticipation mechanism is the one that gives the smallest threshold value H from which we have convergence to the highest fixed point $\bar{h}^2$. Conversely, the pessimistic anticipation mechanism is the one that gives the largest threshold value.

3.4. **Conclusion.** It is worthwhile to note that, whatever the type of evolution equation for the human capital (traditional or anticipative) and for any selection mechanisms, the result can always be expressed by an assertion of the type: if the initial capital h_0 is smaller than a threshold value H, then the human capital will converge to the low fixed point $\bar{h}_1$, while if the initial capital h_0 is larger than a threshold value H, then the human capital will converge to the high fixed point $\bar{h}_2$. The values of the fixed points $\bar{h}_1$ and $\bar{h}_2$ do not depend on the type of evolution equation and of the selection mechanism. Only the threshold value H depends on the type of evolution equation and of the selection mechanism.

4. Appendix - Complementary results

Proof or Proposition 5: The first interesting remark is that the two new thresholds satisfy to $h_o^{\#} < h^{\#} < h_p^{\#}$ where $h^{\#}$ is the human capital threshold of the traditional case in the previous section. We will comment this remark at the end of the paper. In particular, it is interesting to know what happens if we do not satisfy (13). We will address the two other cases $e_1^{\gamma(\delta-1)} e_2^{-\gamma\delta} > A\big(h^{\#}\big)^{\delta-1}$ and $A\big(h^{\#}\big)^{\delta-1} > e_1^{-\gamma\delta} e_2^{\gamma(\delta-1)}$ in [4].

We discuss now of 1) and 2) in the proposition. From (11) we obtain a function $f(h_{t+1}) = \frac{1}{A} h_t^{-\delta}$. If the average human capital h_t at period t is lower than the threshold $h_o^{\#}$ (i.e case 1)), then $\frac{1}{A} h_t^{-\delta} > \frac{e_2^{\gamma}}{h^{\#}}$ i.e there exists a unique solution $h_{t+1} = A e_1^{\gamma} h_t^{\delta}$ which will be the lower economy at which we will certainly converge. Similar argument can be used to justify the case 2).

Stability: The question is stating with average initial human capital h_0 in one of the two intervals w.r.t (14), do we remain in the same interval-region (lower or higher economy). For example, given $h_0 > 0$ such that $\frac{1}{A} h_0^{-\delta} > \frac{e_2^{\gamma}}{h^{\#}}$, we want to know if still we have $\frac{1}{A} h_1^{-\delta} > \frac{e_2^{\gamma}}{h^{\#}}$ (i.e we stay in the same uni-valued region). Since $h_1 = A(e_1^{\gamma}) h_0^{\delta} = \frac{e_1^{\gamma} h^{\#}}{e_2^{\gamma}}$ (see definition (2)), we want $\frac{1}{A}\left(h^{\#}\right)^{1-\delta} e_1^{-\gamma\delta} e_2^{\gamma(\delta-1)} > 1$, which is obviously true since that we have $A < \left(h^{\#}\right)^{1-\delta} (e_1^{\gamma})^{-\delta} (e_2^{\gamma})^{\delta-1}$. Now, if $\frac{1}{A} h_0^{-\delta} < \frac{e_1^{\gamma}}{h^{\#}}$, then $\frac{1}{A} h_1^{-\delta} < \frac{e_1^{\gamma}}{h^{\#}}$ since $A > \left(h^{\#}\right)^{1-\delta} (e_2^{\gamma})^{-\delta} (e_1^{\gamma})^{\delta-1}$.

More general, for any t, if $h_0 > \left(e_2^{\gamma} h^{\#}\right)/e_1^{\gamma}$ then $e_2^{\gamma} = h_t^{1-\delta}/A$ iff $h_t = (A e_2^{\gamma})^{\frac{1}{1-\delta}}$. This is true iff $(A(e_2^{\gamma}))^{\frac{1}{1-\delta}} > \left(e_2^{\gamma} h^{\#}\right)/e_1^{\gamma}$. But this is equivalent to $A > \left(h^{\#}\right)^{1-\delta} (e_2^{\gamma})^{-\delta} (e_1^{\gamma})^{\delta-1}$. From another side, if $h_0 < \frac{e_1^{\gamma} h^{\#}}{e_2^{\gamma}}$, then $A < \left(h^{\#}\right)^{1-\delta} e_1^{-\gamma\delta} e_2^{\gamma(\delta-1)}$. In conclusion, the economy surely converges to a steady in the important case of (13). We finally notice the equvalence of (13) with (14) as we have

$$\begin{cases} \bar{h}^1 < \left(\dfrac{h^\#}{Ae_2^\gamma}\right)^{\frac{1}{\delta}} & \Leftrightarrow \quad (Ae_1^\gamma)^{\frac{1}{1-\delta}} < \left(\dfrac{h^\#}{Ae_2^\gamma}\right)^{\frac{1}{\delta}} \quad \Leftrightarrow \quad Ae_1^{\gamma\delta} e_2^{\gamma(1-\delta)} < \left(h^\#\right)^{1-\delta}, \\ \bar{h}^2 > \left(\dfrac{h^\#}{Ae_1^\gamma}\right)^{\frac{1}{\delta}} & \Leftrightarrow \quad (Ae_2^\gamma)^{\frac{1}{1-\delta}} > \left(\dfrac{h^\#}{Ae_1^\gamma}\right)^{\frac{1}{\delta}} \quad \Leftrightarrow \quad Ae_1^{\gamma(1-\delta)} e_2^{\gamma\delta} > \left(h^\#\right)^{1-\delta}. \end{cases}$$

Lemma 10 (Stability)**.** *Under the hypothesis*

$$\frac{e_1^\gamma}{h^\#} < \frac{1}{A} h_0^{-\delta} < \frac{e_2^\gamma}{h^\#} \tag{19}$$

we have the following two assertions:

1. *At the first step, if we choose the solution $h_1 = Ae_1^\gamma h_0^\delta$, then we have convergence to the unique fixed point in the region $h_{t+1} = Ae_1^\gamma h_t^\delta$.*
2. *If we choose the solution $h_1 = Ae_2^\gamma h_0^\delta$, then we have convergence to the unique fixed point in the region $h_{t+1} = Ae_2^\gamma h_t^\delta$.*

Proof - We suppose (19) true, then we have the two cases:

1. Given h_0, if $h_1 = Ae_1^\gamma h_0^\delta$, the question is: do we still have $h_1 < h_0$? We use the hypothesis (13) as follows:

$$\begin{array}{rccc} & A & < & (e_2^\gamma)^{\delta-1}(e_1^\gamma)^{-\delta}(h^\#)^{1-\delta} \\ \Leftrightarrow & (h^\#)^{1-\delta} & > & (e_2^\gamma)^{1-\delta}(e_1^\gamma)^{\delta} A \\ \Leftrightarrow & (h^\#)^{1-\delta} & > & (e_2^\gamma A)^{1-\delta}(Ae_1^\gamma)^{\delta} \\ \Leftrightarrow & h^\# & > & (e_2^\gamma A)(A(e_1^\gamma))^{\frac{\delta}{1-\delta}} \\ \Leftrightarrow & \left(\dfrac{h^\#}{e_2^\gamma A}\right) & > & (Ae_1^\gamma)^{\frac{\delta}{1-\delta}}. \end{array} \tag{20}$$

   Now, from the hypothesis (19), we have $h_0^\delta > \frac{h^\#}{e_2^\gamma A}$. Then from (20) we have $h_0^\delta > (Ae_1^\gamma)^{\frac{\delta}{1-\delta}}$, hence $h_0^{\frac{1}{1-\delta}} > h_1^{\frac{1}{1-\delta}}$.
2. Using the same arguments, if we choose the solution $h_1 = Ae_2^\gamma h_0^\delta$, then we find $h_0 < h_1$ and then we have the convergence to the unique fixed point in the region $h_{t+1} = Ae_2^\gamma h_t^\delta$.

■

Lemma 11. *We suppose that the hypothesis (13) is satisfied. Then we have the two following assertions:*
(a) If $h_0 > h_c^\#$, then the solution given by $h_{t+1} = Ae_2^\gamma h_t^\delta$ is the one of minimal norm: $\|h_{t+1} - h_0\| = min$. Moreover, this solution converges to the unique fixed point $\bar{h}_2$.

(b) If $h_c^{\#} > h_0$, then $\|h_{t+1} - h_0\| = min$, and moreover $(h_t)_t$ converges to the unique fixed point $\bar{h}_1$.

Proof - Suppose that (13) holds. There is two different possibilities for the first step: $h_{1,1} = Ae_1^\gamma h_0^\delta$ and $h_{1,2} = Ae_2^\gamma h_0^\delta$. From Lemma 10 we have $h_{1,1} < h_0 < h_{1,2}$. Suppose that $h_0 > h_c^{\#}$, then

$$\begin{array}{lcll} & h_0 & > & h_c^{\#} \\ \Leftrightarrow & h_0^{1-\delta} & > & A\,\frac{(e_1^\gamma + e_2^\gamma)}{2} \\ \Leftrightarrow & 2h_0 & > & A\,(e_1^\gamma + e_2^\gamma)\,h_0^\delta \\ \Leftrightarrow & 2h_0 - h_{1,1} - h_{1,2} & > & 0 \end{array}$$

that is $h_0 - h_{1,1} > h_{1,2} - h_0$. So $h_{1,2}$ is the closest to h_0. We easily generalize to any t. We do the same analysis for (b). ■

Remark 12. *We have to be sure that the set of possible solutions in Lemma 11 is non empty. This is satisfied iff*

$$\begin{array}{lcll} & h^{\#} & > & Ae_1^\gamma \left(\frac{A\,(e_1^\gamma + e_2^\gamma)}{2}\right)^{\frac{\delta}{1-\delta}} \\ \Leftrightarrow & (h^{\#})^{1-\delta} & > & Ae_1^{\gamma(1-\delta)} \left(\frac{e_1^\gamma + e_2^\gamma}{2}\right)^{\delta} \\ \Leftrightarrow & \dfrac{A}{(h^{\#})^{1-\delta}} & < & e_1^{\gamma(\delta-1)} \left(\frac{e_1^\gamma + e_2^\gamma}{2}\right)^{-\delta} \end{array}$$

which is still true iff

$$\begin{array}{lcll} & e_1^{-\gamma\delta} e_2^{\gamma(\delta-1)} & < & e_1^{\gamma(\delta-1)} (\frac{e_1^\gamma + e_2^\gamma}{2})^{-\delta} \\ \Leftrightarrow & e_1^{\gamma(1-2\delta)} & < & e_2^{\gamma(1-\delta)} (\frac{e_1^\gamma + e_2^\gamma}{2})^{-\delta} \\ \Leftrightarrow & \left(\frac{e_1}{e_2}\right)^{\gamma(1-\delta)} & < & \left(\frac{1 + e_2^\gamma / e_1^\gamma}{2}\right)^{-\delta} \end{array}$$

which means that the following function

$$\psi(X) = \left(\frac{1+X}{2}\right)^{-\delta} X^{1-\delta} - 1 \tag{21}$$

is non negative, which is obviously true.

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