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PUBLIC INVESTMENT, DISTRIBUTIVE POLITICS AND ECONOMIC GROWTH

Francisca Guedes de Oliveira

Universidade Católica Portuguesa (Porto)

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Francisca Guedes de Oliveira
Universidade Católica Portuguesa
Faculdade de Economia e Gestão

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Abstract

This paper develops on a Solow type of model where the government is introduced as a decision maker. Additionally, this paper introduces consumer decisions and assumes that individuals can be differentiated by their relative factor endowment (labor and private capital). The results indicate that the economy's growth rate has an inverted U-shape relationship with the tax rate on private capital τ. They also indicate that the tax rate has a positive relation with the amount of money government spend on consumption, θ, (rather than on investment in public capital). The paper also concludes that the choice of the tax rate will be above the optimal level and hence the potential growth rate will not be achieved. Taking the analysis further, it can be assumed that voters will try to correct lower tax rates of public investment by choosing an higher tax rate. This tax rate will be higher if society is more disparate in terms of income distribution. However, by reducing θ, τ automatically decreases thus bringing us closer to the optimum.

Finally, the conclusion from a public policy perspective is that there is a negative relationship between the chosen tax rate and public investment and that this relationship is highly sensitive to the model parameters.

JEL: A, H, O11, O43

Keywords: growth, income distribution, government budget, government efficiency

1 Introduction

In recent literature there are several papers that approach issues related to the effects of income distribution in economy (for example [1] or [32]). Other papers relate income distribution and policy but usually address the problem in the reverse perspective: how does policy affect income distribution and poorness ([17], [19]). Some literature focus on the relationship between distribution and economic performance. In [27] reference is made to the specific channels through which income distribution affects growth[1]. There is also research that deals with the specific relation between income distribution and education in an empirical perspective (see [14]) or in a more theoretical framework (see [13]). Income equality and economic performance are clearly important subjects in the literature (even though it is not a recent subject, see for example [18]).

There are several papers that link income inequality and economic growth. In [29] the authors find evidence that there is a negative association between income inequality and growth. The authors state that due to income inequality the political deciders produce policies that tax activities that could promote growth (namely investment). Additionally in [7] the authors show that the growth rate falls with the gap between rich and poor. The paper [1] also addresses the issue of the relationship between income distribution and economic growth. In this paper the authors develop a model of endogenous growth where the government has a constructive role in the sense that they provide a productive good. The consumers distinguish themselves through the relative endowment of labor and capital. Individuals will chose the tax rate on capital. Applying the median voter theorem the authors conclude that the more unequal the income distribution is the further will the tax rate be from the tax rate that maximizes growth. So there is a negative relation between income inequality and slow growth. Our paper goes a step further

[1]In the cited paper the authors conclude that there is a strong link between income distribution and social/political instability and income distirbution and education/fertility decisions. They conclude that there is less support of the link between income distirbution and fiscal policy.

and tries to see if governments can, via public investment in public capital, interfere in this relation by reducing the tax rate that maximizes the utility of the median voter.

There is a relatively extensive literature concerning the importance of government investment, the composition of public expenditures and growth[2]. In [17] the authors claim that "Governments can accelerate economic development through their decisions on public expenditures" a conclusion supported by the authors in [31]. In [4] the authors look for the relationship between public capital and economic growth by trying to find the ratio of public to private capital that maximizes growth. They argue that the decrease in this ratio in the late years in the United States is probably responsible for the low rate of productivity growth[3]. The link between public capital and private factors productivity is also addressed, in an empirical perspective, by [15] and restated by [12] and [2]. In [31] the authors survey the link between public investment and economic growth and find evidence in the literature that there is a growing consensus that public capital represents "the wheels - if not the engine - of economic activity". This idea is reinforced by the position of the authors in [3] by claiming the importance of the quantity of public capital required for economic growth. Our model (that is based on the one presented in [1]) is an extension of the model presented in [26] and fits in the literature of endogenous growth models such as [6] and [5].

Our starting point is that empirically there is some evidence of a relation between income distribution government quality and government size. In [25] several measures of government efficiency are built by constructing the ratio between the output of a given sector (for example drop out rate in the case of education or infant mortality rate in the case of the health department) and the amount of money governments spend in that precise sector. Using one of these ratios (Mortality rate infant/Public health expenditures) and the gini index we constructed a chart (Appendix A) that gives us an idea of the positive relation between income inequality and government inefficiency (the higher the gini index is the higher our ratio is, i.e., the less efficient

[2]This literature follows closely [5]. See also [31] for a critical survey on public capital and economic growth.

[3]This conclusion is also referred by [2] where the authors establish a link between underinvestment in public capital and low productivty growth.

governments are). Using general government expenditures as a measure of government size we can also plot government size against income inequality (Appendix A). We can broadly conclude that there is a negative relation between government size and income inequality[4].

We can say that the paper is linked with two streams of literature: income distribution and growth and public investment and growth trying to bridge the two. In the next section we present a model of endogenous growth where we have the government as a decision maker. The government has to decide, after the voters choose the level of the tax rate on capital, how much resources should be devoted to public consumption and how many resources should be spent on investment in public capital, which in the line with [26] allows the government to produce quality. The link between quality and growth has been addressed in the literature (see for example [11], [21], [20] or [9]) . The consumers have different capital endowments and have to vote on the tax rate. There is a single good in the economy which is produced using private capital and the consumption good and the capital good produced by the government. In section three we solve the decentralized problem of maximizing the utility and find out the steady state growth rate and the tax rate that maximizes it. In section four we analyse the solution if the government was to choose the tax rate that maximizes the utility of consumer i. We then see what will the policy choice be under majority voting and according with the median voter theorem. We analyze how the government intervention will reduce the majority choice of the tax rate and do some sensitivity analysis to the parameters in the model. In section six we conclude.

2 The Model

The model presented has some close similarities with [26]. The definition of the production function and of the government uses for the tax revenues are basically the same. One first difference has to do with the tax base. In [26] the tax is on income. In the present paper we introduce the consumer side

[4]The data used for both charts is from World Development Indicators 2002.

and we distinguish consumers based on their relative factor ownership. In this case the tax is on private capital.

Consider an economy where output is linear in capital and public services taken together, with the following aggregate production function:

$$y_t = AK_{pt}^{\alpha} L_t^{1-\alpha} (H_t q_t)^{\beta} \tag{1}$$

Taking the return on capital and labor:

$$\frac{\partial y_t}{\partial k_{pt}} = A\alpha K_{pt}^{\alpha-1} L_t^{1-\alpha} (H_t q_t)^{\beta} \tag{2}$$

$$\frac{\partial y_t}{\partial L_t} = A(1-\alpha) K_{pt}^{\alpha} L_t^{-\alpha} (H_t q_t)^{\beta} \tag{3}$$

To finance spending on public services, the government uses a distortionary tax on private capital income, τ. The budget is balanced every instant so: $g_t = \tau k_{pt}$.

There are two uses of tax revenues, according to:

$$\begin{aligned} H_t &= \theta \tau K_{pt} && (4) \\ \dot{K}_{gt} + \delta K_{gt} &= (1-\theta)\tau K_{pt} && (5) \\ q_t &= \left(\frac{K_{gt}}{L_t}\right)^{\psi} && (6) \end{aligned}$$

We can se that H_t is basically a consumption good in the sense that governments spend a given percentage, θ, to deliver H_t.

The idea behind government quality, q_t, is that for a government to be efficient it needs to accumulate, i.e., efficiency or quality take time to build. we do not assume a linear relation between quality and per capita capital because we want to introduce the idea of saturation, i.e., governments can accumulate capital that allows them to be more efficient but this accumulation becomes less and less productive. In [25] there was empirical evidence that in fact there was a positive and strong relation between public capital and government efficiency. The results where quite striking and robust once they survive the inclusion of control variables such has GDP or even government spending.

Rewriting the partial derivatives assuming that labor is supplied inelastically, which allow us to set the economy's aggregate labor endowment to 1, we get:

$$\begin{aligned}\frac{\partial y_t}{\partial k_{pt}} &= A\alpha k_{pt}^{\alpha-1}(\theta\tau k_{pt}k_{gt}^{\psi})^{\beta} = A\alpha(\theta\tau)^{\beta}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} \quad (7)\\ \frac{\partial y_t}{\partial k_{pt}} &= r_t = r(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} \quad (8)\end{aligned}$$

$$\begin{aligned}\frac{\partial y_t}{\partial L_t} &= A(1-\alpha)k_{pt}^{\alpha}(\theta\tau k_{pt}k_{gt}^{\psi})^{\beta} \quad (9)\\ \frac{\partial y_t}{\partial L_t} &= A(1-\alpha)(\theta\tau)^{\beta}k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta} = w(\tau,\theta)k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta} \quad (10)\end{aligned}$$

Where $r(\tau,\theta)$ is $A\alpha(\theta\tau)^{\beta}$ and $w(\tau,\theta)$ is $A(1-\alpha)(\theta\tau)^{\beta}$ both expressions depend on technological parameters (A, α and β) and on government choices (through θ and τ). $r(\tau,\theta)$ is a positive function of all the parameters while $w(\tau,\theta)$ is a negative function of α (and a positive function of all the other parameters). Both marginal productivites are positive functions of the tax rate and of θ. They also depend positively on public capital. The wage rate is also increasing in the private capital stock and assuming that $\alpha+\beta<1$ the rate of return on private capital depends negatively on private capital.

Labor and private capital income net of taxes are given by[5]:

$$\begin{aligned}y^{k_p} &= \left[A\alpha\left(\theta\tau\right)^{\beta}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}-\tau\right]k_{pt} \quad (11)\\ y^{L} &= \left[A(1-\alpha)(\theta\tau)^{\beta}k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta}\right] \quad (12)\end{aligned}$$

Each individual is indexed by a relative factor endowment $\sigma^i = \frac{L_t^i}{\left(k_{pt}^i/k_{pt}\right)}$ where L_i is the labor endowment of individual i.

[5]For the national income identity to be satisfied, it is necessary that: $y^{k_p}+y^L+g=y \Longleftrightarrow \left[A\alpha\left(\theta\tau\right)^{\beta}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}-\tau\right]k_{pt}+\left[A(1-\alpha)(\theta\tau)^{\beta}k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta}\right]$ $+\tau k_{pt} = y \Longleftrightarrow A\alpha\left(\theta\tau\right)^{\beta}k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta}-\tau k_{pt}+A(1-\alpha)(\theta\tau)^{\beta}k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta}+\tau k_{pt} = y \Longleftrightarrow$ $A(\theta\tau)^{\beta}k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta}=y$ $c.q.d.$.

2.1 Individual Income and Inequality

We consider an economy where if σ is high the individual is capital poor and if σ is low than individual i is capital rich. In a perfect egalitarian society we would have $\sigma^i = 1$ $\forall i$, since everybody would have the exact same amount of private capital. In the real world we have that the median citizen has $\sigma^m < 1$.

Individual income:

$$y_t^i = w(\tau,\theta)k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta}L_t^i + \left[r(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - \tau\right]k_{pt}^i \Longleftrightarrow \quad (13)$$

$$y_t^i = w(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\sigma^i k_{pt}^i + \left[r(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - \tau\right]k_{pt}^i \quad (14)$$

3 Decentralized Problem

3.1 Individual Maximization for given τ and θ

We assume that all individuals have the same utility function. Consumers solve the following problem:

$$\underset{s.t.:}{Max}U^i = \int e^{-\rho t}\log c^i dt \quad (15)$$

$$\dot{k}_p^i = w(\tau,\theta)k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta}L_t^i + \left[r(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - \tau\right]k_{pt}^i - c^i \quad (16)$$

We have the following Lagrangian:

$$Lag = \int e^{-\rho t}\{\log c_t^i + \lambda_t[w(\tau,\theta)k_{pt}^{\alpha+\beta}k_{gt}^{\psi\beta}L_t^i + \left(r(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - \tau\right)k_{pt}^i - \quad (17)$$

$$-c_t^i - \dot{k}_{pt}^i]\}dt \Longleftrightarrow$$

$$Lag = \int e^{-\rho t}\{\log c_t^i + \lambda_t \left[w(\tau,\theta) k_{pt}^{\alpha+\beta} k_{gt}^{\psi\beta} L_t^i + \left(r(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} - \tau \right) k_{pt}^i - c_t^i \right] -$$

$$-(\lambda_t \rho - \dot{\lambda}) k_{pt}^i\} dt$$

Solving the FOC:

$$\frac{\partial Lag}{\partial c_t^i} = 0 \Longleftrightarrow \frac{1}{c_t^i} = \lambda_t \tag{18}$$

This relation reports us to the familiar facts that marginal utility of consumption equals marginal utility of income and that marginal utility of consumption decreases with consumption.

$$\begin{aligned} \frac{\partial Lag}{\partial k_{pt}^i} &= 0 \Longleftrightarrow \lambda_t \left[r(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} - \tau \right] - \lambda_t \rho + \dot{\lambda} = 0 \Longleftrightarrow & (19) \\ -\frac{\dot{\lambda}}{\lambda_t} &= r(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} - \tau - \rho & (20) \end{aligned}$$

This equation says that households choose consumption as to equal the rate of return on capital to the rate of time preference, plus the tax rate plus the rate of decrease of the marginal utility of consumption.

From [18] we can write, after some algebra, the Euler equation:

$$\frac{\dot{c}^i}{c_t^i} = A\alpha(\theta\tau)^\beta k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} - \tau - \rho = \gamma \tag{21}$$

Which as we know tells us that the growth rate of consumption equals the rate of return net of time discount and taxes.

In a balanced growth path we must have:

$$\begin{aligned} \frac{\partial \left(\frac{\dot{c}^i}{c_t^i} \right)}{\partial t} &= 0 \Longleftrightarrow & (22) \\ r(\tau,\theta)(\alpha+\beta-1) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} \frac{\dot{k}_p}{k_{pt}} + r(\tau,\theta)\psi\beta k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} \frac{\dot{k}_g}{k_{gt}} &= 0 & (23) \end{aligned}$$

If we rule out corner solutions we will have:

$$\psi\beta\frac{\dot{k}_g}{k_{gt}} = (1-\alpha-\beta)\frac{\dot{k}_p}{k_{pt}} \tag{24}$$

Proposition 1 *If we assume constant returns*[6] *to scale on private and public capital in the production function we will have, along a balanced growth path:* $\frac{\dot{k}_g}{k_{gt}} = \frac{\dot{k}_p}{k_{pt}}$

Rewriting (15) we have:

$$\dot{k}_p^i = w(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\sigma^i k_{pt}^i + \left[r(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - \tau\right]k_{pt}^i - c^i \tag{25}$$

Which leads us to:

$$\frac{\dot{k}_p^i}{k_{pt}^i} = w(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\sigma^i + \left[r(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - \tau\right] - \frac{c_t^i}{k_{pi}} \tag{26}$$

$$\frac{\dot{k}_p^i}{k_{pt}^i} = k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\left[w(\tau,\theta)\sigma^i + r(\tau,\theta)\right] - \tau - \frac{c_t^i}{k_{pi}} \tag{27}$$

In a balanced growth path we have:

$$\frac{\partial\frac{\dot{k}_p^i}{k_{pt}^i}}{\partial t} = 0 \tag{28}$$

Which leads to the following result[7]:

$$\frac{\dot{k}_p^i}{k_{pt}^i} = \frac{\dot{c}^i}{c_t^i} \tag{29}$$

What we have is that the private capital growth rate is independent of any individual characteristics and it is independent of the individual factor

[6] Note that we have to assume that $1-\alpha-\beta > 0$ otherwise one of the growth rates would be negative. Under constant or diminuishing returns this assumption is guaranteed:

Constant or diminuishing returns would imply: $\alpha+\beta+\psi\beta \leq 1 \Longleftrightarrow \alpha+\beta \leq 1-\psi\beta \Longrightarrow \alpha+\beta < 1$.

[7] Appendix B

endowment. This allows us to conclude that the growth rate of individual private capital is equal to the growth rate of the economy's private capital

$$\frac{\dot{k}_p^i}{k_{pt}^i} = \frac{\dot{k}_p}{k_{pt}} = \gamma \tag{30}$$

This last equation assures us that the identity of the median voter will always be the same because σ^i will remain constant.

We can write:

$$\frac{\dot{k}_p^i}{k_{pt}^i} = \frac{\dot{c}^i}{c_t^i} = \frac{\dot{k}_p}{k_p} = \frac{\psi\beta}{1-\alpha-\beta}\frac{\dot{k}_g}{k_{gt}} = A\alpha(\theta\tau)^\beta k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - \tau - \rho = \gamma \tag{31}$$

To assure that we are in a balanced growth path we must impose constant returns to scale, this follows directly from (24) and (4)

$$\begin{aligned} \dot{k}_{gt} + \delta k_{gt} &= (1-\theta)\tau k_{pt} \Longleftrightarrow & (32)\\ \frac{\dot{k}_{gt}}{k_{gt}} + \delta &= (1-\theta)\tau\frac{k_{pt}}{k_{gt}} & (33) \end{aligned}$$

If we are in a balanced growth path we know that $\frac{\partial\left(\dot{k}_{gt}/k_{gt}\right)}{\partial t} = 0$ and we can rewrite the previous equation:

$$\begin{aligned} \frac{\partial\left(\frac{\dot{k}_{gt}}{k_{gt}}\right)}{\partial t} &= (1-\theta)\tau\frac{\dot{k}_{pt}k_{gt} - \dot{k}_{gt}k_{pt}}{k_{gt}^2} \Longleftrightarrow & (34)\\ 0 &= \frac{\dot{k}_{pt}k_{gt} - \dot{k}_{gt}}{k_{gt}^2} \Longleftrightarrow & (35)\\ \frac{\dot{k}_{pt}}{k_{pt}} &= \frac{\dot{k}_{gt}}{k_{gt}} \Longleftrightarrow & (36) \end{aligned}$$

From (24):

$$\begin{aligned} \frac{\psi\beta}{1-\alpha-\beta}\frac{\dot{k}_g}{k_{gt}} &= \frac{\dot{k}_{gt}}{k_{gt}} \Longleftrightarrow & (37)\\ \psi\beta &= 1-\alpha-\beta \Longleftrightarrow & (38)\\ \psi\beta+\alpha+\beta &= 1 & (39) \end{aligned}$$

Which means: $\frac{\dot{k}_p^i}{k_{pt}^i} = \frac{\dot{c}^i}{c_t^i} = \frac{\dot{k}_p}{k_p} = \frac{\dot{k}_g}{k_{gt}} = A\alpha(\theta\tau)^\beta k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - \tau - \rho = \gamma$

3.2 Government Choice of τ

Now instead of considering τ and θ constants we are interested in seeing what happens to the economy (in particular to the growth rate) if we vary each of them at a time. To see the effect of variations in τ and θ in the growth rate we first have to see what happens to k_{pt} and k_{gt} when these parameters change. If we consider k_{p0} and k_{g0} to be, respectively, the stock of private capital and the stock of public capital at the beginning of the balanced growth path, we can write:

$$k_{pt} = k_{p0}e^{\gamma t} \tag{40}$$
$$k_{gt} = k_{g0}e^{\gamma t} \tag{41}$$

And so:

$$\frac{\partial k_{pt}}{\partial \tau} = k_{p0}\frac{\partial \gamma}{\partial \tau}te^{\gamma t} \tag{42}$$
$$\frac{\partial k_{gt}}{\partial \tau} = k_{g0}\frac{\partial \gamma}{\partial \tau}te^{\gamma t} \tag{43}$$

$$\frac{\partial k_{pt}}{\partial \theta} = k_{p0}\frac{\partial \gamma}{\partial \theta}te^{\gamma t} \tag{44}$$
$$\frac{\partial k_{gt}}{\partial \theta} = k_{g0}\frac{\partial \gamma}{\partial \theta}te^{\gamma t} \tag{45}$$

We can now find the tax rate that maximizes the economy's growth rate:

$$\frac{\partial \gamma}{\partial \tau} = A\alpha\beta\theta^{\beta}\tau^{\beta-1}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} + A\alpha\left(\theta\tau\right)^{\beta}\left(\alpha+\beta-1\right)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\frac{\left(\partial k_{pt}/\partial \tau\right)}{k_{pt}} +$$

$$+A\alpha\left(\theta\tau\right)^{\beta}\left(\psi\beta\right)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\frac{\left(\partial k_{gt}/\partial \tau\right)}{k_{gt}} - 1 = 0 \tag{46}$$

Remember that we must have constant returns to scale which means that: $\psi\beta = 1 - \alpha - \beta$

$$A\alpha\beta\theta^{\beta}\tau^{\beta-1}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - A\alpha\left(\theta\tau\right)^{\beta}\left(\psi\beta\right)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\left(\frac{k_{p0}\frac{\partial\gamma}{\partial\tau}te^{\gamma t}}{k_{p0}e^{\gamma t}} - \frac{k_{g0}\frac{\partial\gamma}{\partial\tau}te^{\gamma t}}{k_{g0}e^{\gamma t}}\right) = 1 \tag{47}$$

$$\Longleftrightarrow$$

$$A\alpha\beta\theta^{\beta}\tau^{\beta-1}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} - A\alpha\left(\theta\tau\right)^{\beta}\left(\psi\beta\right)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\left(\frac{\partial\gamma}{\partial\tau}t - \frac{\partial\gamma}{\partial\tau}t\right) = 1 \tag{48}$$

$$\tau^{*} = \left[A\alpha\beta\theta^{\beta}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\right]^{\frac{1}{1-\beta}} \tag{49}$$

We can see that, given k_{pt} and k_{gt} the growth rate has an inverted U-shape relation with τ[8].

So we have a tax rate that maximizes the economy's growth rate and that has a positive monotonous relation with k_{gt} and with θ and a negative monotonous relation with k_{pt}.

We can also find out the relation between γ and θ

$$\frac{\partial\gamma}{\partial\theta} = A\alpha\beta\tau^{\beta}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\theta^{\beta-1} \tag{52a}$$

It is easy to see that the growth rate has a monotonous positive relation with θ.

[8]We can also see that τ^{*} is time invariant:

$$\frac{\partial\tau^{*}}{\partial t} = \frac{1}{1-\beta}\left[A\alpha\beta\theta^{\beta}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\right]^{\frac{1}{1-\beta}-1}\left\{A\alpha\beta\theta^{\beta}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\left[\left(\alpha+\beta-1\right)\right]\frac{\dot{k}_{p}}{k_{pt}} + \psi\beta\frac{\dot{k}_{g}}{k_{gt}}\right\} \tag{50}$$

According to (24) we can easliy see that:

$$\frac{\partial\tau^{*}}{\partial t} = 0 \tag{51}$$

4 Centralized Problem

We are now interested in seeing what is the individual $i\prime s$ preferred policy in what concerns τ and θ. We are going to address this issue by seeing what would the tax rate and θ be if the government was interested in maximizing individual $i\prime s$ well being or we can think of it as a centralized problem where individual i makes the decisions.

Lets first see what is the instantaneous level of consumption along the optimal path.

We know from (31) that:

$$\dot{k}_p^i = (A\alpha(\tau)^{\beta} k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} - \tau - \rho) k_{pt}^i \tag{53}$$

Replacing (53) in (15) we will have:

$$\left(r(\tau,\theta)^{\beta} k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} - \tau - \rho\right) k_{pt}^i = w(\tau,\theta) k_{pt}^{\alpha+\beta} k_{gt}^{\psi\beta} L_t^i + \left[r(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} - \tau\right] k_{pt}^i - c_t^i$$

$$\begin{aligned} c^i &= w(\tau,\theta) k_{pt}^{\alpha+\beta} k_{gt}^{\psi\beta} L_t^i + \rho k_{pt}^i \Longleftrightarrow && (54) \\ c^i &= w(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} \sigma^i k_{pt}^i + \rho k_{pt}^i \Longleftrightarrow && (55) \\ c^i &= \left[w(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} \sigma^i + \rho\right] k_{pt}^i && (56) \end{aligned}$$

Individual i consumes the entire labor income plus a fraction of his capital stock.

The government maximization problem becomes:

$$\begin{aligned} \underset{\tau,\theta}{Max}\ U^i &= \int e^{-\rho t} \log c_t^i dt && (57) \\ s.t.: & \\ c_t^i &= \left[w(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} \sigma^i + \rho\right] k_{pt}^i && (58) \\ \dot{k}_{pt}^i &= \gamma k_{pt}^i && (59) \\ \dot{k}_{pt} &= \gamma k_{pt} && (60) \end{aligned}$$

As in [1] the constrains make clear that the choices of policy affect both the level of consumption and its growth rate. The last restriction is necessary because k$_{pt}$ enters the definition of σ^i.

The Hamiltonian can be written as:

$$Ham = e^{-\rho t} \log\left\{\left[w(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} \sigma^i + \rho\right] k_{pt}^i\right\} + \mu_1 \gamma k_{pt}^i + \mu_2 \gamma k_{pt} \quad (61)$$

The first order conditions are[9]:

$$\frac{\partial Ham}{\partial \tau} = 0 \Longleftrightarrow e^{-\rho t} \frac{w_\tau(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} L^i k_{pt}}{c^i} + \mu_1 \gamma_\tau k_{pt}^i + \mu_2 \gamma_\tau k_{pt} = 0 \quad (62)$$

$$\frac{\partial Ham}{\partial \theta} = 0 \Longleftrightarrow e^{-\rho t} \frac{w_\theta(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} L^i k_{pt}}{c^i} + \mu_1 \gamma_\theta k_{pt}^i + \mu_2 \gamma_\theta k_{pt} = 0 \quad (63)$$

$$\frac{\partial Ham}{\partial k_{pt}^i} = e^{-\rho t} \frac{\rho}{c^i} + \mu_1 \gamma = -\dot{\mu}_1 \quad (64)$$

$$\frac{\partial Ham}{\partial k_{pt}} = e^{-\rho t} \frac{w(\tau,\theta)\,(\alpha+\beta)\, k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} L^i}{c^i} + \mu_1\,(\alpha+\beta-1)\,\gamma \frac{k_{pt}^i}{k_{pt}} + \mu_2((\alpha+\beta-1)\,\gamma + \gamma) = -\dot{\mu}_2 \quad (65)$$

$$e^{-\rho t} \frac{w(\tau,\theta)\,(\alpha+\beta)\, k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} L^i}{c^i} + \mu_1\,(\alpha+\beta-1)\,\gamma \frac{k_{pt}^i}{k_{pt}} + \mu_2(\alpha+\beta)\gamma = -\dot{\mu}_2 \quad (66)$$

[9] We are going to ignore the partial derivatives of k_{pt} and k_{gt} with respect to τ or θ because we have already established that $\frac{(\partial k_{pt}/\partial \tau)}{k_{pt}} = \frac{(\partial k_{gt}/\partial \tau)}{k_{gt}}$ and $\frac{(\partial k_{pt}/\partial \theta)}{k_{pt}} = \frac{(\partial k_{gt}/\partial \theta)}{k_{gt}}$

4.1 Optimal τ

After some considerable amount of algebra (appendix C) and making $k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta} = \pi$, we are left with an expression that gives us, implicitly, the optimal tax rate for individual i:

$$\frac{\left(\tau_i^{1-\beta} - A\alpha\beta\theta^{\beta}\pi\right)\rho}{A\left(1-\alpha\right)\beta\theta^{\beta}\pi\sigma^{i}} + \tau_i^{\beta}\left(\tau_i^{1-\beta} - A\alpha\beta\theta^{\beta}\pi\right)\frac{\alpha+\beta}{\beta} +$$

$$+\left(1-\alpha-\beta\right)\tau_i^{\beta}\left(\tau_i^{1-\beta} - A\alpha\theta^{\beta}\pi\right) = (\alpha+\beta)\rho \qquad (67)$$

The expression on the left side of the above equation (lets call it expression A) is an increasing function of τ while the expression on the right side is a constant, we can easily see that this implies an unique solution for τ_i (see figure 1).

Figure 1

Now rearranging (67) we have:

$$\frac{\left(\tau_i^{1-\beta} - A\alpha\beta\theta^{\beta}\pi\right)\rho}{A\left(1-\alpha\right)\beta\theta^{\beta}\pi} + \sigma^{i}\tau_i^{\beta}\left(\tau_i^{1-\beta} - A\alpha\beta\theta^{\beta}\pi\right)\frac{\alpha+\beta}{\beta} +$$

$$+\sigma^{i}\left(1-\alpha-\beta\right)\tau^{\beta}\left(\tau_{i}^{1-\beta}-A\alpha\theta^{\beta}\pi\right)=\sigma^{i}(\alpha+\beta)\rho \tag{68}$$

Setting $\sigma^{i}=0$ we are in the case of an individual that is pure capitalist we can easily see that in this scenario we will have:

$$\tau_{i}=\tau^{*}=\left[A\alpha\beta\theta^{\beta}k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}\right]^{\frac{1}{1-\beta}} \tag{69}$$

In Appendix C equation [C22] we can see that expression A is decreasing in σ^{i}. So if $\sigma^{0}<\sigma^{1}$ we will have $A^{0}>A^{1}$ and this will imply that $\tau_{0}<\tau_{1}$ (see figure 2)

Figure 2

What we have is that the bigger σ^{i} is the higher will be the tax rate that maximizes individual $i\prime s$ utility.

The exact same reasoning can be applied to θ and we can say that the higher θ is the higher the tax rate that maximizes individual i's utility must be.

We can then write:

Proposition 2 *The less capitalist and individual is the higher will be is preferred tax rate.*

Proposition 3 *The lower the government saving rate* $(1-\theta)$ *is the higher will be individual's i preferred tax rate.*

This last proposition tell us that individuals will try and correct the sub-investment in public capital by demanding an higher tax rate.

4.2 Optimal θ

It is also possible to drive the optimal θ for individual i (Appendix D):

$$\theta^i = \left[\frac{(1-\alpha-\beta)\,\tau - (\alpha+\beta)\,\rho - \frac{\alpha\rho}{(1-\alpha)\sigma^i}}{A\alpha\tau^{\beta}\pi} \right]^{\frac{1}{\beta}} \tag{70}$$

From equation (70) we can easily conclude that θ^i is an increasing function of σ^i and of τ^i.We can state that:

Proposition 4 *The less capitalistic individual i is the lower will is preferred government saving rate be.*

Proposition 5 *The lower the tax rate is the lower will the government saving rate* $(1-\theta)$ *be.*

5 Policy Choice under Majority Voting

Lets remember the following concepts [10]:

Definition 1: A Condorcet Winner is a policy choice that beats any other feasible policy in a pair wise vote

[10] See [28]

Proposition 6 *If all voters have singled peaked policy preferences over a given ordering of policy alternatives, a Condorcet winner always exists and coincides with the median-ranked bliss point.*

The median-voter theorem can be applied in this case if we assume that consumers will only vote one thing at a time (τ or θ). When individuals look at their maximization problem they clearly see that τ affects their income and hence will interfere in the choices made but they don't have a clear perception of the effect of θ. θ is a variable on the government side and it is not explicit in any function from the consumer's view point. Unless the consumer knows precisely how the government works he will not understand the effect of θ. It seems more reasonable to have voters choosing the tax rate rather than the government saving rate. So we will assume that individuals will vote only over τ. With this assumption we have that voting takes place over a single issue, preferences are singled peaked and there exists a monotonic relation between ideal policies (whether is τ or is θ) and individual endowments.

Moreover under (24) we assure that optimal policies and factor endowments are constant over time and so it does not matter the moment in time where voting takes place.

Under these conditions the median-voter theorem assures us that we have (implicitly) a condorcet winner:

$$\frac{\left(\tau_m^{1-\beta} - A\alpha\beta\theta^\beta\pi\right)\rho}{A\left(1-\alpha\right)\beta\theta^\beta\pi\sigma^m} + \tau_i^\beta\left(\tau_m^{1-\beta} - A\alpha\beta\theta^\beta\pi\right)\frac{\alpha+\beta}{\beta} +$$

$$+\left(1-\alpha-\beta\right)\tau_m^\beta\left(\tau_m^{1-\beta} - A\alpha\theta^\beta\pi\right) = (\alpha+\beta)\rho \tag{71}$$

Where the index m indicates that we are talking about the median voter.

We can look at this expression as the reaction function for the median voter, i.e., when individual m knows θ he will react and choose the tax rate that maximizes is utility.

Now in a perfectly egalitarian society $\sigma^i = 1\ \forall i$ because everybody would have the same percentage of private capital. In the real world we have $\sigma^m > 1$

since most voters have relatively more labor[11] than private capital. Concerning factor ownership, the greater the gap between σ^m and 1 the more unequal the society is.

Going back to [13] we can write:

$$y^i = \left\{ w(\theta, \tau)\pi + [r(\theta, \tau) - \tau]\frac{1}{\sigma^i} \right\} k_{pt} L_t^i \tag{72}$$

If L_t^i represents unskilled labor then everybody has basically the same amount of L, which means that individual income will be decreasing on factor endowment. The less capitalistic an individual is the lower will his income be.

We have already seen that the greater the gap between σ^m and 1 the more unequal the society is in what factor ownership is concerned and we know now that this implies:

Proposition 7 *The greater the gap between σ^m and 1 the more unequal the society is in terms of income distribution*[12].

We can also easily conclude that the greater the gap between σ^m and 1 the further τ^m (the actual choice of policy) will be from τ^* and hence the lower the growth rate will be. We can state that:

Proposition 8 *The more unequal the society is in, what income distribution is concerned, the lower the growth rate will be.*

Can the government interfere in the choice of τ by changing its saving rate, i.e., if the government changes the level of θ (1-saving rate) will the distance between the actual chosen τ and the optimal one be smaller?

We know that if θ decreases the tax rate that maximizes the growth rate will also decrease and the tax rate that maximizes the utility of individual i (in our case $i = m$) will also decrease. If the effect is stronger on τ^m than on τ^* we could conclude the following:

[11] We are talking about unskilled labor. This model does not consider human capital.

[12] Remember that 1 is the average factor endowment and that the distance between the average and median gives us a measure of inequality.

Proposition 9 *By increasing its own saving rate, the government can improve the economy's performance by approximating the growth rate to its optimal level.*

The proof can be seen in Appendix E

We can show that the more unequal income distribution is the more apart will τ^m be from τ^* and the stronger the impact on the distance between the actual growth rate and the optimal growth rate will a change in θ produce (since the higher this tax is, so is the variation of τ^i).

Comparing our results with the one obtained in [1] we can see that in this paper the authors consider the maximum distortion, i.e., they consider $\theta = 1$.

From this result we could expect governments to chose $\theta \approx 0$. The reason why this doesn't happen is that a decrease in θ will have a negative effect on the balanced growth path growth rate (52a). What we have is a situation where governments should have to find a θ that guarantees they achieve their goal of maximizing the growth rate. There will be situations where increasing θ will be beneficial (in terms of the growth rate) and others where it will not.

5.1 Sensitivity Analysis

In this section we will try to see how the parameters of the model affect the positive relation between τ and θ.

To calibrate the model we used the following values for the parameters:

A	α	β	π	ρ	σ
1	0.36	0.15	0.4778	0.02	1.1

The values for A, α and ρ where taken from [10]. In [22] The authors suggest that the weight of the public sector in the production function should be somewhere around 0.31 but in [23] they correct this estimate and show evidence that this value should not be larger than 0.15. Knowing α and β, the value of ψ was taken from (37). To evaluate π we used the data from [16] and worked with the median public capital and the median private capital. Finally we had to come up with a plausible value for σ. We had data on wages and salaries and on income from property[13] from 1970 till 2000. We subtracted these two series to the GDP and considered that everything else was capital income. We then calculated the ratio of wages and salaries to capital income and the median value was 1.100124.

The first thing we were able to confirm is that equation (C22) postulates a positive relation between τ on θ. In Figure 3 this relation becomes obvious.

Figure 3

We then, tried to see how did this relation react to changes in β. Lets first recall that β gives us the weight of the public sector in the production function. In figure F2 we have equation (C22) ploted three times: the black line correspondes to $\beta = 0.1$, the red line to $\beta = 0.15$ and the yellow line to

[13]OECD database

$\beta = 0.2$[14]. What we can see is that if the relevance of public inputs increases voters will have a bigger need to correct the public "sub-investment". Given θ increases in β will lead to higer tax rates.

Figure 4

In Figure 5 we were interested in seeing how the relation between the tax rate and public savings was affected by the leve of income inequalities. Once again we ploted the same relation considering three different values of σ (blue $\sigma = 0.1$, red $\sigma = 1.1$ and green $\sigma = 3$). It is easy to see that as societies become more even concerning income distribution there will be a smaller need to correct government choices. With σ closer to 0 (closer to a pure capitalist) the choice of τ will be closer to the optimum and hence smaller than what should be expected in a society with σ far from 1 (far from the average).

[14]Changes in β will necessarily reflect in changes in π we will have:

β	0.1	0.15	0.2
π	0.4431	0.4778	05153

Figure 5

At last we can see how the reaction function of individual i is afected by the time discount rate. Looking at Figure 6 we can see that as the future becomes more important (blue $\rho = 0.009$, red $\rho = 0.02$ and green $\rho = 0.1$) voters will be more willing to pay taxes in order to correct the distortion towards public consumption.

Figure 6

6 Conclusion

We have introduced a model similar to the one presented in [26] and we added the households side of the problem.

Consumers maximize their utility and they distinguish themselves by the relative factor endowment (labor and private capital). σ_i is the relative factor endowment (labor/capital) for individual i and the higher it is the more capital poor individual i is. If σ_i is zero then the individual i is a pure capitalist (he doesn't have any labor income).

Solving the decentralized problem we found the economy's growth rate. This growth rate has an inverted U-shape relationship with the tax rate (which is consistent with the findings of [6] and of [1]). The tax rate that maximizes the growth rate (τ^*) is constant over time and depends on the parameters of the model and also on the government saving rate.

Individual i has one and only one tax rate that maximizes its own utility. The more capital poor an individual i is the higher will is preferred tax rate be and hence the furthest apart from τ^*.

In what the policy choice is concerned (individuals have to choose the tax rate taking the government saving rate as given) we have proven that we have a Condorcet winner and it will be the tax rate that maximizes the utility of the median voter. Being this the case we have also established that the more unequal a society is, in what income distribution is concerned, the further away will the chosen tax rate be form τ^* and hence the lower will the growth rate be. This results are similar to do ones obtained by [1], however we introduced government expenditures composition as a device for interfering in the choice of τ. This new instrument allows the government to interfere in the relation between τ^* and τ_m. Because voters are aware of the excessive amount of public consumption they will prefer to be more taxed in order to correct the sub-investment on the government side. We have concluded that the positive relation between θ and τ is sensitive to the parameters of the model. More weight of the public sector on the production function, a more uneven society (in what income distribution is concerned) and a larger discount rate will lead to larger tax rates (taken the public saving rate as given).

The possibility of a deciding government that manipulates it's saving rate (1-θ) allows for a less harmful effect of income inequality on the economy's performance namely on its growth rate. However this increase can also be harmful to the growth rate once it depends positively on θ. There is some trade-off that can be explored in the sense of achieving an optimal level of public consumption or investment.

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Appendix A - Relation Between Income Inequality and Government Efficiency

Government Quality and Income Inequality

Government Expenditures and Income Inequality

Appendix B - Equality of Growth Rates

$$\frac{\partial(\dot{k}_{pi} \backslash k_{pi})}{\partial t} = \left[w(\tau,\theta)\sigma^i + r(\tau,\theta)\right](\alpha+\beta-1)\, k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} \frac{\dot{k}_p}{k_{pt}} + \tag{B1}$$

$$\left[w(\tau,\theta)\sigma^i + r(\tau,\theta)\right]\alpha\psi k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} \frac{\dot{k}_g}{k_{gt}} - \frac{\dot{c}_i k_p^i - \dot{k}_p^i c^i}{k_p^{i2}} = 0$$

$$\left[w(\tau,\theta)\sigma^i + r(\tau,\theta)\right] k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} \left((\alpha+\beta-1)\frac{\dot{k}_p}{k_{pt}} + \alpha\beta\frac{\dot{k}_g}{k_{gt}}\right) = \frac{\dot{c}_i k_p^i - \dot{k}_p^i c^i}{k_p^{i2}} \Longleftrightarrow \tag{B2}$$

$$0 = \frac{\dot{c}_i k_p^i - \dot{k}_p^i c^i}{k_p^{i2}}$$

From (24) we have

$$\frac{\dot{k}_p^i}{k_{pt}^i} = \frac{\dot{c}^i}{c^i} \tag{B3}$$

Appendix C - Optimal Tax Rate for Individual i

Dividing (62) by k_{pt} :

$$e^{-\rho t}\frac{w_\tau(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i}{c^i}+\mu_1\gamma_\tau\frac{k_{pt}^i}{k_{pt}}+\mu_2\gamma_\tau=0 \tag{C1}$$

$$e^{-\rho t}\frac{w_\tau(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i}{c^i}=-\gamma_\tau\left(\mu_1\frac{k_{pt}^i}{k_{pt}}+\mu_2\right) \tag{C2}$$

From (C1) we have:

$$\frac{1}{c^{i^2}}\left\{-\rho e^{-\rho t}\left(w_\tau(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i\right)+e^{-\rho t}\left[w_\tau(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i\left((\alpha+\beta-1)\frac{\dot{k}_{pg}}{k_{pt}}+\psi\beta\frac{\dot{k}_g}{k_{gt}}\right)\right]\right\} \tag{C3}$$

$$-\frac{1}{c^{i^2}}\left[\dot{c}^i w_\tau(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i\right]+\dot{\mu}_1\gamma_\tau\frac{k_{pt}^i}{k_{pt}}+\dot{\mu}_2\gamma_\tau=0 \Longleftrightarrow$$

$$\frac{-\rho e^{-\rho t}w_\tau(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^ic^i-\dot{c}^i e^{-\rho t}w_\tau(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i}{c^{i^2}}=-\gamma_\tau\left(\dot{\mu}_1\gamma_\tau\frac{k_{pt}^i}{k_{pt}}+\dot{\mu}_2\right) \tag{C4}$$

Making (C4)/(C2) we end up with:

$$-\rho-\frac{\dot{c}^i}{c^i}=\frac{\dot{\mu}_1\gamma_\tau\frac{k_{pt}^i}{k_{pt}}+\dot{\mu}_2}{\mu_1\frac{k_{pt}^i}{k_{pt}}+\mu_2} \tag{C5}$$

From (C2) we have:

$$\mu_1\frac{k_{pt}^i}{k_{pt}}+\mu_2=-\frac{1}{\gamma_\tau}e^{-\rho t}\frac{w_\tau(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i}{c^i} \tag{C6}$$

Lets now multiply (64) by $\frac{k_{pt}^i}{k_{pt}}$:

$$e^{-\rho t}\frac{\rho}{c^i}\frac{k_{pt}^i}{k_{pt}}+\mu_1\gamma\frac{k_{pt}^i}{k_{pt}}=-\dot{\mu}_1\frac{k_{pt}^i}{k_{pt}} \tag{C7}$$

This last expression added to 66:

$$\frac{e^{-\rho t}}{c^i}\left[\rho\frac{k_{pt}^i}{k_{pt}}+w(\tau,\theta)\left(\alpha+\beta\right)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i\right]+$$

$$+\mu_1\left(\alpha+\beta\right)\gamma\frac{k_{pt}^i}{k_{pt}}+\mu_2(\alpha+\beta)\gamma=-\dot{\mu}_1\frac{k_{pt}^i}{k_{pt}}-\dot{\mu}_2\Leftrightarrow \tag{C8}$$

$$\frac{e^{-\rho t}}{c^i}\left[\rho\frac{k_{pt}^i}{k_{pt}}+w(\tau,\theta)\left(\alpha+\beta\right)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i\right]+$$

$$+\left(\alpha+\beta\right)\gamma\left(\mu_1\frac{k_{pt}^i}{k_{pt}}+\mu_2\right)=-\dot{\mu}_1\frac{k_{pt}^i}{k_{pt}}-\dot{\mu}_2\Leftrightarrow$$

$$\frac{\frac{e^{-\rho t}}{c^i}\left[\rho\frac{k_{pt}^i}{k_{pt}}+w(\tau,\theta)\left(\alpha+\beta\right)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i\right]+\left(\alpha+\beta\right)\gamma\left(\mu_1\frac{k_{pt}^i}{k_{pt}}+\mu_2\right)}{\left(-\mu_1\frac{k_{pt}^i}{k_{pt}}-\mu_2\right)}=\frac{-\dot{\mu}_1\frac{k_{pt}^i}{k_{pt}}-\dot{\mu}_2}{\left(-\mu_1\frac{k_{pt}^i}{k_{pt}}-\mu_2\right)}\Leftrightarrow \tag{C9}$$

From (C5)

$$\frac{\frac{e^{-\rho t}}{c^i}\left[\rho\frac{k_{pt}^i}{k_{pt}}+w(\tau,\theta)\left(\alpha+\beta\right)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i\right]}{\left(-\mu_1\frac{k_{pt}^i}{k_{pt}}-\mu_2\right)}-\left(\alpha+\beta\right)\gamma=-\rho-\frac{\dot{c}^i}{c^i} \tag{C10}$$

From (C6)

$$\frac{\frac{e^{-\rho t}}{c^i}\left[\rho\frac{k_{pt}^i}{k_{pt}}+w_\tau(\tau,\theta)\left(\alpha+\beta\right)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i\right]}{\frac{1}{\gamma_\tau}e^{-\rho t}\frac{w_\tau(\tau,\theta)k_{pt}^{\alpha+\beta-1}k_{gt}^{\psi\beta}L^i}{c^i}}-\left(\alpha+\beta\right)\gamma=-\rho-\frac{\dot{c}^i}{c^i}\Leftrightarrow \tag{C11}$$

$$\frac{\gamma_\tau \rho \frac{k_{pt}^i}{k_{pt}} + \gamma_\tau w(\tau,\theta)\left(\alpha+\beta\right) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} L^i}{w_\tau(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} L^i} - \left(\alpha+\beta\right)\gamma = -\rho - \frac{\dot{c}^i}{c^i} \Leftrightarrow \tag{C12}$$

$$\frac{\gamma_\tau \rho \frac{k_{pt}^i}{k_{pt}}}{w_\tau(\tau,\theta) k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} L^i} + \gamma_\tau \left(\alpha+\beta\right) \frac{w(\tau,\theta)}{w_\tau(\tau,\theta)} - \left(\alpha+\beta\right)\gamma = -\rho - \gamma \tag{C13}$$

With:

$$k_{pt}^{\alpha+\beta-1} k_{gt}^{\psi\beta} = \pi \tag{C14}$$

We know that:

$$\gamma = A\alpha\left(\theta\tau\right)^{\beta}\pi - \tau - \rho \tag{C15}$$

$$\gamma_\tau = A\alpha\beta\left(\theta\tau\right)^{\beta}\pi\tau^{-1} - 1 \tag{C16}$$

$$w\left(\tau,\theta\right) = A(1-\alpha)(\theta\tau)^{\beta} \tag{C17}$$

$$w_\tau = A\left(1-\alpha\right)\beta\left(\theta\tau\right)^{\beta}\tau^{-1} \tag{C18}$$

Going back to (C13) and replacing we have:

$$\frac{\left(A\alpha\beta\left(\theta\tau\right)^{\beta}\pi\tau^{-1}-1\right)\rho\frac{k_{pt}^i}{k_{pt}}}{A\left(1-\alpha\right)\beta\left(\theta\tau\right)^{\beta}\tau^{-1}\pi L^i} + \left(A\alpha\beta\left(\theta\tau\right)^{\beta}\pi\tau^{-1}-1\right)\left(\alpha+\beta\right)\frac{1}{\beta\tau^{-1}} -$$

$$-\left(\alpha+\beta\right)\left(A\alpha\left(\theta\tau\right)^{\beta}\pi-\tau-\rho\right) = -\rho - \left(A\alpha\left(\theta\tau\right)^{\beta}\pi-\tau-\rho\right) \Leftrightarrow \tag{C19}$$

$$\frac{\left(A\alpha\beta\left(\theta\tau\right)^{\beta}\pi-\tau\right)\rho}{A\left(1-\alpha\right)\beta\left(\theta\tau\right)^{\beta}\pi\sigma^i} + \left(A\alpha\beta\left(\theta\tau\right)^{\beta}\pi-\tau\right)\frac{\alpha+\beta}{\beta} +$$

$$+\left(1-\alpha-\beta\right)\left(A\alpha\left(\theta\tau\right)^{\beta}\pi-\tau\right)=-\rho+(1-\alpha-\beta)\rho \tag{C20}$$

$$\frac{\left(A\alpha\beta\theta^{\beta}\pi-\tau^{\beta-1}\right)\rho}{A\left(1-\alpha\right)\beta\theta^{\beta}\pi\sigma^{i}}+\tau^{\beta}\left(A\alpha\beta\theta^{\beta}\pi-\tau^{1-\beta}\right)\frac{\alpha+\beta}{\beta}+$$

$$+\left(1-\alpha-\beta\right)\tau^{\beta}\left(A\alpha\theta^{\beta}\pi-\tau^{1-\beta}\right)=(-\alpha-\beta)\rho \tag{C21}$$

$$\frac{\left(\tau_{i}^{1-\beta}-A\alpha\beta\theta^{\beta}\pi\right)\rho}{A\left(1-\alpha\right)\beta\theta^{\beta}\pi\sigma^{i}}+\tau_{i}^{\beta}\left(\tau_{i}^{1-\beta}-A\alpha\beta\theta^{\beta}\pi\right)\frac{\alpha+\beta}{\beta}+$$

$$+\left(1-\alpha-\beta\right)\tau^{\beta}\left(\tau_{i}^{1-\beta}-A\alpha\theta^{\beta}\pi\right)=(\alpha+\beta)\rho \tag{C22}$$

Appendix D - Optimal Theta for individual i

Looking again to the first order conditions of the centralized problem, and dividing (63) by k_{pt} we have:

$$\frac{e^{-\rho t}w_{\theta}(\theta,\tau)\pi L^{i}}{c^{i}}+\mu_1\gamma_{\theta}\frac{k^{i}_{pt}}{k_{pt}}+\mu_2\gamma_{\theta}=0\Leftrightarrow$$

$$\frac{e^{-\rho t}w_{\theta}(\theta,\tau)\pi L^{i}}{c^{i}}=-\gamma_{\theta}[\mu_1\frac{k^{i}_{pt}}{k_{pt}}+\mu_2]\Leftrightarrow \tag{D1}$$

$$\frac{e^{-\rho t}w_{\theta}(\theta,\tau)\pi L^{i}}{\gamma_{\theta}c^{i}}=-\mu_1\frac{k^{i}_{pt}}{k_{pt}}-\mu_2 \tag{D2}$$

From (D1) we obtain:

$$\frac{-\rho e^{-\rho t}w_{\theta}(\theta,\tau)\pi L^{i}-e^{-\rho t}\dot{c}^{i}w_{\theta}(\theta,\tau)\pi L^{i}}{c^{i2}}=-\gamma_{\theta}[\dot{\mu}_1\frac{k^{i}_{pt}}{k_{pt}}+\dot{\mu}_2] \tag{D3}$$

Making (D3)/(D1) we have once again:

$$-\rho-\frac{\dot{c}^{i}}{c^{i}}=\frac{\dot{\mu}_1\gamma_{\tau}\frac{k^{i}_{pt}}{k_{pt}}+\dot{\mu}_2}{\mu_1\frac{k^{i}_{pt}}{k_{pt}}+\mu_2} \tag{D4}$$

Replacing in (C10) by (D2) we have:

$$\begin{aligned}
e^{-\rho t}\frac{\rho\frac{k^{i}_{pt}}{k_{pt}}+w\left(\theta,\tau\right)\left(\alpha+\beta\right)\pi L^{i}}{c^{i}\frac{e^{-\rho t}w_{\theta}(\theta,\tau)\pi L^{i}}{\gamma_{\theta}c^{i}}}-\left(\alpha+\beta\right)\gamma &= -\rho-\gamma\Leftrightarrow && \text{(D5)}\\
\gamma_{\theta}\frac{\rho\frac{k^{i}_{pt}}{k_{pt}}+w\left(\theta,\tau\right)\left(\alpha+\beta\right)\pi L^{i}}{w_{\theta}\left(\theta,\tau\right)\pi L^{i}}-\left(\alpha+\beta\right)\gamma &= -\rho-\gamma\Leftrightarrow && (73)\\
\gamma_{\theta}\frac{\rho}{w_{\theta}\left(\theta,\tau\right)\pi L^{i}}+\gamma_{\theta}\frac{w\left(\theta,\tau\right)\left(\alpha+\beta\right)}{w_{\theta}\left(\theta,\tau\right)}+\left(1-\alpha-\beta\right)\gamma &= -\rho && (74)
\end{aligned}$$

Recalling that:

$$\gamma = A\alpha\left(\theta\tau\right)^{\beta}\pi - \tau - \rho \tag{D6}$$

$$\gamma_{\theta} = A\alpha\beta\left(\theta\tau\right)^{\beta}\pi\theta^{-1} \tag{D7}$$

$$w\left(\tau,\theta\right) = A(1-\alpha)(\theta\tau)^{\beta} \tag{D8}$$

$$w_{\theta} = A\left(1-\alpha\right)\beta\left(\theta\tau\right)^{\beta}\theta^{-1} \tag{D9}$$

Replacing in (73) and after some algebra we have:

$$\theta^{i} = \left[\frac{\left(1-\alpha-\beta\right)\tau - \left(\alpha+\beta\right)\rho - \frac{\alpha\rho}{(1-\alpha)\sigma^{i}}}{A\alpha\tau^{\beta}\pi}\right]^{\frac{1}{\beta}} \tag{D10}$$

Appendix E - Proof of Proposition 9

Proof. Lets recall equation (C22) and lets differentiate this equation in order to find $\partial\tau/\partial\theta$:

$$\frac{(1-\beta)\frac{\partial\tau}{\partial\theta}\tau^{-1}\rho-\beta\theta^{-1}\tau^{1-\beta}\rho}{\beta A(1-\alpha)\theta^{\beta}\pi\sigma^{i}}+\frac{\alpha+\beta}{\beta}\frac{\partial\tau}{\partial\theta}-\beta\left(\alpha+\beta\right)A\alpha^{\beta}\theta^{\beta-1}-$$

$$-\beta\left(\alpha+\beta\right)A\alpha\theta^{\beta}\frac{\partial\tau}{\partial\theta}\tau^{\beta-1}\pi+\left(1-\alpha-\beta\right)\frac{\partial\tau}{\partial\theta}-\left(1-\alpha-\beta\right)A\pi\alpha\left[\beta\theta^{\beta-1}\tau^{\beta}+\beta\frac{\partial\tau}{\partial\theta}\tau^{\beta-1}\theta^{\beta}\right]=0 \tag{E1}$$

$$\frac{\partial\tau}{\partial\theta}\left\{\frac{(1-\beta)\,\rho}{\beta A\,(1-\alpha)\,\pi\theta^{\beta}\sigma^{i}}+\frac{\alpha+\beta}{\beta}-\beta(\alpha+\beta)A\alpha\pi\theta^{\beta}\tau^{\beta-1}+1-\alpha-\beta-\left(1-\alpha-\beta\right)A\alpha\pi\beta\theta^{\beta}\tau^{\beta-1}\right\}$$

$$-\frac{\tau^{1-\beta}\rho}{A\pi\left(1-\alpha\right)\theta^{\beta+1}\sigma^{i}}-\beta\left(\alpha+\beta\right)A\pi\alpha\tau^{\beta}\theta^{\beta-1}-\left(1-\alpha-\beta\right)A\pi\alpha\beta\theta^{\beta-1}\tau^{\beta}=0 \tag{E2}$$

$$\frac{\partial\tau}{\partial\theta}=\frac{\frac{\tau^{1-\beta}\rho}{A\pi(1-\alpha)\theta^{\beta+1}\sigma^{i}}+\beta\left(\alpha+\beta\right)A\pi\alpha\tau^{\beta}\theta^{\beta-1}+\left(1-\alpha-\beta\right)A\pi\alpha\beta\theta^{\beta-1}\tau^{\beta}}{\frac{(1-\beta)\rho}{\beta A(1-\alpha)\pi\theta^{\beta}\sigma^{i}}+\frac{\alpha+\beta}{\beta}-\beta(\alpha+\beta)A\alpha\pi\theta^{\beta}\tau^{\beta-1}+1-\alpha-\beta-\left(1-\alpha-\beta\right)A\alpha\pi\beta\theta^{\beta}\tau^{\beta-1}} \tag{E3}$$

Replacing τ by $k\tau^{*}$ with $k\geq1$ we have, after some algebra:

$$\frac{\partial\tau}{\partial\theta}=\frac{k^{1-\beta}\frac{\alpha\beta\rho}{(1-\alpha)\sigma^{i}}+k^{\beta}\left(A\beta\alpha\pi\right)^{\frac{1}{1-\beta}}\theta^{\frac{2\beta-1}{1-\beta}}}{\frac{(1-\beta)\rho}{\beta A(1-\alpha)\pi\theta^{\beta}\sigma^{i}}+\frac{\alpha+\beta}{\beta}+1-\alpha-\beta-k^{\beta-1}} \tag{E4}$$

Taking yet another derivative (and calling N to the numerator of the previous expression and D to the denominator) we have:

$$\frac{\partial\tau}{\partial\theta\partial k}=\frac{\left[\left(1-\beta\right)k^{-\beta}\frac{\alpha\beta\rho}{(1-\alpha)\sigma^{i}}+\beta k^{\beta-1}\left(A\beta\alpha\pi\right)^{\frac{1}{1-\beta}}\theta^{\frac{2\beta-1}{1-\beta}}\right]D-\left(\beta-1\right)k^{\beta-2}N}{D^{2}} \tag{E5}$$

Knowing that $D^2 > 0$ and seeing that[15] :

$$\left[(1-\beta)\, k^{-\beta}\frac{\alpha\beta\rho}{(1-\alpha)\sigma^i} + \beta k^{\beta-1}\left(A\beta\alpha\pi\right)^{\frac{1}{1-\beta}}\theta^{\frac{2\beta-1}{1-\beta}}\right] D + (1-\beta)\, k^{\beta-2} N > 0 \tag{E6}$$

We conclude that: $\frac{\partial\tau}{\partial\theta\partial k} > 0$. This means that the higher k is, i.e., the further apart we are from τ^* the effect of a change in θ in τ will be stronger. So if we diminish θ we will be diminishing the distance between τ^* and τ^i ■

[15](notice that N is positive and D has to be positive once we have already establish that $\frac{\partial\tau}{\partial\theta} > 0$)